HINTS & SOLUTIONS

EXERCISE - 1
Single Choice

1. $60 = 2^2 \times 3^1 \times 5^1$

Number of divisor = 12

3. $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$\Rightarrow \quad A_\alpha A_\beta = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$

$= \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix} = A_{\alpha+\beta}$

4. $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

$abc = 120 = 2^3 \times 3^1 \times 5^1$

Case - I $= {}^5C_2 \times {}^3C_2 \times {}^3C_2 = 90$

Case - I $= 3 \times ({}^5C_2 \times {}^3C_2 \times {}^3C_2) = 270$

$= 90 + 270 = 360$

5. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1+2 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1+2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1+2+3 \\ 0 & 1 \end{bmatrix}$

on multiplying the matrix we get

$\begin{bmatrix} 1 & 1+2+.....+n \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 378 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \quad n(n+1) = 378 \times 2 \quad \Rightarrow \quad n = 27$

6. **Hint :** $x = 11 - y$ & $x + 5 = y$

9. $A^2 - 2A + I = 0$

$\Rightarrow \quad (A - I)^2 = 0$

$A^n = (A - I + I)^n$

$= {}^nC_0 (A-I)^n + ... + {}^nC_{n-2} (A-I)^2 \cdot I^{n-2} + {}^nC_{n-1} (A-I) \cdot I^{n-1} + {}^nC_n I^n$

$= 0 + 0 + ... + 0 + n(A - I) + I = nA - (n-1)I$

11. $A^T = -A.$ & $A^TA = I$

$\Rightarrow \quad A^2 = -I \quad \Rightarrow A^{4n} = I$

$A^{4n-1} = A^{-1} \quad \Rightarrow \quad A^{4n-1} = A^T$ (A is orthogonal)

17. A is involutary

$\Rightarrow \quad A^2 = I \qquad \Rightarrow \quad A = A^{-1}$

now $A^2 = \left(\frac{A}{2}\right)(2A) = I \quad \Rightarrow \quad 2A = \left(\frac{A}{2}\right)^{-1}$

18. $A = 3 \times 4$; $A' = 4 \times 3$

As A' B is defined $\qquad \Rightarrow$ let order of $B = 3 \times n$

now $BA' = (3 \times n) \times (4 \times 3) \Rightarrow n = 4$

$\therefore$ order of B is 3×4

$\therefore$ order of $B' = 4 \times 3$

order of $B'A = (4\times3) \times (3 \times 4) = 4 \times 4$

22. $AB = \begin{pmatrix} \cos^2\alpha & \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha & \sin^2\alpha \end{pmatrix} \begin{pmatrix} \cos^2\beta & \sin\beta\cos\beta \\ \sin\beta\cos\beta & \sin^2\beta \end{pmatrix}$

$= \begin{pmatrix} \cos^2\alpha\cos^2\beta + \sin\alpha\cos\alpha\sin\beta\cos\beta & \cos^2\alpha\sin\beta\cos\beta + \sin\alpha\cos\alpha\sin^2\beta \\ \cos^2\beta\sin\alpha\cos\alpha + \sin^2\alpha\sin\beta\cos\beta & \sin\alpha\cos\alpha\sin\beta\cos\beta + \sin^2\alpha\sin^2\beta \end{pmatrix}$

$\begin{pmatrix} \cos\alpha\cos\beta\cos(\alpha-\beta) & \cos\alpha\sin\beta\cos(\alpha-\beta) \\ \sin\alpha\cos\beta\cos(\alpha-\beta) & \sin\alpha\sin\beta\cos(\alpha-\beta) \end{pmatrix}$

$\Rightarrow \quad \alpha - \beta$ must be an odd integral multiple of $\pi/2$

$\Rightarrow$ **(C)**

23. $t_r(A) + 2\,t_r(B) = -1$ (from the given matrix)

and $2\,t_r(A) - t_r(B) = 3$ (from the given matrix)

Let $t_r(A) = x$ and $t_r(B) = y$

$x + 2y = -1$

$2x - y = 3$

solving $x = 1$ and $y = -1$

Hence $t_r(A) - t_r(B) = x - y = 2$

24. Obv. A is orthogonal as $a_{11}^2 + a_{12}^2 = 1 = a_{21}^2 + a_{22}^2 = a_{11}^2 + a_{22}^2$

for skew symmetric matrix $a_{ii} = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}$

for symmetric matrix , $A = A^T \Rightarrow \sin\theta = 0 \Rightarrow \theta = n\pi$

Also $\text{adj}A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ and $|A| = 1$

hence $A = A^{-1}$ is possible if $\sin\theta = 0$

25. $A.\ \text{adj}\ A = |A|\ I$

$|A| = xyz - 8x - 3(z-8) + 2(2-2y)$

$|A| = xyz - (87x + 3z + 4y) + 28 \Rightarrow 60 - 20 + 28 = 68$

$\Rightarrow$ **(C)**

26. $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$

hence $\det. A = \sec^2 x$

$\therefore \det A^T = \sec^2 x$

now $f(x) = \det.(A^T A^{-1})$

$= (\det. A^T)(\det. A^{-1})$

$= (\det. A^T)(\det. A)^{-1}$

$= \dfrac{\det.(A^T)}{\det.(A)} = 1$

hence $f(x) = 1$

27. As $A(\theta) = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$

$A(\theta)$ is certainly neither symmetric nor skew symmetric

Further, $A(\pi-\theta) = \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

and $A(\theta)\cdot A(\pi-\theta) = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow A^{-1}(\theta) = A(\pi-\theta)$

$A^2(\theta) = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}\begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$

$= \begin{bmatrix} -\cos 2\theta & \sin 2\theta \\ -\sin 2\theta & -\cos 2\theta \end{bmatrix} \neq A\left(\frac{\pi}{2} - 2\theta\right)$

28. $\begin{vmatrix} 1 & 2 & 2 \\ 1 & 3 & 4 \\ 3 & 4 & k \end{vmatrix} = 0$

$1(3k-16) - 2(k-12) + 2(4-9) = 0$

$3k - 16 - 2k + 24 - 10 = 0$

$k = 2$

29. $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ ....**(i)**

and $A^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ....**(ii)**

Let A be given by $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$;

Hence $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$; $\begin{bmatrix} a-b \\ c-d \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

The first equation gives

$a - b = -1$ ....**(iii)**

and $c - d = 2$ ....**(iv)**

For second equation, $A^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = A\left(A\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$

