3D Geometry(cbse,solutions)

Solutions

1. Let direction cosines of the line be l, m and n.

Given, $\alpha = 90°, \beta = 135°$ and $\gamma = 45°$

Then, $l = \cos\alpha = \cos 90° = 0,$

$m = \cos\beta = \cos 135° = \frac{-1}{\sqrt{2}}$

and $n = \cos\gamma = \cos 45° = \frac{1}{\sqrt{2}}$

Hence, the direction cosines of a line are $0, \frac{-1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$. **(1)**

2. Equation of a line passing through a point with position vector $\vec{a}$ and parallel to a vector $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

Since, line passes through (3, 4, 5)

$\therefore \quad \vec{a} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

Since, line is parallel to $2\hat{i} + 2\hat{j} - 3\hat{k}$

$\therefore \quad \vec{b} = 2\hat{i} + 2\hat{j} - 3\hat{k}$

Equation of line is $\vec{r} = \vec{a} + \lambda\vec{b}$, i.e.

$$\vec{r} = (3\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 2\hat{j} - 3\hat{k})$$

which is the required vector equation. **(1)**

3. Given, line makes equal angles with coordinate axes. Let α, β and γ be the angles made by the line with coordinate axes.

Then, $\alpha = \beta = \gamma \Rightarrow \cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow \quad l = m = n \quad$ ...(i)

$[\because l = \cos\alpha, m = \cos\beta, n = \cos\gamma]$

We know that, $l^2 + m^2 + n^2 = 1$

$\therefore \quad l^2 + l^2 + l^2 = 1 \quad$ [from Eq. (i)]

$\Rightarrow \quad 3l^2 = 1 \Rightarrow l^2 = \frac{1}{3} \Rightarrow l = \pm\frac{1}{\sqrt{3}}$

From Eq. (i), direction cosines of a line are

$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ or $\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$ **(1)**

4. The given line passes through the point A having position vector $\vec{a}_1 = 2\hat{i} - \hat{j} + 4\hat{k}$ and is parallel to the vector $\vec{b} = (\hat{i} + \hat{j} - 2\hat{k})$

$\therefore$ The equation of the given line is

$$\vec{r} = \vec{a}_1 + \lambda\vec{b} \Rightarrow \vec{r} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(\hat{i} + \hat{j} - 2\hat{k}) \quad ...(i)$$

For cartesian equation put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in Eq. (i), we get

$\Rightarrow$ $(x\hat{i} + y\hat{j} + z\hat{k}) = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(\hat{i} + \hat{j} - 2\hat{k})$

$\Rightarrow$ $x\hat{i} + y\hat{j} + z\hat{k} = (2+\lambda)\hat{i} + (\lambda - 1)\hat{j} + (4 - 2\lambda)\hat{k}$

$\Rightarrow$ $x = 2 + \lambda,\ y = \lambda - 1$ and $z = 4 - 2\lambda$

$\Rightarrow$ $\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2} = \lambda$

Hence, $\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$ is the required equation of the given line in cartesian form.

5. Let a line makes angles α, β and γ with the X-axis, Y-axis and Z-axis, respectively.

$\therefore$ $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow$ $\cos^2 90° + \cos^2 60° + \cos^2\gamma = 1$ **(1/2)**

$\Rightarrow$ $0 + \left(\frac{1}{2}\right)^2 + \cos^2\gamma = 1$

$\Rightarrow$ $\frac{1}{4} + \cos^2\gamma = 1$

$\Rightarrow$ $\cos^2\gamma = 1 - \frac{1}{4} = \frac{3}{4}$

$\Rightarrow$ $\cos\gamma = \pm\frac{\sqrt{3}}{2}$

$\therefore$ $\gamma = 30°, 150°$ **(1/2)**

6. Let l, m and n be the direction cosines of the given line. Then, we have

$l = \cos 90° = 0,$

$m = \cos 60° = \frac{1}{2}$

and $n = \cos\theta$

$\because$ $l^2 + m^2 + n^2 = 1$

$\therefore$ $0 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$

$\Rightarrow$ $\cos^2\theta = 1 - \frac{1}{4} = \frac{3}{4}$

$\Rightarrow$ $\cos\theta = \pm\frac{\sqrt{3}}{2}$

[$\because$ $\cos\theta$ cannot be negative as θ is an acute angle]

$\Rightarrow$ $\cos\theta = \cos 30°$

$\therefore$ $\theta = 30°$ **(1)**

7. Given equations of a line is

$5x - 3 = 15y + 7 = 3 - 10z$ ...(i)

Let us first convert the equation in standard form

$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$ ...(ii)

Let us divide Eq. (i) by LCM (coefficients of x, y and z), i.e. LCM (5, 15, 10) = 30

Now, the Eq. (i) becomes

$\frac{5x-3}{30} = \frac{15y+7}{30} = \frac{3-10z}{30}$

$\Rightarrow$ $\frac{5\left(x - \frac{3}{5}\right)}{30} = \frac{15\left(y + \frac{7}{15}\right)}{30} = \frac{-10\left(z - \frac{3}{10}\right)}{30}$

$\Rightarrow$ $\frac{x - \frac{3}{5}}{6} = \frac{y + \frac{7}{15}}{2} = \frac{z - \frac{3}{10}}{-3}$ **(1/2)**

On comparing the above equation with Eq.(ii), we get 6, 2, −3 are the direction ratios of the given line.

Now, the direction cosines of given line are

$\frac{6}{\sqrt{6^2 + 2^2 + (-3)^2}}, \frac{2}{\sqrt{6^2 + 2^2 + (-3)^2}}$ and $\frac{-3}{\sqrt{6^2 + 2^2 + (-3)^2}}$

i.e. $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$. **(1/2)**

8. Given, if a line makes angles α, β, γ with the coordinate axes.

Then, direction cosine of a line are $\cos\alpha, \cos\beta, \cos\gamma$.

$\therefore$ $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$= 1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma$

$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$= 3 - 1 = 2$ [$\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$] **(1)**

9.

> Firstly, consider any point on X-axis be $Q(x, 0, 0)$.
> Then, use the formula for distance of points $R(x_1, y_1, z_1)$ from $S(x_2, y_2, z_2)$
> $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

Given point is $P(a, b, c)$.

Then, the coordinates of the point on X-axis be $(a, 0, 0)$. **(1/2)**

[$\because$ x-coordinate of both points will be same]

$\therefore$ Required distance $= \sqrt{(a-a)^2 + (0-b)^2 + (0-c)^2}$

$= \sqrt{0 + b^2 + c^2} = \sqrt{b^2 + c^2}$ **(1/2)**

10. Given cartesian equation of a line is

$$\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$$

On rewriting the given equation in standard form, we get

$$\frac{x-3}{-5}=\frac{y+4}{7}=\frac{z-3}{2}=\lambda \text{ (let)}$$

$\Rightarrow \quad x=-5\lambda+3,\ y=7\lambda-4$ and $z=2\lambda+3$ **(1/2)**

Now, $x\hat{i}+y\hat{j}+z\hat{k}=(-5\lambda+3)\hat{i}+(7\lambda-4)\hat{j}+(2\lambda+3)\hat{k}$

$=3\hat{i}-4\hat{j}+3\hat{k}+\lambda(-5\hat{i}+7\hat{j}+2\hat{k})$

$\therefore \quad \vec{r}=(3\hat{i}-4\hat{j}+3\hat{k})+\lambda(-5\hat{i}+7\hat{j}+2\hat{k})$

which is the required equation of line in vector form. **(1/2)**

11. The vector equation of a line parallel to Z-axis is $\vec{m}=0\hat{i}+0\hat{j}+\hat{k}$. Then, the required line passes through the point $A(\alpha,\beta,\gamma)$ whose position vector is $\vec{r_1}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{j}$ and is parallel to the vector $\vec{m}=(0\hat{i}+0\hat{j}+\hat{k})$. **(1/2)**

$\therefore$ The equation is $\vec{r}=\vec{r_1}+\lambda\vec{m}$

$=(\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k})+\lambda(0\hat{i}+0\hat{j}+\hat{k})$

$=(\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k})+\lambda(\hat{k})$ **(1/2)**

12. Given equation of line is

$$\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$$

It can be rewritten in standard form as

$$\frac{x-4}{-2}=\frac{y}{6}=\frac{z-1}{-3}$$

Here, DR's of the line are $(-2, 6, -3)$.

$\therefore$ Direction cosines of the line are

$$\frac{-2}{\sqrt{(-2)^2+6^2+(-3)^2}},\ \frac{6}{\sqrt{(-2)^2+6^2+(-3)^2}}$$

and $\dfrac{-3}{\sqrt{(-2)^2+6^2+(-3)^2}}$ i.e. $\dfrac{-2}{\sqrt{49}},\dfrac{6}{\sqrt{49}}$ and $\dfrac{-3}{\sqrt{49}}$

Thus, DC's of line are $\left(-\frac{2}{7},\frac{6}{7},-\frac{3}{7}\right)$ **(1)**

13. We know that, the vector equation of a line passing through a point with position vector $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\vec{r}=\vec{a}+\lambda\vec{b}$, where $\lambda\in R$. Here, $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$. [$\because$ DR's of given line is 1, 2 and −2]

$\therefore$ Required vector equation of line is

$\vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\lambda(\hat{i}+2\hat{j}-2\hat{k})$, where $\lambda\in R$. **(1)**

14.

> If two lines are parallel, then direction ratios of both lines are proportional. Use this result and simplify it.

Given, the required line is parallel to the line

$$\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6} \text{ or } \frac{x+3}{3}=\frac{y-4}{-5}=\frac{z+8}{6}$$

$\therefore$ DR's of both lines are proportional to each other. **(1/2)**

The required equation of the line passing through $(-2, 4, -5)$ having DR's $(3, -5, 6)$ is

$$\frac{x+2}{3}=\frac{y-4}{-5}=\frac{z+5}{6}$$ **(1/2)**

15. Given, DR's of the line are $(2, -1, -2)$.

$\therefore$ Direction cosines of the line are

$$\left(\frac{2}{\sqrt{(2)^2+(-1)^2+(-2)^2}},\frac{-1}{\sqrt{(2)^2+(-1)^2+(-2)^2}},\frac{-2}{\sqrt{(2)^2+(-1)^2+(-2)^2}}\right)$$

$$\left[\because l=\pm\frac{a}{\sqrt{a^2+b^2+c^2}},\ m=\pm\frac{b}{\sqrt{a^2+b^2+c^2}},\ n=\pm\frac{c}{\sqrt{a^2+b^2+c^2}}\right]$$

$$=\left(\frac{2}{\sqrt{4+1+4}},\frac{-1}{\sqrt{4+1+4}},\frac{-2}{\sqrt{4+1+4}}\right)$$

$$=\left(\frac{2}{\sqrt{9}},\frac{-1}{\sqrt{9}},\frac{-2}{\sqrt{9}}\right)=\left(\frac{2}{3},\frac{-1}{3},\frac{-2}{3}\right)$$ **(1)**

16. Clearly, the direction ratios of line joining the points $(1, 0, 0)$ and $(0, 1, 1)$ are $0-1, 1-0$ and $1-0$, i.e. $-1, 1$ and 1 **(1/2)**

$\therefore$ Direction cosines are $\dfrac{-1}{\sqrt{(-1)^2+1^2+1^2}}$, $\dfrac{1}{\sqrt{(-1)^2+1^2+1^2}}$ and $\dfrac{1}{\sqrt{(-1)^2+1^2+1^2}}$

i.e., $\dfrac{-1}{\sqrt{3}},\dfrac{1}{\sqrt{3}}$ and $\dfrac{1}{\sqrt{3}}$ **(1/2)**

17. Do same as Q.No. 10.

[**Ans.** $\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$]

18. Given equation of line can be written as

$$\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+2}{1}.$$

Here, DR's of a line are $(-2, 2, 1)$.

$\therefore$ DC's of line parallel to above line are given by

$$\left(\frac{-2}{\sqrt{(-2)^2 + (2)^2 + (1)^2}}, \frac{2}{\sqrt{(-2)^2 + (2)^2 + (1)^2}}, \frac{1}{\sqrt{(-2)^2 + (2)^2 + (1)^2}}\right)$$

$$= \left(\frac{-2}{\sqrt{4+4+1}}, \frac{2}{\sqrt{4+4+1}}, \frac{1}{\sqrt{4+4+1}}\right)$$

$$= \left(\frac{-2}{\sqrt{9}}, \frac{2}{\sqrt{9}}, \frac{1}{\sqrt{9}}\right) = \left(-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$$

Hence, required DC's of a line parallel to the given line are $\left(-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$. **(1)**

NOTE Before we can use the DR's of a line, first we ensure that coefficients of x, y and z are unity with positive sign.

19. Given equation of line can be written as

$$\frac{x-3}{-1} = \frac{y+2}{-2} = \frac{z-5}{4}$$

$\therefore$ DR's of the line parallel to above line are $(-1, -2, 4)$.

[$\because$ direction ratios of two parallel lines are proportional] **(1)**

20. Do same as Q. No. 9. [**Ans.** 5 units]

21. Do same as Q. No. 10.

[**Ans.** $\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} - 2\hat{k})$]

22. Given line is $5x - 25 = 14 - 7y = 35z$.

$$\Rightarrow \frac{x-5}{1/5} = \frac{2-y}{1/7} = \frac{z}{1/35} \Rightarrow \frac{x-5}{1/5} = \frac{y-2}{-1/7} = \frac{z}{1/35}$$

$\Rightarrow$ Direction ratio of the given line are $\frac{1}{5}, -\frac{1}{7}, \frac{1}{35}$. **(1)**

$\Rightarrow$ Direction ratio of a line parallel to the given line are $\frac{1}{5}, -\frac{1}{7}, \frac{1}{35}$.

$\therefore$ The required equation of a line passing through the point $A(1, 2, -1)$ and parallel to the given line is

$$\frac{x-1}{1/5} = \frac{y-2}{-1/7} = \frac{z+1}{1/35}.$$ **(1)**

23. The equation of line joining the points $P(2, 2, 1)$ and $Q(5, 1, -2)$ is

$$\frac{x-2}{5-2} = \frac{y-2}{1-2} = \frac{z-1}{-2-1}$$

$$\Rightarrow \frac{x-2}{3} = \frac{y-2}{-1} = \frac{z-1}{-3}$$ **(1)**

Since, x-coordinate is 4.

$$\therefore \frac{4-2}{3} = \frac{z-1}{-3} \Rightarrow z = -1$$ **(1)**

24. Given equation of lines can be written in standard form as

$$\frac{x-1}{-3} = \frac{y-2}{\lambda/7} = \frac{z-3}{2} = r_1 \text{ (let)} \quad \text{...(i)}$$

and $$\frac{x-1}{\frac{-3\lambda}{7}} = \frac{y-5}{1} = \frac{z-6}{-5} = r_2 \text{ (let)} \quad \text{...(ii)}$$ **(1)**

These lines will intersect at right angle, if

$$-3\left(\frac{-3\lambda}{7}\right) + \frac{\lambda}{7}(1) + 2(-5) = 0$$

[$\because$ two lines with DR's a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$]

$$\Rightarrow \frac{9\lambda}{7} + \frac{\lambda}{7} = 10 \Rightarrow \frac{10\lambda}{7} = 10 \Rightarrow \lambda = 7,$$

which is the required value of λ. **(1)**

Now, let us check whether the lines are intersecting or not.

Coordinates of any point on line (i) are

$$(-3r_1 + 1, r_1 + 2, 2r_1 + 3)$$

and coordinates of any point on line (ii) are

$$(-3r_2 + 1, r_2 + 5, -5r_2 + 6)$$ **(1)**

Clearly, the line will intersect if

$$(-3r_1 + 1, r_1 + 2, 2r_1 + 3) = (-3r_2 + 1, r_2 + 5, -5r_2 + 6)$$

For some $r_1, r_2 \in R$

$\Rightarrow -3r_1 + 1 = -3r_2 + 1;\ r_1 + 2 = r_2 + 5;$
$2r_1 + 3 = -5r_2 + 6$

$\Rightarrow \quad r_1 = r_2;\ r_1 - r_2 = 3;\ 2r_1 + 5r_2 = 3$

which is not possible simultaneously for any $r_1, r_2 \in R$.

Hence, the lines are not intersecting. **(1)**

25. Do same as Q. No. 24.

[**Ans.** $\lambda = -2$, do not intersect]

26. Given equation of lines are

$$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k}) \quad \text{...(i)}$$

and $$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k}) \quad \text{...(ii)}$$

On comparing Eqs. (i) and (ii) with $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ respectively, we get

$$\vec{a}_1 = 4\hat{i} - \hat{j}, \vec{b}_1 = \hat{i} + 2\hat{j} - 3\hat{k}$$

and $\vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}, \vec{b}_2 = 2\hat{i} + 4\hat{j} - 5\hat{k}$

Here $\vec{a}_2 - \vec{a}_1 = -3\hat{i} + 2\hat{k}$ **(1)**

and $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}$

$$= \hat{i}(-10+12) - \hat{j}(-5+6) + \hat{k}(4-4) = 2\hat{i} - \hat{j}$$

$$\Rightarrow |\vec{b}_1 \times \vec{b}_2| = \sqrt{2^2 + (-1)^2}$$

$$= \sqrt{4+1} = \sqrt{5}$$ **(1)**

Now, the shortest distance between the given lines is given by

$$d = \frac{|(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)|}{|\vec{b}_1 \times \vec{b}_2|}$$

$$= \frac{|(2\hat{i} - \hat{j})\cdot(-3\hat{i} + 2\hat{k})|}{\sqrt{5}}$$ **(1)**

$$= \frac{|-6|}{\sqrt{5}} = \frac{6}{\sqrt{5}} \text{ units}$$ **(1)**

27. Given lines are $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ **(1)**

On comparing the given equations of lines with

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$$

and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$,

we get $x_1 = 1, y_1 = 2, z_1 = 3; a_1 = 2, b_1 = 3, c_1 = 4$

and $x_2 = 2, y_2 = 4, z_2 = 5; a_2 = 3, b_2 = 4, c_2 = 5$ **(1)**

On putting these values in $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$, we get

$$\begin{vmatrix} 2-1 & 4-2 & 5-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$$= 1(15-16) - 2(10-12) + 2(8-9)$$

$$= -1 + 4 - 2 = 1$$

Now,

$$\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}$$

$$= \sqrt{(3\times5 - 4\times4)^2 + (4\times3 - 5\times2)^2 + (2\times4 - 3\times3)^2}$$

$$= \sqrt{(15-16)^2 + (12-10)^2 + (8-9)^2}$$

$$= \sqrt{(-1)^2 + (2)^2 + (-1)^2} = \sqrt{1+4+1} = \sqrt{6}$$ **(1)**

$\therefore$ SD

$$= \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}},$$

$$= \frac{1}{\sqrt{6}} \text{ units,}$$

which is the required shortest distance. **(1)**

28. Given equations of lines are

$$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$$

and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

On comparing with vector form of equation of a line, i.e. $\vec{r} = \vec{a} + \lambda\vec{b}$, we get

$$\vec{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k}$$

and $\vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k}$ **[1/2]**

Now, we determine

$$\vec{b} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$$

$$= \hat{i}(80-56) - \hat{j}(-15-21) + \hat{k}(24+48)$$

$$= 24\hat{i} + 36\hat{j} + 72\hat{k} = 12(2\hat{i} + 3\hat{j} + 6\hat{k})$$ **[1]**

Since, the required line is perpendicular to the given lines. So, it is parallel to $\vec{b}_1 \times \vec{b}_2$. Now, the equation of a line passing through the point $(1, 2, -4)$ and parallel to $24\hat{i} + 36\hat{j} + 72\hat{k}$ or $(2\hat{i} + 3\hat{j} + 6\hat{k})$ is

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$ **[1]**

which is required vector equation of a line.

For cartesian equation, put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, we get

$$x\hat{i} + y\hat{j} + z\hat{k} = (1+2\lambda)\hat{i} + (2+3\lambda)\hat{j} + (-4+6\lambda)\hat{k}$$ **[1/2]**

On comparing the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$, we get

$$x = 1 + 2\lambda,\ y = 2 + 3\lambda \text{ and } z = -4 + 6\lambda$$

$$\Rightarrow \quad \frac{x-1}{2} = \lambda,\ \frac{y-2}{3} = \lambda \text{ and } \frac{z+4}{6} = \lambda$$

$$\therefore \quad \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6} \quad [1]$$

which is the required cartesian equation of a line.

Alternate Method

Let the equation of line passing through (1, 2, –4) is

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda_1(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) \quad \ldots \text{(i)}$$

Since, the line (i) is perpendicular to the given lines

$$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$$

and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

Therefore, we have

$$\Rightarrow \quad (b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) \cdot (3\hat{i} - 16\hat{j} + 7\hat{k}) = 0$$

$$\Rightarrow \quad 3b_1 - 16b_2 + 7b_3 = 0 \quad \ldots\text{(ii)}$$

and $\quad (b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) \cdot (3\hat{i} + 8\hat{j} - 5\hat{k}) = 0$

$$\Rightarrow \quad 3b_1 + 8b_2 - 5b_3 = 0 \quad \ldots\text{(iii)} [1]$$

[$\because$ if two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ are perpendicular, then $\vec{b_1} \cdot \vec{b_2} = 0$.]

