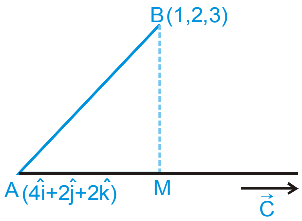
SOLVED EXAMPLES

Ex. 1 Find the distance of the point $B(\hat{i} + 2\hat{j} + 3\hat{k})$ from the line which is passing through $A(4\hat{i} + 2\hat{j} + 2\hat{k})$ and which is parallel to the vector $\vec{C} = 2\hat{i} + 3\hat{j} + 6\hat{k}$.

Sol. $AB = \sqrt{3^2 + 1^2} = \sqrt{10}$



$$AM = \overrightarrow{AB}.\hat{i} = (-3\hat{i} + \hat{k}).\frac{(2\hat{i} + 3\hat{j} + 6\hat{k})}{7}$$

$$= -6 + 6 = 0$$

$BM^2 = AB^2 - AM^2$

So, $BM = AB = \sqrt{10}$

Ex. 2 Find the direction cosines ℓ, m, n of a line which are connected by the relations $\ell + m + n = 0$, $2mn + 2m\ell - n\ell = 0$

Sol. Given, $\ell + m + n = 0$ ..... **(i)**

$2mn + 2m\ell - n\ell = 0$ ..... **(ii)**

From (1), $n = -(\ell + m)$.

Putting $n = -(\ell + m)$ in equation (ii), we get,

$-2m(\ell + m) + 2m\ell + (\ell + m)\ell = 0$

or, $-2m\ell - 2m^2 + 2m\ell + \ell^2 + m\ell = 0$

or, $\ell^2 + m\ell - 2m^2 = 0$

or, $\left(\frac{\ell}{m}\right)^2 + \left(\frac{\ell}{m}\right) - 2 = 0$ [dividing by m^2]

or $\frac{\ell}{m} = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} = 1, -2$

Case I. when $\frac{\ell}{m} = 1$: In this case $m = \ell$

From (1), $2\ell + n = 0$ $\Rightarrow$ $n = -2\ell$

$\therefore$ $\ell : m : n = 1 : 1 : -2$

$\therefore$ Direction ratios of the line are 1, 1, – 2

$\therefore$ Direction cosines are

$$\pm\frac{1}{\sqrt{1^2 + 1^2 + (-2)^2}}, \pm\frac{1}{\sqrt{1^2 + 1^2 + (-2)^2}}, \pm\frac{-2}{\sqrt{1^2 + 1^2 + (-2)^2}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}} \quad \text{or} \quad -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$$

Case II. When $\frac{\ell}{m} = -2$: In this case $\ell = -2m$

From (i), $-2m + m + n = 0 \quad \Rightarrow \quad n = m$

$\therefore \quad \ell : m : n = -2m : m : m$

$= -2 : 1 : 1$

$\therefore$ Direction ratios of the line are $-2, 1, 1$.

$\therefore$ Direction cosines are

$$\pm\frac{-2}{\sqrt{(-2)^2+1^2+1^2}}, \pm\frac{1}{\sqrt{(-2)^2+1^2+1^2}}, \pm\frac{1}{\sqrt{(-2)^2+1^2+1^2}}$$

$\Rightarrow \quad \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \quad$ or $\quad \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}$

Ex. 3 If $\vec{a}, \vec{b}, \vec{c}$ a re three non zero vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$, prove that $\vec{a}, \vec{b}, \vec{c}$ are mutually at right angles and $|\vec{b}| = 1$ and $|\vec{c}| = |\vec{a}|$.

Sol. $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} = \vec{b} \times \vec{c}$

$\Rightarrow \quad \vec{c} \perp \vec{a}, \vec{c} \perp \vec{b}$ and $\vec{a} \perp \vec{b}, \vec{a} \perp \vec{c}$

$\Rightarrow \quad \vec{a} \perp \vec{b}, \vec{b} \perp \vec{c}$ and $\vec{c} \perp \vec{a}$

$\Rightarrow \quad \vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors.

Again, $\quad \vec{a} \times \vec{b} = \vec{c} \quad$ and $\quad \vec{b} \times \vec{c} = \vec{a}$

$\Rightarrow \quad |\vec{a} \times \vec{b}| = |\vec{c}| \quad$ and $\quad |\vec{b} \times \vec{c}| = |\vec{a}|$

$\Rightarrow \quad |\vec{a}||\vec{b}|\sin\frac{\pi}{2} = |\vec{c}| \quad$ and $\quad |\vec{b}||\vec{c}|\sin\frac{\pi}{2} = |\vec{a}| \quad \left(\because \vec{a} \perp \vec{b} \text{ and } \vec{b} \perp \vec{c}\right)$

$\Rightarrow \quad |\vec{a}||\vec{b}| = |\vec{c}| \quad$ and $\quad |\vec{b}||\vec{c}| = |\vec{a}| \quad \Rightarrow \quad |\vec{b}|^2 |\vec{c}| = |\vec{c}|$

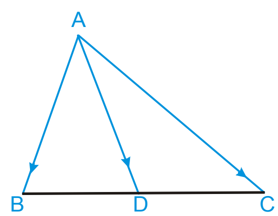
$\Rightarrow \quad |\vec{b}|^2 = 1 \quad \Rightarrow \quad |\vec{b}| = 1$

putting in $\quad |\vec{a}||\vec{b}| = |\vec{c}|$

$\Rightarrow \quad |\vec{a}| = |\vec{c}|$

Ex. 4 D is the mid point of the side BC of a ΔABC, show that $AB^2 + AC^2 = 2(AD^2 + BD^2)$

Sol. We have $\overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DB}$



$\Rightarrow \quad AB^2 = (\overrightarrow{AD} + \overrightarrow{DB})^2$

$\Rightarrow \quad AB^2 = AD^2 + DB^2 + 2\overrightarrow{AD} \cdot \overrightarrow{DB} \quad$ ....(i)

Also we have $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$

$\Rightarrow \quad AC^2 = (\overrightarrow{AD} + \overrightarrow{DC})^2$

$\Rightarrow \quad AC^2 = AD^2 + DC^2 + 2\overrightarrow{AD} \cdot \overrightarrow{DC} \quad$ ....(ii)

Adding (i) and (ii), we get $AB^2 + AC^2 = 2AD^2 + 2BD^2 + 2\overrightarrow{AD} \cdot (\overrightarrow{DB} + \overrightarrow{DC})$

$\Rightarrow \quad AB^2 + AC^2 = 2(AD^2 + BD^2) \quad \because \overrightarrow{DB} + \overrightarrow{DC} = \vec{0}$

Ex. 5 For any vector $\vec{a}$, prove that $|\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2 = 2\,|\vec{a}|^2$

Sol. Let $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$. Then

$$\vec{a}\times\hat{i} = (a_1\hat{i}+a_2\hat{j}+a_3\hat{k}) \times \hat{i} = a_1(\hat{i}\times\hat{i})+a_2(\hat{j}\times\hat{i}) + a_3\,(\hat{k}\times\hat{i}) = -a_2\hat{k}+a_3\hat{j}$$

$\Rightarrow$ $|\vec{a}\times\hat{i}|^2 = a_2^{\,2}+a_3^{\,2}$

$$\vec{a}\times\hat{j} = (a_1\hat{i}+a_2\hat{j}+a_3\hat{k}) \times \hat{j} = a_1\hat{k}-a_3\hat{i}$$

$\Rightarrow$ $|\vec{a}\times\hat{j}|^2 = a^2{}_1+a_3^{\,2}$

$$\vec{a}\times\hat{k} = (a_i\,\hat{i}+a_2\hat{j}+a_3\hat{k})\times\vec{k} = -\,a_i\hat{j}+a_2\hat{i}$$

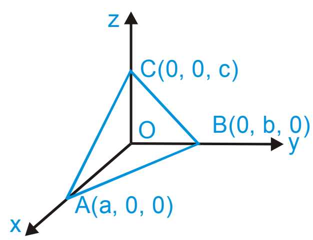
$\Rightarrow$ $|\vec{a}\times\hat{k}|^2 = a_1^{\,2}+a_2^{\,2}$

$\therefore$ $|\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2 = a_2^{\,2}+a_3^{\,3}+a_1^{\,2}+a_3^{\,2}+a_1^{\,2}+a_2^{\,2}$

$$= 2\,(a_1^{\,2}+a_2^{\,2}+a_3^{\,2}) = 2\,|\vec{a}|^2$$

Ex. 6 If a variable plane cuts the coordinate axes in A, B and C and is at a constant distance p from the origin, find the locus of the centroid of the tetrahedron OABC.

Sol. Let $A \equiv (a, 0, 0)$, $B \equiv (0, b, 0)$ and $C \equiv (0, 0, c)$



$\therefore$ Equation of plane ABC is $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$

Now p = length of perpendicular from O to plane **(i)**

$$= \frac{1}{\sqrt{\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2}}} \quad \text{or} \quad p^2 = \frac{1}{\left(\dfrac{1}{a}\right)^2+\left(\dfrac{1}{b}\right)^2+\left(\dfrac{1}{c}\right)^2}$$

Let G(α, β, γ) be the centroid of the tetrahedron OABC, then

$$\alpha = \frac{a}{4},\ \beta = \frac{b}{4},\ \gamma = \frac{c}{4} \qquad \left[\because\ \alpha = \frac{a+0+0+0}{4} = \frac{a}{4}\right]$$

or, $a = 4\alpha,\ b = 4\beta,\ c = 4\gamma$

Putting these values of a, b, c in equation **(ii)**, we get

$$p^2 = \frac{16}{\left(\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}+\dfrac{1}{\gamma^2}\right)} \quad \text{or} \quad \frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2} = \frac{16}{p^2}$$

$\therefore$ locus of (α, β, γ) is $x^{-2}+y^{-2}+z^{-2} = 16\,p^{-2}$

Ex. 7 Find the angle between the lines $x - 3y - 4 = 0$, $4y - z + 5 = 0$ and $x + 3y - 11 = 0$, $2y - z + 6 = 0$.

Sol. Given lines are $\left.\begin{array}{r} x-3y-4=0 \\ 4y-z+5=0 \end{array}\right\}$ ..... **(1)**

and $\left.\begin{array}{r} x+3y-11=0 \\ 2y-z+6=0 \end{array}\right\}$ ..... **(2)**

Let ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 be the direction cosines of lines **(1)** and **(2)** respectively

$\because$ line **(1)** is perpendicular to the normals of each of the planes

$x - 3y - 4 = 0$ and $4y - z + 5 = 0$

$\therefore$ $\ell_1 - 3m_1 + 0.n_1 = 0$ ..... **(3)**

and $0\ell_1 + 4m_1 - n_1 = 0$ ..... **(4)**

Solving equations **(3)** and **(4)**, we get $\dfrac{\ell_1}{3-0} = \dfrac{m_1}{0-(-1)} = \dfrac{n_1}{4-0}$

or, $\dfrac{\ell_1}{3} = \dfrac{m_1}{1} = \dfrac{n_1}{4} = k$ (let).

Since line **(2)** is perpendicular to the normals of each of the planes

$x + 3y - 11 = 0$ and $2y - z + 6 = 0$,

$\therefore$ $\ell_2 + 3m_2 = 0$ ..... **(5)**

and $2m_2 - n_2 = 0$ ..... **(6)**

$\therefore$ $\ell_2 = -3m_2$

or, $\dfrac{\ell_2}{-3} = m_2$

and $n_2 = 2m_2$ or, $\dfrac{n_2}{2} = m_2$.

$\therefore$ $\dfrac{\ell_2}{-3} = \dfrac{m_2}{1} = \dfrac{n_2}{2} = t$ (let).

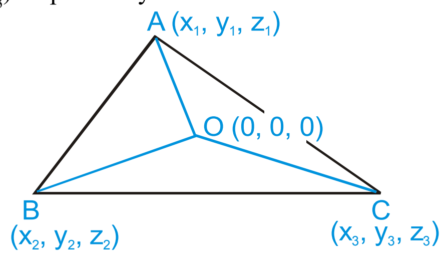
If θ be the angle between lines (1) and (2), then $\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2$

$= (3k)(-3t) + (k)(t) + (4k)(2t) = -9kt + kt + 8kt = 0$

$\therefore$ $\theta = 90°$.

Ex. 8 If two pairs of opposite edges of a tetrahedron are mutually perpendicular, show that the third pair will also be mutually perpendicular.

Sol. Let OABC be the tetrahedron, where O is the origin and co-ordinates of A, B, C are (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, x_3) respectively.



Let $OA \perp BC$ and $OB \perp CA$.

We have to prove that $OC \perp BA$.

Now, direction ratios of OA are $x_1 - 0, y_1 - 0, z_1 - 0$ or, x_1, y_1, z_1

direction ratios of BC are $(x_3 - x_2), (y_3 - y_2), (z_3 - z_2)$.

$\because$ $OA \perp BC$.

$\therefore$ $x_1(x_3 - x_2) + y_1(y_3 - y_2) + z_1(z_3 - z_2) = 0$ ..... **(1)**

Similarly,

$\because$ $OB \perp CA$

$\therefore$ $x_2(x_1 - x_3) + y_2(y_1 - y_3) + z_2(z_1 - z_3) = 0$ ..... **(2)**

Adding equations **(1)** and **(2)**, we get

$x_3(x_1 - x_2) + y_3(y_1 - y_2) + z_3(z_1 - z_2) = 0$

$\therefore$ $OC \perp BA$ ($\because$ direction ratios of OC are x_3, y_3, z_3 and that of BA are $(x_1 - x_2), (y_1 - y_2), (z_1 - z_2)$)

Ex. 9 If $\vec{a}' = \dfrac{\vec{b} \times \vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \vec{b}' = \dfrac{\vec{c} \times \vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \vec{c}' = \dfrac{\vec{a} \times \vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$ then shown that ; $\vec{a} \times \vec{a}' + \vec{b} \times \vec{b}' + \vec{c} \times \vec{c}' = 0$

where $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors.

Sol. Here $\vec{a} \times \vec{a}' = \dfrac{\vec{a} \times (\vec{b} \times \vec{c})}{[\vec{a}\,\vec{b}\,\vec{c}]}$

$$\vec{a} \times \vec{a}' = \frac{(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}$$

Similarly $\vec{b} \times \vec{b}' = \dfrac{(\vec{b}.\vec{a})\vec{c} - (\vec{b}.\vec{c})\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}$ & $\vec{c} \times \vec{c}' = \dfrac{(\vec{c}.\vec{b})\vec{a} - (\vec{c}.\vec{a})\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$

$$\vec{a} \times \vec{a}' + \vec{b} \times \vec{b}' + \vec{c} \times \vec{c}' = \frac{(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} + (\vec{b}.\vec{a})\vec{c} - (\vec{b}.\vec{c})\vec{a} + (\vec{c}.\vec{b})\vec{a} - (\vec{c}.\vec{a})\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]} \quad [\because \vec{a}.\vec{b} = \vec{b}.\vec{a} \text{ etc.}]$$

$= 0$.

Ex. 10 Let $\vec{a} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} + 2\alpha\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \alpha\hat{j} + \hat{k}$. Find the value(s) of α, if any, such that $\left\{\left(\vec{a} \times \vec{b}\right) \times \left(\vec{b} \times \vec{c}\right)\right\} \times \left(\vec{c} \times \vec{a}\right) = \vec{0}$.

Sol. $\left\{\left(\vec{a} \times \vec{b}\right) \times \left(\vec{b} \times \vec{c}\right)\right\} \times \left(\vec{c} \times \vec{a}\right) = \left[\vec{a}\ \vec{b}\ \vec{c}\right] \vec{b} \times \left(\vec{c} \times \vec{a}\right)$

$= \left[\vec{a}\ \vec{b}\ \vec{c}\right] \left\{\left(\vec{a}.\vec{b}\right)\vec{c} - \left(\vec{b}.\vec{c}\right)\vec{a}\right\}$

which vanishes if **(i)** $\left(\vec{a}.\vec{b}\right)\vec{c} = \left(\vec{b}.\vec{c}\right)\vec{a}$ **(ii)** $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$

(i) $\left(\vec{a}\,.\,\vec{b}\right)\ \vec{c}\ =\ \left(\vec{b}\,.\,\vec{c}\right)\ \vec{a}$ leads to the equation 2 α^3 + 10 α + 12 = 0, $\alpha^2 + 6\alpha = 0$ and $6\alpha - 6 = 0$, which do not have a common solution.

(ii) $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$

$$\Rightarrow \quad \begin{vmatrix} \alpha & 2 & -3 \\ 1 & 2\alpha & -2 \\ 2 & -\alpha & 1 \end{vmatrix} = 0 \qquad \Rightarrow \qquad 3\alpha = 2 \qquad \Rightarrow \qquad \alpha = \frac{2}{3}$$

Ex. 11 If $\vec{x}\times\vec{a}+k\vec{x}=\vec{b}$, where k is a scalar and $\vec{a},\vec{b}$ are any two vectors, then determine $\vec{x}$ in terms of $\vec{a},\vec{b}$ and k.

Sol. $\vec{x}\times\vec{a}+k\vec{x}=\vec{b}$ ....**(i)**

Premultiply the given equation vectorially by $\vec{a}$

$\vec{a}\times(\vec{x}\times\vec{a})\ +k\ (\vec{a}\times\vec{x})\ =\ \vec{a}\times\vec{b}$

$\Rightarrow$ $(\vec{a}\,.\,\vec{a})\,\vec{x}-(\vec{a}\,.\,\vec{x})\,\vec{a}+k(\vec{a}\times\vec{x})=\vec{a}\times\vec{b}$ ....**(ii)**

Premultiply **(i)** scalarly by $\vec{a}$

$[\vec{a}\ \vec{x}\ \vec{a}]\ +\ k\,(\vec{a}\,.\,\vec{x})\ =\ \vec{a}\,.\,\vec{b}$

$k(\vec{a}\,.\,\vec{x})=\vec{a}\,.\,\vec{b}$ ....**(iii)**

Substituting $\vec{x}\times\vec{a}$ from **(i)** and $\vec{a}\,.\,\vec{x}$ from **(iii)** in **(ii)** we get

$$\vec{x}\ =\ \frac{1}{a^2+k^2}\left[k\vec{b}+(\vec{a}\times\vec{b})+\frac{(\vec{a}\,.\,\vec{b})}{k}\,\vec{a}\right]$$

Ex. 12 Forces of magnitudes 5, 4, 3 units act on a particle in the directions $2\hat{i}-2\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}+2\hat{k}$ and $-2\hat{i}+\hat{j}-2\hat{k}$ respectively, and the particle gets displaced from the point A whose position vector is $6\hat{i}+2\hat{j}+3\hat{k}$, to the point B whose position vector is $9\hat{i}+7\hat{j}+5\hat{k}$. Find the work done.

Sol. If the forces are $\vec{F}_1,\vec{F}_2,\vec{F}_3$ then $\vec{F}_1\ =\frac{5}{3}(2\hat{i}-2\hat{j}+\hat{k})$; $\vec{F}_2\ =\frac{4}{3}(\hat{i}+2\hat{j}+2\hat{k})$ and $\vec{F}_3\ =\frac{3}{3}(-2\hat{i}+\hat{j}-2\hat{k})$ and hence the sum force $\vec{F}=\vec{F}_1+\vec{F}_2+\vec{F}_3\ =\frac{1}{3}(8\hat{i}+\hat{j}+7\hat{k})$

Displacement vector $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}\ =9\hat{i}+7\hat{j}+5\hat{k}-(6\hat{i}+2\hat{j}+3\hat{k})=3\hat{i}+5\hat{j}+2\hat{k}$

Work done $=\frac{1}{3}(8\hat{i}+\hat{j}+7\hat{k})\,.\,(3\hat{i}+5\hat{j}+2\hat{k})=\frac{1}{3}(24+5+14)=\frac{43}{3}$ units.

Ex. 13 Show that the points A, B, C with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ respectively are the vertices of a right angled triangle. Also find the remaining angles of the triangle.

Sol. We have,

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= (\hat{i}-3\hat{j}-5\hat{k}) - (2\hat{i}-\hat{j}+\hat{k}) = -\hat{i}-2\hat{j}-6\hat{k}$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$= (3\hat{i}-4\hat{j}-4\hat{k}) - (\hat{i}-3\hat{j}-5\hat{k}) = 2\hat{i}-\hat{j}+\hat{k}$

and, $\overrightarrow{CA}$ = Position vector of A – Position vector of C

$= (2\hat{i}-\hat{j}+\hat{k}) - (3\hat{i}-4\hat{j}-4\hat{k}) = -\hat{i}+3\hat{j}+5\hat{k}$

Since $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = (-\hat{i}-2\hat{j}-6\hat{k}) + (2\hat{i}-\hat{j}+\hat{k}) + (-\hat{i}+3\hat{j}+5\hat{k}) = \vec{0}$

So A, B and C are the vertices of a triangle.

Now, $\overrightarrow{BC} \,.\, \overrightarrow{CA} = (2\hat{i}-\hat{j}+\hat{k}) \,.\, (-\hat{i}+3\hat{j}+5\hat{k}) = -2-3+5 = 0$

$\Rightarrow \quad \overrightarrow{BC} \perp \overrightarrow{CA} \qquad \Rightarrow \qquad \angle BCA = \dfrac{\pi}{2}$

Hence ABC is a right angled triangle.

Since A is the angle between the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Therefore

$$\cos A = \frac{\overrightarrow{AB}.\overrightarrow{AC}}{|\overrightarrow{AB}||\overrightarrow{AC}|} = \frac{(-\hat{i}-2\hat{j}-6\hat{k}).(\hat{i}-3\hat{j}-5\hat{k})}{\sqrt{(-1)^2+(-2)^2+(-6)^2}\sqrt{1^2+(-3)^2+(-5)^2}}$$

$$= \frac{-1+6+30}{\sqrt{1+4+36}\sqrt{1+9+25}} = \frac{35}{\sqrt{41}\sqrt{35}} = \sqrt{\frac{35}{41}}$$

$$A4 = \cos^{-1}\sqrt{\frac{35}{41}}$$

$$\cos B = \frac{\overrightarrow{BA}.\overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|} = \frac{(\hat{i}+2\hat{j}+6\hat{k}).(2\hat{i}-\hat{j}+\hat{k})}{\sqrt{1^2+2^2+6^2}\sqrt{2^2+(-1)^2+(1)^2}}$$

$$\Rightarrow \quad \cos B = \frac{2-2+6}{\sqrt{41}\sqrt{6}} = \sqrt{\frac{6}{41}} \qquad \Rightarrow \qquad B = \cos^{-1}\sqrt{\frac{6}{41}}$$

Ex.14 If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors of equal magnitude, prove that $\vec{a}+\vec{b}+\vec{c}$ is equally inclined with vectors $\vec{a}, \vec{b}$ and $\vec{c}$.

Sol. Let $|\vec{a}| = |\vec{b}| = |\vec{c}| = \lambda$ (say). Since $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors, therefore $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$ ....**(i)**

Now, $\left|\vec{a}+\vec{b}+\vec{c}\right|^2 = \vec{a}.\vec{a} + \vec{b}.\vec{b} + \vec{c}.\vec{c} + 2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{c}.\vec{a}$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$ [Using **(i)**]

$= 3\lambda^2$ [$\because |\vec{a}| = |\vec{b}| = |\vec{c}| = \lambda$]

$\therefore \quad |\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}\lambda$ ....**(ii)**

Suppose $\vec{a}+\vec{b}+\vec{c}$ makes angles $\theta_1, \theta_2, \theta_3$ with $\vec{a}, \vec{b}$ and $\vec{c}$ respectively. Then,

$$\cos\theta_1 = \frac{\vec{a}.(\vec{a}+\vec{b}+\vec{c})}{|\vec{a}||\vec{a}+\vec{b}+\vec{c}|} = \frac{\vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}}{|\vec{a}||\vec{a}+\vec{b}+\vec{c}|}$$

$$= \frac{|\vec{a}|^2}{|\vec{a}||\vec{a}+\vec{b}+\vec{c}|} = \frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|} = \frac{\lambda}{\sqrt{3}\lambda} = \frac{1}{\sqrt{3}} \qquad \text{[Using (ii)]}$$

$$\therefore \quad \theta_1 = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Similarly, $\theta_2 = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ and $\theta_3 = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$$\therefore \quad \theta_1 = \theta_2 = \theta_3.$$

Hence, $\vec{a}+\vec{b}+\vec{c}$ is equally inclineded with $\vec{a}, \vec{b}$ and $\vec{c}$

Ex. 15 Let $\vec{u}$ and $\vec{v}$ be unit vectors. If $\vec{w}$ is a vector such that $\vec{w} + (\vec{w} \times \vec{u}) = \vec{v}$, then prove that $|(\vec{u} \times \vec{v}).\vec{w}| \le \frac{1}{2}$ and that the equality holds if and only if $\vec{u}$ is perpendicular to $\vec{v}$.

Sol. $\vec{w} + (\vec{w} \times \vec{u}) = \vec{v}$ ......**(i)**

$\Rightarrow \quad \vec{w} \times \vec{u} = \vec{v} - \vec{w} \quad \Rightarrow \quad (\vec{w} \times \vec{u})^2 = v^2 + w^2 - 2\vec{v}.\vec{w}$

$\Rightarrow \quad 2\vec{v}.\vec{w} = 1 + w^2 - (\vec{u} \times \vec{w})^2$ ......**(ii)**

also taking dot product of **(i)** with $\vec{v}$, we get

$$\vec{w}.\vec{v} + (\vec{w} \times \vec{u}).\vec{v} = \vec{v}.\vec{v}$$

$\Rightarrow \quad \vec{v}.(\vec{w} \times \vec{u}) = 1 - \vec{w}.\vec{v}$ ....**(iii)** $\qquad \{\therefore \vec{v}.\vec{v} = |\vec{v}|^2 = 1\}$

Now ; $\quad \vec{v}.(\vec{w} \times \vec{u}) = 1 - \frac{1}{2}(1 + w^2 - (\vec{u} \times \vec{w})^2)$ (using **(ii)** and **(iii)**)

$$= \frac{1}{2} - \frac{w^2}{2} + \frac{(\vec{u} \times \vec{w})^2}{2} \qquad (\therefore 0 \le \cos^2\theta \le 1)$$

$$= \frac{1}{2}(1 - w^2 + w^2 \sin^2\theta) \qquad \text{......(iv)}$$

as we know ; $\quad 0 \le w^2 \cos^2\theta \le w^2$

$$\therefore \quad \frac{1}{2} \ge \frac{1 - w^2\cos^2\theta}{2} \ge \frac{1 - w^2}{2}$$

$$\Rightarrow \quad \frac{1 - w^2\cos^2\theta}{2} \le \frac{1}{2} \qquad \text{......(v)}$$

from **(iv)** and **(v)**

$$|\vec{v}.(\vec{w} \times \vec{u})| \le \frac{1}{2}$$

Equality holds only when $\cos^2\theta = 0 \quad \Rightarrow \quad \theta = \frac{\pi}{2}$

i.e., $\vec{u} \perp \vec{w} \Rightarrow \vec{u}.\vec{w} = 0 \quad \Rightarrow \quad \vec{w} + (\vec{w} \times \vec{u}) = \vec{v}$

$\Rightarrow \quad \vec{u}\cdot\vec{w} + \vec{u}.(\vec{w} \times \vec{u}) = \vec{u}.\vec{v}$ (taking dot with $\vec{u}$)

$\Rightarrow \quad 0 + 0 = \vec{u}\cdot\vec{v} \quad \Rightarrow \quad \vec{u}\cdot\vec{v} = 0 \quad \Rightarrow \quad \vec{u} \perp \vec{v}$

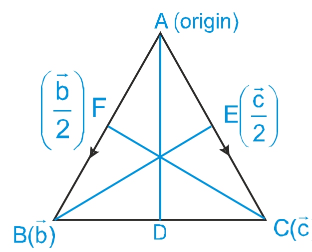
Ex.16 Prove using vectors : If two medians of a triangle are equal, then it is isosceles.

