HINTS & SOLUTIONS

EXERCISE - 1

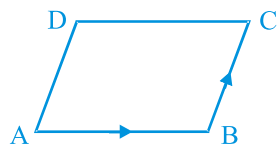
Single Choice

6. $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= 2\hat{i} - 2\hat{j} + 4\hat{k}$

$\overrightarrow{BD} = -\overrightarrow{AB} + \overrightarrow{BC}$

$= -4\hat{i} + 2\hat{j}$



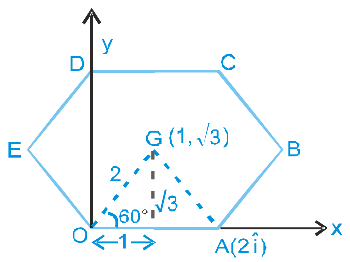
Let Angle between $\overrightarrow{AC}$ & $\overrightarrow{BD}$ is θ

$\therefore \quad \dfrac{\overrightarrow{AC}.\overrightarrow{BD}}{|\overrightarrow{AC}||\overrightarrow{BD}|} = \cos\theta$

$\Rightarrow \quad \cos\theta = \dfrac{-12}{4\sqrt{6}\sqrt{5}} = -\sqrt{\dfrac{3}{10}}$.

$\Rightarrow$ Acute angle between diagonals $= \cos^{-1}\sqrt{\dfrac{3}{10}}$

13.



$G \equiv (\hat{i} + \sqrt{3}\hat{j})$

Let Position vector of P is $\vec{p}$

$\because \quad \overrightarrow{GP} \,||\, \hat{k}$

then $\quad \vec{p} - (\hat{i} + \sqrt{3}\hat{j}) = \lambda\hat{k}$

$\Rightarrow \quad \vec{p} = \hat{i} + \sqrt{3}\,\hat{j} + \lambda\hat{k}$

also $\left|\overrightarrow{OP}\right| = 3$

$\Rightarrow \quad \sqrt{1+3+\lambda^2} = 3$

$\Rightarrow \quad \lambda^2 = 5$

$\Rightarrow \quad \lambda = \pm\sqrt{5} \qquad \Rightarrow \quad \vec{p} = \hat{i} + \sqrt{3}\,\hat{j} \pm \sqrt{5}\,\hat{k}$

For positive Z-axis $\vec{p} = \hat{i} + \sqrt{3}\hat{j} + \sqrt{5}\,\hat{k}$

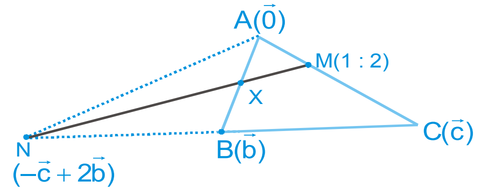
So $\quad \overrightarrow{AP} = \vec{p} - 2\hat{i} = -\hat{i} + \sqrt{3}\hat{j} + \sqrt{5}\,\hat{k}$

17. Position vector of M $\equiv \dfrac{\vec{c}}{3}$

Position vector of N $\equiv (-\vec{c} + 2\vec{b})$

$\therefore$ equation of line BC is $\vec{r} = \vec{b} + \lambda(\vec{b} - \vec{c})$

$\therefore$ equation of line AB is is $\vec{r} = \vec{0} + \mu\vec{b}$



$\therefore$ equation of line MN is $\vec{r} = \dfrac{\vec{c}}{3} + t\left(\dfrac{4\vec{c}}{3} - 2\right)$

$\Rightarrow \quad \mu = -2t, \quad 0 = \dfrac{1}{3} + \dfrac{4}{3}t$

which gives $\mu = \dfrac{1}{2} \quad \Rightarrow$ Position vector of X is $\dfrac{\vec{b}}{2}$.

18. $\vec{a} = \hat{i} + \hat{j}$ & $\vec{b} = 2\hat{i} - \hat{k}$

$\vec{r} \times \vec{a} = \vec{b} \times \vec{a} \quad \Rightarrow \quad (\vec{r} - \vec{b}) \times \vec{a} = 0$

$\Rightarrow \quad \vec{r} = \vec{b} + \lambda\vec{a}$ .....(i)

similarly $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$

$\Rightarrow \quad \vec{r} = \vec{a} + \mu\vec{b}$ .....(ii)

Putting the vector $\vec{a}$ & $\vec{b}$ in (i) & (ii) and equating

we get $2\hat{i} - \hat{k} + \lambda(\hat{i} + \hat{j}) = \hat{i} + \hat{j} + \mu(2\hat{i} - \hat{k})$

$\Rightarrow \quad 2 + \lambda = 1 + 2\mu, \ \lambda = 1, \mu = 1$

$\therefore$ Point of intersection is $3\hat{i} + \hat{j} - \hat{k}$.

20. Equation of plane containing L_1 and parallel to

L_2 is $\begin{vmatrix} x-2 & y-1 & z+1 \\ 1 & 0 & 2 \\ 1 & 1 & -1 \end{vmatrix} = 0$

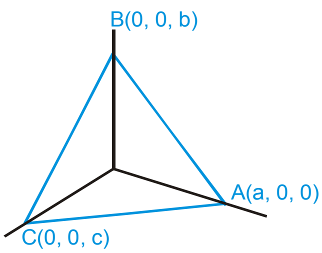
$\Rightarrow \quad 2x - 3y - z = 2$

distance from origin $= \dfrac{2}{\sqrt{14}} = \sqrt{\dfrac{2}{7}}$

21. Let the equation of plane be

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

as (α, β, γ) is centroid



22. L. H. S $= \left(\lambda(\vec{a}+\vec{b})\times\lambda^2\vec{b}\right).\lambda\vec{c}$

$= \lambda^4\left((\vec{a}+\vec{b})\times\vec{b}\right).\vec{c} = \lambda^4[a\ b\ c]$

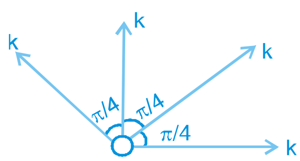
R.H.S. $= (\vec{a}\times(\vec{b}+\vec{c})).\vec{b} = [\vec{a}\ \vec{c}\ \vec{b}]$

$\Rightarrow$ $\lambda^4[a\ b\ c] = -[a\ b\ c]$

$\Rightarrow$ $\lambda^4 = -1$ which is not possible.

23. These forces can be written in terms of vector as

$$k\hat{i},\ \frac{k}{\sqrt{2}}\hat{i}+\frac{k}{\sqrt{2}}\hat{j},\ k\hat{j} \quad \text{and} \quad -\frac{k}{\sqrt{2}}\hat{i}+\frac{k}{\sqrt{2}}\hat{j}$$



Resultant $= k\hat{i}+(k+\sqrt{2}k)\hat{j}$

magnitude $= \sqrt{k^2+(k+\sqrt{2}k)^2} = k\sqrt{4+2\sqrt{2}}$

24. Equation of plane is $\vec{r}.\hat{n} = \dfrac{q}{|\vec{n}|}$

for intercept on x-axis take dot product with $\hat{i}$

$\Rightarrow$ intercept on x-axis $= \dfrac{q}{\hat{i}.\vec{n}}$

25. $\vec{c}.\vec{a} = \vec{a}.(\vec{a}\times\vec{b}) \quad \Rightarrow \quad \vec{c}.\vec{a} = 0 = \vec{c}.\vec{b} \quad = \vec{a}.\vec{b}$

Also $\quad |\vec{a}\times\vec{b}| = |\vec{c}|$

$|\vec{a}||\vec{b}|\sin 90° = |\vec{c}|$

$|\vec{a}|^2 = |\vec{a}| \quad \Rightarrow \quad |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$|3\vec{a}+4\vec{b}+12\vec{c}| = \sqrt{9a^2+16b^2+144c^2} = 13$

$\{\because |\vec{a}|=|\vec{b}|=|\vec{c}|=1\}$

28. From P(f, g, h) the foot of perpendicular on plane yz = (0, g, h),

similarly from P(f, g, h) perpendicular to z x = (f,0,h)

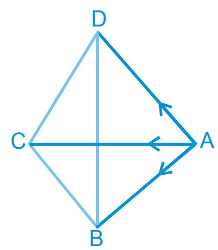
Equation of plane is

$$\begin{vmatrix} x & y & z \\ f & 0 & h \\ 0 & g & h \end{vmatrix} = 0 \quad \Rightarrow \quad \frac{x}{f}+\frac{y}{g}-\frac{z}{h}=0$$

30. $\overline{AD} = -2\hat{i}+2\hat{j}-\hat{k}$

$\overline{AC} = \hat{i}+2\hat{j}+2\hat{k}$

$\overline{AB} = 3\hat{j}+4\hat{k}$



$$\vec{n}_1 = \overline{AD}\times\overline{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 2 & -1 \\ 1 & 2 & 2 \end{vmatrix} = 6\hat{i}+3\hat{j}-6\hat{k}$$

$$= 3\left(2\hat{i}+\hat{j}-2\hat{k}\right)$$

$$\vec{n}_2 = \overline{AC}\times\overline{AB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 0 & 3 & 4 \end{vmatrix} = 2\hat{i}-4\hat{j}+3\hat{k}$$

$$|\vec{n}_1\times\vec{n}_2| = 3\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 2 & -4 & 3 \end{vmatrix} = 3(-5\hat{i}-10\hat{j}-10\hat{k})$$

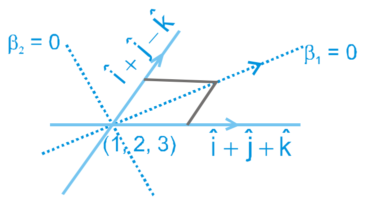
$$\sin\theta = \frac{5}{\sqrt{29}} \qquad \left(\sin\theta = \frac{|\vec{n}_1\times\vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}\right)$$

33. Dr's of bisector

$$\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}+\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}=\lambda(\hat{i}+\hat{j})$$

Hence Dr's are $\lambda, \lambda, 0\ (\lambda \in R)$

Equation of bisector

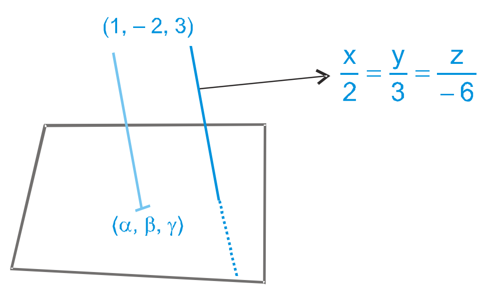


$$\frac{x-1}{\lambda}=\frac{y-2}{\lambda}=\frac{z-3}{0}$$

$$\frac{x-1}{2}=\frac{y-2}{2};\ z-3=0$$

34. $\alpha-1=2\lambda \quad \Rightarrow \quad \alpha=2\lambda+1$

$\beta+2=3\lambda \quad \Rightarrow \quad \beta=3\lambda-2$



$\gamma-3=-6\lambda \quad \Rightarrow \quad \gamma=-6\lambda+3$

$2\lambda+1-3\lambda+2-6\lambda+3=5$

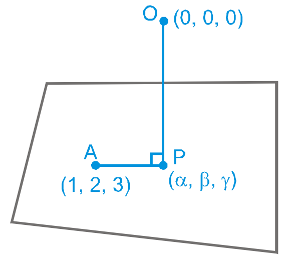
$7\lambda=1 \quad \Rightarrow \quad \lambda=1/7$

$\therefore$ Point on the plane is $\left(\frac{9}{7},-\frac{11}{7},\frac{15}{7}\right)$

$$\text{Distance}=\sqrt{(\alpha-1)^2+(\beta+2)^2+(\gamma-3)^2}$$

$$=\lambda\sqrt{4+9+36}=\frac{1}{7}.7=1$$

36. $OP \perp AP$



$\alpha(\alpha-1)+\beta(\beta-2)+\gamma(\gamma-3)=0$

$\therefore$ Locus of $P(\alpha, \beta, \gamma)$ is

$x^2+y^2+z^2-x-2y-3z=0$

51. $a(x-2)+b(y-3)+6(z-1)=0 \qquad$ .....**(i)**

$2a-2b-3c=0$

$4a+0.b+6c=0$

$$\frac{a}{-12-0}=\frac{b}{-12-12}=\frac{c}{0+8}$$

$$\frac{a}{3}=\frac{b}{6}=\frac{c}{-2}=\lambda \text{ (let)}$$

Put these values of a, b, c in **(i)**

$3(x-2)+6(y-3)-2(z-1)=0$

$3x+6y-2z-22=0$

$$d=\left|\frac{-15-24-16-22}{\sqrt{9+36+4}}\right|=\left|\frac{77}{7}\right|=11$$

54. Let the tetrahedron cut x-axis, y-axis and z-axis at a, b & c respectively.

$$\text{volume}=\frac{1}{6}[a\hat{i}\ b\hat{j}\ c\hat{k}] \quad \text{(Given)}$$

Then $\frac{1}{6}(abc)=64K^3 \qquad$ .....**(i)**

Let centroid be (x_1, y_1, z_1)

$$\therefore x_1=\frac{a}{4},\ y_1=\frac{b}{4},\ z_1=\frac{c}{4}$$

put in **(i)** wet get

$x_1y_1z_1=6K^3$

$\therefore$ Locus is $xyz=6K^3$

The required locus is $xyz=6K^3$

58. $\vec{r}.\vec{n} = d$ ....(i)

$\vec{r} = \vec{r}_0 + t\vec{n}$ ....(ii)

from (i) and (ii)

$(\vec{r}_0 + t\vec{n}).\vec{n} = d \Rightarrow t = \dfrac{d - \vec{r}_0.\vec{n}}{\vec{n}^2}$

substitute the value of 't' in (ii)

$r = \vec{r}_0 + \left(\dfrac{d - \vec{r}_0.\vec{n}}{\vec{n}^2}\right)\vec{n}$

59. $\vec{a} \times \vec{b} = 2(\vec{a} \times \vec{b})$

$\vec{a} \times (\vec{b} - 2\vec{c}) = 0 \Rightarrow \vec{b} - 2\vec{c} = \alpha\vec{a}$

squaring $b^2 + 4c^2 - 4\vec{b}\cdot\vec{c} = \alpha^2 a^2$

$16 + 4 - 4.4.1.\dfrac{1}{4} = \alpha^2 \Rightarrow \alpha = \pm 4$

$\vec{b} = 2\vec{c} \pm 4\vec{a}$

$|\ell| + |\mu| = 6$

60. $(a - b)\vec{x} + (b - c)\vec{y} + (c - a)(\vec{x} \times \vec{y}) = 0$

As $\vec{x}, \vec{y}$ & $(\vec{x} \times \vec{y})$ are non zero, non coplanar vectors, then

$a - b = b - c = c - a = 0$

$\Rightarrow a = b = c$

Hence ΔABC is an equilateral triangle.

Hence, acute angled triangle.

63. $\vec{c}$ is along the vector $\vec{a} \times (\vec{a} \times \vec{b})$

$= (\vec{a}.\vec{b})\vec{a} - (\vec{a}.\vec{a})\vec{b}$

$= (-1)(i + j - k) - 3(i - j + k) = -4i + 2j - 2k$

$\vec{c} = \dfrac{-2i + j - k}{\sqrt{6}}$

$\vec{d} = \dfrac{(\vec{a} \times \vec{c})}{|\vec{a} \times \vec{c}|}$;

$\vec{a} \times \vec{c} = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ -2 & 1 & -1 \end{vmatrix} = -j(-3) + k.3 = 3(j + k)$

$\vec{d} = \dfrac{j + k}{\sqrt{2}}$

65. Equation of plane containing

$\dfrac{x+1}{-3} = \dfrac{y-3}{2} = \dfrac{z+2}{1}$ and point (0, 7, – 7) is

$\begin{vmatrix} x+1 & y-3 & z+2 \\ -3 & 2 & 1 \\ -1 & -4 & 5 \end{vmatrix} = 0$

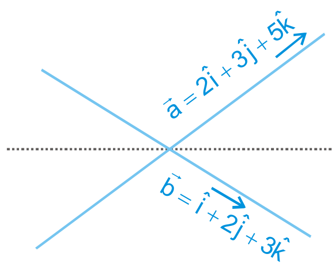
By solving we get

$x + y + z = 0$

68. $\dfrac{x}{2} = \dfrac{y}{3} = \dfrac{z}{5}$ .....(i)

$\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3}$ .....(ii)

$\hat{a} + \hat{b} = \dfrac{2\hat{i} + 3\hat{j} + 5\hat{k}}{\sqrt{38}} + \dfrac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}}$



$\Rightarrow$ (A) and (B) will be incorrect

Let the dr's of line $\perp$ to (1) and (2) be a, b, c

$\Rightarrow 2a + 3b + 5c = 0$ ... (iii)

and $a + 2b + 3c = 0$ ... (iv)

$\therefore \dfrac{a}{9-10} = \dfrac{b}{5-6} = \dfrac{c}{4-3}$

$\Rightarrow \dfrac{a}{-1} = \dfrac{b}{-1} = \dfrac{c}{1} \Rightarrow \dfrac{a}{1} = \dfrac{b}{1} = \dfrac{c}{-1}$

$\therefore$ equation of line passing through (0, 0, 0) and is $\perp$r to the lines (i) and (ii) is

$\dfrac{x}{1} = \dfrac{y}{1} = \dfrac{z}{-1}$

70. $[\vec{a}\ \vec{b}\ \vec{c}]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$

$= \begin{vmatrix} 1 & \cos\theta & \cos\theta \\ \cos\theta & 1 & \cos\theta \\ \cos\theta & \cos\theta & 1 \end{vmatrix}$

EXERCISE - 2

Part # I : Multiple Choice

5. $\vec{a}, \vec{b}, \vec{c}$ are unit vector mutually perpendicular to each other then angle between $\vec{a}+\vec{b}+\vec{c}$ & $\vec{a}$ is given by

$$\cos\theta = \frac{(\vec{a}+\vec{b}+\vec{c}).\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|} = \frac{(\vec{a}+\vec{b}+\vec{c}).\vec{a}}{\sqrt{\vec{a}^2+\vec{b}^2+\vec{c}^2}\,|\vec{a}|}$$

$$= \frac{|\vec{a}|}{\sqrt{\vec{a}^2+\vec{b}^2+\vec{c}^2}}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}} \quad \text{or} \quad \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = \tan^{-1}\sqrt{2}$$

7. $\vec{r} = 2\hat{i} - \hat{j} + 3\hat{k} + \lambda(\hat{i} + \hat{j} + \sqrt{2}\hat{k})$

$$\cos\alpha = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 30°, \quad \cos\beta = \frac{\sqrt{3}}{2} \Rightarrow \beta = 30°,$$

$$\cos\gamma = \frac{\sqrt{2}}{2} \Rightarrow \gamma = 45°$$

By putting the values check options

8. $\vec{r}_1 = \vec{a} - \vec{b} + \vec{c}$ .....(i)

$\vec{r}_2 = \vec{b} + \vec{c} - \vec{a}$ .....(ii)

$\vec{r}_3 = \vec{c} + \vec{a} + \vec{b}$ .....(iii)

$\vec{r} = 2\vec{a} - 3\vec{b} + 4\vec{c}$

If $\vec{r} = \lambda_1\vec{r}_1 + \lambda_2\vec{r}_2 + \lambda_3\vec{r}_3$

then $2\vec{a} - 3\vec{b} + 4\vec{c}$

$= (\lambda_1 - \lambda_2 + \lambda_3)\,\vec{a} + (\lambda_2 - \lambda_1 + \lambda_3)\,\vec{b} + (\lambda_1 + \lambda_2 + \lambda_3)\,\vec{c}$

$\Rightarrow \lambda_1 + \lambda_3 - \lambda_2 = 2$ .....(iv)

$\lambda_2 + \lambda_3 - \lambda_1 = -3$ .....(v)

$\lambda_1 + \lambda_2 + \lambda_3 = 4$ .....(vi)

Solving (iv) (v) & (vi) we get

$\lambda_2 = 1\,;\ \lambda_1 = 7/2\,;\ \lambda_3 = -1/2$

Now check options

9. $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ & $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$

$\Rightarrow \vec{a}\times\vec{b} - \vec{a}\times\vec{c} = \vec{c}\times\vec{d} - \vec{b}\times\vec{d}$

$\Rightarrow \vec{a}\times(\vec{b}-\vec{c}) = (\vec{c}-\vec{b})\times\vec{d}$

$\Rightarrow (\vec{a}-\vec{d})\times(\vec{b}-\vec{c}) = 0$

11. $(\vec{a}\times\vec{b})\times\vec{c} = \vec{a}\times(\vec{b}\times\vec{c})$

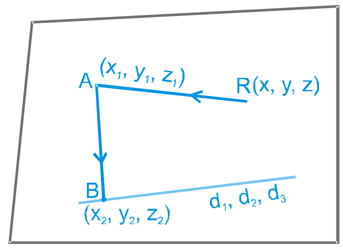
$(\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a} = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

But $\vec{b}.\vec{c} \neq 0$, $\vec{a}.\vec{b} \neq 0$

$\Rightarrow$ $\vec{a}$ & $\vec{c}$ must be parallel.

14. Vectors $\overrightarrow{AR}, \overrightarrow{AB}$ & $\vec{C}$ are coplanar

Equation of the required plane



$\vec{C} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$$

or $$\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$$

16 . Let vector is $\vec{\upsilon} = \lambda_1\hat{a} + \lambda_2\hat{b} + \lambda_3(\hat{a}\times\hat{b})$ also

$$\cos\theta = \frac{\vec{\upsilon}.\hat{a}}{|\vec{\upsilon}||\hat{a}|} = \frac{\vec{\upsilon}.\hat{b}}{|\vec{\upsilon}||\hat{b}|} = \frac{\vec{\upsilon}(\hat{a}\times\hat{b})}{|\vec{\upsilon}||\hat{a}\times\hat{b}|}$$

$\Rightarrow \vec{\upsilon}.\hat{a} = \vec{\upsilon}.\hat{b} = \vec{\upsilon}.(\hat{a}\times\hat{b})$

$[\ |\hat{a}\times\hat{b}| = |\hat{a}|\ |\hat{b}|\ \sin 90° = 1]$

$\Rightarrow \lambda_1 = \lambda_2 = \lambda_3 = \lambda$ (let)

$\therefore\ \vec{\upsilon} = \lambda(\hat{a} + \hat{b} + \hat{a}\times\hat{b})$

$$7|\vec{\upsilon}| = \left|\lambda\sqrt{\hat{a}^2 + \hat{b}^2 + (\hat{a}\times\hat{b})^2 + 2\hat{a}.\hat{b} + 2\hat{b}.(\hat{a}\times\hat{b}) + 2(\hat{a}\times\hat{b}).\hat{a}}\right| = 1$$

$\Rightarrow \left|\lambda\sqrt{1+1+1}\right| = 1 \qquad \Rightarrow \lambda = \pm\frac{1}{\sqrt{3}}$

$\therefore\ \vec{\upsilon} = \pm\frac{1}{\sqrt{3}}(\hat{a} + \hat{b} + \hat{a}\times\hat{b})$

17. Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

it makes equal angle with

$\frac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k}),\ \frac{1}{5}(-4\hat{i} - 3\hat{k}), \hat{j}$ then

$\frac{x - 2y + 2z}{3} = \frac{-4x - 3z}{5} = y$

$4x + 5y + 3z = 0$ ...**(i)**

$x - 5y + 2z = 0$ ...**(ii)**

from **(i)** & **(ii)**

$x = -z$ & $x = -5y$

$\vec{a} = x\left(\hat{i} - \frac{1}{5}\hat{j} - \hat{k}\right)$.

18. $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$

$\Rightarrow (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = \frac{\vec{b}}{\sqrt{2}} + \frac{\vec{c}}{\sqrt{2}}$

$\Rightarrow \left(\vec{a}.\vec{c} - \frac{1}{\sqrt{2}}\right)\vec{b} - \left(\vec{a}.\vec{b} + \frac{1}{\sqrt{2}}\right)\vec{c} = 0$

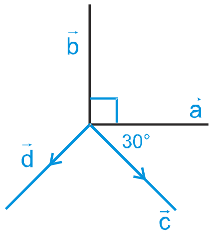
$\therefore \vec{a}.\vec{b} = \frac{-1}{\sqrt{2}}$ & $\vec{a}.\vec{c} = \frac{1}{\sqrt{2}}$

angle between $\vec{a}$ & $\vec{b} = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$

19. If $\lambda = -1$ then $\vec{a} \perp \vec{b}$, $\vec{c} \perp \vec{d}$ and angle between

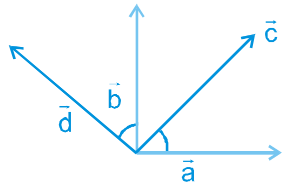
$\vec{a} \times \vec{b}$, $\vec{c} \times \vec{d}$ is π

$\angle$ between $\vec{b}$ and $\vec{d} = 360° - (90° + 90° + 30°) = 150°$

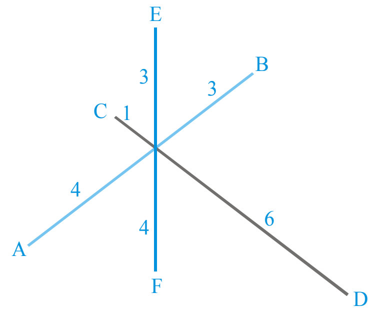


If $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 1$, then following figure is possible

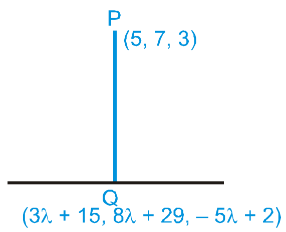
then $\angle$ between $\vec{b}$ and $\vec{d} = 30°$



23. $\frac{3\vec{a} + 4\vec{b}}{7} = \frac{6\vec{c} + \vec{d}}{7} = \frac{4\vec{e} + 3\vec{f}}{7} = \frac{\vec{x}}{7}$



27. d.r's of line are 3, 8, – 5



d.r's of PQ are $3\lambda + 10, 8\lambda + 22, -5\lambda + 2$

$\therefore$ both are perpendidcular

$\therefore (3\lambda + 10)3 + (8\lambda + 22)8 + (-5\lambda + 2)(-5) = 0$

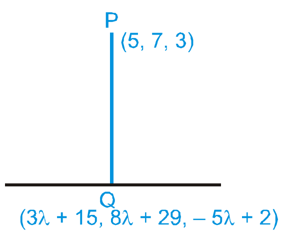
i.e. $\lambda = -2$

$\therefore$ foot is (9, 13, 15), PQ = 14

Since (5, 7, 3), (9, 13, 15) lies on the plane

$9x - 4y - z - 14 = 0$ and $3 \times 9 + 8(-4) + (-5)(-1) = 0$

$\therefore$ equation of the required plane is $9x - 4y - z - 14 = 0$



29. Let any point on line $\frac{x-1}{2} = \frac{y+1}{-3} = z = \lambda$

be $(1 + 2\lambda, -1 - 3\lambda, \lambda)$

$4\sqrt{14} = \sqrt{(1 + 2\lambda - 1)^2 + (-1 - 3\lambda + 1)^2 + \lambda^2}$

$4\sqrt{14} = \sqrt{4\lambda^2 + 9\lambda^2 + \lambda^2}$

$\Rightarrow |\lambda| = 4 \quad \Rightarrow \lambda = \pm 4$

$\therefore$ Points (9, – 13, 4) and (– 7, 11, – 4)

30. Let $\vec r = xi + yj + zk$

then $[\vec r\,\vec b\,\vec c] = 0 \Rightarrow \begin{vmatrix} x & y & z \\ 1 & 2 & -1 \\ 1 & 1 & -2 \end{vmatrix} = 0,$

$-3x + y - z = 0$ ....**(1)**

$\dfrac{\vec r.\vec a}{|\vec a|} = \pm\dfrac{\sqrt{2}}{3} \Rightarrow \dfrac{2x - y + 3}{\sqrt{6}} = \pm\sqrt{\dfrac{2}{3}}$

$2x - y + z = \pm 2$ ...**(2)**

from **(1)** and **(2)** $x = \mp 2$; $y - z = \mp 6$

there fore $\vec r = \mp 2i + yj + (y \pm 6)k$

(A) & (C) are answer

31. The vector parallel to line of intersection of planes is

$\lambda\begin{vmatrix} i & j & k \\ 6 & 4 & -5 \\ 1 & -5 & 2 \end{vmatrix} = -\lambda(17\hat i + 17\hat j + 34\hat k)$

$= \lambda'(\hat i + \hat j + 2\hat k)$ (λ' is scalar)

Now angle between the lines

$\cos\theta = \dfrac{\lambda'(\hat i + \hat j + 2\hat k).(2\hat i - \hat j + \hat k)}{\lambda'\sqrt{6}\times\sqrt{6}} = \dfrac{1}{2}$

$\Rightarrow \theta = \dfrac{\pi}{3}$

33. any such vector = $\lambda\,(\hat a + \hat b)$

$= \lambda\left(\dfrac{7\hat i - 4\hat j - 4\hat k}{9} + \dfrac{-2\hat i - \hat j + 2\hat k}{3}\right)$

$= \dfrac{\lambda}{9}\left[7\hat i - 4\hat j - 4\hat k + 3(-2\hat i - \hat j + 2\hat k)\right]$

$= \dfrac{\lambda}{9}\left[\hat i - 7\hat j + 2\hat k\right]$

$|\vec c| = 5\sqrt{6} \Rightarrow \left|\dfrac{\lambda}{9}\sqrt{1 + 49 + 4}\right| = 5\sqrt{6}$

$\Rightarrow \left|\dfrac{\lambda}{9}\sqrt{54}\right| = 5\sqrt{6}$

$\Rightarrow \lambda = \pm\dfrac{9\times 5\sqrt{6}}{\sqrt{54}} = \pm 15$

$\Rightarrow \vec c = \pm\dfrac{15}{9}(\hat i - 7\hat j + 2\hat k) = \pm\dfrac{5}{3}(\hat i - 7\hat j + 2\hat k)$

35. (A) $\vec a \times [a \times (\vec a \times \vec b)]$

$= \vec a \times [(a.b)\vec a - (\vec a.\vec a)\vec b] = 0 - (\vec a)^2(\vec a \times \vec b)$. False

(B) $\vec a, \vec b, \vec c$ are non-coplanar

$\left.\begin{array}{l} \vec v.\vec a = 0 \\ \vec v.\vec b = 0 \\ \vec v.\vec c = 0 \end{array}\right\} \Rightarrow \vec v.(\vec a + \vec b + \vec c) = 0$

But $\vec a + \vec b + \vec c \neq 0 \Rightarrow \vec v = 0$. i.e. null vector which is true

(C) $\vec a \times \vec b$ & $\vec c \times \vec d$ are perpendicular

so $(\vec a \times \vec b) \times (\vec c \times \vec d) \neq 0$. False

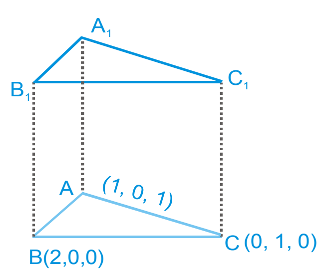
(D) $a' = \dfrac{\vec b \times \vec c}{[a\,b\,c]}, b' = \dfrac{\vec c \times \vec a}{[a\,b\,c]}, c' = \dfrac{\vec a \times \vec b}{[a\,b\,c]}$

is valid only if $\vec a, \vec b, \vec c$ are non coplanar, hence false.