Now, on solving Eqs. (ii) and (iii), we get

$$\frac{b_1}{80-56} = \frac{b_2}{21+15} = \frac{b_3}{24+48} \quad [1]$$

$$\Rightarrow \quad \frac{b_1}{24} = \frac{b_2}{36} = \frac{b_3}{72}$$

$$\Rightarrow \quad \frac{b_1}{2} = \frac{b_2}{3} = \frac{b_3}{6} \text{ [multiplying by 12] } [1]$$

$\Rightarrow b_1 = 2k$, $b_2 = 3k$ and $b_3 = 6k$, for some constant k.

Thus, the required vector equation of line is

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}),$$

where $\lambda = \lambda_1 k$ is any constant. [1]

Now, for cartesian equation do same as in above method.

29. Clearly, the equation of a line joining the points B (0, –1, 3) and C (2, – 3, –1) is

$$\vec{r} = (0\hat{i} - \hat{j} + 3\hat{k}) + \lambda\,[(2-0)\hat{i} + (-3+1)\hat{j} + (-1-3)\,\hat{k}]$$

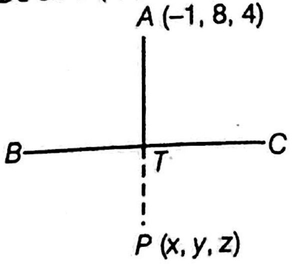
$$\Rightarrow \quad \vec{r} = (-\hat{j} + 3\hat{k}) + \lambda\,(2\hat{i} - 2\hat{j} - 4\hat{k})$$

$$\Rightarrow \quad \vec{r} = (2\lambda)\hat{i} + (-2\lambda - 1)\hat{j} + (-4\lambda + 3)\hat{k}$$

So, any point on line BC is to the form

$$(2\lambda, -2\lambda - 1, -4\lambda + 3) \quad [1]$$

Let foot of the perpendicular drawn from point A to the line BC be T $(2\lambda, -2\lambda - 1, -4\lambda + 3)$.



Now, DR's of line AT is $(2\lambda + 1, -2\lambda - 1 - 8, -4\lambda + 3 - 4)$ or $(2\lambda + 1, 2\lambda - 9, -4\lambda - 1)$.

Since, AT is perpendicular to BC.

$$\therefore \quad 2 \times (2\lambda + 1) + (-2) \times (-2\lambda - 9) + (-4)(-4\lambda - 1) = 0 \quad [1]$$

$$[\because a_1a_2 + b_1b_2 + c_1c_2 = 0]$$

$$\Rightarrow \quad 4\lambda + 2 + 4\lambda + 18 + 16\lambda + 4 = 0$$

$$\Rightarrow \quad 24\lambda + 24 = 0$$

$$\lambda = -1$$

$\therefore$ Coordinates of foot of perpendicular is

$T\,(2 \times (-1)), -2 \times (-1) - 1, -4 \times (-1) + 3)$ or $T(-2, 1, 7)$ [1]

Let $P(x, y, z)$ be the image of a point A with respect to the line BC. So, point T is the mid-point of AP.

$\therefore$ Coordinates of T = Coordinates of mid-point of AP

$$\Rightarrow \quad (-2, 1, 7) = \left(\frac{x-1}{2}, \frac{y+8}{2}, \frac{z+4}{2}\right)$$

On equating the corresponding coordinates, we get

$$-2 = \frac{x-1}{2},\ 1 = \frac{y+8}{2} \text{ and } 7 = \frac{z+4}{2}$$

$$\Rightarrow \quad x = -3,\ y = -6 \text{ and } z = 10$$

Hence, coordinates of the foot of perpendicular is T (–2, 1, 7) and image of the point A is P (–3, – 6, 10). [1]

30. The equation of line through A (0, –1, –1) and B (4, 5, 1) is

$$\frac{x-0}{4-0} = \frac{y+1}{5+1} = \frac{z+1}{1+1},$$

i.e. $$\frac{x}{4} = \frac{y+1}{6} = \frac{z+1}{2} \quad \ldots\text{(i)}$$

and equation of line through $C(3, 9, 4)$ and $D(-4, 4, 4)$ is

$$\frac{x-3}{-4-3}=\frac{y-9}{4-9}=\frac{z-4}{0}$$

i.e., $$\frac{x-3}{-7}=\frac{y-9}{-5}=\frac{z-4}{0} \quad \ldots(ii) \quad (1)$$

We know that, the lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and}$$

$$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2} \text{ will intersect,}$$

if $$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0 \quad (1)$$

∴ The given lines will intersect, if

$$\begin{vmatrix} 3-0 & 9-(-1) & 4-(-1) \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix}=0 \quad (1)$$

Now consider,

$$\begin{vmatrix} 3-0 & 9-(-1) & 4-(-1) \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix}=\begin{vmatrix} 3 & 10 & 5 \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix}$$

$$=3(0+10)-10(0+14)+5(-20+42)$$

$$=30-140+110=0$$

Hence, the given lines intersect. (1)

31. Given lines can be rewritten as

$$\vec{r}=(3\lambda+1)\hat{i}+(1-\lambda)\hat{j}-\hat{k} \quad \ldots(i)$$

and $$\vec{r}=(4+2\mu)\hat{i}+0\hat{j}+(3\mu-1)\hat{k} \quad \ldots(ii)$$

Clearly, any point on line (i) is of the form $P(3\lambda+1, 1-\lambda, -1)$ and any point on line (ii) is of the form $Q(4+2\mu, 0, 3\mu-1)$ (1)

If lines (i) and (ii) intersect, then these points must coincide for some λ and μ.

Consider, $$3\lambda+1=4+2\mu$$

⇒ $$3\lambda-2\mu=3 \quad \ldots(iii)$$

$$1-\lambda=0 \quad \ldots(iv)$$

and $$3\mu-1=-1 \quad \ldots(v) \quad (1)$$

From Eq. (iv), we get $\lambda=1$ and put the value of λ in Eq. (iii), we get

$$3(1)-2\mu=3$$

⇒ $$-2\mu=3-3 \Rightarrow \mu=0$$

On putting the value of μ in Eq. (v), we get

$$3(0)-1=-1 \Rightarrow 0-1=-1$$

⇒ $$-1=-1, \text{ which is true}$$

Hence, both lines intersect each other. (1)

The point of intersection of both lines can be obtained by putting $\lambda=1$ in coordinates of P. So, the point of intersection is $(3+1, 1-1, -1)$, i.e. $(4, 0, -1)$. (1)

32. Given equation of line is

$$\frac{x+2}{2}=\frac{2y-7}{6}=\frac{5-z}{6}$$

This equation can be written as

$$\frac{x+2}{2}=\frac{y-7/2}{3}=\frac{z-5}{-6}$$

So, direction ratios of line are $(2, 3, -6)$. (1)

Now, direction cosines of a line are

$$\left(l=\frac{2}{\sqrt{2^2+3^2+(-6)^2}}, m=\frac{3}{\sqrt{2^2+3^2+(-6)^2}}, n=\frac{-6}{\sqrt{2^2+3^2+(-6)^2}}\right)$$

$$\left[\because l=\frac{\pm a}{\sqrt{a^2+b^2+c^2}}, m=\frac{\pm b}{\sqrt{a^2+b^2+c^2}}, n=\frac{\pm c}{\sqrt{a^2+b^2+c^2}}\right]$$

(1)

⇒ $$l=\frac{2}{\sqrt{49}}, m=\frac{3}{\sqrt{49}}, n=-\frac{6}{\sqrt{49}}$$

So, direction cosines of given line are $\left(\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}\right)$

(1)

Here, DR's of a line parallel to given line are $(2, 3, -6)$. So, the required equation of line passes through the point $A(-1, 2, 3)$ and parallel to given line is

$$\frac{x+1}{2}=\frac{y-2}{3}=\frac{z-3}{-6}. \quad (1)$$

33. If vector form of lines are $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\lambda\vec{b}_2$, then angle between them is

$$\cos\theta=\frac{|\vec{b}_1\cdot\vec{b}_2|}{|\vec{b}_1||\vec{b}_2|}.$$

Given equations of lines are

$$\vec{r}=(2\hat{i}-5\hat{j}+\hat{k})+\lambda(3\hat{i}+2\hat{j}+6\hat{k}) \quad \ldots(i)$$

and $$\vec{r}=(7\hat{i}-6\hat{j}-6\hat{k})+\mu(\hat{i}+2\hat{j}+2\hat{k}) \quad \ldots(ii) \quad (1)$$

On comparing Eqs. (i) and (ii) with vector form of equation of line, i.e.

$\vec{r} = \vec{a} + \lambda \vec{b}$, we get

$\vec{a}_1 = 2\hat{i} - 5\hat{j} + \hat{k}, \vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}$

and $\vec{a}_2 = 7\hat{i} - 6\hat{j} - 6\hat{k}, \vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$ **(1)**

We know that, the angle between two lines is given by

$$\cos\theta = \frac{|\vec{b}_1 \cdot \vec{b}_2|}{|\vec{b}_1||\vec{b}_2|}$$

$$\therefore \quad \cos\theta = \left| \frac{(3\hat{i} + 2\hat{j} + 6\hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{(3)^2 + (2)^2 + (6)^2} \cdot \sqrt{(1)^2 + (2)^2 + (2)^2}} \right| \quad \textbf{(1)}$$

$$\Rightarrow \cos\theta = \left| \frac{3 + 4 + 12}{\sqrt{49} \times \sqrt{9}} \right|$$

$$\Rightarrow \cos\theta = \left| \frac{19}{7 \times 3} \right| \Rightarrow \cos\theta = \frac{19}{21}$$

Hence, the angle between given two lines is

$\theta = \cos^{-1}\left(\frac{19}{21}\right)$. **(1)**

34. Given lines are

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda \text{ (let)} \quad \text{...(i)}$$

and $$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu \text{ (let)} \quad \text{...(ii)}$$

Then, any point on line (i) is of the form

$$P(3\lambda - 1, 5\lambda - 3, 7\lambda - 5) \quad \text{...(iii)}$$

and any point on line (ii) is of the form

$$Q(\mu + 2, 3\mu + 4, 5\mu + 6) \quad \text{...(iv)}$$

If lines (i) and (ii) intersect, then these points must coincide for some λ and μ.

Consider, $3\lambda - 1 = \mu + 2$

$5\lambda - 3 = 3\mu + 4$

and $7\lambda - 5 = 5\mu + 6$

$\Rightarrow$ $3\lambda - \mu = 3$ ...(v)

$5\lambda - 3\mu = 7$ ...(vi)

and $7\lambda - 5\mu = 11$ ...(vii) **(1)**

On multiplying Eq. (v) by 3 and then subtracting Eq. (vi), we get

$$9\lambda - 3\mu - 5\lambda + 3\mu = 9 - 7$$

$$4\lambda = 2$$

$$\Rightarrow \lambda = \frac{1}{2}$$

$\Rightarrow$

On putting the value of λ in Eq. (v), we get

$$3 \times \frac{1}{2} - \mu = 3$$

$$\Rightarrow \frac{3}{2} - \mu = 3$$

$$\Rightarrow \mu = -\frac{3}{2}$$ **(1)**

On putting the values of λ and μ in Eq. (vii), we get

$$7 \times \frac{1}{2} - 5\left(-\frac{3}{2}\right) = 11$$

$$\Rightarrow \frac{7}{2} + \frac{15}{2} = 11 \Rightarrow \frac{22}{2} = 11$$

$\Rightarrow$ $11 = 11$, which is true. **(1)**

Hence, lines (i) and (ii) intersect and their point of intersection is

$$P\left(3 \times \frac{1}{2} - 1, 5 \times \frac{1}{2} - 3, 7 \times \frac{1}{2} - 5\right)$$

[put $\lambda = \frac{1}{2}$ in Eq. (iii)]

i.e. $P\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$. **(1)**

35. Do same as Q. No. 24.

Also, we know that, the equation of a line which passes through the point (x_1, y_1, z_1) with direction ratios a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Since, required line is parallel to line l_1.

So, $a = -3$, $b = \frac{7}{7} = 1$ and $c = 2$

Now, equation of line passing through the point $(3, 2, -4)$ and having direction ratios $(-3, 1, 2)$ is

$$\frac{x-3}{-3} = \frac{y-2}{1} = \frac{z+4}{2}$$

$$\therefore \quad \frac{3-x}{3} = \frac{y-2}{1} = \frac{z+4}{2}$$

36. Do same as Q. No. 28.

[**Ans.** $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(-6\hat{i} - 3\hat{j} + 6\hat{k})$

and $\frac{2-x}{6} = \frac{-y-1}{3} = \frac{z-3}{6}$]

37. Given equations of lines are

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k}) \quad \text{...(i)}$$

and $$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k}) \quad \text{...(ii)}$$

On comparing above equations with vector equation $\vec{r} = \vec{a} + \lambda \vec{b}$, we get

$$\vec{a_1} = \hat{i} + \hat{j}, \vec{b_1} = 2\hat{i} - \hat{j} + \hat{k}$$

and $\vec{a_2} = 2\hat{i} + \hat{j} - \hat{k}, \vec{b_2} = 3\hat{i} - 5\hat{j} + 2\hat{k}$ **(1)**

We know that, the shortest distance between two lines is given by

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right| \quad \text{...(iii)}$$

Now, $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$

$$= \hat{i}(-2+5) - \hat{j}(4-3) + \hat{k}(-10+3)$$

$\Rightarrow \vec{b_1} \times \vec{b_2} = 3\hat{i} - \hat{j} - 7\hat{k}$ ...(iv) **(1)**

and $|\vec{b_1} \times \vec{b_2}| = \sqrt{(3)^2 + (-1)^2 + (-7)^2}$

$= \sqrt{9 + 1 + 49} = \sqrt{59}$ ...(v)

Also, $\vec{a_2} - \vec{a_1} = (2\hat{i} + \hat{j} - \hat{k}) - (\hat{i} + \hat{j})$

$= \hat{i} - \hat{k}$ ...(vi) **(1)**

From Eqs. (iii), (iv), (v) and (vi), we get

$$d = \left| \frac{(3\hat{i} - \hat{j} - 7\hat{k}) \cdot (\hat{i} - \hat{k})}{\sqrt{59}} \right|$$

$$\Rightarrow \quad d = \left| \frac{3 - 0 + 7}{\sqrt{59}} \right| = \frac{10}{\sqrt{59}}$$

Hence, required shortest distance is $\frac{10}{\sqrt{59}}$ units. **(1)**

38. Do same as Q.No. 37. $\left[\textbf{Ans.} \ \frac{9}{\sqrt{171}} \text{ units}\right]$

39. Given equations of lines are

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \quad \text{...(i)}$$

and $$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \quad \text{...(ii)}$$

On comparing above equations with one point form of equation of line, i.e.

$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$, we get

$a_1 = 1, b_1 = -2, c_1 = 1, x_1 = 3, y_1 = 5, z_1 = 7$
and $a_2 = 7, b_2 = -6, c_2 = 1, x_2 = -1,$
$y_2 = -1, z_2 = -1$ **(1)**

We know that, the shortest distance between two lines is given by

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}}$$

$$\therefore \quad d = \frac{\begin{vmatrix} -1-3 & -1-5 & -1-7 \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix}}{\sqrt{(-2+6)^2 + (7-1)^2 + (-6+14)^2}} \quad \textbf{(1)}$$

$$= \frac{\begin{vmatrix} -4 & -6 & -8 \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix}}{\sqrt{(4)^2 + (6)^2 + (8)^2}}$$

$$= \left| \frac{-4(-2+6) + 6(1-7) - 8(-6+14)}{\sqrt{(4)^2 + (6)^2 + (8)^2}} \right|$$

$$= \left| \frac{-4(4) + 6(-6) - 8(8)}{\sqrt{16 + 36 + 64}} \right|$$

$$= \left| \frac{-16 - 36 - 64}{\sqrt{116}} \right| \quad \textbf{(1)}$$

$$= \left| \frac{-116}{\sqrt{116}} \right| = \frac{116}{\sqrt{116}} = \sqrt{116}$$

Hence, the required shortest distance is $\sqrt{116}$ units. **(1)**

40. Given equations of lines are

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

and $$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$$

On comparing with $\vec{r} = \vec{a} + \lambda \vec{b}$, we get

$$a_1 = \hat{i} + 2\hat{j} - 4\hat{k}, \vec{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

and $\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}, \vec{b_2} = 4\hat{i} + 6\hat{j} + 12\hat{k}$ **(1)**

Now, $\vec{a_2} - \vec{a_1} = (3\hat{i} + 3\hat{j} - 5\hat{k}) - (\hat{i} + 2\hat{j} - 4\hat{k})$

$= 2\hat{i} + \hat{j} - \hat{k}$

and $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 4 & 6 & 12 \end{vmatrix}$

$= \hat{i}(36 - 36) - \hat{j}(24 - 24) + \hat{k}(12 - 12) = \vec{0}$ **(1)**

So, both given lines are parallel and

$$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

Then, $\vec{b} \times (\vec{a_2} - \vec{a_1}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}$

$$= \hat{i}(-3-6) - \hat{j}(-2-12) + \hat{k}(2-6)$$

$$= -9\hat{i} + 14\hat{j} - 4\hat{k} \quad \textbf{(1)}$$

Now, required distance between given lines is

$$d = \left|\frac{\vec{b} \times (\vec{a_1} - \vec{a_2})}{|\vec{b}|}\right| = \left|\frac{-9\hat{i} + 14\hat{j} - 4\hat{k}}{\sqrt{(2)^2 + (3)^2 + (6)^2}}\right|$$

$$= \frac{\sqrt{81 + 196 + 16}}{\sqrt{4 + 9 + 36}} = \frac{\sqrt{293}}{\sqrt{49}} = \frac{\sqrt{293}}{7} \text{ units} \quad \textbf{(1)}$$

41. Any line through the point (2, 1, 3) can be written as

$$\frac{x-2}{a} = \frac{y-1}{b} = \frac{z-3}{c} \quad \text{...(i)}$$

where, a, b and c are the direction ratios of line (i).

Now, the line (i) is perpendicular to the lines

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$$

and $\quad \dfrac{x-0}{-3} = \dfrac{y-0}{2} = \dfrac{z-0}{5}.$

Direction ratios of these two lines are (1, 2, 3) and (−3, 2, 5), respectively. **(1)**

We know that, if two lines are perpendicular, then

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$\therefore \quad a + 2b + 3c = 0 \quad$ ...(ii)

and $\quad -3a + 2b + 5c = 0 \quad$ ...(iii)

In Eqs. (ii) and (iii), by cross-multiplication, we get

$$\frac{a}{10-6} = \frac{b}{-9-5} = \frac{c}{2+6}$$

$\Rightarrow \quad \dfrac{a}{4} = \dfrac{b}{-14} = \dfrac{c}{8}$

$\Rightarrow \quad \dfrac{a}{2} = \dfrac{b}{-7} = \dfrac{c}{4} = \lambda$ (say)

$\therefore \quad a = 2\lambda, b = -7\lambda$ **(1)**

and $\quad c = 4\lambda$

On substituting the values of a, b and c in Eq. (i), we get

$$\frac{x-2}{2\lambda} = \frac{y-1}{-7\lambda} = \frac{z-3}{4\lambda} \Rightarrow \frac{x-2}{2} = \frac{y-1}{-7} = \frac{z-3}{4} \quad \textbf{(1)}$$

which is the required cartesian equation of the line.

The vector equation of line which passes through (2, 1, 3) and parallel to the vector $2\hat{i} - 7\hat{j} + 4\hat{k}$ is

$$\vec{r} = 2\hat{i} + \hat{j} + 3\hat{k} + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$$

which is the required vector equation of the line. **(1)**

42. Given equation of line is

$$6x - 2 = 3y + 1 = 2z - 2$$

or $\quad \dfrac{x - 2/6}{1/6} = \dfrac{y + 1/3}{1/3} = \dfrac{z - 2/2}{1/2}$

$\Rightarrow \quad \dfrac{x - 1/3}{1/6} = \dfrac{y + 1/3}{1/3} = \dfrac{z-1}{1/2}$

$\Rightarrow \quad \dfrac{x - 1/3}{1} = \dfrac{y + 1/3}{2} = \dfrac{z-1}{3}$

[on dividing each by 6]

Here, DR's of the line are (1, 2, 3).