$= A\left(\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

This gives $-a + 2b = 1$ ....**(v)**

and $-c + 2d = 0$ ....**(vi)**

(iii) + **(v)** $\Rightarrow$ $b = 0$ and $a = -1$

(iv) + **(vi)** $\Rightarrow$ $d = 2$ and $c = 4$

so the sum $a + b + c + d = 5$

30. Given $A^2 = A$

$I = (I - 0.4A)(I - \alpha A)$

$= I - I\alpha A - 0.4AI + 0.4\alpha A^2$

$= I - A\alpha - 0.4A + 0.4\alpha A$

$= I - A(0.4 + \alpha) + 0.4\alpha A$

hence $0.4\alpha = 0.4 + \alpha \Rightarrow \alpha = -2/3$

EXERCISE - 2

Part # I : Multiple Choice

1. $A^2 = A \Rightarrow |A|^2 = |A| \Rightarrow |A| = 1$

$A(\text{adj } A) = |A|\, I$

$\text{adj}A = A^{-1}$

also $A^2 = A$

$A = I \Rightarrow \text{adj } A = I$

$(\text{adj } A)^2 = I \Rightarrow (\text{adj } A)^2 = \text{adj } A$

4. $A^T = BCD$

$AA^T = ABCD \Rightarrow AA^T = S$

$\Rightarrow AA^T = S^T \Rightarrow S = S^T$

$D^T C^T B^T A^T = ABC \,.\, DAB \,.\, CDA.\, BCD$

$(ABCD)^T = (ABCD)\,(ABCD)\,(ABCD)$

$S^T = S^3 \Rightarrow S = S^3$

$\Rightarrow S^2 = S^4$

11. $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - \begin{bmatrix} 8 & -4 \\ 8 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

$C = A - B \begin{bmatrix} 1-\alpha & 0 \\ 0 & -2 \end{bmatrix}$ is diagonal matrix, $\forall\ \alpha \in R$

12. $|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^2 - \lambda(a + d) + ad - bc = 0$

This is characteristic equation. Comparing with given equation we get

$k = ad - bc = |A|, \quad a + d = 0$

14. For $|AB| = 0 \Rightarrow |A|\cdot|B| = 0$

$\Rightarrow |A| = 0$ or $|B| = 0$

$AA^{-1} = I \Rightarrow |A|\cdot|A|^{-1} = |I| = 1$

$\Rightarrow |A^{-1}| = \dfrac{1}{|A|} = |A|^{-1}$

15. We have $A^2B = A(AB) = AA = A^2$, $B^2A = B(BA) = BB = B^2$, $ABA = A(BA) = AB = A$, and $BAB = B(AB) = BA = B$]

16. Note that A is non singular but B is singular hence only A^{-1} exists

Now $XA = B \Rightarrow X = BA^{-1}$ .....(i)

and $AY = B \Rightarrow Y = A^{-1}B$ .....(ii)

Also $A^{-1} = \dfrac{1}{3}\begin{bmatrix} -1 & 1 \\ -4 & 1 \end{bmatrix}$ now verify

17. $AB = O$

$\therefore |AB| = 0 \Rightarrow |A|\,|B| = 0$

$\therefore \det A \neq 0$

$\therefore A^{-1}$ exist

$\therefore A^{-1}(AB) = A^{-1}(0) = 0$

$IB = 0$

$B = 0 \Rightarrow$ B must be null matrix.

18. $PQ = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow$ **B, C, D**]

19. $A = \begin{bmatrix} x & 1 & 0 \\ 1 & x & 1 \\ 0 & 1 & x \end{bmatrix}$; $x(x^2 - 1) - 1(x) = 6 - 2 = 4$

20. $|A^TA^{-1}| = |A^T|\,|A^{-1}| = |A^T|\dfrac{1}{|A|} = 1$

$\Rightarrow f(x) = 1$

Part # II : Assertion & Reason

3. Given $AB + A + B = O$

$AB + A + B + I = I$

$A(B + I) + (B + I) = I$

$(A + I)(B + I) = I$

$\Rightarrow (A + I)$ and $(B + I)$ are inverse of each other

$\Rightarrow (A + I)(B + I) = (B + I)(A + I)$

$\Rightarrow AB = BA$

4. Let $A=\begin{bmatrix} a & p \\ b & q \\ c & r \end{bmatrix}$ $A^T=\begin{bmatrix} a & b & c \\ p & q & r \end{bmatrix}$

$$AA^T=\begin{bmatrix} a^2+p^2 & ab+pq & ac+pr \\ ab+pq & b^2+q^2 & bc+qr \\ ac+pr & bc+qr & c^2+r^2 \end{bmatrix}$$

$$|AA^T|=\begin{vmatrix} a & p & 0 \\ b & q & 0 \\ c & r & 0 \end{vmatrix}\begin{vmatrix} a & b & c \\ p & q & r \\ 0 & 0 & 0 \end{vmatrix}=0$$

$\Rightarrow$ AA^T is singular.

6. Statement-I :

$$\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} a+2c & b+2d \\ -a-c & -b-d \end{bmatrix}=\begin{bmatrix} a-b & 2a-b \\ c-d & 2c-d \end{bmatrix}$$

$\Rightarrow$ $2c = -b$ & $b = a - d$

$\therefore$ infinite matrix are there.

EXERCISE - 3

Part # II : Comprehension

1. $\det. A=\begin{vmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{vmatrix}$

$=2(-16+15)+1(12-15)+1(-15+20)$

$=-2-3+5=0 \Rightarrow$ A is singular

$\det. B=\begin{vmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{vmatrix}$

$=1(-12+12)+1(12-12)+1(-12+12)$

$=0$

$\Rightarrow$ B is also singular

$\det. C=\begin{vmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{vmatrix}=-1(16-12)-1(-12+9)$

$=-4+3=-1$

$\Rightarrow$ C is non singular again

$$A^2=\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$=\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}=A \Rightarrow \text{A is idempotent}$$

$$B^2=\begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ **(B)** is nilpotent

$$C^2=\begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}\begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}=I$$

$\Rightarrow$ C is involutary

(i) obvious **(B)** as B is nilpotent

(ii) $P = A^3C^2 = A^3 = A \quad \Rightarrow \quad P^2 = A^2 = A$

$\therefore \quad P^2 = P$

ly in B and D

hence BC^2 is not Idempotent.

(iii) Let $X = BC^2 \quad \Rightarrow \quad \det. X = 0$

$Y = A^2C^2 \quad \Rightarrow \quad \det. Y = 0$

$Z = A^2B \quad \Rightarrow \quad \det. Z = 0$

but $W = C^3 \quad \Rightarrow \quad \det. W \neq 0$

hence C^3 has an inverse $\Rightarrow$ **(D)**

2.

