3D-geometry -CBSE

1 Mark Questions

1. If a line makes angles 90°, 135°, 45° with the x, y and z axes respectively, find its direction cosines.
2. Find the vector equation of the line which passes through the point (3, 4, 5) and is parallel to the vector $2\hat{i} + 2\hat{j} - 3\hat{k}$.
3. What are the direction cosines of a line which makes equal angles with the coordinate axes?
4. A line passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of the vector $\hat{i} + \hat{j} - 2\hat{k}$. Find the equation of the line in cartesian form.

5. If a line makes angles 90° and 60°, respectively with the positive directions of X and Y-axes, find the angle which it makes with the positive direction of Z-axis.

6. If a line makes angles 90°, 60° and θ with X, Y and Z-axes respectively, where θ is acute angle, then find θ.

7. The equations of a line is $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line.

8. If a line makes angles α, β, γ with the position direction of coordinate axes, then write the value of $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$.

9. Write the distance of a point $P(a, b, c)$ from X-axis.

10. If the cartesian equation of a line is $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$, then write the vector equation for the line.

11. Write the equation of the straight line through the point (α, β, γ) and parallel to Z-axis. **All India 2014**

12. Find the direction cosines of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

13. Write the vector equation of a line passing through point $(1, -1, 2)$ and parallel to the line whose equation is $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$.

14. Find the cartesian equation of the line which passes through the point $(-2, 4, -5)$ and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.

15. If a line has direction ratios $(2, -1, -2)$, then what are its direction cosines?

16. Write the direction cosines of the line joining the points (1, 0, 0) and (0, 1,1).

17. Write the vector equation of the line given by $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$.

18. Equation of line is $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$.
Find the direction cosines of a line parallel to above line.

19. If the equations of line AB is $\frac{3-x}{1} = \frac{y+2}{-2} = \frac{z-5}{4}$, then write the direction ratios of the line parallel to above line AB.

20. Find the distance of point (2, 3, 4) from X-axis.

21. Write the vector equation of the following line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$.

2 Marks Questions

22. Find the vector equation of the line passing through the point $A(1, 2, -1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$.

23. The x-coordinate of a point on the line joining the points $P(2, 2, 1)$ and $Q(5, 1, -2)$ is 4. Find its z-coordinate

4 Marks Questions

24. Find the value of λ, so that the lines $\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.

25. If the lines $\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{2} = \frac{z-6}{-5}$ are perpendicular, find the value of λ. Hence find whether the lines are intersecting or not.

26. Find the shortest distance between the lines

$$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$$

and $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$.

27. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

28. Find the vector and cartesian equations of the line through the point $(1, 2, -4)$ and perpendicular to the two lines

$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and

$\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

Or Find the equation of a line passing through the point $(1, 2, -4)$ and perpendicular to two lines

$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$

and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

29. Find the coordinates of the foot of perpendicular drawn from the point $A(-1, 8, 4)$ to the line joining the points $B(0, -1, 3)$ and $C(2, -3, -1)$. Hence, find the image of the point A in the line BC.

30. Prove that the line through $A(0, -1, -1)$ and $B(4, 5, 1)$ intersects the line through $C(3, 9, 4)$ and $D(-4, 4, 4)$.

31. Show that the lines

$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and

$\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect. Also, find their point of intersection.

32. Find the direction cosines of the line $\frac{x+2}{2} = \frac{2y-7}{6} = \frac{5-z}{6}$. Also, find the vector equation of the line through the point $A(-1, 2, 3)$ and parallel to the given line.

33. Find the angle between the lines

$$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$$

and $\vec{r} = 7\hat{i} - 6\hat{j} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$.

34. Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also, find their point of intersection.

35. Find the value of p, so that the lines

$l_1 : \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$ and

$l_2 : \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are

perpendicular to each other. Also, find the equation of a line passing through a point $(3, 2, -4)$ and parallel to line l_1.

36. A line passes through the point $(2, -1, 3)$ and is perpendicular to the lines

$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$

and $\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$.