$\therefore$ Direction cosines of the line are

$$\frac{1}{\sqrt{(1)^2 + (2)^2 + (3)^2}}, \frac{2}{\sqrt{(1)^2 + (2)^2 + (3)^2}}, \frac{3}{\sqrt{(1)^2 + (2)^2 + (3)^2}}$$

$$= \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \quad \textbf{(1)}$$

The equation of a line passing through (2, −1, −1) and parallel to the given line is

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z+1}{3} = \lambda \text{ (say)} \quad \textbf{(1)}$$

$$\left[\because \frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}\right]$$

$\Rightarrow \quad x = 2 + \lambda,\ y = -1 + 2\lambda$ and $z = -1 + 3\lambda$

Now, $\quad x\hat{i} + y\hat{j} + z\hat{k} = (2 + \lambda)\hat{i} + (-1 + 2\lambda)\hat{j} + (-1 + 3\lambda)\hat{k}$ **(1)**

$\therefore \quad \vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ which is the required equation of line in vector form. **(1)**

43. Do same as Q. No. 37. **[Ans.** 9 units]

44. Do same as Q. No. 31. **[Ans.** (−1, −6, −12)]

45. The vector equation of line AB is

$$\vec{r} = (-2\hat{i} + 3\hat{j} + 5\hat{k}) + \lambda[(7 + 2)\hat{i} + (0 - 3)\hat{j} + (-1 - 5)\hat{k}]$$

i.e. $\vec{r} = (-2\hat{i} + 3\hat{j} + 5\hat{k}) + \lambda(9\hat{i} - 3\hat{j} - 6\hat{k})$ ...(i)

$\Rightarrow \quad (9\lambda - 2)\hat{i} + (-3\lambda + 3)\hat{j} + (-6\lambda + 5)\hat{k}$

and vector equation of line CD is

$$\vec{r} = (-3\hat{i} - 2\hat{j} - 5\hat{k}) + \mu[(3 + 3)\hat{i} + (4 + 2)\hat{j} + (7 + 5)\hat{k}]$$

i.e. $\vec{r} = (-3\hat{i} - 2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 6\hat{j} + 12\hat{k})$ ...(ii) **(1)**

$\Rightarrow \quad (6\mu - 3)\hat{i} + (6\mu - 2)\hat{j} + (12\mu - 5)\hat{k}$

Clearly, any point on the line (i) is of the form

$$P(9\lambda - 2, -3\lambda + 3, -6\lambda + 5)$$

and any point on the line (ii) is of the form

$$Q(6\mu - 3, 6\mu - 2, 12\mu - 5)$$ **(1)**

If lines (i) and (ii) intersect, then these points must coincide for some λ and μ. **(1/2)**

Consider, $9\lambda - 2 = 6\mu - 3, -3\lambda + 3 = 6\mu - 2$ and $-6\lambda + 5 = 12\mu - 5$

$\Rightarrow \quad 9\lambda - 6\mu = -1$ ...(iii)

$3\lambda + 6\mu = 5$ ...(iv)

and $\quad 6\lambda + 12\mu = 10$ ...(v)

On adding Eqs. (iii) and (iv), we get

$$12\lambda = 4$$

$\Rightarrow \quad \lambda = \frac{1}{3}$

On substituting $\lambda = \frac{1}{3}$ in Eq. (iv), we get

$$6\mu = 5 - 1$$

$\Rightarrow \quad \mu = \frac{4}{6} = \frac{2}{3}$

Now, on substituting $\lambda = \frac{1}{3}$ and $\mu = \frac{2}{3}$ in Eq. (v), we get

$$6 \cdot \frac{1}{3} + 12 \cdot \frac{2}{3} = 10 \Rightarrow 2 + 8 = 10$$

$\Rightarrow \quad 10 = 10$, which is true **(1)**

Thus, AB and CD intersect and their point of intersection is given by

$$P\left(9 \cdot \frac{1}{3} - 2, -3 \cdot \frac{1}{3} + 3, -6 \cdot \frac{1}{3} + 5\right) \text{ i.e. } P(1, 2, 3)$$

Hence proved. (1/2)

46. Do same as Q. No. 37. [**Ans.** lines do not intersect]

Hint : A pair of lines will intersect, if the shortest distance between them is zero.

47. Do same as Q.No. 41. $\left[\textbf{Ans. } \frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}\right]$

48.

> Firstly, convert the given lines in standard form and then use the formula
> $$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}},$$
> to get the required angle.

Given equations of two lines are

$$\frac{-x+2}{-2} = \frac{y-1}{7} = \frac{z+3}{-3}$$

and $$\frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4}$$

Above equations can be written as

$$\frac{x-2}{2} = \frac{y-1}{7} = \frac{z+3}{-3} \quad \text{...(i)}$$

and $$\frac{x+2}{-1} = \frac{y-4}{2} = \frac{z-5}{4} \quad \text{...(ii)}$$ **(1)**

On comparing Eqs. (i) and (ii) with one point form of equation of line

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}, \text{ we get}$$

$$a_1 = 2, b_1 = 7, c_1 = -3$$

and $$a_2 = -1, b_2 = 2, c_2 = 4$$

We know that, angle between two lines is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}}$$ **(1)**

$$\therefore \quad \cos\theta = \frac{(2)(-1) + (7)(2) + (-3)(4)}{\left[\sqrt{(2)^2 + (7)^2 + (-3)^2} \cdot \sqrt{(-1)^2 + (2)^2 + (4)^2}\right]}$$

$$\Rightarrow \quad \cos\theta = \frac{-2 + 14 - 12}{\sqrt{62} \times \sqrt{21}} = \frac{0}{\sqrt{62} \times \sqrt{21}} = 0$$ **(1)**

$$\Rightarrow \quad \cos\theta = \cos\frac{\pi}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{2} \quad \left[\because 0 = \cos\frac{\pi}{2}\right]$$

Hence, the angle between them is $\frac{\pi}{2}$. Therefore, the given pair of lines are perpendicular to each other. **(1)**

NOTE Please be careful while taking DR's of a line, the line should be in symmetrical or in standard form, otherwise there may be chances of error.

49. Firstly, convert both the vector equations in the form $\vec{r} = \vec{a} + \lambda\vec{b}$. Then, apply the shortest distance formula,

i.e. $d = \left| \dfrac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$.

Given equations of lines are

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k} \quad \text{...(i)}$$

and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$ ...(ii)

Firstly, we convert both equations in the vector form as $\vec{r} = \vec{a} + \lambda\vec{b}$ ...(iii)

So, Eq. (i) can be written as

$$\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + t(-\hat{i} + \hat{j} - 2\hat{k}) \quad \text{...(iv)} \quad \textbf{(1)}$$

and Eq. (ii) can be written as

$$\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + s(\hat{i} + 2\hat{j} - 2\hat{k}) \quad \text{...(v)}$$

From Eqs. (iii), (iv) and (v), we get

$$\vec{a_1} = \hat{i} - 2\hat{j} + 3\hat{k},\ \vec{b_1} = -\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{a_2} = \hat{i} - \hat{j} - \hat{k},\ \vec{b_2} = \hat{i} + 2\hat{j} - 2\hat{k}$$

Now, $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$

$$= \hat{i}(-2+4) - \hat{j}(2+2) + \hat{k}(-2-1) \quad \textbf{(1)}$$

$\Rightarrow \quad \vec{b_1} \times \vec{b_2} = 2\hat{i} - 4\hat{j} - 3\hat{k}$

$\therefore \quad |\vec{b_1} \times \vec{b_2}| = \sqrt{(2)^2 + (-4)^2 + (-3)^2}$

$$= \sqrt{4 + 16 + 9} = \sqrt{29}$$

Also, $\vec{a_2} - \vec{a_1} = (\hat{i} - \hat{j} - \hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k})$

$$= \hat{j} - 4\hat{k} \quad \textbf{(1)}$$

We know that, the shortest distance between the lines is given as

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

Hence, required shortest distance,

$$d = \left| \frac{(2\hat{i} - 4\hat{j} - 3\hat{k}) \cdot (\hat{j} - 4\hat{k})}{\sqrt{29}} \right| = \left| \frac{0 - 4 + 12}{\sqrt{29}} \right| = \frac{8}{\sqrt{29}}$$

$\therefore \quad d = \dfrac{8\sqrt{29}}{29}$ units **(1)**

50. Do same as Q. No. 37. $\left[\textbf{Ans. } \dfrac{3\sqrt{2}}{2} \text{ units}\right]$

51. Firstly, determine any point P on the given line and DR's between given point Q and P, using the relation $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$, where (a_1, b_1, c_1) and (a_2, b_2, c_2) are DR's of PQ and given line.

Given equation of line AB is

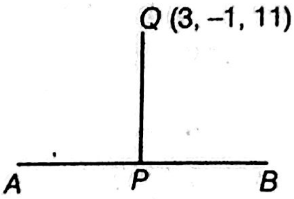
$$\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda \text{ (say)}$$

$\Rightarrow \quad x = 2\lambda,\ y = 3\lambda + 2$

and $\quad z = 4\lambda + 3$ **(1)**

$\therefore$ Any point P on the given line

$$= (2\lambda, 3\lambda + 2, 4\lambda + 3)$$



Let P be the foot of perpendicular drawn from point $Q(3, -1, 11)$ on line AB. Now, DR's of line

$$QP = (2\lambda - 3, 3\lambda + 2 + 1, 4\lambda + 3 - 11) \quad \textbf{(1)}$$

$$= (2\lambda - 3, 3\lambda + 3, 4\lambda - 8)$$

Here, $a_1 = 2\lambda - 3,\ b_1 = 3\lambda + 3,\ c_1 = 4\lambda - 8,$

and $a_2 = 2,\ b_2 = 3,\ c_2 = 4$

Since, $QP \perp AB$

$\therefore$ We have, $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$ ...(i)

$\Rightarrow \quad 2(2\lambda - 3) + 3(3\lambda + 3) + 4(4\lambda - 8) = 0$

$\Rightarrow \quad 4\lambda - 6 + 9\lambda + 9 + 16\lambda - 32 = 0$

$\Rightarrow \quad 29\lambda - 29 = 0 \Rightarrow 29\lambda = 29 \Rightarrow \lambda = 1$ **(1)**

$\therefore$ Foot of perpendicular $P = (2, 3 + 2, 4 + 3)$

$$= (2, 5, 7)$$

Now, equation of perpendicular QP, where $Q(3, -1, 11)$ and $P(2, 5, 7)$, is

$$\frac{x-3}{2-3} = \frac{y+1}{5+1} = \frac{z-11}{7-11}$$

$\left[\text{using two points form of equation of line, i.e. } \dfrac{x - x_1}{x_2 - x_1} = \dfrac{y - y_1}{y_2 - y_1} = \dfrac{z - z_1}{z_2 - z_1}\right]$

$\Rightarrow \quad \dfrac{x-3}{-1} = \dfrac{y+1}{6} = \dfrac{z-11}{-4}$

Now, length of perpendicular QP = distance between points $Q(3, -1, 11)$ and $P(2, 5, 7)$

$$= \sqrt{(2-3)^2 + (5+1)^2 + (7-11)^2}$$

$$\left[\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right]$$

$$= \sqrt{1 + 36 + 16} = \sqrt{53}$$

Hence, length of perpendicular is $\sqrt{53}$. **(1)**

52. Do same as Q. No. 51.

[**Ans.** Length of perpendicular is $\sqrt{24}$.
Coordinates of foot of perpendicular = $(3, -4, -2)$
$\therefore$ Equation of perpendicular is $\frac{x-1}{1} = \frac{y}{-2} = \frac{z}{-1}$.]

53. Given equation of line is

$$\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \lambda \text{ (say)}$$

$$\Rightarrow \quad x = 3\lambda - 2, y = 2\lambda - 1, z = 2\lambda + 3$$

So, we have a point on the line is

$$Q(3\lambda - 2, 2\lambda - 1, 2\lambda + 3) \quad \text{...(i)} \quad \textbf{(1)}$$

Now, given that distance between two points $P(1, 3, 3)$ and $Q(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$ is 5 units, i.e. $PQ = 5$

$$\Rightarrow \sqrt{\left[(3\lambda - 2 - 1)^2 + (2\lambda - 1 - 3)^2 + (2\lambda + 3 - 3)^2\right]} = 5$$

$$\left[\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right]$$

$$\Rightarrow \sqrt{(3\lambda - 3)^2 + (2\lambda - 4)^2 + (2\lambda)^2} = 5 \quad \textbf{(1)}$$

On squaring both sides, we get

$$(3\lambda - 3)^2 + (2\lambda - 4)^2 + (2\lambda)^2 = 25$$

$$\Rightarrow \quad 9\lambda^2 + 9 - 18\lambda + 4\lambda^2 + 16 - 16\lambda + 4\lambda^2 = 25$$

$$\Rightarrow \quad 17\lambda^2 - 34\lambda = 0 \Rightarrow 17\lambda(\lambda - 2) = 0 \quad \textbf{(1)}$$

$$\Rightarrow \text{Either } 17\lambda = 0 \text{ or } \lambda - 2 = 0$$

$$\therefore \quad \lambda = 0 \text{ or } 2$$

On putting $\lambda = 0$ and $\lambda = 2$ in Eq. (i), we get the required point as $(-2, -1, 3)$ or $(4, 3, 7)$. **(1)**

54. (i) Do same as Q. No. 29.

[**Ans.** Foot of perpendicular is $(1, 6, 0)$], Image of P is $(-3, 8, -2)$]

(ii) Length of perpendicular

$$= \sqrt{(5-1)^2 + (4-6)^2 + (2-0)^2}$$

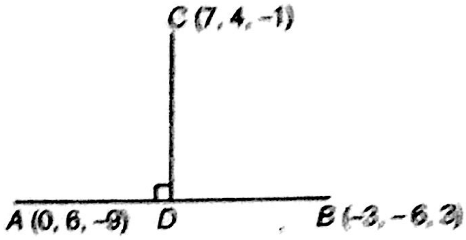
$$= \sqrt{4^2 + 2^2 + 2^2}$$

$$= \sqrt{24} = 2\sqrt{6} \text{ units} \quad \textbf{(1)}$$

55. We know that, equation of line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \quad \text{...(i)} \quad \textbf{(1)}$$

Here, $A(x_1, y_1, z_1) = (0, 6, -9)$
and $B(x_2, y_2, z_2) = (-3, -6, 3)$



$\therefore$ Equation of line AB is given by

$$\frac{x-0}{-3-0} = \frac{y-6}{-6-6} = \frac{z+9}{3+9}$$

$$\Rightarrow \quad \frac{x}{-3} = \frac{y-6}{-12} = \frac{z+9}{12}$$

$$\Rightarrow \quad \frac{x}{-1} = \frac{y-6}{-4} = \frac{z+9}{4} \quad \textbf{(1)}$$

[dividing denominator by 3]

Next, we have to find coordinates of foot of perpendicular D.

Now, let $\frac{x}{-1} = \frac{y-6}{-4} = \frac{z+9}{4} = \lambda$ (say)

$$\Rightarrow \quad x = -\lambda,$$

$$y - 6 = -4\lambda \text{ and } z + 9 = 4\lambda$$

$$\Rightarrow \quad x = -\lambda, y = -4\lambda + 6 \text{ and } z = 4\lambda - 9 \quad \textbf{(1)}$$

Let coordinates of

$$D = (-\lambda, -4\lambda + 6, 4\lambda - 9) \quad \text{...(ii)}$$

Now, DR's of line CD are

$$(-\lambda - 7, -4\lambda + 6 - 4, 4\lambda - 9 + 1)$$

$$= (-\lambda - 7, -4\lambda + 2, 4\lambda - 8)$$

Now, $CD \perp AB$

$$\therefore \quad a_1a_2 + b_1b_2 + c_1c_2 = 0 \quad \textbf{(1)}$$

where, $a_1 = -\lambda - 7, b_1 = -4\lambda + 2$
$c_1 = 4\lambda - 8$ [DR's of line CD]

and $a_2 = -1, b_2 = -4, c_2 = 4$ [DR's of line AB]

$$\Rightarrow (-\lambda - 7)(-1) + (-4\lambda + 2)(-4) + (4\lambda - 8)4 = 0$$

$$\Rightarrow \quad \lambda + 7 + 16\lambda - 8 + 16\lambda - 32 = 0$$

$$\Rightarrow \quad 33\lambda - 33 = 0$$

$$\Rightarrow \quad 33\lambda = 33$$

$$\therefore \quad \lambda = 1 \quad \textbf{(1)}$$

On putting $\lambda = 1$ in Eq. (ii), we get required foot of perpendicular,

$$D = (-1, 2, -5)$$

Also, we have to find equation of line CD, where $C(7, 4, -1)$ and $D(-1, 2, -5)$.

$\therefore$ Required equation of line is

$$\frac{x-7}{-1-7} = \frac{y-4}{2-4} = \frac{z+1}{-5+1} \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \frac{x-7}{-8} = \frac{y-4}{-2} = \frac{z+1}{-4}$$

$$\Rightarrow \frac{x-7}{4} = \frac{y-4}{1} = \frac{z+1}{2} \quad (1)$$

[dividing denominator by −2]

56. Firstly, find the coordinates of foot of perpendicular Q. Then, find the image which is point T by using the fact that Q is the mid-point of line PT. Further, use the formula for equation of line $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$ and distance between two points $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$.

Let T be the image of the point $P(1, 6, 3)$. Q is the foot of perpendicular drawn from the point P on the line AB.

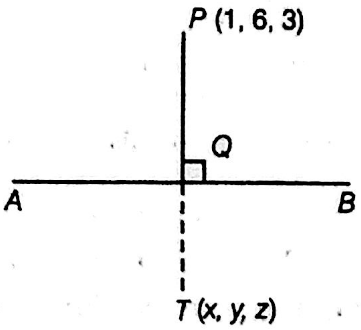
Given equation of line AB is

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} \quad \text{...(i)}$$

Let $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$ (say)

$\Rightarrow x = \lambda, y - 1 = 2\lambda, z - 2 = 3\lambda$

$\Rightarrow x = \lambda, y = 2\lambda + 1, z = 3\lambda + 2$



Then, coordinates of Q

$$= (\lambda, 2\lambda + 1, 3\lambda + 2) \quad \text{...(ii)} \ (1)$$

Now, DR's of line PQ

$$= (\lambda - 1, 2\lambda + 1 - 6, 3\lambda + 2 - 3)$$

$$= (\lambda - 1, 2\lambda - 5, 3\lambda - 1)$$

Since, line $PQ \perp AB$.

Therefore, $a_1a_2 + b_1b_2 + c_1c_2 = 0$,

where $a_1 = \lambda - 1, b_1 = 2\lambda - 5, c_1 = 3\lambda - 1$

and $a_2 = 1, b_2 = 2, c_2 = 3$

$\therefore (\lambda - 1)1 + (2\lambda - 5)2 + (3\lambda - 1)3 = 0$

$\Rightarrow \lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$

$\Rightarrow 14\lambda - 14 = 0$

$\Rightarrow \lambda = 1$ (1)

On putting $\lambda = 1$ in Eq. (ii), we get

$$Q = (1, 2 + 1, 3 + 2) = (1, 3, 5)$$

Let image of a point P be $T(x, y, z)$. Then, Q will be the mid-point of PT.

By using mid-point formula, (1)

Q = mid-point of $P(1, 6, 3)$ and $T(x, y, z)$

$$= \left(\frac{x+1}{2}, \frac{y+6}{2}, \frac{z+3}{2}\right)$$

$$\left[\because \text{mid-point} = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right]$$

But $Q = (1, 3, 5)$

$$\therefore \left(\frac{x+1}{2}, \frac{y+6}{2}, \frac{z+3}{2}\right) = (1, 3, 5)$$

On equating corresponding coordinates, we get

$$\frac{x+1}{2} = 1, \frac{y+6}{2} = 3, \frac{z+3}{2} = 5$$

$\Rightarrow x = 2 - 1, y = 6 - 6, z = 10 - 3$

$\Rightarrow x = 1, y = 0, z = 7$

$\therefore$ Coordinates of $T = (x, y, z) = (1, 0, 7)$

Hence, coordinates of image of point $P(1, 6, 3)$ is $T(1, 0, 7)$. (1)

Now, equation of line joining points $P(1, 6, 3)$ and $T(1, 0, 7)$ is

$$\frac{x-1}{1-1} = \frac{y-6}{0-6} = \frac{z-3}{7-3} \Rightarrow \frac{x-1}{0} = \frac{y-6}{-6} = \frac{z-3}{4} \quad (1)$$

Also, length of segment PT

$$= \sqrt{(1-1)^2 + (6-0)^2 + (3-7)^2}$$

$$= \sqrt{0 + 36 + 16} = \sqrt{52} \text{ units} \quad (1)$$

57. Given equations of lines are

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

and $$\frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12} \quad (1)$$

Now, the vector equation of given lines are

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \quad \text{...(i)}$$

[$\because$ vector form of equation of line is $\vec{r} = \vec{a} + \lambda\vec{b}$]

and $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu\,(4\hat{i} + 6\hat{j} + 12\hat{k})$ ...(ii)

Here, $\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

and $\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$, $\vec{b_2} = 4\hat{i} + 6\hat{j} + 12\hat{k}$

Now, $\vec{a_2} - \vec{a_1} = (3\hat{i} + 3\hat{j} - 5\hat{k}) - (\hat{i} + 2\hat{j} - 4\hat{k})$

$= 2\hat{i} + \hat{j} - \hat{k}$ ...(iii) (1)

and $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 4 & 6 & 12 \end{vmatrix}$

$= \hat{i}\,(36 - 36) - \hat{j}\,(24 - 24) + \hat{k}\,(12 - 12)$

$= 0\hat{i} - 0\hat{j} + 0\hat{k} = \vec{0}$ (1)

$\Rightarrow \vec{b_1} \times \vec{b_2} = \vec{0}$,

i.e. Vector $\vec{b_1}$ is parallel to $\vec{b_2}$.

[$\because$ if $\vec{a} \times \vec{b} = \vec{0}$, then $\vec{a} \parallel \vec{b}$]

Thus, two lines are parallel.