(i) $(A+B)C = (A+B)(A+B)^{-1}(A-B)$

$\Rightarrow \quad (A+B)C = A - B \quad ....(1)$

$$C^T = (A-B)^T\left((A+B)^{-1}\right)^T$$

$$= (A+B)\left((A+B)^T\right)^{-1} \quad \{\text{as } |A+B| \neq 0$$

$\Rightarrow \quad |(A+B)^T| \neq 0 \quad \Rightarrow \quad |A-B| \neq 0\}$

$= (A+B)(A-B)^{-1} \quad ....(2)$

(1) & (2) $C^T(A+B)C = (A+B)(A-B)^{-1}(A-B)$

$= (A+B) \quad ....(3)$ **Ans.**

(ii) taking transpose in (3)

$C^T(A+B)^T(C^T)^T = (A+B)^T$

$C^T(A-B)C = A - B \quad ....(4)$ **Ans.**

(ii) adding (3) and (4)

$C^T[A+B+A-B]C = 2A$

$C^TAC = A$ **Ans.**

EXERCISE - 4

Subjective Type

1. $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \\ 5 & 6 \end{bmatrix} B = \begin{bmatrix} 4 & 5 & 6 \\ 7 & -8 & 2 \end{bmatrix}$

$$AB = \begin{bmatrix} 4+14 & 5-16 & 6+4 \\ -12+28 & 15+32 & 18-10 \\ 20-62 & 25-48 & 30+12 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 18 & -11 & 10 \\ -16 & 47 & 10 \\ 62 & -23 & 42 \end{bmatrix}$$

$$BA = \begin{bmatrix} 4+15+30 & 8-20+36 \\ 7-24+10 & 14+32+12 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 49 & 24 \\ -7 & 58 \end{bmatrix}$$

2. $(I+A) = (I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$\text{L.H.S.} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

R.H.S.

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}\right)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & +\tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos\alpha + \sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha + \tan\frac{\alpha}{2}\cos\alpha \\ -\tan\frac{\alpha}{2}\cos\alpha + \sin\alpha & \sin\alpha\tan\frac{\alpha}{2} + \cos\alpha \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos\alpha + 2\sin\frac{\alpha}{2}\sin\alpha & \sin\frac{\alpha}{2}\left(-2\cos\frac{\alpha}{2} + \frac{\cos\alpha}{\cos\frac{\alpha}{2}}\right) \\ \sin\alpha\left(-\frac{\cos\alpha}{\cos\frac{\alpha}{2}} + 2\cos\frac{\alpha}{2}\right) & \cos\alpha + 2\sin^2\frac{\alpha}{2} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

4. $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}; A^3 = 5A^2 - 6AI + I^2$

$A^2 = \begin{bmatrix} 3 & 3 & 7 \\ 1 & 4 & 4 \\ 3 & 1 & 6 \end{bmatrix}$ $A^3 = \begin{bmatrix} 10 & 9 & 23 \\ 5 & 9 & 14 \\ 9 & 5 & 19 \end{bmatrix}$

$5A^2 + 6A + I = \begin{bmatrix} 10 & 9 & 23 \\ 5 & 9 & 14 \\ 9 & 5 & 19 \end{bmatrix} = A^3$

5. We have, $A - \lambda I$ and $B - \lambda I$ commute

$\Leftrightarrow$ $(A - \lambda I)(B - \lambda I) = (B - \lambda I)(A - \lambda I)$

$\Leftrightarrow$ $AB - \lambda IA - \lambda IB + \lambda^2 I^2 = BA - \lambda BI - \lambda IA + \lambda^2 I^2$

$\Leftrightarrow$ $AB - \lambda A - \lambda B + \lambda^2 I = BA - \lambda B - \lambda A + \lambda^2 I$

$\Leftrightarrow$ $AB = BA$

A and B commute

6. We have,

$AB = A$ and $BA = B$

Now

$AB = A$

$\Rightarrow$ $(AB)A = AA$

[Multiplying both sides on right by A]

$\Rightarrow$ $A(BA) = A^2$ [By associ. of matrix multip]

$\Rightarrow$ $AB = A^2$ [$\because$ BA = B]

$\Rightarrow$ $A = A^2$ [$\because$ AB = A]

Similarly, $B^2 = B$.

7. $\left\{\frac{1}{2}(A - A' + I)\right\}^{-1}$ for $A = \begin{bmatrix} -2 & 3 & 4 \\ 5 & -4 & -3 \\ 7 & 2 & 9 \end{bmatrix}$

$$\frac{1}{2}(A - A^T + I)^{-1} = \left[\frac{1}{2}\begin{bmatrix} 1 & -2 & -3 \\ 2 & 1 & -5 \\ 3 & 5 & 1 \end{bmatrix}\right]^{-1} = \left(\frac{1}{2}B\right)^{-1}$$

$[B] = \frac{-39}{8}$

$$\text{Adj } B = \frac{1}{4}\begin{bmatrix} 26 & -17 & 7 \\ -13 & 10 & -11 \\ 13 & -1 & 5 \end{bmatrix}^T = \frac{1}{4}\begin{bmatrix} 26 & -13 & 13 \\ -17 & 10 & -1 \\ 7 & -11 & 5 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{2}{39}\begin{bmatrix} 26 & -13 & 13 \\ -17 & 10 & -1 \\ 7 & -11 & 5 \end{bmatrix}$$

8. $a^2 + b^2 + c^2 = 1$

$$\Delta = \begin{vmatrix} a^2 + (b^2 + c^2)\cos\phi & ab(1 - \cos\phi) & ac(1 - \cos\phi) \\ ba(1 - \cos\phi) & b^2 + (c^2 + a^2)\cos\phi & bc(1 - \cos\phi) \\ ca(1 - \cos\phi) & cb(1 - \cos\phi) & c^2 + (a^2 + b^2)\cos\phi \end{vmatrix}$$

$$\Delta = \frac{1}{abc}$$

$$\begin{vmatrix} a^3 + a(b^2 + c^2)\cos\phi & b^2a(1 - \cos\phi) & ac^2(1 - \cos\phi) \\ ba^2(1 - \cos\phi) & b^3 + b(c^2 + a^2)\cos\phi & bc^2(1 - \cos\phi) \\ ca^2(1 - \cos\phi) & cb(1 - \cos\phi) & c^3 + c(a^2 + b^2)\cos\phi \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$\frac{abc}{abc}$

$$\begin{vmatrix} a^2 + (b^2 + c^2)\cos\phi & b^2(1 - \cos\phi) & c^2(1 - \cos\phi) \\ a^2(1 - \cos\phi) & b^2 + (c^2 + a^2)\cos\phi & c^2(1 - \cos\phi) \\ a^2(1 - \cos\phi) & b^2(1 - \cos\phi) & c^2 + (a^2 + b^2)\cos\phi \end{vmatrix}$$

$= (a^2 + b^2 + c^2)$

$$\begin{vmatrix} 1 & b^2(1 - \cos\phi) & c^2(1 - \cos\phi) \\ 1 & b^2 + (c^2 + a^2)\cos\phi & c^2(1 - \cos\phi) \\ 1 & b^2(1 - \cos\phi) & c^2 + (a^2 + b^2)\cos\phi \end{vmatrix}$$

$R_2 \to R_2 - R_1$ or $R_3 \to R_3 - R_1$

9. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = \lambda - 3$

$$\Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & \lambda \end{vmatrix} = 2\lambda + \mu - 16$$

$$\Delta_y = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 3 \\ 1 & \mu & \lambda \end{vmatrix} = 4(\lambda - \mu + 7)$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 2 & \mu \end{vmatrix} = (\mu - 10)$$