Obtain its equation in vector and cartesian forms.

37. Find the shortest distance between the lines whose vector equations are

$$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$$

and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$.

38. Find the shortest distance between the two lines whose vector equations are

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ and

$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$.

39. Find the shortest distance between the following lines.

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}, \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

40. Find the distance between the lines l_1 and l_2 given by

$l_1: \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$,

$l_2: \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$.

41. Find the vector and cartesian equations of the line passing through the point (2, 1, 3) and perpendicular to the lines

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$$

and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$.

42. The cartesian equation of a line is $6x - 2 = 3y + 1 = 2z - 2$. Find the direction cosines of the line. Write down the cartesian and vector equations of a line passing through (2, – 1, – 1) which are parallel to the given line.

43. Find the shortest distance between the two lines whose vector equations are

$$\vec{r} = (6\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$$

and $\vec{r} = (-4\hat{i} - \hat{k}) + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$.

44. Show that the lines

$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k});$$

$$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$$

are intersecting. Hence, find their point of intersection.

45. Using vectors, show that the points $A(-2, 3, 5)$, $B(7, 0, -1)$, $C(-3, -2, -5)$ and $D(3, 4, 7)$ are such that AB and CD intersect at the point $P(1, 2, 3)$.

46. Computing the shortest distance between the following pair of lines, determine whether they intersect or not?

$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} - \hat{k});$$

$$\vec{r} = 2\hat{i} - \hat{j} + \mu(\hat{i} - \hat{j} - \hat{k})$$

47. Find the equation of the line passing through the point (–1, 3, – 2) and perpendicular to the lines

$\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.

48. Find the angle between following pair of lines. $\frac{-x+2}{-2} = \frac{y-1}{7} = \frac{z+3}{-3}$

and $\frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4}$

and check whether the lines are parallel or perpendicular.

49. Find the shortest distance between lines whose vector equations are

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$$

$$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}.$$

50. Find shortest distance between the lines

$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and

$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$.

51. Find the equation of the perpendicular from point (3, –1, 11) to line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also, find the coordinates of foot of perpendicular and the length of perpendicular.

52. Find the perpendicular distance of point (1, 0, 0) from the lines $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$. Also, find the coordinates of foot of perpendicular and equation of perpendicular

53. Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point $P(1, 3, 3)$.

54. Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point $P(5, 4, 2)$ to the line

$\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$.

Also, find the image of P in this line.

55. Find the equation of line passing through points $A(0, 6, -9)$ and $B(-3, -6, 3)$. If D is the foot of perpendicular drawn from the point $C(7, 4, -1)$ on the line AB, then find the coordinates of point D and equation of line CD.

56. Find the image of the point $(1, 6, 3)$ on the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, write the equation of the line joining the given points and its image and find the length of segment joining given point and its image.

57. Write the vector equations of following lines and hence find the distance between them.

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$,

$\frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$

58. The points $A(4, 5, 10)$, $B(2, 3, 4)$ and $C(1, 2, -1)$ are three vertices of parallelogram $ABCD$. Find the vector equations of sides AB and BC and also find coordinates of point D.

6 Marks Question

59. Find the vector and cartesian equations of a line passing through (1, 2, –4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

1 Mark Questions

1. Find the distance between the planes $2x - y + 2z = 5$ and $5x - 2.5y + 5z = 20$.

2. Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i} - 3\hat{j} + 6\hat{k}$. **Delhi 2016**

3. Write the sum of intercepts cut off by the plane $\vec{r}\cdot(2\hat{i} + \hat{j} - \hat{k}) - 5 = 0$ on the three axes.

4. Find the vector equation of the plane with intercepts 3, – 4 and 2 on X, Y and Z-axes, respectively

5. Write the equation of a plane which is at a distance of $5\sqrt{3}$ units from origin and the normal to which is equally inclined to coordinate axes.