$\therefore \vec{b} = (2\hat{i} + 3\hat{j} + 6\hat{k})$ ...(iv)

[since, DR's of given lines are proportional] (1)

Since, the two lines are parallel, we use the formula for shortest distance between two parallel lines

$$d = \left| \frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|} \right|$$

$$\Rightarrow d = \left| \frac{(2\hat{i} + 3\hat{j} + 6\hat{k}) \times (2\hat{i} + \hat{j} - \hat{k})}{\sqrt{(2)^2 + (3)^2 + (6)^2}} \right| \quad \text{...(v)}$$

[from Eqs. (iii) and (iv)]

Now, $(2\hat{i} + 3\hat{j} + 6\hat{k}) \times (2\hat{i} + \hat{j} - \hat{k})$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}$

$= \hat{i}\,(-3 - 6) - \hat{j}\,(-2 - 12) + \hat{k}\,(2 - 6)$

$= -9\hat{i} + 14\hat{j} - 4\hat{k}$ (1)

From Eq. (v), we get

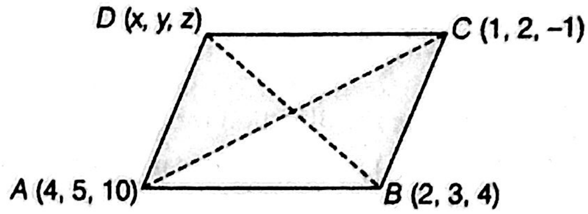
$$d = \left| \frac{-9\hat{i} + 14\hat{j} - 4\hat{k}}{\sqrt{49}} \right| = \frac{\sqrt{(-9)^2 + (14)^2 + (-4)^2}}{7}$$

$$\therefore d = \frac{\sqrt{81 + 196 + 16}}{7} = \frac{\sqrt{293}}{7} \text{ units} \quad (1)$$

58.

> The vector equation of a side of a parallelogram, when two points are given, is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$. Also, the diagonals of a parallelogram intersect each other at mid-point.

Given points are $A\,(4, 5, 10)$, $B\,(2, 3, 4)$ and $C(1, 2, -1)$.



We know that, two points vector form of line is given by $\vec{r} = \vec{a} + \lambda\,(\vec{b} - \vec{a})$ ...(i) (1)

where, $\vec{a}$ and $\vec{b}$ are the position vector of points through which the line is passing through. Here, for line AB position vectors are

$\vec{a} = \vec{OA} = 4\hat{i} + 5\hat{j} + 10\hat{k}$

and $\vec{b} = \vec{OB} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ (1)

Using Eq. (i), the required equation of line AB is

$\vec{r} = (4\hat{i} + 5\hat{j} + 10\hat{k}) + \lambda\,[(2\hat{i} + 3\hat{j} + 4\hat{k}) - (4\hat{i} + 5\hat{j} + 10\hat{k})]$

$\Rightarrow \vec{r} = (4\hat{i} + 5\hat{j} + 10\hat{k}) + \lambda\,(-2\hat{i} - 2\hat{j} - 6\hat{k})$ (1)

Similarly, vector equation of line BC, where $B(2, 3, 4)$ and $C\,(1, 2, -1)$ is

$\vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + \mu\,[(\hat{i} + 2\hat{j} - \hat{k}) - (2\hat{i} + 3\hat{j} + 4\hat{k})]$

$\Rightarrow \vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + \mu\,(-\hat{i} - \hat{j} - 5\hat{k})$ (1)

We know that, mid-point of diagonal BD = Mid-point of diagonal AC

[$\because$ diagonal of a parallelogram bisect each other]

$$\therefore \left(\frac{x+2}{2}, \frac{y+3}{2}, \frac{z+4}{2}\right) = \left(\frac{4+1}{2}, \frac{5+2}{2}, \frac{10-1}{2}\right) \quad (1)$$

On comparing corresponding coordinates, we get

$\frac{x+2}{2} = \frac{5}{2}$, $\frac{y+3}{2} = \frac{7}{2}$ and $\frac{z+4}{2} = \frac{9}{2}$

$\Rightarrow x = 3, y = 4$ and $z = 5$

Hence, coordinates of point $D(x, y, z)$ is $(3, 4, 5)$ and vector equations of sides AB and BC are

$$\vec{r} = (4\hat{i} + 5\hat{j} + 10\hat{k}) + \lambda(-2\hat{i} - 2\hat{j} - 6\hat{k}) \text{ and}$$

$$\vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + \mu(-\hat{i} - 3\hat{j} - 5\hat{k}),$$

respectively. **(1)**

59. Any line through $(1, 2, -4)$ can be written as

$$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c} \quad \ldots\text{(i)}$$ **(1)**

where a, b, c are the direction ratios of line (i).

Now, the line (i) be perpendicular to the lines

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$$ **(1)**

and $$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$ **(1)**

The direction ratios of the above lines are $(3, -16, 7)$ and $(3, 8, -5)$, respectively which are perpendicular to the Eq. (i).

$$\therefore \quad 3a - 16b + 7c = 0 \quad \ldots\text{(ii)}$$

and $$3a + 8b - 5c = 0 \quad \ldots\text{(iii)}$$

By cross-multiplication, we get

$$\frac{a}{80-56} = \frac{b}{21+15} = \frac{c}{24+48}$$ **(1)**

$$\Rightarrow \quad \frac{a}{24} = \frac{b}{36} = \frac{c}{72}$$

$$\Rightarrow \quad \frac{a}{2} = \frac{b}{3} = \frac{c}{6} = \lambda \text{ (say)}$$

$$\Rightarrow \quad a = 2\lambda, b = 3\lambda, c = 6\lambda$$ **(1)**

The equation of required line in cartesian form is

$$\frac{x-1}{2\lambda} = \frac{y-2}{3\lambda} = \frac{z+4}{6\lambda}$$

or $$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

and in vector form is

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}).$$ **(1)**

Solutions

1. Given, $2x - y + 2z = 5$ ...(i)

$\Rightarrow \quad 2x - y + 2z - 5 = 0$

and $\quad 5x - 2.5y + 5z = 20$

$\Rightarrow \quad 5[x - 0.5y + z] = 20$

$\Rightarrow \quad x - \frac{1}{2}y + z = 4$

$\Rightarrow \quad 2x - y + 2z = 8$

$\Rightarrow \quad 2x - y + 2z - 8 = 0$ ...(ii) **(1/2)**

Clearly, planes (i) and (ii) are parallel.

$\therefore$ Distance between two parallel planes,

$$d = \left|\frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}}\right| = \left|\frac{-8-(-5)}{\sqrt{(2)^2 + (-1)^2 + 2^2}}\right|$$

$[\because d_2 = -8, d_1 = -5, a = 2, b = -1 \text{ and } c = 2]$

$$= \left|\frac{-8+5}{\sqrt{4+1+4}}\right| = \left|\frac{-3}{\sqrt{9}}\right| = |-1| = 1$$ **(1/2)**

2. Given, $\vec{n} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $d = 5$

We know that, equation of a plane having distance d from origin and normal vector $\hat{n}$ is

$$\vec{r}\cdot\hat{n} = d$$

$\therefore \quad \vec{r}\cdot\frac{(2\hat{i} - 3\hat{j} + 6\hat{k})}{\sqrt{2^2 + (-3)^2 + (6)^2}} = 5$

$\Rightarrow \quad \vec{r}\cdot(2\hat{i} - 3\hat{j} + 6\hat{k}) = 5\sqrt{49}$

$\Rightarrow \quad \vec{r}\cdot(2\hat{i} - 3\hat{j} + 6\hat{k}) = 35$ **[1]**

3. Given, equation of plane is

$$\vec{r}\cdot(2\hat{i} + \hat{j} - \hat{k}) = 5$$

Put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, we get

$(x\hat{i} + y\hat{j} + z\hat{k})\cdot(2\hat{i} + \hat{j} - \hat{k}) = 5$

$\Rightarrow \quad 2x + y - z = 5$

$\Rightarrow \quad \frac{x}{\frac{5}{2}} + \frac{y}{5} + \frac{z}{-5} = 1$ ...(i) **[1/2]**

On comparing plane (i) with standard equation of plane in intercept form

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, we get

$a = \frac{5}{2}, b = 5 \text{ and } c = -5$

Now, required sum of intercepts cut off by the plane on the three axes $= a + b + c$

$= \frac{5}{2} + 5 - 5 = \frac{5}{2}$ units **(1/2)**

4. The equation of plane having intercepts 3, -4 and 2 is $\quad \frac{x}{3} + \frac{y}{-4} + \frac{z}{2} = 1$

$\Rightarrow 4x - 3y + 6z = 12$, which can be written as

$(x\hat{i} + y\hat{j} + z\hat{k})\cdot(4\hat{i} - 3\hat{j} + 6\hat{k}) = 12$

$\Rightarrow \quad \vec{r}\cdot(4\hat{i} - 3\hat{j} + 6\hat{k}) = 12$

which is the required vector equation of the given. **(1)**

5. Since, the normal to the plane is equally inclined with coordinates axes, therefore its direction cosines are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}}$. **(1/2)**

Now, the required equation of plane is

$$\frac{1}{\sqrt{3}}\cdot x+\frac{1}{\sqrt{3}}\cdot y+\frac{1}{\sqrt{3}}\cdot z=5\sqrt{3}$$

$$\Rightarrow \quad x+y+z=15 \quad \textbf{(1/2)}$$

[$\because$ If l, m and n are DC's of normal to the plane and P is a distance of a plane from origin, then equation of plane is given by $lx+my+nz=p$]

6. Do same as Q. No. 3. $\left[\textbf{Ans. } \frac{5}{2} \text{ units}\right]$

7. Clearly, the vector equation of the line passing through a point with position vector

$\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, is given by $\vec{r}=\vec{a}+\lambda\vec{b}$

$$\Rightarrow \quad \vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda\vec{b} \quad \textbf{(1/2)}$$

Since, the line is perpendicular to the plane

$$\vec{r}\cdot(\hat{i}+2\hat{j}-5\hat{k})+9=0$$

Therefore, $\vec{b}$ will be normal to the plane and so $\vec{b}=\hat{i}+2\hat{j}-5\hat{k}$

Hence, the required equation of line is

$$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}+2\hat{j}-5\hat{k}) \quad \textbf{(1/2)}$$

8. The required plane is passing through the point (a, b, c) whose position vector is $\vec{p}=a\hat{i}+b\hat{j}+c\hat{k}$ and is parallel to the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2$

So, it is normal to the vector

$$\vec{n}=\hat{i}+\hat{j}+\hat{k}$$

Hence, required equation of plane is

$$(\vec{r}-\vec{p})\cdot\vec{n}=0 \Rightarrow \vec{r}\cdot\vec{n}=\vec{p}\cdot\vec{n}$$

$$\Rightarrow \quad \vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=(a\hat{i}+b\hat{j}+c\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})$$

$$\therefore \quad \vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=a+b+c \quad \textbf{(1)}$$

9. The distance from point (x_1, y_1, z_1) to the plane $Ax+By+Cz+D=0$ is $\frac{Ax_1+By_1+Cz_1+D}{\sqrt{A^2+B^2+C^2}}$.

Given equation of plane is

$$2x-3y+6z+21=0 \quad \ldots(i)$$

$\therefore$ Length of the perpendicular drawn from the origin to this plane

$$=\left|\frac{2\cdot 0-3\cdot 0+6\cdot 0+21}{\sqrt{(2)^2+(-3)^2+(6)^2}}\right|$$

$$=\left|\frac{0-0+0+21}{\sqrt{4+9+36}}\right|=\frac{21}{\sqrt{49}}=\frac{21}{7}=3 \text{ units} \quad \textbf{(1)}$$

10. Do same as Q.No. 9. $\left[\textbf{Ans. } \frac{3}{13} \text{ unit}\right]$

11. Firstly, we convert the given equation of plane in intercept form, i.e. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, which cut the X-axis at $(a, 0, 0)$.

Given equation of plane is $2x+y-z=5$.

On dividing both sides by 5, we get

$$\frac{2x}{5}+\frac{y}{5}-\frac{z}{5}=1 \Rightarrow \frac{x}{\left(\frac{5}{2}\right)}+\frac{y}{5}+\frac{z}{(-5)}=1$$

On comparing above equation of plane with the intercept form of equation of plane

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

where, a = x-intercept, b = y-intercept and c = z-intercept

we get, $\quad a=\frac{5}{2}$

i.e. intercept cut-off on X-axis $=\frac{5}{2}$ units **(1)**

12. Do same as Q. No. 9. $\left[\textbf{Ans. } \frac{1}{3} \text{ unit}\right]$

13. Given, line $\frac{x-2}{6}=\frac{y-1}{\lambda}=\frac{z+5}{-4}$ is perpendicular to plane $3x-y-2z=7$.

Therefore, DR's of the line are proportional to the DR's normal to the plane.

$$\therefore \quad \frac{6}{3}=\frac{\lambda}{-1}=\frac{-4}{-2}$$

$$\Rightarrow \quad 2=-\lambda \quad \Rightarrow \quad \lambda=-2 \quad \textbf{(1)}$$

14. The given points are $A(2, 5, -3)$, $B(-2, -3, 5)$ and $C(5, 3, -3)$.

Let $\vec{a}=2\hat{i}+5\hat{j}-3\hat{k}$, $\vec{b}=-2\hat{i}-3\hat{j}+5\hat{k}$ and $\vec{c}=5\hat{i}+3\hat{j}-3\hat{k}$

Now, the vector equation of the plane passing through $\vec{a}, \vec{b}$ and $\vec{c}$ is given by

$$(\vec{r}-\vec{a})\cdot(\vec{AB}\times\vec{AC})=0$$

$$\Rightarrow \quad (\vec{r}-\vec{a})\cdot\{(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})\}=0 \quad \textbf{(1)}$$

Here, $\vec{b} - \vec{a} = -2\hat{i} - 3\hat{j} + 5\hat{k} - 2\hat{i} - 5\hat{j} + 3\hat{k}$

$= -4\hat{i} - 8\hat{j} + 8\hat{k}$

and $\vec{c} - \vec{a} = 5\hat{i} + 3\hat{j} - 3\hat{k} - 2\hat{i} - 5\hat{j} + 3\hat{k} = 3\hat{i} - 2\hat{j}$

$\therefore \quad \{\vec{r} - (2\hat{i} + 5\hat{j} - 3\hat{k})\}$

$\cdot[(-4\hat{i} - 8\hat{j} + 8\hat{k}) \times (3\hat{i} - 2\hat{j})] = 0$ **(1)**

Now, $(-4\hat{i} - 8\hat{j} + 8\hat{k}) \times (3\hat{i} - 2\hat{j}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix}$

$= \hat{i}(0 + 16) - \hat{j}(0 - 24) + \hat{k}(8 + 24)$

$= 16\hat{i} + 24\hat{j} + 32\hat{k}$

$\therefore \{\vec{r} - (2\hat{i} + 5\hat{j} - 3\hat{k})\} \cdot (16\hat{i} + 24\hat{j} + 32\hat{k}) = 0$ ...(i)

This is the required vector equation of plane. **(1)**

For cartesian equation put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in Eq. (i), we get

$\Rightarrow \quad (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (16\hat{i} + 24\hat{i} + 32\hat{k})$

$= (2\hat{i} + 5\hat{j} - 3\hat{k}) \cdot (16\hat{i} + 24\hat{j} + 32\hat{k})$

$\Rightarrow \quad 16x + 24y + 32z = 32 + 120 - 96$

$\Rightarrow \quad 16x + 24y + 32z = 56$

$\Rightarrow \quad 2x + 3y + 4z = 7$

which is the required cartesian equation of plane. **(1)**

15. We have, $\dfrac{x-2}{3} = \dfrac{y+1}{4} = \dfrac{z-2}{12}$ ...(i)

and $\quad x - y + z = 5$ ... (ii)

Let $\quad \dfrac{x-2}{3} = \dfrac{y+1}{4} = \dfrac{z-2}{12} = \lambda$ (say)

$\Rightarrow \quad x = 3\lambda + 2,\ y = 4\lambda - 1,\ z = 12\lambda + 2$ **(1)**

Any point on the line is

$P(3\lambda + 2, 4\lambda - 1, 12\lambda + 2)$ ...(iii)

Since, lines and plane intersect, so point P satisfy the plane.

$\therefore \quad (3\lambda + 2) - (4\lambda - 1) + (12\lambda + 2) = 5$

[put coordinates of P in Eq. (ii)]

$\Rightarrow 3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 = 5$

$\Rightarrow \quad 11\lambda = 0 \Rightarrow \lambda = 0$ **(1)**

Put $\lambda = 0$ in Eq. (iii), we get point of intersection $P(2, -1, 2)$. **(1)**

Now, distance between points $(-1, -5, -10)$ and $(2, -1, 2)$ is given by

$\sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2}$

$\left[\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right]$

$= \sqrt{9 + 16 + 144} = \sqrt{169} = 13$ units **(1)**

16. Let the equation of plane passing through the point $(-1, 2, 0)$ is

$a(x + 1) + b(y - 2) + c(z - 0) = 0$ ...(i)

where, a, b and c are direction ratios of normal to the plane. **(1/2)**

Since, the plane passes through $(2, 2, -1)$, therefore we have

$a(2 + 1) + b(2 - 2) + c(-1 - 0) = 0$

$\Rightarrow \quad 3a + 0 \cdot b - c = 0$ ...(ii) **(1/2)**

Also, the plane is parallel to the line

$\dfrac{x-1}{1} = \dfrac{2y+1}{2} = \dfrac{z+1}{-1}$

or $\quad \dfrac{x-1}{1} = \dfrac{y+\frac{1}{2}}{1} = \dfrac{z+1}{-1}$ **(1)**

Therefore, we have

$a + b - c = 0$ ...(iii)

[$\because$ the normal to the plane will be perpendicular to the line,

i.e. $a_1a_2 + b_1b_2 + c_1c_2 = 0$]

On solving Eqs. (ii) and (iii), we get

$\dfrac{a}{0+1} = \dfrac{b}{-1+3} = \dfrac{c}{3-0}$

$\Rightarrow \quad \dfrac{a}{1} = \dfrac{b}{2} = \dfrac{c}{3} = \lambda$ (say) **(1)**

On substituting the values of a, b and c in Eq. (i), we get the required equation of plane is

$1(x + 1) + 2(y - 2) + 3z = 0$

$\Rightarrow \quad x + 1 + 2y - 4 + 3z = 0$

$\therefore \quad x + 2y + 3z = 3$ **(1)**

17. The equation of a plane passing through a point $(3, 2, 0)$ is

$a(x - 3) + b(y - 2) + c(z - 0) = 0$ ...(i) **(1)**

Since, the above plane contains the line

$\dfrac{x-3}{1} = \dfrac{y-6}{5} = \dfrac{z-4}{4}$

So, the point $(3, 6, 4)$ satisfies the equation of plane (i).

$\therefore \quad a(3 - 3) + b(6 - 2) + c(4 - 0) = 0$

$\Rightarrow \quad 0 + 4b + 4c = 0$

$\Rightarrow \quad b + c = 0$

$\Rightarrow \quad b = -c$ ...(ii) **(1)**

As the plane contains the line, therefore normal to the plane is perpendicular to the line, i.e. $a_1a_2 + b_1b_2 + c_1c_2 = 0$, where, $a_1 = a, b_1 = b, c_1 = c$ and $a_2 = 1, b_2 = 5$ and $c_2 = 4$

$\therefore \quad a \times 1 + b \times 5 + c \times 4 = 0$

$\Rightarrow \quad a + 5b + 4c = 0$

$\Rightarrow \quad a - 5c + 4c = 0$ [from Eq.(ii)]

$\Rightarrow \quad a = c$ **(1)**

On putting the values of a, b and c in Eq.(i), we get

$$c(x-3) - c(y-2) + c(z-0) = 0$$

$\Rightarrow \quad 1(x-3) - 1(y-2) + 1(z-0) = 0$ [divide by c]

$\therefore \quad x - y + z = 1$ **(1)**

18. Given, intercepts on the coordinate axes are $(-6, 3, 4)$, then equation of plane will be

$$\frac{x}{-6} + \frac{y}{3} + \frac{z}{4} = 1 \text{ or } \frac{x}{-6} + \frac{y}{3} + \frac{z}{4} - 1 = 0$$ **(1)**

We know that, the distance of a point (x_1, y_1, z_1) from plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|.$$ **(1)**

$\therefore$ The distance of origin from given plane

$$= \left|\frac{\left(\frac{-1}{6}\right)\cdot 0 + \left(\frac{1}{3}\right)\cdot 0 + \left(\frac{1}{4}\right)\cdot 0 - 1}{\sqrt{\left(\frac{-1}{6}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{4}\right)^2}}\right|$$

$$= \left|\frac{-1}{\sqrt{\frac{1}{36} + \frac{1}{9} + \frac{1}{16}}}\right| = \left|\frac{-1}{\sqrt{\frac{4+16+9}{144}}}\right| = \left|\frac{-1}{\sqrt{\frac{29}{144}}}\right| = \frac{12}{\sqrt{29}}$$ **(1)**

Hence, required length of the perpendicular from origin to plane is $\frac{12}{\sqrt{29}}$ units. **(1)**

19. Given lines can be written as

$$\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$$

and $$\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$$ **(1)**

On comparing both lines with,

$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{b}$ respectively, we get

$x_1 = 5, y_1 = 7, z_1 = -3, a_1 = 4, b_1 = 4, c_1 = -5$ and

$x_2 = 8, y_2 = 4, z_2 = 5, a_2 = 7, b_2 = 1, c_2 = 3$ **(1)**

If given lines are coplanar, then

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$ **(1)**

$$\text{LHS} = \begin{vmatrix} 8-5 & 4-7 & 5+3 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 3 & -3 & 8 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$= 3(12 + 5) + 3(12 + 35) + 8(4 - 28)$

$= 3 \times 17 + 3 \times 47 + 8(-24)$

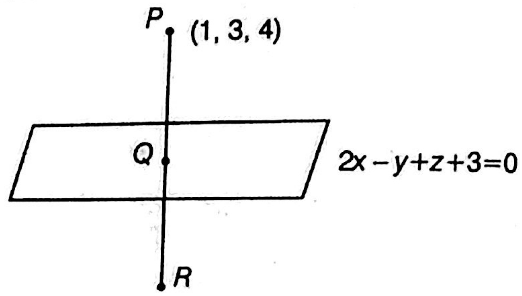
$= 51 + 141 - 192 = 192 - 192 = 0 = \text{RHS}$

Therefore, given lines are coplanar.