(i) $\Delta \neq 0 \therefore \lambda \neq 3$

(ii) infinite solutions $\Delta = 0, \Delta_x = 0, \Delta_y = 0, \Delta_z = 0.$

$\therefore \quad \lambda = 3, \mu = 10$

(iii) $\Delta = 0, \Delta_x, \Delta_y, \Delta_z$

$\therefore \quad \lambda = 3, \mu \neq 10$

10. x, y, z

$x + y + z = 5000$ ......**(i)**

$6x + 7y + 8z = 35800$ ......**(ii)**

$6x + 7y - 8z = 7000$ ......**(iii)**

$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 500 \\ 35800 \\ 7000 \end{bmatrix}$$

$X = A^{-1}B$

$|A| = -16$

$$\text{adj} = \begin{bmatrix} -112 & 96 & 0 \\ 15 & -14 & -1 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$X = A^{-1}B = -\frac{1}{16}\begin{bmatrix} -112 & 15 & 11 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1000 \\ 2200 \\ 1800 \end{bmatrix}, x = 1000, y = 2200, z = 1800$$

14. $A = \begin{vmatrix} 2 & 0 & -\alpha \\ 5 & \alpha & 0 \\ 0 & \alpha & 3 \end{vmatrix}$

$$|A| = \begin{vmatrix} 2 & 0 & -\alpha \\ 5 & \alpha & 0 \\ 0 & \alpha & 3 \end{vmatrix} \neq 0$$

$6\alpha - 5\alpha^2 \neq = 0$

$\alpha(6 - 5\alpha) \neq 0$

$\alpha = 0, 6/5 \qquad \therefore \quad \alpha \in R - \{0, 6/5\}$

$\alpha = 1$

$$A = \begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix} \Rightarrow |A| = 6 - 5 = 1$$

$$\Rightarrow \text{Adj } A = \begin{vmatrix} 3 & -1 & -1 \\ -5 & 6 & 5 \\ 3 & -2 & 2 \end{vmatrix}$$

$$\therefore \quad A^{-1} = \begin{vmatrix} 3 & -1 & -1 \\ -5 & 6 & 5 \\ 5 & -2 & 2 \end{vmatrix}$$

$|A - xI| = 0$

$$|A - xI| = 0) \Rightarrow \begin{vmatrix} 2-x & 0 & -1 \\ 5 & 1-x & 0 \\ 0 & 1 & 3-x \end{vmatrix} = 0$$

$\Rightarrow \quad x^3 - 6x^2 + 11x - 1 = 0$

$A^3 - 6A^2 + 11A = I$

$\Rightarrow \quad A^{-1} = A^2 - 6A + 11.I$

$(a^2 + b^2 + c^2)$

$$\begin{vmatrix} 1 & b^2(1-\cos\phi) & c^2(1-\cos\phi) \\ 0 & (a^2+b^2+c^2)\cos\phi & 0 \\ 0 & 0 & (a^2+b^2+c^2)\cos\phi \end{vmatrix}$$

$= (a^2 + b^2 + c^2)\cos^2\phi = \cos^2\phi$

a, b, c

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

3. $A = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$

$10\,B = \begin{vmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{vmatrix}$

$B = A^{-1}$

$AB = AA^{-1} = I$

$10\,AB = 10I$

(A) $(10B) = 10I$

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 0 & 5-\alpha \\ 0 & 10 & \alpha-5 \\ 0 & 0 & 5+\alpha \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$5 - \alpha = 0$

$\boxed{\alpha = 5}$

4. $A^2 - A + I = 0$

multiplying by A^{-1}

$A^{-1}AA - A^{-1}A + A^{-1}I = 0$

$IA - I + A^{-1} = 0$

$\boxed{A^{-1} = I - A}$

7. $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ $\quad a, b \in N$

$AB = \begin{pmatrix} a & 2b \\ 3a & 4b \end{pmatrix}$

$BA = \begin{pmatrix} a & 2b \\ 3b & 4b \end{pmatrix}$

For AB = BA

$b = a \rightarrow$ their are infinite

Natural number for which a = 6

so Infinite matrix B possible

8. $|A^2| = 25$

$|A|^2 = 25$

$|A| = \pm 5$

$$\begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix} = \pm 5$$

$25\alpha = \pm 5$

$\alpha = \pm \dfrac{1}{5}$

9. $A^2 = I$

$|A^2| = |I|$

$|A^2| = 1$

$|A| = \pm 1$

statement–1 :

If $A \neq I$, $A \neq -I$

but $|A| = \pm 1$

so this statement is true

statement–2 :

Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$|A| = -1$ $\quad$ $tr(A) = 0$

but $A \neq I$, $A \neq -I$

so statement-2 is false

14. $A^T = A$

$B^T = B$

Statement-1 :

$(A(BA))^T = (BA)^T A^T$

$= A^T B^T A^T = A(BA) \rightarrow$ symetric

$((AB)A))^T = A^T B^T A^T = (AB)A \rightarrow$ symetric

Statement - 1 is true

Statement-2 :

$(AB)^T = B^T A^T = BA$

if AB = BA then

$(AB)^T = BA = AB$

Statement.- 2 is true

but Not a correct expalnation.

15. **Statement–1 :** The value of det. of skew sym. matrix of odd order is always zero. So Statement-I. is true.

Statement–II : This st. is not always true depends on the order of matrix.

$|-A| = -|A|$ if order is odd, so Statement—II is wrong.

Statement-I is true and Statement-II is false.

16. Since H is a diagonal matrix.

We know that product of two diagonal matrix is always a diagonal matrix.

So $H^{70} = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \ldots \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$ 70 times

$$= \begin{bmatrix} \omega^{70} & 0 \\ 0 & \omega^{70} \end{bmatrix} = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = H$$

17. $(P^2 + Q^2)\,P = P^3 + Q^2P$ .....**(i)**

$(P^2 + Q^2)\,Q = P^2Q + Q^3$ .....**(ii)**

Equation **(i)** – Equation **(ii)**

$(P^2 + Q^2)(P - Q) = P^3 - Q^3 + Q^2P - P^2Q$

$(P^2 + Q^2)(P - Q) = 0 \quad \because \quad (P \neq Q)$

$P^2 + Q^2 = 0$

So $|P^2 + Q^2| = 0$

18. $A^{-1} = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}^T = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix}$

and $A^{-1}\ A\ U_1 = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ .....**(i)**

$A^{-1}\ A\ U_2 = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$ .....**(ii)**

Eq. **(i)** + **(i)**

$$U_1 + U_2 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

21.

$$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$(1\lambda + 1) - 1(-\lambda^2 + 1) - (\lambda + 1) = 0$

$(\lambda + 1)(\lambda + 1(\lambda - 1) - 1) = 0$

$\lambda = -1$ or 0 or 1

22. $A(\text{adj } A) = |A|\, I_n = AA^T$ [Given]

$|A| = 10a + 3b$

$$A^T = \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$$

$$AA^T \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix} = \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix} = \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$$