36. Volume of prism = Area of base ABC × height

or $3 = \dfrac{\sqrt{6}}{2}\times h$

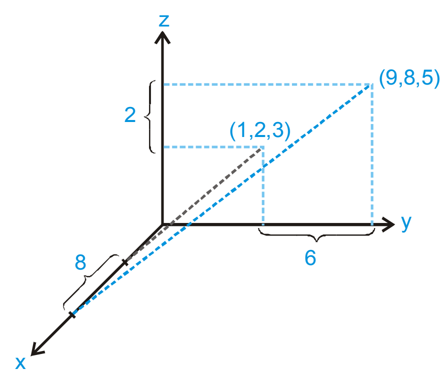
$\Rightarrow h = \sqrt{6}$



Required point A_1 should be just above point A

i.e. line AA_1 is normal to plane ABC and $AA_1 = \sqrt{6}$

41.



Hence, edge length of the parallelopiped

$|x_2 - x_1| = 8$

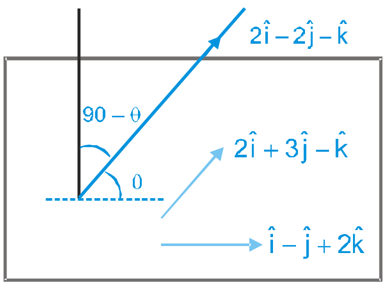
$|y_2 - y_1| = 6$

$|z_2 - z_1| = 2$

42. $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & -1 & 2 \end{vmatrix} = \hat{i}(6-1) - \hat{j}(4+1) + \hat{k}(-2-3)$

$= 5\hat{i} - 5\hat{j} - 5\hat{k}$

$\cos(90 - \theta) = \left|\dfrac{10+10-5}{5\sqrt{3}.\,3}\right|$



$\sin\theta = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = \sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right) = \cot^{-1}(\sqrt{2})$

43. Equation of bisector of plane

$\dfrac{2x - y + 2z + 3}{\sqrt{2^2 + 1^2 + 2^2}} = \pm\dfrac{3x - 2y + 6z + 8}{\sqrt{9 + 4 + 36}}$

$\Rightarrow \dfrac{2x - y + 2z + 3}{3} = \pm\dfrac{(3x - 2y + 6z + 8)}{7}$

$\Rightarrow 14x - 7y + 14z + 21 = \pm(9x - 6y + 18z + 24)$

$\Rightarrow 5x - y - 4z = 3$ and

$23x - 13y + 32z + 45 = 0$

47. Let normal vector n_1 perpendicular to plane determining $\hat{i}, \hat{j} + \hat{k}$ is

$n_1 = \hat{i} \times (\hat{i} + \hat{j}) = \hat{k}$

similarly $n_2 = (\hat{i} - \hat{j}) \times (\hat{i} - \hat{k}) = \hat{i} + \hat{j} + \hat{k}$

Now vector parallel to intersection of plane $= \vec{n}_2 \times \vec{n}_1$

$= \vec{k} \times (\hat{i} + \hat{j} + \hat{k}) = -(\hat{j} - \hat{i}) \Rightarrow \dfrac{x-0}{1-0} = \dfrac{y-0}{1-0} = \dfrac{z-0}{1-0}$

Angle between $\lambda(-\hat{j} + \hat{i})$ and $(\hat{i} - 2\hat{j} + 2\hat{k})$

$\cos\theta = \dfrac{\lambda(-\hat{j} + \hat{i}).(\hat{i} - 2\hat{j} + 2\hat{k})}{\lambda\sqrt{2} \times 3} = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \theta = \dfrac{\pi}{4}$ or $\theta = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$

Part # II : Assertion & Reason

2. **Statement-I** Equation of plane is

$(\vec{r} - \vec{a}).(\vec{b} \times \vec{c}) = 0$ ....**(1)**

$\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ satisfies above equation

Hence True

Statement-II is also true & explain statement I

3. **Statement - I**

$A(\vec{a})$ & $B(\vec{b})$

$\overrightarrow{PA}\,.\,\overrightarrow{PB} \le 0$, then locus of P is sphere having diameter $|(\vec{a} - \vec{b})|$

volume $= \dfrac{4}{3}\pi\left|\dfrac{\vec{a} - \vec{b}}{2}\right|^3 = \dfrac{\pi}{6}|\vec{a} - \vec{b}|^2 \cdot |\vec{a} - \vec{b}|$

$= \dfrac{\pi}{6}(\vec{a}^2 + \vec{b}^2 - 2\vec{a}.\vec{b})\,|\vec{a} - \vec{b}|$

Hence true.

Statement - II : Diameter of sphere subtend acute angle at point P then point P moves out side the sphere having radius r.

5. $[\vec{d}\ \vec{b}\ \vec{c}]\,\vec{a} + [\vec{d}\ \vec{c}\ \vec{a}]\,\vec{b} + [\vec{d}\ \vec{a}\ \vec{b}]\,\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\,\vec{d}$

$= ([\vec{b}\ \vec{c}\ \vec{d}]\,\vec{a} - [\vec{b}\ \vec{c}\ \vec{a}]\,\vec{d}) + ([\vec{d}\ \vec{a}\ \vec{b}]\,\vec{c} - [\vec{d}\ \vec{a}\ \vec{c}]\,\vec{b})$

$= (\vec{b} \times \vec{c}) \times (\vec{a} \times \vec{d}) + (\vec{d} \times \vec{a}) \times (\vec{c} \times \vec{b})$

$= (\vec{d} \times \vec{a}) \times (\vec{b} \times \vec{c}) - (\vec{d} \times \vec{a}) \times (\vec{b} \times \vec{c}) = \vec{0}$

8. Let the coordinates of A, B, C, D be A(1, 0, 0), B(1, 1, 0), C(0, 1, 0) and D(0, 0, 0)

so that coordinates of A_1, B_1, C_1 are $A_1(1,0,1)$, $B_1(1,1,1)$, $C_1(0, 1, 1)$ & $D_1(0, 0, 1)$

The coordinates of midpoint of B_1A_1 is

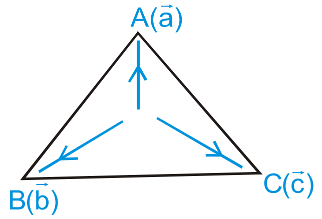
$P\left(1, \dfrac{1}{2}, 1\right)$ and that of B_1C_1 is $Q\left(\dfrac{1}{2}, 1, 1\right)$

Equation of the plane PBQ is $2x + 2y + z = 4$

Its distance from D(0, 0, 0) is $\frac{4}{3}$

So Statement-1 is false and Statement-2 is clearly true.

10.



$$I \equiv \frac{a\vec{a} + b\vec{b} + c\vec{c}}{a + b + c}$$

13. plane P_1 is $\perp$ to $\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 1 & 0 & -2 \end{vmatrix} = 2\hat{i} + \hat{j} + \hat{k}$

and plane P_2 is $\perp$ to $\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -2 \\ 2 & -1 & -3 \end{vmatrix} = -2\hat{i} - \hat{j} - \hat{k}$

$\Rightarrow$ $\vec{a} \,||\, \vec{b}$ $\Rightarrow$ P_1 & P_2 are parallel

also L is parallel to $\vec{c} = \hat{i} - \hat{j} - \hat{k}$

also $\vec{a}.\vec{c} = 0$ & $\vec{b}.\vec{c} = 0$

but it is not essential that if P_1 & P_2 are parallel to L then P_1 & P_2 must be parallel.

So Statement-II is not a correct explanation of Statement-I.

14. Statement - I

$\vec{a} = \hat{i}, \vec{b} = \hat{j}$ & $\vec{c} = \hat{i} + \hat{j}$

$\vec{c} = \vec{a} + \vec{b}$ linearly dependent

$\vec{a}$ & $\vec{b}$ are linearly independent

Hence true.

Statement - II :

$\vec{a}$ & $\vec{b}$ are linearly dependent

$\vec{a} = t\vec{b}$

then $\vec{c} = \lambda\vec{a} + \mu\vec{b}$ which is linearly dependent.

EXERCISE - 3

Part # I : Matrix Match Type

1. (A) If P is a point inside the triangle such that area($\Delta PAB + \Delta PBC + \Delta PCA$) $=$ area (ΔABC)

Then P is centroid.

(B) $\vec{V} = \overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$

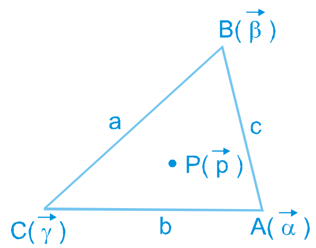
$0 = \vec{a} - \vec{p} + \vec{b} - \vec{p} + \vec{c} - \vec{p}$

$\vec{p} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$ which is centroid.

(C) $\vec{P} = (BC)\overrightarrow{PA} + (CA)\overrightarrow{PB} + (AB)\overrightarrow{PC} = 0$

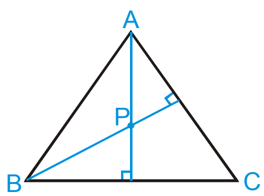
$a(\vec{\alpha} - \vec{p}) + b(\vec{\beta} - \vec{p}) + c(\vec{\gamma} - \vec{p}) = 0$

$\Rightarrow \vec{p} = \frac{a\vec{\alpha} + b\vec{\beta} + c\vec{\gamma}}{a + b + c}$



which is incentre.

(D) From fig.



$\overrightarrow{PA} \cdot \overrightarrow{CB} = 0$

$\overrightarrow{PB}.\overrightarrow{AC} = 0$

$\Rightarrow$ P is orthocentre.

2. (A) Vector parallel to line of intersection of the plane is $(\hat{i} + \hat{j}) \times (\hat{j} + \hat{k}) = \hat{k} - \hat{j} + \hat{i}$

equation of line whose dr's, are (1, – 1, 1) and passing through (0, 0, 0) is

$x = -y = z$

(B) Similarly $(\hat{i} \times \hat{j}) = \hat{k}$.

Hence dr's = (0, 0, 1)

and passing through the point (2, 3, 0)

$\therefore$ Equation of line $\dfrac{x-2}{0} = \dfrac{y-3}{0} = \dfrac{z}{1}$

(C) Similarly $\hat{i}\, x(\hat{j} + \hat{k}) = \hat{k} - \hat{j}$

dr's = (0, –1, 1)

Equation of line $\dfrac{x-2}{0} = \dfrac{y-2007}{-1} = \dfrac{z+2004}{1}$

because $x = 2$ & $y + z = 3$

so $y = 2007$, $z = -2004$ satisfy above equation

(D) $x = 2, x + y + z = 3$

$y + z = 1$

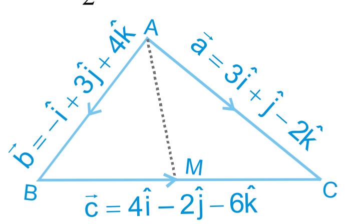
same as part C

we get $\dfrac{x-2}{0} = \dfrac{y}{-1} = \dfrac{z-1}{1}$

3. (A).

here $\vec{a} = \vec{b} + \vec{c}$

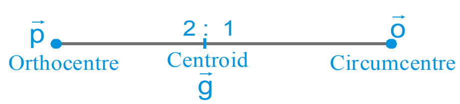
$\overrightarrow{AM} = \dfrac{1}{2}(\vec{a} + \vec{b})$



$= \dfrac{1}{2}[2\hat{i} + 4\hat{j} + 2\hat{k}] = \hat{i} + 2\hat{j} + \hat{k}$

$\Rightarrow \lambda = \sqrt{6}$

(B)



(C) Area $= |\vec{a} \times \vec{b}| = |(\vec{p} + 2\vec{q}) \times (2\vec{p} + \vec{q})|$

$= |\vec{p} \times \vec{q} + 4\vec{q} \times \vec{p}| = |3\vec{p} \times \vec{q}| = 3 \times \dfrac{1}{2} = \dfrac{3}{2}$

(D) $\vec{u} + \vec{v} + \vec{w} = 0$

$\Rightarrow |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 + 2(\vec{u}.\vec{v}) + 2(\vec{v}.\vec{w}) + 2(\vec{w}.\vec{u}) = 0$

$\Rightarrow 9 + 16 + 25 + 2 \quad [\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}] = 0$

$\Rightarrow \sqrt{|\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}|} = 5$

Part # II : Comprehension

Comprehension # 2

1. Equation of the second plane is $-x + 2y - 3z + 5 = 0$

$2(-1) + 3.2 + (-4)(-3) > 0$

$\therefore$ O lies in obtuse angle.

$(2 \times 1 + 3(-2) - 4 \times 3 + 7)(-1 + 2(-2) - 3 \times 3 + 5)$

$= (2 - 6 - 12 + 7)(-1 - 4 - 9 + 5) > 0$

$\therefore$ P lies in obtuse angle.

2. $1 \times 2 + 2 \times 1 - 3 \times 3 < 0$

$\therefore$ O lies in acute angle.

Also

$(2 + 2(-1) - 3(2) + 5)(2 \times 2 - 1 + 3 \times 2 + 1) = (-1)(10) < 0$

$\therefore$ P lies in obtuse angle.

3. $1 - 4 - 9 < 0$

$\therefore$ O lies in acute angle.

Further

$(1 + 4 - 6 + 2)(1 - 4 + 6 + 7) > 0$

$\therefore$ The point P lies in acute angle.

Comprehension # 5

1. We have : $\vec{a}' = \lambda(\vec{b} \times \vec{c})$, $\vec{b}' = \lambda(\vec{c} \times \vec{a})$ and

$\vec{c}' = \lambda(\vec{a} \times \vec{b})$, where $\lambda = \dfrac{1}{[\vec{a}\ \vec{b}\ \vec{c}]}$

$\vec{b} \times \vec{b}' = \vec{b} \times \lambda(\vec{c} \times \vec{a}) = \lambda\{\vec{b} \times (\vec{c} \times \vec{a})\}$

$= \lambda\{(\vec{b}.\vec{a})\vec{c} - (\vec{b}.\vec{c})\vec{a}\}$

and

$\vec{c} \times \vec{c}' = \vec{c} \times \lambda(\vec{a} \times \vec{b}) = \lambda\{\vec{c} \times (\vec{a} \times \vec{b})\}$

$= \lambda\{(\vec{c}.\vec{b})\vec{a} - (\vec{c}.\vec{a})\vec{b}\}$

$\therefore \vec{a} \times \vec{a}' + \vec{b} \times \vec{b}' + \vec{c} \times \vec{c}'$

$= \lambda\{(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}\} + \lambda\{(\vec{b}.\vec{a})\vec{c} - (\vec{b}.\vec{c})\vec{a}\}$
$+ \lambda\{(\vec{c}.\vec{b})\vec{a} - (\vec{c}.\vec{a})\vec{b}\}$

$= \lambda[(\vec{a}.\vec{c})\,\vec{b} - (\vec{a}.\vec{b})\,\vec{c} + (\vec{b}.\vec{a})\,\vec{c} - (\vec{b}.\vec{c})\,\vec{a} + (\vec{c}.\vec{b})\,\vec{a} - (\vec{c}.\vec{a})\,\vec{b}]$

$= \lambda[(\vec{a}.\vec{c})\,\vec{b} - (\vec{a}.\vec{b})\,\vec{c} + (\vec{a}.\vec{b})\,\vec{c} - (\vec{b}.\vec{c})\,\vec{a} + (\vec{b}.\vec{c})\,\vec{a} - (\vec{a}.\vec{c})\,\vec{b}]$

$= \lambda\vec{0} = \vec{0}$

2. $\vec{a}' \times \vec{b}' = \dfrac{(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})}{[\vec{a}\vec{b}\vec{c}]^2} = \dfrac{\vec{c}}{[\vec{a}\vec{b}\vec{c}]}$

$\therefore \quad \vec{a}'\times\vec{b}' + \vec{b}'\times\vec{c}' + \vec{c}'\times\vec{a}' = \dfrac{\vec{a}+\vec{b}+\vec{c}}{[\vec{a}\vec{b}\vec{c}]}$

so $\lambda = 1$

3. $(a' \times b') \times (b' \times c') = \dfrac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]^2}$

$\left[\dfrac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]^2}\ \dfrac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]^2}\ \dfrac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]^2}\right] = \dfrac{[\vec{a}\vec{b}\vec{c}]^2}{[\vec{a}\vec{b}\vec{c}]^6} = [\vec{a}\vec{b}\vec{c}]^{-4}$

$\therefore \quad n = -4$

Comprehension # 6

A (2, 1, 0), B (1, 0, 1)

C (3, 0, 1) and D(0, 0, 2)

1. Equation of plane ABC

$$\begin{vmatrix} x-2 & y-1 & z \\ 1 & 1 & -1 \\ 2 & 0 & 0 \end{vmatrix} = 0 \quad \Rightarrow \quad y + z = 1$$

2. Equation of $L = 2\hat{k} + \lambda(\overrightarrow{AB}\times\overrightarrow{AC})$

so $L = 2\hat{k} + \lambda(\hat{j}+\hat{k})$

3. Equation of plane ABC

$y + z - 1 = 0$

distance from (0, 0, 2) is $= \dfrac{2-1}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

Comprehension # 7

Vector $\vec{p} = \hat{i} + \hat{j} + \hat{k}$, $\vec{q} = 2\hat{i} + 4\hat{j} - \hat{k}$,

$\vec{r} = \hat{i} + \hat{j} + 3\hat{k}$

1. (A) $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 1 & 1 & 3 \end{vmatrix} = 13 - 7 - 2 = 4 \neq 0$

Hence non coplanar; so linearly independent

(B) In triangle, let length of sides of triangle are a, b, c then triangle is formed if sum of two sides is greater than the third side. Check yourself.

(C) $(\vec{q}-\vec{r}).\vec{p}$

$= (i + 3j - 4k).(i + j + k) = 1 + 3 - 4 = 0$

Hence true.

2. $((\vec{p}\times\vec{q})\times\vec{r}) = u\vec{p} + v\vec{q} + w\vec{r}$

$(\vec{p}.\vec{r})\vec{q} - (\vec{q}.\vec{r})\vec{p} = u\vec{p} + v\vec{q} + w\vec{r}$

By solving $\vec{p}.\vec{r}$ & $\vec{q}.\vec{r}$, we get

$5\vec{q} - 3\vec{p} + 0\vec{r} = u\vec{p} + v\vec{q} + w\vec{r}$

compare

$u + v + w = 5 - 3 + 0 = 2.$

3. $\vec{s}$ is unit vector

$(\vec{p}.\vec{s})\,(\vec{q}\times\vec{r}) + (\vec{q}.\vec{s})\,(\vec{r}\times\vec{p}) + \vec{r}.\vec{s}\,(\vec{p}\times\vec{q})$

$\vec{q}\times\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & -1 \\ 1 & 1 & 3 \end{vmatrix} = 13\hat{i} - 7\hat{j} - 2\hat{k}$

$\vec{r}\times\vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = -2\hat{i} + 2\hat{j}$

$\vec{p}\times\vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 4 & -1 \end{vmatrix} = -5\hat{i} + 3\hat{j} + 2\hat{k}$

Let $\vec{s} = \hat{i}$

Putting the value we get

$13\hat{i} - 7\hat{j} - 2\hat{k} + 2(-2\hat{i} + 2\hat{j}) + (-5\hat{i} + 3\hat{j} + 2\hat{k})$

$= 13\hat{i} - 7\hat{j} - 2\hat{k} - 4\hat{i} + 4\hat{j} - 5\hat{i} + 3\hat{j} + 2\hat{k}$

$= 4\hat{i} + 0\hat{j} + 0\hat{k} = 4\hat{i}$

Magnitude = 4.

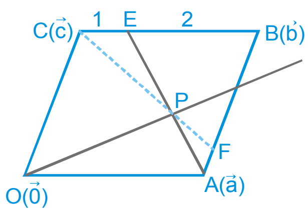
Comprehension # 8

$E = \frac{2\vec{c}+\vec{b}}{3}$

equation of OP $\vec{r} = \lambda\left(\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{c}}{|\vec{c}|}\right)$ ...**(1)**

Let P divide EA in $\mu : 1$

$P\left[\frac{\mu\vec{a}+\frac{2\vec{c}+\vec{b}}{3}}{\mu+1}\right]$



P lies on (1)

$\frac{\mu\vec{a}+\frac{2\vec{c}+\vec{b}}{3}}{\mu+1} = \lambda\left(\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{c}}{|\vec{c}|}\right)$

$\vec{a}+\vec{c}=\vec{b}$

$\frac{\mu\vec{a}+\frac{3\vec{c}+\vec{a}}{3}}{\mu+1} = \lambda\left(\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{c}}{|\vec{c}|}\right)$

Comparing coefficient of $\vec{a}$ and $\vec{c}$

$\frac{\mu+\frac{1}{3}}{\mu=1} = \frac{\lambda}{|\vec{a}|}$ ...**(2)**

and $\frac{1}{\mu+1} = \frac{\lambda}{|\vec{c}|}$ ...**(3)**

divided **(2)** by **(3)** $\mu+\frac{1}{3} = \frac{|\vec{c}|}{|\vec{a}|}$

$\mu = \frac{|\vec{c}|}{|\vec{a}|}-\frac{1}{3}$

Put in **(3)** $\frac{1}{\frac{|c|}{|a|}+\frac{2}{3}} = \frac{\lambda}{|\vec{c}|}$

$\lambda = \frac{3|\vec{a}||\vec{c}|}{c|\vec{c}|+2|\vec{a}|}$

So position vector of P

$\vec{r} = \frac{3|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left(\frac{a}{|\vec{a}|}+\frac{c}{|\vec{c}|}\right)$

Now for solution of 4

equation of AB,

$\vec{r} = \vec{a}+\lambda(\vec{b}-\vec{a}) = \vec{a}+\lambda(\vec{c})$ ...**(4)**

equation of CP, $\vec{r} = \vec{c}+\mu$

$\left(\frac{3|c|\vec{a}}{3|\vec{c}|+2|\vec{a}|}+\frac{3|a|\vec{c}}{3|c|+2|\vec{a}|}-\vec{c}\right)$

$\vec{r} = \vec{c}+\mu\left[\frac{3|c|\vec{a}+3|a|c-3|c|c-2|a|\vec{c}}{3|c|+2|a|}\right]$

$r = \vec{c}+\mu\left[\frac{3|c|\vec{a}+|a|\vec{c}-3|c|\vec{c}}{3|c|+2|a|}\right]$ ..**(5)**

Comparing **(4)** and **(5)**

$\lambda = 1+\frac{\mu|a|-3\mu|c|}{3|c|+2|a|}$

$\lambda = \frac{3|\vec{c}|+2|\vec{a}|+\mu|a|-3\mu|\vec{c}|}{3|\vec{c}|+2|\vec{a}|}$ ...**(6)**

$\mu = \frac{3|\vec{c}|+2|\vec{a}|}{3|\vec{c}|}$

Put value of μ in equation **(6)**

$\lambda = 1+\frac{\mu(|\vec{a}|-3|\vec{c}|)}{3|\vec{c}|+2|\vec{a}|}$

$\lambda = 1+\frac{|a|-3|\vec{c}|}{3|c|} = \frac{1}{3}\frac{|a|}{|c|}$

So position vector of F is $= \vec{a}+\frac{1}{3}\frac{|a|}{|c|}\vec{c}$

Solution – 5

$\vec{A}\,\vec{F}$ = p.v. of F – p.v. of A $= \vec{a}+\frac{1}{3}\frac{|a|}{|c|}\vec{c}-\vec{a}$

$= \frac{1}{3}\frac{|a|}{|c|}\vec{c}$

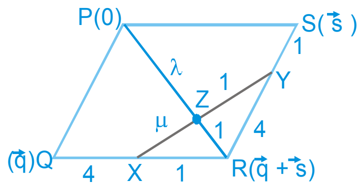
EXERCISE - 4
Subjective Type

1. $\overrightarrow{QX} = 4\overrightarrow{XR}$

$\overrightarrow{RY} = 4\overrightarrow{YS}$

Let $\vec{p}$ be origin

& $R(\vec{q}+\vec{s})$



from figure

$$\text{P.V. of } X = \frac{4(\vec{q}+\vec{s})+\vec{q}}{5} = \frac{5\vec{q}+4\vec{s}}{5}$$

$$\text{P.V. of } Y = \frac{4\vec{s}+\vec{q}+\vec{s}}{5} = \frac{5\vec{s}+\vec{q}}{5}$$

Now Let Z divides PR in ratio $\lambda:1$

Now Let Z divides XY in ratio $\mu:1$

$$\text{P.V. of } Z = \frac{\lambda(\vec{q}+\vec{s})}{\lambda+1} \quad \text{(from PR)}$$

$$\text{P.V. of } Z = \frac{\frac{\mu(5\vec{s}+\vec{q})}{5}+\frac{5\vec{q}+4\vec{s}}{5}}{\mu+1} \quad \text{(from XY)}$$

equating both Z then we get

$$\frac{\lambda}{\lambda+1} = \frac{\mu+5}{5(\mu+1)} \quad \text{.......(i)}$$

$$\frac{\lambda}{\lambda+1} = \frac{5\mu+4}{5(\mu+1)} \quad \text{......(ii)}$$

from (i) & (ii), $\mu = \frac{1}{4}$ & $\lambda = \frac{21}{4}$

$$\text{So P.V. of } Z = \frac{\frac{21}{4}}{\frac{21}{4}+1}(\vec{q}+\vec{s})$$

$$= \frac{21}{25}(\vec{q}+\vec{s}) = \frac{21}{25}\overrightarrow{PR}$$

2. PVs of vertex P,Q,R,S are (Let) $\vec{0}, \vec{a}, \vec{b}+\vec{a}, \vec{b}$

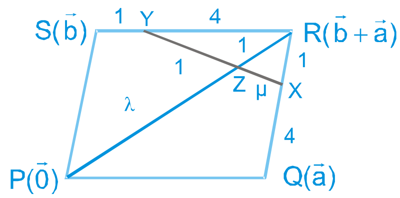
using section rule PVs of

$$X \equiv \frac{4(\vec{b}+\vec{a})+\vec{a}}{5} \quad \text{and} \quad Y \equiv \frac{(\vec{b}+\vec{a})+4\vec{b}}{5}$$

again Let $\frac{PZ}{ZR} = \lambda$ and $\frac{XZ}{YZ} = \mu$

PVs of point Z may be given as

$$\frac{\lambda(\vec{b}+\vec{a})+\vec{0}}{\lambda+1} \text{ \& also as } \frac{\mu\left(\vec{b}+\frac{\vec{a}}{5}\right)+1\left(\vec{a}+\frac{4\vec{b}}{5}\right)}{\mu+1}$$



Equating both answers and coefficient of $\vec{a}$ & $\vec{b}$

(they are representing non collinear vectors $\overrightarrow{PQ}$ & $\overrightarrow{PS}$)

$$\frac{\lambda}{\lambda+1} = \frac{\mu+\left(\frac{1}{5}\right)}{\mu+1} \quad \text{and} \quad \frac{\lambda}{\lambda+1} = \frac{\left(\frac{4\mu}{5}\right)+1}{\mu+1}$$

Solving these equations gives $\lambda = \frac{21}{4}$

3. After rotation equation of plane is new position will be

$\ell x + my + a'z = 0$ ....(1)

Let angle between (1) and $\ell x + my = 0$

is θ, then

$$\cos\theta = \frac{\ell^2+m^2}{\sqrt{\ell^2+m^2}\sqrt{\ell^2+m^2+a'^2}}$$

Solving we get

$a'^2 = (\ell^2+m^2)\tan^2\theta$

$\Rightarrow \quad a' = \pm\sqrt{(\ell^2+m^2)}\tan\theta$

Equation is $\ell x + my \pm z\sqrt{(\ell^2+m^2)}\tan\theta = 0$

4. $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = r$ (Let) ... (1)

$\Rightarrow$ $(2r+1, 3r+2, 4r+3)$ represents any point on (1)

$\frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1}$ ... (2)

To find point of intersection of (1) and (2)

$$\frac{2r+1-4}{5}=\frac{3r+2-1}{2}=\frac{4r+3}{1}$$

$\Rightarrow \quad \dfrac{2r-3}{5}=\dfrac{3r+1}{2}=\dfrac{4r+3}{1}$

$\Rightarrow \quad 4r-6=15r+5$

$\Rightarrow \quad 11r=-11 \quad \Rightarrow \quad r=-1$

$\therefore$ point of intersection of **(1)** and **(2)** is $(-1,-1,-1)$

$\vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(3\hat{i}-\hat{j}) \qquad$ ... **(1)**

$\vec{r}=(4\hat{i}-\hat{k})+\mu(2\hat{i}+3\hat{k}) \qquad$ ... **(2)**

For their point of intersection

$3\lambda+1=4+2\mu \quad \Rightarrow 3\lambda-2\mu-3=0 \qquad$ ... **(3)**

$1-\lambda=0 \quad \Rightarrow \lambda=1 \qquad$ ... **(4)**

and $-1=-1+3\mu \quad \mu=0$

$\therefore$ point of intersection is $(4, 0, -1)$

$\therefore$ required distance

$$=\sqrt{(4+1)^2+1+0}=\sqrt{25+1}=\sqrt{26}$$

5. $\left|(\vec{a}.\vec{d})(\vec{b}\times\vec{c})+(\vec{b}.\vec{d})(\vec{c}\times\vec{a})+(\vec{c}.\vec{d})(\vec{a}\times\vec{b})\right|$

$\left|(\vec{a}.\vec{d})(\vec{b}\times\vec{c})-(\vec{c}.\vec{d})(\vec{b}\times\vec{a})+(\vec{b}.\vec{d})(\vec{c}\times\vec{a})\right|$

$\left|\vec{b}\times[(\vec{a}.\vec{d})\vec{c}-(\vec{c}.\vec{d})\vec{a}]+(\vec{b}.\vec{d})(\vec{c}\times\vec{a})\right|$

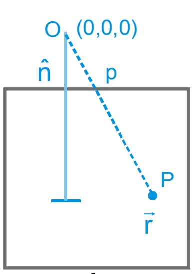
$\left|\vec{b}\times\{(\vec{a}\times\vec{c})\times\vec{d}]+(\vec{b}.\vec{d})(\vec{c}\times\vec{a})\right|$

$=\left|(\vec{b}.\vec{d})(\vec{a}\times\vec{c})-\{\vec{b}.(\vec{a}\times\vec{c})\}\vec{d}-(\vec{b}.\vec{d})(\vec{a}\times\vec{c})\right|$

$=\left|[\vec{b}\ \vec{a}\ \vec{c}]\vec{d}\right|=[\vec{b}\ \vec{a}\ \vec{c}]\ |\vec{d}| \qquad \because\ |\vec{d}|=1$

$=\left[\vec{b}\quad\vec{a}\quad\vec{c}\right]$ Proved.