Hence proved (1)

20. Given, position vector of point is $(\hat{i} + 3\hat{j} + 4\hat{k})$. So, coordinates of point P are $(1, 3, 4)$ and vector equation of plane is $\vec{r}\cdot(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$, then cartesian equation of plane is $2x - y + z + 3 = 0$.

Let Q be the foot of perpendicular from P on the plane.



(1)

Since, PQ is perpendicular to the plane. Hence, DR's of PQ will be $(2, -1, 1)$.

So, the equation of PQ will be

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda \text{ (say)}$$

Coordinates of $Q = (2\lambda + 1, -\lambda + 3, \lambda + 4)$, also Q lies on plane, so it will satisfy the equation of plane.

$\therefore \quad 2(2\lambda + 1) - 1(-\lambda + 3) + (\lambda + 4) + 3 = 0$

$\Rightarrow \quad 4\lambda + 2 + \lambda - 3 + \lambda + 4 + 3 = 0$

$\Rightarrow \quad 6\lambda = -6 \Rightarrow \lambda = -1$ **(1)**

So, coordinates of Q will be $(-2+1, 1+3, -1+4)$, i.e. $(-1, 4, 3)$.

Let $R(x, y, z)$ be the image of a point P, then point Q will be the mid-point of PR.

Therefore, coordinates of Q will be

$$\left(\frac{x+1}{2}, \frac{y+3}{2}, \frac{z+4}{2}\right) \equiv (-1, 4, 3)$$ **(1)**

On comparing corresponding coordinates, we get

$\frac{x+1}{2} = -1 \Rightarrow x + 1 = -2 \Rightarrow x = -3$,

$\frac{y+3}{2} = 4 \Rightarrow y + 3 = 8 \Rightarrow y = 5$

and $\frac{z+4}{2} = 3 \Rightarrow z + 4 = 6 \Rightarrow z = 2$

Hence, required coordinates of image point R is $(-3, 5, 2)$.

(1)

21. The required plane passes through two points $P(2,1,-1)$ and $Q(-1,3,4)$.

Let $\vec{a}$ and $\vec{b}$ be the position vectors of points P and Q, respectively.

Then, $\vec{a}=2\hat{i}+\hat{j}-\hat{k}$

and $\vec{b}=-\hat{i}+3\hat{j}+4\hat{k}$

Now, $\overrightarrow{PQ}=\vec{b}-\vec{a}=(-\hat{i}+3\hat{j}+4\hat{k})-(2\hat{i}+\hat{j}-\hat{k})$
$=-3\hat{i}+2\hat{j}+5\hat{k}$ **(1)**

Let $\vec{n}_1$ be the normal vector to the given plane, $x-2y+4z=10$, then $\vec{n}_1=\hat{i}-2\hat{j}+4\hat{k}$.

Let $\vec{n}$ be the normal vector to the required plane. Then,

$$\vec{n}=\vec{n}_1\times\overrightarrow{PQ}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 4\\ -3 & 2 & 5\end{vmatrix}$$

$$=\hat{i}(-10-8)-\hat{j}(5+12)+\hat{k}(2-6)$$

$$=-18\hat{i}-17\hat{j}-4\hat{k}$$ **(1)**

The required plane passes through a point having position vector $\vec{a}=2\hat{i}+\hat{j}-\hat{k}$ and normal vector $\vec{n}=-18\hat{i}-17\hat{j}-4\hat{k}$. So, its vector equation is

$(\vec{r}-\vec{a})\cdot\vec{n}\Rightarrow\vec{r}\cdot\vec{n}=\vec{a}\cdot\vec{n}$ **(1)**

$\Rightarrow \vec{r}\cdot(-18\hat{i}-17\hat{j}-4\hat{k})=(2\hat{i}+\hat{j}-\hat{k})\cdot(-18\hat{i}-17\hat{j}-4\hat{k})$

$\Rightarrow \vec{r}\cdot(-18\hat{i}-17\hat{j}-4\hat{k})=-36-17+4$

$\therefore \vec{r}\cdot(18\hat{i}+17\hat{j}+4\hat{k})=49$ **(1)**

22. Given equation of line is

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}=\lambda \text{ (say)}$$

$\Rightarrow x=3\lambda+2, y=4\lambda-1, z=2\lambda+2$

Then, $[(3\lambda+2),(4\lambda-1),(2\lambda+2)]$ be any point on the given line. **(1)**

Since, line intersect the plane, therefore any point on the given line $(3\lambda+2, 4\lambda-1, 2\lambda+2)$ lies on the plane $x-y+z-5=0$.

$\therefore (3\lambda+2)-(4\lambda-1)+(2\lambda+2)-5=0$

$\Rightarrow 3\lambda+2-4\lambda+1+2\lambda+2-5=0$

$\Rightarrow \lambda=0$ ...(i) **(1)**

$\therefore$ Point of intersection of the line and the plane

$=(3\times0+2, 4\times0-1, 2\times0+2)=(2,-1,2)$ **(1/2)**

Let ϕ be the angle between line and plane.

Then,
$$\sin\phi=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Here, $a_1=3, b_1=4, c_1=2; a_2=1, b_2=-1, c_2=1$.

$$\therefore \sin\phi=\left|\frac{(3)(1)+4(-1)+2(1)}{\sqrt{9+16+4}\sqrt{1+1+1}}\right|$$

$$\Rightarrow \sin\phi=\left|\frac{3-4+2}{\sqrt{29}\sqrt{3}}\right|=\frac{1}{\sqrt{87}}\Rightarrow\phi=\sin^{-1}\frac{1}{\sqrt{87}}$$ **(1½)**

which is the required angle.

23.

> Firstly, use the intersection equation of planes $\vec{r}\cdot\vec{n}_1=d_1$ and $\vec{r}\cdot\vec{n}_2=d_2$ is $\vec{r}(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$. Further, use the relation $(\vec{n}_1+\lambda\vec{n}_2)\cdot\vec{n}=0$ as above plane is perpendicular to the plane $\vec{r}\cdot\vec{n}=d$.

The intersection equation of planes $\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})=4$ and $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})=-5$ is

$$\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})+\lambda[\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k}]=4-\lambda5$$

[$\because$ intesection of two planes is $\vec{r}_1+\lambda\vec{r}_2=d_1+\lambda d_2$]

$\Rightarrow \vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k}+2\lambda\hat{i}+\lambda\hat{j}-\lambda\hat{k})=4-5\lambda$

$\Rightarrow \vec{r}\cdot[(1+2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3-\lambda)\hat{k}]=4-5\lambda$ ...(i)

Here, $\vec{n}_1=(1+2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3-\lambda)\hat{k}$ **(1)**

Since, the required plane is perpendicular to the plane $\vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k})+8=0$.

$\therefore \vec{n}_2\cdot\vec{n}_1=0$, where $\vec{n}_2=5\hat{i}+3\hat{j}-6\hat{k}$

$\Rightarrow (5\hat{i}+3\hat{j}-6\hat{k})\cdot[(1+2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3-\lambda)\hat{k}]=0$ **(1)**

$\therefore 5(1+2\lambda)+3(2+\lambda)-6(3-\lambda)=0$

$\Rightarrow 5+10\lambda+6+3\lambda-18+6\lambda=0$

$\Rightarrow 19\lambda-7=0\Rightarrow\lambda=\frac{7}{19}$

On putting $\lambda=\frac{7}{19}$ in Eq. (i), we get the equation of plane

$$\vec{r}\cdot\left[\left(1+\frac{14}{19}\right)\hat{i}+\left(2+\frac{7}{19}\right)\hat{j}+\left(3-\frac{7}{19}\right)\hat{k}\right]=4-\frac{35}{19}$$

$$\therefore \vec{r}\cdot\left[\frac{33}{19}\hat{i}+\frac{45}{19}\hat{j}+\frac{50}{19}\hat{k}\right]=\frac{41}{19}$$

$$\Rightarrow \vec{r}=(33\hat{i}+45\hat{j}+50\hat{k})=41$$ **(2)**

24. Let the given points be $A(3, -4, -5)$ and $B(2, -3, 1)$.

The equation of line AB is

$$\frac{x-3}{2-3} = \frac{y+4}{-3+4} = \frac{z+5}{1+5}$$ **(1)**

$$\Rightarrow \quad \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda \text{ (say)}$$

$$\Rightarrow \quad x = -\lambda + 3;\ y = \lambda - 4$$

and $$z = 6\lambda - 5$$

So, any point on line AB is of the form

$$(3-\lambda, \lambda - 4, 6\lambda - 5)$$

Let $P(3-\lambda, \lambda-4, 6\lambda-5)$ be the point where the line crosses the plane $2x + y + z = 7$. **(1/2)**

Clearly, P will satisfy the equation of plane. **(1/2)**

We have,

$$2(3-\lambda) + (\lambda - 4) + (6\lambda - 5) = 7$$

$$\Rightarrow \quad 6 - 2\lambda + \lambda - 4 + 6\lambda - 5 - 7 = 0$$

$$\Rightarrow \quad 5\lambda - 10 = 0 \Rightarrow \lambda = 2$$

Thus, the coordinates of required point are

$(3-2, 2-4, 12-5)$, i.e., $(1, -2, 7)$ **(1)**

25. Do same as Q.No. 24. $\left[\textbf{Ans. } \left(\frac{13}{5}, \frac{23}{5}, 0\right)\right]$

26. Do same as Q.No. 24. **[Ans.** $(5, -6, -17)$**]**

27.

> Firstly, write the required equation of plane as $(x + 3y + 6) + \lambda(3x - y - 4z) = 0$.
> Then, convert the above equation in general form of plane which is $ax + by + c + d = 0$.
> Finally, use the formula for distance from a point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$ is $d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$.

Let the required equation of plane passing through the intersection of planes $x + 3y + 6 = 0$ and $3x - y - 4z = 0$ be

$$(x + 3y + 6) + \lambda(3x - y - 4z) = 0 \quad \text{...(i)}$$

Above equation can be written as

$$x + 3y + 6 + 3\lambda x - \lambda y - 4\lambda z = 0$$

$$\Rightarrow \quad x(1 + 3\lambda) + y(3 - \lambda) - 4\lambda z + 6 = 0 \quad \text{...(ii)}$$ **(1)**

which is the general form of equation of plane.

Also, given that perpendicular distance of plane (i) from origin, i.e. $(0,0,0)$ is unity, i.e. one.

$$\therefore \quad \left|\frac{(1+3\lambda)(0) + (3-\lambda)(0) - 4\lambda(0) + 6}{\sqrt{(1+3\lambda)^2 + (3-\lambda)^2 + (-4\lambda)^2}}\right| = 1$$

[$\because$ distance of point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by

$$d = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

here, $a = 1 + 3\lambda, b = 3 - \lambda, c = -4\lambda$, $(x_1, y_1, z_1) = (0, 0, 0)$] **(1)**

$$\Rightarrow \quad \left|\frac{6}{\sqrt{1 + 9\lambda^2 + 6\lambda + 9 + \lambda^2 - 6\lambda + 16\lambda^2}}\right| = 1$$

$$\Rightarrow \quad \frac{6}{\sqrt{26\lambda^2 + 10}} = 1 \Rightarrow 6 = \sqrt{26\lambda^2 + 10}$$

On squaring both sides, we get

$$36 = 26\lambda^2 + 10$$

$$\Rightarrow \quad 26\lambda^2 = 26 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$$ **(1)**

Now, on putting $\lambda = 1$ in Eq. (i), we get

$$x + 3y + 6 + 3x - y - 4z = 0$$

$$\Rightarrow \quad 4x + 2y - 4z + 6 = 0$$

$$\Rightarrow \quad 2x + y - 2z + 3 = 0$$

[divide by 2]...(iii)

Again, on putting $\lambda = -1$ in Eq. (i), we get

$$x + 3y + 6 - 3x + y + 4z = 0$$

$$\Rightarrow \quad -2x + 4y + 4z + 6 = 0$$

$$\Rightarrow \quad x - 2y - 2z - 3 = 0 \quad \text{[divide by } -2]\text{...(iv)}$$

Hence, required equations of the plane are $2x + y - 2z + 3 = 0$ and $x - 2y - 2z - 3 = 0$. **(1)**

28. Equation of plane passing through the point $A(1, 2, 1)$ is given as

$$a(x-1) + b(y-2) + c(z-1) = 0 \quad \text{...(i)}$$ **(1)**

[$\because$ equation of plane passing through (x_1, y_1, z_1) having DR's a, b, c is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$]

Now, DR's of line PQ, where $P(1, 4, 2)$ and $Q(2, 3, 5)$ are $(2-1, 3-4, 5-2)$, i.e. $(1, -1, 3)$.

Since, plane (i) is perpendicular to line PQ.

$\therefore$ DR's of plane (i) are $(1, -1, 3)$,

[$\because$ DR's normal to the plane are proportional]

i.e. $$a = 1, b = -1, c = 3$$ **(1)**

On putting values of a, b and c in Eq. (i), we get the required equation of plane as

$$\Rightarrow \quad 1(x-1) - 1(y-2) + 3(z-1) = 0$$

$$\Rightarrow \quad x - 1 - y + 2 + 3z - 3 = 0$$

$$x - y + 3z - 2 = 0 \quad \text{...(ii)}$$

Now, the given equation of line is

$$\frac{x+3}{2}=\frac{y-5}{-1}=\frac{z-7}{-1} \quad \ldots\text{(iii)} \ \textbf{(1)}$$

DR's of this line are $(2,-1,-1)$ and passing point $(-3,5,7)$.

DR's of normal to the plane (ii) are $(1,-1,3)$.

Now, we check whether the line is perpendicular to the plane.

Here, $2(1)-1(-1)-1(3)=2+1-3=0$

[by using $a_1 a_2+b_1 b_2+c_1 c_2=0$]

So, line (iii) is parallel to plane (i).

$\therefore$ Required distance = Distance of the point $(-3,5,7)$ from the plane (ii)

$$\Rightarrow d=\left|\frac{(-3)(1)+(5)(-1)+7(3)-2}{\sqrt{(1)^2+(-1)^2+(3)^2}}\right|$$

$\left[\because \text{ distance of the point } (x_1, y_1, z_1) \text{ to the plane } ax+by+cz+d=0 \text{ is } d=\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}\right]$

$$=\left|\frac{-3-5+21-2}{\sqrt{1+1+9}}\right|$$

$$=\left|\frac{11}{\sqrt{11}}\right|=\left|\frac{(\sqrt{11})^2}{\sqrt{11}}\right|$$

$$=\sqrt{11} \text{ units} \quad \textbf{(1)}$$

29. The equation of any plane passing through (x_1, y_1, z_1) is

$$a(x-x_1)+b(y-y_1)+c(z-z_1)=0.$$

This plane is parallel to the line

$$\frac{x-x_2}{a_1}=\frac{y-y_2}{b_1}=\frac{z-z_2}{c_1}.$$

$\therefore$ Normal to the plane is perpendicular to the line, i.e. $aa_1+bb_1+cc_1=0$. Use these results and solve it.

Equation of plane passing through the point $A(0,0,0)$ is

$$a(x-0)+b(y-0)+c(z-0)=0$$

$$\Rightarrow ax+by+cz=0 \quad \ldots\text{(i)} \textbf{(1)}$$

[using one point form of plane $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$]

Given, the plane (i) passes through the point $B(3,-1,2)$.

$\therefore$ Put $x=3, y=-1$ and $z=2$ in Eq. (i), we get

$$3a-b+2c=0 \quad \ldots\text{(ii)}$$

Also, the plane (i) is parallel to the line

$$\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$$

$$\therefore \quad a(1)+b(-4)+c(7)=0$$

[if plane is parallel to the line, then normal to the plane is perpendicular to the line, i.e. $a_1a_2+b_1b_2+c_1c_2=0$].

$$\Rightarrow a-4b+7c=0 \quad \ldots\text{(iii)} \ \textbf{(1)}$$

On solving Eqs. (ii) and (iii) by cross multiplication, we get

$$\frac{a}{(-1)\times(7)-(2)\times(-4)}=\frac{b}{(2)\times(1)-(3)\times(7)}=\frac{c}{3\times(-4)-(-1)\times(1)}$$

$$\Rightarrow \frac{a}{-7+8}=\frac{b}{2-21}=\frac{c}{-12+1}$$

$$\Rightarrow \frac{a}{1}=\frac{b}{-19}=\frac{c}{-11}=\lambda \text{ (say)}$$

$\therefore \quad a=\lambda, b=-19\lambda, c=-11\lambda$ **(1)**

On substituting the values of a, b, c in Eq. (i), we get

$$\lambda(x)+(-19\lambda)y+(-11\lambda)z=0$$

$$\Rightarrow x-19y-11z=0$$

This is the required equation of the plane. **(1)**

30. Let the given points are $A(2,2,-1)$, $B(3,4,2)$ and $C(7,0,6)$.

Let $\vec{a}=2\hat{i}+2\hat{j}-\hat{k}, \vec{b}=3\hat{i}+4\hat{j}+2\hat{k}$

and $\vec{c}=7\hat{i}+6\hat{k}$ **(1)**

Now, the vector equation of the line passing through $\vec{a}, \vec{b}$ and $\vec{c}$ is given by

$$(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]=0$$

$$\Rightarrow \vec{b}-\vec{a}=(3\hat{i}+4\hat{j}+2\hat{k})-(2\hat{i}+2\hat{j}-\hat{k})=\hat{i}+2\hat{j}+3\hat{k}$$

$$\Rightarrow \vec{c}-\vec{a}=(7\hat{i}+6\hat{k})-(2\hat{i}+2\hat{j}-\hat{k})=5\hat{i}-2\hat{j}+7\hat{k}$$ **(1)**

The required equation of the plane is

$$[\vec{r}-(2\hat{i}+2\hat{j}-\hat{k})]\cdot[(\hat{i}+2\hat{j}+3\hat{k})\times(5\hat{i}-2\hat{j}+7\hat{k})]=0$$

$$\Rightarrow [\vec{r}-(2\hat{i}+2\hat{j}-\hat{k})]\cdot\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 5 & -2 & 7\end{vmatrix}=0$$

$$\Rightarrow [\vec{r} - (2\hat{i} + 2\hat{j} - \hat{k})]$$
$$\cdot [\hat{i}\,(14 + 6) - \hat{j}\,(7 - 15) + \hat{k}\,(-2 - 10)] = 0$$
$$\Rightarrow \quad [\vec{r} - (2\hat{i} + 2\hat{j} - \hat{k})] \cdot (20\hat{i} + 8\hat{j} - 12\hat{k}) = 0$$
$$\Rightarrow \vec{r} \cdot (20\hat{i} + 8\hat{j} - 12\hat{k}) = (2\hat{i} + 2\hat{j} - \hat{k}) \cdot (20\hat{i} + 8\hat{j} - 12\hat{k})$$
$$\Rightarrow \vec{r} \cdot (20\hat{i} + 8\hat{j} - 12\hat{k}) = 40 + 16 + 12 = 68$$
$$\Rightarrow \vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = 17,$$

which is the required vector equation. **(1)**

For cartesian equation put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\Rightarrow \quad (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = 17$$
$$\Rightarrow \quad 5x + 2y - 3z = 17 \text{ or } 5x + 2y - 3z - 17 = 0$$ **(1)**

Now, the equation of any plane parallel to above plane is

$$5x + 2y - 3z + k = 0$$

If it passes through (4, 3, 1), then

$$5(4) + 2(3) - 3 + k = 0 \Rightarrow k = -23$$ **(1)**

Thus, the equation of plane is

$$5x + 2y - 3z - 23 = 0$$

Hence, required vector equation of plane is

$$\vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) - 23 = 0$$ **(1)**

31. The required plane passes through the point $A(-1, 3, -4)$ and contains the line $\vec{r} = (\hat{i} + \hat{j}) + \lambda\,(\hat{i} + 2\hat{j} - \hat{k})$ which passes through $B(1, 1, 0)$ and is parallel to the vector $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. Thus, required plane passes through two points $A(-1, 3, -4)$ and $B(1, 1, 0)$ and is parallel to the vector $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. **(1)**

Let $\vec{n}$ be the normal vector to the required plane. Then, $\vec{n}$ is perpendicular to both $\vec{b}$ and $\overrightarrow{AB}$.