6. Find the sum of the intercepts cut off by the plane $2x + y - z = 5$, on the coordinate axes.

7. Write the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane $\vec{r}\cdot(\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$

8. Write the vector equation of the plane passing through the point (a, b, c) and parallel to the plane $\vec{r}\cdot(\hat{i} + \hat{j} + \hat{k}) = 2$.

9. Find the length of the perpendicular drawn from the origin to the plane $2x - 3y + 6z + 21 = 0$.

10. Find distance of the plane $3x - 4y + 12z = 3$ from the origi.

11. Write the intercept cut-off by plane $2x + y - z = 5$ on X-axis.

12. Write the distance of following plane from origin. $2x - y + 2z + 1 = 0$

13. Find the value of λ, such that the line $\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{-4}$ is perpendicular to the plane $3x - y - 2z = 7$.

4 Marks Questions

14. Find the cartesian and vector equations of the plane passing through the points $A(2, 5, -3)$, $B(-2, -3, 5)$ and $C(5, 3, -3)$.

15. Find the distance between the point $(-1, -5, -10)$ and the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$.

16. Find the equation of the plane passing through the points $(-1, 2, 0)$, $(2, 2, -1)$ and parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$.

17. Find the equation of a plane which passes through the point (3, 2, 0) and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$.

18. A plane makes intercepts –6, 3, 4 respectively on the coordinate axes. Find the length of the perpendicular from the origin on it.

19. Show that the lines $\frac{5-x}{-4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{2y-8}{2} = \frac{z-5}{3}$ are coplanar.

20. Find the image of the point having position vector $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $\vec{r}\cdot(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$.

21. Find the vector equation of the plane through the points $(2, 1, -1)$ and $(-1, 3, 4)$ and perpendicular to the plane $x - 2y + 4z = 10$.

22. Find the coordinates of the point, where the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$ intersects the plane $x-y+z-5=0$. Also, find the angle between the line and the plane.

23. Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4=0$ and $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5=0$ and which is perpendicular to the plane $\vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k})+8=0$.

24. Find the coordinates of the point where the line through the points $(3,-4,-5)$ and $(2,-3,1)$ crosses the plane $2x+y+z=7$.

25. Find the coordinates of the point where the line through the points $A(3,4,1)$ and $B(5,1,6)$ crosses the XY-plane.

26. Find the coordinates of the point where the line through the points $(3,-4,-5)$ and $(2,-3,1)$ crosses the plane $3x+2y+z+14=0$.

27. Find the equation of plane(s) passing through the intersection of planes $x+3y+6=0$ and $3x-y-4z=0$ and whose perpendicular distance from origin is unity.

28. Find the equation of plane passing through the point $A(1,2,1)$ and perpendicular to the line joining points $P(1,4,2)$ and $Q(2,3,5)$ Also, find distance of this plane from the line $\frac{x+3}{2}=\frac{y-5}{-1}=\frac{z-7}{-1}$

29. Find the cartesian equation of the plane passing through points $A(0,0,0)$ and $B(3,-1,2)$ and parallel to the line $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$.

6 Marks Questions

30. Find the vector and cartesian equations of the plane passing through the points $(2,2,-1)$, $(3,4,2)$ and $(7,0,6)$. Also find the vector equation of a plane passing through $(4,3,1)$ and parallel to the plane obtained above.

31. Find the vector equation of the plane that contains the lines $\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+2\hat{j}-\hat{k})$ and the points $(-1,3,-4)$. Also, find the length of the perpendicular drawn from the point $(2,1,4)$ to the plane, thus obtained.

32. Find the vector and cartesian equations of the plane passing through the points having position vectors $\hat{i}+\hat{j}-2$ $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$. Write the equation of a plane passing through a point $(2,3,7)$ and parallel to the plane obtained above. Hence, find the distance between the two parallel planes.

33. Find the equation of the line passing through $(2,-1,2)$ and $(5,3,4)$ and of the plane passing through $(2,0,3)$, $(1,1,5)$ and $(3,2,4)$. Also, find their point of intersection.