Sol. Let ABC be a triangle and let BE and CF be two equal medians. Taking A as the origin, let the position vectors of B and C be $\vec{b}$ and $\vec{c}$ respectively. Then,

P.V. of E = $\frac{1}{2}\vec{c}$ and P.V. of F = $\frac{1}{2}\vec{b}$

$\therefore \quad \overrightarrow{BE} = \frac{1}{2}(\vec{c} - 2\vec{b})$

$\overrightarrow{CF} = \frac{1}{2}(\vec{b} - 2\vec{c})$



Now, $BE = CF \quad \Rightarrow \quad |\overrightarrow{BE}| = |\overrightarrow{CF}|$

$\Rightarrow \quad |\overrightarrow{BE}|^2 = |\overrightarrow{CF}|^2 \quad \Rightarrow \quad \left|\frac{1}{2}(\vec{c} - 2\vec{b})\right|^2 = \left|\frac{1}{2}(\vec{b} - 2\vec{c})\right|^2$

$\Rightarrow \quad \frac{1}{4}|\vec{c} - 2\vec{b}|^2 = \frac{1}{4}|\vec{b} - 2\vec{c}|^2 \Rightarrow \quad |\vec{c} - 2\vec{b}|^2 = |\vec{b} - 2\vec{c}|^2$

$\Rightarrow \quad (\vec{c} - 2\vec{b}) \cdot (\vec{c} - 2\vec{b}) = (\vec{b} - 2\vec{c}) \cdot (\vec{b} - 2\vec{c})$

$\Rightarrow \quad \vec{c}.\vec{c} - 4\vec{b}.\vec{c} + 4\vec{b}.\vec{b} = \vec{b}.\vec{b} - 4\vec{b}.\vec{c} + 4\vec{c}.\vec{c}$

$\Rightarrow \quad |\vec{c}|^2 - 4\vec{b}.\vec{c} + 4|\vec{b}|^2 = |\vec{b}|^2 - 4\vec{b}.\vec{c} + 4|\vec{c}|^2$

$\Rightarrow \quad 3|\vec{b}|^2 = 3|\vec{c}|^2 \quad \Rightarrow \quad |\vec{b}|^2 = |\vec{c}|^2$

$\Rightarrow \quad AB = AC$

Hence triangle ABC is an isosceles triangle.

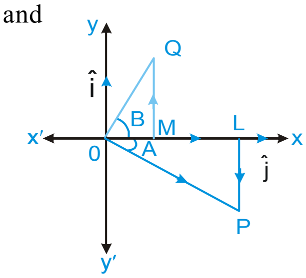
Ex.17 Using vectors : Prove that cos (A + B) = cos A cos B – sin A sin B

Sol. Let OX and OY be the coordinate axes and let $\hat{i}$ and $\hat{j}$ be unit vectors along OX and OY respectively. Let $\angle XOP = A$ and $\angle XOQ = B$. Drawn $PL \perp OX$ and $QM \perp OX$.

Clearly angle between $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ is A + B

In ΔOLP, OL = OP cos A and LP = OP sin A. Therefore $\overrightarrow{OL} = (OP \cos A)\,\hat{i}$ and

$\overrightarrow{LP} = (OP \sin A)\left(-\hat{j}\right)$



Now, $\overrightarrow{OL} + \overrightarrow{LP} = \overrightarrow{OP}$

$\Rightarrow \quad \overrightarrow{OP} = OP[(\cos A)\,\hat{i} - (\sin A)\,\hat{j}]$ ......(i)

In ΔOMQ, OM = OQ cos B and MQ = OQ sin B.

Therefore, $\overrightarrow{OM} = (OQ \cos B)\,\hat{i}$, $\overrightarrow{MQ} = (OQ \sin B)\,\hat{j}$

Now, $\overrightarrow{OM} + \overrightarrow{MQ} = \overrightarrow{OQ}$

$\Rightarrow$ $\overrightarrow{OQ} = OQ[(\cos B)\hat{i} + (\sin B)\hat{j}]$ ......**(ii)**

From **(i)** and **(ii)**, we get

$\overrightarrow{OP} \cdot \overrightarrow{OQ} = OP\,[(\cos A)\ \hat{i} - (\sin A)\,\hat{j}]\,.\,OQ\,[(\cos B)\ \hat{i} + (\sin B)\ \hat{j}]$

$= OP\,.\,OQ\,[\cos A \cos B - \sin A \sin B]$

But, $\overrightarrow{OP} \cdot \overrightarrow{OQ} = |\overrightarrow{OP}|\ |\overrightarrow{OQ}|\ \cos(A + B) = OP\,.\,OQ \cos(A + B)$

$\therefore$ $OP\,.\,OQ \cos(A + B) = OP\,.\,OQ\,[\cos A \cos B - \sin A \sin B]$

$\Rightarrow$ $\cos(A + B) = \cos A \cos B - \sin A \sin B$

Ex. 18 A point $A(x_1, y_1)$ with abscissa $x_1 = 1$ and a point $B(x_2, y_2)$ with ordinate $y_2 = 11$ are given in a rectangular cartesian system of co-ordinates OXY on the part of the curve $y = x^2 - 2x + 3$ which lies in the first quadrant. Find the scalar product of $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

Sol. Since (x_1, y_1) and (x_2, y_2) lies on $y = x^2 - 2x + 3$.

$\therefore$ $y_1 = x_1^2 - 2x_1 + 3$

$y_1 = 1^2 - 2(1) + 3$ (as ; $x_1 = 1$)

$y_1 = 2$

so the co-ordinates of A(1, 2)

Also, $y_2 = x_2^2 - 2x_2 + 3$

$11 = x_2^2 - 2x_2 + 3 \Rightarrow x_2 = 4,\ x_2 \neq -2$ (as B lie in 1st quadrant)

$\therefore$ co-ordinates of B (4, 11).

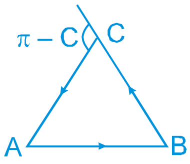
Hence, $\overrightarrow{OA} = \hat{i} + 2\hat{j}$ and $\overrightarrow{OB} = 4\hat{i} + 11\hat{j}$

$\Rightarrow$ $\overrightarrow{OA} \cdot \overrightarrow{OB} = 4 + 22 = 26.$

Ex. 19 Prove that in any triangle ABC

(i) $c^2 = a^2 + b^2 - 2ab \cos C$ **(ii)** $c = b\cos A + a\cos B$.

Sol. **(i)** In ΔABC, $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$



$\Rightarrow$ $\overrightarrow{BC} + \overrightarrow{CA} = -\overrightarrow{AB}$ ......**(i)**

Squaring both sides

$(\overrightarrow{BC})^2 + (\overrightarrow{CA})^2 + 2(\overrightarrow{BC}).\overrightarrow{CA} = (\overrightarrow{AB})^2$

$\Rightarrow$ $a^2 + b^2 + 2(\overrightarrow{BC}.\overrightarrow{CA}) = c^2$

$\Rightarrow$ $c^2 = a^2 + b^2 + 2ab\cos(\pi - C)$

$\Rightarrow$ $c^2 = a^2 + b^2 - 2ab\cos C$

(ii) $(\overrightarrow{BC} + \overrightarrow{CA}).\overrightarrow{AB} = -\overrightarrow{AB}.\overrightarrow{AB}$

$\overrightarrow{BC}\,.\,\overrightarrow{AB} + \overrightarrow{CA}\,.\,\overrightarrow{AB} = -c^2$

$-ac\cos B - bc\cos A = -c^2$

$a\cos B + b\cos A = c.$

Ex. 20 Through a point P(h, k, ℓ) a plane is drawn at right angles to OP to meet the coordinate axes in A, B and C. If OP = p, show that the area of ΔABC is $\dfrac{p^5}{2|hk\ell|}$.

Sol. $OP = \sqrt{h^2 + k^2 + \ell^2} = p$

Direction cosines of OP are $\dfrac{h}{\sqrt{h^2+k^2+\ell^2}}, \dfrac{k}{\sqrt{h^2+k^2+\ell^2}}, \dfrac{\ell}{\sqrt{h^2+k^2+\ell^2}}$

Since OP is normal to the plane, therefore, equation of the plane will be,

$$\frac{h}{\sqrt{h^2+k^2+\ell^2}}x + \frac{k}{\sqrt{h^2+k^2+\ell^2}}y + \frac{\ell}{\sqrt{h^2+k^2+\ell^2}}z = \sqrt{h^2+k^2+\ell^2}$$

or, $hx + ky + \ell z = h^2 + k^2 + \ell^2 = p^2$

$\therefore$ $A \equiv \left(\dfrac{p^2}{h}, 0, 0\right)$, $B \equiv \left(0, \dfrac{p^2}{k}, 0\right)$, $C \equiv \left(0, 0, \dfrac{p^2}{\ell}\right)$

Now area of ΔABC, $\Delta^2 = A_{xy}^2 + A_{yz}^2 + A_{zx}^2$

Now A_{xy} = area of projection of ΔABC on xy-plane = area of ΔAOB

$$= \text{Mod of } \frac{1}{2}\begin{vmatrix} \frac{p^2}{h} & 0 & 1 \\ 0 & \frac{p^2}{k} & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{2}\frac{p^4}{|hk|}$$

Similarly, $A_{yz} = \dfrac{1}{2}\dfrac{p^4}{|k\ell|}$ and $A_{zx} = \dfrac{1}{2}\dfrac{p^4}{|\ell h|}$

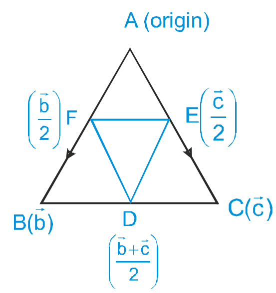
$\therefore$ $\Delta^2 = \dfrac{1}{4}\dfrac{p^8}{h^2k^2} + \dfrac{1}{4}\dfrac{p^8}{k^2\ell^2} + \dfrac{1}{4}\dfrac{p^8}{h^2\ell^2} = \dfrac{p^{10}}{4h^2k^2\ell^2}$

or $\Delta = \dfrac{p^5}{2|hk\ell|}$

Ex. 21 If D, E, F are the mid-points of the sides of a triangle ABC, prove by vector method that area of ΔDEF = $\dfrac{1}{4}$ (area of ΔABC)

Sol. Taking A as the origin, let the position vectors of B and C be $\vec{b}$ and $\vec{c}$ respectively. Then the position vectors of D, E and F are $\dfrac{1}{2}(\vec{b}+\vec{c})$, $\dfrac{1}{2}\vec{c}$ and $\dfrac{1}{2}\vec{b}$ respectively.

Now, $\overline{DE} = \dfrac{1}{2}\vec{c} - \dfrac{1}{2}(\vec{b}+\vec{c}) = \dfrac{-\vec{b}}{2}$



and $\quad \overrightarrow{DF} = \frac{1}{2}\vec{b} - \frac{1}{2}\left((\vec{b}+\vec{c})\right) = \frac{-\vec{c}}{2}$

$\therefore$ Vector area of $\Delta DEF = \frac{1}{2}\left(\overrightarrow{DE}\times\overrightarrow{DF}\right) = \frac{1}{2}\left(\frac{-\vec{b}}{2}\times\frac{-\vec{c}}{2}\right)$

$$= \frac{1}{8}(\vec{b}\times\vec{c}) = \frac{1}{4}\left\{\frac{1}{2}(\overrightarrow{AB}\times\overrightarrow{AC})\right\} = \frac{1}{4}(\text{vector area of } \Delta ABC)$$

Hence area of $\Delta DEF = \frac{1}{4}$ area of ΔABC.

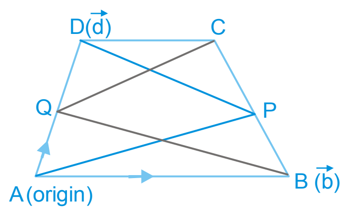
Ex. 22 P, Q are the mid-points of the non-parallel sides BC and AD of a trapezium ABCD. Show that $\Delta APD = \Delta CQB$.

Sol. Let $\overrightarrow{AB} = \vec{b}$ and $\overrightarrow{AD} = \vec{d}$

Now DC is parallel to AB $\Rightarrow$ there exists a scalar t such that $\overrightarrow{DC} = t\,\overrightarrow{AB} = t\,\vec{b}$

$\therefore \quad \overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC} = \vec{d} + t\,\vec{b}$

The position vectors of P and Q are $\frac{1}{2}(\vec{b}+\vec{d}+t\,\vec{b})$ and $\frac{1}{2}\vec{d}$ respectively.



Now $\quad 2\Delta\,\overrightarrow{APD} = \overrightarrow{AP}\times\overrightarrow{AD}$

$$= \frac{1}{2}(\vec{b}+\vec{d}+t\,\vec{b})\times\vec{d} = \frac{1}{2}(1+t)(\vec{b}\times\vec{d})$$

Also $\quad 2\Delta\,\overrightarrow{CQB} = \overrightarrow{CQ}\times\overrightarrow{CB} = \left[\frac{1}{2}\vec{d}-(\vec{d}+t\vec{b})\right]\times[\vec{b}-(\vec{d}+t\,\vec{b})]$

$$= \left[-\frac{1}{2}\vec{d}-t\,\vec{b}\right]\times\left[-\vec{d}+(1-t)\,\vec{b}\right] = -\frac{1}{2}(1-t)(\vec{d}\times\vec{b})+t(\vec{b}\times\vec{d})$$

$$= \frac{1}{2}(1-t+2t)(\vec{b}\times\vec{d}) = \frac{1}{2}(1+t)(\vec{b}\times\vec{d}) = 2\Delta\overrightarrow{APD}$$ Hence Prove.

Ex. 23 If 'a' is real constant and A, B, C are variable angles and $\sqrt{a^2-4}\tan A + a\tan B + \sqrt{a^2+4}\tan C = 6a$ then find the least value of $\tan^2 A + \tan^2 B + \tan^2 C$

Sol. The given relation can be re-written as ;

$$(\sqrt{a^2-4}\,\hat{i} + a\hat{j} + \sqrt{a^2+4}\,\hat{k})\cdot(\tan A\hat{i} + \tan B\hat{j} + \tan C\hat{k}) = 6a$$

$\Rightarrow \quad \sqrt{(a^2-4)+a^2+(a^2+4)}.\ \sqrt{\tan^2 A+\tan^2 B+\tan^2 C}\ .\cos\theta = 6a$

(as, a.b = |a| |b| cos θ)

$\Rightarrow \quad \sqrt{3}\,a\,.\sqrt{\tan^2 A+\tan^2 B+\tan^2 C}\cos\theta = 6a$

$\Rightarrow \quad \tan^2 A + \tan^2 B + \tan^2 C = 12\sec^2\theta$ ......**(i)**

also, $\quad 12\sec^2\theta \geq 12 \quad$ (as, $\sec^2\theta \geq 1$) ......**(ii)**

from **(i)** and **(ii)**,

$\tan^2 A + \tan^2 B + \tan^2 C \geq 12$

$\therefore$ least value of $\tan^2 A + \tan^2 B + \tan^2 C = 12$.

Ex.24 Let $\vec{u}$ and $\vec{v}$ are unit vectors and $\vec{w}$ is a vector such that $(\vec{u}\times\vec{v})+\vec{u} = \vec{w}$ and $\vec{w}\times\vec{u}=\vec{v}$ then find the value of $[\vec{u}\ \vec{v}\ \vec{w}]$.

Sol. Given $(\vec{u}\times\vec{v})+\vec{u} = \vec{w}$ and $\vec{w}\times\vec{u}=\vec{v}$

$\Rightarrow$ $(\vec{u}\times\vec{v})+\vec{u} \times \vec{u} = \vec{w}\times\vec{u}$

$\Rightarrow$ $(\vec{u}\times\vec{v}) \times \vec{u}+\vec{u}\times\vec{u} = \vec{v}$ (as $\vec{w}\times\vec{u}=\vec{v}$)

$\Rightarrow$ $(\vec{u}\,.\,\vec{u})\ \vec{v}-(v\,.\,\vec{u})\ \vec{u}+\vec{u}\times\vec{u}=\vec{v}$ (using $\vec{u}\,.\,\vec{u}=1$ and $\vec{u}\times\vec{u}=0$, since unit vector)

$\Rightarrow$ $\vec{v}-(\vec{v}\,.\,\vec{u})\,\vec{u}=\vec{v}$ $\Rightarrow$ $(\vec{u}\,.\,\vec{v})\,\vec{u}=\vec{0}$

$\Rightarrow$ $\vec{u}\,.\,\vec{v}=0$ (as $\vec{u}\neq 0$) ....**(i)**

Now $\vec{u}\,.\,(\vec{v}\times\vec{w})$

$= \vec{u}\,.\,(\vec{v}\times((\vec{u}\times\vec{v})+\vec{u}))$ (given $\vec{w}=(\vec{u}\times\vec{v})+u$)

$= \vec{u}\,.\,(\vec{v}\times(\vec{u}\times\vec{v})+\vec{v}\times\vec{u}) = \vec{u}\,.\,((\vec{v}\,.\,\vec{v})\,\vec{u}-(\vec{v}\,.\,\vec{u})\,\vec{v}+\vec{v}\times\vec{u})$

$= \vec{u}\,.\,(|\vec{v}|^2\ u-0+\vec{v}\times\vec{u})$ (as $\vec{u}\,.\,\vec{v}=0$ from **(i)**)

$= |\vec{v}|^2\ (\vec{u}\,.\,\vec{u}) - \vec{u}\,.\,(\vec{v}\times\vec{u})$

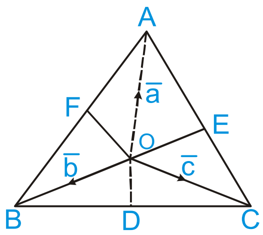
$= |\vec{v}|^2\ |\vec{u}|^2 - 0$ (as $[\vec{u}\ \vec{v}\ \vec{u}]=0$)

$=1$ (as $|\vec{u}|=|\vec{v}|=1$)

$\therefore$ $[\vec{u}\ \vec{v}\ \vec{w}]=1$

Ex.25 In any triangle, show that the perpendicular bisectors of the sides are concurrent.

Sol. Let ABC be the triangle and D, E and F are respectively middle points of sides BC, CA and AB. Let the perpendicular bisectors of BC and CA meet at O. Join OF. We are required to prove that OF is $\perp$ to AB. Let the position vectors of A, B, C with O as origin of reference be $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.



$$\therefore \quad \overrightarrow{OD} = \frac{1}{2}(\vec{b}+\vec{c}),\ \overrightarrow{OE} = \frac{1}{2}(\vec{c}+\vec{a}) \text{ and } \overrightarrow{OF} = \frac{1}{2}(\vec{a}+\vec{b})$$

Also $\overrightarrow{BC} = \vec{c}-\vec{b}$, $\overrightarrow{CA} = \vec{a}-\vec{c}$ and $\overrightarrow{AB} = \vec{b}-\vec{a}$

Since OD $\perp$ BC

$\Rightarrow$ $\frac{1}{2}(\vec{b}+\vec{c})\,.\,(\vec{c}-\vec{b})=0$

$\Rightarrow$ $b^2 = c^2$ ....**(i)**

Similarly OE $\perp$ CA

$\Rightarrow$ $\frac{1}{2}(\vec{c}+\vec{a})\,.\,(\vec{a}-\vec{c})=0$

$\Rightarrow$ $a^2 = c^2$ ....**(ii)**

from **(i)** and **(ii)** we have $a^2 - b^2 = 0$

$\Rightarrow$ $(\vec{a}+\vec{b})\,.\,(\vec{b}-\vec{a})=0$ $\Rightarrow$ $\frac{1}{2}(\vec{b}+\vec{a})\,.\,(\vec{b}-\vec{a})=0$ $\Rightarrow$ $\overline{OF}\perp\overline{AB}$

Ex. 26 Find the locus of a point, the sum of squares of whose distances from the planes : $x - z = 0$, $x - 2y + z = 0$ and $x + y + z = 0$ is 36

Sol. Given planes are $x - z = 0$, $x - 2y + z = 0$ and, $x + y + z = 0$

Let the point whose locus is required be $P(\alpha,\beta,\gamma)$. According to question

$$\frac{|\alpha-\gamma|^2}{2}+\frac{|\alpha-2\beta+\gamma|^2}{6}+\frac{|\alpha+\beta+\gamma|^2}{3}=36$$

or $3(\alpha^2+\gamma^2-2\alpha\gamma)+\alpha^2+4\beta^2+\gamma^2-4\alpha\beta-4\beta\gamma+2\alpha\gamma+2(\alpha^2+\beta^2+\gamma^2+2\alpha\beta+2\beta\gamma+2\alpha\gamma)=36\times 6$

or $6\alpha^2 + 6\beta^2 + 6\gamma^2 = 36 \times 6$

or $\alpha^2 + \beta^2 + \gamma^2 = 36$

Hence, the required equation of locus is $x^2 + y^2 + z^2 = 36$

Ex. 27 A, B, C, D are four points in space. using vector methods, prove that $AC^2 + BD^2 + AD^2 + BC^2 \geq AB^2 + CD^2$ what is the implication of the sign of equality.

Sol. Let the position vector of A, B, C, D be $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ respectively then

$$AC^2 + BD^2 + AD^2 + BC^2 = (\vec{c}-\vec{a}).(\vec{c}-\vec{a}) + (\vec{d}-\vec{b}).(\vec{d}-\vec{b}) + (\vec{d}-\vec{a}).(\vec{d}-\vec{a}) + (\vec{c}-\vec{b}).(\vec{c}-\vec{b})$$

$$= |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a}.\vec{c} + |\vec{d}|^2 + |\vec{b}|^2 - 2\vec{d}.\vec{b} + |\vec{d}|^2 + |\vec{a}|^2 - 2\vec{a}.\vec{d} + |\vec{c}|^2 + |\vec{b}|^2 - 2\vec{b}.\vec{c}$$

$$= |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b} + |\vec{c}|^2 + |\vec{d}|^2 - 2\vec{c}.\vec{d} + |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + |\vec{d}|^2$$

$$+ 2\vec{a}.\vec{b} + 2\vec{c}.\vec{d} - 2\vec{a}.\vec{c} - 2\vec{b}.\vec{d} - 2\vec{a}.\vec{d} - 2\vec{b}.\vec{c}$$

$$= (\vec{a}-\vec{b}).(\vec{a}-\vec{b}) + (\vec{c}-\vec{d}).(\vec{c}-\vec{d}) + (\vec{a}+\vec{b}-\vec{c}-\vec{d})^2$$

$$= AB^2 + CD^2 + (\vec{a}+\vec{b}-\vec{c}-\vec{d}).(\vec{a}+\vec{b}-\vec{c}-\vec{d}) \geq AB^2 + CD^2$$

$\Rightarrow$ $AC^2 + BD^2 + AD^2 + BC^2 \geq AB^2 + CD^2$

for the sign of equality to hold, $\vec{a}+\vec{b}-\vec{c}-\vec{d} = 0$

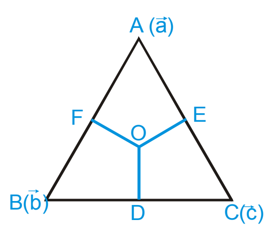
$$\vec{a}-\vec{c} = \vec{d}-\vec{b}$$

$\Rightarrow$ $\overline{AC}$ and $\overline{BD}$ are collinear, the four points A, B, C, D are collinear

Ex. 28 Prove that the right bisectors of the sides of a triangle are concurrent.

Sol. Let the right bisectors of sides BC and CA meet at O and taking O as origin, let the position vectors of A, B and C be taken as $\vec{a}, \vec{b}, \vec{c}$ respectively. Hence the mid-points D, E, F are

$$\frac{\vec{b}+\vec{c}}{2}, \frac{\vec{c}+\vec{a}}{2}, \frac{\vec{a}+\vec{b}}{2}$$



$\because$ $\overrightarrow{OD} \perp \overrightarrow{BC}$, $\quad\therefore\quad \frac{\vec{b}+\vec{c}}{2}.(\vec{c}-\vec{b}) = 0$

i.e. $b^2 = c^2$

Again since $\overrightarrow{OE} \perp \overrightarrow{CA}$, $\quad\therefore\quad \frac{\vec{c}+\vec{a}}{2}.(\vec{a}-\vec{c}) = 0$

or $a^2 = c^2 \quad \therefore \quad a^2 = b^2 = c^2$ .... **(i)**

Now we have to prove that $\overrightarrow{OF}$ is also $\perp$ to $\overrightarrow{AB}$ which will be true if $\frac{\vec{a}+\vec{b}}{2}.(\vec{b}-\vec{a})=0$

i.e. $b^2 = a^2$

which is true by **(i)**

Ex. 29 If P be any point on the plane $\ell x + my + nz = p$ and Q be a point on the line OP such that $OP \,.\, OQ = p^2$, show that the locus of the point Q is $p(\ell x + my + nz) = x^2 + y^2 + z^2$.

Sol. Let $P \equiv (\alpha, \beta, \gamma)$, $Q \equiv (x_1, y_1, z_1)$

Direction ratios of OP are α, β, γ and direction ratios of OQ are x_1, y_1, z_1.

Since O, Q, P are collinear, we have

$$\frac{\alpha}{x_1} = \frac{\beta}{y_1} = \frac{\gamma}{z_1} = k \text{ (say)} \qquad \text{..... (1)}$$

As P (α, β, γ) lies on the plane $\ell x + my + nz = p$,

$$\ell\alpha + m\beta + n\gamma = p \text{ or } \quad k(\ell x_1 + my_1 + nz_1) = p \qquad \text{..... (2)}$$

Given $OP \,.\, OQ = p^2$

$$\therefore \quad \sqrt{\alpha^2+\beta^2+\gamma^2}\,\sqrt{x_1^2+y_1^2+z_1^2} = p^2$$

or, $$\sqrt{k^2(x_1^2+y_1^2+z_1^2)}\,\sqrt{x_1^2+y_1^2+z_1^2} = p^2$$

or, $$k(x_1^2+y_1^2+z_1^2) = p^2 \qquad \text{..... (3)}$$

On dividing **(2)** by **(3)**, we get $\frac{\ell x_1 + my_1 + nz_1}{x_1^2+y_1^2+z_1^2} = \frac{1}{p}$

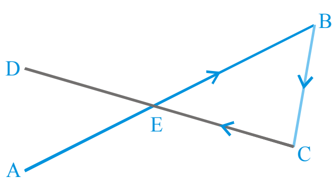
or, $$p(\ell x_1 + my_1 + nz_1) = x_1^2+y_1^2+z_1^2$$

Hence the locus of point Q is $p(\ell x + my + nz) = x^2 + y^2 + z^2$.