Consequently, it is parallel to $\overrightarrow{AB} \times \vec{b}$. **(1)**

Let $\vec{n}_1 = \overrightarrow{AB} \times \vec{b}$. Then,

$$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 4 \\ 1 & 2 & -1 \end{vmatrix} = -6\hat{i} + 6\hat{j} + 6\hat{k}$$

Let $\vec{\alpha}$ be the position vector of A. Then,

$$\vec{\alpha} = -\hat{i} + 3\hat{j} - 4\hat{k}$$ **(1)**

Clearly, the required plane passes through $\vec{\alpha} = -\hat{i} + 3\hat{j} - 4\hat{k}$ and it is perpendicular to $\vec{n}_1 = -6\hat{i} + 6\hat{j} + 6\hat{k}$. **(1)**

So, its vector equation is

$$(\vec{r} - \vec{\alpha}) \cdot \vec{n}_1 = 0 \quad \text{or} \quad \vec{r} \cdot \vec{n}_1 = \vec{\alpha} \cdot \vec{n}_1$$
$$\Rightarrow \quad \vec{r} \cdot (-6\hat{i} + 6\hat{j} + 6\hat{k})$$
$$= (-\hat{i} + 3\hat{j} - 4\hat{k}) \cdot (-6\hat{i} + 6\hat{j} + 6\hat{k})$$
$$\Rightarrow \quad \vec{r} \cdot (-6\hat{i} + 6\hat{j} + 6\hat{k}) = 6 + 18 - 24$$
$$\Rightarrow \quad \vec{r} \cdot (-6\hat{i} + 6\hat{j} + 6\hat{k}) = 0 \Rightarrow \vec{r} \cdot (-\hat{i} + \hat{j} + \hat{k}) = 0$$ **(1)**

The length of perpendicular from $P(2, 1, 4)$ to the above plane is given by

$$d = \left| \frac{(2\hat{i} + \hat{j} + 4\hat{k}) \cdot (-\hat{i} + \hat{j} + \hat{k})}{\sqrt{(-1)^2 + (1)^2 + (1)^2}} \right|$$
$$= \frac{|-2 + 1 + 4|}{\sqrt{(-1)^2 + (1)^2 + (1)^2}} = \frac{3}{\sqrt{3}} = \sqrt{3} \text{ units}$$ **(1)**

32. Let $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} + \hat{k}$

Then the vector equation of a plane passing through $\vec{a}$, $\vec{b}$ and $\vec{c}$ is given by

$$(\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$$
$$\Rightarrow \{\vec{r} - (\hat{i} + \hat{j} - 2\hat{k})\} \cdot [\{(2\hat{i} - \hat{j} + \hat{k}) - (\hat{i} + \hat{j} - 2\hat{k})\}$$
$$\times \{(\hat{i} + 2\hat{j} + \hat{k}) - (\hat{i} + \hat{j} - 2\hat{k})\}] = 0$$
$$\Rightarrow \quad \{\vec{r} - (\hat{i} + \hat{j} - 2\hat{k})\} \cdot [(\hat{i} - 2\hat{j} + 3\hat{k}) \times (\hat{j} + 3\hat{k})] = 0$$

Now, $(\hat{i} - 2\hat{j} + 3\hat{k}) \times (\hat{j} + 3\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 0 & 1 & 3 \end{vmatrix}$ **(1)**

$$= \hat{i}\,(-6 - 3) - \hat{j}\,(3 - 0) + \hat{k}(1 - 0)$$
$$= -9\hat{i} - 3\hat{j} + \hat{k}$$
$$\therefore \quad \{\vec{r} - (\hat{i} + \hat{j} - 2\hat{k})\} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) = 0$$
$$\Rightarrow \quad \vec{r} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) = (\hat{i} + \hat{j} - 2\hat{k}) \cdot (-9\hat{i} - 3\hat{j} + \hat{k})$$ **(1)**
$$\Rightarrow \quad \vec{r} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) = -9 - 3 - 2$$
$$\Rightarrow \quad \vec{r} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) = -14$$
$$\Rightarrow \vec{r} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) + 14 = 0$$ **(1)**

For cartesian equation of that plane put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\Rightarrow \quad (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) + 14 = 0$

$\Rightarrow \quad -9x - 3y + z + 14 = 0$

$\Rightarrow \quad 9x + 3y - z - 14 = 0 \quad \ldots(i)$

Now, any plane parallel to the given plane is

$9x + 3y - z + K = 0 \quad \ldots(ii)$ (1)

If it is passes through (2, 3, 7), then

$9(2) + 3(3) - 7 + K = 0$

$\Rightarrow \quad 18 + 9 - 7 + K = 0$

$\Rightarrow \quad K = -20$

Hence, required equation of the plane is

$9x + 3y - z - 20 = 0$ (1)

Now, we have equation of two parallel planes given by

$9x + 3y - z - 14 = 0$ and $9x + 3y - z - 20 = 0$

$\therefore$ Distance between these two planes

$$= \left| \frac{-20 - (-14)}{\sqrt{9^2 + 3^2 + (-1)^2}} \right| = \frac{6}{\sqrt{91}}$$ (1)

33. We know that the equation of a line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Here, $(x_1, y_1, z_1) = (2, -1, 2)$ and $(x_2, y_2, z_2) = (5, 3, 4)$

So, the equation of the line passing through $A(2, -1, 2)$ and $B(5, 3, 4)$ is

$$\frac{x-2}{5-2} = \frac{y+1}{3+1} = \frac{z-2}{4-2}$$

$$\Rightarrow \quad \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$$

The general equation of a plane passing through (2, 0, 3) is

$a(x-2) + b(y-0) + c(z-3) = 0 \quad \ldots(i)$

It will pass through $B(1, 1, 5)$ and $C(3, 2, 4)$, if

$a(1-2) + b(1-0) + c(5-3) = 0$

$\Rightarrow \quad -a + b + 2c = 0$

$\Rightarrow \quad a - b - 2c = 0 \quad \ldots(ii)$

and $a(3-2) + b(2-0) + c(4-3) = 0$

$\Rightarrow \quad a + 2b + c = 0 \quad \ldots(iii)$

On solving Eqs. (ii) and (iii) by cross-multiplication, we get

$$\frac{a}{-1+4} = \frac{b}{-2-1} = \frac{c}{2+1}$$

$$\Rightarrow \quad \frac{a}{3} = \frac{b}{-3} = \frac{c}{3} = \lambda \text{ (say)}$$

$\Rightarrow \quad a = 3\lambda, b = -3\lambda$ and $c = 3\lambda$

Substituting the values of a, b and c in Eq. (i), we get

$3\lambda(x-2) - 3\lambda(y-0) + 3\lambda(z-3) = 0$

$\Rightarrow \quad x - 2 - y + z - 3 = 0$

$\Rightarrow \quad x - y + z = 5$

which is the required equation of plane.

Now, the coordinates of any point on the line

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = r \text{ (say)}$$

are $\quad x = 3r + 2, y = 4r - 1, z = 2r + 2 \quad \ldots(iv)$

If it lies on the plane $x - y + z = 5$ then

$3r + 2 - 4r + 1 + 2r + 2 = 5 \Rightarrow r = 0$

Substituting the value of $r = 0$ in Eq. (iv), we get

$x = 3 \times 0 + 2, y = 4 \times 0 - 1, z = 2 \times 0 + 2$

$\Rightarrow \quad x = 2, y = -1, z = 2$

Hence, the point of intersection are $(2, -1, 2)$.

34. Given equations of line and plane are

$\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$

$= (2 + 3\lambda)\hat{i} + (-1 + 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k} \quad \ldots(i)$

and $\quad \vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$ (1)

For point of intersection of line and plane, the point $\vec{r}$ satisfy the equation of plane.

$\therefore [(2 + 3\lambda)\hat{i} + (-1 + 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}] \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$

$\Rightarrow (3\lambda + 2) - (4\lambda - 1) + (2\lambda + 2) = 5$

$\Rightarrow \quad 3\lambda + 2 - 4\lambda + 1 + 2\lambda + 2 = 5 \Rightarrow \lambda = 0$ (1)

On putting $\lambda = 0$ in Eq.(i), we get

$\vec{r} = (2 + 0)\hat{i} + (-1 + 0)\hat{j} + (2 + 0)\hat{k} = 2\hat{i} - \hat{j} + 2\hat{k}$

Thus, intersection point of the line and the plane is $(2, -1, 2)$. (1)

Now, the required distance

$$PQ = \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2}$$

$$\left[\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right]$$ (1)

$= \sqrt{9 + 16 + 144} = \sqrt{144 + 25}$

$= \sqrt{169} = 13$ units

Hence, the required distance is 13 units. (2)

35. Suppose the required line is parallel to vector $\vec{b}$ which is given by $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$.

The position vector of the point (1, 2, 3) is

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$

The equation of line passing through (1, 2, 3) and parallel to $\vec{b}$ is given by

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) \quad \text{...(i) (1)}$$

The equation of the given planes are

$$\vec{r}\cdot(\hat{i} - \hat{j} + 2\hat{k}) = 5 \quad \text{...(ii)}$$

and $$\vec{r}\cdot(3\hat{i} + \hat{j} + \hat{k}) = 6 \quad \text{...(iii) (1)}$$

The line in Eq. (i) and plane in Eq. (ii) are parallel.

Therefore, the normal to the plane of Eq. (ii) is perpendicular to the given line.

$$\therefore \quad (\hat{i} - \hat{j} + 2\hat{k})\cdot(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) = 0 \quad \text{(1)}$$

$$\Rightarrow \quad b_1 - b_2 + 2b_3 = 0 \quad \text{...(iv)}$$

Similarly, from Eqs. (i) and (iii), we get

$$(3\hat{i} + \hat{j} + \hat{k})\cdot(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) = 0$$

$$\Rightarrow \quad 3b_1 + b_2 + b_3 = 0 \quad \text{...(v) (1)}$$

On solving Eqs. (iv) and (v) by cross-multiplication, we get

$$\frac{b_1}{(-1)\times1 - 1\times2} = \frac{b_2}{2\times3 - 1\times1} = \frac{b_3}{1\times1 - 3(-1)}$$

$$\Rightarrow \quad \frac{b_1}{-3} = \frac{b_2}{5} = \frac{b_3}{4} \quad \text{(1)}$$

Therefore, the direction ratios of $\vec{b}$ are (–3,5,4).

$$\therefore \quad \vec{b} = -3\hat{i} + 5\hat{j} + 4\hat{k} \quad [\because \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}]$$

On substituting the value of $\vec{b}$ in Eq. (i), we get

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k}) \quad \text{...(i)}$$

which is the equation of the required line.

Any point on line (i) is

$$(1 - 3\lambda, 2 + 5\lambda, 3 + 4\lambda)$$

For this line (i) to intersect the plane

$$\vec{r}(2\hat{i} + \hat{j} + \hat{k}) = 4$$

We have,

$$(1 - 3\lambda)(2) + (2 + 5\lambda)1 + (3 + 4\lambda)1 = 4$$

$$\Rightarrow \quad 2 - 6\lambda + 2 + 5\lambda + 3 + 4\lambda = 4$$

$$\Rightarrow 7 + 3\lambda = 4 \Rightarrow 3\lambda = -3 \Rightarrow \lambda = -1$$

$\therefore$ Required point of intersection is

$(1 - 3(-1), 2 + 5(-1), 3 + 4(-1))$, i.e. $(4, -3, -1)$ **(1)**

36. Any plane through the line of intersection of the two given plane is

$$[\vec{r}\cdot(2\hat{i} - 3\hat{j} + 4\hat{k}) - 1] + \lambda[\vec{r}(\hat{i} - \hat{j}) + 4] = 0$$

$$\Rightarrow \quad \vec{r}\cdot[(2 + \lambda)\hat{i} - (3 + \lambda)\hat{j} + 4\hat{k}] = 1 - 4\lambda \quad \text{...(i)}$$

If this plane is perpendicular to the plane $\vec{r}\cdot(2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$,

Then, $2(2 + \lambda) + (3 + \lambda) + 4 = 0$

$$\Rightarrow \quad 3\lambda + 11 = 0 \Rightarrow \lambda = -\frac{11}{3} \quad \text{(1)}$$

Put $\lambda = -\frac{11}{3}$ in Eq. (i), we get the required equation of the plane is

$$\vec{r}\cdot(-5\hat{i} + 2\hat{j} + 12\hat{k}) = 47$$

and given that equation of line is

$$x - 1 = 2y - 4 = 3z - 12$$

$$\Rightarrow \quad \frac{x-1}{1} = \frac{y-2}{1/2} = \frac{z-4}{1/3} \quad \text{(1)}$$

In vector form, equation of line is

$$\hat{i} + 2\hat{j} + 4\hat{k} + \lambda\left(\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{3}\hat{k}\right) \quad \text{(1)}$$

This line $\vec{r} = \hat{i} + 2\hat{j} + 4\hat{k} + \lambda\left(\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{3}\hat{k}\right)$ passes through a point with position vector $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$ and parallel to the vector $\vec{b} = \hat{i} + \frac{1}{2}\hat{j} + \frac{1}{3}\hat{k}$.

The plane $\vec{r}\cdot(-5\hat{i} + 2\hat{j} + 12\hat{k}) = 47$ contains the given line if

(i) it passes through $\hat{i} + 2\hat{j} + 4\hat{k}$

(ii) it is parallel to the line

We have, $(\hat{i} + 2\hat{j} + 4\hat{k})(-5\hat{i} + 2\hat{j} + 12\hat{k})$

$$= -5 + 4 + 48 = 47 \quad \text{(1)}$$

So, the plane passes through the point $\hat{i} + 2\hat{j} + 4\hat{k}$

and, $\left(\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{3}\hat{k}\right)(-5\hat{i} + 2\hat{j} + 12\hat{k})$

$$= -5 + 1 + 4 = 0 \quad \text{(1)}$$

Therefore, the plane is parallel to the line.

Hence, the plane contains the given line. **(1)**

37. Let the equation of the variable plane is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \quad \text{...(i)}$$

Since, above plane (i) meets the X-axis, Y-axis and Z-axis at the point $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, c)$, respectively and let (α, β, γ) be the coordinates of the centroid of ΔABC.

Then, $\alpha = \frac{a+0+0}{3}$, $\beta = \frac{0+b+0}{3}$ **(1)**

and $\gamma = \frac{0+0+c}{3}$

$\Rightarrow \quad \alpha = \frac{a}{3}, \beta = \frac{b}{3}$ and $\gamma = \frac{c}{3}$ **(1)**

$\Rightarrow \quad a = 3\alpha, b = 3\beta$ and $c = 3\gamma$ ...(ii) **(1)**

$\therefore 3p$ = length of the perpendicular from (0, 0, 0) to the plane (i)

$$\Rightarrow \quad 3p = \frac{\left|\frac{0}{a}+\frac{0}{b}+\frac{0}{c}-1\right|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$$

$$\Rightarrow \quad 3p = \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$$

$$\Rightarrow \quad \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{9p^2} \quad \textbf{(1)}$$

$$\Rightarrow \quad \frac{1}{9\alpha^2}+\frac{1}{9\beta^2}+\frac{1}{9\gamma^2}=\frac{1}{9p^2} \quad \text{[using Eq. (ii)]}$$

$$\Rightarrow \quad \frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=\frac{1}{p^2} \quad \textbf{(1)}$$

Hence, the locus of the centroid is

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$$ **Hence proved. (1)**

38. The equation of a plane passing through the point $L(2, 2, 1)$ is

$a(x-2)+b(y-2)+c(z-1)=0$ ...(i) **(1)**

Also, it is passes through the points $M(3, 0, 1)$ and $N(4, -1, 0)$, respectively.

$\therefore a(3-2)+b(0-2)+c(1-1)=0$

$\Rightarrow \quad a-2b=0$

$\Rightarrow \quad a=2b$ ...(ii) **(1/2)**

and $a(4-2)+b(-1-2)+c(0-1)=0$

$\Rightarrow \quad 2a-3b-c=0$

$\Rightarrow \quad 2(2b)-3b-c=0$ [from Eq. (ii)] **(1/2)**

$\Rightarrow \quad b-c=0$

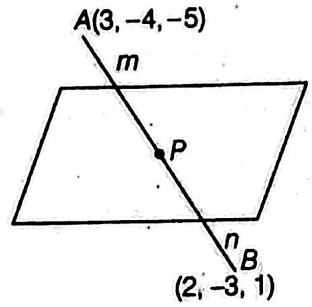
$\Rightarrow \quad c=b$

On putting $a=2b$ and $c=b$ in Eq. (i), we get

$2b(x-2)+b(y-2)+b(z-1)=0$

$\Rightarrow \quad 2x-4+y-2+z-1=0$ [divide by b]

$\Rightarrow \quad 2x+y+z=7$ ...(iii) **(1)**



Let the point P divide the line joining points A and B in the ratio $m:n$.

Then, coordinates of P are

$$P\left(\frac{2m+3n}{m+n}, \frac{-3m-4n}{m+n}, \frac{m-5n}{m+n}\right) \quad \textbf{(1)}$$

Since, the line crosses the plane at point P. So, the coordinates of point P satisfy the equation of plane $2x+y+z=7$.

$$\therefore \quad 2\left(\frac{2m+3n}{m+n}\right)+\left(\frac{-3m-4n}{m+n}\right)+\left(\frac{m-5n}{m+n}\right)=7$$

$\Rightarrow 4m+6n-3m-4n+m-5n=7m+7n$

$\Rightarrow \quad 2m-3n=7m+7n$

$\Rightarrow \quad -5m=10n$

$\Rightarrow \quad m=-2n$ ...(iv) **(1)**

Now, the coordinates of P are

$$\left(\frac{2\times(-2n)+3n}{-2n+n}, \frac{-3\times(-2n)-4n}{-2n+n}, \frac{-2n-5n}{-2n+n}\right)$$

i.e. $\left(\frac{-n}{-n}, \frac{2n}{-n}, \frac{-7n}{-n}\right)$ or $(1, -2, 7)$

From Eq. (iv),

$$m=-2n \Rightarrow \frac{m}{n}=-2$$

Hence, P divides the line joining points A and B externally in the ratio 2:1. **(1)**

NOTE If the ratio is negative, then it means that the point divides the line externally.

39. (i) Do same as Q.No. 24. $\left[\text{Ans.}\left(\frac{17}{3}, 0, \frac{23}{3}\right)\right]$

(ii) Let θ be the angle between the line AB and XZ plane.

Then, $\sin\theta = \left|\dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + C_2^2}}\right|$, where a_1, b_1, c_1 are DR's of line AB and a_2, b_2, c_2 are DR's of normal to plane XZ. **(1)**

Here, $a_1 = 2, b_1 = -3, c_1 = 5$ and $a_2 = 0, b_2 = 1, c_2 = 0$ **(1/2)**

$$\therefore \sin\theta = \left|\frac{2\cdot 0 + (-3)\cdot 1 + 5\cdot 0}{\sqrt{2^2 + (-3)^2 + 5^2}\sqrt{0^2 + 1^2 + 0^2}}\right|$$

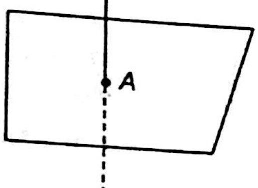
$$= \left|\frac{-3}{\sqrt{4 + 9 + 25}}\right| = \frac{3}{\sqrt{38}}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{3}{\sqrt{38}}\right)$$ **(1/2)**

40. Given, a point P with position vector $2\hat{i} + 3\hat{j} + 4\hat{k}$ and the plane

$\vec{r}\cdot(2\hat{i} + \hat{j} + 3\hat{k}) - 26 = 0$, or $2x + y + 3z = 26$

Let A be the foot of perpendicular. Then, PA is the normal to the plane and so its DR's are 2,1 and 3.



(1)

Now, the equation of perpendicular line PA is

$$\frac{x-2}{2} = \frac{y-3}{1} = \frac{z-4}{3} = \lambda \text{ (say)}$$

$\Rightarrow \quad x = 2\lambda + 2,\ y = \lambda + 3$ and $z = 3\lambda + 4$

$\Rightarrow$ Coordinates of any point on PA is of the form $(2\lambda + 2, \lambda + 3, 3\lambda + 4)$. **(1)**

$\therefore$ Coordinates of A are $(2\lambda + 2, \lambda + 3, 3\lambda + 4)$ for some λ. **(1/2)**

Since, A lies on the plane, therefore we have

$2(2\lambda + 2) + (\lambda + 3) + 3(3\lambda + 4) = 26$ **(1/2)**

$\Rightarrow \quad 4\lambda + 4 + \lambda + 3 + 9\lambda + 12 = 26$

$\Rightarrow \quad 14\lambda + 19 = 26 \Rightarrow 14\lambda = 7 \Rightarrow \lambda = \dfrac{1}{2}$ **(1/2)**

So, the coordinates of foot of perpendicular are

$\left(2\cdot\dfrac{1}{2} + 2, \dfrac{1}{2} + 3, 3\cdot\dfrac{1}{2} + 4\right)$ i.e. $\left(3, \dfrac{7}{2}, \dfrac{11}{2}\right)$.

and therefore its position vector is $3\hat{i} + \dfrac{7}{2}\hat{j} + \dfrac{11}{2}\hat{k}$ **(1/2)**

Now, the required perpendicular distance

$$= \sqrt{(3-2)^2 + \left(\frac{7}{2} - 3\right)^2 + \left(\frac{11}{2} - 4\right)^2}$$

$$= \sqrt{1 + \frac{1}{4} + \frac{9}{4}} = \sqrt{\frac{7}{2}} \text{ units}$$

Now, let $P'(x, y, z)$ be the image of point P in the plane.