$\Rightarrow 15a - 2b = 0 \quad \Rightarrow a = \dfrac{2b}{15}$ & $10a + 3b = 13$

$\Rightarrow a = \dfrac{13 - 3b}{10}$

$\Rightarrow \dfrac{2b}{15} = \dfrac{13 - 3b}{10} \quad \Rightarrow 4b = 39 - 9b$

$\Rightarrow 13b = 39 \quad \Rightarrow b = 3$

$\Rightarrow a\dfrac{2}{15} \times 3 = \dfrac{6}{15} = \dfrac{2}{5} \quad \Rightarrow 5a = 2$

$\therefore\ 5a + b = 2 + 3 = 5$

Part # II : IIT-JEE ADVANCED

3. $|M - I| = |M - M M^T|$

$|M - I| = |M| \, |I - M|$

$\Rightarrow |M - I| = |I - M|$

$\Rightarrow |M - I| = (-1)^3 |M - I|$

$\Rightarrow |M - I| = 0$

4. AX = U

$$\Rightarrow \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} f \\ g \\ h \end{bmatrix}$$

has infinitely many solutions.

$\Rightarrow |A| = 0$

$\Rightarrow (c - d)(ab - 1) = 0$

& (adj A) U = 0

$$\begin{bmatrix} bc - bd & -c & d \\ d - c & ac & -ad \\ 0 & 1 - ab & ab - 1 \end{bmatrix} \begin{bmatrix} f \\ g \\ h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} fbc - fbd - gc + dh \\ fd - fc + agc - adh \\ g - abg + adh - h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow fd - fc + agc - agh = 0$ .....**(i)**

BX = V

$$\Rightarrow \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$$

$|B| = a(dh - gc) + fc - fd = 0$ (from **(i)**)

$\therefore$ system can't have unique solution

Now X = (adj B)V

$$= \begin{bmatrix} dh - gc & g - h & c - d \\ fc & ah - f & -ac \\ -fd & -g + af & ad \end{bmatrix} \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

if afd $\neq 0 \Rightarrow$ (adj B) V $\neq 0$

$\therefore$ adfd $\neq 0$ then BX = V is inconsistent

5. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} \Rightarrow |A| = 6$

$$A^{-1} \Rightarrow \frac{\text{adj}A}{|A|} = \frac{1}{6}\begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix}$$

$$A^{-1} = \frac{1}{6}[A^2 + cA + dI]$$

$$\frac{1}{6}\begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$= \frac{1}{6}\left\{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix} + \begin{bmatrix} c & 0 & 0 \\ 0 & c & c \\ 0 & -2c & 4c \end{bmatrix} + \begin{bmatrix} d & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & d \end{bmatrix}\right\}$$

on comparing we get

$-1 = 5 + c \Rightarrow c = -6$

$1 = 14 + 4c + d \Rightarrow 1 = 14 - 24 + d$

$d = 11$

6. $PP^T = I$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \text{ \& so on}$$

$Q = PAP^T$

$Q^2 = (PAP^T)(PAP^T) = PA^2P^T$

$Q^{2005} = PA^{2005}P^T$

$x = P^T (PA^{2005}P^T)P$

$$\Rightarrow x = A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

10. (C) (i) If A is symmetric, $A^T = A$

$\Rightarrow \begin{bmatrix} a & b \\ c & a \end{bmatrix} = \begin{bmatrix} a & c \\ b & a \end{bmatrix}$

$\Rightarrow$ b = c

If A is skew symmetric, $A^T = -A$

$\Rightarrow \begin{bmatrix} a & b \\ c & a \end{bmatrix} = \begin{bmatrix} -a & -c \\ -b & -a \end{bmatrix}$

$\Rightarrow$ a = 0, b + c = 0

$\because$ b, c $\geq$ 0

$\Rightarrow$ a = 0, b = 0, c = 0

Now, $\det(A) = a^2 - bc$

$= a^2 - b^2$

($\because$ b = c for A being symmetric or skew symmetric or both)

= (a – b)(a + b) is divisible by p.

Let $(a - b)(a + b) = \lambda p$, $\lambda \in I$

Range of (a + b) is 0 to 2p – 2 which includes only one multiple of p i.e. p

$\therefore$ a + b = p & a – b $\in$ I

$\Rightarrow$ possible number of pairs of a & b will be p – 1.

Also, range of (a – b) is 1 – p to p – 1 which includes only one multiple of p i.e. 0

$\therefore$ a – b = 0 & a + b $\in$ I

$\Rightarrow$ Possible number of pairs of a & b will be p.

Hence total number of A in T_p will be

p + p – 1 = 2p – 1

(iii) Total number of A in $T_p = p^3$

when a $\neq$ 0 & det(A) is divisible by p, then number of A will be $(p-1)^2$

When a = 0 & det(A) is divisible by p, then number of A will be 2p–1.

So, total number of A for which det(A) is divisible by p

$= (p-1)^2 + 2p - 1$

$= p^2$

So number of A for which det(A) is not divisible by p

$= p^3 - p^2$

11. (Comment : Although 3 × 3 skew symmetric matrices can never be non-singular. Therefore the information given in question is wrong. Now if we consider only non singular skew symmetric matrices M & N, then the solution is-)

Given $M^T = -M$

$N^T = -N$

$MN = NM$

according to question $M^2N^2(M^TN)^{-1}(MN^{-1})^T$

$= M^2N^2N^{-1}(M^T)^{-1}(N^{-1})^TM^T$

$= M^2N^2N^{-1}(-M)^{-1}(N^T)^{-1}(-M)$

$\left[\begin{array}{l} MN = NM \\ (MN)^{-1} = (NM)^{-1} \\ N^{-1}M^{-1} = M^{-1}N^{-1} \end{array}\right.$

$= -M^2NM^{-1}N^{-1}M$

$= -M^2NN^{-1}M^{-1}M = -M^2$

12. $\begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix}$

$= 1 - c\omega - a(\omega - \omega^2 c) = (1 - c\omega) - a\omega(1 - c\omega)$

$= (1 - c\omega)(1 - a\omega)$

for non singular matrix

$c \neq \dfrac{1}{\omega}$ & $a \neq \dfrac{1}{\omega}$

$\Rightarrow c \neq \omega^2,\ a \neq \omega^2$

$\Rightarrow$ a & c must be ω & b can be ω or ω^2

$\therefore$ total matrices = 2

14. $|Q| = \begin{vmatrix} 2^2a_{11} & 2^3a_{12} & 2^4a_{13} \\ 2^3a_{21} & 2^4a_{22} & 2^5a_{23} \\ 2^4a_{31} & 2^5a_{32} & 2^6a_{33} \end{vmatrix}$

$\Rightarrow |Q| = 2^2.2^3.2^4.\begin{vmatrix} a_{11} & 2a_{12} & 2^2a_{13} \\ a_{21} & 2a_{22} & 2^2a_{23} \\ a_{31} & 2a_{32} & 2^2a_{33} \end{vmatrix}$