Ex. 30 A, B, C and D are four points such that $\overrightarrow{AB} = m(2\hat{i}-6\hat{j}+2\hat{k})$, $\overrightarrow{BC} = (\hat{i}-2\hat{j})$ and $\overrightarrow{CD} = n(-6\hat{i}+15\hat{j}-3\hat{k})$. Find the conditions on the scalars m and n so that $\overrightarrow{CD}$ intersects $\overrightarrow{AB}$ at some point E. Also find the area of the triangle BCE.

Sol. $\overrightarrow{AB} = m(2\hat{i}-6\hat{j}+2\hat{k})$, $\overrightarrow{BC} = (\hat{i}-2\hat{j})$

$\overrightarrow{CD} = n(-6\hat{i}+15\hat{j}-3\hat{k})$



If AB and CD intersect at E, then $\overrightarrow{EB} = p\overrightarrow{AB}$, $\overrightarrow{CE} = q\overrightarrow{CD}$

where both p and q are positive quantities less than 1

Now we know that $\overrightarrow{EB} + \overrightarrow{BC} + \overrightarrow{CE} = \overrightarrow{EE} = 0$

$$\therefore \quad p\overrightarrow{AB} + \overrightarrow{BC} + q\overrightarrow{CD} = 0 \qquad \{\text{by } \mathbf{(i)}\}$$

or $$pm(2\hat{i}-6\hat{j}+2\hat{k}) + (\hat{i}-2\hat{j}) + q.n(-6\hat{i}+15\hat{j}-3\hat{k}) = 0$$

Since $\hat{i}, \hat{j}, \hat{k}$ are non-coplanar, the above relation implies that if $x\hat{i} + y\hat{j} + z\hat{k} = 0$, then $x = 0$, $y = 0$ and $z = 0$

$\therefore \quad 2mp + 1 - 6qn = 0, \; -6pm - 2 + 15qn = 0$

$2pm - 3qn = 0$

Solving these for pm and qn, we get

$$pm = \frac{1}{2},\ qn = \frac{1}{3} \qquad \therefore \qquad p = \frac{1}{2m},\ q = \frac{1}{3n}$$

$$\therefore \qquad 0 < \frac{1}{2m} \le 1,\ 0 \le \frac{1}{3n} \le 1 \quad \text{or} \quad m \ge \frac{1}{2},\ n \ge \frac{1}{3}$$

Again area of $\Delta BCE = \frac{1}{2}\left|\overrightarrow{EC} \times \overrightarrow{EB}\right| = \frac{1}{2}\left|-q\overrightarrow{CD} \times p\,\overrightarrow{AB}\right| = \frac{1}{2}pqnm\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 15 & -3 \\ 2 & -6 & 2 \end{vmatrix}$

Put $\quad pm = \frac{1}{2},\ qn = \frac{1}{3}$

$$= \frac{1}{2}.\frac{1}{2}.\frac{1}{3}|\ 12\hat{i} + 6\hat{j} - 6\hat{k}| = \frac{1}{12}.6\sqrt{6} = \frac{1}{2}\sqrt{6}$$

Ex. 31 $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-coplanar unit vectors such that angle between any two is α. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = \ell\vec{a} + m\vec{b} + n\vec{c}$, then determine ℓ, m, n in terms of α.

Sol. $a^2 = b^2 = c^2 = 1$, $[abc] \ne 0$

$$\vec{a}\,.\,\vec{b} = \vec{b}\,.\,\vec{c} = \vec{c}\,.\,\vec{a} = \cos\alpha \qquad \text{.........(i)}$$

Multiply both sides of given relation scalarly by $\vec{a}$, $\vec{b}$ and $\vec{c}$, we get

$$0 + [\vec{a}\,\vec{b}\,\vec{c}\,] = \ell.1 + (m + n)\cos\alpha \qquad \text{.........(ii)}$$

$$0 = m + (n + \ell)\cos\alpha \qquad \text{.........(iii)}$$

$$[\vec{a}\,\vec{b}\,\vec{c}\,] + 0 = (\ell + m)\cos\alpha + n \qquad \text{.........(iv)}$$

Adding, we get

$$2[\vec{a}\,\vec{b}\,\vec{c}\,] = (\ell + m + n) + 2(\ell + m + n)\cos\alpha$$

or $\qquad 2[\vec{a}\,\vec{b}\,\vec{c}\,] = (\ell + m + n)(1 + 2\cos\alpha) \qquad$ ....(v)

From (ii), $(m + n) = \dfrac{[\vec{a}\,\vec{b}\,\vec{c}] - \ell}{\cos\alpha}$

Putting in (v), we get $\quad 2[\vec{a}\,\vec{b}\,\vec{c}] = \left\{\ell + \dfrac{[\vec{a}\,\vec{b}\,\vec{c}] - \ell}{\cos\alpha}\right\}(1 + 2\cos\alpha)$

or $\qquad [\vec{a}\,\vec{b}\,\vec{c}]\left\{2 - \dfrac{1 + 2\cos\alpha}{\cos\alpha}\right\} = \ell\left(1 - \dfrac{1}{\cos\alpha}\right)(1 + 2\cos\alpha)$

$$\therefore \qquad \ell = \frac{[\vec{a}\,\vec{b}\,\vec{c}]}{(1 + 2\cos\alpha)(1 - \cos\alpha)} = n \qquad \{\text{as above}\}$$

and $\qquad m = -(n + \ell)\cos\alpha = \dfrac{-2[\vec{a}\,\vec{b}\,\vec{c}]\cos\alpha}{(1 + 2\cos\alpha)(1 - \cos\alpha)}$

Thus the values of ℓ, m, n depend on $[\vec{a}\,\vec{b}\,\vec{c}\,]$

Hence we now find the value of scalar $[\vec{a}\,\vec{b}\,\vec{c}\,]$ in terms of α.

Now $[\vec{a}\,\vec{b}\,\vec{c}\,]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} = \begin{vmatrix} 1 & \cos\alpha & \cos\alpha \\ \cos\alpha & 1 & \cos\alpha \\ \cos\alpha & \cos\alpha & 1 \end{vmatrix}$ (Apply $C_1 + C_2 + C_3$)

$= (1 + 2\cos\alpha)\begin{vmatrix} 1 & \cos\alpha & \cos\alpha \\ 1 & 1 & \cos\alpha \\ 1 & \cos\alpha & 1 \end{vmatrix}$ (Apply $R_2 - R_1$ and $R_3 - R_1$)

$\therefore \quad [\vec{a}\,\vec{b}\,\vec{c}\,]^2 = (1 + 2\cos\alpha)(1 - \cos\alpha)^2$

$\therefore \quad \dfrac{[\vec{a}\,\vec{b}\,\vec{c}\,]}{1 - \cos\alpha} = \sqrt{1 + 2\cos\alpha}$

Putting in the value of ℓ, m, n we have $\ell = \dfrac{1}{\sqrt{(1 + 2\cos\alpha)}} = n, m = \dfrac{-2\cos\alpha}{\sqrt{(1 + 2\cos\alpha)}}$

Ex.32 Find the image of the point P (3, 5, 7) in the plane $2x + y + z = 0$.

Sol. Given plane is $2x + y + z = 0$ ..... **(1)**

$P \equiv (3, 5, 7)$

Direction ratios of normal to plane (1) are 2, 1, 1

Let Q be the image of point P in plane **(1)**. Let PQ meet plane (1) in R

then PQ $\perp$ plane (1)

Let $\quad R \equiv (2r + 3, r + 5, r + 7)$

Since R lies on plane (1)

$\therefore \quad 2(2r + 3) + r + 5 + r + 7 = 0$

or, $\quad 6r + 18 = 0 \quad \therefore \quad r = -3$

$\therefore \quad R \equiv (-3, 2, 4)$

Let $\quad Q \equiv (\alpha, \beta, \gamma)$

Since R is the middle point of PQ

$\therefore \quad -3 = \dfrac{\alpha + 3}{2} \Rightarrow \alpha = -9$

$2 = \dfrac{\beta + 5}{2} \Rightarrow \beta = -1$

$4 = \dfrac{\gamma + 7}{2} \Rightarrow \gamma = 1 \qquad \therefore \quad Q = (-9, -1, 1).$

Ex.33 Vectors $\vec{x}, \vec{y}$ and $\vec{z}$ each of magnitude $\sqrt{2}$, make angles of 60° with each other. If $\vec{x} \times (\vec{y} \times \vec{z}) = \vec{a}$, $\vec{y} \times (\vec{z} \times \vec{x}) = \vec{b}$ and $\vec{x} \times \vec{y} = \vec{c}$, then find $\vec{x}, \vec{y}$ and $\vec{z}$ in terms of $\vec{a}, \vec{b}$ and $\vec{c}$.

Sol. $\vec{x}.\vec{y} = \sqrt{2}\sqrt{2}\cos 60° = 1 = \vec{y}.\vec{z} = \vec{z}.\vec{x}$ ...... **(i)**

Also $x^2 = y^2 = z^2 = 2$

Again $\vec{a} = (\vec{x}.\vec{z})\vec{y} - (\vec{x}.\vec{y})\vec{z} = \vec{y} - \vec{z}$ {by **(i)**}

$\therefore$ $\vec{a} = \vec{y} - \vec{z}, \vec{b} = \vec{z} - \vec{x}$ ....**(ii)**

Now $\vec{a} \times \vec{c} = (\vec{y} - \vec{z}) \times (\vec{x} \times \vec{y}) = \vec{y} \times (\vec{x} \times \vec{y}) - \vec{z} \times (\vec{x} \times \vec{y})$

$= [(\vec{y}\,.\,\vec{y})\vec{x} - (\vec{y}\,.\,\vec{x})\vec{y}] - [(\vec{z}\,.\,\vec{y})\vec{x} - (\vec{z}\,.\,\vec{x})\vec{y}] = (2\vec{x} - \vec{y}) - (\vec{x} - \vec{y})$ {by **(i)**}

or $\vec{a} \times \vec{c} = \vec{x}$

Similarly, $\vec{b} \times \vec{c} = \vec{y}$

Now $\vec{z} = \vec{y} - \vec{a}$ or $\vec{z} = \vec{b} + \vec{x}$ {by **(ii)**}

$\therefore$ $\vec{z} = (\vec{b} \times \vec{c} - \vec{a})$ or $\vec{b} + (\vec{a} \times \vec{c})$

Ex.34 The plane $x - y - z = 4$ is rotated through 90° about its line of intersection with the plane $x + y + 2z = 4$. Find its equation in the new position.

Sol. Given planes are $x - y - z = 4$ ..... **(1)**

and $x + y + 2z = 4$ ..... **(2)**

Since the required plane passes through the line of intersection of planes **(1)** and **(2)**

$\therefore$ its equation may be taken as

$x + y + 2z - 4 + k\,(x - y - z - 4) = 0$

or $(1 + k)x + (1 - k)y + (2 - k)z - 4 - 4k = 0$ ..... **(3)**

Since planes (1) and (3) are mutually perpendicular,

$\therefore$ $(1 + k) - (1 - k) - (2 - k) = 0$

or, $1 + k - 1 + k - 2 + k = 0$ or $k = \frac{2}{3}$

Putting $k = \frac{2}{3}$ in equation (3), we get $5x + y + 4z = 20$

Ex.35 If the planes $x - cy - bz = 0$, $cx - y + az = 0$ and $bx + ay - z = 0$ pass through a straight line, then find the value of $a^2 + b^2 + c^2 + 2abc$.

Sol. Given planes are $x - cy - bz = 0$ ..... **(1)**

$cx - y + az = 0$ ..... **(2)**

$bx + ay - z = 0$ ..... **(3)**

Equation of any plane passing through the line of intersection of planes **(1)** and **(2)** may be taken as $x - cy - bz + \lambda\,(cx - y + az) = 0$

or, $x\,(1 + \lambda c) - y\,(c + \lambda) + z\,(-b + a\lambda) = 0$ ..... **(4)**

If planes **(3)** and **(4)** are the same, then equations **(3)** and **(4)** will be identical.

$$\therefore \quad \underset{\textbf{(i)}}{\frac{1+c\lambda}{b}} = \underset{\textbf{(ii)}}{\frac{-(c+\lambda)}{a}} = \underset{\textbf{(iii)}}{\frac{-b+a\lambda}{-1}}$$

From **(i)** and **(ii)**, $a + ac\lambda = -bc - b\lambda$

or, $\lambda = -\dfrac{(a+bc)}{(ac+b)}$ ..... **(5)**

From **(ii)** and **(iii)**,

$c + \lambda = -ab + a^2\lambda$ or $\lambda = \dfrac{-(ab+c)}{1-a^2}$ ..... **(6)**

From **(5)** and **(6)**, we have $\dfrac{-(a+bc)}{ac+b} = \dfrac{-(ab+c)}{(1-a^2)}$.

or, $a - a^3 + bc - a^2bc = a^2bc + ac^2 + ab^2 + bc$

or, $a^2bc + ac^2 + ab^2 + a^3 + a^2bc - a = 0$

or, $a^2 + b^2 + c^2 + 2abc = 1$.

This is the equation of the required plane.

Ex.36 Find direction ratios of normal to the plane which passes through the point (1, 0, 0) and (0, 1, 0) which makes angle $\pi/4$ with $x + y = 3$.

Sol. The plane by intercept form is $\dfrac{x}{1} + \dfrac{y}{1} + \dfrac{z}{c} = 1$

d.r.'s of normal are $1, 1, \dfrac{1}{c}$ and of given plane are 1, 1, 0.

$$\therefore \quad \cos\frac{\pi}{4} = \frac{1\,.\,1 + 1\,.\,1 + 0\cdot\frac{1}{c}}{\sqrt{1+1+\frac{1}{c^2}}\,\sqrt{1+1+0}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2+\frac{1}{c^2}}\,\sqrt{2}} \quad \Rightarrow \quad 2 + \frac{1}{c^2} = 4 \quad \Rightarrow \quad c = \frac{1}{\sqrt{2}}$$

$\therefore$ d.r.'s are $1, 1, \sqrt{2}$

Ex.37 Find the equation of the plane passing through (1, 2, 0) which contains the line

$\dfrac{x+3}{3} = \dfrac{y-1}{4} = \dfrac{z-2}{-2}$.

Sol. Equation of any plane passing through (1, 2, 0) may be taken as

$a(x-1) + b(y-2) + c(z-0) = 0$ ..... **(i)**

where a, b, c are the direction ratios of the normal to the plane. Given line is

$$\frac{x+3}{3} = \frac{y-1}{4} = \frac{z-2}{-2} \quad \text{..... (ii)}$$

If plane **(1)** contains the given line, then

$$3a + 4b - 2c = 0 \quad \text{..... (iii)}$$

Also point (– 3, 1, 2) on line **(2)** lies in plane **(1)**

$\therefore$ $a(-3-1) + b(1-2) + c(2-0) = 0$

or, $-4a - b + 2c = 0$ ..... **(iv)**

Solving equations **(iii)** and **(iv)**, we get $\frac{a}{8-2} = \frac{b}{8-6} = \frac{c}{-3+16}$

or, $\frac{a}{6} = \frac{b}{2} = \frac{c}{13} = k$ (say). ..... **(v)**

Substituting the values of a, b and c in equation (1), we get

$$6(x-1) + 2(y-2) + 13(z-0) = 0.$$

or, $6x + 2y + 13z - 10 = 0$. This is the required equation.

Ex. 38 If $\vec{x} \times \vec{y} = \vec{a}$, $\vec{y} \times \vec{z} = \vec{b}$, $\vec{x} \,.\, \vec{b} = \gamma$, $\vec{x} \,.\, \vec{y} = 1$ and $\vec{y} \,.\, \vec{z} = 1$, then find $\vec{x}$, $\vec{y}$ and $\vec{z}$ in terms of $\vec{a}$, $\vec{b}$ and γ.

Sol. $\vec{x} \times \vec{y} = \vec{a}$ .... **(i)**

$\vec{y} \times \vec{z} = \vec{b}$ .... **(ii)**

Also $\vec{x} \,.\, \vec{b} = \gamma$, $\vec{x} \,.\, \vec{y} = 1$, $\vec{y} \,.\, \vec{z} = 1$ .... **(iii)**

We have to make use of the relations given above.

From **(i)**

$\vec{x} \,.\, (\vec{x} \times \vec{y}) = \vec{x} \,.\, \vec{a}$

$\therefore$ $\vec{x} \,.\, \vec{a} = 0$ $\quad\because\quad$ $[\vec{x}\ \vec{x}\ \vec{y}] = 0$

Similarly $\vec{y} \,.\, \vec{a} = 0$, $\vec{y} \,.\, \vec{b} = 0$, $\vec{z} \,.\, \vec{b} = 0$ .... **(iv)**

Multiplying **(i)** vectorially by $\vec{b}$,

$\vec{b} \times (\vec{x} \times \vec{y}) = \vec{b} \times \vec{a}$ or $(\vec{b} \,.\, \vec{y})\vec{x} - (\vec{b} \,.\, \vec{x})\vec{y} = \vec{b} \times \vec{a}$

or $0 - \gamma\vec{y} = -(\vec{a} \times \vec{b})$ $\quad\therefore\quad$ $\vec{y} = \frac{(\vec{a} \times \vec{b})}{\gamma}$ .... **(v)**

by using relations is **(iii)** and **(iv)**.

Again multiplying **(i)** vectorially by $\vec{y}$,

$(\vec{x} \times \vec{y}) \times \vec{y} = \vec{a} \times \vec{y}$ or $(\vec{x} \,.\, \vec{y})\vec{y} - (\vec{y} \,.\, \vec{y})\vec{x} = \vec{a} \times \vec{y}$

$\vec{y} - \vec{a} \times \vec{y} = |\vec{y}|^2 \vec{x}$ {by **(iii)**}

$\therefore$ $\vec{x} = \frac{1}{|\vec{y}|^2}[\vec{y} - \vec{a} \times \vec{y}]$

where $\vec{y} = \frac{\vec{a} \times \vec{b}}{\gamma}$ {by **(v)**}

Hence x is known in terms of $\vec{a}, \vec{b}$ and γ.

Again multiplying **(ii)** vectorially by $\vec{y}$, we get

$$(\vec{y} \times \vec{z}) \times \vec{y} = \vec{b} \times \vec{y} \quad \text{or} \quad |\vec{y}|^2 \vec{z} - (\vec{y} \cdot \vec{z})\vec{y} = \vec{b} \times \vec{y} \quad \text{or} \quad |\vec{y}|^2 \vec{z} = \vec{b} \times \vec{y} + \vec{y} \quad \{\text{by } \mathbf{(iii)}\}$$

or $$z = \frac{1}{|\vec{y}|^2}[\vec{b} \times \vec{y} + \vec{y}]$$

where $\vec{y}$ is given by **(v)** .... **(vi)**

Results **(v)** and **(vi)** give the values of $\vec{x}, \vec{y}$ and $\vec{z}$ in terms of $\vec{a}, \vec{b}$ and γ.

Ex.39 Find the equation of the sphere if it touches the plane $\vec{r}.(2\hat{i} - 2\hat{j} - \hat{k}) = 0$ and the position vector of its centre is $3\hat{i} + 6\hat{j} - 4\hat{k}$

Sol. Given plane is $\vec{r}.(2\hat{i} - 2\hat{j} - \hat{k}) = 0$ ..... **(1)**

Let H be the centre of the sphere, then $\overrightarrow{OH} = 3\hat{i} + 6\hat{j} - 4\hat{k} = \vec{c}$ (say)

Radius of the sphere = length of perpendicular from H to plane (1)

$$= \frac{|\vec{c}.(2\hat{i} - 2\hat{j} - \hat{k})|}{|2\hat{i} - 2\hat{j} - \hat{k}|} = \frac{|(3\hat{i} + 6\hat{j} - 4\hat{k}).(2\hat{i} - 2\hat{j} - \hat{k})|}{|2\hat{i} - 2\hat{j} - \hat{k}|} = \frac{|6 - 12 + 4|}{3} = \frac{2}{3} = a \text{ (say)}$$

Equation of the required sphere is $|\vec{r} - \vec{c}| = a$

or $$|x\hat{i} + y\hat{j} + z\hat{k} - (3\hat{i} + 6\hat{j} - 4\hat{k})| = \frac{2}{3}$$

or $$|(x - 3)\vec{i} + (y - 6)\vec{j} + (z + 4)\vec{k}|^2 = \frac{4}{9}$$

or $$(x - 3)^2 + (y - 6)^2 + (z + 4)^2 = \frac{4}{9}$$

or $9(x^2 + y^2 + z^2 - 6x - 12y + 8z + 61) = 4$

or $9x^2 + 9y^2 + 9z^2 - 54x - 108y + 72z + 545 = 0$

Ex.40 Find the equation of the sphere passing through the points (3, 0, 0), (0, – 1, 0), (0, 0, – 2) and whose centre lies on the plane $3x + 2y + 4z = 1$

Sol. Let the equation of the sphere be

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ ..... **(i)**

Let $A \equiv (3, 0, 0), B \equiv (0, -1, 0), C \equiv (0, 0, -2)$

Since sphere **(i)** passes through A, B and C,

$\therefore$ $9 + 6u + d = 0$ ..... **(ii)**

$1 - 2v + d = 0$ ..... **(iii)**

$4 - 4w + d = 0$ ..... **(iv)**

Since centre (– u, – v, – w) of the sphere lies on plane

$3x + 2y + 4z = 1$

$\therefore$ $-3u - 2v - 4w = 1$ ..... **(v)**

(ii) – (iii) $\Rightarrow$ $6u + 2v = -8$ ..... **(vi)**

(iii) – (iv) $\Rightarrow$ $-2v + 4w = 3$ ..... **(vii)**

From **(vi)**, $u = \dfrac{-2v-8}{6}$ ..... **(viii)**

From **(vii)**, $4w = 3 + 2v$ ..... **(ix)**

Putting the values of u, v and w in **(v)**, we get $\dfrac{2v+8}{2} - 2v - 3 - 2v = 1$

$\Rightarrow$ $2v + 8 - 4v - 6 - 4v = 2$ $\Rightarrow$ $v = 0$

From **(viii)**, $u = \dfrac{0-8}{6} = -\dfrac{4}{3}$

From **(ix)**, $4w = 3$ $\therefore$ $w = \dfrac{3}{4}$

From **(iii)**, $d = 2v - 1 = 0 - 1 = -1$

From **(i)**, equation of required sphere is $x^2 + y^2 + z^2 - \dfrac{8}{3}x + \dfrac{3}{2}z - 1 = 0$

or $6x^2 + 6y^2 + 6z^2 - 16x + 9z - 6 = 0$

Exercise # 1

[Single Correct Choice Type Questions]

1. If $\vec{a} + \vec{b}$ is along the angle bisector of $\vec{a}$ & $\vec{b}$ then -

(A) $\vec{a}$ & $\vec{b}$ are perpendicular **(B)** $|\vec{a}| = |\vec{b}|$

(C) angle between $\vec{a}$ & $\vec{b}$ is 60° **(D)** $|\vec{a}| \neq |\vec{b}|$

2. If ABCDEF is a regular hexagon and if $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = \lambda\overrightarrow{AD}$, then λ is -

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

3. If the vector $\vec{b}$ is collinear with the vector $\vec{a} = \left(2\sqrt{2}, -1, 4\right)$ and $|\vec{b}| = 10$, then:

(A) $\vec{a} \pm \vec{b} = 0$ **(B)** $\vec{a} \pm 2\vec{b} = 0$ **(C)** $2\vec{a} \pm \vec{b} = 0$ **(D)** none

4. The plane XOZ divides the join of (1, –1, 5) and (2, 3, 4) in the ratio $\lambda : 1$, then λ is -

(A) –3 **(B)** –1/3 **(C)** 3 **(D)** 1/3

5. Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{k}$. The point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is :

(A) $-\hat{i} + \hat{j} + 2\hat{k}$ **(B)** $3\hat{i} - \hat{j} + \hat{k}$ **(C)** $3\hat{i} + \hat{j} - \hat{k}$ **(D)** $\hat{i} - \hat{j} - \hat{k}$

6. The vectors $\overrightarrow{AB} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\overrightarrow{BC} = -\hat{i} + 2\hat{k}$ are the adjacent sides of a parallelogram ABCD then the angle between the diagonals is -

(A) $\cos^{-1}\left(\sqrt{\frac{1}{85}}\right)$ **(B)** $\pi - \cos^{-1}\left(\sqrt{\frac{49}{85}}\right)$ **(C)** $\cos^{-1}\left(\frac{1}{2\sqrt{2}}\right)$ **(D)** $\cos^{-1}\left(\sqrt{\frac{3}{10}}\right)$

7. The value of $\left[\left(\vec{a} + 2\vec{b} - \vec{c}\right)\left(\vec{a} - \vec{b}\right)\left(\vec{a} - \vec{b} - \vec{c}\right)\right]$ is equal to the box product:

(A) $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ **(B)** $2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ **(C)** $3\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ **(D)** $4\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

8. Let ABCD be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the area of triangles ABC, ACD and ADB be 3, 4 and 5 sq. units respectively. Then the area of the triangle BCD, is -

(A) $5\sqrt{2}$ **(B)** 5 **(C)** $\frac{5}{\sqrt{2}}$ **(D)** $\frac{5}{2}$

9. If $|\vec{a}| = 5$, $|\vec{a} - \vec{b}| = 8$ and $|\vec{a} + \vec{b}| = 10$, then $|\vec{b}|$ is equal to :

(A) 1 **(B)** $\sqrt{57}$ **(C)** 3 **(D)** none of these

10. The values of a, for which the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $a\hat{i} - 3\hat{j} + \hat{k}$ respectively are the vertices of a right angled triangle with $C = \frac{\pi}{2}$ are -

(A) –2 and 1 **(B)** 2 and –1 **(C)** 2 and 1 **(D)** –2 and –1

11. The co-ordinates of the centre and the radius of the circle $x + 2y + 2z = 15$, $x^2 + y^2 + z^2 - 2y - 4z = 11$ are

(A) $(4, 3, 1), \sqrt{5}$ **(B)** $(3, 4, 1), \sqrt{6}$ **(C)** $(1, 3, 4), \sqrt{7}$ **(D)** none of these

12. Which one of the following statement is INCORRECT ?

(A) If $\vec{n}.\vec{a} = 0$, $\vec{n}.\vec{b} = 0$ and $\vec{n}.\vec{c} = 0$ for some non zero vector $\vec{n}$, then $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

(B) there exist a vector having direction angles $\alpha = 30°$ and $\beta = 45°$

(C) locus of point in space for which $x = 3$ and $y = 4$ is a line parallel to the z-axis whose distance from the z-axis is 5

(D) In a regular tetrahedron OABC where 'O' is the origin, the vector $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$ is perpendicular to the plane ABC.