6. **(i)** Projection of OP on $\hat{n}$



$\vec{r}\,.\,\hat{n}=p$

(ii) $\vec{r}\,.\,\vec{a}-p+\lambda(\vec{r}.\,\vec{b}-q)=0$

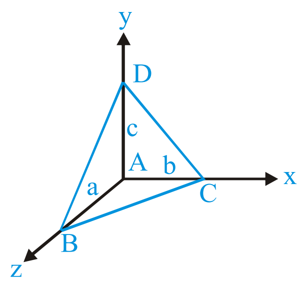
$\vec{r}=\vec{0}$

$\therefore \quad -p-\lambda q=0 \quad \lambda=-\dfrac{p}{q}$

$\vec{r}\,.\,\vec{a}-p-\dfrac{p}{q}(\vec{r}.\,\vec{b}-q)=0$

$\vec{r}\,.(\vec{a}q-p\vec{b})=0$

7.



Area of Δ ABC $\Rightarrow \dfrac{1}{2}ab=x \qquad$ ...**(i)**

Area of Δ ABC $\Rightarrow \dfrac{1}{2}bc=y \qquad$ ...**(ii)**

Area of Δ ACD $\Rightarrow \dfrac{1}{2}ac=z \qquad$ ....**(iii)**

Area of ΔBCD $=\dfrac{1}{2}\sqrt{a^2b^2+b^2c^2+c^2a^2}$

$=\dfrac{1}{2}\times 2\sqrt{x^2+y^2+z^2}$

$=\sqrt{x^2+y^2+z^2}$

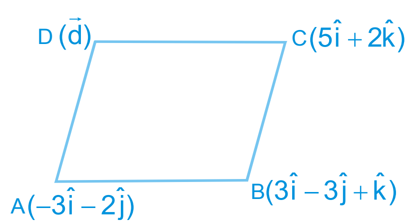
8. **(a)** $(3\hat{i}-3\hat{j}+\hat{k}+\vec{d}) \equiv 2\hat{i}-2\hat{j}+2\hat{k}$

$\Rightarrow \quad \vec{d}=-\hat{i}+\hat{j}+\hat{k}$

(b) $\overrightarrow{AB}=6\hat{i}-\hat{j}+\hat{k}$

$\overrightarrow{AC}=8\hat{i}+2\hat{j}+2\hat{k}$

$\Rightarrow \quad \left|\overrightarrow{AC}\right|=\sqrt{64+4+4}=\sqrt{72}$



Required vector is $\frac{\sqrt{72}}{\sqrt{38}}(6\hat{i}-\hat{j}+\hat{k})$

$$=\frac{6}{\sqrt{19}}(6\hat{i}-\hat{j}+\hat{k})$$

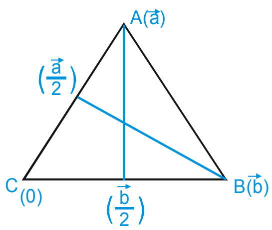
(c) $\overrightarrow{BD}=-4\hat{i}+4\hat{j}$

$$\cos\theta=\frac{\overrightarrow{AC}.\overrightarrow{BD}}{|\overrightarrow{AC}||\overrightarrow{BD}|}=\frac{-32+8}{\sqrt{72}\sqrt{32}}=\frac{-24}{6\sqrt{2}\cdot4\sqrt{2}}$$

$$=-\frac{1}{2}$$

$$\Rightarrow\quad \theta=\frac{2\pi}{3}$$

9. Let origin be C



Given $\left|\vec{a}-\frac{\vec{b}}{2}\right|=\left|\vec{b}-\frac{\vec{a}}{2}\right|$ (medians are equal)

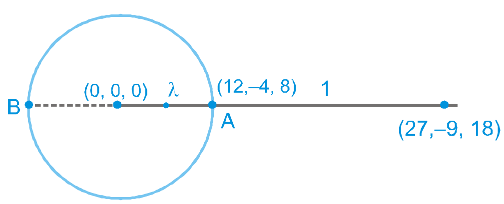
$$\Rightarrow\quad \vec{a}^2+\frac{\vec{b}^2}{4}-\vec{a}.\vec{b}=\vec{b}^2+\frac{\vec{a}^2}{4}-\vec{a}.\vec{b}$$

$$\frac{3\vec{a}^2}{4}=\frac{3}{4}\vec{b}^2\quad\Rightarrow\quad|\vec{a}|=|\vec{b}|$$

10. $A\left(\frac{27\lambda+12}{\lambda+1},\frac{-9\lambda-4}{\lambda+1},\frac{18\lambda+8}{\lambda+1}\right)$

Which lies on the sphere

$$\therefore\left(\frac{27\lambda+12}{\lambda+1}\right)^2+\left(\frac{-9\lambda-4}{\lambda+1}\right)^2+\left(\frac{18\lambda+8}{\lambda+1}\right)^2=504$$



Solving above we get $9\lambda^2=4\quad \lambda=\pm\frac{2}{3}$

11. Let point on line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ .....(1)

are $(3+2\lambda, 3+\lambda, \lambda)$

Equation of line which pass through origin is

$$\frac{x-0}{3+2\lambda}=\frac{y-0}{3+\lambda}=\frac{z-0}{\lambda}\quad .....(2)$$

Angle between (1) & (2)

$$\cos\frac{\pi}{3}=\frac{(3+2\lambda)2+(3+\lambda)1+\lambda\times1}{\sqrt{(3+2\lambda)^2+(3+\lambda)^2+\lambda^2}\sqrt{2^2+1^2+1}}$$

Solving we get

$$\lambda^2+3\lambda+2=0$$

$$\Rightarrow\quad \lambda=-1,-2$$

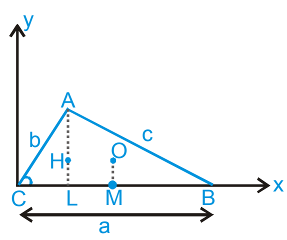
Putting the value of λ in equation (2)

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}\text{ or }\frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$$

12. M is mid point of CB, also OM = R cosA

$\Rightarrow$ PV's of circumcentre O is $\equiv\left(\frac{a}{2}\hat{i}+R\cos A\hat{j}\right)$

again CL = bcosC and HL = 2RcosB cosC



$\Rightarrow$ PV's of orthocentre H is

$$\equiv(b\cos C\,\hat{i}+2R\cos B\cos C\,\hat{j})$$

Distance between points O & H

$$\equiv\left|\left(\frac{a}{2}-b\cos C\right)\hat{i}+\left(R\cos A-2R\cos B\cos C\right)\hat{j}\right|$$

$$=\sqrt{(R\sin A-2R\sin B\cos C)^2+(R\cos A-2R\cos B\cos C)^2}$$

$$=\sqrt{\begin{array}{l}\sin^2A+4\sin^2B\cos^2C-4\sin A\sin B\cos C+\cos^2A\\+4\cos^2B\cos^2C-4\cos A\cos B\cos C\end{array}}$$

$$=R\sqrt{1+4\cos^2C-4\cos C(\sin A\sin B+\cos A\cos B)}$$

$= R\sqrt{1+4\cos^2 C - 4\cos C\cos(A-B)}$

$= R\sqrt{1+4\cos^2 C + 4\cos(A+B)\cos(A-B)}$

$= R\sqrt{1+4\cos^2 C + 4\cos^2 A - 4\sin^2 B}$

$= R\sqrt{1-8\cos A\ \cos B\ \cos C}$

13. $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$[\vec{a}\,\vec{b}\,\vec{c}]$ is written as $\begin{vmatrix} \vec{a}.\hat{i} & \vec{a}.\hat{j} & \vec{a}.\hat{k} \\ \vec{b}.\hat{i} & \vec{b}.\hat{j} & \vec{b}.\hat{k} \\ \vec{c}.\hat{i} & \vec{c}.\hat{j} & \vec{c}.\hat{k} \end{vmatrix}$

Now $\quad \{(n\vec{a}+\vec{b})\times(n\vec{b}+\vec{c})\}.(n\vec{c}+\vec{a})$

$= \{n^2(\vec{a}\times\vec{b}) + n(\vec{a}\times\vec{c}) + \vec{b}\times\vec{c}\}\,.\,(n\vec{c}+\vec{a})$

$= n^3[\vec{a}\,\vec{b}\,\vec{c}] + [\vec{b}\,\vec{c}\,\vec{a}]$

$= (n^3+1)\ [\vec{a}\,\vec{b}\,\vec{c}]$

14. $\vec{w} + (\vec{w}\times\vec{u}) = \vec{v}$ ... **(1)**

Dot **(1)** with $\vec{v}$

$\vec{w}\,.\,\vec{v} + [v\,w\,u] = 1$ ... **(2)**

Dot **(1)** with $\vec{u}$

$\vec{w}.\vec{u} + 0 = \vec{v}.\vec{u}$ ... **(3)**

cross **(1)** with $\vec{u}$

$\vec{u}\times\vec{w} + (\vec{u}.\vec{u})\vec{w} - (\vec{u}.\vec{w})\vec{u} = \vec{u}\times\vec{v}$

Using **(3)** we get

$\vec{u}\times\vec{w} + \vec{w} - (\vec{v}.\vec{u})\vec{u} = \vec{u}\times\vec{v}$

$[v\,u\,w] + (\vec{v}.\vec{w}) - (\vec{v}.\vec{u})^2 = 0$

Using **(2)** we get

$[v\,u\,w] + 1 - [v\ w\ u] - (\vec{u}.\vec{v})^2 = 0$

$2[u\ v\ w] = 1 - (\vec{u}.\vec{v})^2$

$[u\ v\ w]_{max} = \frac{1}{2}$

when $\vec{u}.\vec{v} = \vec{0} \quad \Rightarrow \quad \vec{u}\perp\vec{v}$

15. Angular point OABC are (0, 0, 0), (0, 0, 2), (0, 4, 0) & (6, 0, 0)

Let centre of sphere be (r, r, r)

Equation of plane passing ABC is

$\frac{x}{6} + \frac{y}{4} + \frac{z}{2} = 1$

$r = \left|\frac{\frac{r}{6}+\frac{r}{4}+\frac{r}{2}-1}{\sqrt{\frac{1}{6^2}+\frac{1}{4^2}+\frac{1}{2^2}}}\right|$

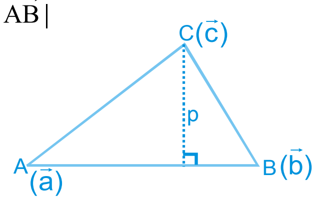
$7r = \pm(11r-12)$

$r = \frac{2}{3}$, $r = 3$ (not satisfied)

16. (a) Let $\perp$ distance of $\vec{c}$ from line joining $\vec{a}$ and $\vec{b}$ is p.

Now $\quad \Delta = \frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right| = \frac{1}{2}\left|\overrightarrow{AB}\right|\times p$

$\Rightarrow \quad p = \frac{|\overrightarrow{AB}\times\overrightarrow{AC}|}{|\overrightarrow{AB}|}$



$= \frac{|(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})|}{|\vec{b}-\vec{a}|} = \frac{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}{|\vec{b}-\vec{a}|}$

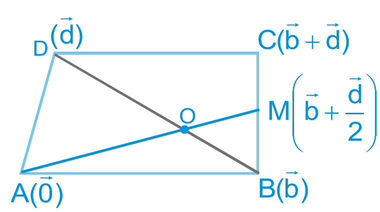
(b) Equation of line AM is

$\vec{r} = \lambda\left(\vec{b}+\frac{\vec{d}}{2}\right)$

Equation of line BD is

$\vec{r} = \vec{b} + \mu(\vec{d}-\vec{b})$

to obtain point of intersection



$\lambda\left(\vec{b}+\frac{\vec{d}}{2}\right) = \vec{b} + \mu(\vec{d}-\vec{b})$

$\Rightarrow \quad \lambda = 1-\mu \ \& \ \frac{\lambda}{2} = \mu$

$\Rightarrow \quad \lambda = 1 - \dfrac{\lambda}{2} \quad$ or $\quad \lambda = \dfrac{2}{3}$

hence point O is $\dfrac{2}{3}\left(\vec{b} + \dfrac{\vec{d}}{2}\right)$

Area OMCD = Area OMC + Area OCD

$$= \frac{1}{2}\left|\frac{1}{3}\left(\vec{b}+\frac{\vec{d}}{2}\right)\times\left(\frac{\vec{b}}{3}+\frac{2\vec{d}}{3}\right)\right| + \frac{1}{2}\left|\left(\frac{\vec{b}}{3}+\frac{2\vec{d}}{3}\right)\times\left(\frac{-2}{3}\vec{b}+\frac{2}{3}\vec{d}\right)\right|$$

$$= \frac{1}{2}\left|\frac{1}{9}\left(\vec{b}\times 2\vec{d}+\frac{\vec{d}}{2}\times\vec{b}\right)\right| + \frac{1}{2}\left|\frac{1}{9}(\vec{b}\times 2\vec{d} - 4\vec{d}\times\vec{b})\right|$$

$$= \frac{1}{18}\left|\frac{3}{2}\vec{b}\times\vec{d}\right| + \frac{1}{18}|6\vec{b}\times\vec{d}| = \frac{1}{18}\times\frac{15}{2}|\vec{b}\times\vec{d}|$$

$$= \frac{15}{18\times 2}\times 12 = 5 \text{ sq. units}$$

$$= \frac{1}{18}\left|\frac{3}{2}\vec{b}\times\vec{d}\right| + \frac{1}{18}|6\vec{b}\times\vec{d}| = \frac{1}{18}\times\frac{15}{2}|\vec{b}\times\vec{d}|$$

$$= \frac{15}{18\times 2}\times 12 = 5 \text{ sq. units}$$

17. Let $|\vec{u}| = \lambda$

$$\vec{u} = \frac{\lambda}{2}(\hat{i} + \sqrt{3}\,\hat{j})$$

Given $\left|\dfrac{\lambda}{2}(\hat{i}+\sqrt{3}\,\hat{j}) - \hat{i}\right|^2 = \lambda\left|\dfrac{\lambda}{2}(\hat{i}+\sqrt{3}\,\hat{j}) - 2\hat{i}\right|$

$$\left(\left(\frac{\lambda}{2}-1\right)^2 + \frac{3\lambda^2}{4}\right)^2 = \lambda^2\left(\left(\frac{\lambda-4}{2}\right)^2 + \frac{3\lambda^2}{4}\right)$$

$(4\lambda^2 - 4\lambda + 4)^2 = 16\lambda^2(\lambda^2 - 2\lambda + 4)$

$(\lambda^2 - \lambda + 1)^2 = \lambda^2(\lambda^2 - 2\lambda + 4)$

solving we get $\lambda = \dfrac{-2 \pm \sqrt{4+4}}{2} = -1 \pm \sqrt{2}$

But $\lambda > 0$

$\Rightarrow \lambda = \sqrt{2} - 1$

$\therefore \quad a = 2, \; b = 1$

18. For linearly dependent vectors

$\ell(i - 2j + 3k) + m(-2i + 3j - 4k) + n(i - j + xk) = 0$

$\ell - 2m + n = 0, \; -2\ell + 3m - n = 0$

$3\ell - 4m + nx = 0$

$$\therefore \begin{vmatrix} 1 & -2 & 1 \\ -2 & 3 & -1 \\ 3 & -4 & x \end{vmatrix} = 0 \text{ is } x = 1$$

20. (i) $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\,\vec{b} - (\vec{a}.\vec{b})\,\vec{c}$

$\Rightarrow \quad 10\vec{b} - 3\vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$

$p = 0, \; q = +10, \; r = -3$

[$\vec{a}, \vec{b}, \vec{c}$ are non coplanar]

(ii) $(\vec{a}\times\vec{b}) \times (\vec{a}\times\vec{c})\,.\,\vec{d}$

$= \{((\vec{a}\times\vec{b}).\vec{c})\,\vec{a} - ((\vec{a}\times\vec{b}).\vec{a})\,\vec{c}\}\cdot\vec{d}$

$= [\vec{a}\;\vec{b}\;\vec{c}]\;\vec{a}.\vec{d} - 0 = 20 \times (-5) = -100$

21. $\pm\hat{i}$

22. vectors $\vec{a}, \vec{b}$ & $\vec{c}$ are non coplanar so are the vectors

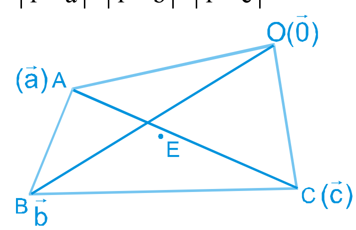
$\vec{a}\times\vec{b}, \; \vec{b}\times\vec{c}$

Let position vector of circumcentre

$\vec{r} \equiv x(\vec{a}\times\vec{b}) + y(\vec{b}\times c) + z(\vec{c}\times\vec{a})$

also $\quad OE = AE = EB = EC$

$\Rightarrow \quad |\vec{r}| = |\vec{r} - \vec{a}| = |\vec{r} - \vec{b}| = |\vec{r} - \vec{c}|$



or $\quad \vec{r}^2 = \vec{r}^2 + \vec{a}^2 - 2\vec{r}.\vec{a}$

$= \vec{r}^2 + \vec{b}^2 - 2\vec{r}.\vec{b} = \vec{r}^2 + \vec{c}^2 - 2\vec{r}.\vec{c}^2$

$\Rightarrow \quad 2\vec{r}.\vec{a} = \vec{a}^2, \quad 2\vec{r}.\vec{b} = \vec{b}^2, \quad 2\vec{r}.\vec{c} = \vec{c}^2$

or $\quad 2y[\vec{a}\;\vec{b}\;\vec{c}] = \vec{a}^2 \quad \Rightarrow \quad y = \dfrac{\vec{a}^2}{2[\vec{a}\;\vec{b}\;\vec{c}]}$

23. $\vec{\alpha} = \hat{i} + a\hat{j} + a^2\hat{k}$

$\vec{\beta} = \hat{i} + b\hat{j} + b^2\hat{k}$

$\vec{\gamma} = \hat{i} + c\hat{j} + c^2\hat{k}$

$\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are non coplanar

$$\therefore \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0$$

$\Rightarrow$ (a – b) (b – c) (c – a) ≠ 0 $\Rightarrow$ a ≠ b ≠ c

If α_1, β_1 & γ_1 are coplanar

$$\text{Then} \begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & b_1 & b_1^2 \\ 1 & c_1 & c_1^2 \end{vmatrix} = 0$$

$\Rightarrow$ $a_1 = b_1 = c_1$

$$\text{Given} \begin{vmatrix} (a_1 - a)^2 & (a_1 - b)^2 & (a_1 - c)^2 \\ (b_1 - a)^2 & (b_1 - b)^2 & (b_1 - c)^2 \\ (c_1 - a)^2 & (c_1 - b)^2 & (c_1 - c)^2 \end{vmatrix} = 0$$

$\Rightarrow$ $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$, we get

$$(a_1 - b_1)(b_1 - c_1)\begin{vmatrix} a_1 + b_1 - 2a & a_1 + b_1 - 2b & a_1 + b_1 - 2c \\ b_1 + c_1 - 2a & b_1 + c_1 - 2b & b_1 + c_1 - 2c \\ (c_1 - a)^2 & (c_1 - b)^2 & (c_1 - c)^2 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$

$$\Rightarrow (a_1 - b_1)(b_1 - c_1)\begin{vmatrix} a_1 - c_1 & a_1 - c_1 & a_1 - c_1 \\ b_1 + c_1 - 2a & b_1 + c_1 - 2b & b_1 + c_1 - 2c \\ (c_1 - a)^2 & (c_1 - b)^2 & (c_1 - c^2) \end{vmatrix} = 0$$

$\Rightarrow$ $(a_1 - b_1)(b_1 - c_1)(c_1 - a_1)$

$$\begin{vmatrix} 1 & 1 & 1 \\ b_1 + c_1 - 2a & b_1 + c_1 - 2b & b_1 + c_1 - 2c \\ (c_1 - a)^2 & (c_1 - b)^2 & (c_1 - c)^2 \end{vmatrix} = 0$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$\Rightarrow$ $(a_1 - b_1)(b_1 - c_1)(c_1 - a_1)$

$$\underbrace{\begin{vmatrix} 0 & 0 & 1 \\ 2(b-a) & 2(c-b) & b_1 + c_1 - 2c \\ a^2 - b^2 - 2c_1(a-b) & b^2 - c^2 - 2c_1(b-c) & (c_1 - c)^2 \end{vmatrix}}_{\Delta} = 0$$

$(a_1 - b_1)(b_1 - c_1)(c_1 - a_1)\,\Delta = 0$

$\Rightarrow$ $(a_1 - b_1)(b_1 - c_1)(c_1 - c_1) = 0$ [$\Delta \neq 0$]

$\Rightarrow$ $a_1 = b_1 = c_1$

$\Rightarrow$ $\vec{\alpha}_1, \vec{\beta}_1, \vec{\gamma}_1$ are coplanar

24. $\ell + m + n = 0$ ......**(1)**

$\ell^2 + m^2 = n^2$ ......**(2)**

Put $n = -(\ell + m)$ in (2)

$\ell^2 + m^2 = \ell^2 + m^2 + 2\ell m$

$\Rightarrow$ $\ell m = 0$

(i) if $\ell = 0$; $m \neq 0$ then from (1) $m = -n$

$\therefore \frac{\ell}{0} = \frac{m}{1} = \frac{n}{-1}$

$\therefore$ direction cosine are : $0, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$

(ii) if $\ell \neq 0$; $m = 0$, then from (1), $\ell = -n$

$\therefore \frac{\ell}{1} = \frac{m}{0} = \frac{n}{-1}$

$\therefore$ direction cosine are : $\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}$

Let θ be the angle between the lines

$\therefore \cos\theta = 0 + 0 + \frac{1}{2}$

$\cos\theta = \frac{1}{2}$ $\Rightarrow$ $\theta = \frac{\pi}{3}$

25. $|\vec{r} + b\vec{s}|$ is minimum

Let $f(b) = \sqrt{\vec{r}^2 + b^2\vec{s}^2 + 2\vec{r}.b\vec{s}}$

for maxima & minima

$$f'(b) = \frac{2b\vec{s}^2 + 2\vec{r}.\vec{s}}{\sqrt{\vec{r}^2 + b^2\vec{s}^2 + 2b\vec{r}.\vec{s}}} = 0$$

$b = -\frac{\vec{r}.\vec{s}}{\vec{s}^2}$

$$|b\vec{s}|^2 + |\vec{r} + b\vec{s}|^2 = b^2\vec{s}^2 + \vec{r}^2 + b^2\vec{s}^2 + 2b\,\vec{r}.\vec{s}$$

$$= 2b^2\vec{s}^2 + \vec{r}^2 - 2b^2\vec{s}^2 = |\vec{r}|^2$$

26. Angle between two vectors

$$= \frac{1\times1+(-1)(1)+(1)(-1)}{\sqrt{3}\,\sqrt{3}} = -\frac{1}{3}$$

Hence obtuse angle between them.

Vector along acute angle bisector

$$=\lambda\left[\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}-\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}\right]$$

$$\frac{2\lambda}{\sqrt{3}}\left[-\hat{j}+\hat{k}\right]=t(\hat{j}-\hat{k})$$

hence equation of acute angle bisector

$$= (\hat{i}+2\hat{j}+3\hat{k})+t(\hat{j}-\hat{k})$$

27. Line : $\frac{x-1}{2}=\frac{y+2}{-3}=\frac{z}{5}$

Plane : $x-y+z+2=0$

The vector perpendicular to required plane is

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -3 & 5\\ 1 & -1 & 2\end{vmatrix}=2\hat{i}+3\hat{j}+\hat{k}$$

Now equation of plane passing through (1, – 2, 0)

and perpendicular to $2\hat{i}+3\hat{j}+\hat{k}$

$$(x-1)2+(y+2)3+(z-0)1=0$$

$\Rightarrow$ $2x+3y+z+4=0$

28. $L_1 : \frac{x}{0}=\frac{y}{b}=\frac{z-c}{-c}=r$

$L_2 : \frac{x}{a}=\frac{y}{0}=\frac{z+c}{c}=\ell$

Dr's of AB are $-a\ell, br, -cr-c\ell+2c$

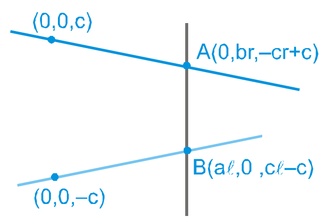
AB is perpendicular to both the lines

$\therefore$ $0(-a\ell)+b.\,br+(-c)(-cr-c\ell+2c)=0$

$(b^2+c^2)r+c^2\ell=2c^2$ .....**(1)**

and $a(-a\ell)+0(br)+c(-cr-c\ell+2c)=0$

$-(a^2+c^2)\ell-c^2r+2c^2=0$



$(a^2+c^2)\ell+c^2r=2c^2$ .....**(2)**

from (1) & (2)

$$\ell=\frac{2b^2c^2}{a^2b^2+b^2c^2+c^2a^2},\quad r=\frac{2a^2c^2}{a^2b^2+b^2c^2+c^2a^2}$$

$$A\left(0,\frac{2a^2bc^2}{a^2b^2+b^2c^2+c^2a^2},c\left(\frac{a^2b^2+b^2c^2-c^2a^2}{a^2b^2+b^2c^2+c^2a^2}\right)\right)$$

$$B\left(\frac{2ab^2c^2}{a^2b^2+b^2c^2+c^2a^2},0,c\left(\frac{b^2c^2-a^2b^2-c^2a^2}{a^2b^2+b^2c^2+c^2a^2}\right)\right)$$

$$4d^2=\frac{4a^2b^4c^4}{(a^2b^2+b^2c^2+c^2a^2)^2}+\frac{4a^4b^2c^4}{(a^2b^2+b^2c^2+c^2a^2)^2}+\frac{4c^2(a^4b^4)}{(a^2b^2+b^2c^2+c^2a^2)^2}$$

$$\frac{1}{d^2}=\frac{(a^2b^2+b^2c^2+c^2a^2)^2}{a^2b^4c^4+a^4b^2c^4+a^4b^4c^2}=\frac{a^2b^2+b^2c^2+c^2a^2}{a^2b^2c^2}$$

$$\frac{1}{d^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$$

29. Given $\overrightarrow{OP_{n-1}}+\overrightarrow{OP_{n+1}}=\frac{3}{2}\overrightarrow{OP_n}$ $n=2,3$

(a) Let P_1 & P_2 be $\left(t_1,\frac{1}{t_1}\right)$ & $\left(t_2,\frac{1}{t_2}\right)$

for $n=2$

$$\overrightarrow{OP_1}+\overrightarrow{OP_3}=\frac{3}{2}\overrightarrow{OP_2}$$

$$\Rightarrow \overrightarrow{OP_3}=\frac{3}{2}\left(t_2\hat{i}+\frac{1}{t_2}\hat{j}\right)-t_1 i-\frac{1}{t_1}\hat{j}$$

or $\overrightarrow{OP_3}=\left(\frac{3}{2}t_2-t_1\right)\hat{i}+\left(\frac{3}{2t_2}-\frac{1}{t_1}\right)\hat{j}$

Point $P_3=\left(\frac{3t_2-2t_1}{2},\frac{3t_1-2t_2}{2t_1t_2}\right)$

which does not lie on xy = 1

(b) Let P_1 & P_3 on circle $x^2 + y^2 = 1$ are $(\cos\alpha, \sin\alpha)$, $(\cos\beta, \sin\beta)$

For $n = 2$, $\overrightarrow{OP_1} + \overrightarrow{OP_3} = \frac{3}{2}\overrightarrow{OP_2}$

$$\overrightarrow{OP_2} = \frac{2}{3}\{(\cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}) + (\cos\beta\,\hat{i} + \sin\beta\,\hat{j})\}$$

$$\overrightarrow{OP_2} = \frac{2}{3}\{(\cos\alpha + \cos\beta)\hat{i} + (\sin\alpha + \sin\beta)\hat{j}\}$$

As P_2 lies on the circle then

$$|\overrightarrow{OP_2}| = 1$$

$$\frac{4}{9}\{(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2\} = 1$$

$$2 + 2\cos(\alpha - \beta) = \frac{9}{4}$$

$$\Rightarrow \cos(\alpha - \beta) = \frac{1}{8}$$

$$\overrightarrow{OP_4} = \frac{3}{2}\overrightarrow{OP_3} - \frac{2}{3}\left(\overrightarrow{OP_1} + \overrightarrow{OP_3}\right)$$

$$= \frac{5}{6}\overrightarrow{OP_3} - \frac{2}{3}\overrightarrow{OP_1}$$

$$= \left(\frac{5}{6}\cos\alpha - \frac{2}{3}\cos\beta\right)\hat{i} + \left(\frac{5}{6}\sin\alpha - \frac{2}{3}\sin\beta\right)\hat{j}$$

$$|\overrightarrow{OP_4}|^2 = \frac{25}{36} + \frac{4}{9} - 2.\frac{5}{6}.\frac{2}{3}\cos(\alpha - \beta) = 1$$

$\Rightarrow$ P_4 lies on $x^2 + y^2 = 1$

30. $3\hat{i} + 3\hat{k}$

31. a **(i)** $\overrightarrow{AB} = 3\hat{i} - \hat{j} - \hat{k}$ $\quad \overrightarrow{AC} = 4\hat{i} + 2\hat{j} + 4\hat{k}$

$\overrightarrow{AD} = 2\hat{i} + 2\hat{j}$

$$V = \frac{1}{6}\begin{vmatrix} 3 & -1 & -1 \\ 4 & 2 & 4 \\ 2 & 2 & 0 \end{vmatrix} = 6 \text{ cubic unit}$$

a **(ii)** Equation of line AB is

$$\vec{r} = \hat{j} + 2\hat{k} + \lambda\ (3\hat{i} - \hat{j} - \hat{k})$$

Equation of Line CD is

$$\vec{r} = 4\hat{i} + 3\hat{j} + 6\hat{k} + \mu(-2\hat{i} - 4\hat{k})$$

Shortest distance $= \dfrac{(a_2 - a_1).(b_1 \times b_2)}{|b_1 \times b_2|}$

$$= \frac{[(4\hat{i} + 3\hat{j} + 6\hat{k}) - (\hat{j} + 2\hat{k})].[(3\hat{i} - \hat{j} - \hat{k}) \times (-2\hat{i} - 4\hat{k})]}{|(3\hat{i} - \hat{j} - \hat{k}) \times (2\hat{i} - 4\hat{k})|}$$

$$= \frac{[4\hat{i} + 2\hat{j} + 4\hat{k}]\cdot[4\hat{i} + 14\hat{j} - 2\hat{k}]}{|4\hat{i} + 14\hat{j} - 2\hat{k}|}$$

$$= \frac{16 + 28 - 8}{\sqrt{16 + 196 + 4}} = \frac{36}{\sqrt{216}} = \frac{26}{2\sqrt{54}} = \frac{18}{3\sqrt{6}} = \sqrt{6}$$

(b) $\overrightarrow{AD} = -2\hat{i} + 2\hat{j} - \hat{k}$, $\overrightarrow{AC} = \hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore$ vector perpendicular to the face ADC is