34. Find the distance of the point $P(-1,-5,-10)$ from the point of intersection of the line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}-\hat{j}+\hat{k})=5$.

35. Find the vector equation of the line passing through the point $(1,2,3)$ and parallel to the planes $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=5$ and $\vec{r}\cdot(3\hat{i}+\hat{j}+\hat{k})=6$. Also, find the point of intersection of the line, thus obtained with the plane $\vec{r}\cdot(2\hat{i}+\hat{j}+\hat{k})=4$

36. Find the equation of the plane through the line of intersection of $\vec{r}\cdot(2\hat{i}-3\hat{j}+4\hat{k})=1$ and $\vec{r}\cdot(\hat{i}-\hat{j})+4=0$ and perpendicular to the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+8=0$. Hence, find whether the plane thus obtained contains the line $x-1=2y-4=3z-12$.

37. A variable plane which remains at a constant distance $3p$ from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of ΔABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$.

38. Find the coordinates of the point P where the line through $A(3,-4,-5)$ and $B(2,-3,1)$ crosses the plane passing through three points $L(2,2,1)$, $M(3,0,1)$ and $N(4,-1,0)$. Also, find the ratio in which P divides the line segment AB.

39. Find the coordinates of the point where the line through the points $A(3,4,1)$ and $B(5,1,6)$ crosses the XZ-plane. Also, find the angle which this line makes with the XZ-plane.

40. Find the position vector of the foot of perpendicular and the perpendicular distance from the point P with position vector $2\hat{i}+3\hat{j}+4\hat{k}$ to the plane $\vec{r}\cdot(2\hat{i}+\hat{j}+3\hat{k})-26=0$. Also, find image of P in the plane.

41. Find the equation of the plane which contains the line of intersection of the planes $\vec{r}\cdot(\hat{i}-2\hat{j}+3\hat{k})-4=0$ and $\vec{r}\cdot(-2\hat{i}+\hat{j}+\hat{k})+5=0$ and whose intercept on X-axis is equal to that of on Y-axis.

42. Find the equation of the plane which contains the line of intersection of the planes $x+2y+3z-4=0$ and $2x+y-z+5=0$ and whose x-intercept is twice its z-intercept. Hence write the vector equation of a plane passing through the point $(2,3,-1)$ and parallel to the plane obtained above.

43. If lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then find the value of k and hence, find the equation of the plane containing these lines.

44. Find the distance of the point $P(3,4,4)$ from the point, where the line joining the points $A(3,-4,-5)$ and $B(2,-3,1)$ intersects the plane $2x+y+z=7$.

45. Find the distance of the point $(1,-2,3)$ from the plane $x-y+z=5$ measured parallel to the line whose direction cosines are proportional to $(2,3,-6)$.

46. Find the equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ which is perpendicular to the plane $x-y+z=0$. Then, find the distance of plane thus obtained from the point $A(1,3,6)$.

47. Find the equation of the plane passing through the line of intersection of the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0$ and parallel to X-axis.

48. Find the distance between the point $(7,2,4)$ and the plane determined by the points $A(2,5,-3)$, $B(-2,-3,5)$ and $C(5,3,-3)$.

49. Find the equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$, which is perpendicular to the plane $x-y+z=0$. Also, find the distance of the plane obtained above, from the origin.

50. Find the distance of the point $(2,12,5)$ from the point of intersection of the line $\vec{r}=2\hat{i}-4\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}-2\hat{j}+\hat{k})=0$

51. Find the equation of the plane that contains the point $(1,-1,2)$ and is perpendicular to both the planes $2x+3y-2z=5$ and $x+2y-3z=8$. Hence, find the distance of point $P(-2,5,5)$ from the plane obtained above.

52. Find the distance of the point $P(-1, -5, -10)$ from the point of intersection of the line joining the points $A(2, -1, 2)$ and $B(5, 3, 4)$ with the plane $x - y + z = 5$.