$= 2^2.2^3.2^4|P|.2^3$

$= 2^2.2^3.2^4.2.2^3 = 2^{13}$

15. $P^T = 2P + I$

$\Rightarrow$ $P = 2P^T + I$

$\Rightarrow$ $P = 2(2P + I) + I$

$\Rightarrow$ $P = 4P + 3I$

$\Rightarrow$ $P = -I$

$\Rightarrow$ $PX = -X$

16. $|adjP| = \begin{vmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{vmatrix}$

$\Rightarrow$ $|P|^2 = 4$

$\Rightarrow$ $|P| = \pm 2$

19. $\left(\frac{P}{K}\right).Q = 1$

$\therefore$ $Q = \left(\frac{P}{K}\right)^{-1}$

Comparing q_{23}, we get

$$\frac{-K}{8} = \frac{-K(3a+4)}{(12a+20)}$$

$\alpha = -1$

Also, $|P| . |Q| = K^3$

$\therefore$ $(12a+20)\frac{K^2}{2} = K^3$

$K = 6a + 10 = 4$

Hence **(B)**, **(C)** are correct.

20. $P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 48 & 8 & 1 \end{bmatrix}$

$P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 48 & 8 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 96 & 12 & 1 \end{bmatrix}$

$\therefore$ $P^n = \begin{bmatrix} 1 & 0 & 0 \\ 4n & 1 & 0 \\ 8(n^2+n) & 4n & 1 \end{bmatrix}$

$\therefore$ $P^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ 8 \times 50(51) & 200 & 1 \end{bmatrix}$

$P^{50} - Q = I$

$\therefore$ Equate we get

$200 - q_{21} = 0 \Rightarrow q_{21} = 200$

$400 \times 51 - q_{31} = 0$

$q_{31} = 400 \times 51$

$200 - q_{32} = 0 \Rightarrow q_{32} = 200$

$$\frac{q_{31} - q_{32}}{q_{21}} = \frac{400 \times 51 + 200}{200} = 2(51) + 1 = 103$$

MOCK TEST

1. (D)

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix}$$

$$BA = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a+3b & 2a+4b \\ c+3d & 2c+4d \end{bmatrix}$$

if $AB = BA$, then $a + 2c = a + 3b$

$\Rightarrow 2c = 3b$ $\Rightarrow$ $b \neq 0$

$b + 2d = 2a + 4b$

$\Rightarrow 2a - 2d = -3b$

$$\frac{a-d}{3b-c} = \frac{-\frac{3}{2}b}{3b-\frac{3}{2}b} = -1$$

2. $AA' = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}$

$AA' = \begin{bmatrix} 11 & -1 \\ -1 & 5 \end{bmatrix}$ symmetrix

3. (A)

$(B^T AB)^T = B^T A^T (B^T)^T = B^T A^T B$

$= B^T AB$ iff A is symmetric

$\therefore$ $B^T AB$ is symmetric iff A is symmetric

Also $(B^T AB)^T = B^T A^T B = (-B) A^T B$

$\therefore$ $B^T AB$ is not skew symmetric if B is skew symmetric

4. (C)

$AB = A, BA = B \Rightarrow A^2 = A$ and $B^2 = B$

$(A + B)^2 = A^2 + B^2 + AB + BA$

$= A + B + A + B = 2(A + B)$

$(A + B)^3 = (A + B)^2 (A + B)$

$= 2(A + B)^2 = 2^2 (A + B)$

$\therefore$ $(A + B)^7 = 2^6 (A + B) = 64 (A + B)$

5. (B)

$A^3 = 0$

$(I + A + A^2)(I - A) = I - A^3 = I$

$\therefore$ $I + A + A^2 = (I - A)^{-1}$

6. $A = \begin{bmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{bmatrix}$

$|A| = (3x+\lambda)\begin{vmatrix} 1 & x & x \\ 1 & x+\lambda & x \\ 1 & x & x+\lambda \end{vmatrix}$

by $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$ $= (3x+\lambda)\begin{vmatrix} 1 & x & x \\ 1 & \lambda & -\lambda \\ 0 & 0 & \lambda \end{vmatrix}$

$= (3x+\lambda)(\lambda^2)$

$|A| \neq 0$ for non-singular matrix

$\therefore$ $3x + \lambda \neq 0, \lambda \neq 0$

7. Let $X = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$ $x^n = 0$ $\forall\, n \geq 2$

$\Rightarrow A = X + I$

$A^n = (X + I)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} I \ldots\ldots\ldots\ldots + {}^nC_{n-2} x^2 +$

${}^nC_{n-1} + {}^nC_n I = n\,x + I = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$

$$\lim_{n\to\infty} \frac{1}{n}\begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$$

8. (A)

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}(-1)$$

$$= \frac{1}{2}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

10. (C)

S_1 : $A^T = A$

$((A^{-1})^{-1})^{-1} = A^{-1}$

$(A^{-1})^T = (A^T)^{-1} = A^{-1}$

So $((A^{-1})^{-1})^{-1}$ is also symmetric

S_2 : Obvious

S_3 : Obvious

S_4 : $|A| \neq 0, |B| \neq 0 \quad \Rightarrow \quad |AB| \neq 0$

So AB is invertible

12. (A, B, D)

$$A^2 - 4A - 5I_3 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$-4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & -8 & -8 \\ -8 & -4 & -8 \\ -8 & -8 & -4 \end{bmatrix} + \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$\therefore \quad A^2 - 4A - 5I_3 = 0$

or $\quad A^{-1}A^2 - 4A^{-1}A - 5A^{-1}I_3 = 0$

or $\quad (A^{-1}A)A - 4I_3 - 5A^{-1} = 0$

or $\quad IA - 4I_3 - 5A^{-1} = 0$

$\therefore \quad A^{-1} = \frac{1}{5}(A - 4I_3)$

Also, $\quad |A^2| = \begin{vmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{vmatrix} = 9(81-64) - 8(72-64) + 8(64-72)$

$= 9 \times 17 - 8 \times 8 + 8 \times (-8) = 133 - 128 = 5 \neq 0$

$\therefore \quad A^2$ is invertible

and $A^3 = A \cdot A^2 = A.(4A - 5I_3) = 4A^2 - 5A$

$$= \begin{bmatrix} 36 & 32 & 32 \\ 32 & 36 & 32 \\ 32 & 32 & 36 \end{bmatrix} + \begin{bmatrix} -5 & -10 & -10 \\ -10 & -5 & -10 \\ -10 & -10 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 31 & 22 & 22 \\ 22 & 31 & 22 \\ 22 & 22 & 31 \end{bmatrix}$$

$\therefore \quad |A^3| \neq 0$

$\therefore \quad A^3$ is invertible.

13. Taking $C_3 \rightarrow C_3 - (C_1\alpha - C_2)$

we get $|A| = \begin{vmatrix} a & b & 0 \\ b & c & 0 \\ 2 & 1 & -2\alpha+1 \end{vmatrix} = (1 - 2\alpha)(ac - b^2)$

$\therefore \quad$ non-invertible if $\alpha = \frac{1}{2}$ and if a, b, c are in G.P.