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 2 & -1 \\ 1 & 2 & 2 \end{vmatrix} = 6\hat{i} + 3\hat{j} - 6\hat{k}$$

$$\overrightarrow{AB} = 3\hat{j} + 4\hat{k}$$

$\therefore$ A vector perpendicular to the face ABC is

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 4 \\ 1 & 2 & 2 \end{vmatrix} = -2\hat{i} + 4\hat{j} - 3\hat{k}$$

$\therefore$ acute angle between the two faces is given by

$$\cos\theta = \left|\frac{-12 + 12 + 18}{\sqrt{36 + 9 + 36}\sqrt{4 + 16 + 9}}\right| = \frac{2}{\sqrt{29}}$$

$\therefore$ $\tan\theta = \dfrac{5}{2}$ $\qquad$ $\therefore$ $\theta = \tan^{-1}\dfrac{5}{2}$

32. $\overrightarrow{OP} = \hat{i} + 2\hat{j} + 2\hat{k}$

after rotation of $\overrightarrow{OP}$, let new vector is $\overrightarrow{OP}\,'$

Now $\overrightarrow{OP}$, $\hat{i}$, $\overrightarrow{OP}\,'$ will be coplanar

So $\overrightarrow{OP}\,' = |\overrightarrow{OP}|\dfrac{(\overrightarrow{OP} \times \hat{i}) \times \overrightarrow{OP}}{|(\overrightarrow{OP} \times \hat{i}) \times \overrightarrow{OP}|}$ $\quad \left[\because |\overrightarrow{OP}| = |\overrightarrow{OP}\,'|\right]$

But $(\overrightarrow{OP} \times \hat{i}) \times \overrightarrow{OP} = 8\hat{i} - 2\hat{j} - 2\hat{k}$

$$\Rightarrow \overrightarrow{OP}\,' = \frac{3(8\hat{i} - 2\hat{j} - 2\hat{k})}{2 \times 3\sqrt{2}}$$

or $\overrightarrow{OP}\,' = \dfrac{4}{\sqrt{2}}\hat{i} - \dfrac{1}{\sqrt{2}}\hat{j} - \dfrac{1}{\sqrt{2}}\hat{k}$

33. $\vec a\times\vec b-\vec c\times\vec b+\vec c\times\vec a-\vec c\times\vec c$

$(\vec a-\vec c)\times\vec b+\vec c\times(\vec a-\vec c)=0$

$(\vec a-\vec c)\times(\vec b-\vec c)=0$

$\vec{CA}\times\vec{CB}=0$ $\therefore$ $\vec{BC}$ is || to $\vec{AC}$

$\vec{BC}=\pm 14\left(\frac{2\hat i-3\hat j+6\hat k}{7}\right)=\pm(4\hat i-6\hat j+12\hat k)$

34. O (0,0), A(1,0) & B (–1, 0)

Let P (x,y)

$\overline{PA}=(1-x)\hat i-y\hat j$

$\overline{PB}=-(1+x)\hat i-y\hat j$

$\overline{PA}\cdot\overline{PB}+3\,\overline{OA}.\overline{OB}=0$

$\Rightarrow$ $(x^2-1)+y^2-3=0$

$x^2+y^2=4$ ... **(1)**

$|\overline{PA}||\overline{PB}|=\sqrt{(x-1)^2+y^2}\ \sqrt{(x+1)^2+y^2}$

$=\sqrt{5-2x}\cdot\sqrt{5+2x}$

$=\sqrt{25-4x^2}$, $x\in(-2,2)$ (from **(1)**)

so M = 5, m = 3

$\Rightarrow$ $M^2+m^2=25+9=34$

35. Let the plane is

$(2x+3y-z)+1+\lambda(x+y-2z+3)=0$ ....**(1)**

$(2+\lambda)x+(3+\lambda)y-(1+2\lambda)z+1+3\lambda=0$

$3(2+\lambda)-(3+\lambda)+2(1+2\lambda)=0$

$6\lambda+5=0$ $\Rightarrow$ $\lambda=-5/6$

Putting value of λ in **(1)**

$7x+13y+4z-9=0$

Now image of (1, 1, 1) in plane π is

$\frac{x-1}{7}=\frac{y-1}{13}=\frac{z-1}{4}=-2\left(\frac{7+13+4-9}{49+169+16}\right)$

$\Rightarrow$ $\frac{x-1}{7}=\frac{y-1}{13}=\frac{z-1}{4}=-\frac{15}{117}$

$x=\frac{12}{117}$, $y=\frac{-78}{117}$, $z=\frac{57}{117}$

36. $\lambda=-2\pm\sqrt{29}$

37. Equation of plane passing through (1, 1, 1) is

$a(x-1)+b(y-1)+c(z-1)=0$ ... **(1)**

$\because$ it passes through (1, – 1, 1) and (– 7, – 3, – 5)

$\therefore$ $a.0-2.b+0.c=0$ $\Rightarrow$ $b=0$

and $-8a-4b-6c=0$

$4a+2b+3c=0$ $\because$ $b=0$

$\therefore$ $4a+3c=0$ $\Rightarrow$ $c=-\frac{4a}{3}$

$\therefore$ dr's of normal to the plane are $1, 0-\frac{4}{3}$

and dr's of the normal to the x-z plane are 0, 1, 0

$\therefore$ $\cos\theta=\left|\frac{0+0+0}{\sqrt{\Sigma a^2}\sqrt{\Sigma a_1^2}}\right|=0$ $\therefore$ $\theta=\frac{\pi}{2}$

38. $\vec x\times\vec a+(\vec x.\vec b)\vec a=\vec c$ ....**(i)**

taking cross product with $\vec b$:

$(\vec x\times\vec a)\times\vec b+(\vec x.\vec b)(\vec a\times\vec b)=\vec c\times\vec b$

$(\vec x.\vec b)\vec a-(\vec a.\vec b)\vec x+(\vec x.\vec b)(\vec a\times\vec b)=\vec c\times\vec b$ ....**(ii)**

Now taking dot product with $\vec a$ in **(i)**

$(\vec x.\vec b)a^2=\vec a.\vec c$

$\vec x.\vec b=\frac{\vec a.\vec c}{a^2}$

$\frac{(\vec a.\vec c)}{a^2}\vec a-(\vec a.\vec b)\vec x+\frac{\vec a.\vec c}{a^2}(\vec a\times\vec b)=\vec c\times\vec b$

$\frac{1}{(\vec a.\vec b)}\left[\frac{(\vec a.\vec c)}{a^2}\vec a+\frac{\vec a.\vec c}{a^2}\left(\vec a\times\vec b\right)-\vec c\times\vec b\right]=\vec x$

$\vec x=\frac{1}{(\vec a.\vec b)}\left[\frac{\vec a.\vec c}{a^2}\left(\vec a-\vec b\times\vec a\right)+\vec b\times\vec c\right]$

39. $SD=\frac{(\hat i-\hat j+2\hat k-4\hat i+\hat j)\cdot[(\hat i+2\hat j-3\hat k)\times(2\hat i+4\hat j-5\hat k)]}{|(\hat i+2\hat j-3\hat k)\times(2\hat i+4\hat j-5\hat k)|}$

$=\left|\frac{(-3\hat i+2\hat k).(2\hat i-\hat j)}{|2\hat i-\hat j|}\right|=\frac{6}{\sqrt5}$

40. $x = \dfrac{\dfrac{\vec{a}\times\vec{b}}{\gamma} - \vec{a}\times\dfrac{\vec{a}\times\vec{b}}{\gamma}}{\left(\dfrac{\vec{a}\times\vec{b}}{\gamma}\right)^2}$;

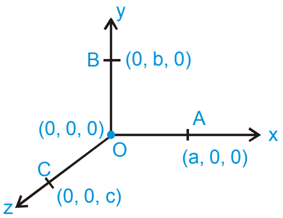
$y = \dfrac{\vec{a}\times\vec{b}}{\gamma}$; $z = \dfrac{\dfrac{\vec{a}\times\vec{b}}{\gamma} + \vec{b}\times\dfrac{\vec{a}\times\vec{b}}{\gamma}}{\left(\dfrac{\vec{a}\times\vec{b}}{\gamma}\right)^2}$

41. Let the required point be $P(\alpha, \beta, \gamma)$

$OP = PA = PB = PC$

$\therefore$ $OP^2 = PA^2 = PB^2 = PC^2$

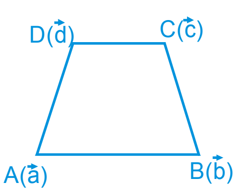
$\alpha^2 + \beta^2 + \gamma^2 = (\alpha - a)^2 + \beta^2 + \gamma^2 = \alpha^2 + (\beta - b)^2 + y^2 = \alpha^2 + \beta^2 + (\gamma - c)^2$



$\therefore$ $\alpha = \dfrac{a}{2}$; $\beta = \dfrac{b}{2}$; $\gamma = \dfrac{c}{2}$

$\therefore$ required point is $\left(\dfrac{a}{2}, \dfrac{b}{2}, \dfrac{c}{2}\right)$

42.



In cyclic quadrilateral

tanA + tan C = 0

$\Rightarrow \dfrac{|\overrightarrow{AB}\times\overrightarrow{AD}|}{\overrightarrow{AB}.\overrightarrow{AD}} + \dfrac{|\overrightarrow{CB}\times\overrightarrow{CD}|}{\overrightarrow{CB}.\overrightarrow{CD}} = 0$

$\Rightarrow \dfrac{|(\vec{b}-\vec{a})\times(\vec{d}-\vec{a})|}{(\vec{b}-\vec{a}).(\vec{d}-\vec{a})} + \dfrac{|(\vec{b}-\vec{c})\times(\vec{d}-\vec{c})|}{(\vec{b}-\vec{c}).(\vec{d}-\vec{c})} = 0$

$\Rightarrow \dfrac{|\vec{a}\times\vec{b}+\vec{b}\times\vec{d}+\vec{d}\times\vec{a}|}{(\vec{b}-\vec{a}).(\vec{d}-\vec{a})} + \dfrac{|\vec{b}\times\vec{c}+\vec{c}\times\vec{d}+\vec{d}\times\vec{b}|}{(\vec{b}-\vec{c}).(\vec{d}-\vec{c})} = 0$

43. $\therefore$ $3.1 - 2.4 + 5\times 1 = 0$, line is parallel to the plane

$\therefore$ reflection of line will also have same direction ratios i.e. 3, 4, 5

Also mirror image of (1, 2, 3) will be on required line.

$\dfrac{x-1}{1} = \dfrac{y-2}{-2} = \dfrac{z-3}{1} = -2\left(\dfrac{1-4+3-6}{1^2+1^2+(-2)^2}\right)$

$(x, y, z) = (3, -2, 5)$

$\therefore$ equation of straight line $\dfrac{x-3}{3} = \dfrac{y+2}{4} = \dfrac{z-5}{5}$

44. Planes are $x - 2y + z = 1$ ....(i)

$x + 2y - 2z = 5$ ....(ii)

$2x + 2y + z = -6$ ....(iii)

Add (i) + (ii) + (iii)

$4x + 2y = 0 \Rightarrow y = -2x$ ....(iv)

From equations (iii) – (i)

$x + 4y = -7$ ....(v)

from (iv) and (v) we get

$x = 1, y = -2$

Put in (i) we get $z = -4$

So point of intersection is $(1, -2, -4)$

45. $2r + 1 - (3r + 2) + 2(4r + 3) + 2 = 0$

$7r + 7 = 0 \Rightarrow r = -1$

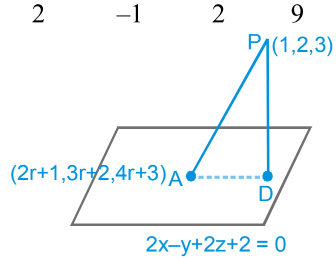
$\therefore$ $A(-1, -1, -1)$

required line will be projection of given line in the plane

foot of $\perp$ of P will be on D

$\dfrac{x-1}{2} = \dfrac{y-2}{-1} = \dfrac{z-3}{2} = -\left(\dfrac{2.1-2+2.3+2}{2^2+(-1)^2+2^2}\right)$

$\dfrac{x-1}{2} = \dfrac{y-2}{-1} = \dfrac{z-3}{2} = \dfrac{-8}{9}$



$x = \dfrac{-7}{9}$; $y = \dfrac{26}{9}$; $z = \dfrac{11}{9}$

$\dfrac{x+1}{2/9} = \dfrac{y+1}{35/9} = \dfrac{z+1}{20/9}$

$\dfrac{x+1}{2} = \dfrac{y+1}{35} = \dfrac{z+1}{20}$

46. $\vec{x} + \vec{c} \times \vec{y} = \vec{a}$ ......**(i)**

$\vec{y} + \vec{c} \times \vec{x} = \vec{b}$ .....**(ii)**

$\Rightarrow \quad \vec{y} = \vec{b} - \vec{c} \times \vec{x}$ put in **(i)**

$\vec{x} + \vec{c} \times \vec{b} - \vec{c} \times (\vec{c} \times \vec{x}) = \vec{a}$

$\vec{x} - (\vec{c}.\vec{x})\,\vec{c} + (\vec{c}.\vec{c})\vec{x} = \vec{a} - \vec{c} \times \vec{b}$

$(1 + c^2)\ \vec{x} = \vec{a} - \vec{c} \times \vec{b} + (\vec{c}.\vec{x})\,\vec{c}$ ......**(iii)**

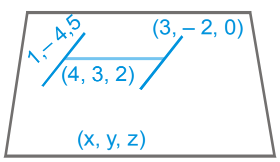
Taking both side dot product with $\vec{c}$ in equation **(i)**

We get $\vec{x}.\vec{c} = \vec{a}.\vec{c}$, (put in **(iii)**)

$$\vec{x} = \frac{a + (\vec{a}.\vec{c})\vec{c} + \vec{b} \times \vec{c}}{1 + \vec{c}^2}$$

Putting in **(ii)**, we get $\vec{y} = \dfrac{\vec{b} + (\vec{c}.\vec{b})\vec{c} + \vec{a} \times \vec{c}}{1 + (\vec{c})^2}$

47. $\dfrac{x-4}{1} = \dfrac{y-3}{-4} = \dfrac{z-2}{5}$ .... **(1)**



$\dfrac{x-3}{1} = \dfrac{y+2}{1/\lambda} = \dfrac{z-0}{1/\mu}$ .... **(2)**

Equation of the plane is

$$\begin{vmatrix} x-3 & y+2 & z \\ 1 & 5 & 2 \\ 1 & -4 & 5 \end{vmatrix} = 0$$

$(x-3)(25+8) - (y+2)(5-2) + z(-4-5) = 0$

$33x - 99 - 3y - 6 - 9z = 0$

$33x - 3y - 9z - 105 = 0$

$11x - y - 3z = 35$

48. $\vec{a} = \sqrt{3}\,i - \hat{j}$, $\vec{b} = \dfrac{1}{2}\hat{i} + \dfrac{\sqrt{3}}{2}\hat{j}$

$\Rightarrow \quad \vec{a}.\vec{b} = 0$

$\vec{x}.\vec{y} = 0$ (given)

$(\vec{a} + (q^2 - 3)\vec{b}).(-p\vec{a} + q\vec{b}) = 0$

$\Rightarrow \quad p = \dfrac{q(q^2 - 3)}{4} = f(q)$

for monotonocity

$p' = 3q^2 - 3$

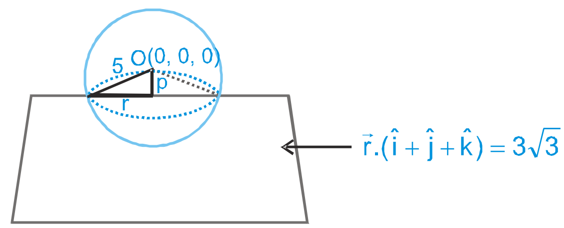
if p' < 0 then f(q) is decreasing

$\Rightarrow \quad (q-1)(q+1) < 0$

$\Rightarrow \quad -1 < q < 1$

Decreasing for $q \in (-1, 1)$, $q \neq 0$

49.



$x + y + z - 3\sqrt{3} = 0$

$$p = \left|\frac{-3\sqrt{3}}{\sqrt{3}}\right| = 3$$

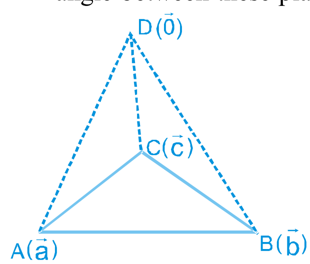
$\Rightarrow \quad r = 4$

50. (a) Since tetrahedron is regular AB = BC = AC = DC and angle between two adjcant side = $\pi/3$ consider planes ABD and DBC

vector, normal to plane ABD is = $\vec{a} \times \vec{b}$

vector, normal to plane DBC is = $\vec{b} \times \vec{c}$

angle between these planes is angle between



vectors $(\vec{a} \times \vec{b})$ & $(\vec{b} \times \vec{c})$

$$\Rightarrow \cos\theta = \frac{(\vec{a}\times\vec{b}).(\vec{b}\times\vec{c})}{|\vec{a}\times\vec{b}||\vec{b}\times\vec{c}|} = \frac{-\frac{1}{4}|\vec{b}|^2|\vec{a}||\vec{c}|}{\frac{3}{4}|\vec{a}||\vec{b}|^2|\vec{c}|} = -\frac{1}{3}$$

Since acute angle is required $\theta = \cos^{-1}\left(\dfrac{1}{3}\right)$

(b) circum-radius $\equiv$ distance of circum centre from any of the vertex

$\equiv$ distance of $\frac{\vec{a}+\vec{b}+\vec{c}}{4}$ from vertex D $(\vec{0})$ [tetrahedron is regular]

Circumradius

$$=\frac{1}{4}\left|\vec{a}+\vec{b}+\vec{c}\right|=\frac{1}{4}\sqrt{\vec{a}^2+\vec{b}^2+\vec{c}^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})}$$

$$=\frac{1}{4}\sqrt{k^2+k^2+k^2+2\left(\frac{k^2}{2}+\frac{k^2}{2}+\frac{k^2}{2}\right)}$$

$$=\frac{1}{4}\sqrt{6k^2}=\sqrt{\frac{3}{8}}\,k$$

$$\frac{r}{R}=\frac{1}{3} \quad \Rightarrow r=\frac{R}{3}=\frac{k}{\sqrt{24}}$$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

6. We have,

$\vec{u}.\hat{n}=0$ and $\vec{v}.\hat{n}=0$

$\Rightarrow \hat{n}\perp\vec{u}$ and $\hat{n}\perp\vec{v}$

$\Rightarrow \hat{n}=\pm\frac{\vec{u}\times\vec{v}}{|\vec{u}\times\vec{v}|}$

Now, $\vec{u}\times\vec{v}=(\hat{i}+\hat{j})\times(\hat{i}-\hat{j})=-2\hat{k}$

$\therefore \hat{n}=\pm\hat{k}$

Hence, $|\vec{w}.\hat{n}|=|(\hat{i}+2\hat{j}+3\hat{k}).(\pm\hat{k})|=3$

7. We have,

$\vec{F}$ = Total force = $7\hat{i}+2\hat{j}-4\hat{k}$

$\vec{d}$ = Displacement vector = $4\hat{i}+2\hat{j}-2\hat{k}$

$\Rightarrow$ Work done = $\vec{F}.\vec{d}=(28+4+8)$ units
= 40 units

8. Let D be the mid-point of BC. Then,

$$\overrightarrow{AD}=\frac{\overrightarrow{AB}+\overrightarrow{AC}}{2}$$

$\Rightarrow |\overrightarrow{AD}|=4\hat{i}+\hat{j}+4\hat{k}$

$\Rightarrow |\overrightarrow{AD}|=\sqrt{16+1+16}=\sqrt{33}$

Hence, required length = $\sqrt{33}$ units.

9. We have,

$\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|=\vec{0} \quad \Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2=0$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

$\Rightarrow 1+4+9+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

$\Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-7$

11. We have,

$(\vec{u}+\vec{v}-\vec{w}).(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})$

$=(\vec{u}+\vec{v}-\vec{w}).(\vec{u}\times\vec{v}-\vec{u}\times\vec{w}-\vec{v}\times\vec{v}+\vec{v}\times\vec{w})$

$=(\vec{u}+\vec{v}-\vec{w}).(\vec{u}\times\vec{v}-\vec{u}\times\vec{w}+\vec{v}\times\vec{w})$

$=\vec{u}.(\vec{u}\times\vec{v})-\vec{u}.(\vec{u}\times\vec{w})\times\vec{u}+(\vec{v}\times\vec{w})$

$+\vec{v}.(\vec{u}\times\vec{v})-\vec{v}.(\vec{u}\times\vec{w})+\vec{v}.(\vec{v}\times\vec{w})$

$-\vec{w}.(\vec{u}\times\vec{v})+\vec{w}.(\vec{u}\times\vec{w})-\vec{w}.(\vec{v}\times\vec{w})$

$=\vec{u}.(\vec{v}\times\vec{w})-\vec{v}.(\vec{u}\times\vec{w})-\vec{w}.(\vec{u}\times\vec{v})$

$=[\vec{u}\,\vec{v}\,\vec{w}]-[\vec{v}\,\vec{u}\,\vec{w}]-[\vec{w}\,\vec{u}\,\vec{v}]$

$=[\vec{u}\,\vec{v}\,\vec{w}]+[\vec{u}\,\vec{v}\,\vec{w}]-[\vec{u}\,\vec{v}\,\vec{w}]$

$=[\vec{u}\,\vec{v}\,\vec{w}]=\vec{u}.(\vec{v}\times\vec{w})$

12. It is given that

$\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$

$\Rightarrow$ $\vec{a}+2\vec{b}=\lambda\vec{c}$ and $\vec{b}+3\vec{c}=\mu\vec{a}$ for some scalar λ and μ.

$\Rightarrow$ $\vec{b}+3\vec{c}=\mu(\lambda\vec{c}-2\vec{b})$

$\Rightarrow$ $(2\mu+1)\vec{b}+(3-\mu\lambda)\vec{c}=\vec{0}$

$\Rightarrow$ $2\mu+1=0$ and $3-\mu\lambda=0$

$\Rightarrow$ $\mu=-\frac{1}{2}$ ' $\lambda=-6$ $\left[\begin{array}{c}\because \vec{b}\text{ and }\vec{c}\\ \text{are non-collinear}\end{array}\right]$

$\therefore$ $\vec{a}+2\vec{b}=\lambda\vec{c}$

$\Rightarrow$ $\vec{a}+2\vec{b}=-6\vec{c}$ $\Rightarrow$ $\vec{a}+2\vec{b}+6\vec{c}=\vec{0}$

14. Let $\vec{\alpha}=\vec{a}+2\vec{b}+3\vec{c},\ \beta=\lambda\vec{b}+4\vec{c}$ and $\vec{\gamma}=(2\lambda-1)\vec{c}$.

Then, $[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}]=\begin{vmatrix}1 & 2 & 3\\ 0 & \lambda & 4\\ 0 & 0 & (2\lambda-1)\end{vmatrix}[\vec{a}\,\vec{b}\,\vec{c}]$

$\Rightarrow$ $[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}]=\lambda(2\lambda-1)\,[\vec{a}\,\vec{b}\,\vec{c}]$

$\Rightarrow$ $[\alpha\ \vec{\beta}\ \vec{\gamma}]=0$, if $\lambda=0,\ \frac{1}{2}$ $\quad[\because\ [\vec{a}\,\vec{b}\,\vec{c}]\neq 0]$

Hence, $\vec{\alpha},\vec{\beta},\vec{\gamma}$ are non-coplanar for all values of λ except two values 0 and $\frac{1}{2}$.

16. $(a\times b)\times c=1/3|b|\,|c|\,a$

$\Rightarrow$ $(a.c)b-(b.c)a=1/3|b||c|\,a$

$\Rightarrow$ $(a.c)\,b=\left\{(b.c)+\frac{1}{3}|b||c|\right\}a$

$\Rightarrow$ $(a.c)b=|b|\,|c|\left\{\cos\theta+\frac{1}{3}\right\}a$

As a and b are not parallel, a.c=0 and $\cos\theta+\frac{1}{3}=0$

$\Rightarrow$ $\cos\theta=-\frac{1}{3}$. Hence $\sin\theta=\frac{2\sqrt{2}}{3}$

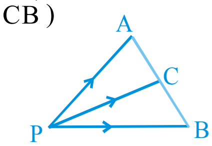
17. $\overrightarrow{PA}+\overrightarrow{PB}=(\overrightarrow{PA}+\overrightarrow{AC})+(\overrightarrow{PB}+\overrightarrow{BC})-(\overrightarrow{AC}+\overrightarrow{BC})$

$=\overrightarrow{PC}+\overrightarrow{PC}-(\overrightarrow{AC}-\overrightarrow{CB})$

$=2\overrightarrow{PC}-0$

$(\because\ \overrightarrow{AC}=\overrightarrow{CB})$

$\therefore$ $\overrightarrow{PA}+\overrightarrow{PB}=2\overrightarrow{PC}$



21. $[abc]=\begin{vmatrix}1 & 0 & -1\\ x & 1 & 1-x\\ y & x & 1+x-y\end{vmatrix}=\begin{vmatrix}1 & 0 & 0\\ x & 1 & 1\\ y & x & 1+x\end{vmatrix}=\begin{vmatrix}1 & 1\\ x & 1+x\end{vmatrix}=1$

22. $(\vec{a}\times\vec{b})\times\vec{c}=\vec{a}\times(\vec{b}\times\vec{c})$

$\Rightarrow$ $(\vec{a}.\vec{c})\vec{b}-(\vec{b}.\vec{c})\vec{a}=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$

$\Rightarrow$ $(\vec{b}.\vec{c})\vec{a}=(\vec{a}.\vec{b})\vec{c}$

So that $\vec{a}$ is parallel to $\vec{c}$

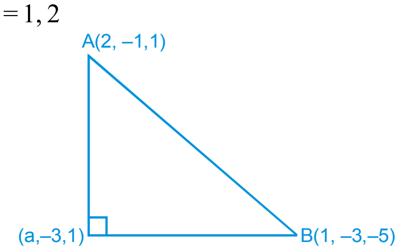
24. $AC\perp BC$

$\therefore$ dr's of AC and BC will be (2–a,2,0) and (1–a,0,–6)

So that $(2-a)(1-a)+2\times 0+0\times(-6)=0$

$\Rightarrow$ $a^2-3a+2=0$

$\therefore$ $a=1,2$



29. $[3\vec{u}\ \ p\vec{v}\ \ p\vec{w}]-[p\vec{v}\ \ \vec{w}\ \ q\vec{u}]-[2\vec{w}\ q\vec{v}\ q\vec{u}]=0$

$3p^2[\vec{u}\ \vec{v}\ \vec{w}]-pq[\vec{v}\ \ \vec{w}\ \ \vec{u}]-2q^2[\vec{w}\ \ \vec{v}\ \ \vec{u}]=0$

$(3p^2-pq+2q^2).[\vec{u}\ \ \vec{v}\ \ \vec{w}]=0$

$3p^2-pq+2q^2=0$

has exactly one solution

$p=q=0$

30. $(\vec{a}\times\vec{b})+\vec{c}=0$

$(\vec{a}\times\vec{b})=-\vec{c}$

$\Rightarrow \vec{a}\times(\vec{a}\times\vec{b})=-\vec{a}\times\vec{c}$

$\Rightarrow (\vec{a}.\vec{b})\vec{a}-|\vec{a}|^2\,\vec{b}=-\vec{a}\times\vec{c}$

$\Rightarrow 3(\hat{j}-\hat{k})-2\vec{b}=-(-2i-j-k)$

$(\vec{a}\times\vec{c}=-2i-j-k)$

$\Rightarrow 2\vec{b}=(-2i+2j-4k)$

$\Rightarrow \vec{b}=-i+j-2k$

31. Give $\vec{a} \perp \vec{b}$, $\vec{a} \perp \vec{c}$ & $\vec{b} \perp \vec{c}$

so $\vec{a}.\vec{c} = 0$ & $\vec{b}.\vec{c} = 0$

$\Rightarrow \lambda - 1 + 2\mu = 0$ & $2\lambda + 4 + \mu = 0$

$\Rightarrow \lambda = -3$ & $\mu = 2$

32. a.b. ≠ 0

a.d = 0

b × c = b × d

a × (b × c) = a × (b × d)

(a.c)b – (a.b)c – (a.c)b – (a.b)d {a.d=0}

$\Rightarrow$ (a.b)d = (a.b)c (a.c)b (divide by a.b)

$$\boxed{d = c - \frac{(a.c)}{(a.b)} b}$$

33. $\vec{a}.\vec{b} = 0$ and |a| = |b| = 1

$(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b}) = (\vec{a} \times \vec{b}) \times \vec{a} + (\vec{a} \times \vec{b}) \times 2\vec{b}$

$= -\left[\vec{a} \times (\vec{a} \times \vec{b}) + 2\vec{b} \times (\vec{a} \times \vec{b})\right]$

$= -\left[(\vec{a}.\vec{b})\vec{a} - (\vec{a}\,.\,\vec{a})\vec{b} + 2(\vec{b}\,.\vec{b})\vec{a} - 2(\vec{b}\,.\,\vec{a})\vec{b}\right]$

$= -\left[0 - \vec{b} + 2\vec{a} + 0\right] = \left[\vec{b} - 2\vec{a}\right]$

$\therefore (2\vec{a} - \vec{b}) . \left[(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})\right]$

$= (2\vec{a} - \vec{b}) . (\vec{b} - 2\vec{a})$

$= -4a^2 - b^2 + 4\,\vec{a}\,.\,\vec{b} = -5$

34. $\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} = 0$

p(qr – 1) – (r – 1) + (1 – q) = 0

pqr – p – r + 1 + 1 – q = 0

pqr – (p + r + q) + 2 = 0

pqr – (p + r + q) = –2

35. Let

$a + 3b = \lambda\,\vec{c}$

add $6\vec{c}$ both side

$\vec{a} + 3\vec{b} + 6\vec{c} = (\lambda + 6)\vec{c}$

Let

$\vec{b} + 2\vec{c} = \mu\vec{a}$

$3b + 6\vec{c} = 3\mu\,\vec{a}$

add $\vec{a}$ both side

$\vec{a} + 3\vec{b} + 6\vec{c} = (3\mu + 1)\vec{a}$

Hence $(\lambda + 6)\vec{c} = (3\mu + 1)\vec{a}$

But given $\vec{a}$ and $\vec{c}$ are non coliner

Hence $\lambda + 6 = 3\mu + 1 = 0$

so $\vec{a} + 3\vec{b} + 6\vec{c} = \vec{0}$

36. $\vec{c}.\vec{d} = 0$

$\Rightarrow (\hat{a} + 2\hat{b}).(5\hat{a} - 4\hat{b}) = 0$

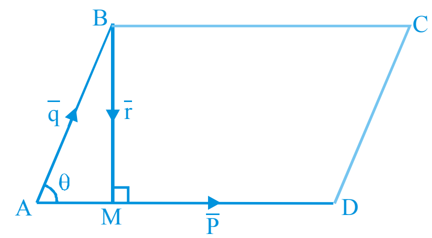
$\Rightarrow 5 - 8 + 6\,\hat{a}\,.\,\hat{b} = 0$

$\Rightarrow \hat{a}\,.\,\hat{b} = 1/2$

$\Rightarrow \cos\theta = 1/2$

$\Rightarrow \theta = \frac{\pi}{3}$

37.