Then, A will be mid-point of PP'.

$$\therefore \quad \left(3, \frac{7}{2}, \frac{11}{2}\right) = \left(\frac{2+x}{2}, \frac{3+y}{2}, \frac{4+z}{2}\right)$$

$$\Rightarrow 3 = \frac{2+x}{2}; \frac{7}{2} = \frac{3+y}{2}; \frac{11}{2} = \frac{4+z}{2}$$

$\Rightarrow x = 4; y = 4$ and $z = 7$

Thus, the coordinates of the image of the point P are $(4, 4, 7)$. **(1)**

41. Given equation of planes are

$\vec{r}\cdot(\hat{i} - 2\hat{j} + 3\hat{k}) = 4$ and $\vec{r}\cdot(-2\hat{i} + \hat{j} + \hat{k}) = -5$

On comparing these with $\vec{r}\cdot\vec{n} = d$, we get

$\vec{n_1} = \hat{i} - 2\hat{j} + 3\hat{k}, d_1 = 4,$

$\vec{n_2} = -2\hat{i} + \hat{j} + \hat{k}$ and $d_2 = -5$ **(1)**

Now, the equation of the plane which contains the intersection of the given planes is

$$\vec{r}\cdot(\vec{n_1} + \lambda\vec{n_2}) = d_1 + \lambda d_2$$

$\Rightarrow \vec{r}\cdot[\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(-2\hat{i} + \hat{j} + \hat{k})] = 4 - 5\lambda$ **(1)**

$\Rightarrow \vec{r}\cdot[(1 - 2\lambda)\hat{i} + (-2 + \lambda)\hat{j} + (3 + \lambda)\hat{k}] = 4 - 5\lambda$ ...(i)

Also, given intercept on X and Y-axes are same.

$$\therefore \quad \frac{4 - 5\lambda}{1 - 2\lambda} = \frac{4 - 5\lambda}{-2 + \lambda}$$ **(1)**

$\Rightarrow \quad 1 - 2\lambda = -2 + \lambda$

$\Rightarrow \quad 3\lambda = 3$

$\Rightarrow \quad \lambda = 1$ **(1)**

On putting $\lambda = 1$ in Eq. (i), we get

$\vec{r}\cdot[(1 - 2)\hat{i} + (-2 + 1)\hat{j} + (3 + 1)\hat{k}] = 4 - 5\times 1$

$\therefore \quad \vec{r}\cdot(-\hat{i} - \hat{j} + 4\hat{k}) = -1,$

which is the required equation of plane. **(2)**

42. Given equation of planes are

$x + 2y + 3z - 4 = 0$ ...(i)

and $\quad 2x + y - z + 5 = 0$ ...(ii)

Clearly, the equation of plane which contain the line of intersection of planes (i) and (ii), is

$$(x+2y+3z-4)+\lambda(2x+y-z+5)=0 \quad \textbf{(1)}$$

$$\Rightarrow (1+2\lambda)x+(2+\lambda)y+(3-\lambda)z+5\lambda-4=0 \quad \ldots(iii)$$

This equation can be written in intercept form as

$$\frac{x}{\frac{4-5\lambda}{1+2\lambda}}+\frac{y}{\frac{4-5\lambda}{2+\lambda}}+\frac{z}{\frac{4-5\lambda}{3-\lambda}}=1 \quad \textbf{(1/2)}$$

Since, it is given that the x-intercept of plane (iii) is twice its z-intercept.

$$\therefore \quad \frac{4-5\lambda}{1+2\lambda}=2\left(\frac{4-5\lambda}{3-\lambda}\right) \quad \textbf{(1)}$$

$$\Rightarrow \quad 3-\lambda=2+4\lambda$$

$$\Rightarrow \quad 5\lambda=1 \Rightarrow \lambda=\frac{1}{5} \quad \textbf{(1/2)}$$

So, the required equation of plane is

$$\left(1+\frac{2}{5}\right)x+\left(2+\frac{1}{5}\right)y+\left(3-\frac{1}{5}\right)z=4-5\cdot\frac{1}{5}$$

$$\Rightarrow \quad \frac{7}{5}x+\frac{11}{5}y+\frac{14}{5}z=\frac{15}{5}$$

$$\Rightarrow \quad 7x+11y+14z=15 \quad \ldots(iv)\ \textbf{(1/2)}$$

Clearly, the DR's of normal to the plane, which is parallel to plane (iv), are 7, 11 and 14. **(1/2)**

$\therefore$ The vector equation of a plane passing through the (2, 3, −1) and parallel to the plane (iv), is

$$[\vec{r}-(2\hat{i}+3\hat{j}-\hat{k})]\cdot(7\hat{i}+11\hat{j}+14\hat{k})=0 \quad \textbf{(1)}$$

$$\Rightarrow \vec{r}\cdot(7\hat{i}+11\hat{j}+14\hat{k})=(2\hat{i}+3\hat{j}-\hat{k})\cdot(7\hat{i}+11\hat{j}+14\hat{k})$$

$$\Rightarrow \vec{r}\cdot(7\hat{i}+11\hat{j}+14\hat{k})=14+33-14$$

$$\Rightarrow \vec{r}\cdot(7\hat{i}+11\hat{j}+14\hat{k})=33$$

which is the required equation. **(1)**

43. Given equation of lines are

$$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4} \quad \ldots(i)$$

and $$\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1} \quad \ldots(ii)$$

Since, these lines intersect, therefore the shortest distance between them will be zero.

Now, on comparing these lines with

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$$

and $$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2},$$

we get $x_1=1,\ y_1=-1,\ z_1=1$

$x_2=3,\ y_2=k,\ z_2=0$

$a_1=2,\ b_1=3,\ c_1=4$

$a_2=1,\ b_2=2,\ c_2=1$ **(1)**

Since, two lines are intersect, so shortest distance = 0.

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} 3-1 & k+1 & 0-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix}=0 \quad \textbf{(1)}$$

$$\Rightarrow \quad 2(3-8)-(k+1)(2-4)-1(4-3)=0$$

$$\Rightarrow \quad 2(-5)-(k+1)(-2)-1(1)=0$$

$$\Rightarrow \quad -10+2(k+1)-1=0 \Rightarrow 2(k+1)=11$$

$$\therefore \quad k=\frac{11}{2}-1=\frac{9}{2} \quad \textbf{(1)}$$

Now, let the required equation of plane be $a(x-1)+b(y+1)+c(z-1)=0$. ...(iii)

[equation of plane $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$]

where, a, b and c are direction ratios of normal and (1, −1, 1) is the point on the line (i).

As, plane contains the intersecting lines, so normal to the plane is perpendicular to both the lines.

$\therefore \quad 2a+3b+4c=0$ and $a+2b+c=0$ **(1)**

$[\because a_1a_2+b_1b_2+c_1c_2=0]$

$$\Rightarrow \quad \frac{a}{3-8}=\frac{b}{4-2}=\frac{c}{4-3}$$

$$\Rightarrow \quad \frac{a}{-5}=\frac{b}{2}=\frac{c}{1} \quad \textbf{(1)}$$

Thus, the required equation of the plane is

$$-5(x-1)+2(y+1)+1(z-1)=0$$

$$\Rightarrow \quad -5x+5+2y+2+z-1=0$$

$$\Rightarrow \quad 5x-5-2y-2-z+1=0$$

$$\therefore \quad 5x-2y-z=6 \quad \textbf{(1)}$$

44. The direction ratios of line joining $A(3,-4,-5)$ and $B(2,-3,1)$ are $[(2-3),(-3+4),(1+5)]$, i.e. $(-1,1,6)$. **(1)**

Now, the equation of line passing through $(3, -4, -5)$ and having DR's $(-1, 1, 6)$ is given by

$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$$

$$\left[\because \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}\right]$$

Now, let $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$ (say)

$\Rightarrow \quad x = -\lambda + 3,\ y = \lambda - 4$ and $z = 6\lambda - 5$ **(1)**

Thus, the general point on the line is given by $(3-\lambda, \lambda-4, 6\lambda-5)$. **(1)**

Since, line intersect the plane $2x + y + z = 7$. So, general point on the line $(3-\lambda, \lambda-4, 6\lambda-5)$ satisfy the equation of plane.

$\therefore \quad 2(3-\lambda) + \lambda - 4 + 6\lambda - 5 = 7$

$\Rightarrow \quad 6 - 2\lambda + \lambda - 4 + 6\lambda - 5 = 7 \Rightarrow 5\lambda = 10$

$\therefore \quad \lambda = 2$ **(1)**

So, the point of intersection of line and plane is $(3-2, 2-4, 6\times 2-5)$, i.e. $(1, -2, 7)$. **(1)**

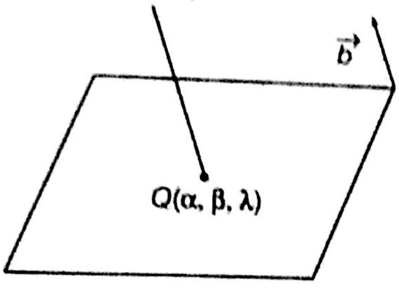
Now, distance between $(3, 4, -4)$ and $(1, -2, 7)$ is given by $\sqrt{(3-1)^2 + (4+2)^2 + (4-7)^2}$

$$= \sqrt{4+36+9} = \sqrt{49} = 7 \text{ units}$$ **(1)**

45. Let $P(1, -2, 3)$ be the given point and $Q(\alpha, \beta, \gamma)$ be the point on the given plane

$$x - y + z = 5 \quad \ldots(i)$$

Such that PQ is parallel to given line whose direction ratios are $(2, 3, -6)$.



(1)

Now, $\overrightarrow{PQ}$ = Position vector of Q − Position vector of P

$$= (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k})$$

$$= (\alpha - 1)\hat{i} + (\beta + 2)\hat{j} + (\gamma - 3)\hat{k}$$ **(1)**

Since, $\overrightarrow{PQ}$ is parallel to $\vec{b}$.

$$\therefore \quad \frac{\alpha-1}{2} = \frac{\beta+2}{3} = \frac{\gamma-3}{-6} = \lambda \text{ (say)}$$

$\Rightarrow \quad \alpha = 2\lambda + 1, \beta = 3\lambda - 2$ and $\gamma = -6\lambda + 3 \quad \ldots$(ii) **(1)**

Since, the point $Q(\alpha, \beta, \gamma)$ lies on the plane (i), so it satisfies.

$$\alpha - \beta + \gamma = 5$$

$\therefore$

$\Rightarrow \quad (2\lambda + 1) - (3\lambda - 2) + (-6\lambda + 3) = 5$

$\Rightarrow \quad -7\lambda + 6 = 5$

$\Rightarrow \quad \lambda = \frac{1}{7}$ **(1)**

Now, put $\lambda = \frac{1}{7}$ in Eq. (ii), we get

$$\alpha = 2\times\frac{1}{7} + 1, \beta = 3\times\frac{1}{7} - 2 \text{ and } \gamma = -6\times\frac{1}{7} + 3$$

$$\Rightarrow \alpha = \frac{9}{7}, \beta = \frac{-11}{7} \text{ and } \gamma = \frac{15}{7}$$

Hence, coordinates of Q are $\left(\frac{9}{7}, \frac{-11}{7}, \frac{15}{7}\right)$. **(1)**

$\therefore$ Required distance

$$PQ = \sqrt{\left(\frac{9}{7}-1\right)^2 + \left(\frac{-11}{7}+2\right)^2 + \left(\frac{15}{7}-3\right)^2}$$

$$= \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{-6}{7}\right)^2}$$

$$= \frac{1}{7}\sqrt{4+9+36} = \frac{7}{7} = 1 \text{ unit}$$ **(1)**

46. Equation of any plane through the line of intersection of the given planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ is

$$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$$

$\Rightarrow \quad (1 + 2\lambda)x + (1 + 3\lambda)y$

$$+ (1 + 4\lambda)z - 1 - 5\lambda = 0 \quad \ldots(i)$$ **(1)**

The direction ratios (a_1, b_1, c_1) of the plane are $[(2\lambda + 1), (3\lambda + 1), (4\lambda + 1)]$.

Also, given that the plane, i.e. Eq. (i) is perpendicular to the plane $x - y + z = 0$, whose direction ratios (a_2, b_2, c_2) are $(1, -1, 1)$.

Then, **(1)**

$$\therefore \quad a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$\Rightarrow \quad 1(1 + 2\lambda) - 1(1 + 3\lambda) + 1(1 + 4\lambda) = 0$

$\Rightarrow \quad 1 + 2\lambda - 1 - 3\lambda + 1 + 4\lambda = 0$

$\Rightarrow \quad 3\lambda = -1 \Rightarrow \lambda = -\frac{1}{3}$ **(1)**

On substituting the value of λ in Eq. (i), we get the equation of required plane as

$$\left(1 - \frac{2}{3}\right)x + \left(1 - \frac{3}{3}\right)y + \left(1 - \frac{4}{3}\right)z - 1 + \frac{5}{3} = 0$$

$\Rightarrow \quad \frac{1}{3}x - \frac{1}{3}z + \frac{2}{3} = 0$

$\Rightarrow \quad x - z + 2 = 0$ **(1)**

Now, we know that, distance between a point $P(x_1, y_1, z_1)$ and plane $Ax + By + Cz = D$ is

$$d = \frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A^2 + B^2 + C^2}}.$$ **(1)**

Here, point is $A(1, 3, 6)$ and the plane is $x - z + 2 = 0$.

$\therefore$ Required distance,

$$d = \frac{|1 - 6 + 2|}{\sqrt{(1)^2 + (-1)^2}}$$

$$= \frac{|-3|}{\sqrt{2}} = \frac{3}{\sqrt{2}} \text{ units} \quad (1)$$

47. Given equation of planes are

$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = -4$

Equation of plane passing through the intersection of above two planes is

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) + \vec{r} \cdot \lambda (2\hat{i} + 3\hat{j} - \hat{k}) = 1 + \lambda(-4)$$

$$[\because \vec{r}_1 + \lambda r_2 = d_1 + \lambda d_2]$$

$$\Rightarrow \vec{r} \cdot [(\hat{i} + \hat{j} + \hat{k}) + \lambda (2\hat{i} + 3\hat{j} - \hat{k})] = 1 - 4\lambda \quad (1)$$

$$\Rightarrow \vec{r} \cdot [(1 + 2\lambda)\hat{i} + (1 + 3\lambda)\hat{j} + (1 - \lambda)\hat{k}] = 1 - 4\lambda \quad \text{...(i)}$$

Here, $\vec{n} = (1 + 2\lambda)\hat{i} + (1 + 3\lambda)\hat{j} + (1 - \lambda)\hat{k}$ (1)

We know that, direction cosines of X-axis is 1, 0, 0. (1)

Since, the required plane is parallel to X-axis, therefore normal of the plane (i) is perpendicular to the X-axis.

$$(1 + 2\lambda) \cdot (1) + (1 + 3\lambda) \cdot (0) + (1 - \lambda) \cdot (0) = 0$$

$$\Rightarrow [(1 + 2\lambda)\hat{i} + (1 + 3\lambda)\hat{j} + (1 - \lambda)\hat{k}] \cdot (\hat{i}) = 0$$

$$\Rightarrow 1 + 2\lambda = 0$$

$$\Rightarrow \lambda = -\frac{1}{2} \quad (1)$$

On putting $\lambda = -\frac{1}{2}$ in Eq. (i), we get

$$\vec{r} \cdot \left[\left(1 - \frac{2}{2}\right)\hat{i} + \left(1 - \frac{3}{2}\right)\hat{j} + \left(1 + \frac{1}{2}\right)\hat{k}\right] = 1 + \frac{4}{2} \quad (1)$$

$$\Rightarrow \vec{r} \cdot [0\hat{i} - \frac{1}{2}\hat{j} + \frac{3}{2}\hat{k}] = 3$$

$$\Rightarrow \vec{r} \cdot [-\hat{j} + 3\hat{k}] = 3 \times 2$$

$$\therefore \vec{r} \cdot [\hat{j} - 3\hat{k}] + 6 = 0 \quad (1)$$

48. Do same as Q. No. 14.

We get required equation of plane

$$2x + 3y + 4z - 7 = 0 \quad \text{...(i) (1)}$$

Now, distance between the plane (i) and the point (7, 2, 4) is

$$d = \left|\frac{2(7) + 3(2) + 4(4) - 7}{\sqrt{(2)^2 + (3)^2 + (4)^2}}\right| \quad (1)$$

$\left[\because\right.$ distance between the plane $ax + by + cz + d = 0$ and the point (x_1, y_1, z_1) is $\left.\left|\frac{ax_1 + by_1 + cz_1 - d}{\sqrt{a^2 + b^2 + c^2}}\right|\right]$

$$= \left|\frac{14 + 6 + 16 - 7}{\sqrt{4 + 9 + 16}}\right| = \frac{29}{\sqrt{29}} = \sqrt{29} \text{ units} \quad (1)$$

49. Do same as Q. No. 46. [**Ans.** $x - z + 2 = 0, \sqrt{2}$ units]

50. Do some as Q. No. 34. [**Ans.** 13 units]

51. (i)

> The equation of any plane passing through (x_1, y_1, z_1) is
>
> $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$
>
> and if it is perpendicular to the planes
>
> $a_1x + b_1y + c_1z + d_1 = 0$ and
>
> $a_2x + b_2y + c_2z + d_2 = 0$.
>
> Then, $aa_1 + bb_1 + cc_1 = 0$
>
> and $aa_2 + bb_2 + cc_2 = 0$.
>
> Use these results and solve it.

Equation of plane passing through point (1, −1, 2) is given by

$$a(x - 1) + b(y + 1) + c(z - 2) = 0 \quad \text{...(i) (1)}$$

Now, given that plane (i) is perpendicular to planes

$$2x + 3y - 2z = 5 \quad \text{...(ii)}$$

and

$$x + 2y - 3z = 8 \quad \text{...(iii)}$$

We know that, when two planes

$$a_1x + b_1y + c_1z = d_1$$

and $a_2x + b_2y + c_2z = d_2$ are perpendicular, then

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \quad (1)$$

$\therefore$ $2a + 3b - 2c = 0$ [from Eqs. (i) and (ii)]

and $a + 2b - 3c = 0$ [from Eqs. (i) and (iii)]

$$\Rightarrow 2a + 3b = 2c \quad \text{...(iv)}$$

and

$$a + 2b = 3c \quad \text{...(v) (1)}$$

On multiplying Eq. (v) by 2 and subtracting it from Eq. (iv), we get

$$\begin{array}{r} 2a + 3b = 2c \\ \underline{-2a \pm 4b = -6c} \\ -b = -4c \end{array} \Rightarrow b = 4c$$

On putting $b = 4c$ in Eq. (v), we get

$$a + 8c = 3c \Rightarrow a = -5c$$

Now, on putting $a = -5c$ and $b = 4c$ in Eq.(i), we get the required equation of plane as

$$-5c(x-1) + 4c(y+1) + c(z-2) = 0$$

$\Rightarrow$ $$-5(x-1) + 4(y+1) + (z-2) = 0$$

[dividing both sides by c]

$\Rightarrow$ $$-5x + 5 + 4y + 4 + z - 2 = 0$$

$\therefore$ $$5x - 4y - z - 7 = 0$$ **(1)**

(ii) The distance of point $P(-2, 5, 5)$ from the plane obtained $= \left|\frac{5(-2) - 4(5) - (5) - 7}{\sqrt{25+16+1}}\right|$

$$= \left|\frac{-42}{\sqrt{42}}\right| = \sqrt{42} \text{ units}$$ **(1)**

52. Do same as Q. No. 44. **[Ans.** 13 units**]**

53. Let equation of plane through $(1, 2, -4)$ be

$$a(x-1) + b(y-2) + c(z+4) = 0 \quad ...(i)$$

Given lines are

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

and $$\vec{r} = \hat{i} - 3\hat{j} + 5\hat{k} + \mu(\hat{i} + \hat{j} - \hat{k})$$ **(1)**

The cartesian equations of given lines are

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6} \text{ and } \frac{x-1}{1} = \frac{y+3}{1} = \frac{z-5}{-1}$$ **(1)**

Since, the required plane (i) is parallel to the given lines, so normal to the plane is perpendicular to the given lines.