14. (A, B, C)

$\because \quad |A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix} = 3(-3+4) - 2(-3+4) + 0 = 1$

$\because \quad \text{adj}(\text{adj } A) = |A|^{3-2} A = A$ and $|\text{adj}(\text{adj } A)| = |A| = 1$

Also, $\quad |\text{adj } A| = |A|^{3-1} = |A|^2 = 1^2 = 1$

15. (A, D)

$$A^2 = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 3A$$

$A^3 = A^2A = 3A.A = 3A^2 = 3.(3A) = 9A$ and $|A| = 0$

$\therefore \quad A^{-1}$ does not exist

16. (D)

Statement-1 is false

$\because \quad A = [a_{ij}]_{n \times n}$ where $a_{ij} = 0, i \geq j$

$\therefore \quad |A| = 0$ hence A is singular inverse of A is not defined

Statement-2 $|A| = 0$

$\therefore \quad$ inverse of A is not defined.

17. $adj(adj\,A) = |A|^{n-2} A$

Here, $n = 3$

$\therefore$ $(adj)(adj\,A) = |A|\,A$ ....**(i)**

Now, $|A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix}$

$= 3(-3+4) + 3(2) + 4(-2) = 1$

From eq. **(i)**,

$adj(adj)\,A = A$

18. (A)

$$A(\alpha) = \begin{bmatrix} f_1(\alpha) & f_2(\alpha) & f_3(\alpha) \\ f_4(\alpha) & f_5(\alpha) & f_6(\alpha) \\ f_7(\alpha) & f_8(\alpha) & f_9(\alpha) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x - \alpha$ is a factor of $f_1(x)$, $f_2(x)$ .... $f_9(x)$

$f(x) = (x - \alpha)\,\phi(x)$

$f(\alpha) = 0 \Rightarrow x - \alpha$ is a factor of $f(x)$

19. In a unit matrix of order $n \times n$

Number of non-zero rows $= n$

$\therefore$ Rank is n. The rank of a non singular matrix of order $n \times n$ is n

$\therefore$ For non singular $|A| \neq 0$.

20. $A = \begin{pmatrix} 0 & -c & b \\ c & 0 & a \\ -b & -a & 0 \end{pmatrix}$

$\because$ $A = -A'$ ($\because$ A is skew symmetric)

$\therefore$ $\det(A) = \det(-A')$

$= -\det(A')$

$= -\det A$

$\therefore$ $\det A = 0$

$\because$ $\det A' = \det(-A')$ is not true

$\therefore$ $\det(-A') = (-1)^3 \det(A') = -\det A'$

21. (A) → (q), (B) → (s, t), (C) → (r), (D) → (p)

(A) $A^2 = -I$ $\therefore$ A is of even order

(B) $(I + A)^n = C_0 I^n + C_1 I A + C_2 I A^2 + \ldots\ldots\ldots + C_n I A^n$

$= C_o I + C_1 A + C_2 A + \ldots\ldots\ldots + C_n A$

$= I + (2^n - 1) A$

$\therefore$ $\lambda = 2^n - 1$

(C) $A^2 = A$ and $B = I - A$

$AB + BA + I - (I + A^2 - 2A)$

$= AB + BA - A + 2A = AB + BA + A$

$= A(I - A) + (I - A)A + A$

$= A - A + A - A + A = A$

(D) $A^* = A$, $B^* = B$

$(AB - BA)^* = B^* A^* - A^* B^* = BA - AB$

22. (A) $A * B = \frac{1}{2}(AB + BA) = \frac{1}{2}(BA + AB) = B * A$

$A * A = \frac{1}{2}(A.A + A.A) = A^2$

$A * I = \frac{1}{2}(AI + IA) = \frac{1}{2}(A + A) = A$

Now, $A * (B + C) = \frac{1}{2}\{A(B + C) + (B + C)A\}$

(B) $A * B = \frac{1}{2}(AB' + A'B) = \frac{1}{2}(A'B + AB')$

$= \frac{1}{2}(BA' + B'A) = B * A(P)$

$A * A = \frac{1}{2}(AA' + A'A) \neq A^2$

$A * (B + C) = \frac{1}{2}(A(B + C)' + A'(B + C))$

$= \frac{1}{2}(A(B' + C') + A'(B + C))$

$= \frac{1}{2}(AB' + A'B) + \frac{1}{2}(AC' + A'C)$

$= A * B + A * C$ (Q)

$A * I = \frac{1}{2}(AI' + A'I) = \frac{1}{2}(A + A') \neq A$ or O

(C) $A * B = \frac{1}{2}(AB - BA) - \frac{1}{2}(BA - AB) = -(B * A)$

$$A * (B + C) = \frac{1}{2}\{A(B + C) - (B + C)A\}$$

$$= \frac{1}{2}(AB - BA) + \frac{1}{2}(AC - CA)$$

$$= A * B + A * C(Q)$$

$$A * A = \frac{1}{2}(A^2 - A^2) = O \neq A^2$$

$$A * I = \frac{1}{2}(AI - IA) = \frac{1}{2}(A - A) = O(T)$$

23.

1. A is singular

$$\therefore \quad |A| = 0 \quad \Rightarrow \quad \begin{vmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{vmatrix} = 0$$

$\Rightarrow$ $1(40 - 40) - 3(20 - 24) + (\lambda + 2)(10 - 12) = 0$

$\Rightarrow$ $12 - 2\lambda - 4 = 0$

$\Rightarrow$ $\lambda = 4$

$$\therefore \quad A + B = \begin{pmatrix} 4 & 5 & 10 \\ 5 & 6 & 13 \\ 5 & 6 & 14 \end{pmatrix}$$

$\Rightarrow$ $tr(A + B) = 4 + 6 + 14 = 24$

2. $2A + 3B = \begin{pmatrix} 11 & 12 & 2\lambda+16 \\ 13 & 14 & 31 \\ 12 & 13 & 32 \end{pmatrix}$

$|2A + 3B| = 0$

we get $\lambda = \frac{17}{2}$

$\Rightarrow$ $2\lambda = 17$

3. For $\lambda = 3$

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 3 & 2 & 4 \\ 3 & 2 & 5 \\ 2 & 1 & 4 \end{pmatrix}$$

$\because$ $tr(AB) + tr(BA) = 2tr(AB)$

$\because$ $tr(AB) = C_{11} + C_{22} + C_{33} = 119$

$\therefore$ $\frac{1}{7}(tr(AB) + tr(BA)) = \frac{1}{7}(238) = 34$

24.

1. $adj(adj\ A) = |A|^{n-2} A = |A|\ A = 2A$

2. $|adj(adj)\ A| = |A|^{(n-1)^2} = |A|^4 = 2^4 = 16$

3. $|adj\ B| = |B|^{n-1} = |B|^2 = 24^2 = 576$

25.