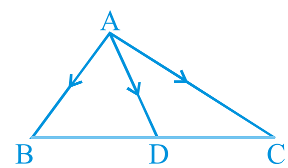
$\bar{q} + \bar{r} = \overline{AM}$

$\Rightarrow \bar{r} = -\bar{q} + \overline{AM}$

$\Rightarrow \bar{r} = -\bar{q} + \frac{\bar{p}.\bar{q}}{|\bar{p}|^2}\bar{p}$

$\Rightarrow \bar{r} = -\bar{q} + \left(\frac{\bar{p}.\bar{q}}{\bar{p}.\bar{p}}\right)\bar{p}$

38.



$\overrightarrow{AD} = \frac{\overline{AB} + \overline{AC}}{2} = 4\hat{j} - \hat{j} + 4\hat{k}$

$|\overrightarrow{AD}| = \sqrt{33}$

43. $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix}=0$

$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix}=0 \quad \Rightarrow \quad \begin{vmatrix} 0 & 0 & -1 \\ 2 & 1+k & -k \\ k+2 & 1 & 1 \end{vmatrix}=0$

$k^2+3k=0 \quad \Rightarrow \quad k(k+3)=0 \quad \Rightarrow \quad k=0 \text{ or } -3$

45. Let $\vec{n}_1$ and $\vec{n}_2$ be the vectors normal to the faces OAB and ABC. Then,

$$\vec{n}_1 = \overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 5\hat{i}-\hat{j}-3\hat{k}$$

and, $$\vec{n}_2 = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix} = \hat{i}-5\hat{j}-3\hat{k}$$

If θ is the angle between the faces OAB and ABC, then

$$\cos\theta = \frac{\vec{n}_1 . \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$$

$$\Rightarrow \cos\theta = \frac{5+5+9}{\sqrt{25+1+9}\sqrt{1+25+9}} = \frac{19}{35}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{19}{35}\right)$$

46. $\ell_1 - am_1 = 0$ and $cm_1 - n_1 = 0 \Rightarrow \frac{\ell_1}{a} = \frac{m_1}{1} = \frac{n_1}{c}$

Also $\ell_2 - a'm_2 = 0$ and $c'm_2 - n_2 = 0$

$$\Rightarrow \frac{\ell_2}{a'} = \frac{m_2}{1} = \frac{n_2}{c'}$$

$\therefore \quad \ell_1\ell_2 + m_1m_2 + n_1n_2 = aa' + cc' + 1 = 0$

47. Here, $\ell = \cos\theta,\ m = \cos\beta,\ n = \cos\theta,\ (\because \ell = n)$

Now, $\ell^2 + m^2 + n^2 = 1 \Rightarrow 2\cos^2\theta + \cos^2\beta = 1$

$\Rightarrow$ Given, $\sin^2\beta = 3\sin^2\theta \Rightarrow 2\cos^2\theta = 3\sin^2\theta$

$5\cos^2\theta = 3, \quad \therefore \cos^2\theta = \frac{3}{5}$

48. Given plane are $2x+y+2z-8=0$

or $4x+2y+4z-16=0$ ..... (i)

and $4x+2y+4z+5=0$ ..... (ii)

Distance between two parallel planes

$$= \left|\frac{-16-5}{\sqrt{4^2+2^2+4^2}}\right| = \frac{21}{6} = \frac{7}{2}$$

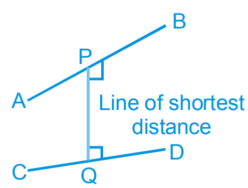
49. Let the two lines be AB and CD having equation

$$\frac{x}{1} = \frac{y+a}{1} = \frac{z}{1} = \lambda \text{ and } \frac{x+a}{2} = \frac{y}{1} = \frac{z}{1} = \mu$$

then $P \equiv (\lambda, \lambda - a, \lambda)$ and $Q = (2\mu - a, \mu, \mu)$

So according to question,

$$\frac{\lambda-2\mu+a}{2} = \frac{\lambda-a-\mu}{1} = \frac{\lambda-\mu}{2}$$



$\Rightarrow \mu = a$ and $\lambda = 3a$

$\therefore \quad P \equiv (3a, 2a, 3a)$

and $Q \equiv (a, a, 0)$

50. We have, $\frac{x-1}{1} = \frac{y+3}{-\lambda} = \frac{z-1}{\lambda} = s$

and $\frac{x-0}{1/2} = \frac{y-1}{1} = \frac{z-2}{-1} = t$

Since, lines are coplanar then

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} -1 & 4 & 1 \\ 1 & -\lambda & \lambda \\ 1/2 & 1 & -1 \end{vmatrix}=0$$

On solving, $\lambda = -2$

52. Angle between line and normal to plane is

$\cos\left(\frac{\pi}{2}-\theta\right) = \frac{1\times2-2\times1+2\sqrt{\lambda}}{3\times\sqrt{5+\lambda}}$, where θ is the angle between line and plane

$$\Rightarrow \sin\theta = \frac{1\times2+2\times(-1)+2\sqrt{\lambda}}{3\times\sqrt{5+\lambda}} \Rightarrow \frac{1}{3} = \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}}$$

$$\Rightarrow \frac{1}{3} = \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}} \Rightarrow \lambda = \frac{5}{3}$$

53. The lines are $\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ and $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$

Since, $a_1a_2 + b_1b_2 + c_1c_2 = 6 - 24 + 18 = 0$

$\Rightarrow \theta = 90°$

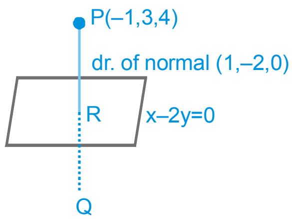
58. Equation of line PQ is

$$\frac{x+1}{1} = \frac{y-3}{-2} = \frac{z-4}{0} = \lambda$$

For some suitable value of λ, co-ordinates of point $Q(\lambda - 1, 3 - 2\lambda, 4)$

R is the mid point of P and Q.

$\therefore \quad R \equiv \left(\frac{\lambda-2}{2}, \frac{6-2\lambda}{2}, 4\right)$



$R \equiv \left(\frac{\lambda}{2} - 1, 3 - \lambda, 4\right)$

It satisfies $x - 2y = 0$

$\Rightarrow \lambda = \frac{14}{5}$

$\therefore \quad Q = \left(\frac{2}{5}, \frac{1}{5}, 4\right)$

59. If direction cosines of L be ℓ, m, n then

$2\ell + 3m + n = 0$

$\ell + 3m + 2n = 0$

Solving, we get, $\frac{\ell}{3} = \frac{m}{-3} = \frac{n}{3}$

$\therefore \ \ell : m : n = \frac{1}{\sqrt{3}} : -\frac{1}{\sqrt{3}} : \frac{1}{\sqrt{3}} \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

60. $\ell = \cos\frac{\pi}{4}$, $m = \cos\frac{\pi}{4}$

we know that $\ell^2 + m^2 + n^2 = 1$

$\frac{1}{2} + \frac{1}{2} + n^2 = 1 \Rightarrow n = 0$

Hence angle with positive direction of z-axis is $\frac{\pi}{2}$

64. Line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ .... **(1)**

Plane $x + 3y - \alpha z + \beta = 0$ .... **(2)**

Point (2, 1, –2) put in **(2)**

$2 + 3 + 2\alpha + \beta = 0$

$\Rightarrow 2\alpha + \beta = -5$

Now $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$3 - 15 - 2\alpha = 0$

$-12 - 2\alpha = 0$

$\alpha = -6$

$-12 + \beta = -5$

$\beta = 7$

$\alpha = -6, \beta = 7$

65. Proj. of a vector ($\vec{r}$) on x–axis $= |\vec{r}|\,\ell$

on y–axis $= |\vec{r}|\, m$

on z–axis $= |\vec{r}|\, n$

$6 = 7\ell, \Rightarrow \ell = \frac{6}{7}$ similarly $m = -\frac{3}{7}$, $n = \frac{2}{7}$

66. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ...**(i)**

$\alpha = 45°, \beta = 120°$

Put in equation **(i)**

$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1$

$\Rightarrow \cos^2\gamma = \frac{1}{4}$

$\Rightarrow \gamma = 60°$

67. Mirror image of B(1, 3, 4) in plane x–y+z = 5

$$\frac{x-1}{1} = \frac{y-3}{-1} = \frac{z-4}{1} = -2\frac{(1-3+4-5)}{1+1+1} = 2$$

$\Rightarrow x = 3, y = 1, z = 6$

$\therefore$ mirror image of B (1, 3, 4) is A (3, 1, 6)

statement-1 is correct

statement-2 is true but it is not the correct explanation.

68. $\frac{x}{1} = \frac{y-1}{2} = \frac{z-3}{\lambda}$ equation of line

equation of plane $x + 2y + 3z = 4$

$\sin\theta = \frac{1+4+3\lambda}{\sqrt{14}\sqrt{1+4+\lambda^2}}$

$\Rightarrow \lambda = \frac{2}{3}$

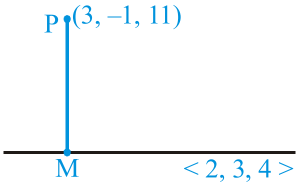
69. $1(1-1)+2(0-6)+3(7-3)$

$= 0 - 12 + 12 = 0$

mid point AB (1, 3, 5)

lies on $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$

70.



M(2r, 3r + 2, 4r + 3)

Dr's of PM < 2r – 3, 3r + 3, 4r – 8 >

$2(2r-3)+3(3r+3)+4(4r-8)=0$

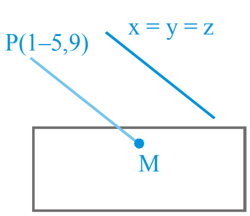
$29r-29=0$

$r = 1$

M(2, 5, 7)

Distance PM $=\sqrt{1+36+16}=\sqrt{53}$

71.



eq^n. of a line ‖ to x = y = z and passing through (1, –5, 9) is

$\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=r$

Let is meets plane at M(r+1, r–5, r+9)

Put in equation of plane

$x - y + z = 5$

$r + 1 - r + 5 + r + 9 = 5$

$r = -10$

Hence M (–9, –15, –1)

Distance PM $=\sqrt{100+100+100}=10\sqrt{3}$

72. Equation of plane parallel to

$x - 2y + 2z - 5 = 0$ is $x - 2y + 2z = k$

or $\frac{x}{3}-\frac{2}{3}y+\frac{2}{3}z=\frac{K}{3}$

$\left|\frac{K}{3}\right| = 1$

$\Rightarrow K = \pm 3$

$\therefore$ Equation of required plane is

$x - 2y + 2z \pm 3 = 0$

73. $\begin{vmatrix} 3-1 & K+1 & 0-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 2 & K+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow 2K - 9 = 0$

$\Rightarrow K = \frac{9}{2}$

74. $4x + 2y + 4z + 5 = 0$

$4x + 2y + 4z - 16 = 0$

$\Rightarrow d = \left|\frac{21}{\sqrt{36}}\right| = \frac{7}{2}$

75. $\Rightarrow (\vec{a}-\vec{b}).(\vec{c}\times\vec{d}) = 0$

$\Rightarrow \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow (1 + 2k) + (1 + k^2) - (2 - k) = 0$

$\Rightarrow k^2 + 3k = 0 \begin{smallmatrix} 0 \\ -3 \end{smallmatrix}$

82. $l(3) + m(-2) - (-4) = 9$

$3l - 2m = 5$ .....(i)

$3l - 2m - 3 = 0$

$2l - m = 3$ .....(ii)

$4l - 2m = 6$ .....(iii)

(iii) – (i)

$l = 1$

$m = -1$ $\quad l^2 + m^2 = 2$

83. $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\sqrt{3}}{2}(\vec{b}+\vec{c})$

$\Rightarrow$ $(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}=\frac{\sqrt{3}}{2}\vec{b}+\frac{\sqrt{3}}{2}\vec{c}$

$\Rightarrow$ $\vec{a}.\vec{c}=\frac{\sqrt{3}}{2}$ and $\vec{a}.\vec{b}=-\frac{\sqrt{3}}{2}$

$\Rightarrow$ Angle between $\vec{a}$ & $\vec{c}$ $=30^{o}$

$\vec{a}$ & $\vec{c}$ $=150^{o}=\frac{5\pi}{6}$

84. Equation of line : $\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=\lambda$

Any point is $(\lambda+1,\lambda-5,\lambda+9)$

It lies on plane

$\Rightarrow$ $(\lambda+1)-(\lambda-5)+(\lambda+9)=5$

$\Rightarrow$ $\lambda+1-\lambda+5+\lambda+9=5$

$\Rightarrow$ $\lambda=-10$

$\therefore$ Point is $(-9,-15,-1)$, another is $(1,-5,9)$

Distance $=\sqrt{100+100+100}=10\sqrt{3}$

Part # II : IIT-JEE ADVANCED

1. **(b)** Given that $\vec{a},\vec{b},\vec{c},\vec{d}$ are vectors such that

$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=0$ .....**(1)**

P_1 is the plane determined by vectors $\vec{a}$ and $\vec{b}$

$\therefore$ Normal vectors $\vec{n}_1$ to P_1 will be given by

$\vec{n}_1=\vec{a}\times\vec{b}$

Similarly P_2 is the plane determined by vectors $\vec{c}$ and $\vec{d}$

$\therefore$ Normal vector $\vec{n}_2$ to P_2 will be given by

$\vec{n}_2=\vec{c}\times\vec{d}$

Substituting the values of $\vec{n}_1$ and $\vec{n}_2$ in equation **(1)** we get $\vec{n}_1\times\vec{n}_2=0$

$\Rightarrow$ $\vec{n}_1 || \vec{n}_2$

and hence the planes will also be parallel to each other.

Thus angle between the planes = 0.

3. **(a)** $\hat{a},\hat{b},\hat{c}$ are unit vectors.

$\therefore$ $\hat{a}\cdot\hat{a}=\hat{b}\cdot\hat{b}=\hat{c}\cdot\hat{c}=1$

Now, $x=|\hat{a}-\hat{b}|^2+|\hat{b}-\hat{c}|^2+|\hat{c}-\hat{a}|^2$

$=\hat{a}.\hat{a}+\hat{b}.\hat{b}-2\hat{a}.\hat{b}+\hat{b}.\hat{b}+\hat{c}.\hat{c}-$

$2\hat{b}.\hat{c}+\hat{c}.\hat{c}+\hat{a}.\hat{a}-2\hat{c}.\hat{a}$

$=6-2(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a})$ ......**(1)**

Also $|\hat{a}+\hat{b}+\hat{c}|\geq 0$

$\Rightarrow$ $\hat{a}.\hat{a}+\hat{b}.\hat{b}+\hat{c}.\hat{c}+2(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a})\geq 0$

$\Rightarrow$ $3+2(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a})\geq 0$

$\Rightarrow$ $-2(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a})\leq 3$

$\Rightarrow$ $6-2(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a})\leq 9$ ... **(2)**

From **(1)** and **(2)**, $x\leq 9$

$\therefore$ x does not exceed 9

5. Given data is insufficient to uniquely determine the three vectors as there are only 6 equations involving 9 variables.

$\therefore$ We can obtain infinitely many set of three vectors, $\vec{v}_1, \vec{v}_2, \vec{v}_3$, satisfying these conditions.

From the given data, we get

$\vec{v}_1.\vec{v}_1 = 4 \quad \Rightarrow \quad |\vec{v}_1| = 2$

$\vec{v}_2.\vec{v}_2 = 2 \quad \Rightarrow \quad |\vec{v}_2| = \sqrt{2}$

$\vec{v}_3.\vec{v}_3 = 29 \quad \Rightarrow \quad |\vec{v}_3| = \sqrt{29}$

Also $\quad \vec{v}_1.\vec{v}_2 = -2$

$\Rightarrow \quad |\vec{v}_1|\,|\vec{v}_2|\cos\theta = -2$

[where θ is the angle between $\vec{v}_1$ and $\vec{v}_2$]

$\Rightarrow \quad \cos\theta = \dfrac{-1}{\sqrt{2}} \quad \Rightarrow \quad \theta = 135°$

Now since any two vectors are always coplanar, let us suppose that $\vec{v}_1$ and $\vec{v}_2$ are in x–y plane. Let $\vec{v}_1$ is along the positive direction of x-axis then $\vec{v}_1 = 2\hat{i}$. $[\because |\vec{v}_1| = 2]$

As $\vec{v}_2$ makes an angle 135° with $\vec{v}_1$ and ies in x–y plane, also keeping in mind

$|\vec{v}_2| = \sqrt{2}$ we obtain

$\vec{v}_2 = -\hat{i} \pm \hat{j}$

Again let, $\vec{v}_3 = \alpha\hat{i} + \beta\hat{i} + \gamma\hat{k}$

$\because \quad \vec{v}_3.\vec{v}_1 = 6 \quad \Rightarrow \quad 2\alpha = 6 \quad \Rightarrow \quad \alpha = 3$

and $\vec{v}_3.\vec{v}_2 = -5 \quad \Rightarrow \quad -\alpha \pm \beta = -5 \quad \Rightarrow \quad \beta = \pm 2$

Also $|\vec{v}_3| = \sqrt{29} \Rightarrow \alpha^2 + \beta^2 + \gamma^2 = 29$

$\Rightarrow \quad \gamma = \pm 4$

Hence $\quad \vec{v}_3 = 3\hat{i} \pm 2\hat{j} \pm 4\hat{k}$

Thus, $\quad \vec{v}_1 = 2\hat{i};\ \vec{v}_2 = -\hat{i} \pm \hat{j};\ \vec{v}_3 = 3\hat{i} \pm 2\hat{j} \pm 4\hat{k}$

are some possible answers.

6. $\vec{A}(t)$ is parallel to $\vec{B}(t)$ for some $t \in [0,1]$ if and only if

$\dfrac{f_1(t)}{g_1(t)} = \dfrac{f_2(t)}{g_2(t)}$ for some $t \in [0,1]$

or $\quad f_1(t).g_2(t) = f_2(t).g_1(t)$ for some $t \in [0,1]$

Let $\quad h(t) = f_1(t).g_2(t) - f_2(t).g_1(t)$

$h(0) = f_1(0).g_2(0) - f_2(0).g_1(0)$

$\quad = 2 \times 2 - 3 \times 3 = -5 < 0$

$h(1) = f_1(1).g_2(1) - f_2(1).g_1(1)$

$\quad = 6 \times 6 - 2 \times 2 = 32 > 0$

Since h is a continuous function, and $h(0).h(1) < 0$

$\Rightarrow$ there is some $t \in [0,1]$ for which $h(t) = 0$

i.e., $\vec{A}(t)$ and $\vec{B}(t)$ are parallel vectors for this t.

8. Given that, $\quad \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

where a_r, b_r, c_r, $r = 1,2,3$ are all non negative real numbers.

Also $\displaystyle\sum_{r=1}^{3} (a_r + b_r + c_r) = 3L$

To prove $V \le L^3$ Where V is vol. of parallelopiped formed by the vectors $\vec{a}, \vec{b}$ and $\vec{c}$

$\therefore$ We have $V = [\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$\Rightarrow \quad V = (a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2) - (a_1b_3c_2 + a_2b_1c_3 + a_3b_2c_1) \quad$ ....**(1)**

Now we know that AM $\ge$ GM

$\therefore \dfrac{(a_1 + b_1 + c_1) + (a_2 + b_2 + c_2) + (a_3 + b_3 + c_3)}{3}$

$\ge [(a_1 + b_1 + c_1)\,(a_2 + b_2 + c_2)\,(a_3 + b_3 + c_3)]^{1/3}$

$\Rightarrow \quad \dfrac{3L}{3} \ge [(a_1+b_1+c_1)\,(a_2+b_2+c_2)\,(a_3+b_3+c_3)]^{1/3}$

$\Rightarrow \quad L^3 \ge (a_1+b_1+c_1)\,(a_2 + b_2 + c_2)\,(a_3 + b_3 + c_3)$

$= a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 + 24$ more such terms

$\geq a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2$

$[\because\ a_r, b_r, c_r \geq 0 \text{ or } r = 1,2,3]$

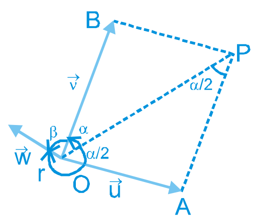
$\geq (a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2)$

$-(a_1b_3c_2 + a_2b_1c_3 + a_3b_2c_1)$ [same reason]

$= V$ from (1)

Thus, $L^3 \geq V$ Hence Proved

10. Given that u, ν, ω are three non coplanar unit vectors. Angle between $\vec{u}$ and $\vec{v}$ is α, between $\vec{v}$ and $\vec{\omega}$ is β and between $\vec{\omega}$ and $\vec{u}$ it is γ. In fig. $\overline{OA}$ and $\overline{OB}$ represent $\vec{u}$ and $\vec{v}$. Let P be a pt. on angle bisector of ∠AOB such that OAPB is a parallelogram.



Also ∠POA = ∠BOP = α/2

∴ ∠APO = ∠BOP = α/2 (Alternate angles)

∴ In Δ OAP, OA = AP

∴ $\overline{OP} = \overline{OA} + \overline{AP} = \vec{u} + \vec{v}$

∴ $\vec{O}P = \dfrac{\vec{u}+\vec{v}}{|\vec{u}+\vec{v}|}$ i.e. $\vec{x} = \dfrac{\vec{u}+\vec{v}}{|\vec{u}+\vec{v}|}$

But $|\vec{u}+\vec{v}|^2 = (\vec{u}+\vec{v}).(\vec{u}+\vec{v})$

$= 1 + 1 + 2\,\vec{u}.\vec{v}$

$[\because\ |\vec{u}| = |\vec{v}| = 1]$

$= 2 + 2\cos\alpha = 4\cos^2 \alpha/2.$

∴ $|\vec{u}+\vec{v}| = 2\cos\alpha/2$

$\Rightarrow\ \vec{x} = \dfrac{1}{2}\sec(\alpha/2)\,(\vec{u}+\vec{v})$

Similarly, $\vec{y} = \dfrac{1}{2}\sec(\beta/2)\,(\vec{v}+\vec{\omega})$

$\vec{z} = \dfrac{1}{2}\sec(\gamma/2)(\vec{\omega}+\vec{u})$

Now consider $[\vec{x}\times\vec{y}\ \ \vec{y}\times\vec{z}\ \ \vec{z}\times\vec{x}]$

$= (\vec{x}\times\vec{y}).[(\vec{y}\times\vec{z})\times(\vec{z}\times\vec{x})]$

$= (\vec{x}\times\vec{y}).[\{(\vec{y}\times\vec{z}).\vec{x}\}\vec{z} - \{(\vec{y}\times\vec{z}).\vec{z}\}\vec{x}]$

[Using def^n of vector triple product,]

$= (\vec{x}\times\vec{y}).[[\vec{x}\,\vec{y}\,\vec{z}]\vec{z} - 0]$

$= [\vec{x}\,\vec{y}\,\vec{z}]\,[\vec{x}\,\vec{y}\,\vec{z}]$ $[\because\ [\vec{y}\,\vec{z}\,\vec{z}] = 0]$

$= [\vec{x}\,\vec{y}\,\vec{z}]^2$ ...**(i)**

Also $[\vec{x}\,\vec{y}\,\vec{z}] = \Big[\dfrac{1}{2}\sec\dfrac{\alpha}{2}(\vec{u}+\vec{v})\ \ \dfrac{1}{2}\sec\dfrac{\beta}{2}$

$(\vec{v}+\vec{\omega})\ \ \dfrac{1}{2}\sec(\gamma/2)(\vec{w}+\vec{u}))\Big]$

$= \dfrac{1}{8}\sec(\alpha/2)\sec(\beta/2)\sec(\gamma/2)\,[\vec{u}+\vec{v}\ \ \vec{v}+\vec{\omega}\ \ \vec{\omega}+\vec{u}]$

$= \dfrac{1}{8}\sec(\alpha/2)\sec(\beta/2)\sec(\gamma/2)$

$[(\vec{u}+\vec{v}).(\vec{v}+\vec{\omega})\times(\vec{\omega}+\vec{u})]$

$= \dfrac{1}{8}\sec(\alpha/2)\sec(\beta/2)\sec(\gamma/2)$

$[(\vec{u}+\vec{v}).(\vec{v}\times\vec{\omega}+\vec{v}\times\vec{u}+\vec{\omega}\times\vec{u})]$

$= \dfrac{1}{8}\sec(\alpha/2)\sec(\beta/2)\sec(\gamma/2)\,[\vec{u}.\vec{v}\times\vec{\omega}+\vec{v}.\vec{\omega}\times\vec{u}]$

$(\because\ [\vec{a}\,\vec{b}\,\vec{c}] = 0$ when ever any two vectors are same)

$= \dfrac{1}{8}\sec(\alpha/2)\sec(\beta/2)\sec(\gamma/2)\ 2\,[\vec{u}\,\vec{v}\,\vec{\omega}]$

$= \dfrac{1}{4}\sec(\alpha/2)\sec(\beta/2)\sec(\gamma/2)\ 2\,[\vec{u}\,\vec{v}\,\vec{\omega}]$

∴ $[\vec{x}\,\vec{y}\,\vec{z}]^2 = \dfrac{1}{16}[\vec{u}\,\vec{v}\,\vec{\omega}]^2\sec^2\alpha/2\ \sec^2\beta/2\ \sec^2\gamma/2$ ....**(ii)**

From **(i)** and **(ii)**,

$[\vec{x}\times\vec{y}\ \ \vec{y}\times\vec{z}\ \ \vec{z}\times\vec{x}]$

$= \dfrac{1}{16}[\vec{u}\,\vec{v}\,\vec{\omega}]^2\ \ \sec^2\alpha/2\ \sec^2\beta/2\ \sec^2\gamma/2.$

12. Given that $\vec{a} \neq \vec{b} \neq \vec{c} \neq \vec{d}$

Such that $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$ ...**(i)**

$\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ ...**(ii)**

To prove that $(\vec{a}-\vec{d}).(\vec{b}-\vec{c}) \neq 0$

Subtracting equation **(ii)** from **(i)** we get

$$\vec{a} \times (\vec{c} - \vec{b}) = (\vec{b} - \vec{c}) \times \vec{d}$$

$\Rightarrow$ $\vec{a} \times (\vec{c} - \vec{b}) = \vec{d} \times (\vec{c} - \vec{b})$

$\Rightarrow$ $\vec{a} \times (\vec{c} - \vec{b}) - \vec{d} \times (\vec{c} - \vec{b}) = 0$

$\Rightarrow$ $(\vec{a} - \vec{d}) \times (\vec{c} - \vec{b}) = 0$ $\quad\Rightarrow$ $\vec{a} - \vec{d} \parallel \vec{c} - \vec{b}$

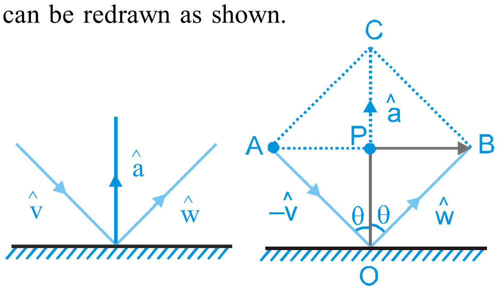
[$\because$ $\vec{a} - \vec{d} \neq 0,\ \vec{c} - \vec{b} \neq 0$ as all distinct]

$\Rightarrow$ Angle between $\vec{a} - \vec{d}$ and $\vec{c} - \vec{b}$ is either 0 or 180°.

$\Rightarrow$ $(\vec{a} - \vec{d})\cdot(\vec{c} - \vec{b}) = |\vec{a} - \vec{d}||\vec{c} - \vec{b}|$

[cos 0° or cos 180°] ≠ 0 as $\vec{a}, \vec{d}, \vec{c}, \vec{b}$ all are different.

14. Given that incident ray is along $\hat{v}$, reflected ray is along $\hat{w}$ and normal is along $\hat{a}$, outwards. The given figure can be redrawn as shown.



We know that incident ray, reflected ray and normal lie in a plane, and angle of incidence = angle of reflection. Therefore $\hat{a}$ will be along the angle bisector of $\hat{w}$ and $-\hat{v}$, i.e.,

$$\hat{a} = \frac{\hat{w} + (-\hat{v})}{|\hat{w} - \hat{v}|} \qquad \text{...(i)}$$

[$\because$ angle bisector will along a vector dividing in same ratio as the ratio of the sides forming that angle.]

But $\hat{a}$ is a unit vector

where $|\hat{w} - \hat{v}|$ = OC = 2OP

$= 2\,|\hat{w}|\cos\theta = 2\cos\theta$

Substituting this value in equation **(i)** we get

$$\hat{a} = \frac{\hat{w} - \hat{v}}{2\cos\theta}$$

$\therefore$ $\hat{w} = \hat{v} + (2\cos\theta)\,\hat{a}$

$= \hat{v} - 2(\hat{a}\cdot\hat{v})\,\hat{a}$ $\quad$ [$\because$ $\hat{a}\cdot\hat{v} = -\cos\theta$].

15. **(b)** Normal to plane P_1 is

$\vec{n}_1 = (2\hat{j} + 3\hat{k}) \times (4\hat{j} - 3\hat{k}) = -18\hat{i}$

Normal to plane P_2 is

$\vec{n}_2 = (\hat{j} - \hat{k}) \times (3\hat{i} + 3\hat{j}) = 3\hat{i} - 3\hat{j} - 3\hat{k}$

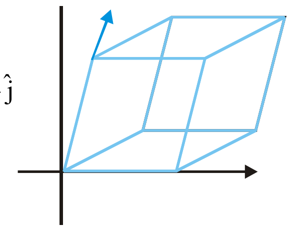
$\therefore$ $\vec{A}$ is parallel to $\pm(\vec{n}_1 \times \vec{n}_2) = \pm(-54\hat{j} + 54\hat{k})$

Now, angle between $\vec{A}$ and $2\hat{i} + \hat{j} - 2\hat{k}$ is given by

$$\cos\theta = \pm\frac{(-54\hat{j} + 54\hat{k}).(2\hat{i} + \hat{j} - 2\hat{k})}{54\sqrt{2}.3} = \pm\frac{1}{\sqrt{2}}$$

$\theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$.

19. Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$



$\vec{a} = \hat{i}$ then $\vec{b} = \frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$

so $x = \frac{1}{2}$

& $\frac{x}{2} + \frac{y\sqrt{3}}{2} = \frac{1}{2}$

$\Rightarrow$ $y\sqrt{3} = \frac{1}{2}$ $\quad\therefore$ $y = 1/2\sqrt{3}$

also $x^2 + y^2 + z^2 = 1$

$\Rightarrow$ $z^2 = 2/3 \Rightarrow z = \pm\sqrt{2/3}$

$$\text{so volume} = \begin{vmatrix} 1 & 0 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 1/2 & 1/2\sqrt{3} & \pm\sqrt{2/3} \end{vmatrix} = 1/\sqrt{2}$$

Alternative

volume = $|\vec{a}\cdot(\vec{b}\times\vec{c})|$

$$\sqrt{\begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c} \end{vmatrix}} = \sqrt{\begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix}} = \frac{1}{\sqrt{2}}$$

20. $|\overline{OP}| = |\hat{a}\cos t + \hat{b}\sin t|$

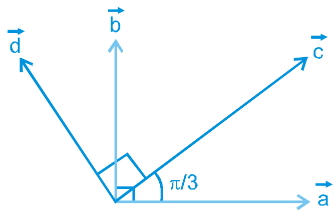
$= (\cos^2 t + \sin^2 t + 2\sin t\cos t\ \hat{a}\cdot\hat{b})^{1/2}$

$= (1 + \sin 2t\,\hat{a}\cdot\hat{b})^{1/2}$

$\therefore$ $|\overrightarrow{OP}|_{max} = (1 + \hat{a}\cdot\hat{b})^{1/2}$, $\quad$ when $t = \frac{\pi}{4}$

$$\text{Now } \hat{u} = \frac{\hat{a} + \hat{b}}{\sqrt{2}\frac{|\hat{a} + \hat{b}|}{\sqrt{2}}} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$$

21. $(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d}) = 1$ ...**(1)**



Let $\vec{a} \wedge \vec{b} = \alpha$

$\vec{a} \wedge \vec{b} = \beta$

angle between plane of $(\vec{a}, \vec{b})$ & $(\vec{c}, \vec{d})$ be θ

equation **(1)** becomes

$\sin\alpha \, . \, \sin\beta \cos\theta = 1$

$\Rightarrow \alpha = \frac{\pi}{2}, \beta = \frac{\pi}{2}, \theta = 0$

$\Rightarrow \vec{b}$ & $\vec{d}$ are non-parallel.

22. (A) $2 \sin^2 \theta + \sin^2 2\theta = 2$

$\sin^2 \theta + 2 \sin^2 \theta \cos^2 \theta = 1$

$t + 2t(1 - t) = 1$

$t + 2t - 2t^2 = 1$

$2t^2 - 3t + 1 = 0$

$(2t - 1)(t - 1) = 0$

$t = 1, 1/2$

$\sin^2 \theta = 1, 1/2$

(B) $\frac{6x}{\pi} = I_1$ & $\frac{3x}{\pi} = I_2$

$\Rightarrow x = \frac{I_1 \pi}{6} = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \pi.$

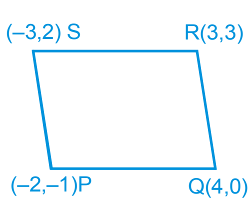
(C) $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

(D) $\vec{a} + \vec{b} + \sqrt{3}\vec{c} = 0$

$\Rightarrow a^2 + b^2 + 2\vec{a}.\vec{b} = 3c^2 \Rightarrow 2 + 2\cos\theta = 3$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}.$

23. Ans. (A)



$\overrightarrow{PQ} = 6\hat{i} + \hat{j}$

$\overrightarrow{SR} = 6\hat{i} + \hat{j}$

$\therefore \overrightarrow{PQ} = \overrightarrow{SR}$

$\overrightarrow{PS} = -\hat{i} + 3\hat{j}$

$\overrightarrow{QR} = -\hat{i} + 3\hat{j}$

$\therefore \overrightarrow{PS} = \overrightarrow{QR}$

But $\overrightarrow{PQ}.\overrightarrow{PS} = -6 + 3 = -3 \neq 0$ & $|\overrightarrow{PQ}| \neq |\overrightarrow{PS}|$

$\Rightarrow$ PQRS is a parallelogram but neither a rhombus nor a rectangle.

24. $(2\vec{a} + \vec{b}).[(\vec{a} \times \vec{b}) \times \vec{a} - 2(\vec{a} \times \vec{b}) \times \vec{b}]$

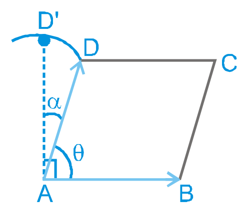
$= (2\vec{a} + \vec{b}).[a^2\vec{b} - (\vec{a}.\vec{b})\vec{a} - 2\{(\vec{a}.\vec{b})\vec{b} - b^2\vec{a}\}]$

$= (2\vec{a} + \vec{b}).[a^2\vec{b} + 2b^2\vec{a}]$; as $\vec{a}.\vec{b} = 0$

$= (2\vec{a} + \vec{b}).[2\vec{a} + \vec{b}]$ as $[a^2 = b^2 = 1]$

$\Rightarrow 4a^2 + b^2 = 5$

25. Let θ be the angle between $\overrightarrow{AB}$ and $\overrightarrow{AD}$



$\Rightarrow \theta + \alpha = 90°$

$\Rightarrow \alpha = 90° - \theta$

$\Rightarrow \cos\alpha = \sin\theta$ ....**(i)**

Now, $\cos\theta = \frac{\overrightarrow{AB}.\overrightarrow{AD}}{|\overrightarrow{AB}||\overrightarrow{AD}|} = \frac{8}{9}$

$\Rightarrow \cos\theta = \frac{\sqrt{17}}{9}$ from **(i)**.

26. (a) $\vec{v} = x\vec{a} + y\vec{b}$

$= \hat{i}(x + y) + \hat{j}(x - y) + \hat{k}(x + y)$ ....**(i)**

Given, $\vec{v}.\hat{c} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{x + y - x + y - x - y}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$y - x = 1$

$\Rightarrow x - y = -1$ ....**(ii)**

using **(ii)** in **(i)** we get

$\vec{v} = (x + y)\hat{i} - \hat{j} + (x + y)\hat{k}$

(b) $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ $\quad\quad \vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{c} = \hat{i} + \hat{j} + \hat{k}$

$\vec{v} = \lambda((\vec{a} \times \vec{b}) \times \vec{c}) = \lambda((\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a})$

$\vec{v} = \lambda[4(\hat{i} + 2\hat{j} + \hat{k}) - 4(\hat{i} + \hat{j} + 2\hat{k})]$

$\vec{v} = 4\lambda(\hat{j} - \hat{k})$

(c) $\vec{a} = -\hat{i} - \hat{k}$

$\vec{b} = -\hat{i} + \hat{j}$

$\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$

Taking cross product by $\vec{a}$

$(\vec{r} \times \vec{b}) \times \vec{a} = (\vec{c} \times \vec{b}) \times \vec{a}$

$\Rightarrow$ $(\vec{r}.\vec{a})\vec{b} - (\vec{b}.\vec{a})\vec{r} = (\vec{c}.\vec{a})\vec{b}) - (\vec{b}.\vec{a})\vec{c}$

$\Rightarrow$ $0 - \vec{r} = (-1-3)(-\hat{i} + \hat{j}) - (1)(\hat{i} + 2\hat{j} + 3\hat{k})$

$\vec{r} = -3\hat{i} + 6\hat{j} + 3\hat{k}$

$\vec{r}.\vec{b} = 3 + 6 = 9$

27. (a) $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9$

$\Rightarrow$ $6 - 2\Sigma\vec{a}.\vec{b} = 9$

$\Rightarrow$ $\Sigma\vec{a}.\vec{b} = -\dfrac{3}{2}$ $\quad$ **...(1)**

$|\vec{a} + \vec{b} + \vec{c}|^2 \ge 0$

$\Sigma\vec{a}^2 + 2\Sigma\vec{a}.\vec{b} \ge 0$

$\Sigma\vec{a}.\vec{b} \ge -\dfrac{3}{2}$

for equality $|\vec{a} + \vec{b} + \vec{c}| = 0$

$\Rightarrow$ $\vec{a} + \vec{b} + \vec{c} = 0$

$5\vec{b} + 5\vec{c} = -5\vec{a}$

$2\vec{a} + 5\vec{b} + 5\vec{c} = -3\vec{a}$

$|2\vec{a} + 5\vec{b} + 5\vec{c}| = 3|\vec{a}| = 3$

(b) $(\vec{a} + \vec{b}) \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = 0$

$\Rightarrow$ $\vec{a} + \vec{b} = \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

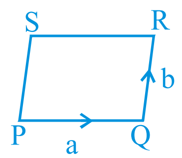
$|\vec{a} + \vec{b}| = \sqrt{29} \Rightarrow |\lambda| = 1$

$\vec{a} + \vec{b} = (2\hat{i} + 3\hat{j} + 4\hat{k})$

$(\vec{a} + \vec{b}).(-7\hat{i} + 2\hat{j} + 3\hat{k})$

$= -14 + 6 + 12 = 4$

28. $\vec{a} + \vec{b} = \overline{PR}$ & $\vec{a} - \vec{b} = \overline{QS}$



$\vec{a} = \dfrac{\overline{PR} + \overline{QS}}{2}$ & $\vec{b} = \dfrac{\overline{PR} - \overline{QS}}{2}$

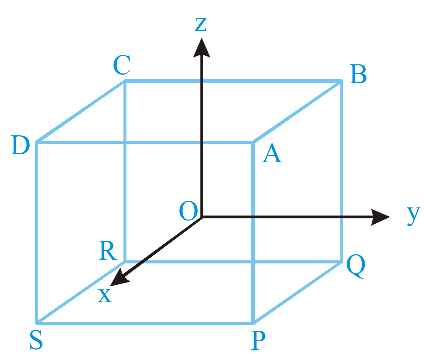
$\vec{a} = 2\hat{i} - \hat{j} - 3\hat{k}$ & $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

$$\text{Volume} = \begin{vmatrix} 2 & -1 & -3 \\ 1 & 2 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

$2(4) + (3-1) - 3(2-2)$

$8 + 2 = 10$

29.



O is at the centre of cube ABCDPQRS

The 8 vectors will represent

$\overline{OA}, \overline{OB} \ldots\ldots \overline{OD}, \overline{OP}, \ldots\ldots \overline{OS}$

any three out of these 8 will be coplanar when two of them are collinear. There are 4 pairs of collinear vectors

$\overline{OA}$ & $\overline{OR}$, $\overline{OB}$ & $\overline{OS}$, $\overline{OC}$ & $\overline{OP}$, $\overline{OD}$ & $\overline{OQ}$

(it will generate 4 × 6 = 24 set of coplanar vectors) rest of the combination of 3 vectors will form three edges of a tetrahedron so they will be not coplanar.

So number of non-coplanar vectors

${}^8C_3 - 4.6 = 32$

30. **(P)** Given $[\vec{a}\ \vec{b}\ \vec{c}] = 2$

$[2(\vec{a}\times\vec{b})\ 3(\vec{b}\times\vec{c})\ (\vec{c}\times\vec{a})]$

$= 6[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = 6[\vec{a}\ \vec{b}\ \vec{c}]^2 = 24$

(Q) Given $[\vec{a}\ \vec{b}\ \vec{c}] = 5$

$[3(\vec{a}+\vec{b})\ \ (\vec{b}+\vec{c})\ \ 2(\vec{c}+\vec{a})]$

$= 12[\vec{a}\ \vec{b}\ \vec{c}] = 60$

(R) Given $\frac{1}{2}|\vec{a}\times\vec{b}| = 20 \Rightarrow |\vec{a}\times\vec{b}| = 40$

$\left|\frac{1}{2}(2\vec{a}+3\vec{b})\times(\vec{a}-\vec{b})\right| = \frac{1}{2}\left|0+3\vec{b}\times\vec{a}-2\vec{a}\times\vec{b}\right|$

$= \frac{1}{2}|-5\vec{a}\times\vec{b}| = \frac{5}{2}|\vec{a}\times\vec{b}| = \frac{5}{2}.40 = 100$

(S) Given $|\vec{a}\times\vec{b}| = 30$

$|(\vec{a}+\vec{b})\times\vec{a}| = |0+\vec{b}\times\vec{a}| = 30$

33. Let the equation of the plane ABCD be $ax + by + cz + d = 0$, the point A'' be (α, β, γ) and the height of the parallelopiped ABCDA'B'C'D' be h.

$\Rightarrow \frac{|a\alpha + b\beta + c\gamma + d|}{\sqrt{a^2+b^2+c^2}} = 90\%.h$

$\Rightarrow a\alpha + b\beta + c\gamma + d = \pm 0.9h\sqrt{a^2+b^2+c^2}$

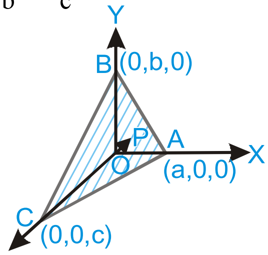
$\therefore$ locus is $ax, + by + cz + d = \pm 0.9h\sqrt{a^2+b^2+c^2}$

$\therefore$ locus of A'' is a plane parallel to the plane ABCD.

36. As $\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$ cuts the coordinate axes at A(a, 0, 0), B(0, b, 0), C(0, 0, c)

and its distance from origin = 1

$\Rightarrow \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} = 1$



or $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} = 1$ .....**(1)**

where P is centroid of Δ

$\therefore P(x, y, z) = \left(\frac{a+0+0}{3}, \frac{0+b+0}{3}, \frac{0+0+c}{3}\right)$

$\Rightarrow x = \frac{a}{3}, y = \frac{b}{3}, z = \frac{c}{3}$ ... **(2)**

Thus, from (1) and (2)

$\frac{1}{9x^2}+\frac{1}{9y^2}+\frac{1}{9z^2} = 1$

or $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2} = 9 = K$

$\therefore K = 9$

37. Equation of plane containing the line,

$2x - y + z - 3 = 0$ and $3x + y + z = 5$ is

$(2x - y + z - 3) + \lambda(3x + y + z - 5) = 0$

$\Rightarrow (2+3\lambda)x + (\lambda-1)y + (\lambda+1)z - 3 - 5\lambda = 0$

Since distance of plane from (2, 1, –1) to above plane is $1/\sqrt{6}$

$\therefore \left|\frac{6\lambda+4+\lambda-1-\lambda-1-3-5\lambda}{\sqrt{(3\lambda+2)^2+(\lambda-1)^2+(\lambda+1)^2}}\right| = \frac{1}{\sqrt{6}}$

$\Rightarrow 6(\lambda-1)^2 = 11\lambda^2 + 12\lambda + 6$

$\Rightarrow \lambda = 0, -\frac{24}{5}$

$\therefore$ Equation of planes are,

$2x-y+z-3 = 0$ and $62x+29y+19z-105 = 0$

39. **(A)** Solving the two equations, say i.e.,

$x + y = |a|$ and $ax - y = 1$, we get

$x = \frac{|a|+1}{a+1} > 0$ and $y = \frac{|a|-1}{a+1} > 0$

when $a + 1 > 0$; we get $a > 1$

$\therefore a_0 = 1$

(B) We have, $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\Rightarrow \vec{a}.\hat{k} = \gamma$

Now, $\hat{k}\times(\hat{k}\times\vec{a}) = (\hat{k}.\vec{a})\hat{k} - (\hat{k}.\hat{k})\vec{a}$

$= \gamma\hat{k} - (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k})$

$\Rightarrow \quad \alpha\hat{i} + \beta\hat{j} = 0$

$\Rightarrow \quad \alpha = \beta = 0$

Also $\alpha + \beta + \gamma = 2$

$\Rightarrow \quad \gamma = 2.$

(C) $\left|\int_0^1 \left(1 - y^2\right) dy\right| + \left|\int_0^1 \left(y^2 - 1\right) dy\right|$

$= 2\int_0^1 (1 - y^2)dy = \frac{4}{3}$

Also $\left|\int_0^1 \sqrt{1 - x}dx\right| + \left|\int_{-1}^0 \sqrt{1 + x}dx\right|$

$= 2\int_0^1 \sqrt{1 - x}dx = \frac{4}{3}$

(D) $\sin A \sin B \sin C + \cos A \cos B$

$\leq \sin A \sin B + \cos A \cos B = \cos(A - B)$

$\Rightarrow \quad \cos(A - B) \geq 1$

$\Rightarrow \quad \cos(A - B) = 1$

$\Rightarrow \quad \sin C = 1.$

40. **(A)** $\sum_{i=1}^{\infty} \tan^{-1}\left(\frac{1}{2i^2}\right) = t.$

$\Rightarrow \quad \sum_{i=1}^{\infty} \tan^{-1}\left(\frac{2}{4i^2 - 1 + 1}\right)$

$= \sum_{i=1}^{\infty} \tan^{-1}\left\{\frac{(2i+1) - (2i-1)}{1 + (2i-1)(2i+1)}\right\}$

$= (\tan^{-1}3 - \tan^{-1}1) + (\tan^{-1}5 - \tan^{-1}3) + ...$

$+ \{(\tan^{-1}(2n+1) - \tan^{-1}(2n-1))$

$\therefore \quad t = \lim_{n\to\infty} (\tan^{-1}(2n+1) - \tan^{-1} 1)$

$= \lim_{n\to\infty} \tan^{-1}\left(\frac{2n}{1 + 2n + 1}\right) = \frac{\pi}{4}$

$\therefore \quad \tan t = 1.$

(B) We have, $\cos\theta_1 = \dfrac{1 - \tan^2\frac{\theta_1}{2}}{1 + \tan^2\frac{\theta_1}{2}} = \dfrac{a}{b + c}$

$\Rightarrow \quad \tan^2\left(\frac{\theta_1}{2}\right) = \frac{b + c - a}{b + c + a}$

Also, $\cos\theta_3 = \dfrac{1 - \tan^2\frac{\theta_3}{2}}{1 + \tan^2\frac{\theta_3}{2}} = \dfrac{c}{a + b}$

$\Rightarrow \quad \tan^2\frac{\theta_3}{2} = \frac{a + b - c}{a + b + c}$

$\Rightarrow \quad \tan^2\frac{\theta_1}{2} + \tan^2\frac{\theta_3}{2}$

$= \frac{2b}{a + b + c} = \frac{2b}{3b} = \frac{2}{3}$

{as, a, b, c are in AP

$\Rightarrow \quad 2b = a + c$}

(C) Line through (0, 1, 0) and perpendicular to plane $x + 2y + 2z = 0$ is given by

$\frac{x - 0}{1} = \frac{y - 1}{2} = \frac{z - 0}{2} = r$

$\therefore$ P(r, 2r + 1, 2r) be the foot of perpendicular on the straight line then

$r \cdot 1 + (2r + 1) \cdot 2 + (2r) \cdot 2 = 0$

$\Rightarrow \quad r = -\frac{2}{9}$

$\therefore \quad P\left(-\frac{2}{9}, \frac{5}{9}, -\frac{4}{9}\right)$

$\therefore$ Required perpendicular distance

$= \sqrt{\frac{4 + 25 + 16}{81}} = \frac{\sqrt{5}}{3}$ unit.

41. Let $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= \frac{-1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$

(A) If $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$

$\Rightarrow \quad (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

$\Rightarrow \quad \Delta = 0$ and $a = b = c \neq 0$

$\Rightarrow$ the equation represent identical planes.

(B) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$

$\Rightarrow \quad \Delta = 0$

Since all the three planes pass through (1,1,1)

So equation of the line of intersection of these plane will be $\frac{x - 0}{1 - 0} = \frac{y - 0}{1 - 0} = \frac{z - 0}{1 - 0}$

(C) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$

$\Rightarrow$ $\Delta \neq 0$

$\Rightarrow$ the equations represent planes meeting at only one point i.e. (0,0,0)

(D) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$

$\Rightarrow$ $a = b = c = 0$

$\Rightarrow$ the equations represent whole of the three dimensional space.

43. Dr's of

$L_1 = 0, -4, -4$

$L_2 = 0, -2, -2$

$L_3 = 0, 2, 2$

So all the three lines are parallel

Hence St.-I is false

$$\text{Now } \Delta = \begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -3 & 3 \end{vmatrix} = 0$$

so there will be no solution.

Hence St.-II is true.

Paragraph for Question 44 to 46

44. $L_1 : \dfrac{x+1}{3} = \dfrac{y+2}{1} = \dfrac{z+1}{2}$

$L_2 : \dfrac{x-2}{1} = \dfrac{y+2}{2} = \dfrac{z-3}{3}$

a vector perpendicular to L_1 & L_2 will be

$$= \begin{vmatrix} i & j & k \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -i - 7j + 5k$$

Hence unit vector $= \dfrac{-i - 7j + 5k}{5\sqrt{3}}$

45. Shortest distance

$$= (3i - 4k).\frac{(-i - 7j + 5k)}{5\sqrt{3}} = \frac{17}{5\sqrt{3}}$$

46. Eq. of plane $-(x + 1) - 7(y + 2) + 5(z + 1) = 0$

$x + 7y - 5z + 10 = 0$

distance from (1, 1, 1) $= \dfrac{1 + 7 - 5 + 10}{5\sqrt{3}} = \dfrac{13}{5\sqrt{3}}$

47. Let DC's be $(\cos\alpha, \cos\alpha, \cos\alpha)$

$3\cos^2\alpha = 1$

$\cos\alpha = \dfrac{1}{\sqrt{3}}$

Line PQ is $\dfrac{x-2}{1/\sqrt{3}} = \dfrac{y+1}{1/\sqrt{3}} = \dfrac{z-2}{1/\sqrt{3}} = \lambda$

$Q\left(\dfrac{\lambda}{\sqrt{3}} + 2, \dfrac{\lambda}{\sqrt{3}} - 1, \dfrac{\lambda}{\sqrt{3}} + 2\right)$

Putting in plane

$\dfrac{2\lambda}{\sqrt{3}} + 4 + \dfrac{\lambda}{\sqrt{3}} - 1 + \dfrac{\lambda}{\sqrt{3}} + 2 = 9$

$\dfrac{4\lambda}{\sqrt{3}} = 4$

$\lambda = \sqrt{3}$

$Q = (3, 0, 3)$

$(PQ)^2 = 1+1+1$

$PQ = \sqrt{3}$

48. Let Q be $(1 - 3\mu, \mu - 1, 5\mu + 2)$

$\Rightarrow$ $\overrightarrow{PQ} = (-3\mu - 2)\hat{i} + (\mu - 3)\hat{j} + (5\mu - 4)\hat{k}$

$\Rightarrow$ $\overrightarrow{PQ}.\hat{n} = 0$ (where $\hat{n}$ is $\perp^{er}$ to plane)

$\Rightarrow$ $(-3\mu - 2)1 + (\mu - 3).(-4) + (5\mu - 4)3 = 0$

$\Rightarrow$ $\mu = \dfrac{1}{4}$.

49. (A) $f(x) = xe^{\sin x} - \cos x$

$f(0) = -1$

$f(\pi/2) = \dfrac{\pi}{2}e$

$f'(x) = xe^{\sin x}\cos x + e^{\sin x} > 0$

(B) $\begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow$ $k(k-4) - 4c + 8 - 2k = 0$

$\Rightarrow$ $k^2 - 4k + 8 - 2k = 0$

$\Rightarrow$ $k^2 - 6k + 8 = 0$

$\Rightarrow$ $k = 2, 4$

(C) $|x-1|+|x-2|+|x+1|+|x+2|=4k$

$4k = 8, 12, 16, 20$ { modulus denotes the distance of x from –2, –1, 1, 2

$\therefore k = 2, 3, 4, 5.$

(D) $\frac{dy}{y+1} = dx$

$\ln(y+1) = ke^x$

$y+1 = ke^x$

$y+1 = 2 = k$

$y+1 = 2e^x$

$y = (2e^x - 1)$

$y(\ln 2) = 3.$

50. Normal vector to the plane containing the lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

$$\hat{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = 8\hat{i} - \hat{j} - 10\hat{k}$$

Let direction ratios of required plane be a, b, c.

Now $8a - b - 10c = 0$

and $2a + 3b + 4c = 0$

($\because$ plane contains the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$)

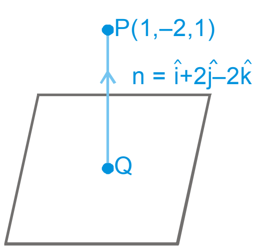
$\Rightarrow \frac{a}{1} = \frac{b}{-2} = \frac{c}{1}$

$\Rightarrow$ equation of plane is $x - 2y + z = d$

$\because$ plane contains the line, which passes through origin, hence origin lies on a plane.

$\Rightarrow$ equation of required plane is $x - 2y + z = 0$.

51. $\because \left|\frac{1-4-2-\alpha}{3}\right| = 5$



$\Rightarrow \alpha = 10, -20$

$\Rightarrow \alpha = 10 \because \alpha > 0$

Now, let $Q(\alpha, \beta, \gamma)$ be the foot of perpendicular from P to the plane $x + 2y - 2z = 10$

Equation of line PQ is

$$\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-1}{-2} = r \quad \text{(Let)}$$

$\Rightarrow \alpha = r + 1, \beta = 2r - 2$ and $\gamma = -2r + 1$

$\because$ Q lies in the plane

$\therefore (r + 1) + 2(2r - 2) - 2(-2r + 1) = 10$

$\Rightarrow r = \frac{5}{3}$

foot of the perpendicular is $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$

52. Plane containing the line

Direction ratio's of normal to the plane :

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$$

Hence equation of plane $1(x-1) - 2(y-2) + 1(z-3) = 0$

i.e. $x - 2y + z = 0$

As given plane must be parallel $\Rightarrow A = 1$

& distance between the planes $\left|\frac{d-0}{\sqrt{1^2+2^2+1^2}}\right| = \sqrt{6}$

$|d| = 6$

53. (A) $P(\lambda+2, -2\lambda+1, \lambda-1)$

$$Q\left(2k + \frac{8}{3}, -k-3, k+1\right)$$

$3\lambda + 6 = a(6k+8)$ ......**(i)**

$-2\lambda + 1 = a(-k - 3)$ ......**(ii)**

$2\lambda - 2 = 2a(k + 1)$ ......**(iii)**

(ii) + **(iii)** $\Rightarrow -1 = ak - a$

$k = \frac{a-1}{a}$ ......**(iv)**

Put the value of k in equation **(iii)**

$\Rightarrow \lambda = 2a$ ......**(v)**

Put the values of λ & k in equation **(i)**

$6a + 6 = a\left(\frac{6a-6}{a} + 8\right) \Rightarrow 6 = 6a - 6 + 8a$

$\Rightarrow a = \frac{3}{2}$

Put the value of a in equation **(iv)** & **(v)**

$$k = \frac{\frac{3}{2}-1}{\frac{3}{2}} = \frac{1}{3} \quad \& \quad \lambda = 3$$

$$P(5, -5, 2) \ \& \ Q\left(\frac{10}{3}, \frac{-10}{3}, \frac{4}{3}\right)$$

$$d = \sqrt{\left(5-\frac{10}{3}\right)^2 + \left(5-\frac{10}{3}\right)^2 + \left(\frac{2}{3}\right)^2}$$

$$= \sqrt{\frac{25}{9}+\frac{25}{9}+\frac{4}{9}}$$

$$\Rightarrow d = \sqrt{6} \quad \Rightarrow \quad d^2 = 6$$

(B) $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \tan^{-1}\left(\frac{3}{4}\right)$

$$\tan^{-1}\left(\frac{(x+3)-(x-3)}{1+(x^2-9)}\right) = \tan^{-1}\left(\frac{3}{4}\right)$$

$\Rightarrow\ 1 + x^2 - 9 = 8 \quad \Rightarrow \quad x^2 = 16$

$\Rightarrow\ x = \pm 4$

(C) $\mu b^2 + 4\vec{b}.\vec{c} = 0$

$b^2 - \vec{a}.\vec{c} + \vec{b}.\vec{c} = 0$

$b^2 - \left(\mu\vec{b} + 4\vec{c}\right).\vec{c} + \vec{b}.\vec{c}$

$= b^2 + \vec{b}.\vec{c}\left(1-\mu\right) - 4c^2 = 0$

$b^2 - \frac{\mu}{4}b^2\left(1-\mu\right) = 4c^2$

$b^2\left(4-\mu+\mu^2\right) = 16c^2$ ....**(i)**

$4b^2 + 8\vec{b}.\vec{c} + 4c^2 = b^2 + a^2$

$3b^2 - 2\mu b^2 + 4c^2 = \left(\mu\vec{b} + 4\vec{c}\right)^2$

$3b^2 - 2\mu b^2 + 4c^2 = \mu^2 b^2 + 8\mu\vec{b}.\vec{c} + 16c^2$

$b^2\left(3-2\mu-\mu^2\right) = 12c^2 - 2\mu^2 \times b^2$

$b^2(3 - 2\mu + \mu^2) = 12c^2$ ....**(ii)**

$$\frac{4-\mu+\mu^2}{3-2\mu+\mu^2} = \frac{4}{3}$$

$12 - 3\mu + 3\mu^2 = 12 - 8\mu + 4\mu^2$

$\mu^2 - 5\mu = 0$

$\mu = 0, 5$

(D) $I = \frac{2}{\pi}\int_{-\pi}^{\pi} \frac{\sin\frac{9x}{2}}{\sin\frac{x}{2}}dx$

$$I = \frac{4}{\pi}\int_0^{\pi} \frac{\sin\frac{9x}{2}}{\sin\frac{x}{2}}dx \qquad \text{....... (i)}$$

$$I = \frac{4}{\pi}\int_0^{\pi} \frac{\cos\frac{9x}{2}}{\cos\frac{x}{2}}dx \qquad \text{....... (ii)}$$

(i) + **(ii)**

$$I = \frac{2}{\pi}\int_0^{\pi} \frac{\sin 5x}{\sin\frac{x}{2}\cos\frac{\pi}{2}}dx = \frac{4}{\pi}\int_0^{\pi}\frac{\sin 5x}{\sin x}dx$$

$f(x) = f(\pi - x)$

$$I = \frac{8}{\pi}\int_0^{\pi/2} \frac{\sin 5x}{\sin x}dx \qquad \text{....... (i)}$$

$$I = \frac{8}{\pi}\int_0^{\pi/2} \frac{\cos 5x}{\cos x}dx \qquad \text{....... (ii)}$$

(i) + **(ii)**

$$I = \frac{4}{\pi}\int_0^{\pi/2} \frac{\sin 6x}{\sin x \cos x}dx$$

$$I = \frac{8}{\pi}\int_0^{\pi/2} \frac{\sin 6x}{\sin 2x}dx = \frac{8}{\pi}\int_0^{\pi/2}(3 - 4\sin^2 2x)dx$$

$$= \frac{8}{\pi}\int_0^{\pi/2} 3 - 2(1 - \cos 2x)dx$$

$$= \frac{8}{\pi}\int_0^{\pi/2}(1 + 2\cos 2x)dx = \frac{8}{\pi}\times\frac{\pi}{2} = 4$$

54. (a) Line QR : $\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-5}{1} = \lambda$

Any point on line QR :

$(\lambda + 2, 4\lambda + 3, \lambda + 5)$

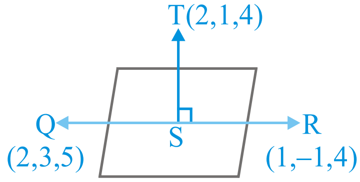
$\therefore$ Point of intersection with plane :

$5\lambda + 10 - 16\lambda - 12 - \lambda - 5 = 1$

$\Rightarrow\ \lambda = -\dfrac{2}{3}$

$\therefore\ P\left(\dfrac{4}{3}, \dfrac{1}{3}, \dfrac{13}{3}\right)$

Also



$\because\ TQ = TR = \sqrt{5}$

$\Rightarrow$ S is the mid-point of QR

$\Rightarrow\ S\left(\dfrac{3}{2}, 1, \dfrac{9}{2}\right) \Rightarrow PS = \dfrac{1}{\sqrt{2}}$ units

(b) Let required plane be

$(x + 2y + 3z - 2) + \lambda(x - y + z - 3) = 0$

$\because$ plane is at a distance $\dfrac{2}{\sqrt{3}}$ from the point $(3,1,-1)$.

$$\Rightarrow \left|\frac{(3+2-3-2)+\lambda(3-1-1-3)}{\sqrt{(1+\lambda)^2+(2-\lambda)^2+(3+\lambda)^2}}\right| = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \lambda^2 = \frac{(1+\lambda)^2+(2-\lambda)^2+(3+\lambda)^2}{3}$$

$\Rightarrow 3\lambda^2 = 3\lambda^2 + 2\lambda - 4\lambda + 6\lambda + 14$

$\Rightarrow \lambda = -\dfrac{7}{2}$

$\therefore$ required plane is $(x + 2y + 3z - 2)$

$$+\left(-\frac{7}{2}\right)(x - y + z - 3) = 0$$

$\Rightarrow 5x - 11y + z = 17$

(c) $(1, -1, 0); (-1, -1, 0)$

For coplanarity of lines

$$\begin{vmatrix} 2 & 0 & 0 \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix} = 0 \quad \Rightarrow \quad 2(k^2 - 4) = 0$$

$\Rightarrow k = \pm 2$

for $k = 2$

Normal vector $\vec{n} = \hat{j} - \hat{k}$

$\therefore$ Required plane : $y - z = \lambda$

$\because$ Passes through $(1, -1, 0)$

$\Rightarrow \lambda = -1$

$\therefore\ y - z = -1$

for $k = -2$

$\vec{n} = \hat{j} + \hat{k}$

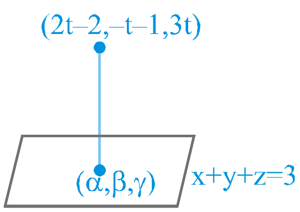
$\therefore$ Required plane : $y + z = \lambda$

$\because$ Passes through $(1, -1, 0)$

$\Rightarrow \lambda = -1$

$\therefore\ y + z = -1$

55. $\dfrac{\alpha - 2t + 2}{1} = \dfrac{\beta + t + 1}{1} = \dfrac{\gamma - 3t}{1} = k$



$\alpha = k + 2t - 2$

$\beta = k - t - 1$

$\gamma = k + 3t$

$\alpha + \beta + \gamma = 3$

$k = \dfrac{6 - 4t}{3}$

$\alpha = \dfrac{6 - 4t}{3} + 2t - 2 = \dfrac{2t}{3}$

$\beta = \dfrac{6 - 4t}{3} - t - 1 = \dfrac{3 - 7t}{3}$

$\gamma = \dfrac{6 - 4t}{3} + 3t = \dfrac{5t + 6}{3}$

$\Rightarrow \dfrac{3\alpha}{2} = \dfrac{3\beta - 3}{-7} = \dfrac{3\gamma - 6}{5}$

$\Rightarrow \dfrac{x}{2} = \dfrac{y - 3}{-7} = \dfrac{z - 2}{5}$

56. $\ell_1 : \vec{r} = (3, -1, 4) + (1, 2, 2)t$

$\ell_2 : \vec{r} = (3, 3, 2) + (2, 2, 1)s$

vector perpendicular to ℓ_1 and ℓ_2 :

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -2\hat{i} + 3\hat{j} - 2\hat{k}$$

∴ Equation of line ℓ : $\vec{r} = 0 + (-2, 3, -2)\lambda$

Point of intersection of ℓ_1 and ℓ :

$3 + t = -2\lambda$

$-1 + 2t = 3\lambda$.