53. Find the vector and cartesian forms of the equation of the plane passing through the point $(1, 2, -4)$ and parallel to the lines

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

and $\vec{r} = \hat{i} - 3\hat{j} + 5\hat{k} + \mu(\hat{i} + \hat{j} - \hat{k})$. Also, find the distance of the point $(9, -8, -10)$ from the plane thus obtained.

54. Show that the lines
$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and
$\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ are coplanar. Also, find the equation of the plane containing them.

55. Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z+2}{-6}$.

56. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also, find the equation of the plane containing these lines.

57. Find the equation of the plane passing through the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 3\hat{j}) - 6 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$, whose perpendicular distance from origin is unity.

58. Find the coordinates of the point, where the line through $(3, -4, -5)$ and $(2, -3, 1)$ crosses the plane, passing through the points $(2, 2, 1)$, $(3, 0, 1)$ and $(4, -1, 0)$.

59. Find the vector equation of the plane passing through the three points with position vectors $\hat{i} + \hat{j} - 2\hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$. Also, find the coordinates of the point of intersection of this plane and the line $\vec{r} = 3\hat{i} - \hat{j} - \hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$.

60. Find the equation of plane determined by points $A(3, -1, 2)$, $B(5, 2, 4)$ and $C(-1, -1, 6)$ and hence, find the distance between plane and point $(6, 5, 9)$.

61. Find the equation of the plane passing through the point $(3, -3, 1)$ and perpendicular to the line joining the points $(3, 4, -1)$ and $(2, -1, 5)$. Also, find the coordinates of foot of perpendicular, the equation of perpendicular line and the length of perpendicular drawn from origin to the plane.

62. Find the vector equation of the plane that contains the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$. Also, find the length of perpendicular drawn from the point $(2, 1, 4)$ to the plane thus obtained.

63. If the lines $\frac{x-1}{-3} = \frac{y-2}{-2k} = \frac{z-3}{2}$ and $\frac{x-1}{k} = \frac{y-2}{1} = \frac{z-3}{5}$ are perpendicular, find the value of k and hence find the equation of plane containing these lines.

64. Find the length and foot of perpendicular from point $P(7, 14, 5)$ to plane $2x + 4y - z = 2$. Also, find the image of point P in the plane.

65. Find the equation of plane passing through the line of intersection of planes $2x + y - z = 3$ and $5x - 3y + 4z + 9 = 0$ and parallel to line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$.

66. Find the equation of plane passing through the point $(-1, 3, 2)$ and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 5$.

67. Find the equation of plane passing through point $(1, 1, -1)$ and perpendicular to planes $x + 2y + 3z - 7 = 0$ and $2x - 3y + 4z = 0$.

68. Find the vector and cartesian equation of a plane containing the two lines

$\vec{r} = (2\hat{i} + \hat{j} - 3\hat{k}) + \lambda\,(\hat{i} + 2\hat{j} + 5\hat{k})$ and

$\vec{r} = (3\hat{i} + 3\hat{j} + 2\hat{k}) + \mu\,(3\hat{i} - 2\hat{j} + 5\hat{k})$.

Also, show that the line

$\vec{r} = (2\hat{i} + 5\hat{j} + 2\hat{k}) + P\,(3\hat{i} - 2\hat{j} + 5\hat{k})$ lies in the plane.

69. Find the equation of plane passing through the point $(1, 2, 1)$ and perpendicular to line joining points $(1, 4, 2)$ and $(2, 3, 5)$. Also, find the coordinates of foot of the perpendicular and the perpendicular distance of the point $(4, 0, 3)$ from above found plane.

70. Find the equation of plane passing through point $P(1, 1, 1)$ and containing the line $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda\,(3\hat{i} - \hat{j} - 5\hat{k})$. Also, show that plane contains the line $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu\,(\hat{i} - 2\hat{j} - 5\hat{k})$.

71. Find the coordinates of the foot of perpendicular and the perpendicular distance of point $P(3, 2, 1)$ from the plane $2x - y + z + 1 = 0$. Also, find image of the point in the plane.