13. OABCDE is a regular hexagon of side 2 units in the XY−plane in the I[st] quadrant . O being the origin and OA taken along the X−axis. A point P is taken on a line parallel to Z−axis through the centre of the hexagon at a distance of 3 units from O in the positive Z direction. Then vector $\overrightarrow{AP}$ is:

(A) $-\hat{i} + 3\hat{j} + \sqrt{5}\hat{k}$ **(B)** $\hat{i} - \sqrt{3}\hat{j} + 5\hat{k}$ **(C)** $-\hat{i} + \sqrt{3}\hat{j} + \sqrt{5}\hat{k}$ **(D)** $\hat{i} + \sqrt{3}\hat{j} + \sqrt{5}\hat{k}$

14. If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = \hat{i}-\hat{j}+\hat{k}$, $\vec{c} = \hat{i}+2\hat{j}-\hat{k}$, then the value of $\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} =$

(A) 2 **(B)** 4 **(C)** 16 **(D)** 64

15. Let $\vec{a}, \vec{b}, \vec{c}$ be vectors of length 3, 4, 5 respectively. Let $\vec{a}$ be perpendicular to $\vec{b} + \vec{c}$, $\vec{b}$ to $\vec{c} + \vec{a}$ and $\vec{c}$ to $\vec{a} + \vec{b}$. Then $|\vec{a} + \vec{b} + \vec{c}|$ is equal to :

(A) $2\sqrt{5}$ **(B)** $2\sqrt{2}$ **(C)** $10\sqrt{5}$ **(D)** $5\sqrt{2}$

16. Consider the following 5 statements

(I) There exists a plane containing the points (1, 2, 3) and (2, 3, 4) and perpendicular to the vector $\vec{V}_1 = \hat{i} + \hat{j} - \hat{k}$

(II) There exist no plane containing the point (1, 0, 0); (0, 1, 0); (0, 0, 1) and (1, 1, 1)

(III) If a plane with normal vector $\vec{N}$ is perpendicular to a vector $\vec{V}$ then $\vec{N} \cdot \vec{V} = 0$

(IV) If two planes are perpendicular then every line in one plane is perpendicular to every line on the other plane

(v) Let P_1 and P_2 are two perpendicular planes. If a third plane P_3 is perpendicular to P_1 then it must be either parallel or perpendicular or at an angle of 45° to P_2.

Choose the correct alternative.

(A) exactly one is false **(B)** exactly 2 are false **(C)** exactly 3 are false **(D)** exactly four are false

17. Taken on side $\overrightarrow{AC}$ of a triangle ABC, a point M such that $\overrightarrow{AM} = \frac{1}{3}\overrightarrow{AC}$. A point N is taken on the side $\overrightarrow{CB}$ such that $\overrightarrow{BN} = \overrightarrow{CB}$, then for the point of intersection X of $\overrightarrow{AB}$ and $\overrightarrow{MN}$ which of the following holds good?

(A) $\overrightarrow{XB} = \frac{1}{3}\overrightarrow{AB}$ **(B)** $\overrightarrow{AX} = \frac{1}{3}\overrightarrow{AB}$ **(C)** $\overrightarrow{XN} = \frac{3}{4}\overrightarrow{MN}$ **(D)** $\overrightarrow{XM} = 3\,\overrightarrow{XN}$

18. Let $\vec{a} = \hat{i} + \hat{j}$ & $\vec{b} = 2\hat{i} - \hat{k}$. The point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ & $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is -

(A) $-\hat{i} + \hat{j} + \hat{k}$ **(B)** $3\hat{i} - \hat{j} + \hat{k}$ **(C)** $3\hat{i} + \hat{j} - \hat{k}$ **(D)** $\hat{i} - \hat{j} - \hat{k}$

19. Consider a tetrahedron with faces f_1, f_2, f_3, f_4. Let $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ be the vectors whose magnitudes are respectively equal to the areas of f_1, f_2, f_3, f_4 and whose directions are perpendicular to these faces in the outward direction. Then,

(A) $\vec{a}_1 + \vec{a}_2 + \vec{a}_3 + \vec{a}_4 = \vec{0}$ **(B)** $\vec{a}_1 + \vec{a}_3 = \vec{a}_2 + \vec{a}_4$

(C) $\vec{a}_1 + \vec{a}_2 = \vec{a}_3 + \vec{a}_4$ **(D)** none

20. Let L_1 be the line $\vec{r}_1 = 2\hat{i} + \hat{j} - \hat{k} + \lambda(\hat{i} + 2\hat{k})$ and let L_2 be the line $\vec{r}_2 = 3\hat{i} + \hat{j} + \mu(\hat{i} + \hat{j} - \hat{k})$.
Let Π be the plane which contains the line L_1 and is parallel to L_2. The distance of the plane Π from the origin is -

(A) $\sqrt{2/7}$ **(B)** $1/7$ **(C)** $\sqrt{6}$ **(D)** none of these

21. A plane meets the coordinate axes in A, B, C and (α, β, γ) is the centroid of the triangle ABC, then the equation of the plane is

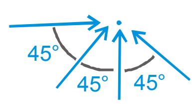
(A) $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$ **(B)** $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$ **(C)** $\frac{3x}{\alpha} + \frac{3y}{\beta} + \frac{3z}{\gamma} = 1$ **(D)** $\alpha x + \beta y + \gamma z = 1$

22. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number then $\left[\lambda(\vec{a} + \vec{b})\ \ \lambda^2\vec{b}\ \ \lambda\vec{c}\right] = \left[\vec{a}\ \ \vec{b} + \vec{c}\ \ \vec{b}\right]$ for -

(A) exactly two values of λ **(B)** exactly three values of λ

(C) no value of λ **(D)** exactly one value of λ

23. Four coplanar forces are applied at a point O. Each of them is equal to k and the angle between two consecutive forces equals 45° as shown in the figure. Then the resultant has the magnitude equal to :



(A) $k\sqrt{2 + 2\sqrt{2}}$ **(B)** $k\sqrt{3 + 2\sqrt{2}}$ **(C)** $k\sqrt{4 + 2\sqrt{2}}$ **(D)** none

24. The intercept made by the plane $\vec{r} \cdot \vec{n} = q$ on the x-axis is -

(A) $\frac{q}{\hat{i} \cdot \vec{n}}$ **(B)** $\frac{\hat{i} \cdot \vec{n}}{q}$ **(C)** $(\hat{i} \cdot \vec{n})q$ **(D)** $\frac{q}{|\vec{n}|}$

25. If $\vec{a} \times \vec{b} = \vec{c}$, $\vec{b} \times \vec{c} = \vec{a}$, then find value of $|3\vec{a} + 4\vec{b} + 12\vec{c}|$ if $\vec{a}, \vec{b}, \vec{c}$ are vectors of same magnitude.

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14

26. Volume of the tetrahedron whose vertices are represented by the position vectors , A (0, 1, 2) ; B (3, 0, 1) ; C (4, 3, 6) & D (2, 3, 2) is -

(A) 3 (B) 6 (C) 36 (D) none

27. The equation of the plane passing through the point (1, – 3, – 2) and perpendicular to planes $x + 2y + 2z = 5$ and $3x + 3y + 2z = 8$, is

(A) $2x - 4y + 3z - 8 = 0$ (B) $2x - 4y - 3z + 8 = 0$
(C) $2x + 4y + 3z + 8 = 0$ (D) None of these

28. If from the point P(f, g, h) perpendiculars PL, PM be drawn to yz and zx planes then the equation to the plane OLM is

(A) $\frac{x}{f} + \frac{y}{g} + \frac{z}{h} = 0$ (B) $\frac{x}{f} + \frac{y}{g} - \frac{z}{h} = 0$

(C) $\frac{x}{f} - \frac{y}{g} + \frac{z}{h} = 0$ (D) $-\frac{x}{f} + \frac{y}{g} + \frac{z}{h} = 0$

29. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are linearly independent vectors, then which one of the following set of vectors is linearly dependent ?

(A) $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$ (B) $\vec{a}-\vec{b}$, $\vec{b}-\vec{c}$, $\vec{c}-\vec{a}$ (C) $\vec{a} \times \vec{b}$, $\vec{b} \times \vec{c}$, $\vec{c} \times \vec{a}$ (D) none

30. The sine of angle formed by the lateral face ADC and plane of the base ABC of the tetrahedron ABCD where $A \equiv (3, -2, 1)$; $B \equiv (3, 1, 5)$; $C \equiv (4, 0, 3)$ and $D \equiv (1, 0, 0)$ is -

(A) $\frac{2}{\sqrt{29}}$ (B) $\frac{5}{\sqrt{29}}$ (C) $\frac{3\sqrt{3}}{\sqrt{29}}$ (D) $\frac{-2}{\sqrt{29}}$

31. Given the points $A(-2, 3, -4)$, $B(3, 2, 5)$, $C(1, -1, 2)$ & $D(3, 2, -4)$. The projection of the vector $\vec{AB}$ on the vector $\vec{CD}$ is -

(A) $\frac{22}{3}$ (B) $-\frac{21}{4}$ (C) $-\frac{47}{7}$ (D) –47

32. Given the vertices A (2, 3, 1), B (4, 1, –2), C (6, 3, 7) & D (–5, –4, 8) of a tetrahedron. The length of the altitude drawn from the vertex D is -

(A) 7 (B) 9 (C) 11 (D) none

33. Equation of the angle bisector of the angle between the lines $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$ & $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{-1}$ is :

(A) $\frac{x-1}{2} = \frac{y-2}{2}$; $z - 3 = 0$ (B) $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$

(C) $x - 1 = 0$; $\frac{y-2}{1} = \frac{z-3}{1}$ (D) None of these

34. The distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel to the line, $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$, is:

(A) 1 (B) 6/7 (C) 7/6 (D) None of these

35. The line which contains all points (x, y, z) which are of the form $(x, y, z) = (2, -2, 5) + \lambda(1, -3, 2)$ intersects the plane $2x - 3y + 4z = 163$ at P and intersects the YZ plane at Q. If the distance PQ is $a\sqrt{b}$, where $a, b \in N$ and $a > 3$ then $(a + b)$ equals -

(A) 23 (B) 95 (C) 27 (D) none of these

36. A variable plane passes through a fixed point (1, 2, 3). The locus of the foot of the perpendicular drawn from origin to this plane is:

(A) $x^2 + y^2 + z^2 - x - 2y - 3z = 0$ (B) $x^2 + 2y^2 + 3z^2 - x - 2y - 3z = 0$
(C) $x^2 + 4y^2 + 9z^2 + x + 2y + 3 = 0$ (D) $x^2 + y^2 + z^2 + x + 2y + 3z = 0$

37. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be non-zero vectors such that $\vec{a}$ and $\vec{b}$ are non-collinear & satisfies $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$.

If θ is the angle between the vectors $\vec{b}$ and $\vec{c}$ then $\sin\theta$ equals -

(A) $\frac{2}{3}$ (B) $\sqrt{\frac{2}{3}}$ (C) $\frac{1}{3}$ (D) $\frac{2\sqrt{2}}{3}$

38. A, B, C & D are four points in a plane with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ & $\vec{d}$ respectively such that

$\left(\vec{a} - \vec{d}\right).\left(\vec{b} - \vec{c}\right) = \left(\vec{b} - \vec{d}\right).\left(\vec{c} - \vec{a}\right) = 0$. Then for the triangle ABC, D is its:

(A) incentre (B) circumcentre (C) orthocentre (D) centroid

39. A plane passes through the point P(4, 0, 0) and Q(0, 0, 4) and is parallel to the y-axis. The distance of the plane from the origin is -

(A) 2 (B) 4 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

40. Let $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that $\vec{r}_1 = \vec{a} - \vec{b} + \vec{c}$, $\vec{r}_2 = \vec{b} + \vec{c} - \vec{a}$, $\vec{r}_3 = \vec{c} + \vec{a} + \vec{b}$, $\vec{r} = 2\vec{a} - 3\vec{b} + 4\vec{c}$.

If $\vec{r} = \lambda_1\vec{r}_1 + \lambda_2\vec{r}_2 + \lambda_3\vec{r}_3$, then the values of λ_1, λ_2 and λ_3 respectively are

(A) 7, 1, –4 (B) 7 / 2, 1, –1 / 2 (C) 5 / 2, 1, 1/2 (D) –1 / 2, 1, 7 / 2

41. The vertices of a triangle are A(1, 1, 2), B(4, 3, 1) and C(2, 3, 5). A vector representing the internal bisector of the angle A is :

(A) $\hat{i} + \hat{j} + 2\hat{k}$ (B) $2\hat{i} - 2\hat{j} + \hat{k}$ (C) $2\hat{i} + 2\hat{j} - \hat{k}$ (D) $2\hat{i} + 2\hat{j} + \hat{k}$

42. A, B, C, D be four points in a space and if, $|\overrightarrow{AB} \times \overrightarrow{CD} + \overrightarrow{BC} \times \overrightarrow{AD} + \overrightarrow{CA} \times \overrightarrow{BD}| = \lambda$ (area of triangle ABC)

then the value of λ is -

(A) 4 (B) 2 (C) 1 (D) none of these

43. For a non zero vector $\vec{A}$ if the equations $\vec{A}.\vec{B} = \vec{A}.\vec{C}$ and $\vec{A} \times \vec{B} = \vec{A} \times \vec{C}$ hold simultaneously, then:

(A) $\vec{A}$ is perpendicular to $\vec{B} - \vec{C}$ (B) $\vec{A} = \vec{B}$
(C) $\vec{B} = \vec{C}$ (D) $\vec{C} = \vec{A}$

44. The distance between the parallel planes given by the equations, $\vec{r}\,.\,(2\hat{i} - 2\hat{j} + \hat{k}) + 3 = 0$ and $\vec{r}\,.\,(4\hat{i} - 4\hat{j} + 2\hat{k}) + 5 = 0$ is -

(A) 1/2 **(B)** 1/3 **(C)** 1/4 **(D)** 1/6

45. If $\vec{b}$ and $\vec{c}$ are two non-collinear vectors such that $\vec{a} \parallel (\vec{b} \times \vec{c})$, then $(\vec{a} \times \vec{b})\,.\,(\vec{a} \times \vec{c})$ is equal to

(A) $\vec{a}^2\,(\vec{b}\,.\,\vec{c})$ **(B)** $\vec{b}^2\,(\vec{a}\,.\,\vec{c})$ **(C)** $\vec{c}^2\,(\vec{a}\,.\,\vec{b})$ **(D)** none of these

46. Unit vector perpendicular to the plane of the triangle ABC with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ of the vertices A, B, C, is (where Δ is the area of the triangle ABC).

(A) $\dfrac{\left(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right)}{\Delta}$ **(B)** $\dfrac{\left(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right)}{2\Delta}$

(C) $\dfrac{\left(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right)}{4\Delta}$ **(D)** none of these

47. If the volume of the parallelopiped whose conterminous edges are represented by $-12\hat{i} + \lambda\hat{k},\ 3\hat{j} - \hat{k},\ 2\hat{i} + \hat{j} - 15\hat{k}$ is 546, then λ equals-

(A) 3 **(B)** 2 **(C)** –3 **(D)** –2

48. The reflection of the point (2, –1, 3) in the plane $3x - 2y - z = 9$ is :

(A) $\left(\dfrac{26}{7}, \dfrac{15}{7}, \dfrac{17}{7}\right)$ **(B)** $\left(\dfrac{26}{7}, \dfrac{-15}{7}, \dfrac{17}{7}\right)$ **(C)** $\left(\dfrac{15}{7}, \dfrac{26}{7}, \dfrac{-17}{7}\right)$ **(D)** $\left(\dfrac{26}{7}, \dfrac{17}{7}, \dfrac{-15}{7}\right)$

49. If the plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}(k)$ with x-axis, then k is equal to -

(A) $\dfrac{\sqrt{3}}{2}$ **(B)** $\dfrac{2}{7}$ **(C)** $\dfrac{\sqrt{2}}{3}$ **(D)** 1

50. A line makes angles α, β, γ with the coordinate axes. If $\alpha + \beta = 90^\circ$, then $\gamma =$

(A) 0 **(B)** 90º **(C)** 180º **(D)** None of these

51. Given the vertices A (2, 3, 1), B (4, 1, –2), C (6, 3, 7) & D (–5, –4, 8) of a tetrahedron. The length of the altitude drawn from the vertex D is:

(A) 7 **(B)** 9 **(C)** 11 **(D)** none of these

52. If $\vec{a} + 5\vec{b} = \vec{c}$ and $\vec{a} - 7\vec{b} = 2\vec{c}$, then-

(A) $\vec{a}$ and $\vec{c}$ are like but $\vec{b}$ and $\vec{c}$ are unlike vectors

(B) $\vec{a}$ and $\vec{b}$ are unlike vectors and so also $\vec{a}$ and $\vec{c}$

(C) $\vec{b}$ and $\vec{c}$ are like but $\vec{a}$ and $\vec{b}$ are unlike vectors

(D) $\vec{a}$ and $\vec{c}$ are unlike vectors and so also $\vec{b}$ and $\vec{c}$

53. The straight lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-1}{2}=\frac{y-2}{2}=\frac{z-3}{-2}$ are

(A) Parallel lines **(B)** Intersecting at 60º
(C) Skew lines **(D)** Intersecting at right angle

54. A variable plane forms a tetrahedron of constant volume $64K^3$ with the coordinate planes and the origin, then locus of the centroid of the tetrahedron is -

(A) $x^3+y^3+z^3=6K^2$ **(B)** $xyz=6k^3$ **(C)** $x^2+y^2+z^2=4K^2$ **(D)** $x^{-2}+y^{-2}+z^{-2}=4k^{-2}$

55. The locus represented by xy + yz = 0 is

(A) A pair of perpendicular lines **(B)** A pair of parallel lines
(C) A pair of parallel planes **(D)** A pair of perpendicular planes

56. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-coplanar and $\vec{p}$, $\vec{q}$, $\vec{r}$ are reciprocal vectors to $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively, then $(\ell\vec{a}+m\vec{b}+n\vec{c}).(\ell\vec{p}+m\vec{q}+n\vec{r})$ is equal to : (where ℓ, m, n are scalars)

(A) $\ell^2+m^2+n^2$ **(B)** $\ell m+mn+n\ell$ **(C)** 0 **(D)** none of these

57. Let $\vec{a}=x\hat{i}+12\hat{j}-\hat{k}$, $\vec{b}=2\hat{i}+2x\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{k}$. If the ordered set $\left[\vec{b}\ \vec{c}\ \vec{a}\right]$ is left handed, then :

(A) $x\in(2,\infty)$ **(B)** $x\in(-\infty,-3)$ **(C)** $x\in(-3,2)$ **(D)** $x\in\{-3,2\}$

58. The expression in the vector form for the point $\vec{r}_1$ of intersection of the plane $\vec{r}\cdot\vec{n}=d$ and the perpendicular line $\vec{r}=\vec{r}_0+t\vec{n}$ where t is a parameter given by -

(A) $\vec{r}_1=\vec{r}_0+\left(\frac{d-\vec{r}_0\cdot\vec{n}}{\vec{n}^2}\right)\vec{n}$ **(B)** $\vec{r}_1=\vec{r}_0-\left(\frac{\vec{r}_0\cdot\vec{n}}{\vec{n}^2}\right)\vec{n}$

(C) $\vec{r}_1=\vec{r}_0-\left(\frac{\vec{r}_0\cdot\vec{n}-d}{|\vec{n}|}\right)\vec{n}$ **(D)** $\vec{r}_1=\vec{r}_0+\left(\frac{\vec{r}_0\cdot\vec{n}}{|\vec{n}|}\right)\vec{n}$

59. If 3 non zero vectors $\vec{a},\vec{b},\vec{c}$ are such that $\vec{a}\times\vec{b}=2(\vec{a}\times\vec{c})$, $|\vec{a}|=|\vec{c}|=1$; $|\vec{b}|=4$ the angle between $\vec{b}$ and $\vec{c}$ is $\cos^{-1}\frac{1}{4}$ then $\vec{b}=\ell\vec{c}+\mu\vec{a}$ where $|\ell|+|\mu|$ is -

(A) 6 **(B)** 5 **(C)** 4 **(D)** 0

60. If $\vec{x}$ & $\vec{y}$ are two non collinear vectors and a, b, c represent the sides of a ΔABC satisfying $(a-b)\vec{x}+(b-c)\vec{y}+(c-a)(\vec{x}\times\vec{y})=0$ then ΔABC is -

(A) an acute angle triangle **(B)** an obtuse angle triangle
(C) a right angle triangle **(D)** a scalene triangle

61. If a plane cuts off intercepts OA = a, OB = b, OC = c from the coordinate axes (where 'O' is the origin), then the area of the triangle ABC is equal to

(A) $\frac{1}{2}\sqrt{b^2c^2+c^2a^2+a^2b^2}$ (B) $\frac{1}{2}(bc+ca+ab)$

(C) $\frac{1}{2}abc$ (D) $\frac{1}{2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}$

62. If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are three non-coplanar vectors then $(\vec{A}+\vec{B}+\vec{C}).[(\vec{A}+\vec{B})\times(\vec{A}+\vec{C})]$ equals -

(A) 0 (B) $[\vec{A}\ \vec{B}\ \vec{C}]$ (C) $2[\vec{A}\ \vec{B}\ \vec{C}]$ (D) $-[\vec{A}\ \vec{B}\ \vec{C}]$

63. If $\vec{a}=i+j-k$, $\vec{b}=i-j+k$, $\vec{c}$ is a unit vector such that $\vec{c}.\vec{a}=0$, $[\vec{c}\vec{a}\vec{b}]=0$ then a unit vector $\vec{d}$ perpendicular to both $\vec{a}$ and $\vec{c}$ is

(A) $\frac{1}{\sqrt{6}}(2i-j+k)$ (B) $\frac{1}{2}(j+k)$ (C) $\frac{1}{\sqrt{2}}(i+j)$ (D) $\frac{1}{\sqrt{2}}(i+k)$

64. The equation of a plane which passes through (2, – 3, 1) & is perpendicular to the line joining the points (3, 4, – 1) & (2, – 1, 5) is given by:

(A) $x+5y-6z+19=0$ (B) $x-5y+6z-19=0$

(C) $x+5y+6z+19=0$ (D) $x-5y-6z-19=0$

65. The equation of the plane containing the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and the point (0, 7, –7) is -

(A) $x+y+z=1$ (B) $x+y+z=2$ (C) $x+y+z=0$ (D) none of these

66. Equation of plane which passes through the point of intersection of lines $\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and at greatest distance from the point (0, 0, 0) is :

(A) $4x+3y+5z=25$ (B) $4x+3y+5z=50$

(C) $3x+4y+5z=49$ (D) $x+7y-5z=2$

67. If the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$, $\frac{x-1}{3}=\frac{y-2}{-1}=\frac{z-3}{4}$ and $\frac{x+k}{3}=\frac{y-1}{2}=\frac{z-2}{h}$ are concurrent then

(A) $h=-2, k=-6$ (B) $h=\frac{1}{2}, k=2$ (C) $h=6, k=2$ (D) $h=2, k=\frac{1}{2}$

68. Consider the lines $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$, then the equation of the line which

(A) bisects the angle between the lines is $\frac{x}{3}=\frac{y}{3}=\frac{z}{8}$

(B) bisects the angle between the lines is $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$

(C) passes through origin and is perpendicular to the given lines is $x = y = -z$

(D) none of these

69. The coplanar points A, B, C, D are $(2-x, 2, 2)$, $(2, 2-y, 2)$, $(2, 2, 2-z)$ and $(1, 1, 1)$ respectively, then

(A) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$

(B) $x+y+z=1$

(C) $\frac{1}{1-x}+\frac{1}{1-y}+\frac{1}{1-z}=1$

(D) none of these

70. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non-coplanar unit vectors equally inclined to one another at an acute angle θ. Then $\left|[\vec{a}\ \vec{b}\ \vec{c}]\right|$ in terms of θ is equal to:

(A) $(1+\cos\theta)\sqrt{\cos 2\theta}$

(B) $(1+\cos\theta)\sqrt{1-2\cos 2\theta}$

(C) $(1-\cos\theta)\sqrt{1+2\cos\theta}$

(D) none of these

Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

1. ABCD is a parallelogram. E and F be the middle points of the sides AB and BC, then -

(A) DE trisect AC **(B)** DF trisect AC

(C) DE divide AC in ratio 2 : 3 **(D)** DF divide AC in ratio 3 : 2

2. Consider the plane $\vec{r}.\vec{n}_1 = d_1$ and $\vec{r}.\vec{n}_2 = d_2$, then which of the follwoing are true -

(A) they are perpendicular if $\vec{n}_1.\vec{n}_2 = 0$

(B) angle between them is $\cos^{-1}\left(\frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right)$

(C) normal form of the equation of plane are $\vec{r}.\vec{n}_1 = \frac{d_1}{|\vec{n}_1|}$ & $\vec{r}.\vec{n}_2 = \frac{d_2}{|\vec{n}_2|}$

(D) none of these

3. The vector $\frac{1}{3}\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$ is:

(A) a unit vector **(B)** makes an angle $\frac{\pi}{3}$ with the vector $2\hat{i} - 4\hat{j} + 3\hat{k}$

(C) parallel to the vector $-\hat{i} + \hat{j} - \frac{1}{2}\hat{k}$ **(D)** perpendicular to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

4. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is magnitude $\sqrt{2/3}$ is -

(A) $2\hat{i} + 3\hat{j} - 3\hat{k}$ **(B)** $2\hat{i} + 3\hat{j} + 3\hat{k}$ **(C)** $-2\hat{i} - 5\hat{j} + \hat{k}$ **(D)** $2\hat{i} + \hat{j} + 5\hat{k}$

5. $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitude then angle between $\vec{a} + \vec{b} + \vec{c}$ and $\vec{a}$ is -

(A) $\cos^{-1}\left(\frac{1}{3}\right)$ **(B)** $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ **(C)** $\pi - \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ **(D)** $\tan^{-1}\sqrt{2}$

6. If $\vec{z}_1 = a\hat{i} + b\hat{j}$ and $\vec{z}_2 = c\hat{i} + d\hat{j}$ are two vectors in $\hat{i}$ and $\hat{j}$ system, where $|\vec{z}_1| = |\vec{z}_2| = r$ and $\vec{z}_1 . \vec{z}_2 = 0$, then $\vec{w}_1 = a\hat{i} + c\hat{j}$ and $\vec{w}_2 = b\hat{i} + d\hat{j}$ satisfy:

(A) $|\vec{w}_1| = r$ **(B)** $|\vec{w}_2| = r$ **(C)** $\vec{w}_1 . \vec{w}_2 = 0$ **(D)** none of these

7. If the line $\vec{r} = 2\hat{i} - \hat{j} + 3\hat{k} + \lambda(\hat{i} + \hat{j} + \sqrt{2}\,\hat{k})$ makes angles α, β, γ with xy, yz and zx planes respectively then which one of the following are not possible ?