1. (A)

Possible combinations are (1, 2, 6), (1, 3, 5), (1, 8, 9), (2, 3, 4), (2, 7, 9), (3, 6, 9), (3, 7, 8), (4, 5, 9), (4, 6, 8), (5, 6, 7)

Hence total 10 combinations are possible.

2. (B)

In a matrix $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ a_{22} must be a multiple of 3

(3, 6 or 9) because from the above possible combinations only 3, 6 & 9 are repeated four times in arow or column or diagonal.

3. (A)

Clearly maximum value of sum of the diagonal elements is 18 which is called the trace or the matrix A.

26. (4)

$A'A = I$

$\therefore$ $|A'A| = |I|$ $\Rightarrow$ $|A| = \pm 1$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \pm 1$$

$\Rightarrow$ $3abc - a^3 - b^3 - c^3 = \pm 1$

$\Rightarrow$ $a^3 + b^3 + c^3 = 2$ or 4

27. (1)

$$I+A=\begin{pmatrix}1&0\\0&1\end{pmatrix}+\begin{pmatrix}1&2\\1&1\end{pmatrix}=\begin{pmatrix}2&2\\1&2\end{pmatrix}$$

and $\quad I-A=\begin{pmatrix}1&0\\0&1\end{pmatrix}-\begin{pmatrix}1&2\\1&1\end{pmatrix}=\begin{pmatrix}0&-2\\-1&0\end{pmatrix}$

Now, $\quad |I-A|=\begin{vmatrix}0&-2\\-1&0\end{vmatrix}=0-2=-2$

$$\text{adj}\,(I-A)=\begin{pmatrix}0&2\\1&0\end{pmatrix}$$

$$(I-A)^{-1}=\begin{pmatrix}0&-1\\-\frac{1}{2}&0\end{pmatrix}$$

$$\therefore\quad \phi(A)=(I+A)\,(I-A)^{-1}=\begin{pmatrix}2&2\\1&2\end{pmatrix}\begin{pmatrix}0&-1\\-\frac{1}{2}&0\end{pmatrix}$$

$$=\begin{pmatrix}-1&-2\\-1&-1\end{pmatrix}=-A$$

28. (1)

$$\frac{|\text{adj}B|}{|C|}=\frac{|\text{adj}(\text{adj}A)|}{|5A|}=\frac{|A|^{(3-1)^2}}{5^3\,|A|}=\frac{|A|^3}{125}$$

Now $\quad |A|=5$

$$\therefore\quad \frac{|\text{adj}B|}{|C|}=1$$

29. $P^2=\begin{bmatrix}\cos\frac{\pi}{9}&\sin\frac{\pi}{9}\\-\sin\frac{\pi}{9}&\cos\frac{\pi}{9}\end{bmatrix}=\begin{bmatrix}\cos\frac{\pi}{9}&\sin\frac{\pi}{9}\\-\sin\frac{\pi}{9}&\cos\frac{\pi}{9}\end{bmatrix}$

$$=\begin{bmatrix}\cos^2\frac{\pi}{9}-\sin^2\frac{\pi}{9}&\cos\frac{\pi}{9}\sin\frac{\pi}{9}+\sin\frac{\pi}{9}\cos\frac{\pi}{9}\\-\sin\frac{\pi}{9}\cos\frac{\pi}{9}-\cos\frac{\pi}{9}\sin\frac{\pi}{9}&-\sin^2\frac{\pi}{9}+\cos^2\frac{\pi}{9}\end{bmatrix}$$

$$=\begin{bmatrix}\cos\frac{2\pi}{9}&\sin\frac{2\pi}{9}\\-\sin\frac{2\pi}{9}&\cos\frac{2\pi}{9}\end{bmatrix}$$

$$P^3=\begin{bmatrix}\cos\frac{2\pi}{9}&\sin\frac{2\pi}{9}\\-\sin\frac{2\pi}{9}&\cos\frac{2\pi}{9}\end{bmatrix}\begin{bmatrix}\cos\frac{\pi}{9}&\sin\frac{\pi}{9}\\-\sin\frac{\pi}{9}&\cos\frac{\pi}{9}\end{bmatrix}$$

$$P^3=\begin{bmatrix}\cos\frac{3\pi}{9}&\sin\frac{3\pi}{9}\\-\sin\frac{3\pi}{9}&\cos\frac{3\pi}{9}\end{bmatrix}=\begin{bmatrix}\frac{1}{2}&\frac{\sqrt{3}}{2}\\-\frac{\sqrt{3}}{2}&\frac{1}{2}\end{bmatrix}$$

$$P^6=P^3\cdot P^3=\begin{bmatrix}\frac{1}{2}&\frac{\sqrt{3}}{2}\\-\frac{\sqrt{3}}{2}&\frac{1}{2}\end{bmatrix}\begin{bmatrix}\frac{1}{2}&\frac{\sqrt{3}}{2}\\-\frac{\sqrt{3}}{2}&\frac{1}{2}\end{bmatrix}=\begin{bmatrix}-\frac{1}{2}&\frac{\sqrt{3}}{2}\\-\frac{\sqrt{3}}{2}&\frac{1}{2}\end{bmatrix}$$

$$\alpha P^6+\beta P^3+\alpha I=0$$

i.e. $\begin{bmatrix}-\frac{\alpha}{2}+\frac{\beta}{2}+\gamma&\frac{\sqrt{3}}{2}\alpha+\frac{\sqrt{3}}{2}\beta\\-\frac{\sqrt{3}}{2}\alpha-\frac{\sqrt{3}}{2}\beta&-\frac{\alpha}{2}+\frac{\beta}{2}+\gamma\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$

$\alpha=-\beta\ \ \beta=-\gamma\quad\Rightarrow\quad\alpha=\gamma$

$\Rightarrow\quad \alpha+\beta=0\ ;\ \beta+\gamma=0\ ;\ \alpha-\gamma=0$

$\Rightarrow\quad (\alpha^2+\beta^2+\gamma^2)^o=1$

30. Given $\sum_{k=1}^{4}a_{ik}=1\ \ \forall\, i\in\{1,2,3,4\}$

Let's consider $\qquad B=A^2\ \ B=[\,bij]_{4\times4}$

$$bij=\sum_{k=1}^{4}a_{ik}\,.\,a_{kj}$$

Sum of elements of i^{th} row in A^2 is

$$\sum_{J=1}^{4}b_{ij}=\sum_{j=1}^{4}\sum_{k=1}^{4}a_{ik}.a_{kj}=\sum_{K=1}^{4}\sum_{J=1}^{4}a_{ik}.a_{kj}=\sum_{K=1}^{4}a_{ik}\sum_{j=1}^{4}a_{kj}$$

$$\because\quad \sum_{j=1}^{4}a_{kj}=1=\sum_{K=1}^{4}a_{ik}=1$$

$\Rightarrow\quad$ Sum of elements in a row of A^2 is 1.

$\Rightarrow\quad$ Similarly sum of elements in a row of A^{10} is 1

$\Rightarrow\quad$ Sum of all elements = 4