Objective Questions

(For Complete Chapter)

1 Mark Questions

1. The direction cosines of the line joining the points $(4, 3, -5)$ and $(-2, 1, -8)$ are

(a) $\left(\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right)$ (b) $\left(\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}\right)$

(c) $\left(\frac{6}{7}, \frac{3}{7}, \frac{2}{7}\right)$ (d) None of these

2. If the direction cosines of a line are $\left(\frac{1}{c}, \frac{1}{c}, \frac{1}{c}\right)$, then

(a) $0 < c < 1$ (b) $c > 2$

(c) $c = \pm\sqrt{2}$ (d) $c = \pm\sqrt{3}$

3. The point of intersection of lines $\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1}$ and $\frac{x+3}{-36} = \frac{y-3}{2} = \frac{z-6}{4}$ is

(a) $(5, 7, -2)$ (b) $(-3, 3, 6)$

(c) $(2, 10, 4)$ (d) $\left(21, \frac{5}{3}, \frac{10}{3}\right)$

4. Equation of the line passing through $(2, -1, 1)$ and parallel to the line $\frac{x-5}{4} = \frac{y+2}{-3} = \frac{z}{5}$ is

(a) $\frac{x-2}{4} = \frac{y+1}{-3} = \frac{z-1}{5}$

(b) $\frac{x-2}{4} = \frac{y+1}{3} = \frac{z-1}{5}$

(c) $\frac{x-2}{-4} = \frac{y+1}{-3} = \frac{z-1}{5}$

(d) None of the above

5. The angle between the lines $x = 1, y = 2$ and $y = -1, z = 0$ is

(a) $30°$ (b) $60°$ (c) $90°$ (d) $0°$

6. If the lines $\frac{1-x}{3} = \frac{y-2}{2\alpha} = \frac{z-3}{2}$ and $\frac{x-1}{3\alpha} = y - 1 = \frac{6-z}{5}$ are perpendicular, then the value of α is

(a) $\frac{-10}{7}$ (b) $\frac{10}{7}$ (c) $\frac{-10}{11}$ (d) $\frac{10}{11}$

7. The equation of the plane perpendicular to Z-axis and passing through $(2, -3, 5)$ is

(a) $x - 2 = 0$ (b) $y + 3 = 0$

(c) $z - 5 = 0$ (d) $2x - 3y + 5z + 4 = 0$

8. The intercepts of the plane $2x - 3y + 4z = 12$ on the coordinate axes are given by

(a) $3, -2, 1.5$ (b) $6, -4, 3$

(c) $6, -4, -3$ (d) $2, -3, 4$

9. The distance of the plane $6x - 3y + 2z - 14 = 0$ from the origin is

(a) 2 (b) 1 (c) 14 (d) 8

10. The line joining the points $(1, 1, 2)$ and $(3, -2, 1)$ meets the plane $3x + 2y + z = 6$ at the point

(a) $(1, 1, 2)$ (b) $(3, -2, 1)$

(c) $(2, -3, 1)$ (d) $(3, 2, 1)$

11. The equation of the plane containing the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$, where

(a) $ax_1 + by_1 + cz_1 = 1$ (b) $a/l = b/m = c/n$

(c) $lx_1 + my_1 + nz_1 = 0$ (d) $al + bm + cn = 0$

12. The equation of the plane containing the lines $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x}{2} = \frac{y-2}{-1} = \frac{z+1}{3}$ is

(a) $8x - y + 5z - 8 = 0$ (b) $8x + y - 5z - 7 = 0$

(c) $x - 8y + 3z + 6 = 0$ (d) $8x + y - 5z + 7 = 0$

13. If the plane $3x + y + 2z + 6 = 0$ is parallel to the line $\frac{3x-1}{2b} = 3 - y = \frac{z-1}{a}$, then the value of $3a + 3b$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 3 (d) 4

Objective Questions

(For Complete Chapter)

1 Mark Questions

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(c) $\left(\frac{6}{7}, \frac{3}{7}, \frac{2}{7}\right)$ (d) None of these

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