(A) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$ and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(B) $\tan^2\alpha + \tan^2\beta + \tan^2\gamma = 7$ and $\cot^2\alpha + \cot^2\beta + \cot^2\gamma = 5/3$

(C) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1$ and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 2$

(D) $\sec^2\alpha + \sec^2\beta + \sec^2\gamma = 10$ and $\mathrm{cosec}^2\alpha + \mathrm{cosec}^2\beta + \mathrm{cosec}^2\gamma = 14/3$

8. Let $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that $\vec{r}_1 = \vec{a} - \vec{b} + \vec{c}$, $\vec{r}_2 = \vec{b} + \vec{c} - \vec{a}$, $\vec{r}_3 = \vec{c} + \vec{a} + \vec{b}$, $\vec{r} = 2\vec{a} - 3\vec{b} + 4\vec{c}$. If $\vec{r} = \lambda_1\vec{r}_1 + \lambda_2\vec{r}_2 + \lambda_3\vec{r}_3$, then -

(A) $\lambda_1 = 7$ **(B)** $\lambda_1 + \lambda_3 = 3$ **(C)** $\lambda_1 + \lambda_2 + \lambda_3 = 4$ **(D)** $\lambda_3 + \lambda_2 = 2$

9. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then the vectors $\vec{a} - \vec{d}$ and $\vec{b} - \vec{c}$ are:

(A) collinear **(B)** linearly independent **(C)** perpendicular **(D)** parallel

10. The points A(5, –1, 1), B(7, –4, 7), C(1, –6, 10) and D(–1, –3, 4) are the vertices of a -

(A) parallelogram **(B)** rectangle **(C)** rhombus **(D)** square

11. If $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$, where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are any three vectors such that $\vec{a}.\vec{b} \neq 0$, $\vec{b}.\vec{c} \neq 0$ then $\vec{a}$ and $\vec{c}$ are -

(A) perpendicular **(B)** parallel **(C)** non collinear **(D)** linearly dependent

12. A line passes through a point A with position vector $3\hat{i} + \hat{j} - \hat{k}$ and is parallel to the vector $2\hat{i} - \hat{j} + 2\hat{k}$. If P is a point on this line such that AP = 15 units, then the position vector of the point P is/are

(A) $13\hat{i} + 4\hat{j} - 9\hat{k}$ **(B)** $13\hat{i} - 4\hat{j} + 9\hat{k}$ **(C)** $7\hat{i} - 6\hat{j} + 11\hat{k}$ **(D)** $-7\hat{i} + 6\hat{j} - 11\hat{k}$

13. Taken on side $\overrightarrow{AC}$ of a triangle ABC, a point M such that $\overrightarrow{AM} = \frac{1}{3}\overrightarrow{AC}$. A point N is taken on the side $\overrightarrow{CB}$ such that $\overrightarrow{BN} = \overrightarrow{CB}$ then, for the point of intersection X of $\overrightarrow{AB}$ & $\overrightarrow{MN}$ which of the following holds good ?

(A) $\overrightarrow{XB} = \frac{1}{3}\overrightarrow{AB}$ **(B)** $\overrightarrow{AX} = \frac{1}{2}\overrightarrow{AB}$ **(C)** $\overrightarrow{XN} = \frac{3}{4}\overrightarrow{MN}$ **(D)** $\overrightarrow{XM} = 3\overrightarrow{XN}$

14. Equation of the plane passing through A(x_1, y_1, z_1) and containing the line

$\frac{x - x_2}{d_1} = \frac{y - y_2}{d_2} = \frac{z - z_2}{d_3}$ is

(A) $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$

(B) $\begin{vmatrix} x - x_2 & y - y_2 & z - z_2 \\ x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$

(C) $\begin{vmatrix} x - d_1 & y - d_2 & z - d_3 \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = 0$

(D) $\begin{vmatrix} x & y & z \\ x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$

15. The equation of the plane which contains the lines $\vec{r} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = \hat{i} + 2\hat{j} - \hat{k} + \mu(\hat{i} + \hat{j} + 3\hat{k})$ must be -

(A) $\vec{r}.(7\hat{i} - 4\hat{j} - \hat{k}) = 0$ **(B)** $7(x - 1) - 4(y - 2) - (z + 1) = 0$

(C) $\vec{r}.(\hat{i} + 2\hat{j} - \hat{k}) = 0$ **(D)** $\vec{r}.(\hat{i} + \hat{j} + 3\hat{k}) = 0$

16. $\hat{a}$ and $\hat{b}$ are two given unit vectors at right angle. The unit vector equally inclined with $\hat{a}$, $\hat{b}$ and $\hat{a} \times \hat{b}$ will be :

(A) $-\frac{1}{\sqrt{3}}\left(\hat{a} + \hat{b} + \hat{a} \times \hat{b}\right)$ **(B)** $\frac{1}{\sqrt{3}}\left(\hat{a} + \hat{b} + \hat{a} \times \hat{b}\right)$

(C) $\frac{1}{\sqrt{3}}\left(\hat{a} + \hat{b} - \hat{a} \times \hat{b}\right)$ **(D)** $-\frac{1}{\sqrt{3}}\left(\hat{a} + \hat{b} - \hat{a} \times \hat{b}\right)$

17. A vector which makes equal angles with the vectors $\frac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k})$, $\frac{1}{5}(-4\hat{i} - 3\hat{k})$, $\hat{j}$ is -

(A) $5\hat{i} + \hat{j} + 5\hat{k}$ **(B)** $-5\hat{i} + \hat{j} + 5\hat{k}$ **(C)** $5\hat{i} - \hat{j} - 5\hat{k}$ **(D)** $5\hat{i} + \hat{j} - 5\hat{k}$

18. If $\vec{a}$, $\vec{b}$ & $\vec{c}$ are non coplanar unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$, then the angle between -

(A) $\vec{a}$ & $\vec{b}$ is $\frac{3\pi}{4}$ **(B)** $\vec{a}$ & $\vec{b}$ is $\frac{\pi}{4}$ **(C)** $\vec{a}$ & $\vec{c}$ is $\frac{3\pi}{4}$ **(D)** $\vec{a}$ & $\vec{c}$ is $\frac{\pi}{4}$

19. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = \lambda$ and $\vec{a}.\vec{c} = \frac{\sqrt{3}}{2}$, then

(A) $\vec{a}, \vec{b}, \vec{c}$ are coplanar if $\lambda = 1$ **(B)** Angle between $\vec{b}$ and $\vec{d}$ is 30° if $\lambda = -1$

(C) angle between $\vec{b}$ and $\vec{d}$ is 150° if $\lambda = -1$ **(D)** If $\lambda = 1$ then angle between $\vec{b}$ and $\vec{c}$ is 60°

20. If P_1, P_2, P_3 denotes the perpendicular distances of the plane $2x - 3y + 4z + 2 = 0$ from the parallel planes $2x - 3y + 4z + 6 = 0$, $4x - 6y + 8z + 3 = 0$ and $2x - 3y + 4z - 6 = 0$ respectively, then -

(A) $P_1 + 8P_2 - P_3 = 0$ **(B)** $P_3 = 16P_2$

(C) $8P_2 = P_1$ **(D)** $P_1 + 2P_2 + 3P_3 = \sqrt{29}$

21. If unit vectors $\hat{i}$ & $\hat{j}$ are at right angles to each other and $\vec{p} = 3\hat{i} + 4\hat{j}$, $\vec{q} = 5\hat{i}$, $4\vec{r} = \vec{p} + \vec{q}$ and $2\vec{s} = \vec{p} - \vec{q}$, then

(A) $\left|\vec{r} + k\vec{s}\right| = \left|\vec{r} - k\vec{s}\right|$ for all real *k* **(B)** $\vec{r}$ is perpendicular to $\vec{s}$

(C) $\vec{r} + \vec{s}$ is perpendicular to $\vec{r} - \vec{s}$ **(D)** $\left|\vec{r}\right| = \left|\vec{s}\right| = \left|\vec{p}\right| = \left|\vec{q}\right|$

22. If a line has a vector equation $\vec{r} = 2\hat{i} + 6\hat{j} + \lambda\left(\hat{i} - 3\hat{j}\right)$, then which of the following statements hold good?

(A) the line is parallel to $2\hat{i} + 6\hat{j}$ **(B)** the line passes through the point $3\hat{i} + 3\hat{j}$

(C) the line passes through the point $\hat{i} + 9\hat{j}$ **(D)** the line is parallel to XY-plane

23. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{e}, \vec{f}$ are position vectors of 6 points A, B, C, D, E & F respectively such that $3\vec{a} + 4\vec{b} = 6\vec{c} + \vec{d} = 4\vec{e} + 3\vec{f} = \vec{x}$, then -

(A) $\overrightarrow{AB}$ is parallel to $\overrightarrow{CD}$

(B) line AB, CD and EF are concurrent

(C) $\frac{\vec{x}}{7}$ is position vector of the point dividing CD in ratio 1 : 6

(D) A, B, C, D, E & F are coplanar

24. If $|(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})| = 1$ where $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are unit vectors and $\vec{a}.\vec{c} = 0$ then

(A) $\vec{a}, \vec{b}, \vec{c}$ are non coplanar

(B) $\vec{b}, \vec{c}, \vec{d}$ are non coplanar

(C) If $[\vec{a}\ \vec{b}\ \vec{d}] = \frac{1}{2}$, then acute angle between $\vec{c}$ and $\vec{d}$ is 60°

(D) $\vec{b}, \vec{d}$ are perpendicular

25. A plane meets the coordinate axes in A, B, C such that the centroid of the triangle ABC is the point $(1, r, r^2)$. The plane passes through the point (4, –8, 15) if r is equal to -

(A) –3 **(B)** 3 **(C)** 5 **(D)** –5

26. Indicate the correct order statements -

(A) The lines $\frac{x-4}{-3} = \frac{y+6}{-1} = \frac{z+6}{-1}$ and $\frac{x-1}{-1} = \frac{y-2}{-2} = \frac{z-3}{2}$ are orthogonal

(B) The planes $3x - 2y - 4z = 3$ and the plane $x - y - z = 3$ are orthogonal.

(C) The function $f(x) = \ell n(e^{-2} + e^{x})$ is monotonic increasing $\forall\ x \in R$.

(D) If g is the inverse of the function, $f(x) = \ell n(e^{-2} + e^{x})$ then $g(x) = \ell n(e^{x} - e^{-2})$

27. Let a perpendicular PQ be drawn from P (5, 7, 3) to the line $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ where Q is the foot of perpendicular, then

(A) Q is (9, 13, – 15)

(B) PQ = 14

(C) the equation of plane containing PQ and the given line is $9x - 4y - z - 14 = 0$

(D) none of these

28. Read the following statement carefully and identify the true statement -

(a) Two lines parallel to a third line are parallel.

(b) Two lines perpendicular to a third line are parallel.

(c) Two lines parallel to a plane are parallel.

(d) Two lines perpendicular to a plane are parallel.

(e) Two lines either intersect or are parallel.

(A) a & b **(B)** a & d **(C)** d & e **(D)** a

29. The coordinates of a point on the line $\frac{x-1}{2} = \frac{y+1}{-3} = z$ at a distance $4\sqrt{14}$ from the point (1, –1, 0) are-

(A) (9, –13, 4) **(B)** $(8\sqrt{14}+1, -12\sqrt{14}-1, 4\sqrt{14})$

(C) (–7, 11, –4) **(D)** $(-8\sqrt{14}+1, 12\sqrt{14}-1, -4\sqrt{14})$

30. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\frac{\sqrt{2}}{3}$ is

(A) $2i + 3j - 3k$ **(B)** $2i + 3j + 3k$ **(C)** $-2i - j + 5k$ **(D)** $i - 5j + 3k$

31. Let $6x+4y-5z=4$, $x-5y+2z=12$ and $\frac{x-9}{2}=\frac{y+4}{-1}=\frac{z-5}{1}$ be two lines then-

(A) the angle between them must be $\frac{\pi}{3}$ **(B)** the angle between them must be $\cos^{-1}\frac{5}{6}$

(C) the plane containing them must be $x+y-z=0$ **(D)** they are non-coplanar

32. The vector $\frac{1}{3}(2\hat{i}-2\hat{j}+\hat{k})$ is -

(A) unit vector **(B)** makes an angle $\pi/3$ with vector $2\hat{i}-4\hat{j}+3\hat{k}$

(C) parallel to the vector $-\hat{i}+\hat{j}-(1/2)\hat{k}$ **(D)** perpendicular to the vector $3\hat{i}+2\hat{j}-2\hat{k}$

33. The vector $\vec{c}$, directed along the internal bisector of the angle between the vectors $\vec{a}=7\hat{i}-4\hat{j}-4\hat{k}$ and $\vec{b}=-2\hat{i}-\hat{j}+2\hat{k}$ with $|\vec{c}|=5\sqrt{6}$, is :

(A) $\frac{5}{3}(\hat{i}-7\hat{j}+2\hat{k})$ **(B)** $\frac{5}{3}(\hat{i}+7\hat{j}-2\hat{k})$ **(C)** $\frac{5}{3}(-\hat{i}+7\hat{j}-2\hat{k})$ **(D)** $\frac{5}{3}(-\hat{i}-7\hat{j}+2\hat{k})$

34. The lines $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{\lambda}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z+1}{-1}$ are -

(A) coplanar for all λ

(B) coplanar for $\lambda = 19/3$

(C) if coplanar then intersect at $\left(-\frac{1}{5},-\frac{2}{5},-\frac{4}{5}\right)$

(D) intersect at $\left(\frac{1}{2},-\frac{1}{2},-1\right)$

35. Identify the statement (s) which is/are incorrect ?

(A) $\vec{a}\times\left[\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right]=\left(\vec{a}\times\vec{b}\right)\left(\vec{a}^2\right)$

(B) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non coplanar vectors and $\vec{v}.\vec{a}=\vec{v}.\vec{b}=\vec{v}.\vec{c}=0$ then $\vec{v}$ must be a null vector

(C) If $\vec{a}$ and $\vec{b}$ lie in a plane normal to the plane containing the vectors $\vec{c}$ and $\vec{d}$ then $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=0$

(D) If $\vec{a},\vec{b},\vec{c}$ and $\vec{a}',\vec{b}',\vec{c}'$ are reciprocal system of vectors then $\vec{a}.\vec{b}'+\vec{b}.\vec{c}'+\vec{c}.\vec{a}'=3$

36. The volume of a right triangular prism $ABCA_1B_1C_1$ is equal to 3. If the position vectors of the vertices of the base ABC are A(1, 0, 1), B(2,0, 0) and C(0, 1, 0), then position vectors of the vertex A_1 can be:

(A) (2, 2, 2) **(B)** (0, 2, 0) **(C)** (0, –2, 2) **(D)** (0, –2, 0)

37. If a vector $\vec{r}$ of magnitude $3\sqrt{6}$ is collinear with the bisector of the angle between the vectors $\vec{a} = 7\hat{i} - 4\hat{j} - 4\hat{k}$ & $\vec{b} = -2\hat{i} - \hat{j} + 2\hat{k}$, then $\vec{r} =$

(A) $\hat{i} - 7\hat{j} + 2\hat{k}$ **(B)** $\hat{i} + 7\hat{j} - 2\hat{k}$ **(C)** $\dfrac{13\hat{i} - \hat{j} - 10\hat{k}}{\sqrt{5}}$ **(D)** $\hat{i} - 7\hat{j} - 2\hat{k}$

38. The plane containing the lines $\vec{r} = \vec{a} + t\vec{a}\,'$ and $\vec{r} = \vec{a}\,' + s\vec{a}$ -

(A) must be parallel to $\vec{a} \times \vec{a}\,'$

(B) must be the perpendicular to $\vec{a} \times \vec{a}\,'$

(C) must be $[\vec{r},\ \vec{a},\ \vec{a}\,'] = 0$

(D) $(\vec{r} - \vec{a}).(\vec{a} \times \vec{a}\,') = 0$

39. Vector of length 3 unit which is perpendicular to $\hat{i} + \hat{j} + \hat{k}$ and lies in the plane of $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j}$, is

(A) $\dfrac{3}{\sqrt{6}}(\hat{i} - 2\hat{j} + \hat{k})$ **(B)** $\dfrac{3}{\sqrt{6}}(2\hat{i} - \hat{j} - \hat{k})$ **(C)** $\dfrac{3}{\sqrt{114}}(7\hat{i} - 8\hat{j} + \hat{k})$ **(D)** $\dfrac{3}{\sqrt{114}}(-7\hat{i} + 8\hat{j} - \hat{k})$

40. If two pairs of opposite edges of a tetrahedron are perpendicular then -

(A) the third is also perpendicular

(B) the third pair is inclined at 60°

(C) the third pair is inclined at 45°

(D) (B), (C) are false

41. A parallelopiped is formed by planes drawn through the points (1, 2, 3) and (9, 8, 5) parallel to the coordinate planes then which of the following is the length of an edge of this rectangular parallelopiped -

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8

42. The acute angle that the vector $2\hat{i} - 2\hat{j} + \hat{k}$ makes with the plane contained by the two vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $\hat{i} - \hat{j} + 2\hat{k}$ is given by:

(A) $\cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ **(B)** $\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ **(C)** $\tan^{-1}\left(\sqrt{2}\right)$ **(D)** $\cot^{-1}\left(\sqrt{2}\right)$

43. The equation of a plane bisecting the angle between the plane $2x - y + 2z + 3 = 0$ and $3x - 2y + 6z + 8 = 0$ is -

(A) $5x - y - 4z - 45 = 0$

(B) $5x - y - 4z - 3 = 0$

(C) $23x - 13y + 32z + 45 = 0$

(D) $23x - 13y + 32z + 5 = 0$

44. If a, b, c are different real numbers and $a\hat{i} + b\hat{j} + c\hat{k}$; $b\hat{i} + c\hat{j} + a\hat{k}$ & $c\hat{i} + a\hat{j} + b\hat{k}$ are position vectors of three non-collinear points A, B & C then -

(A) centroid of triangle ABC is $\dfrac{a+b+c}{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$

(B) $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the three vectors

(C) perpendicular from the origin to the plane of triangle ABC meet at centroid

(D) triangle ABC is an equilateral triangle.

45. If $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are linearly independent set of vectors and $K_1\vec{a} + K_2\vec{b} + K_3\vec{c} + K_4\vec{d} = \vec{0}$, then

(A) $K_1+K_2+K_3+K_4=0$
(B) $K_1+K_3=K_2+K_4=0$
(C) $K_1+K_4=K_2+K_3=0$
(D) none of these

46. If A $(\bar{a})$; B $(\bar{b})$; C $(\bar{c})$ and D $(\bar{d})$ are four points such that $\bar{a} = -2\hat{i} + 4\hat{j} + 3\hat{k}$; $\bar{b} = 2\hat{i} - 8\hat{j}$; $\bar{c} = \hat{i} - 3\hat{j} + 5\hat{k}$;

$\bar{d} = 4\hat{i} + \hat{j} - 7\hat{k}$, d is the shortest distance between the lines AB and CD, then

(A) d = 0, hence AB and CD intersect
(B) $d = \dfrac{[\overrightarrow{AB}\ \overrightarrow{CD}\ \overrightarrow{BD}]}{|\overrightarrow{AB} \times \overrightarrow{CD}|}$
(C) AB and CD are skew lines and $d = \dfrac{23}{13}$
(D) $d = \dfrac{[\overrightarrow{AB}\ \overrightarrow{CD}\ \overrightarrow{AC}]}{|\overrightarrow{AB} \times \overrightarrow{CD}|}$

47. A non-zero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}$, $\hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}$, $\hat{i}-\hat{k}$,. The possible angle between $\vec{a}$ and $\hat{i} - 2\hat{j} + 2\hat{k}$ is -

(A) $\pi/3$ **(B)** $\pi/4$ **(C)** $\pi/6$ **(D)** $3\pi/4$

48. The planes $2x - 3y - 7z = 0$, $3x - 14y - 13z = 0$ and $8x - 31y - 33z = 0$

(A) pass through origin
(B) intersect in a common line
(C) form a triangular prism
(D) none of these

49. If ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 are DCs of the two lines inclined to each other at an angle θ, then the DCs of the bisector of the angle between these lines are-

(A) $\dfrac{\ell_1+\ell_2}{2\sin\theta/2}, \dfrac{m_1+m_2}{2\sin\theta/2}, \dfrac{n_1+n_2}{2\sin\theta/2}$
(B) $\dfrac{\ell_1+\ell_2}{2\cos\theta/2}, \dfrac{m_1+m_2}{2\cos\theta/2}, \dfrac{n_1+n_2}{2\cos\theta/2}$
(C) $\dfrac{\ell_1-\ell_2}{2\sin\theta/2}, \dfrac{m_1-m_2}{2\sin\theta/2}, \dfrac{n_1-n_2}{2\sin\theta/2}$
(D) $\dfrac{\ell_1-\ell_2}{2\cos\theta/2}, \dfrac{m_1-m_2}{2\cos\theta/2}, \dfrac{n_1-n_2}{2\cos\theta/2}$

50. Points that lie on the lines bisecting the angle between the lines $\dfrac{x-2}{2} = \dfrac{y-3}{3} = \dfrac{z-6}{6}$ and $\dfrac{x-2}{3} = \dfrac{y-3}{6} = \dfrac{z-6}{2}$ are -

(A) (7, 12, 14) **(B)** (0, –3, 14) **(C)** (1, 0, 10) **(D)** (–3, –6, –2)

Part # II [Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

1. **Statement-I :** The volume of a parallelopiped whose co-terminous edges are the three face diagonals of a given parallelopiped is double the volume of given parallelopied.

Statement-II : For any vectors $\vec{a}, \vec{b}, \vec{c}$ we have $[\vec{a}+\vec{b}\ \vec{b}+\vec{c}\ \vec{c}+\vec{a}] = 2[\vec{a}\,\vec{b}\,\vec{c}]$

2. **Statement - I :** If a plane contains point A($\vec{a}$) and is parallel to vectors $\vec{b}$ and $\vec{c}$, then its vector equation is $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$, where λ & μ are parameters and $\vec{b} \nparallel \vec{c}$.

Statement - II : If three vectors are co-planar, then any one can be expressed as the linear combination of other two.

3. **Statement-I :** Let $A(\vec{a})$ & $B(\vec{b})$ be two points in space. Let $P(\vec{r})$ be a variable point which moves in space such that $\overrightarrow{PA}.\overrightarrow{PB} \le 0$, such a variable point traces a three-dimensional figure whose volume is given by $\frac{\pi}{6}\left\{\vec{a}^2 + \vec{b}^2 - 2\vec{a}.\vec{b}\right\}.|\vec{a}-\vec{b}|$

Statement-II : Diameter of sphere subtends acute angle at any point inside the sphere & its volume is given by $\frac{4}{3}\pi r^3$, where 'r' is the radius of sphere.

4. **Statement - I :** If ax + by + cz = $\sqrt{a^2+b^2+c^2}$ be a plane and (x_1, y_1, z_1) and (x_2, y_2, z_2) be two points on this plane then $a(x_1 - x_2) + b(y_1 - y_2) + c(z_1 - z_2) = 0$.

Statement - II : If two vectors $p_1\hat{i} + p_2\hat{j} + p_3\hat{k}$ and $q_1\hat{i} + q_2\hat{j} + p_3\hat{k}$ are orthogonal then $p_1q_1 + p_2q_2 + p_3q_3 = 0$.

5. **Statement-1 :** If $\vec{a}, \vec{b}, \vec{c}$ are non–coplanar and $\vec{d}$ is any vector, then

$$[\vec{d}\ \vec{b}\ \vec{c}]\,\vec{a} + [\vec{d}\ \vec{c}\ \vec{a}]\vec{b} + [\vec{d}\ \vec{a}\ \vec{b}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d} = \vec{0}$$

Statement-II : Any vector in three dimension can be written as linear combination of three non–coplanar vectors.

6. **Statement - I :** If the lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are coplanar then

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} x_2 & y_2 & z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

Statement - II : If the two lines are coplanar then shortest distance between them is zero.

7. **Statement-I :** Let $\vec{a}, \vec{b}, \vec{c}$ be there non-coplanar vectors. Let $\vec{p}_1$ be perpendicular to plane of $\vec{a}$ & $\vec{b}$, $\vec{p}_2$ perpendicular to plane $\vec{b}$ & $\vec{c}$, $\vec{p}_3$ perpendicular to plane of $\vec{c}$ & $\vec{a}$ then $\vec{p}_1, \vec{p}_2$ & $\vec{p}_3$ are non-coplanar.

Statement-II : $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = [\vec{a}\,\vec{b}\,\vec{c}]^2$

8. **Statement - I :** $ABCDA_1B_1C_1D_1$ is a cube of edge 1 unit. P and Q are the mid points of the edges B_1A_1, and B_1C_1 respectively. Then the distance of the vertex D from the plane PBQ is $\frac{8}{3}$.

Statement - II : Perpendicular distance of point (x_1, y_1, z_1) from the plane $ax + by + cz + d = 0$ is given by $\left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$.

9. **Statement - I :** If $2a + 3b + 6c = 14$, where $a, b \,\&\, c \in R$, then the minimum value of $a^2 + b^2 + c^2$ is 4.

Statement - II : The perpendicular distance of the plane $px + qy + rz = 1$ from origin is $\frac{1}{\sqrt{p^2 + q^2 + r^2}}$.

10. **Statement-I :** If I is incentre of ΔABC then $|\overrightarrow{BC}|\overrightarrow{IA} + |\overrightarrow{CA}|\overrightarrow{IB} + |\overrightarrow{AB}|\overrightarrow{IC} = \vec{0}$

Statement-II : In a triangle, if position vector of vertices are $\vec{a}, \vec{b}, \vec{c}$, then position vector of incentre is $\frac{\vec{a} + \vec{b} + \vec{c}}{3}$

11. **Statement-I :** If $\vec{r} = \vec{a} + \lambda\vec{b}$ & $\vec{r} = \vec{p} + \mu\vec{d}$ be two lines such that $\vec{b} = t\vec{d}$ & $\vec{a} - \vec{p} = s\vec{b}$ where λ, μ t & s be non-zero scalars then the two lines have unique point of intersection.

Statement-II : Two non-parallel coplanar lines have unique point of intersection.

12. Consider following two planes

$$P_1 \equiv [\vec{r} - \vec{p} \;\; \vec{a} \;\; \vec{b}] = 0$$

$$P_2 \equiv [\vec{r} - \vec{p} \;\; \vec{c} \;\; \vec{d}] = 0$$

such that $|(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})| \neq 0$ & let $\vec{x}$ be any vector in space.