$4 + 2t = -2\lambda$.

On solving we get $\lambda = -1$, $t = -1$

∴ Point of intersection of ℓ_1 & ℓ : P(2, –3, 2)

A point on ℓ_2 at distance of $\sqrt{17}$ from P :

⇒ $(1 + 2s)^2 + (6 + 2s)^2 + s^2 = 17$

⇒ $s = -\frac{10}{9}$; $s = -2$

for above s, point will be (B), (D)

57. $L_1 : \frac{x-5}{0} = \frac{y}{3-\alpha} = \frac{z}{-2}$

$L_2 : \frac{x-\alpha}{0} = \frac{y}{-1} = \frac{z}{2-\alpha}$

for lines to be coplanar

$$\begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$$

⇒ $(5 - \alpha)((3 - \alpha)(2-\alpha) - 2) = 0$

⇒ $(5 - \alpha)(\alpha^2 - 5\alpha + 4) = 0$

⇒ $\alpha = 1, 4, 5$

58. For point of intersection of L_1 and L_2

$$\begin{cases} 2\lambda + 1 = \mu + 4 \\ -\lambda = \mu - 3 \\ \lambda - 3 = 2\mu - 3 \end{cases} \Rightarrow \mu = 1$$

⇒ point of intersction is (5, –2, –1)

Now, vector normal to the plane is

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix} = -16(\hat{i} - 3\hat{j} - 2\hat{k})$$

Let equation of required plane be

$x - 3y - 2z = \alpha$

∵ it passes through (5, –2, –1)

∴ $\alpha = 13$

⇒ equation of plane is $x - 3y - 2z = 13$

69. Direction of OQ ≡ (3, 3, 0)

Direction of OS ≡ $\left(\frac{3}{2}, \frac{3}{2}, 3\right)$

$$\cos\theta = \frac{3\times\frac{3}{2} + 3\times\frac{3}{2}}{\sqrt{3^2+3^2}\sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + 3^2}}$$

$$= \frac{1}{\sqrt{3}}$$

∴ Hence (A) wrong.

For option B

$$\text{Normal of plane } \overrightarrow{OQ} \times \overrightarrow{OS} = \pm\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 0 \\ \frac{3}{2} & \frac{3}{2} & 3 \end{vmatrix}$$

$$= \pm\left(9\hat{i} - 9\hat{j}\right)$$

Equation of plane passing origin is $\vec{r}.\vec{n} = 0$

∴ $\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(9\hat{i} - 9\hat{j}\right) = 0$

⇒ $x - y = 0$

For (C) Perpendicular from P(3, 0, 0) to $x - y = 0$

$$= \left|\frac{3-0}{\sqrt{1^2+1^2}}\right| = \frac{3}{\sqrt{2}}$$

Equation of RS is $\frac{x-0}{\frac{3}{2}-0} = \frac{y-3}{\frac{3}{2}-3} = \frac{z-0}{3-0}$

$$\frac{x}{\frac{3}{2}} = \frac{y-3}{-\frac{3}{2}} = \frac{z}{3}$$

Angle between line RS and OR

$$\cos\theta = \frac{0+3\left(-\frac{3}{2}\right)+0}{\sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{3}{2}\right)^2+3^2}\sqrt{3^2}} = \frac{1}{\sqrt{6}}$$

Distance = OT = OR sin θ

$$= 3\sqrt{1-\frac{1}{6}} = 3\sqrt{\frac{5}{6}} = \sqrt{\frac{15}{2}}$$

70. Let image (x, y, z)

$$\frac{x-3}{1} = \frac{y-1}{-1} = \frac{z-7}{1} = -2\left(\frac{3-1+7-3}{1^2+1^2+1^2}\right)$$

P(x, y, z) = (–1, 5, 3)

Plane passing through P(–1, 5, 3) is

$a(x+1)+b(y-5)+c(z-3)=0$ ....(i)

Given (0, 0, 0) satisfy

$\Rightarrow$ $a-5b-3c=0$ ....(ii)

and $a\times 1+b\times 2+c\times 1=0$

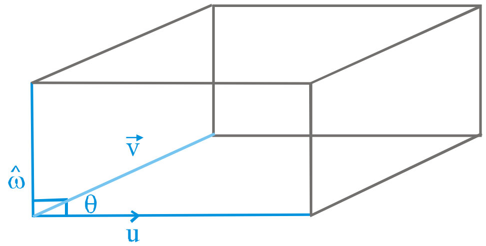
$a+2b+c=0$ ....(iii)

from (ii) and (iii) $\frac{a}{1}=\frac{b}{-4}=\frac{c}{7}$

put in (i) $(x+1)-4(y-5)+7(z-3)=0$

$x-4y+7z=0$

71.



Given condition $\hat{\omega}$ is perpendicular to $\hat{u}\times\hat{v}$

As $|\hat{u}\times\hat{v}|=1$ and angle between u and v can change

$\Rightarrow$ infinitely many choice for such v.

$\vec{w}$ is $\perp \vec{u}$

$\Rightarrow$ $u_1+u_2+u_3=0$

If $\vec{u}$ in xy plane

$\Rightarrow$ $u_3=0$.

$\Rightarrow$ $|u_1|=|u_2|$

MOCK TEST (VECTOR)

1. $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = [\vec{a}\ \vec{b}\ \vec{c}]^2$

$$= ((\vec{a}\times\vec{b}).\vec{c})^2 = (ab\sin\theta\,\vec{c}.\vec{c})^2 = \frac{a^2b^2}{4} = \frac{1}{4}$$

$(a_1^2+a_2^2+a_3^2)\ (b_1^2+b_2^2+b_3^2)$

2. **(D)**

Volume of the parallelopiped formed by $\vec{a}'$, $\vec{b}'$, $\vec{c}'$ is 4

$\therefore$ Volume of the parallelopiped formed by $\vec{a}$, $\vec{b}$, $\vec{c}$ is $\frac{1}{4}$

$$\vec{b}\times\vec{c} = \frac{(\vec{c}'\times\vec{a}')\times\vec{c}}{4} = \frac{1}{4}\vec{a}'$$

$$\therefore\ |\vec{b}\times\vec{c}| = \frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}}$$

$$\therefore\ \text{length of altitude} = \frac{1}{4}\times 2\sqrt{2} = \frac{1}{\sqrt{2}}.$$

3. A vector along the angle bisector

$$= \hat{a}+\hat{b} = \frac{(-4\hat{i}+3\hat{k})}{5} + \frac{(14\hat{i}+2\hat{j}-5\hat{k})}{15}$$

$$= \frac{-12\hat{i}+9\hat{k}+14\hat{i}+2\hat{j}-5\hat{k}}{15} = \frac{2(\hat{i}+\hat{j}+2\hat{k})}{15}$$

$$\therefore\ \vec{d} = \hat{i}+\hat{j}+2\hat{k}$$

4. **(C)**

$\vec{r}.\vec{a}=0$, $|\vec{r}\times\vec{b}| = |\vec{r}|\ |\vec{b}|$ and $|\vec{r}\times\vec{c}| = |\vec{r}||\vec{c}|$

$\Rightarrow$ $\vec{r}\perp\vec{a},\vec{b},\vec{c}$

$\therefore$ $\vec{a},\vec{b},\vec{c}$ are coplaner.

$\therefore$ $[\vec{a}\ \vec{b}\ \vec{c}]=0$

5. $|\overrightarrow{AC}|^2 = |2\overrightarrow{AB}|^2$

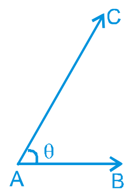
$\Rightarrow$ $|4\hat{i}+(4x-2)\hat{j}+2\hat{k}|^2 = 4\,|(\hat{i}+x\hat{j}+3\hat{k})|^2$

$\Rightarrow$ $16+(4x-2)^2+4 = 4(1+x^2+9)$

$\Rightarrow$ $20+16x^2+4-16x = 4+4x^2+36$

$\Rightarrow$ $12x^2-16x-16=0$

$\Rightarrow$ $3x^2-4x-4=0$



$\Rightarrow$ $x = 2, -\dfrac{2}{3}$ .....**(i)**

angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is

$$\cos\theta = \frac{\overrightarrow{AB}.\overrightarrow{AC}}{|\overrightarrow{AB}|\,|\overrightarrow{AC}|} = \frac{\overrightarrow{AB}.\overrightarrow{AC}}{2\,|\overrightarrow{AB}|^2}$$

$$\Rightarrow \frac{11}{14} = \frac{(\hat{i}+x\hat{j}+3\hat{k}).(4\hat{i}+(4x-2)\hat{j}+2\hat{k})}{2(1+x^2+9)}$$

$$= \frac{4+x(4x-2)+6}{2x^2+20}$$

$\Rightarrow$ $11x^2 + 110 = 70 + 28x^2 - 14x$

$\Rightarrow$ $17x^2 - 14x - 40 = 0$

$\therefore$ $x = 2, -\dfrac{20}{17}$ ....**(ii)**

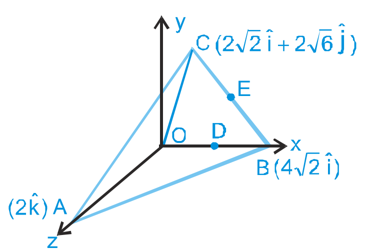
from **(i)** and **(ii)**

$x = 2$

6. (D)

$$\vec{a}' = \frac{\vec{b}\times\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]} = \frac{\hat{i}+\hat{j}-\hat{k}}{2}$$

7. $\overrightarrow{AD} = 2\sqrt{2}\,\hat{i} - 2\hat{k}$



$\overrightarrow{OE} = 3\sqrt{2}\,\hat{i} + \sqrt{6}\,\hat{j}$

$$\cos\theta = \frac{12}{\sqrt{12}\sqrt{24}}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

8. (B)

Let $\vec{r} = \ell(\vec{b}\times\vec{c}) + m(\vec{c}\times\vec{a}) + n(\vec{a}\times\vec{b})$

$\vec{r}.\vec{a} = \ell\,[\vec{a}\;\vec{b}\;\vec{c}]$

$\Rightarrow$ $\ell = 1$

similarly $m = 2$, $n = 3$

$\therefore$ $\vec{r} = (\vec{b}\times\vec{c}) + 2(\vec{c}\times\vec{a}) + 3(\vec{a}\times\vec{b})$

9. $\displaystyle\sum_{i=1}^{n-1} \overrightarrow{OA_i}\times\overrightarrow{OA_{i+1}}$

$= \overrightarrow{OA_1}\times\overrightarrow{OA_2} + \overrightarrow{OA_2}\times\overrightarrow{OA_3}$ ....+ $\overrightarrow{OA_{n-1}}\times\overrightarrow{OA_n}$

$= (n-1)\;(\overrightarrow{OA_1}\times\overrightarrow{OA_2})$

$= (1-n)\;(\overrightarrow{OA_2}\times\overrightarrow{OA_1})$

10. (B)

S_1: $\vec{a}$ and $\lambda\vec{a}$ are parallel vectors.

S_2: $\vec{a}\cdot\vec{b}$ may take negative values also.

S_3: $|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})| = |-(\vec{a}\times\vec{b})+(\vec{b}\times\vec{a})| = 2\,|\vec{b}\times\vec{a}|$

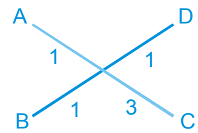
S_4: $(\vec{a}\times\vec{b})^2 = (\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{b}) = \vec{a}\cdot(\vec{b}\times(\vec{a}\times\vec{b}))$

$= \vec{a}\cdot\left((\vec{b}.\vec{b})\,\vec{a} - (\vec{a}.\vec{b})\,\vec{b}\right)$

$= (\vec{a}\cdot\vec{a})\,(\vec{b}\cdot\vec{b}) - (\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{a})$

11. $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$

sum of coefficient = 0 $\Rightarrow$ $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar



Also $2\vec{b} + 2\vec{d} = 3\vec{a} + \vec{c}$

$$\Rightarrow \frac{\vec{b}+\vec{d}}{2} = \frac{3\vec{a}+\vec{c}}{4}$$

12. (A, B, D)

$(\lambda-1)\,(\vec{a}_1 - \vec{a}_2) + \mu\,(\vec{a}_2 + \vec{a}_3) + \gamma\,(\vec{a}_3 + \vec{a}_4 - 2\vec{a}_2) +$

$\vec{a}_3 + \delta\vec{a}_4 = \vec{0}$

i.e. $(\lambda-1)\,\vec{a}_1 + (1-\lambda+\mu-2\gamma)\,\vec{a}_2 + (\mu+\gamma+1)\,\vec{a}_3$

$+ (\gamma+\delta)\,\vec{a}_4 = \vec{0}$

Since $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ are linearly independent

$\therefore$ $\lambda - 1 = 0,\ 1-\lambda+\mu-2\gamma = 0,\ \mu+\gamma+1 = 0$ and $\gamma+\delta = 0$

i.e. $\lambda = 1,\ \mu = 2\gamma,\ \mu+\gamma+1 = 0,\ \gamma+\delta = 0$

i.e. $\lambda = 1,\ \mu = -\dfrac{2}{3},\ \gamma = -\dfrac{1}{3},\ \delta = \dfrac{1}{3}$

13. $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = (xy+yz+zx)$

$\{(y-z)\hat{i}+(z-x)\hat{j}+(x-y)\hat{k}\}$

$\therefore$ all the option are correct

14. (A, C, D)

(A) $\vec{a}\times\left[\vec{a}\times(\vec{a}\times\vec{b})\right] = \vec{a}\times\left[(\vec{a}.\vec{b})\vec{a}-(\vec{a}.\vec{a})\vec{b}\right]$

$= -(\vec{a}.\vec{a})(\vec{a}\times\vec{b})$

$\therefore$ (A) is not correct

(B) $\vec{v}.\vec{a}=\vec{0} \Rightarrow \vec{v}=\vec{0}$ or $\vec{v}\perp\vec{a}$

$\vec{v}.\vec{b}=\vec{0} \Rightarrow \vec{v}=\vec{0}$ or $\vec{v}\perp\vec{b}$

$\vec{v}.\vec{c}=\vec{0} \Rightarrow \vec{v}=\vec{0}$ or $\vec{v}\perp\vec{c}$

$\therefore$ $\vec{v}=\vec{0}$ or $\vec{v}\perp\vec{a},\vec{b},\vec{c}$

$\therefore$ $\vec{v}=\vec{0}$

(C) $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d}) = \vec{0}$

$\therefore$ statement is incorrect

(D) $\vec{a}\times\vec{b}' + \vec{b}.\vec{c}' + \vec{c}.\vec{a}' = 0.$

(Property of reciprocal system)

(D) is incorrect

15. Since $\vec{a}$ makes obtuse angle with z-axis

$\therefore$ $\dfrac{\sin 2\alpha}{\sqrt{1+9+\sin^2 2\alpha}} < 0$ i.e. $\sin 2\alpha < 0$

$\therefore$ either $\dfrac{\pi}{2} < \alpha < \pi$ or $\dfrac{3\pi}{2} < \alpha < 2\pi$ .....**(i)**

since $\vec{b}$ and $\vec{c}$ are orthogonal

$\therefore$ $\tan^2\alpha - \tan\alpha - 6 = 0$

i.e. $\tan\alpha = 3, -2$ ......**(ii)**

from **(i)** and **(ii)**, we get

$\tan\alpha = -2$

$\therefore$ $\alpha = \pi - \tan^{-1} 2$ or $\alpha = 2\pi - \tan^{-1} 2$

16. (B)

Both the statements are correct but statement–2 is not correct explanation of statement–1 because vectors $\vec{b}, \vec{c}, \vec{d}$ in statement–1 are coplanar.

17. (D)

Statement–1 is false and Statement–2 is true.

Since $\vec{a}.(\vec{b}\times\vec{c}) = 0$

$\therefore$ $\vec{a},\vec{b},\vec{c}$ are coplanar

18. (B)

Statement-I is correct and Statement – II is correct but Statement – II is not correct explanation of Statement – I

19. (A)

$3\vec{a}-2\vec{b}+5\vec{c}-6\vec{d} = (2\vec{a}-2\vec{b})+(-5\vec{a}+5\vec{c})+(6\vec{a}-6\vec{d})$

$= -2\overrightarrow{AB}+5\overrightarrow{AC}-6\overrightarrow{AD}=\vec{0}$

$\therefore$ $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ are linearly dependent,

Hence by statement-2, the statement-1 is true.

20. (D)

Statement - 1 $\vec{b}_1 = \left(\dfrac{(2\hat{i}+\hat{j}-3\hat{k}).(3\hat{i}-\hat{j})}{|3\hat{i}-\hat{j}|}\right)\dfrac{3\hat{i}-\hat{j}}{|3\hat{i}-\hat{j}|}$

$= \dfrac{3\hat{i}}{2} - \dfrac{\hat{j}}{2}$

$\therefore$ $\vec{b}_2 = 2\hat{i}+\hat{j}-3\hat{k}-\dfrac{3\hat{i}}{2}+\dfrac{\hat{j}}{2} = \dfrac{\hat{i}}{2}+\dfrac{3\hat{j}}{2}-3\hat{k}$

$\therefore$ statement is false

Statement - 2 is true

21. (A) → (t), (B) → (p), (C) → (q), (D) → (s)

(A) $\vec{a}+\vec{b} = \hat{j}$ and $2\vec{a}-\vec{b} = 3\hat{i}+\dfrac{\hat{j}}{2}$

$\therefore$ $\vec{a} = \hat{i}+\dfrac{\hat{j}}{2}$, $\vec{b} = -\hat{i}+\dfrac{\hat{j}}{2}$

$\therefore$ $\cos\theta = -\dfrac{3}{5}$

(B) $|\vec{a}+\vec{b}+\vec{c}| = \sqrt{6}$

$\Rightarrow$ $\vec{a}^2+\vec{b}^2+\vec{c}^2+2\vec{a}\cdot\vec{b}+2\vec{b}\cdot\vec{c}+2\vec{c}\cdot\vec{a} = 6$

$\therefore$ $|\vec{a}| = 1$

(C) $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = -2\hat{i}-14\hat{j}-10\hat{k}$

$\therefore$ Area $= 5\sqrt{3}$

(D) $\vec{a}$ is perpendicular $\vec{b}+\vec{c} \Rightarrow \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=0$ .... **(i)**

$\vec{b}$ is perpendicular $\vec{a}+\vec{c}$

$\Rightarrow \vec{b}\cdot\vec{c}+\vec{b}\cdot\vec{c}=0$ .... **(ii)**

$\vec{c}$ is perpendicular $\vec{a}+\vec{b}$

$\Rightarrow \vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}=0$ .... **(iii)**

from **(i)**, **(ii)** and **(iii)** we get

$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$

$\therefore \ |\vec{a}+\vec{b}+\vec{c}|=7$

22. (A) → (s), (B) → (p), (C) → (r), (D) → (t)

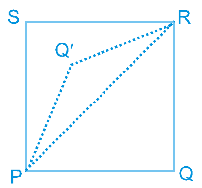
(A) $\overrightarrow{OA}=\hat{i}+2\hat{j}+3\hat{k}$, $\overrightarrow{OB}=-2\hat{i}+\hat{j}-4\hat{k}$,

$\overrightarrow{OC}=3\hat{i}+4\hat{j}-2\hat{k}$

$$\text{Area}=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|=\frac{\sqrt{1218}}{2}$$

(B) $((\vec{a}\times\vec{b})\times\vec{c}).\vec{d}+((\vec{b}\times\vec{c})\times\vec{a}).\vec{d}+((\vec{c}\times\vec{a})\times\vec{b}).\vec{d}=0$

(C)



taking P as origin position vector of Q, R and S are $P\hat{i}$, $P\hat{i}+P\hat{j}$, $P\hat{j}$

equations of PQ′ and RS are $\vec{r}=t(\hat{i}+\hat{j}+\sqrt{2}\hat{k})$,

$\vec{r}=P\hat{i}+P\hat{j}+\lambda\hat{i}$

$\therefore$ shortest distance $=\dfrac{2P}{\sqrt{6}}$

$\therefore \ k=2$

(D) $(\vec{a}.\vec{c})(\vec{b}.\vec{d})-(\vec{b}.\vec{c})(\vec{a}.\vec{d})=21$

23.

1. (B)

$$\vec{a}_1=\left[(2\hat{i}+3\hat{j}-6\hat{k})\cdot\frac{(2\hat{i}-3\hat{j}+6\hat{k})}{7}\right]\frac{2\hat{i}-3\hat{j}+6\hat{k}}{7}$$

$$=\frac{-41}{49}(2\hat{i}-3\hat{j}+6\hat{k})$$

$$\vec{a}_2=\frac{-41}{49}\left((2\hat{i}-3\hat{j}+6\hat{k})\cdot\frac{(-2\hat{i}+3\hat{j}+6\hat{k})}{7}\right)$$

$$\frac{(-2\hat{i}+3\hat{j}+6\hat{k})}{7}$$

$$=\frac{-41}{(49)^2}(-4-9+36)(-2\hat{i}+3\hat{j}+6\hat{k})$$

$$=\frac{943}{49^2}(2\hat{i}-3\hat{j}-6\hat{k})$$

2. (A)

$$\vec{a}_1.\vec{b}=\frac{-41}{49}(2\hat{i}-3\hat{j}+6\hat{k}).(2\hat{i}-3\hat{j}+6\hat{k})=-41$$

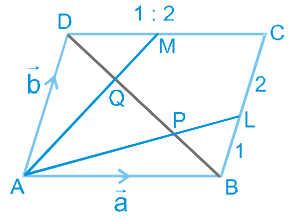
3. (C)

$\vec{a}$, $\vec{a}_1$, $\vec{b}$ are coplanar, because $\vec{a}_1$, $\vec{b}$ are collinear.

24. $\overrightarrow{BL}=\dfrac{1}{3}\vec{b}$

$\therefore \ \overrightarrow{AL}=\vec{a}+\dfrac{1}{3}\vec{b}$

Let $\overrightarrow{AP}=\lambda\,\overrightarrow{AL}$ and P divides DB in the ratio $\mu:1-\mu$



Then $\overrightarrow{AP}=\lambda\vec{a}+\dfrac{\lambda}{3}\vec{b}$ ....**(i)**

Also $\overrightarrow{AP}=\mu\vec{a}+(1-\mu)\vec{b}$ ....**(ii)**

from **(i)** and **(ii)** $\lambda\vec{a}+\dfrac{\lambda}{3}\vec{b}=\mu\vec{a}+(1-\mu)\vec{b}$

$\therefore \ \lambda=\mu$ and $\dfrac{\lambda}{3}=1-\mu$

$\therefore \ \lambda=\dfrac{3}{4}$

$\therefore$ P divides AL in the ratio 3 : 1 and P divides DB in the ratio 3 : 1

similarly Q divides DB in the ratio 1 : 3

thus $DQ=\dfrac{1}{4}DB$ and $PB=\dfrac{1}{4}DB$

$\therefore \ PQ=\dfrac{1}{2}DB$ i.e. $PQ:DB=1:2$

25

1. (A)

The diagonals are

$\vec{d}_1 = 3\hat{a} - 2\hat{b} + 2\hat{c} + (-\hat{a} - 2\hat{c}) = 2\hat{a} - 2\hat{b}$

$\vec{d}_2 = 3\hat{a} - 2\hat{b} + 2\hat{c} - (-\hat{a} - 2\hat{c}) = 4\hat{a} - 2\hat{b} + 4\hat{c}$

Angle between them $= \cos^{-1} \dfrac{\vec{d}_1 . \vec{d}_2}{|\vec{d}_1|.|\vec{d}_2|}$

$= \cos^{-1}\left(\dfrac{8+4}{2\sqrt{2}\,(6)}\right) = \cos^{-1}\dfrac{1}{\sqrt{2}} = \dfrac{\pi}{4}$

2. (D)

$\vec{x} + \vec{y} = 2\hat{b} - 3\hat{c}$ and $\vec{y} + \vec{z} = -2\hat{a} + 3\hat{b} - 3\hat{c}$

$\therefore \quad (\vec{x}+\vec{y}) \times (\vec{y}+\vec{z}) = \begin{vmatrix} \hat{a} & \hat{b} & \hat{c} \\ 0 & 2 & -3 \\ -2 & 3 & -3 \end{vmatrix} = 3\hat{a} + 6\hat{b} + 4\hat{c}$

$\therefore$ requied unit vector $= \dfrac{3\hat{a} + 6\hat{b} + 4\hat{c}}{\sqrt{61}}$

3. (A)

$\begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ x & -1 & 2 \end{vmatrix} = 0$

$\Rightarrow \; 2(4-1) + 3(2+x) + 4(-1-2x) = 0 \;\Rightarrow\; x = \dfrac{8}{5}$

4. (C)

$\vec{r} \times \vec{x} = \vec{y} \times \vec{x} \;\Rightarrow\; (\vec{r} - \vec{y}) \times \vec{x} = \vec{0}$

$\Rightarrow \; \vec{r} = \vec{y} + \lambda \vec{x}$

$\vec{r} \times \vec{y} = \vec{x} \times \vec{y} \Rightarrow (\vec{r} - \vec{x}) \times \vec{y} = \vec{0} \;\Rightarrow\; \vec{r} = \vec{x} + \mu \vec{y}$

$\vec{y} + \lambda \vec{x} = \vec{x} + \mu \vec{y}$

$(2\vec{a} - \vec{b}) + \lambda(\vec{a} + \vec{b}) = (\vec{a} + \vec{b}) + \mu(2\vec{a} - \vec{b})$

$\Rightarrow \; 2 + \lambda = 1 + 2\mu, \; -1 + \lambda = 1 - \mu$

$\Rightarrow \; \mu = 1, \lambda = 1$

The point of intersection is $3\vec{a}$

5. (B)

$\hat{a} \times \hat{b} = \hat{c} \;\Rightarrow\; \hat{c}.\hat{a} \times \hat{b} = \hat{c}.\hat{c} = 1$

$\Rightarrow \; \hat{a}\,.(\hat{b} \times \hat{c}) + \hat{b}\,.(\hat{c} \times \hat{a}) + \hat{c}\,.(\hat{a} \times \hat{b}) = 3$

26. (50)

$V_1 = [\vec{a}\;\vec{b}\;\vec{c}\,] \qquad V_2 = \dfrac{1}{2}[\vec{a}\;\vec{b}\;\vec{c}\,]$

$V_3 = \dfrac{1}{6}[\vec{a}\;\vec{b}\;\vec{c}\,]$

$V_1 : V_2 : V_3 = 1 : \dfrac{1}{2} : \dfrac{1}{6}$

$= 6 : 3 : 1$

$V_1 = \begin{vmatrix} 1 & -1 & -6 \\ 1 & -1 & 4 \\ 2 & -5 & 3 \end{vmatrix} = 1(-3+20) + 1(3-8) - 6(-5+2)$

$= 17 - 5 + 18 = 30$

$\therefore \; V_1 + V_2 + V_3 = 30 + 15 + 5 = 50$

27. $V = \dfrac{1}{6}[\vec{a}\;\vec{b}\;\vec{c}\,]$

The centroid are $\dfrac{\vec{a}+\vec{b}}{3}, \dfrac{\vec{b}+\vec{c}}{3}, \dfrac{\vec{c}+\vec{a}}{3}, \dfrac{\vec{a}+\vec{b}+\vec{c}}{3}$

$\therefore \quad V' = \dfrac{1}{6}\left[\dfrac{\vec{c}-\vec{a}}{3} \;\; \dfrac{\vec{c}-\vec{b}}{3} \;\; \dfrac{\vec{c}}{3}\right] = \dfrac{1}{6 \times 27}[\vec{a}\;\vec{b}\;\vec{c}\,]$

$= \dfrac{1}{27} V$

$\therefore \quad k = 27$

28. (2)

$\Sigma[\vec{p} \times \{(\vec{x} - \vec{q}) \times \vec{p}\}] = \vec{0}$

$\Rightarrow \; \Sigma[\vec{p} \times (\vec{x} \times \vec{p})] - \Sigma[\vec{p} \times (\vec{q} \times \vec{p})] = \vec{0}$

$\Rightarrow \; \Sigma \vec{p}^{\,2}\,\vec{x} - \Sigma(\vec{p}\,.\,\vec{x})\,\vec{p} - \Sigma \vec{p}^{\,2}\,\vec{q} + \Sigma(\vec{p}.\vec{q})\,\vec{p} = \vec{0}$

$\Rightarrow \; 3\vec{p}^{\,2}\,\vec{x} - \vec{p}^{\,2}\,\vec{x} - \vec{p}^{\,2}\,(\vec{p} + \vec{q} + \vec{r}) = \vec{0}$

$\Rightarrow \; 2\vec{p}^{\,2}\,\vec{x} = \vec{p}^{\,2}\,(\vec{p} + \vec{q} + \vec{r})$

$\Rightarrow \; \vec{x} = \dfrac{1}{2}(\vec{p} + \vec{q} + \vec{r})$

29. Equation of line L_1 is $7\hat{i} + 6\hat{j} + 2\hat{k} + l\,(-3\hat{i} + 2\hat{j} + 4\hat{k})$

Equation of line L_2 is $5\hat{i} + 3\hat{j} + 4\hat{k} + \mu\,(2\hat{i} + \hat{j} + 3\hat{k})$

$\overline{CD} = 2\hat{i} + 3\hat{j} - 2\hat{k} + \lambda(-3\hat{i} + 2\hat{j} + 4\hat{k}) - \mu\,(2\hat{i} + \hat{j} + 3\hat{k})$

since it is parallel to $2\hat{i} - 2\hat{j} - \hat{k}$

$\therefore \quad \dfrac{2 - 3\lambda - 2\mu}{2} = \dfrac{3 + 2\lambda - \mu}{-2} = \dfrac{-2 + 4\lambda - 3\mu}{-1}$

$\therefore\ \lambda = 2\ ,\ \mu = 1$

$\therefore\ \overrightarrow{CD} = -6\hat{i}+6\hat{j}+3\hat{k}$

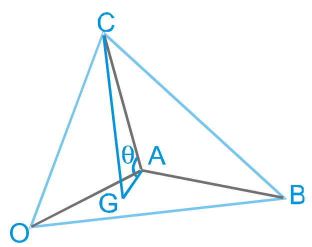
$\therefore\ |\overrightarrow{CD}| = 9$

30. (13)

Let OABC be the tetrahedron. Let G be the centroid of the face OAB, then $GA = \frac{1}{\sqrt{3}}AC$.