Statement-I : $\vec{x}.\{(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})\} = 0 \Rightarrow \vec{x}.\{t_1\vec{a} + t_2\vec{b}\} = 0, \forall\, t_1, t_2 \in R$

Statement-II : $\vec{x}.\{t_1\vec{a} + t_2\vec{b}\} = 0 \;\forall\, t_1, t_2 \in R \Rightarrow \vec{x}.\{(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})\} = 0$.

13. Consider planes $P_1 : (\vec{r} - \hat{i}).\{(\hat{i} - \hat{j} - \hat{k}) \times (\hat{i} - 2\hat{k})\} = 0$ and $P_2 : (\vec{r} - (2\hat{i} - \hat{j} - \hat{k})).\{(\hat{i} - 2\hat{k}) \times (2\hat{i} - \hat{j} - 3\hat{k})\} = 0$

and line L : $\vec{r} = 5\hat{i} + \lambda(\hat{i} - \hat{j} - \hat{k})$

Statement-I : P_1 & P_2 are parallel planes.

Statement-II : L is parallel to both P_1 & P_2.

14. **Statement-I :** If $\vec{a} = \hat{i}, \vec{b} = \hat{j}$ and $\vec{c} = \hat{i} + \hat{j}$, then $\vec{a}$ and $\vec{b}$ are linearly independent but $\vec{a}, \vec{b}$ and $\vec{c}$ are linearly dependent.

Statement-II : If $\vec{a}$ and $\vec{b}$ are linearly dependent and $\vec{c}$ is any vector, then $\vec{a}$, $\vec{b}$ and $\vec{c}$ are linearly dependent.

15. **Statement-I :** If α,β,γ are the angles which a half ray makes with the positive directions of the axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

Statement-II : If ℓ,m,n are the direction cosines of a line then $\ell^2 + m^2 + n^2 = 1$.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **one or more** statement(s) in **Column-II**.

1. **Column-I** **Column-II**

(A) ABC is a triangle. If P is a point inside the ΔABC such that areas of the triangle PBC, PCA and PAB, all are equal, then with respect to the ΔABC, P is its **(p)** centroid

(B) If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the three non-collinear points A, B and C respectively such that the vector $\vec{V} = \overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$ is a null vector, then with respect to the ΔABC, P is its **(q)** orthocentre

(C) If P is a point inside the ΔABC such that the vector $\vec{R} = (BC)(\overrightarrow{PA}) + (CA)(\overrightarrow{PB}) + (AB)(\overrightarrow{PC})$ is a null vector, then with respect to the ΔABC, P is its **(r)** incentre

(D) If P is a point in the plane of the triangle ABC such that the scalar product $\overrightarrow{PA}.\overrightarrow{CB}$ and $\overrightarrow{PB}.\overrightarrow{AC}$ vanishes, then with respect to the ΔABC, P is its **(s)** circumcentre

2. Match the following pair of planes with their lines of intersections :

Column-I **Column-II**

(A) $x + y = 0 = y + z$ **(p)** $\frac{x-2}{0} = \frac{y-2007}{-1} = \frac{z+2004}{1}$

(B) $x = 2, y = 3$ **(q)** $\frac{x-2}{0} = \frac{y}{-1} = \frac{z-1}{1}$

(C) $x = 2, y + z = 3$ **(r)** $x = -y = z$

(D) $x = 2, x + y + z = 3$ **(s)** $\frac{x-2}{0} = \frac{y-3}{0} = \frac{z}{1}$

3. **Column – I** **Column – II**

(A) If the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = 4\hat{i} - 2\hat{j} - 6\hat{k}$ constitute the sides of a ΔABC and length of the median bisecting the vector $\vec{c}$ is λ, then λ^2 **(p)** 2

(B)	Let $\vec{p}$ is the position vector of the orthocentre and $\vec{g}$ is the position vector of the centroid of the triangle ABC, where circumcentre is the origin. If $\vec{p} = K\vec{g}$, then K is equal to :	**(q)**	3
(C)	Twice of the area of the parallelogram constructed on the vectors $\vec{a} = \vec{p} + 2\vec{q}$ and $\vec{b} = 2\vec{p} + \vec{q}$, where $\vec{p}$ and $\vec{q}$ are unit vectors containing an angle of 30º, is :	**(r)**	6
(D)	Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ are vector such that $\vec{u} + \vec{v} + \vec{w} = \vec{0}$. If $\lvert\vec{u}\rvert = 3$, $\lvert\vec{v}\rvert = 4$, $\lvert\vec{w}\rvert = 5$ then $\sqrt{\lvert\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}\rvert}$ is	**(s)**	5

4. Let a, b, c be vectors then -

	Column-I	**Column-II**
(A)	$[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}]$	**(p)** $\lvert\vec{b}\rvert^2 [\vec{a}\ \vec{c}\ \vec{b}]$
(B)	$[(\vec{a} \times \vec{b}) \times (\vec{a} \times \vec{c})].\vec{b}$	**(q)** $(\vec{a}.\vec{b})[\vec{a}\ \vec{b}\ \vec{c}]$
(C)	$[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}]$	**(r)** $2[\vec{a}\ \vec{b}\ \vec{c}]$
(D)	$\vec{b}.\{(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{b})\}$	**(s)** $[\vec{a}\ \vec{b}\ \vec{c}]^2$

5. Consider three planes

$P_1 \equiv 2x + y + z = 1$

$P_2 \equiv x - y + z = 2$

$P_3 \equiv \alpha x - y + 3z = 5$

The three planes intersects each other at point P on XOY plane and at point Q on YOZ plane. O is the origin.

	Column-I	**Column-II**
(A)	The value of α is	**(p)** 1
(B)	The length of projection of PQ on x-axis is	
(C)	If the co-ordinates of point R situated at a minimum distance from point 'O' on the line PQ are (a, b, c), then value of 7a + 14b + 14c is	**(q)** 2 **(r)** 4
(D)	If the area of ΔPOQ is $\sqrt{\frac{a}{b}}$, then value of a – b is	**(s)** 3

6.

	Column – I	**Column – II**
(A)	The distance of the point (1, 3, 4) from the plane $2x - y + z = 3$ measured parallel to the line $\frac{x}{2} = \frac{y}{-1} = \frac{z}{-1}$ is	**(p)** 0
(B)	The distance of the point P(3, 8, 2) from the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-2}{3}$ measured parallel to the plane $3x + 2y - 2z + 17 = 0$ is	**(q)** 7

(C) The points (0, –1, –1), (4, 5, 1), (3, 9, 4) and (–4, 4, k) are coplanar, then k = (r) 4

(D) In ΔABC the mid points of the sides AB, BC and CA are respectively (ℓ, 0, 0) (0, m ,0) and (0, 0, n). (s) 8

Then $\dfrac{AB^2+BC^2+CA^2}{\ell^2+m^2+n^2}$ is equal to

7. Consider the following four pairs of lines in column–I and match them with one or more entries in column–II

Column-I **Column-II**

(A) $L_1 : x=1+t, y=t, z=2-5t$ (p) non coplanar lines

$L_2 : \vec{r} = (2, 1, -3) + \lambda\ (2,2,-10)$

(B) $L_1 : \dfrac{x-1}{2} = \dfrac{y-3}{2} = \dfrac{z-2}{-1}$ (q) lines lie in a unique plane

$L_2 : \dfrac{x-2}{1} = \dfrac{y-6}{-1} = \dfrac{z+2}{3}$

(C) $L_1 : x=-6t, y=1+9t, z=-3t$ (r) infinite planes containing both the lines

$L_2 : x=1+2s, y=4-3s, z=s$

(D) $L_1 : \dfrac{x}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{3}$ (s) lines are not intersecting

$L_2 : \dfrac{x-3}{-4} = \dfrac{y-2}{-3} = \dfrac{z-1}{2}$

8. **Column-I** **Column-II**

(A) Let $\vec{a} = \hat{i}+\hat{j}$ & $\vec{b} = 2\hat{i}-\hat{k}$. If the point of intersection of the lines $\vec{r}\times\vec{a} = \vec{b}\times\vec{a}$ & $\vec{r}\times\vec{b} = \vec{a}\times\vec{b}$ is 'P', then $\ell^2(OP)$ (where O is the origin) is (p) 0

(B) If $\vec{a} = \hat{i}+2\hat{j}+3\hat{k}$, $\vec{b} = 2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c} = 3\hat{i}+2\hat{j}+\hat{k}$ and $\vec{a}\times(\vec{b}\times\vec{c})$ is equal to $x\vec{a}+y\vec{b}+z\vec{c}$, then x + y + z is equal to (q) 5

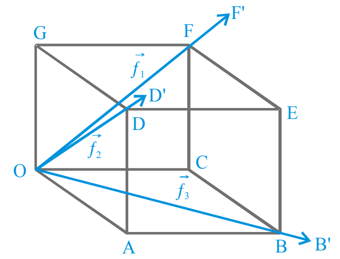
(C) The number of values of x for which the angle between the vectors $\vec{a} = x^9\hat{i}+(x^3-1)\hat{j}+2\hat{k}$ & $\vec{b} = (x^3-1)\hat{i}+x\hat{j}+\frac{1}{2}\hat{k}$ is obtuse (r) 7

(D) Let $P_1 \equiv 2x-y+z=7$ & $P_2 \equiv x+y+z=2$. If P be a point that lies on P_1, P_2 and XOY plane, Q be the point that lies on P_1, P_2 and YOZ plane and R be the point that lies on P_1, P_2 & XOZ plane, then [Area of triangle PQR] (where [.] is greatest integer function) (s) 11

Part # II [Comprehension Type Questions]

Comprehension # 1

Three forces $\vec{f}_1, \vec{f}_2$ & $\vec{f}_3$ of magnitude 2, 4 and 6 units respectively act along three face diagonals of a cube as shown in figure. Let P_1 be a parallelopiped whose three co-terminus edges be three vectors $\vec{f}_1, \vec{f}_2$ & $\vec{f}_3$. Let the joining of mid-points of each pair of opposite edges of parallelopiped P_1 meet in point X.



On the basis of above information, answer the following questions :

1. The magnitude of the resultant of the three forces is -

(A) 5 **(B)** 10 **(C)** 15 **(D)** none of these

2. The volume of the parallelopiped P_1 is -

(A) $48\sqrt{2}$ **(B)** $96\sqrt{2}$ **(C)** $24\sqrt{2}$ **(D)** $50\sqrt{2}$

3. $\ell(OX)$ is equal to -

(A) 5 **(B)** 1.5 **(C)** 2 **(D)** 2.5

Comprehension # 2

Let $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ be two planes, where $d_1, d_2 > 0$. Then origin lies in acute angle if $a_1a_2 + b_1b_2 + c_1c_2 < 0$ and origin lies in obtuse angle if $a_1a_2 + b_1b_2 + c_1c_2 > 0$.
Further point (x_1, y_1, z_1) and origin both lie either in acute angle or in obtuse angle ,
if $(a_1x_1 + b_1y_1 + c_1z_1 + d_1)(a_2x_1 + b_2y_1 + c_2z_1 + d_2) > 0$, one of (x_1, y_1, z_1) and origin lie in acute angle and the other in obtuse angle, if $(a_1x_1 + b_1y_1 + c_1z_1 + d_1)(a_2x_1 + b_2y_1 + c_2z_1 + d_2) < 0$

1. Given the planes $2x + 3y - 4z + 7 = 0$ and $x - 2y + 3z - 5 = 0$, if a point P is $(1, -2, 3)$ and O is origin, then

(A) O and P both lie in acute angle between the planes
(B) O and P both lie in obtuse angle between the planes
(C) O lies in acute angle, P lies in obtuse angle.
(D) O lies in obtue angle, P lies in acute angle.

2. Given the planes $x + 2y - 3z + 5 = 0$ and $2x + y + 3z + 1 = 0$. If a point P is $(2, -1, 2)$ and O is origin, then

(A) O and P both lie in acute angle between the planes
(B) O and P both lie in obtuse angle between the planes
(C) O lies in acute angle, P lies in obtuse angle.
(D) O lies in obtue angle, P lies in acute angle.

3. Given the planes $x + 2y - 3z + 2 = 0$ and $x - 2y + 3z + 7 = 0$, if the point P is $(1, 2, 2)$ and O is origin, then

(A) O and P both lie in acute angle between the planes
(B) O and P both lie in obtuse angle between the planes
(C) O lies in acute angle, P lies in obtuse angle.
(D) O lies in obtue angle, P lies in acute angle.

Comprehension # 3

Consider a triangular pyramid ABCD the position vectors of whose angular point are A(3, 0, 1); B(–1, 4, 1); C(5, 2, 3) and D(0, –5, 4). Let G be the point of intersection of the medians of the triangle BCD.

On the basis of above information, answer the following questions :

1. The length of the vector $\overrightarrow{AG}$ is-

(A) $\sqrt{17}$ **(B)** $\frac{\sqrt{51}}{3}$ **(C)** $\frac{\sqrt{51}}{9}$ **(D)** $\frac{\sqrt{59}}{4}$

2. Area of the triangle ABC in sq. units is-

(A) 24 **(B)** $8\sqrt{6}$ **(C)** $4\sqrt{6}$ **(D)** none of these

3. The length of the perpendicular from the vertex D on the opposite face is -

(A) $\frac{14}{\sqrt{6}}$ **(B)** $\frac{2}{\sqrt{6}}$ **(C)** $\frac{3}{\sqrt{6}}$ **(D)** none of these

4. Equation of the plane ABC is -

(A) $x+y+2z=5$ **(B)** $x-y-2z=1$ **(C)** $2x+y-2z=4$ **(D)** $x+y-2z=1$

Comprehension # 4

Three points A(1, 1, 4), B(0, 0, 5) & C(2, –1, 0) forms a plane. P is a point lying on the line $\vec{r}=\hat{i}+3\hat{j}+\lambda(\hat{i}+\hat{j}+\hat{k})$.

The perpendicular distance of point P from plane ABC is $\frac{2\sqrt{6}}{3}$.

'Q' is a point inside the tetrahedron PABC such that resultant of vectors $\overrightarrow{AQ}$, $\overrightarrow{BQ}$, $\overrightarrow{CQ}$ & $\overrightarrow{PQ}$ is a null vector.

On the basis of above information, answer the following questions :

1. Co-ordinates of point 'P' is -

(A) (2, 4, 1) **(B)** (1, 3, 0) **(C)** (4, 6, 3) **(D)** (7, 9, 6)

2. Volume of tetrahedron PABC is -

(A) $\frac{4\sqrt{81}}{9}$ **(B)** $\frac{2\sqrt{81}}{9}$ **(C)** $\frac{\sqrt{81}}{9}$ **(D)** $\frac{6\sqrt{81}}{9}$

3. Co-ordinates of point 'Q' is -

(A) $\left(\frac{5}{4}, 1, \frac{5}{2}\right)$ **(B)** (5, 1, 5) **(C)** $\left(\frac{5}{2}, 1, \frac{5}{4}\right)$ **(D)** $\left(\frac{5}{4}, 5, \frac{5}{2}\right)$

Comprehension # 5

If $\vec{a}, \vec{b}, \vec{c}$ & $\vec{a}', \vec{b}', \vec{c}'$ are two sets of non-coplanar vectors such that $\vec{a}.\vec{a}'=\vec{b}.\vec{b}'=\vec{c}.\vec{c}'=1$, then the two systems are called Reciprocal System of vectors and $\vec{a}'=\frac{\vec{b}\times\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{b}'=\frac{\vec{c}\times\vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$ and $\vec{c}'=\frac{\vec{a}\times\vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$.

1. Find the value of $\vec{a}\times\vec{a}'+\vec{b}\times\vec{b}'+\vec{c}\times\vec{c}'$.

(A) $\vec{0}$ **(B)** $\vec{a}+\vec{b}+\vec{c}$ **(C)** $\vec{a}-\vec{b}+\vec{c}$ **(D)** $\vec{a}+\vec{b}-\vec{c}$

2. Find value of λ such that $\vec{a}'\times\vec{b}'+\vec{b}'\times\vec{c}'+\vec{c}'\times\vec{a}'=\lambda\dfrac{\vec{a}+\vec{b}+\vec{c}}{[\vec{a}\vec{b}\vec{c}]}$.

(A) -1 **(B)** 1 **(C)** 2 **(D)** -2

3. If $[(a'\times b')\times(b'\times c')\ \ (b'\times c')\times(c'\times a')\ \ (c'\times a')\times(a'\times b')]=[abc]^{n}$, then find n.

(A) $n=-4$ **(B)** $n=4$ **(C)** $n=-3$ **(D)** $n=3$

Comprehension # 6

Given four points A(2, 1, 0); B(1, 0, 1); C(3, 0, 1) and D(0, 0, 2). The point D lies on a line L orthogonal to the plane determined by the point A, B, and C

On the basis of above information, answer the following questions :

1. Equation of the plane ABC is -

(A) $x+y+z-3=0$ **(B)** $y+z-1=0$ **(C)** $x+z-1=0$ **(D)** $2y+z-1=0$

2. Equation of the line L is -

(A) $\vec{r}=2\hat{k}+\lambda(\hat{i}+\hat{k})$ **(B)** $\vec{r}=2\hat{k}+\lambda(2\hat{j}+\hat{k})$

(C) $\vec{r}=2\hat{k}+\lambda(\hat{j}+\hat{k})$ **(D)** none of these

3. Perpendicular distance of D from the plane ABC, is -

(A) $\sqrt{2}$ **(B)** $\dfrac{1}{2}$ **(C)** 2 **(D)** $\dfrac{1}{\sqrt{2}}$

Comprehension # 7

Consider three vectors $\vec{p}=\hat{i}+\hat{j}+\hat{k}$, $\vec{q}=2\hat{i}+4\hat{j}-\hat{k}$ and $r=\hat{i}+\hat{j}+3\hat{k}$ and let $\vec{s}$ be a unit vector.

On the basis of above information, answer the following questions :

1. $\vec{p}$, $\vec{q}$ and $\vec{r}$ are -

(A) linearly dependent

(B) can form the sides of a possible triangle

(C) such that the vector $(\vec{q}-\vec{r})$ is orthogonal to $\vec{p}$

(D) such that each one of these can be expressed as a linear combination of the other two

2. If $(\vec{p}\times\vec{q})\times\vec{r}=u\vec{p}+v\vec{q}+w\vec{r}$, then (u+v+w) equals to -

(A) 8 **(B)** 2 **(C)** –2 **(D)** 4

3. The magnitude of the vector $(\vec{p}.\vec{s})(\vec{q}\times\vec{r})+(\vec{q}.\vec{s})\ (\vec{r}\times\vec{p})+(\vec{r}.\vec{s})(\vec{p}\times\vec{q})$ is -

(A) 4 **(B)** 8 **(C)** –2 **(D)** 2

Comprehension # 8

In a parallelogram OABC, vectors $\vec{a}, \vec{b}, \vec{c}$ are respectively the position vectors of vertices A, B, C with reference to O as origin. A point E is taken on the side BC which divides it in the ratio of 2 : 1 internally. Also, the line segment AE intersect the line bisecting the angle O internally in point P. If CP, when extended meets AB in point F. Then

1. The position vector of point P, is

(A) $\dfrac{3|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left\{\dfrac{\vec{a}}{|\vec{a}|}+\dfrac{\vec{c}}{|\vec{c}|}\right\}$ **(B)** $\dfrac{|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left\{\dfrac{\vec{a}}{|\vec{a}|}+\dfrac{\vec{c}}{|\vec{c}|}\right\}$

(C) $\dfrac{2|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left\{\dfrac{\vec{a}}{|\vec{a}|}+\dfrac{\vec{c}}{|\vec{c}|}\right\}$ **(D)** none of these

2. The position vector of point F, is

(A) $\vec{a}+\dfrac{1}{3}\dfrac{|\vec{a}|}{|\vec{c}|}\vec{c}$ **(B)** $\vec{a}+\dfrac{|\vec{a}|}{|\vec{c}|}\vec{c}$ **(C)** $\vec{a}+\dfrac{2|\vec{a}|}{|\vec{c}|}\vec{c}$ **(D)** $\vec{a}-\dfrac{|\vec{a}|}{|\vec{c}|}\vec{c}$

3. The vector $\overrightarrow{AF}$, is given by

(A) $-\dfrac{|\vec{a}|}{|\vec{c}|}\vec{c}$ **(B)** $\dfrac{|\vec{a}|}{|\vec{c}|}\vec{c}$ **(C)** $\dfrac{2|\vec{a}|}{|\vec{c}|}\vec{c}$ **(D)** $\dfrac{1}{3}\dfrac{|\vec{a}|}{|\vec{c}|}\vec{c}$

Comprehension # 9

If a line passes through P (x_1, y_1, z_1) and having Dr's a, b, c, then the equation of line is $\dfrac{x-x_1}{a}=\dfrac{y-y_1}{b}=\dfrac{z-z_1}{c}$ and equation of plane perpendicular to it and passing through P is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$.
Further equation of plane through the intersection of the two planes
$a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is
$(a_1x+b_1y+c_1z+d_1)+k(a_2x+b_2y+c_2z+d_2)=0$

On the basis of above information, answer the following questions :

1. The distance of the point (1, –2, 3) from the plane x – y + z = 5 measured parallel to the line $\dfrac{x}{2}=\dfrac{y}{3}=\dfrac{z-3}{-4}$ is -

(A) $\dfrac{1}{5}\sqrt{21}$ **(B)** $\dfrac{1}{5}\sqrt{29}$ **(C)** $\dfrac{1}{5}\sqrt{13}$ **(D)** $\dfrac{2}{\sqrt{5}}$

2. The equation of the plane through (0, 2, 4) and containing the line $\dfrac{x+3}{3}=\dfrac{y-1}{4}=\dfrac{z-2}{-2}$ is -

(A) $x-2y+4z-12=0$ **(B)** $5x+y+9z-38=0$
(C) $10x-12y-9z+60=0$ **(D)** $7x+5y-3z+2=0$

3. The plane x – y – z = 2 is rotated through 90° about its line of intersection with the plane x + 2y + z = 2. Then equation of this plane in new position is -
(A) $5x+4y+z-10=0$ **(B)** $4x+5y+3z=0$ **(C)** $2x+y+2z=9$ **(D)** $3x+4y-5z=9$

Exercise # 4 [Subjective Type Questions]

1. Points X & Y are taken on the sides QR & RS, respectively of a parallelogram PQRS, so that $\overrightarrow{QX} = 4\overrightarrow{XR}$ & $\overrightarrow{RY} = 4\overrightarrow{YS}$. The line XY cuts the line PR at Z. Prove that $\overrightarrow{PZ} = \left(\frac{21}{25}\right)\overrightarrow{PR}$.

2. Points X and Y are taken on the sides QR and RS, respectively of a parallelogram PQRS, so that QX = 4XR and RY = 4YS. The line XY cuts the line PR at Z. Find the ratio PZ : ZR.

3. The plane $\ell x + my = 0$ is rotated about its line of intersection with the plane z = 0 through an angle θ. Prove that the equation to the plane in new position is $\ell x + my \pm z\sqrt{\ell^2 + m^2}\tan\theta = 0$

4. Find the distance between points of intersection of

Lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ & $\frac{x-4}{5} = \frac{y-1}{2} = z$

and Lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ & $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$

5. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and $\vec{d}$ is a unit vector, then find the value of, $|(\vec{a}.\vec{d})(\vec{b}\times\vec{c}) + (\vec{b}.\vec{d})(\vec{c}\times\vec{a}) + (\vec{c}.\vec{d})(\vec{a}\times\vec{b})|$ independent of $\vec{d}$.

6.(i) If $\hat{n}$ is the unit vector normal to a plane and p be the length of the perpendicular from the origin to the plane, find the vector equation of the plane.

(ii) Find the equation of the plane which contains the origin and the line of intersection of the planes $\vec{r}.\vec{a} = p$ and $\vec{r}.\vec{b} = q$

7. Let ABCD be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the area of triangles ABC, ACD and ADB be denoted by x, y and z sq. units respectively. Find the area of the triangle BCD.

8. If the three successive vertices of a parallelogram have the position vectors as, A (–3, –2, 0); B (3, –3, 1) and C (5, 0, 2). Then find

(A) position vector of the fourth vertex D

(B) a vector having the same direction as that of $\overrightarrow{AB}$ but magnitude equal to $\overrightarrow{AC}$

(C) the angle between $\overrightarrow{AC}$ and $\overrightarrow{BD}$.

9. Two medians of a triangle are equal, then using vector show that the triangle is isosceles.

10. Find the ratio in which the sphere $x^2 + y^2 + z^2 = 504$ divides the line joining the points (12, – 4, 8) and (27, – 9, 18)

11. Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at an angle of $\frac{\pi}{3}$

12. In triangle ABC using vector method show that the distance between the circumcentre and the orthocentre is $R\sqrt{1 - 8\cos A\ \cos B\ \cos C}$, where R is the circumradius of the triangle ABC.

13. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$; $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then show that the value of the scalar triple product $\left[n\vec{a}+\vec{b} \quad n\vec{b}+\vec{c} \quad n\vec{c}+\vec{a}\right]$ is $(n^3 + 1)\begin{vmatrix} \vec{a}.\hat{i} & \vec{a}.\hat{j} & \vec{a}.\hat{k} \\ \vec{b}.\hat{i} & \vec{b}.\hat{j} & \vec{b}.\hat{k} \\ \vec{c}.\hat{i} & \vec{c}.\hat{j} & \vec{c}.\hat{k} \end{vmatrix}$

14. Let $\vec{u}$ & $\vec{v}$ be unit vectors. If $\vec{w}$ is a vector such that $\vec{w} + \left(\vec{w} \times \vec{u}\right) = \vec{v}$, then prove that $\left|\left(\vec{u} \times \vec{v}\right).\vec{w}\right| \le \frac{1}{2}$ and the equality holds if and only if $\vec{u}$ is perpendicular to $\vec{v}$.

15. The position vectors of the four angular points of a tetrahedron OABC are (0, 0, 0); (0, 0, 2); (0, 4, 0) and (6, 0, 0) respectively. A point P inside the tetrahedron is at the same distance 'r' from the four plane faces of the tetrahedron. Find the values of 'r'.

16.(A) Show that the perpendicular distance of the point $\vec{c}$ from the line joining $\vec{a}$ and $\vec{b}$ is

$$\frac{\left|\vec{b}\times\vec{c} + \vec{c}\times\vec{a} + \vec{a}\times\vec{b}\right|}{\left|\vec{b}-\vec{a}\right|}.$$

(B) Given a parallelogram ABCD with area 12 sq. units. A straight line is drawn through the mid point M of the side BC and the vertex A which cuts the diagonal BD at a point 'O'. Use vectors to determine the area of the quadrilateral OMCD.