Then $\cos\theta = \frac{GA}{CA} = \frac{1}{\sqrt{3}}$

$\therefore\ \cos^2\theta = \frac{1}{3}$



$\therefore\ a = 1$ and $b = 3$

$\therefore\ 10a + b = 13$

MOCK TEST (3-D)

1. Any pt. on line is $(3\lambda+2, 2\lambda-1, 1-\lambda)$

but it lies on the curve $xy = c^2$ & $z = 0$

$\Rightarrow\ (3\lambda+2)(2\lambda-1) = c^2$ & $1-\lambda = 0$

$\Rightarrow\ (3\lambda+2)(2\lambda-1) = c^2$ & $\lambda = 1$

$\Rightarrow\ c^2 = 5 \quad \Rightarrow \quad c = \pm\sqrt{5}$

2. (A)

Foot of perpendicular from point $A(\vec{a})$ on the plane

$\vec{r}.\vec{n} = d$ is $\vec{a} + \frac{(d-\vec{a}.\vec{n})}{|\vec{n}|^2}\vec{n}$

$\therefore$ Equation of line parallel to $\vec{r} = \vec{a} + \lambda\vec{b}$ in the plane $\vec{r}\,.\,\vec{n} = d$ is given by

$$\vec{r} = \vec{a} + \frac{(d-\vec{a}.\vec{n})}{|\vec{n}|^2}\vec{n} + \lambda\vec{b}$$

3. Position of pt. after t hours is (2t, – 4t, 4t)

Position of pt. after 10 hours is (20, – 40, 40)

Distance from origin

$= \sqrt{(20)^2+(-40)^2+(40)^2} = 60$ km

4. (D)

$P_1 = P_2 = 0$, $P_2 = P_3 = 0$ and $P_3 = P_1 = 0$ are lines of intersection of the three planes P_1, P_2 and P_3.

As $\vec{n}_1$, $\vec{n}_2$ and $\vec{n}_3$ are non-coplanar, planes P_1, P_2 and P_3 will intersect at unique point. So the given lines will pass through a fixed point.

5. Let $\overrightarrow{OA} = \ell_1\hat{i}+m_1\hat{j}+n_1\hat{k}$, $\overrightarrow{OB} = \ell_2\hat{i}+m_2\hat{j}+n_2\hat{k}$ and $\overrightarrow{OC} = \ell_3\hat{i}+m_3\hat{j}+n_3\hat{k}$ be mutually perpendicular vectors.

Let $\overrightarrow{OP} = \ell\hat{i}+m\hat{j}+n\hat{k}$ be equally inclined to $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$.

Then

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \Sigma\ell_1\hat{i}+\Sigma m_1\hat{j}+\Sigma n_1\hat{k}$

$|\overrightarrow{OP}|^2 = (\Sigma\ell_1)^2+(\Sigma m_1)^2+(\Sigma n_1)^2$

$= 3 + 2\Sigma(\ell_1\ell_2+m_1m_2+n_1n_2) = 3$

$\therefore \quad |\overrightarrow{OP}| = \sqrt{3}$

$\therefore \quad \hat{OP} = \frac{\ell_1 + \ell_2 + \ell_3}{\sqrt{3}}\hat{i} + \frac{m_1 + m_2 + m_3}{\sqrt{3}}\hat{j} + \frac{n_1 + n_2 + n_3}{\sqrt{3}}\hat{k}$

6. (A)

Let θ be the required angle then θ will be the angle between $\vec{a}$ and $\vec{b}+\vec{c}$ ($\vec{b}+\vec{c}$ lies along the angular bisector of $\vec{a}$ and $\vec{b}$)

$$\cos\theta = \frac{\vec{a}.(\vec{b}+\vec{c})}{|\vec{a}||\vec{b}+\vec{c}|}$$

$$= \frac{2\cos\alpha}{\sqrt{2+2\cos\alpha}} = \frac{\cos\alpha}{\cos\frac{\alpha}{2}}$$

$$\theta = \cos^{-1}\left(\frac{\cos\alpha}{\cos\alpha/2}\right)$$

7. Circle passing through A (1, 0, 0) ; B(0, 1, 0) and C(0, 0, 1) will be greatest circle of sphere

⇒ circumcentre of Δ ABC will be centre of circle as well as of sphere, but since ΔABC is equailatral

∴ centre of the sphere is centroid of the ΔABC

∴ centre of the sphere is $D\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

Also radius AD = BD = CD $= \sqrt{\frac{6}{9}}$

Equation. of sphere

$$\left(x-\frac{1}{3}\right)^2 + \left(y-\frac{1}{3}\right)^2 + \left(z-\frac{1}{3}\right)^2 = \frac{6}{9}$$

⇒ $9(x^2+y^2+z^2) - 6(x+y+z) + 3 = 6$

⇒ $3(x^2+y^2+z^2) - 2(x+y+z) - 1 = 0$

⇒ $3\sum x^2 - 2\sum x - 1 = 0$

8. (A)

A(1, 1, 1), B(2, 3, 5), C(– 1, 0, 2) directions ratios of AB are <1, 2, 4>

direction ratios of AC are <– 2, – 1, 1>

∴ direction ratios of normal to plane ABC are <2, – 3, 1>

∴ Equation of the plane ABC is $2x - 3y + z = 0$

Let the equation of the required plane be $2x - 3y + z = k$,

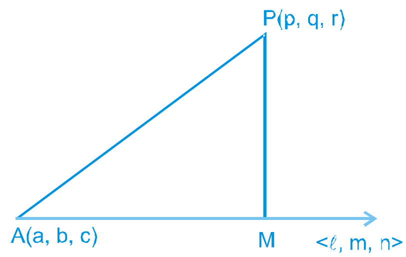
then $\left|\frac{k}{\sqrt{4+9+1}}\right| = 2$

$k = \pm 2\sqrt{14}$

∴ Equation of the required plane is $2x - 3y + z + 2\sqrt{14} = 0$

9. Consider $\overrightarrow{AP} \times (\ell\hat{i} + m\hat{j} + n\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ p-a & q-b & r-c \\ \ell & m & n \end{vmatrix}$

$= \sum (n(q-b) - m(r-c))\,\hat{i}$



$\therefore \quad MP^2 = \left|\overrightarrow{AP} \times (\ell\hat{i} + m\hat{j} + n\hat{k})\right|^2$

$= \sum \{n(q-b) - m(r-c)\}^2$

10. (A)

S_1 : true by definition

S_2 : false (because by the given condition, at least one point may lie on the plane)

S_3 : true (Standard result)

S_4 : True shortest distance $= \left|\frac{-11-3}{\sqrt{9+36+4}}\right| = 2$

11. (A, B, C)

$x + y + z - 1 = 0$

$4x + y - 2z + 2 = 0$

∴ direction ratios of the line are <–3, 6, –3 >

i.e. <1, –2, 1>

Let z = k, then x = k – 1, y = 2 – 2k

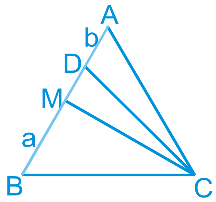
i.e. (k – 1, 2 – 2k, k) is any point on the line

∴ (–1, 2, 0), (0, 0, 1) and $\left(-\frac{1}{2}, 1, \frac{1}{2}\right)$ are points on the line

∴ (A), (B) and (C) are correct options

12. $\overrightarrow{CD} = \dfrac{a\vec{a} + b\vec{b}}{a+b}$

$\overrightarrow{CM} = \dfrac{\vec{a} + \vec{b}}{2}$



$\therefore$ area (Δ CDM) $= \dfrac{1}{2}|\overrightarrow{CD} \times \overrightarrow{CM}|$

$= \dfrac{1}{4} \times \dfrac{1}{(a+b)} \left|(a\vec{a} + b\vec{b}) \times (\vec{a} + \vec{b})\right|$

$= \dfrac{1}{4(a+b)} |(a-b)(\vec{a} \times \vec{b})|$

$= \dfrac{1}{2} \cdot \dfrac{a-b}{a+b} \cdot \dfrac{1}{2} |\vec{a} \times \vec{b}| = \dfrac{1}{2} \dfrac{a-b}{a+b}$ ar (ΔABC)

$\therefore \dfrac{ar(\Delta\, CDM)}{ar(\Delta\, ABC)} = \dfrac{1}{2} \dfrac{a-b}{a+b} = \dfrac{1}{2} \tan \dfrac{A-B}{2} \cot \dfrac{A+B}{2}$

13. (A , B)

$3x - 6y + 2z + 5 = 0$ ....**(i)**

$-4x + 12y - 3z + 3 = 0$ ....**(ii)**

$\dfrac{3x - 6y + 2z + 5}{\sqrt{9 + 36 + 4}} = \dfrac{-4x + 12y - 3z + 3}{\sqrt{16 + 144 + 9}}$

bisects the angle between the planes that contains the origin.

$13(3x - 6y + 2z + 5) = 7(-4x + 12y - 3z + 3)$

$39x - 78y + 26z + 65 = -28x + 84y - 21z + 21$

$67x - 162y + 47z + 44 = 0$ ....**(iii)**

Further $3 \times (-4) + (-6)(12) + 2 \times (-3) < 0$

$\therefore$ origin lies in acute angle

14. (B)

S_1 : Since $\begin{vmatrix} 1-1 & 7+2 & -4-3 \\ 2 & 5 & 7 \\ 1 & 2 & 3 \end{vmatrix} = \begin{vmatrix} 0 & 9 & -7 \\ 2 & 5 & 7 \\ 1 & 2 & 3 \end{vmatrix} = 16 \neq 0$

S_2 : by the given condition

$\dfrac{4+2}{3} = \dfrac{13-3}{5} = \dfrac{1-5}{2}$ i.e. $2 = 2 = -2$

Which is not true

S_3 : Let $S \equiv x^2 + y^2 + z^2 - 2x - 4y - 2z + 2 = 0$

Then $S_1 = 4 + 1 + 1 - 4 - 4 - 2 + 2 = -2 < 0$

$\therefore$ Statement is false

S_4 : Let $< a, b, c >$ be direction ratios of the line, then

$a + b + c = 0$

$4a + b - 2c = 0$

i.e. $\dfrac{a}{-2-1} = \dfrac{b}{4+2} = \dfrac{c}{1-4}$

i.e. $\dfrac{a}{-3} = \dfrac{b}{6} = \dfrac{c}{-3}$ i.e. $\dfrac{a}{1} = \dfrac{b}{-2} = \dfrac{c}{1}$

$\therefore$ Statement is true

15. (A, B)

Equation of required plane is

$\ell x + my + \lambda z = 0$ ....**(i)**

angle between **(i)** & $\ell x + my = 0$ is α.

$\Rightarrow \cos\alpha = \dfrac{\ell^2 + m^2}{\sqrt{\ell^2 + m^2}\sqrt{\ell^2 + m^2 + \lambda^2}}$

$\Rightarrow \cos^2\alpha = \dfrac{\ell^2 + m^2}{\ell^2 + m^2 + \lambda^2}$

$\Rightarrow \lambda = \pm\sqrt{\ell^2 + m^2}\,\tan\alpha$

Hence equation of plane is

$\ell x + my \pm z\sqrt{\ell^2 + m^2}\,\tan\alpha = 0$

16. (A)

S_1 : (1, 2, –1) is a point on the line and $11 + 3 - 14 = 0$

$\therefore$ The point lies on the plane $11x - 3z - 14 = 0$

Further $3 \times 11 + 11(-3) = 0$

$\therefore$ The line lies in the plane

S_2 : obviously true

17. (A)

Statement -I $\overrightarrow{PA} \cdot \overrightarrow{PB} = 9 > 0$

$\therefore$ P is exterior to the sphere

Statement -II is true (standard result)

18. (D)

Statement - II : $\vec{r}\times(\hat{i}+2\hat{j}-3\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 2 & -3 \end{vmatrix}$

$= \hat{i}(-3y-2z) - \hat{j}(-3x-z) + \hat{k}(2x-y)$

$\therefore\ -3y-2z=2,\ 3x+z=-1,\ 2x-y=0$

i.e. $-6x-2z=2,\ 3x+z=-1$

$\therefore$ straight line $2x-y=0$, $3x+z=-1$

Statement -I : $\vec{r}\times(2\hat{i}-\hat{j}+3\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 2 & -1 & 3 \end{vmatrix}$

$= \hat{i}(3y+z) - \hat{j}(3x-2z) + \hat{k}(-x-2y)$

$\therefore\ 3y+z=3,\ 3x-2z=0,\ -x-2y=1$

$3x-2(3-3y)=0$

$\Rightarrow\ 3x+6y=6$

$\Rightarrow\ x+2y=2$

Now $x+2y=-1,\ x+2y=2$ are parallel planes

$\therefore\ \vec{r}\times(2\hat{i}-\hat{j}+3\hat{k}) = 3\hat{i}+\hat{k}$ is not a straight line

19. (A)

$$\sin\theta = \left|\frac{2-3+2}{\sqrt{4+9+4\sqrt{3}}}\right| = \frac{1}{\sqrt{51}}$$

$\therefore$ Statement-1 is true, Statement- 2 is true by definition

20. (B)

Statement - 1

$3y-4z=5-2k$

$-2y+4z=7-3k$

$\therefore\ x=k,\ y=12-5k,\ z=\frac{31-13k}{4}$ is a point on the line

for all real values of k.

Statement is true

Statement - 2

direction ratios of the straight line are

$< bc'-kbc,\ kac-ac',\ 0 >$

direction ratios of normal to be plane $< 0, 0, 1 >$

Now $0\times(bc'-kbc)+0\times(kac-ac')+1\times 0=0$

$\therefore$ the straight line is parallel to the plane

$\therefore$ statement is true but does not explain statement - 1

21. (A) → (s,t), (B) → (p,t), (C) → (q), (D) → (r)

(A) Both the lines pass through the point (7, 11, 15)

(B) < 2, 3, 4 > are direction ratios of both the lines. Also the point (1, 2, 3) is common to both

$\therefore$ The lines are conicident.

(C) < 5, 4 –2 > are direction ratios of both the lines

$\therefore$ The lines are parallel.

Also $x=2+5\lambda,\ y=-3+4\lambda,\ z=5-2\lambda$

$$\therefore\ \frac{2+5\lambda-7}{5} = \frac{-3+4\lambda-1}{4} = \frac{5-2\lambda-2}{-2}$$

i.e. $\lambda-1=\lambda-1=\frac{3-2\lambda}{-2}$

$\therefore$ no value of λ

Thus the lines are parallel and different.

(D) < 2, 3, 5 > and < 3, 2, 5 > are direction ratios of first and 2nd line respectively.

$\therefore$ The lines are not parallel.

$x=3+2\lambda,\quad y=-2+3\lambda,\ z=4+5\lambda$

$x=3+3\mu,\quad y=-2+2\mu,\ z=7+5\mu$

are parametric equations of the lines.

Solving $3+2\lambda=3+3\mu$ and $-2+3\lambda=2+2\mu$

we get $\lambda=\frac{12}{5},\ \mu=\frac{8}{5}$

Now substituting these values in $4+5\lambda=7+5\mu$

we get $4+12=7+8$ i.e. $16=15$ which is not true.

$\therefore$ The lines do not intersect

Hence the lines are skew.

22. (A) → (s) (B) → (p) (C) → (t) (D) → (q)

(A) Let the foot of perpendicular be $Q(2+2\lambda,\ 1+3\lambda,\ 2+4\lambda)$

$\therefore\ 2(2\lambda+1)+3(3\lambda-1)+4(4\lambda-1)=0$

$29\lambda=5 \quad\Rightarrow\quad \lambda=\frac{5}{29}$

$\therefore$ Foot $=\left(\frac{68}{29},\frac{44}{29},\frac{78}{29}\right)$ $\quad\therefore$ (A) → (s)

(B) Let the image be the point (a, b, c), then from previous solution

$$\frac{1+a}{2}=\frac{68}{29},\ \frac{2+b}{2}=\frac{44}{29} \text{ and } \frac{3+c}{2}=\frac{78}{29}$$

i.e. $a=\frac{107}{29},\ b=\frac{30}{29}$ and $c=\frac{68}{29}$ $\quad\therefore$ (B) → (p)

(C) $\dfrac{x-2}{2}=\dfrac{y-3}{3}=\dfrac{z-5}{-4}=-\dfrac{4+9-20+17}{4+9+16}=\dfrac{-10}{29}$

$\therefore\quad a=\dfrac{38}{29},\ b=\dfrac{57}{29}$ and $c=\dfrac{185}{29}\quad\therefore\quad (C)\to(t)$

(D) $\dfrac{x-2}{3}=\dfrac{y-5}{-2}=\dfrac{z-1}{4}=-2\left(\dfrac{6-10+4-5}{29}\right)=\dfrac{10}{29}$

$x=2+\dfrac{30}{29}=\dfrac{88}{29},\ y=5-\dfrac{20}{29}=\dfrac{125}{29},\ z=1+\dfrac{40}{29}=\dfrac{69}{29}$

$\therefore\quad (D)\to(q)$

23.

1. (D)

shortest distance between both lines

$$=\left|\frac{\begin{vmatrix}3-2 & 1-1 & 0+1\\ 1 & 0 & 2\\ 1 & 1 & -1\end{vmatrix}}{\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 0 & 2\\ 1 & 1 & -1\end{vmatrix}}\right|=\left|\frac{1(0-2)-0(-1-2)+1(1-0)}{-2\hat{i}+3\hat{j}+\hat{k}}\right|$$

$$=\frac{1}{\sqrt{4+9+1}}=\frac{1}{\sqrt{14}}$$

2. (B)

Equation of plane P

$$\begin{vmatrix}x-2 & y-1 & z+1\\ 1 & 0 & 2\\ 1 & 1 & -1\end{vmatrix}=0$$

$-2(x-2)+(y-1)(3)+(z+1)=0$

$-2x+3y+z+2=0$

$2x-3y-z-2=0$

Now image of point 0(0, 0, 0) in plane P

$\dfrac{x-0}{2}=\dfrac{y-0}{-3}=\dfrac{z-0}{-1}=\dfrac{-2(-2)}{4+9+1}$

$\dfrac{x}{2}=\dfrac{y}{-3}=\dfrac{z}{-1}=\dfrac{4}{14}$

Image point $\left(\dfrac{4}{7},\dfrac{-6}{7},\dfrac{-2}{7}\right)$

3. (C)

$0(0,0,0), A(1,0,0), B\left(0,\dfrac{-2}{3},0\right), C(0,0,-2)$

volume of tetrahetron OABC $=\dfrac{1}{6}\left|[\overrightarrow{OA}\ \overrightarrow{OB}\ \overrightarrow{OC}]\right|$

$$=\frac{1}{6}\begin{vmatrix}1 & 0 & 0\\ 0 & -2/3 & 0\\ 0 & 0 & -2\end{vmatrix}=\frac{4}{18}=\frac{2}{9}\text{ cu unit}$$

24.

1. $\Delta=\dfrac{1}{2}\left|\overline{AB}\times\overline{AC}\right|=\dfrac{1}{2}\left|\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & 1 & 0\\ -2 & 0 & 2\end{vmatrix}\right|=\left|\hat{i}+2\hat{j}+\hat{k}\right|=\sqrt{6}$

2. $H(\alpha,\beta,\gamma)\Rightarrow AH\perp BC, BH\perp CA$

$\Rightarrow\ \dfrac{\alpha}{1}=\dfrac{\beta}{2}=\dfrac{\gamma}{1}$

H lies on the plane $\dfrac{x}{2}+y+\dfrac{z}{2}=1$

$\Rightarrow\ \gamma=\dfrac{1}{3}$

3. $H\left(\dfrac{1}{3},\dfrac{2}{3},\dfrac{1}{3}\right), G\left(\dfrac{2}{3},\dfrac{1}{3},\dfrac{2}{3}\right)\Rightarrow S\left(\dfrac{5}{6},\dfrac{1}{6},\dfrac{5}{6}\right)$

$\Rightarrow$ y-coordinates is $\dfrac{1}{6}$

4. $P(x,y,z)$

$\Rightarrow\ x^2+y^2+z^2=(x-2)^2+y^2+z^2$

$=x^2+(y-1)^2+z^2=x^2+y^2+(z-2)^2$

$\Rightarrow\ x=1,\ y=\dfrac{1}{2},\ z=1$

$P\left(1,\dfrac{1}{2},1\right),\quad A(2,0,0)\quad\Rightarrow\ AP=\dfrac{3}{2}$

25. $u\ell+vm+wn=0$

$a\ell^2+bm^2+cn^2=0$

$a\ell^2+bm^2+c\left\{-\dfrac{(4\ell+vm)}{w}\right\}^2=0$

$\Rightarrow\ (aw^2+cu^2)\ell^2+(bw^2+cv^2)m^2+2\,cuv\,\ell m=0$

$\Rightarrow \quad (aw^2 + cu^2)\left(\frac{\ell}{m}\right)^2 + (bw^2 + cv^2) + 2\,cuv\left(\frac{\ell}{m}\right) = 0$ ....(i)

put u = v = w = 1 in equation , then

$(a+c)\left(\frac{\ell}{m}\right)^2 + 2c\left(\frac{\ell}{m}\right) + (b+c) = 0$

similarly $(a+b)\left(\frac{m}{n}\right)^2 + 2a\left(\frac{m}{n}\right) + (c+a) = 0$

and $(b+c)\left(\frac{n}{\ell}\right)^2 + 2b\left(\frac{n}{\ell}\right) + (a+b) = 0$ ....(ii)

From equation **(ii)** $\frac{n_1}{\ell_1}.\frac{n_2}{\ell_2} = \frac{a+b}{b+c}$

similarly $\frac{\ell_1\ell_2}{b+c} = \frac{m_1m_2}{c+a} = \frac{n_1n_2}{a+b}$ ....(iii)

$\therefore \quad \frac{m_1m_2}{\ell_1\ell_2} = \frac{c+a}{b+c}$

From equation **(iii)**

$\frac{\ell_1\ell_2}{b+c} = \frac{m_1m_2}{c+a} = \frac{n_1n_2}{a+b} = \frac{\ell_1\ell_2 + m_1m_2 + n_1n_2}{(b+c)+(b+c)+(a+b)}$

$\because$ lines are perpendicular

$\therefore \quad \ell_1\ell_2 + m_1m_2 + n_1n_2 = 0$

then (b + c) + (c + a) + (a + b) must be zero

$2a + 2b + 2c = 0 \quad \Rightarrow \quad a + b + c = 0$

26. Let p.v. of P be $(\vec{p})$ & that of A, B, C be $\vec{a}$, $\vec{b}$, $\vec{c}$ with respect to origin 'O'.

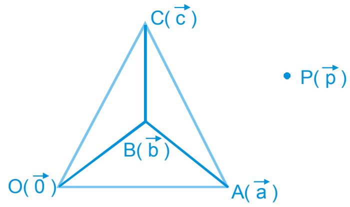
$PA^2 + PB^2 + PC^2 + PO^2 = 4p^2 + 3 - 2\,\vec{p}.(\vec{a}+\vec{b}+\vec{c})$

$\{\because\ |\vec{a}| = |\vec{b}| = |\vec{c}| = 1\}$

For above to be minimum $\vec{p}.(\vec{a}+\vec{b}+\vec{c})$ should be maximum

Which is $= |\vec{p}||\vec{a}+\vec{b}+\vec{c}|$

Further $|\vec{a}+\vec{b}+\vec{c}| = \sqrt{a^2+b^2+c^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})} = 3$



Hence $\quad PA^2 + PB^2 + PC^2 + PO^2 = 4p^2 + 3 - 2\,p.\,3$

$= \left(2p - \frac{3}{2}\right)^2 + \frac{3}{4}$

Whose least value is $\frac{3}{4}$, when $|\vec{p}| = \frac{3}{4}$

& $\vec{p} \,||\, \vec{a} + \vec{b} + \vec{c}$

27. (0)

Let equation of a plane containing the line be

$\ell(x-1) + m(y+2) + nz = 0$

then $2\ell - 3m + 5n = 0$ and $\ell - m + n = 0$

$\therefore \quad \frac{\ell}{2} = \frac{m}{3} = \frac{n}{1}$

$\therefore$ the plane is $2(x-1) + 3(y+2) + z = 0$

i.e. $2x + 3y + z + 4 = 0$

$\therefore \quad a = 2, b = -3, c = 1$

28. $\cos\theta = \frac{\ell m + mn + n\ell}{\ell^2 + m^2 + n^2}$ ....(i)

$x^3 + x^2 - 4x - 4 = 0$

$\ell + m + n = -1$

$\ell m + mn + n\ell = -4$

$(\ell + m + n)^2 = \ell^2 + m^2 + n^2 + 2(-4)$

$\Rightarrow \quad \ell^2 + m^2 + n^2 = 1 + 8 = 9$

$\therefore \quad \cos\theta = -\frac{4}{9}$

$\therefore$ acute angle between the lines is $\cos^{-1}\frac{4}{9}$

29. (3)

$\vec{r} = 3\hat{i} + 8\hat{j} + 3\hat{k} + \lambda(3\hat{i} - \hat{j} + \hat{k})$ ....(i)

$\vec{r} = -3\hat{i} - 7\hat{j} + 6\hat{k} + \mu(-3\hat{i} + 2\hat{j} + 4\hat{k})$ ....(ii)

Let L and M be points on the line **(i)** and **(ii)** respectively

So that LM is perpendicular to both the lines.

Let position vector of L be

$3\hat{i} + 8\hat{j} + 3\hat{k} + \lambda_0(3\hat{i} - \hat{j} + \hat{k})$

and the position vector of M be

$-3\hat{i} - 7\hat{j} + 6\hat{k} + \mu_0(-3\hat{i} + 2\hat{j} + 4\hat{k})$

then $\overrightarrow{LM} = -6\hat{i} - 15\hat{j} + 3\hat{k} - \lambda_0(3\hat{i} - \hat{j} + \hat{k})$

$+\mu_0(-3\hat{i} + 2\hat{j} + 4\hat{k})$

since $\overrightarrow{LM}$ is perpendicular to both the lines (i) and (ii)

$\therefore \quad \overrightarrow{LM} \cdot (3\vec{i} - \vec{j} + \vec{k}) = 0$ and $\overrightarrow{LM} \cdot (-3\vec{i} + 2\vec{j} + 4\vec{k}) = 0$

Thus $-18 + 15 + 3 - \lambda_0 (9 + 1 + 1) + \mu_0 (-9 - 2 + 4) = 0$

i.e. $-11\lambda_0 - 7\mu_0 = 0$ ......(iii)

and $18 - 30 + 12 - \lambda_0 (-9 - 2 + 4) + \mu_0 (9 + 4 + 16) = 0$

i.e. $7\lambda_0 + 29\mu_0 = 0$ ......(iv)

from (iii) and (iv) we get

$\lambda_0 = \mu_0 = 0$

$\therefore \quad \overrightarrow{LM} = -6\vec{i} - 15\vec{j} + 3\vec{k}$

$\therefore \quad |\overrightarrow{LM}| = \sqrt{36 + 225 + 9} = \sqrt{270} = 3\sqrt{30}$

position vector of L is $3\vec{i} + 8\vec{j} + 3\vec{k}$

$\therefore$ equation of the line of shortest distance (LM) is

$\vec{r} = 3\vec{i} + 8\vec{j} + 3\vec{k} + \lambda(-6\vec{i} - 15\vec{j} + 3\vec{k})$

30. Let A be the point $(2\lambda + 1, 4\lambda + 3, 3\lambda + 2)$

so that AP is parallel to the given plane.

Then $3(2\lambda + 1 - 3) + 2(4\lambda + 3 - 8) - 2(3\lambda + 2 - 2) = 0$

$\Rightarrow \quad 8\lambda = 16$

$\therefore \quad \lambda = 2$

Therefore, A is (5, 11, 8)

$PA = \sqrt{(5-3)^2 + (11-8)^2 + (8-2)^2}$

$= \sqrt{4 + 9 + 36} = 7$