17. Let $\vec{u}$ be a vector on rectangular coordinate system with sloping angle 60°. Suppose that $|\vec{u}-\hat{i}|$ is geometric mean of $|\vec{u}|$ and $|\vec{u}-2\hat{i}|$ where $\hat{i}$ is the unit vector along x-axis then $|\vec{u}|$ has the value equal to $\sqrt{a}-\sqrt{b}$ where $a, b \in N$. Find the value $(a + b)^3 + (a - b)^3$.

18. Find value of $x \in R$ for which the vectors $\vec{a} = (1, -2, 3)$, $\vec{b} = (-2, 3, -4)$, $\vec{c} = (1, -1, x)$ form a linearly dependent system.

19. Through a point P(f, g, h), a plane is drawn at right angles to OP where 'O' is the origin, to meet the coordinate axes in A, B, C. Prove that the area of the triangle ABC is $\frac{r^5}{2fgh}$ where OP = r.

20. Examine for coplanarity of the following sets of points

(A) $4\hat{i} + 8\hat{j} + 12\hat{k}, 2\hat{i} + 4\hat{j} + 6\hat{k}, 3\hat{i} + 5\hat{j} + 4\hat{k}, 5\hat{i} + 8\hat{j} + 5\hat{k}$.

(B) $3\vec{a} + 2\vec{b} - 5\vec{c}, 3\vec{a} + 8\vec{b} + 5\vec{c}, -3\vec{a} + 2\vec{b} + \vec{c}, \vec{a} + 4\vec{b} - 3\vec{c}$.

21. If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + 2\hat{k}$ & $\vec{c} = -\hat{i} + 2\hat{j} - \hat{k}$, find a unit vectors normal to the vectors $\vec{a} + \vec{b}$ and $\vec{b} - \vec{c}$.

22. Show that the circumcentre of the tetrahedron OABC is given by $\frac{\vec{a}^2\left(\vec{b}\times\vec{c}\right) + \vec{b}^2\left(\vec{c}\times\vec{a}\right) + \vec{c}^2\left(\vec{a}\times\vec{b}\right)}{2\left[\vec{a}\,\vec{b}\,\vec{c}\right]}$, where $\vec{a}, \vec{b}$ & $\vec{c}$ are the position vectors of the points A, B, C respectively relative to the origin 'O'.

23. Let $\begin{vmatrix} (a_1-a)^2 & (a_1-b)^2 & (a_1-c)^2 \\ (b_1-a)^2 & (b_1-b)^2 & (b_1-c)^2 \\ (c_1-a)^2 & (c_1-b)^2 & (c_1-c)^2 \end{vmatrix} = 0$ and if the vectors $\vec{\alpha} = \hat{i} + a\hat{j} + a^2\hat{k}$; $\vec{\beta} = \hat{i} + b\hat{j} + b^2\hat{k}$; $\vec{\gamma} = \hat{i} + c\hat{j} + c^2\hat{k}$ are non coplanar, show that the vectors $\vec{\alpha}_1 = \hat{i} + a_1\hat{j} + a_1^2\hat{k}$; $\vec{\beta}_1 = \hat{i} + b_1\hat{j} + b_1^2\hat{k}$ and $\vec{\gamma}_1 = \hat{i} + c_1\hat{j} + c_1^2\hat{k}$ are coplanar.

24. Find the angle between the lines whose direction cosines are given by $\ell + m + n = 0$ and $\ell^2 + m^2 = n^2$.

25. If $\vec{r}$ and $\vec{s}$ are nonzero constant vectors and the scalar b is chosen such that $|\vec{r} + b\vec{s}|$ is minimum, then show that the value of $|b\vec{s}|^2 + |\vec{r} + b\vec{s}|^2$ is equal to $|\vec{r}|^2$.

26. If $\vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda(\hat{i}-\hat{j}+\hat{k})$ and $\vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \mu(\hat{i}+\hat{j}-\hat{k})$ are two lines, then find the equation of acute angle bisector of two lines.

27. Find the equation of the plane containing the straight line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z}{5}$ are perpendicular to the plane $x - y + z + 2 = 0$

28. If '2d' be the shortest distance between the lines $\frac{y}{b} + \frac{z}{c} = 1; x = 0$ and $\frac{x}{a} - \frac{z}{c} = 1; y = 0$ then prove $\frac{1}{d^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$.

29. Given four points P_1, P_2, P_3 and P_4 on the coordinate plane with origin O which satisfy the condition

$$\overrightarrow{OP_{n-1}} + \overrightarrow{OP_{n+1}} = \frac{3}{2}\overrightarrow{OP_n}, \ n = 2, 3$$

(A) If P_1, P_2 lie on the curve $xy = 1$, then prove that P_3 does not lie on this curve.

(B) If P_1, P_2, P_3 lie on the circle $x^2 + y^2 = 1$, then prove that P_4 lies on this circle.

30. Find the point R in which the line AB cuts the plane CDE, where position vectors of points A, B, C, D, E are respectively $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$, $\vec{c} = -4\hat{j} + 4\hat{k}$, $\vec{d} = 2\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{e} = 4\hat{i} + \hat{j} + 2\hat{k}$.

31. (A) The position vectors of the four angular points of a tetrahedron are :
$A(\hat{j}+2\hat{k})$, $B(3\hat{i}+\hat{k})$, $C(4\hat{i}+3\hat{j}+6\hat{k})$ and $D(2\hat{i}+3\hat{j}+2\hat{k})$. Find :

(i) the volume of the tetrahedron ABCD.

(ii) the shortest distance between the lines AB and CD.

(B) The position vectors of the angular points of a tetrahedron are
$A(3\hat{i}-2\hat{j}+\hat{k})$, $B(3\hat{i}+\hat{j}+5\hat{k})$, $C(4\hat{i}+3\hat{k})$ and $D(\hat{i})$.
Then find the acute angle between the lateral face ADC and the base face ABC.

32. The vector $\overrightarrow{OP} = \hat{i} + 2\hat{j} + 2\hat{k}$ turns through a right angle, passing through the positive x-axis on the way. Find the vector in its new position.

33. Position vectors of A, B, C are given by $\vec{a}, \vec{b}, \vec{c}$ where $\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a} = 0$. If $\vec{AC} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ then find $\vec{BC}$ if $BC = 14$.

34. Given three points on the xy plane O(0, 0), A(1, 0) and B(–1, 0). Point P is moving on the plane satisfying the condition $\left(\overrightarrow{PA}.\overrightarrow{PB}\right) + 3\left(\overrightarrow{OA}.\overrightarrow{OB}\right) = 0$. If the maximum and minimum values of $\left|\overrightarrow{PA}\right|\left|\overrightarrow{PB}\right|$ are M and m respectively then find the values of $M^2 + m^2$.

35. Find the π-plane passing through the points of intersection of the planes $2x + 3y - z + 1 = 0$ and $x + y - 2z + 3 = 0$ and is perpendicular to the plane $3x - y - 2z = 4$. Also find the image of point (1, 1, 1) in π–plane .

36. Find the value of λ such that a, b, c are all non-zero and $(-4\hat{i} + 5\hat{j})a + (3\hat{i} - 3\hat{j} + \hat{k})b + (\hat{i} + \hat{j} + 3\hat{k})c = \lambda\ (a\hat{i} + b\hat{j} + c\hat{k})$

37. Find the angle between the plane passing through points (1, 1, 1), (1, – 1, 1), (–7, –3, –5) & x–z plane.

38. Solve for $\vec{x}$: $\vec{x} \times \vec{a} + (\vec{x}.\vec{b})\vec{a} = \vec{c}$, where $\vec{a}$ and $\vec{c}$ are non zero non collinear and $\vec{a}.\vec{b} \neq 0$

39. Find the shortest distance between the lines :

$\vec{r} = (4\hat{i} - \hat{j}) + \lambda\ (\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu\ (2\hat{i} + 4\hat{j} - 5\hat{k})$

40. If $\vec{x} \times \vec{y} = \vec{a},\ \vec{y} \times \vec{z} = \vec{b},\ \vec{x}.\vec{b} = \gamma,\ \vec{x}.\vec{y} = 1$ and $\vec{y}.\vec{z} = 1$, then find $\vec{x},\ \vec{y}$ & $\vec{z}$ in terms of $\vec{a},\ \vec{b}$ & γ.

41. Find the coordinates of the point equidistant from the point (a, 0, 0), (0, b, 0), (0, 0, c) and (0, 0, 0).

42. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are position vectors of the vertices of a cyclic quadrilateral ABCD prove that :

$$\frac{\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{d} + \vec{d} \times \vec{a}\right|}{(\vec{b} - \vec{a}).(\vec{d} - \vec{a})} + \frac{\left|\vec{b} \times \vec{c} + \vec{c} \times \vec{d} + \vec{d} \times \vec{b}\right|}{(\vec{b} - \vec{c}).(\vec{d} - \vec{c})} = 0$$

43. The reflection of line $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{5}$ about the plane $x - 2y + z - 6 = 0$ is

44. Find the point where the line of intersection of the planes $x - 2y + z = 1$ and $x + 2y - 2z = 5$, intersects the plane $2x + 2y + z + 6 = 0$

45. A line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ intersects the plane $2x - y + 2z + 2 = 0$ at point A. Find equation of the straight line passing through A lying in the given plane and at minimum inclination with the given line

46. Solve the simultaneous vector equations for the vectors $\vec{x}$ and $\vec{y}$. $\vec{x} + \vec{c} \times \vec{y} = \vec{a}$ and $\vec{y} + \vec{c} \times \vec{x} = \vec{b}$ where $\vec{c}$ is a non zero vector.

47. Find the equation of the plane containing parallel lines $(x - 4) = \frac{3-y}{4} =$ and $(x - 3) = \lambda\ (y + 2) = \mu z$

48. Let = i – j and = + and = + $(q^2 - 3)$, = – p + q . If ⊥ , then express p as a function of q, say p = f(q), ($p \neq 0$ and $q \neq 0$) and find the intervals of monotonicity of f(q).

49. Find the radius of the circular section of the sphere $\left|\ \right| = 5$ by the plane, . = 3

50. **(A)** Prove that the acute angle between the plane faces of a regular tetrahedron is arc cos (1/3).

(B) Find the circum–radius and in–radius of a regular tetrahedron in terms of the length k of each edge.

VECTOR

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. Let , and be three non-zero vectors such that is a unit vector perpendicular to both . If the angle between is , then is equal to:

(A) 0 **(B)** 1

(C) **(D)**

2. If = , = & = . Then altitude of the parallelopiped formed by the vectors , , having base formed by & is (, , and , , are reciprocal systems of vectors)

(A) 1 **(B)** **(C)** **(D)**

3. The vectors $= -4 \quad + 3$, $= 14 \quad + 2 \quad - 5$ are co-initial. The vector which is bisecting the angle between the vectors and and is having the magnitude , is

(A) $+ \quad + 2$ **(B)** **(C)** **(D)** none

4. Let be four non – zero vectors such that $= 0$, = , = ,

then [a b c] =

(A) |a| |b| |c| **(B)** – |a| |b| |c| **(C)** 0 **(D)** none of these

5. The vector is rotated through an angle of $\cos^{-1}$ and doubled in magnitude, then it becomes . The value of 'x' is:

(A) – **(B)** **(C)** **(D)** 2

6. If = , = , = , then in the reciprocal system of vectors of the vectors , , , reciprocal of vector is

(A) **(B)** **(C)** **(D)**

7. The base of the pyramid AOBC is an equilateral triangle OBC with each side equal to $4\sqrt{2}$. 'O' is the origin of reference, AO is perpendicular to the plane of Δ OBC and $|\overrightarrow{AO}| = 2$. Then the cosine of the angle between the skew straight lines one passing through A and the mid point of OB and the other passing through 'O' and the mid point of BC is :

(A) $-\frac{1}{\sqrt{2}}$ (B) 0 (C) $\frac{1}{\sqrt{6}}$ (D) $\frac{1}{\sqrt{2}}$

8. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non coplanar vectors and $\vec{r}$ be any vector in space such that $\vec{r}.\vec{a} = 1$, $\vec{r}.\vec{b} = 2$ and $\vec{r}.\vec{c} = 3$. If $[\vec{a}\ \vec{b}\ \vec{c}] = 1$, then $\vec{r}$ is equal to -

(A) $\vec{a} + 2\vec{b} + 3\vec{c}$ (B) $(\vec{b}\times\vec{c}) + 2(\vec{c}\times\vec{a}) + 3(\vec{a}\times\vec{b})$

(C) $(\vec{b}.\vec{c})\vec{a} + 2(\vec{c}.\vec{a})\vec{b} + 3(\vec{a}.\vec{b})\vec{c}$ (D) None of these

9. If in a plane A_1, A_2, A_3,...... A_n are the vertices of a regular polygon with n sides and O is its centre then $\sum_{i=1}^{n-1}(\overrightarrow{OA_i}\times\overrightarrow{OA_{i+1}})$

(A) $(1-n)(\vec{OA_2}\times\vec{OA_1})$ (B) $(n-1)(\vec{OA_2}\times\vec{OA_1})$ (C) $n(\vec{OA_2}\times\vec{OA_1})$ (D) none

10. S_1 : If $\lambda \in R$, $\lambda \neq 0$ and $\vec{a} \neq \vec{0}$, then vectors $\vec{a}$ and $\lambda\vec{a}$ are non-parallel vectors.

S_2 : minimum value of $\vec{a}\ .\ \vec{b}$ is 0

S_3 : If $\vec{a}$ and $\vec{b}$ are non-collinear vectors, then $|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})|$ is equal to twice the area of the parallelogram formed by $\vec{a}$ and $\vec{b}$.

S_4 : $(\vec{a}\times\vec{b})^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{b}.\vec{a} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} \end{vmatrix}$

(A) TTFF (B) FFTT (C) FTFT (D) TTTF

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the position vectors of the points A, B, C and D respectively in three dimensional space no three of A, B, C, D are collinear and satisfy the relation $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = \vec{0}$, then :

(A) A, B, C and D are coplanar

(B) The line joining the points B and D divides the line joining the point A and C in the ratio 2 : 1.

(C) The line joining the points A and C divides the line joining the points B and D in the ratio 1 : 1.

(D) The four vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are linearly dependents.

12. In a four - dimensional space where unit vectors along axes are $\hat{i}, \hat{j}, \hat{k}$ and $\hat{\ell}$ and $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ are four non-zero vectors such that no vector can be expressed as linear combination of others and $(\lambda - 1)(\vec{a}_1 - \vec{a}_2) + \mu(\vec{a}_2 + \vec{a}_3) + \gamma(\vec{a}_3 + \vec{a}_4 - 2\vec{a}_2) + \vec{a}_3 + \delta\vec{a}_4 = \vec{0}$ then

(A) $\lambda = 1$, (B) $\mu = -\frac{2}{3}$ (C) $\gamma = \frac{2}{3}$ (D) $\delta = \frac{1}{3}$

13. If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$, $\vec{b} = y\hat{i} + z\hat{j} + x\hat{k}$ and $\vec{c} = z\hat{i} + x\hat{j} + y\hat{k}$, then $\vec{a} \times (\vec{b} \times \vec{c})$ is

(A) parallel to $(y-z)\hat{i} + (z-x)\hat{j} + (x-y)\hat{k}$ **(B)** orthogonal to $\hat{i} + \hat{j} + \hat{k}$

(C) orthogonal to $(y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$ **(D)** orthogonal to $x\hat{i} + y\hat{j} + z\hat{k}$

14. Identify the statement(s) which is/are incorrect ?

(A) $\vec{a}\ \text{x}\left[\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right] = \left(\vec{a} \times \vec{b}\right)\left(\vec{a}^2\right)$

(B) If $\vec{a}, \vec{b}, \vec{c}$ are non-zero, non coplanar vectors, and $\vec{v}.\vec{a} = \vec{v}.\vec{b} = \vec{v}.\vec{c} = 0$ then $\vec{v}$ must be a null vector

(C) If $\vec{a}$ and $\vec{b}$ lie in a plane normal to the plane containing the vectors $\vec{a} \times \vec{b}$, $\vec{c} \times \vec{d}$; $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are non-zero vector,

then $\left(\vec{a} \times \vec{b}\right) \text{x} \left(\vec{c} \times \vec{d}\right) = \vec{0}$

(D) If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vectors then $\vec{a}.\vec{b}' + \vec{b}.\vec{c}' + \vec{c}.\vec{a}' = 3$

15. The value(s) of $\alpha \in [0, 2\pi]$ for which vector $\vec{a} = \hat{i} + 3\hat{j} + (\sin 2\alpha)\hat{k}$ makes an obtuse angle with the z-axis and the vectors $\vec{b} = (\tan\alpha)\hat{i} - \hat{j} + 2\sqrt{\sin\frac{\alpha}{2}}\,\hat{k}$ and $\vec{c} = (\tan\alpha)\hat{i} + (\tan\alpha)\hat{j} - 3\sqrt{\operatorname{cosec}\frac{\alpha}{2}}\,\hat{k}$ are orthogonal, is/are:

(A) $\tan^{-1} 3$ **(B)** $\pi - \tan^{-1} 2$ **(C)** $\pi + \tan^{-1} 3$ **(D)** $2\pi - \tan^{-1} 2$

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I :** If $\vec{a} = 3\hat{i} + \hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{c} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{d} = 2\hat{i} - \hat{j}$, then there exist real numbers α, β, γ such that $\vec{a} = \alpha\vec{b} + \beta\vec{c} + \gamma\vec{d}$

Statement-II : $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are four vectors in a 3 – dimensional space. If $\vec{b}, \vec{c}, \vec{d}$ are non-coplanar, then there exist real numbers α, β, γ such that $\vec{a} = \alpha\vec{b} + \beta\vec{c} + \gamma\vec{d}$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

17. **Statement-I :** Let $A(\vec{a})$, $B(\vec{b})$ and $C(\vec{c})$ be three points such that $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = -\hat{i} + 7\hat{j} - 5\hat{k}$, then OABC is a tetrahedron.

Statement-2 : Let $A(\vec{a})$, $B(\vec{b})$ and $C(\vec{c})$ be three points such that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non-coplanar. then OABC is a tetrahedron, where O is the origin.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

18. **Statement-I :** Let $\vec{a} = 2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b} = 4\hat{i}+6\hat{j}-2\hat{k}$, then $\vec{a}\times\vec{b} = \vec{0}$

Statement-II : If $\vec{a} \neq \vec{0}$, $\vec{b} \neq \vec{0}$ and $\vec{a}$ and $\vec{b}$ are non-collinear vectors, then $\vec{a}\times\vec{b} = ab\sin\theta\ \hat{n}$, where θ is the smaller angle between the vectors $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is unit vector such that $\vec{a}, \vec{b}, \hat{n}$ taken in this order form right handed orientation

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

19. **Statement-I :** Let $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ are position vectors of four points A, B, C & D and $3\vec{a}-2\vec{b}+5\vec{c}-6\vec{d}=\vec{0}$, then points A, B, C and D are coplanar.

Statement-II : Three non zero, linearly dependent co-initial vectors ($\overrightarrow{PQ}, \overrightarrow{PR}$ & $\overrightarrow{PS}$) are coplaner.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

20. **Statement-1 :** Let $\vec{a} = 3\hat{i}-\hat{j}$, $\vec{b} = 2\hat{i}+\hat{j}-3\hat{k}$. If $\vec{b} = \vec{b}_1+\vec{b}_2$ such that $\vec{b}_1$ is collinear with $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$ is possible, then $\vec{b}_2 = \hat{i}+3\hat{j}-3\hat{k}$.

Statement-2 : If $\vec{a}$ and $\vec{b}$ are non-zero, non-collinear vectors, then $\vec{b}$ can be expressed as $\vec{b} = \vec{b}_1+\vec{b}_2$, where $\vec{b}_1$ is collinear with $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21. **Match the column**

Column - I		Column - I	
(A)	If $\vec{a}+\vec{b} = \hat{j}$ and $2\vec{a}-\vec{b} = 3\hat{i}+\frac{\hat{j}}{2}$, then modulus of cosine of the angle between $\vec{a}$ and $\vec{b}$ is	**(p)**	1
(B)	If $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = \lvert\vec{c}\rvert$, angle between each pair of vectors is $\frac{\pi}{3}$ and $\lvert\vec{a}+\vec{b}+\vec{c}\rvert = \sqrt{6}$, then $\lvert\vec{a}\rvert =$	**(q)**	$5\sqrt{3}$
(C)	Area of the parallelogram whose diagonals represent the vectors $3\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}-3\hat{j}+4\hat{k}$ is	**(r)**	2
(D)	If $\vec{a}$ is perpendicular to $\vec{b}+\vec{c}$, $\vec{b}$ is perpendicular to $\vec{c}+\vec{a}$, $\vec{c}$ is perpendicular to $\vec{a}+\vec{b}$, $\lvert\vec{a}\rvert = 2$, $\lvert\vec{b}\rvert = 3$ and $\lvert\vec{c}\rvert = 6$, then $\lvert\vec{a}+\vec{b}+\vec{c}\rvert =$	**(s)**	7
		(t)	$\frac{3}{5}$

22. **Match the column**

Column – I		Column – II	
(A)	The area of the triangle whose vertices are the points with ractangular cartesian coordinates (1, 2, 3), (–2, 1, – 4), (3, 4, – 2) is	**(p)**	0
(B)	The value of $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d}) + (\vec{b}\times\vec{c}).(\vec{a}\times\vec{d}) + (\vec{c}\times\vec{a}).(\vec{b}\times\vec{d})$ is	**(q)**	28
(C)	A square PQRS of side length P is folded along the diagonal PR so that the point Q reaches at Q′ and planes PRQ′ and PRS are perpendicular to one another, the shortest distance between PQ′ and RS is $\frac{kP}{\sqrt{6}}$, then k =	**(r)**	2
(D)	$\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}-4\hat{k}$, $\vec{c}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{d}=3\hat{i}+2\hat{j}+\hat{k}$ then $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})$ =	**(s)**	$\frac{\sqrt{1218}}{2}$
		(t)	21

SECTION - V : COMPREHENSION TYPE

23. **Read the following comprehension carefully and answer the questions.**

Let $\vec{a} = 2\hat{i}+3\hat{j}-6\hat{k}$, $\vec{b} = 2\hat{i}-3\hat{j}+6\hat{k}$ and $\vec{c} = -2\hat{i}+3\hat{j}+6\hat{k}$. Let $\vec{a}_1$ be projection of $\vec{a}$ on $\vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1$ on $\vec{c}$, then

1. $\vec{a}_2$ =

(A) $\frac{943}{49}(2\hat{i}-3\hat{j}-6\hat{k})$ **(B)** $\frac{943}{49^2}(2\hat{i}-3\hat{j}-6\hat{k})$

(C) $\frac{943}{49}(-2\hat{i}+3\hat{j}+6\hat{k})$ **(D)** $\frac{943}{49^2}(-2\hat{i}+3\hat{j}+6\hat{k})$

2. $\vec{a}_1 . \vec{b}$ =

(A) –41 **(B)** $-\frac{41}{7}$ **(C)** 41 **(D)** 287

3. Which of the following is true

(A) $\vec{a}$ and $\vec{a}_2$ are collinear **(B)** $\vec{a}_1$ and $\vec{c}$ are collinear

(C) $\vec{a}$, $\vec{a}_1$, $\vec{b}$ are coplanar **(D)** $\vec{a}$, $\vec{a}_1$, $\vec{a}_2$ are coplanar

24. Read the following comprehension carefully and answer the questions.

ABCD is a parallelogram . L is a point on BC which divides BC in the ratio 1 : 2. AL intersects BD at P. M is a point on DC which divides DC in the ratio 1 : 2 and AM intersects BD in Q .

1. Point P divides AL in the ratio

(A) 1 : 2 **(B)** 1 : 3 **(C)** 3 : 1 **(D)** 2 : 1

2. Point Q divides DB in the ratio

(A) 1 : 2 **(B)** 1 : 3 **(C)** 3 : 1 **(D)** 2 : 1

3. PQ : DB =

(A) 2 : 3 **(B)** 1 : 3 **(C)** 1 : 2 **(D)** 3 : 4

25. Read the following comprehension carefully and answer the questions.

Three vector $\hat{a}$, $\hat{b}$ and $\hat{c}$ are forming a right handed system, if $\hat{a}\times\hat{b}=\hat{c}$, $\hat{b}\times\hat{c}=\hat{a}$, $\hat{c}\times\hat{a}=\hat{b}$, then answer the following question

1. If vector $3\hat{a}-2\hat{b}+2\hat{c}$ and $-\hat{a}-2\hat{c}$ are adjacent sides of a parallelogram, then an angle between the diagonals is

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{3}$ **(C)** $\frac{\pi}{2}$ **(D)** $\frac{2\pi}{3}$

2. If $\vec{x}=\hat{a}+\hat{b}-\hat{c}$, $\vec{y}=-\hat{a}+\hat{b}-2\hat{c}$, $\vec{z}=-\hat{a}+2\hat{b}-\hat{c}$, then a unit vector normal to the vectors $\vec{x}+\vec{y}$ and $\vec{y}+\vec{z}$ is

(A) $\vec{a}$ **(B)** $\vec{b}$ **(C)** $\vec{c}$ **(D)** none of these

3. Vectors $2\hat{a}-3\hat{b}+4\hat{c}$, $\hat{a}+2\hat{b}-\hat{c}$ and $x\hat{a}-\hat{b}+2\hat{c}$ are coplanar, then x =

(A) $\frac{8}{5}$ **(B)** $\frac{5}{8}$ **(C)** 0 **(D)** 1

4. Let $\vec{x} = \vec{a}+\vec{b}$, $\vec{y} = 2\vec{a} - \vec{b}$, then the point of intersection of straight lines $\vec{r}\times\vec{x} = \vec{y}\times\vec{x}$, $\vec{r}\times\vec{y} = \vec{x}\times\vec{y}$ is

(A) $2\vec{b}$ **(B)** $3\vec{b}$ **(C)** $3\vec{a}$ **(D)** $2\vec{a}$

5. $\hat{a}\,.(\hat{b}\times\hat{c}) + \hat{b}\,.(\hat{c}\times\hat{a}) + \hat{c}\,.(\hat{a}\times\hat{b})$ is equal to

(A) 1 **(B)** 3 **(C)** 0 **(D)** – 12

SECTION - VI : INTEGER TYPE

26. If V_1, V_2, V_3 are volumes of parallelopiped, triangular prism and tetrahedron respectively. The three coterminus edges of all three figures are the vectors

$\hat{i} - \hat{j} - 6\hat{k}$, $\hat{i} - \hat{j} + 4\hat{k}$ and $2\hat{i} - 5\hat{j} + 3\hat{k}$, then find sum of the volumes V_1, V_2 and V_3 .

27. If V be the volume of a tetrahedron and V' be the volume of the tetrahedron formed by the centroids and $V = kV'$ then find the value of k.

28. If $\vec{p}$, $\vec{q}$, $\vec{r}$ are mutually perpendicular vectors, where $|\vec{p}| = |\vec{q}| = |\vec{r}|$ and

$\vec{p} \times \{(\vec{x} - \vec{q}) \times \vec{p}\} + \vec{q} \times \{(\vec{x} - \vec{r}) \times \vec{q}\} + \vec{r} \times \{(\vec{x} - \vec{p}) \times \vec{r}\} = \vec{0}$ and $\vec{x} = \frac{1}{\lambda}(\vec{p} + \vec{q} + \vec{r})$, then find the value of λ.

29. Line L_1 is parallel to vector $\vec{\alpha} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ and passes through a point A(7, 6, 2) and line L_2 is parallel to a vector $\vec{\beta} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through a point B(5, 3, 4). Now a line L_3 parallel to a vector $\vec{r} = 2\hat{i} - 2\hat{j} - \hat{k}$ intersects the lines L_1 and L_2 at points C and D respectively, then find $|\overrightarrow{CD}|$

30. In a regular tetrahedron let θ be the angle between any edge and a face not containing the edge. If $\cos^2\theta = \frac{a}{b}$ where a, b $\in I^+$ also a and b are coprime, then find the value of $10a + b$

MOCK TEST

3-DIMENSIONAL

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. The line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the curve $xy = c^2$, $z = 0$ if c is equal to :

(A) ± 1 **(B)** $\pm \frac{1}{3}$ **(C)** $\pm \sqrt{5}$ **(D)** none of these

2. Equation of the straight line in the plane $\vec{r} \cdot \vec{n} = d$ which is parallel to $\vec{r} = \vec{a} + \lambda \vec{b}$ and passes through the foot of perpendicular drawn from the point $P(\vec{a})$ to the plane $\vec{r} \cdot \vec{n} = d$ is (where $\vec{n} \cdot \vec{b} = 0$)

(A) $\vec{r} = \vec{a} + \left(\frac{d - \vec{a}\cdot\vec{n}}{\vec{n}^2}\right)\vec{n} + \lambda\vec{b}$ **(B)** $\vec{r} = \vec{a} + \left(\frac{d - \vec{a}\cdot\vec{n}}{\vec{n}}\right)\vec{n} + \lambda\vec{b}$

(C) $\vec{r} = \vec{a} + \left(\frac{\vec{a}\cdot\vec{n} - d}{\vec{n}^2}\right)\vec{n} + \lambda\vec{b}$ **(D)** $\vec{r} = \vec{a} + \left(\frac{\vec{a}\cdot\vec{n} - d}{\vec{n}}\right)\vec{n} + \lambda\vec{b}$

3. The equation of motion of a point in space is x = 2t, y = – 4t, z = 4t where t measured in hours and the co-ordinates of moving point in kilometers, then the distance of the point from the starting point O (0, 0, 0) in 10 hours is :

(A) 20 km **(B)** 40 km **(C)** 60 km **(D)** 55 km

4. If $P_1 : \vec{r}.\vec{n}_1 - d_1 = 0$, $P_2 : \vec{r}.\vec{n}_2 - d_2 = 0$ and $P_3 : \vec{r}.\vec{n}_3 - d_3 = 0$ are three planes and $\vec{n}_1$, $\vec{n}_2$ and $\vec{n}_3$ are three non-coplanar vectors then, the three lines $P_1 = 0, P_2 = 0$; $P_2 = 0, P_3 = 0$ and $P_3 = 0, P_1 = 0$ are

(A) parallel lines **(B)** coplanar lines **(C)** coincident lines **(D)** concurrent lines

5. The direction cosines of a line equally inclined to three mutually perpendicular lines having D.C.'s as ℓ_1, m_1, n_1 ; ℓ_2, m_2, n_2 ; ℓ_3, m_3, n_3 are

(A) $\ell_1 + \ell_2 + \ell_3, m_1 + m_2 + m_3, n_1 + n_2 + n_3$ **(B)** $\frac{\ell_1 + \ell_2 + \ell_3}{\sqrt{3}}, \frac{m_1 + m_2 + m_3}{\sqrt{3}}, \frac{n_1 + n_2 + n_3}{\sqrt{3}}$

(C) $\frac{\ell_1 + \ell_2 + \ell_3}{3}, \frac{m_1 + m_2 + m_3}{3}, \frac{n_1 + n_2 + n_3}{3}$ **(D)** none of these

6. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three unit vectors equally inclined to each other at an angle α. Then the angle between $\vec{a}$ and plane of $\vec{b}$ and $\vec{c}$ is

(A) $\theta = \cos^{-1}\left(\frac{\cos\alpha}{\cos\frac{\alpha}{2}}\right)$ **(B)** $\theta = \sin^{-1}\left(\frac{\cos\alpha}{\cos\frac{\alpha}{2}}\right)$

(C) $\theta = \cos^{-1}\left(\frac{\sin\frac{\alpha}{2}}{\sin\alpha}\right)$ **(D)** $\theta = \sin^{-1}\left(\frac{\sin\frac{\alpha}{2}}{\sin\alpha}\right)$

7. The equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0) and (0, 0, 1) and having radius as small as possible is

(A) $3\sum x^2 - 2\sum x - 1 = 0$ **(B)** $\sum x^2 - \sum x - 1 = 0$

(C) $3\sum x^2 - 2\sum x + 1 = 0$ **(D)** $\sum x^2 - \sum x + 1 = 0$

8. Let A(1, 1, 1), B(2, 3, 5), C(–1, 0, 2) be three points, then equation of a plane parallel to the plane ABC which is at a distance 2 from origin, is

(A) $2x - 3y + z + 2\sqrt{14} = 0$ **(B)** $2x - 3y + z - \sqrt{14} = 0$

(C) $2x - 3y + z + 2 = 0$ **(D)** $2x - 3y + z - 2 = 0$

9. The square of the perpendicular distance of a point P (p, q, r) from a line through A(a, b, c) and whose direction cosines are ℓ, m, n is

(A) $\Sigma\{(q-b)n - (r-c)m\}^2$ **(B)** $\Sigma\{(q+b)n - (r+c)m\}^2$

(C) $\Sigma\{(q-b)n + (r-c)m\}^2$ **(D)** none of these

10. S_1 : Radius of great circle of section of a plane with a sphere is equal to the radius of the sphere.

S_2 : Points (x_1, y_1, z_1) and (x_2, y_2, z_2) lies on the same side of the plane $ax + by + cz + d = 0$ if $(ax_1 + by_1 + cz_1 + d)(ax_2 + by_2 + cz_2 + d) \geq 0$

S_3 : Point (x_1, y_1, z_1) lies inside of the sphere $x^2 + y^2 + z^2 - 2ux - 2vy + 2wz + d = 0$ If $x_1^2 + y_1^2 + z_1^2 < 2ux_1 + 2uy_1 - 2wz_1 - d$

S_4 : Shortest distance between the planes $3x + 6y - 2z - 11 = 0$ and $3x + 6y - 2z + 3 = 0$ is 2 units.

(A) TFTT **(B)** FFTT **(C)** TTFF **(D)** FTFT

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. The equation of the line $x + y + z - 1 = 0$, $4x + y - 2z + 2 = 0$ written in the symmetrical form is

(A) $\frac{x+1}{1} = \frac{y-2}{-2} = \frac{z-0}{1}$ **(B)** $\frac{x}{1} = \frac{y}{-2} = \frac{z-1}{1}$

(C) $\frac{x+1/2}{1} = \frac{y-1}{-2} = \frac{z-1/2}{1}$ **(D)** $\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z-2}{2}$

12. In a ΔABC, let M be the mid point of segment AB and let D be the foot of the bisector of $\angle$C. Then the value of $\frac{ar(\Delta\, CDM)}{ar(\Delta\, ABC)}$ is:

(A) $\frac{1}{4}\,\frac{a-b}{a+b}$ **(B)** $\frac{1}{2}\,\frac{a-b}{a+b}$

(C) $\frac{1}{2}\tan\frac{A-B}{2}\cot\frac{A+B}{2}$ **(D)** $\frac{1}{4}\cot\frac{A-B}{2}\tan\frac{A+B}{2}$

13. Consider the planes $3x - 6y + 2z + 5 = 0$ and $4x - 12y + 3z = 3$. The plane $67x - 162y + 47z + 44 = 0$ bisects that angle between the given planes which

(A) contains origin **(B)** is acute **(C)** is obtuse **(D)** none of these

14. S_1 : Plane containing the line $\frac{x-1}{2} = \frac{y+2}{5} = \frac{z-3}{7}$ and a straight line parallel to the line whose d.r. are $\langle 1, 2, 3 \rangle$ contains the point (1, 7, – 4)

S_2 : The point (4, 13, 5) lies on the line $\frac{x+2}{3} = \frac{y-3}{5} = \frac{1-z}{2}$

S_3 : Point (2, 1, 1) lies on a tangent plane to the sphere $x^2 + y^2 + z^2 - 2x - 4y - 2z + 2 = 0$

S_4 : Direction ratios of the line $x + y + z - 7 = 0$, $4x + y - 2z + 7 = 0$ are <1, –2, 1>

(A) TFTT (B) FFFT (C) TTFF (D) FTFT

15. The plane $\ell x + my = 0$ is rotated about its line of intersection with the plane z = 0, through an angle α, then equation of plane in its new position may be

(A) $\ell x + my + z\sqrt{\ell^2 + m^2}\ \tan\alpha = 0$ (B) $\ell x + my - z\sqrt{\ell^2 + m^2}\ \tan\alpha = 0$

(C) data is not sufficient (D) None of these

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement -I :** line $\frac{x-1}{3} = \frac{y-2}{11} = \frac{z+1}{11}$ lies in the plane $11x - 3z - 14 = 0$

Statement -II : A straight line lies in a plane if the line is parallel to the plane and a point of the line lies in the plane.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

17. **Statement -I :** Let A($\vec{i} + \vec{j} + \vec{k}$) and B($\vec{i} - \vec{j} + \vec{k}$) be two points, then point P $(2\vec{i} + 3\vec{j} + \vec{k})$ lies exterior to the sphere with AB as one of its diameters.

Statement -II : If A and B are any two points and P is a point in space such that $\vec{PA} \cdot \vec{PB} > 0$, then the point P lies exterior to the sphere with AB as one of its diameters.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

18. **Statement -I :** If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then equation $\vec{r} \times (2\hat{i} - \hat{j} + 3\hat{k}) = 3\hat{i} + \hat{k}$ represents a straight line.

Statement -II : If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then equation $\vec{r} \times (\hat{i} + 2\hat{j} - 3\hat{k}) = 2\hat{i} - \hat{j}$ represents a st. line

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

19. **Statement-I :** Let θ be the angle between the line $\frac{x-2}{2} = \frac{y-1}{-3} = \frac{z+2}{-2}$ and the plane $x+y-z=5$.

Then $\theta = \sin^{-1}\frac{1}{\sqrt{51}}$

Statement-II : Angle between a st. line and a plane is the complement of angle between the line and normal to the plane.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

20. **Statement - I :** A point on the straight line $2x + 3y - 4z = 5$ and $3x - 2y + 4z = 7$ can be determined by taking $x = k$ and then solving the two equations for y and z, where k is any real number.

Statement - II : If $c' \neq kc$, then the straight line $ax + by + cz + d = 0$, $kax + kby + c'z + d' = 0$, does not intersect the plane $z = \alpha$, where α is any real number.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21.

Column – I		Column – II	
	The lines		
(A)	$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-5}{5}$ are	**(p)**	coincident
(B)	$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-3}{2} = \frac{y-5}{3} = \frac{z-7}{4}$ are	**(q)**	Parallel and different
(C)	$\frac{x-2}{5} = \frac{y+3}{4} = \frac{5-z}{2}$ and $\frac{x-7}{5} = \frac{y-1}{4} = \frac{z-2}{-2}$ are	**(r)**	skew
(D)	$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z-4}{5}$ and $\frac{x-3}{3} = \frac{y-2}{2} = \frac{z-7}{5}$ are	**(s)**	Intersecting in a point
		(t)	coplanar

22.

Column – I		Column – II	
(A)	Foot of perpendicular drawn from point (1, 2, 3) to the line $\frac{x-2}{2} = \frac{y-1}{3} = \frac{z-2}{4}$ is	**(p)**	$\left(\frac{107}{29}, \frac{30}{29}, \frac{69}{29}\right)$
(B)	Image of line point (1, 2, 3) in the line $\frac{x-2}{2} = \frac{y-1}{3} = \frac{z-2}{4}$ is	**(q)**	$\left(\frac{88}{29}, \frac{125}{29}, \frac{69}{29}\right)$

(C)	Foot of perpendicular from the point (2, 3, 5) to the plane $2x + 3y - 4z + 17 = 0$ is	**(r)**	$\left(\frac{107}{29}, \frac{125}{29}, \frac{185}{29}\right)$
(D)	Image of the point (2, 5, 1) in the plane	**(s)**	$\left(\frac{68}{29}, \frac{44}{29}, \frac{78}{29}\right)$
	$3x - 2y + 4z - 5 = 0$ is	**(t)**	$\left(\frac{38}{29}, \frac{57}{29}, \frac{185}{29}\right)$

SECTION - V : COMPREHENSION TYPE

23. **Read the following comprehension carefully and answer the questions.**

Consider two lines $L_1 : \frac{x-2}{1} = \frac{y-1}{0} = \frac{z+1}{2}$ and $L_2 : \frac{x-3}{1} = \frac{y-1}{1} = \frac{z-0}{-1}$. Let P be the plane which contains the line L_1 and is parallel to L_2 and intersecting coordinates axes at A,B,C respectively -

1. The shortest distance between the lines L_1 and L_2 is -

(A) $\frac{1}{\sqrt{5}}$ **(B)** $\frac{1}{\sqrt{6}}$ **(C)** $\frac{1}{\sqrt{8}}$ **(D)** $\frac{1}{\sqrt{14}}$

2. Image of origin in the plane P , is -

(A) $\left(\frac{2}{7}, \frac{-3}{7}, \frac{-1}{7}\right)$ **(B)** $\left(\frac{4}{7}, \frac{-6}{7}, \frac{-2}{7}\right)$ **(C)** $\left(\frac{-2}{7}, \frac{3}{7}, \frac{1}{7}\right)$ **(D)** $\left(\frac{-4}{7}, \frac{6}{7}, \frac{2}{7}\right)$

3. Volume of the tetrahedron OABC (where 'O' is origin) is -

(A) $\frac{2}{3}$ **(B)** $\frac{4}{9}$ **(C)** $\frac{2}{9}$ **(D)** $\frac{4}{3}$

24. **Read the following comprehension carefully and answer the questions.**

The vertices of ΔABC are A(2, 0, 0), B(0, 1, 0), C(0, 0, 2). Its orthocentre is H and circumcentre is S. P is a point equidistant from A, B, C and the origin O.

1. The area of ΔABC is

(A) $\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** $\sqrt{6}$ **(D)** 3

2. The z-coordinate of H is

(A) 1 **(B)** $\frac{1}{2}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{3}$

3. The y-coordinate of S is

(A) $\frac{5}{6}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{2}$

4. PA =

(A) 1 **(B)** $\sqrt{2}$ **(C)** $\sqrt{\frac{3}{2}}$ **(D)** $\frac{3}{2}$

25. **Read the following comprehension carefully and answer the questions.**

Suppose direction cosines of two lines are given by $u\ell + vm + wn = 0$ and $a\ell^2 + bm^2 + cn^2 = 0$, where u, v, w, a, b, c are arbitrary constants and ℓ, m, n are direction cosines of the lines.

1. For $u = v = w = 1$, both lines satisfies the relation -

(A) $(b+c)\left(\frac{n}{\ell}\right)^2 + 2b\frac{n}{\ell} + (a+b) = 0$ **(B)** $(c+a)\left(\frac{\ell}{m}\right)^2 + 2c\frac{\ell}{m} + (b+c) = 0$

(C) $(a+b)\left(\frac{m}{n}\right)^2 + 2a\frac{m}{n} + (c+a) = 0$ **(D)** All of the above

2. For $u = v = w = 1$, If $\frac{n_1n_2}{\ell_1\ell_2} = \frac{a+b}{b+c}$, then

(A) $\frac{m_1m_2}{\ell_1\ell_2} = \frac{b+c}{c+a}$ **(B)** $\frac{m_1m_2}{\ell_1\ell_2} = \frac{c+a}{b+c}$ **(C)** $\frac{m_1m_2}{\ell_1\ell_2} = \frac{a+b}{c+a}$ **(D)** $\frac{m_1m_2}{\ell_1\ell_2} = \frac{c+a}{a+b}$

3. For $u = v = w = 1$ and if lines are perpendicular, then -

(A) $a + b + c = 0$ **(B)** $ab + bc + ca = 0$ **(C)** $ab + bc + ca = 3abc$ **(D)** $ab + bc + ca = abc$

SECTION - VI : INTEGER TYPE

26. Let OABC is a regular tetrahedron and P is any point in space. If edge length of tetrahedron is 1 unit, find the least value of $PA^2 + PB^2 + PC^2 + PO^2$.

27. If equation of the plane through the straight line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z}{5}$ and perpendicular to the plane $x - y + z + 2 = 0$ is $ax - by + cz + 4 = 0$, then find the value of $a + b + c$

28. Find the acute angle between the lines $\frac{x-1}{\ell} = \frac{y+1}{m} = \frac{z}{n}$ and $\frac{x+1}{m} = \frac{y-3}{n} = \frac{z-1}{\ell}$ where $\ell > m > n$, and ℓ, m, n are the roots of the cubic equation $x^3 + x^2 - 4x = 4$.

29. The shortest distance between the lines given by $\vec{r} = 3\vec{i} + 8\vec{j} + 3\vec{k} + \lambda(3\vec{i} - \vec{j} + \vec{k})$ and $\vec{r} = -3\vec{i} - 7\vec{j} + 6\vec{k} + \mu(-3\vec{i} + 2\vec{j} + 4\vec{k})$ is $\lambda\sqrt{30}$, then find the value of λ.

30. Find the distance of the point P (3, 8, 2) from the line $\frac{1}{2}(x-1) = \frac{1}{4}(y-3) = \frac{1}{3}(z-2)$ measured parallel to the plane $3x + 2y - 2z + 15 = 0$.

ANSWER KEY

EXERCISE - 1

1. B	**2.** D	**3.** C	**4.** D	**5.** C	**6.** D	**7.** C	**8.** A	**9.** B	**10.** C	**11.** C	**12.** B	**13.** C
14. C	**15.** D	**16.** D	**17.** C	**18.** C	**19.** A	**20.** A	**21.** A	**22.** C	**23.** C	**24.** A	**25.** C	**26.** B
27. A	**28.** B	**29.** B	**30.** B	**31.** C	**32.** C	**33.** A	**34.** A	**35.** A	**36.** A	**37.** D	**38.** C	**39.** D
40. B	**41.** D	**42.** A	**43.** C	**44.** D	**45.** A	**46.** B	**47.** C	**48.** B	**49.** B	**50.** B	**51.** C	**52.** A
53. D	**54.** B	**55.** D	**56.** A	**57.** C	**58.** A	**59.** A	**60.** A	**61.** A	**62.** D	**63.** B	**64.** A	**65.** C
66. B	**67.** D	**68.** C	**69.** A	**70.** C								

EXERCISE - 2 : PART # I

1. AB	**2.** AB	**3.** ACD	**4.** AC	**5.** BD	**6.** ABC	**7.** ABD	**8.** BC	**9.** AD
10. AC	**11.** BD	**12.** BD	**13.** BC	**14.** AB	**15.** AB	**16.** AB	**17.** BC	**18.** AD
19. ABCD	**20.** ABCD	**21.** ABC	**22.** BCD	**23.** BC	**24.** AC	**25.** BC	**26.** CD	**27.** BC
28. BD	**29.** AC	**30.** AC	**31.** AC	**32.** ACD	**33.** AC	**34.** BC	**35.** ACD	**36.** AD
37. AC	**38.** BCD	**39.** CD	**40.** AD	**41.** ACD	**42.** BD	**43.** BC	**44.** ABCD	**45.** ABC
46. BCD	**47.** BD	**48.** AB	**49.** BC	**50.** ABCD				

PART - II

1. A	**2.** C	**3.** C	**4.** A	**5.** A	**6.** C	**7.** A	**8.** D	**9.** A	**10.** C	**11.** D	**12.** D	**13.** B
14. B	**15.** A											

EXERCISE - 3 : PART # I

1. $A \to p$ $B \to p$ $C \to r$ $D \to q$

2. $A \to r$ $B \to s$ $C \to p$ $D \to q$

3. $A \to r$ $B \to q$ $C \to q$ $D \to s$

4. $A \to r$ $B \to q$ $C \to s$ $D \to p$

5. $A \to r$ $B \to p$ $C \to r$ $D \to s$

6. $A \to p$ $B \to q$ $C \to r$ $D \to s$

7. $A \to r$ $B \to q$ $C \to q,s$ $D \to p,s$

8. $A \to s$ $B \to r$ $C \to p$ $D \to p$

PART - II

Comprehension # 1 : **1.** B **2.** C **3.** A

Comprehension # 2 : **1.** B **2.** C **3.** A

Comprehension # 3 : **1.** B **2.** C **3.** A **4.** D

Comprehension # 4 : **1.** A **2.** B **3.** A

Comprehension # 5 : **1.** A **2.** B **3.** A

Comprehension # 6 : **1.** B **2.** C **3.** D

Comprehension # 7 : **1.** C **2.** B **3.** A

Comprehension # 8 : **1.** A **2.** A **3.** D

Comprehension # 9 : **1.** B **2.** C **3.** A

EXERCISE - 5 : PART # I

1. 4 **2.** 4 **3.** 2 **4.** 2 **5.** 3 **6.** 1 **7.** 4 **8.** 4 **9.** 3 **10.** 4 **11.** 3 **12.** 4 **13.** 1
14. 3 **15.** 3 **16.** 4 **17.** 1 **18.** 3 **19.** 1 **20.** 2 **21.** 4 **22.** 3 **23.** 3 **24.** 4 **25.** 4 **26.** 4
27. 4 **28.** 4 **29.** 3 **30.** 1 **31.** 1 **32.** 2 **33.** 3 **34.** 1 **35.** 4 **36.** 4 **37.** 3 **38.** 3 **39.** 4
40. 3 **41.** 3 **42.** 4 **43.** 4 **44.** 4 **45.** 2 **46.** 1 **47.** 3 **48.** 3 **49.** 2 **50.** 1 **51.** 4 **52.** 1
53. 2 **54.** 3 **55.** 2 **56.** 2 **57.** 4 **58.** 4 **59.** 1 **60.** 4 **61.** 1 **62.** 3 **63.** 1 **64.** 4 **65.** 1
66. 3 **67.** 2 **68.** 3 **69.** 4 **70.** 4 **71.** 2 **72.** 2 **73.** 4 **74.** 3 **75.** 3 **76.** 1 **77.** 4 **78.** 1
79. 3 **80.** 1 **81.** 2 **82.** 3 **83.** 3 **84.** 1

PART - II

1. **A.** B **B.** A **C.** A **3.** **A.** B **B.** C **5.** $\vec{v}_1 = 2\hat{i}$, $\vec{v}_2 = -\hat{i} \pm \hat{j}$, $\vec{v}_3 = 3\hat{i} \pm 2\hat{j} \pm 4\hat{k}$
12. **A.** B **B.** C **9.** D **11.** **A.** C **B.** A **13.** B **14.** $\hat{w} = \hat{v} - 2(\hat{a}.\hat{v})\hat{a}$
15. **A.** A **B.** B,D **16.** C **17.** B **18.** C **19.** A **20.** A **21.** C
22. **A** $\rightarrow$ q, s **B** $\rightarrow$ p, r, s, t **C** $\rightarrow$ T, **D** $\rightarrow$ R **23.** A **24.** 5 **25.** B **26.** **A.** C **B.** A, D **C.** 9
27. **A.** 3 **B.** C **28.** C **29.** 5 **30.** C **31.** **A.** $x + y - 2z = 3$ **B.** 6, 5, –2 **32.** B
34. $\frac{9}{2}$ cubic unit **36.** D **37.** $2x - y + z - 3 = 0, 62x + 29y + 19z - 105 = 0$ **38.** D
39. **A** $\rightarrow$ s **B** $\rightarrow$ p **C** $\rightarrow$ q, r **D** $\rightarrow$ s **40.** **A** $\rightarrow$ q **B** $\rightarrow$ s **C** $\rightarrow$ r
41. **A.** $\rightarrow$ r **B** $\rightarrow$ q **C** $\rightarrow$ p **D** $\rightarrow$ s **42.** D **43.** D **44.** B **45.** D **46.** C **47.** C
48. A **49.** **A.** $\rightarrow$ P **B** $\rightarrow$ Q, S **C** $\rightarrow$ Q, R, S, T **D** $\rightarrow$ R **50.** C **51.** A **52.** 6
53. **A** $\rightarrow$ t **B** $\rightarrow$ p,r **C** $\rightarrow$ q,s, **D** $\rightarrow$ r **54.** **A.** A **B.** A **C.** B,C **55.** D **56.** B, D
57. A,D **58.** A **59.** C **60.** ABC **61.** 4 **62.** A **63.** ACD **64.** BD **65.** AB
66. **A** $\rightarrow$ p, q **B** $\rightarrow$ p, q **C** $\rightarrow$ p, q, s, t **D** $\rightarrow$ q, t
67. **A** $\rightarrow$ p, r, s **B** $\rightarrow$ p **C** $\rightarrow$ p, q **D** $\rightarrow$ s, t **68.** 9 **69.** BCD **70.** A **71.** B, C

MOCK TEST

VECTOR

1. C **2.** A **3.** A **4.** C **5.** D **6.** D **7.** D **8.** B **9.** A **10.** B **11.** A, C, D **12.** A, B, D
13. A, B, C, D **14.** A, C, D **15.** B, D **16.** B **17.** D **18.** B **19.** A **20.** D
21. A $\rightarrow$ t B $\rightarrow$ p C $\rightarrow$ q D $\rightarrow$ s **22.** A $\rightarrow$ s B $\rightarrow$ p C $\rightarrow$ r D $\rightarrow$ t
23. **1.** B **2.** A **3.** C **24.** **1.** C **2.** B **3.** C **25.** **1.** A **2.** D **3.** A **4.** C **5.** B **26.** 50
27. 27 **28.** 2 **29.** 9 **30.** 13

3-DIMENSIONAL

1. C **2.** A **3.** C **4.** D **5.** B **6.** A **7.** A **8.** A **9.** A **10.** A **11.** A, B, C **12.** B, C

13. A, B **14.** B **15.** A, B **16.** A **17.** A **18.** D **19.** A **20.** B

21. $A \rightarrow s,t$ $B \rightarrow p,t$ $C \rightarrow q$ $D \rightarrow r$ **22.** $A \rightarrow s$ $B \rightarrow p$ $C \rightarrow t$ $D \rightarrow q$

23. **1.** D **2.** B **3.** C **24.** **1.** C **2.** D **3.** C **4.** D **25.** **1.** D **2.** B **3.** A **26.** $\frac{3}{4}$

27. 0 **28.** $\cos^{-1}\frac{4}{9}$ **29.** 3 **30.** 7