$\therefore$ $$2a + 3b + 6c = 0$$

and $$a + b - c = 0$$ **(1)**

For solving these two equations by cross-multiplication, we get

$$\frac{a}{-3-6} = \frac{b}{6+2} = \frac{c}{2-3} \Rightarrow \frac{a}{-9} = \frac{b}{8} = \frac{c}{-1} = \lambda \text{ (say)}$$

$\therefore$ $a = -9\lambda, b = 8\lambda, c = -\lambda$

On putting values of a, b and c in Eq. (i), we get $-9\lambda(x-1) + 8\lambda(y-2) - \lambda(z+4) = 0$

$\therefore$ Equation of plane in cartesian form is

$$-9\lambda(x-1) + 8\lambda(y-2) - \lambda(z+4) = 0$$

$\Rightarrow$ $$-9x + 9 + 8y - 16 - z - 4 = 0$$

$\Rightarrow$ $$9x - 8y + z + 11 = 0$$ **(1)**

Now, vector form of plane is

$$\vec{r} \cdot (9\hat{i} - 8\hat{j} + \hat{k}) = -11$$ **(1)**

Also, distance of $(9, -8, -10)$ from the above plane

$$= \left|\frac{9(9) - 8(-8) + 1(-10) + 11}{\sqrt{9^2 + (-8)^2 + 1^2}}\right| = \left|\frac{81 + 64 - 10 + 11}{\sqrt{81 + 64 + 1}}\right|$$

$$\left[\therefore D = \left|\frac{Ax + by + Cz + D}{\sqrt{A^2 + B^2 + C^2}}\right|\right]$$

$$= \left|\frac{146}{\sqrt{146}}\right| = \sqrt{146} \text{ units}$$ **(1)**

54. Given lines are $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$

and $$\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$$

On comparing both equations of lines with $\vec{r} = \vec{a} + \lambda\vec{b}$ respectively, we get

$$\vec{a_1} = \hat{i} + \hat{j} - \hat{k}, \vec{b_1} = 3\hat{i} - \hat{j}$$

and $$\vec{a_2} = 4\hat{i} - \hat{k}, \vec{b_2} = 2\hat{i} + 3\hat{k}$$ **(1)**

Now, $$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix}$$

$$= \hat{i}(-3-0) - \hat{j}(9-0) + \hat{k}(0+2)$$

$$= -3\hat{i} - 9\hat{j} + 2\hat{k}$$ **(1)**

and $\vec{a_2} - \vec{a_1} = (4\hat{i} - \hat{k}) - (\hat{i} + \hat{j} - \hat{k}) = 3\hat{i} - \hat{j}$ **(1)**

Now, $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = (3\hat{i} - \hat{j}) \cdot (-3\hat{i} - 9\hat{j} + 2\hat{k})$

$$= -9 + 9 = 0$$

Hence, given lines are coplanar. **(1)**

Now, cartesian equations of given lines are

$$\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$$

and $$\frac{x-4}{2} = \frac{y-0}{0} = \frac{z+1}{3}.$$

Then, equation of plane containing them is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$\Rightarrow$ $$\begin{vmatrix} x-1 & y-1 & z+1 \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = 0$$ **(1)**

$\Rightarrow$ $(x-1)(-3-0) - (y-1)(9-0) + (z+1)(0+2) = 0$

$\Rightarrow$ $$-3x + 3 - 9y + 9 + 2z + 2 = 0$$

$\therefore$ $$3x + 9y - 2z = 14$$ **(1)**

55. Do same as Q.No. 45. [Ans. 1 unit]

56. Do same as Q.No.54. [Ans. $x - 2y + z = 0$]

57. Do same as Q. No. 27.

[Ans. $\vec{r}\cdot(2\hat{i} + \hat{j} - 2\hat{k}) - 3 = 0$ and $\vec{r}\cdot(-\hat{i} + 2\hat{j} + 2\hat{k}] - 3 = 0$]

58. Do same as Q. No. 38. [Ans. $(1, -2, 7)$]

59. Do same as Q. No. 32. (1½)

We get required equation of plane

$$\vec{r}\,(-9\hat{i} - 3\hat{j} + \hat{k}) + 14 = 0 \quad \text{...(i)}$$

Also, given equation of line is

$$\vec{r} = 3\hat{i} - \hat{j} - \hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k}) \quad \text{...(ii)}$$

This intersect the plane (i), so

$$[(3+2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (-1 + \lambda)\hat{k}]\cdot(9\hat{i} - 3\hat{j} + \hat{k}) + 14 = 0$$

$\Rightarrow \quad -9(3+2\lambda) - 3(-1 - 2\lambda) + (-1 + \lambda) + 14 = 0$

$\Rightarrow \quad -27 - 18\lambda + 3 + 6\lambda - 1 + \lambda + 14 = 0$

$\Rightarrow \quad -11\lambda - 11 = 0$

$\Rightarrow \quad \lambda = -1$ (1½)

On putting $\lambda = -1$ in Eq. (ii), the required point of intersection is

$$\vec{r} = 3\hat{i} - \hat{j} - \hat{k} - 1(2\hat{i} - 2\hat{j} + \hat{k})$$

$\therefore \quad \vec{r} = \hat{i} + \hat{j} - 2\hat{k}$ (1)

Thus, intersection point of the line and the plane is $(1, 1, -2)$.

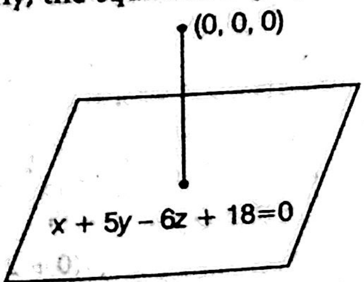
60. Do same as Q. No. 48.

[Ans. $3x - 4y + 3z - 19 = 0$; $\frac{6}{\sqrt{34}}$ units]

61. (i) Do same as Q. No. 28.

[Ans. $x + 5y - 6z + 18 = 0$] (2)

(ii) Clearly, the equation of perpendicular line is



$$\frac{x-0}{1} = \frac{y-0}{5} = \frac{z-0}{-6}$$

$$\Rightarrow \quad \frac{x}{1} = \frac{y}{5} = \frac{z}{-6} \quad (1)$$

Clearly, any point on the above line will be of the form $(\lambda, 5\lambda, -6\lambda)$.

$\therefore$ The coordinates of foot of perpendicular will be $(\lambda, 5\lambda, -6\lambda)$ for some λ. (1)

Since, the foot of perpendicular lie on the plane, therefore we have

$\lambda + 5(5\lambda) - 6(-6\lambda) + 18 = 0$

$\Rightarrow \quad 26\lambda + 36\lambda + 18 = 0$

$\Rightarrow \quad 62\lambda = -18$

$\Rightarrow \quad \lambda = -\frac{9}{31}$

So, the coordinates of foot of perpendicular are $\left(\frac{-9}{31}, \frac{-45}{31}, \frac{54}{31}\right)$ (1)

Now, the length of perpendicular

$$= \sqrt{\left(\frac{-9}{31} - 0\right)^2 + \left(\frac{-45}{31} - 0\right)^2 + \left(\frac{54}{31} - 0\right)^2}$$

$$= \sqrt{\frac{5022}{31^2}} = \sqrt{\frac{162}{31}}$$

$$= 9\sqrt{\frac{2}{31}} \text{ units} \quad (1)$$

62. Given equation of lines are

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda\,(\hat{i} + 2\hat{j} - \hat{k}) \quad \text{...(i)}$$

and $\quad \vec{r} = (\hat{i} + \hat{j}) + \mu\,(-\hat{i} + \hat{j} - 2\hat{k}) \quad$ ...(ii)

On comparing these equations with standard equation of line, $\vec{r} = \vec{a} + \lambda\vec{b}$, we get

$\vec{a}_1 = \hat{i} + \hat{j}$, $\vec{a}_2 = \hat{i} + \hat{j}$, $\vec{b}_1 = \hat{i} + 2\hat{j} - \hat{k}$ and

$\vec{b}_2 = -\hat{i} + \hat{j} - 2\hat{k}$ (1)

Now, the equation of plane containing both the lines is given by

$$(\vec{r} - \vec{a}_1)\cdot\vec{n} = 0$$

where, $\vec{n}$ is normal to both the lines. (1)

Clearly, $\quad \vec{n} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix}$ (1)

$= \hat{i}(-4+1) - \hat{j}\,(-2+1)\hat{j} + \hat{k}(1 + 2)$

$= -3\hat{i} + 3\hat{j} + 3\hat{k}$ (1/2)

$\therefore$ The required equation of plane is

$$[\vec{r}-(\hat{i}+\hat{j})]\cdot(-3\hat{i}+3\hat{j}+3\hat{k})=0 \quad \textbf{(1/2)}$$

$$\Rightarrow \vec{r}\cdot(-3\hat{i}+3\hat{j}+3\hat{k})=(\hat{i}+\hat{j})\cdot(-3\hat{i}+3\hat{j}+3\hat{k}) \quad \textbf{(1/2)}$$

$$\Rightarrow \vec{r}\cdot(-3\hat{i}+3\hat{j}+3\hat{k})=-3+3=0$$

$$\Rightarrow \vec{r}\cdot(-\hat{i}+\hat{j}+\hat{k})=0 \quad \textbf{(1/2)}$$

Now, the length of perpendicular from the point (2, 1, 4) to the above plane

$$=\frac{|(2\hat{i}+\hat{j}+4\hat{k})\cdot(-\hat{i}+\hat{j}+\hat{k})|}{\sqrt{(-1)^2+1^2+1^2}}$$

$$=\frac{|-2+1+4|}{\sqrt{1+1+1}}$$

$$=\frac{3}{\sqrt{3}}=\sqrt{3}\text{ units} \quad \textbf{(1)}$$

63. Given equation of lines are

$$\frac{x-1}{-3}=\frac{y-2}{-2k}=\frac{z-3}{2} \quad \text{...(i)}$$

and $$\frac{x-1}{k}=\frac{y-2}{1}=\frac{z-3}{5} \quad \text{...(ii)}$$

Since, the lines (i) and (ii) are perpendicular, therefore $(-3)\cdot k+(-2k)\cdot(1)+2.5=0$

$$\Rightarrow -3k-2k+10=0$$

$$\Rightarrow 5k=10$$

$$\Rightarrow k=2$$

(ii) Do same as Q. No. 43.

[**Ans.** $-22x+19y+5z=31$]

64. Do same as Q. No. 40.

[**Ans.** $\sqrt{189}$ units, (1, 2, 8) and (−5, −10, 11)]

65. Given equations of planes are

$$2x+y-z-3=0 \quad \text{...(i)}$$

and $$5x-3y+4z+9=0 \quad \text{...(ii)} \textbf{ (1)}$$

Let the required equation of plane which passes through the line of intersection of planes (i) and (ii) be

$$(2x+y-z-3)+\lambda(5x-3y+4z+9)=0 \quad \text{...(iii)}$$

$$\Rightarrow x(2+5\lambda)+y(1-3\lambda)+z(-1+4\lambda)+(-3+9\lambda)=0 \quad \text{...(iv)} \textbf{ (1)}$$

Here, DR's of plane are $(2+5\lambda, 1-3\lambda, -1+4\lambda)$. Also, given that the plane (iv) is parallel to the line, whose equation is

$$\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$$

DR's of the line are (2, 4, 5).

Since, the plane is parallel to the line. Therefore, normal to the plane is perpendicular to the line,

i.e. $a_1a_2+b_1b_2+c_1c_2=0$ **(1)**

Here, $a_1=2+5\lambda, b_1=1-3\lambda, c_1=-1+4\lambda$

and $a_2=2, b_2=4, c_2=5$

$$\therefore (2+5\lambda)2+(1-3\lambda)4+(-1+4\lambda)5=0$$

$$\Rightarrow 4+10\lambda+4-12\lambda-5+20\lambda=0$$

$$\Rightarrow 18\lambda+3=0$$

$$\Rightarrow \lambda=-\frac{3}{18}=-\frac{1}{6} \quad \textbf{(1½)}$$

On putting $\lambda=-\frac{1}{6}$ in Eq. (iii), we get the required equation of plane as

$$(2x+y-z-3)-\frac{1}{6}(5x-3y+4z+9)=0$$

$$\Rightarrow 12x+6y-6z-18-5x+3y-4z-9=0$$

$$\therefore 7x+9y-10z-27=0 \quad \textbf{(1½)}$$

66. Do same as Q. No. 51. [**Ans.** $7x-8y+3z+25=0$]

67. Do same as Q. No. 51. [**Ans.** $7x+2y-7z-26=0$]

68. Given equations of lines are

$$\vec{r}=(2\hat{i}+\hat{j}-3\hat{k})+\lambda(\hat{i}+2\hat{j}+5\hat{k}) \quad \text{...(i)}$$

and $$\vec{r}=(3\hat{i}+3\hat{j}+2\hat{k})+\mu(3\hat{i}-2\hat{j}+5\hat{k}) \quad \text{...(ii)}$$

On comparing Eqs. (i) and (ii) with the vector equation of line $\vec{r}=\vec{a}+\lambda\vec{b}$ respectively, we get

$$\vec{a}_1=2\hat{i}+\hat{j}-3\hat{k}, \vec{b}_1=\hat{i}+2\hat{j}+5\hat{k}$$

and $$\vec{a}_2=3\hat{i}+3\hat{j}+2\hat{k}, \vec{b}_2=3\hat{i}-2\hat{j}+5\hat{k} \quad \textbf{(1)}$$

Now, the required plane which contains the lines (i) and (ii) will pass through $\vec{a}_1=2\hat{i}+\hat{j}-3\hat{k}$.

Also, the required plane has $\vec{b}_1$ and $\vec{b}_2$ parallel to it.

$\therefore$ The normal vector to the required plane is

$$\vec{n}=\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 5\\ 3 & -2 & 5\end{vmatrix}$$

$$=\hat{i}(10+10)-\hat{j}(5-15)+\hat{k}(-2-6)$$

$$=20\hat{i}+10\hat{j}-8\hat{k} \quad \textbf{(1)}$$

$\therefore$ The vector equation of required plane is given by $(\vec{r}-\vec{a})\cdot\vec{n}=\vec{0}$.

$$\Rightarrow \vec{r}\cdot\vec{n}=\vec{a}_1\cdot\vec{n} \quad [\because \vec{a}=\vec{a}_1]$$

$$\Rightarrow \vec{r}\cdot(20\hat{i}+10\hat{j}-8\hat{k})=(2\hat{i}+\hat{j}-3\hat{k})\cdot(20\hat{i}+10\hat{j}-8\hat{k}) \quad \textbf{(1)}$$

$\Rightarrow \vec{r}\cdot(20\hat{i}+10\hat{j}-8\hat{k}) = 40+10+24 = 74$

$\Rightarrow \quad \vec{r}\cdot(10\hat{i}+5\hat{j}-4\hat{k}) = 37$ [dividing by 2]...(iii)

which is the required equation of plane.
Also, its cartesian equation is given by

$$10x + 5y - 4z = 37 \qquad \textbf{(1)}$$

[$\because$ vector form of plane $\vec{r}\cdot(a_1\hat{i}+a_2\hat{j}+a_3\hat{k}) = d$ can be written in its cartesian form as $a_1x + a_2y + a_3z = d$]

Now, we have to show that the line

$$\vec{r} = (2\hat{i}+5\hat{j}+2\hat{k}) + P(3\hat{i}-2\hat{j}+5\hat{k}) \qquad \text{...(iv)}$$

lies in the plane (iii). **(1)**

The above line will lie on plane (iii) when it passes through the point $\vec{a} = 2\hat{i}+5\hat{j}+2\hat{k}$ of line (iv) and it is parallel to line (iv).

$\therefore \quad \vec{a}\cdot(10\hat{i}+5\hat{j}-4\hat{k}) = 37$

$\Rightarrow \quad (2\hat{i}+5\hat{j}+2\hat{k})\cdot(10\hat{i}+5\hat{j}-4\hat{k}) = 37$

$\Rightarrow \quad 20+25-8 = 37$

$\Rightarrow \quad 37 = 37$

$\Rightarrow \vec{a}$ lies on plane whose equation is given by Eq. (iii) and $(3\hat{i}-2\hat{j}+5\hat{k})\cdot(10\hat{i}+5\hat{j}-4\hat{k})$
$= 30-10-20-0$

Therefore, the plane is parallel to the line.
Hence, line (iv) lies on the plane (iii). **(1)**

69. Do same as Q. No. 61.
[**Ans.** $x - y + 3z - 2 = 0$, $(3, 1, 0)$ and $\sqrt{11}$ units]

70. Equation of plane passing through point $P(1, 1, 1)$ is given by

$$a(x-1) + b(y-1) + c(z-1) = 0 \qquad \text{....(i)} \ \textbf{(1)}$$

[$\because$ equation of plane passing through (x_1, y_1, z_1) is given as $a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$]

Given equation of line is

$$\vec{r} = (-3\hat{i}+\hat{j}+5\hat{k}) + \lambda(3\hat{i}-\hat{j}-5\hat{k}) \qquad \text{...(ii)}$$

DR's of the line are $(3, -1, -5)$ and the line passes through point $(-3, 1, 5)$.
Now, as the plane (i) contains line (ii), so

$$a(-3-1) + b(1-1) + c(5-1) = 0$$

[as plane contains a line, it means point of line lies on a plane.]

$\Rightarrow \quad -4a + 4c = 0 \qquad \text{...(iii)} \ \textbf{(1)}$

$\Rightarrow \quad 4a = 4c$

$\Rightarrow \quad a = c$

Since, plane contains a line, so normal to the plane is perpendicular to the line.

$\therefore \quad 3a - b - 5c = 0 \qquad \text{...(iv)}$

[$\because aa_1 + bb_1 + cc_1 = 0$]

On putting $a = c$ in Eq. (iv), we get

$3c - b - 5c = 0 \qquad \textbf{(1)}$

$\Rightarrow \quad -b - 2c = 0$

$\Rightarrow \quad b = -2c$

On putting $a = c$ and $b = -2c$ in Eq. (i), we get the required equation of plane as

$$c(x-1) - 2c(y-1) + c(z-1) = 0$$

On dividing both sides by c, we get

$x - 1 - 2y + 2 + z - 1 = 0$

$\Rightarrow \quad x - 2y + z = 0 \qquad \text{...(v)} \ \textbf{(1½)}$

Now, we have to show that the above plane (v) contains the line

$$\vec{r} = (-\hat{i}+2\hat{j}+5\hat{k}) + \mu(\hat{i}-2\hat{j}-5\hat{k}) \qquad \text{...(vi)}$$

Vector equation of plane (v) is

$$\vec{r}\cdot(\hat{i}-2\hat{j}+\hat{k}) = 0 \qquad \text{...(vii)}$$

The plane (vii) will contains line (vi), if
(i) it passes through $-\hat{i}+2\hat{j}+5\hat{k}$
(ii) it is parallel to the line
We have, $(-\hat{i}+2\hat{j}+5\hat{k})\cdot(\hat{i}-2\hat{j}+\hat{k})$

$= -1-4+5 = 0 \quad [\because \vec{a}\cdot\vec{n} = d]$

So, the plane passes through the point $-\hat{i}+2\hat{j}+5\hat{k}$) and $(\hat{i}-2\hat{j}-5\hat{k})\cdot(\hat{i}-2\hat{j}+\hat{k}) = 0$

$[\because \vec{b_1}\cdot\vec{b_2} = 0]$

$\Rightarrow \quad (1)(1) - 2(-2) - 5(1) = 0$

$\Rightarrow \quad 1 + 4 - 5 = 0$

$\Rightarrow \quad 0 = 0$, which is true.

Hence, the plane contains the given line. **(1½)**

71. Do same as Q. No. 40.

[**Ans.** Foot of perpendicular $= (1, 3, 0)$, perpendicular distance $= \sqrt{6}$ units, image point $= (-1, 4, -1)$]

Solutions Objective

1. (*a*) Let the points be

$P = (4, 3, -5)$ and $Q = (-2, 1, -8)$.

Now, $|PQ| = \sqrt{(-2-4)^2 + (1-3)^2 + (-8+5)^2}$

$= \sqrt{36 + 4 + 9} = \sqrt{49} = 7$

$\therefore$ DC's of line PQ are

$$l = \frac{x_2 - x_1}{|PQ|}, m = \frac{y_2 - y_1}{|PQ|} \text{ and } n = \frac{z_2 - z_1}{|PQ|}$$

$$\therefore \quad l = \frac{6}{7}, m = \frac{2}{7}, n = \frac{3}{7}$$

2. (*d*) Since, DC's of a line are $\left(\frac{1}{c}, \frac{1}{c}, \frac{1}{c}\right)$

$$\therefore \left(\frac{1}{c}\right)^2 + \left(\frac{1}{c}\right)^2 + \left(\frac{1}{c}\right)^2 = 1 \Rightarrow c^2 = 3 \Rightarrow c = \pm\sqrt{3}$$

3. (*d*) Let $\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1} = k$ [say]

Any point on the line is $P(3k+5, -k+7, k-2)$.
To determine the intersection point, equation second satisfying the point

$$P(3k+5, -k+7, k-2).$$

$$\therefore \quad \frac{3k+5+3}{-36} = \frac{-k+7-3}{2} = \frac{k-2-6}{4}$$

$$\Rightarrow \quad \frac{3k+8}{-36} = \frac{-k+4}{2} \Rightarrow k = \frac{16}{3}$$

$\therefore$ Point is P is $\left(21, \frac{5}{3}, \frac{10}{3}\right)$

4. (*a*) Equation of line passing through (x_1, y_1, z_1) and parallel to the line $\frac{x-x_2}{a} = \frac{y-y_2}{b} = \frac{z-z_2}{c}$ is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

$\therefore$ Equation of line is $\frac{x-2}{4} = \frac{y+1}{-3} = \frac{z-1}{5}$.

5. (*c*) Given lines are

$$\frac{x-1}{0} = \frac{y-2}{0} = \frac{z}{1} \text{ and } \frac{x}{1} = \frac{y+1}{0} = \frac{z}{0}$$

$$\therefore \cos\theta = 0\cdot1 + 0\cdot0 + 1\cdot0 = 0 \quad \Rightarrow \quad \theta = 90°$$

6. (*a*) Given lines can be rewritten as

$$\frac{x-1}{-3} = \frac{y-2}{2\alpha} = \frac{z-3}{2} \text{ and } \frac{x-1}{3\alpha} = \frac{y-1}{1} = \frac{z-6}{-5}$$

Since, lines are perpendicular.

$$\therefore \quad a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow \quad (-3)(3\alpha) + 2\alpha(1) + 2(-5) = 0$$

$$\Rightarrow \quad -9\alpha + 2\alpha - 10 = 0$$

$$\therefore \quad \alpha = -\frac{10}{7}$$

7. (*c*) The equation of the plane perpendicular to Z-axis and passing through (x_1, y_1, z_1) is $z - z_1 = 0$.

$$\therefore \quad z - 5 = 0$$

8. (*b*) Plane can be rewritten as $\frac{x}{6} + \frac{y}{-4} + \frac{z}{3} = 1$

So, the intercepts are 6, − 4, 3.

9. (*a*) The distance from origin (0, 0, 0) to the plane $6x - 3y + 2z - 14 = 0$ is

$$d = \frac{|6(0) - 3(0) + 2(0) - 14|}{\sqrt{36+9+4}} = 2$$

10. (*b*) The straight line joining the points (1, 1, 2) and (3, − 2, 1) is $\frac{x-1}{2} = \frac{y-1}{-3} = \frac{z-2}{-1} = r$ [say]

Then, the point is $(2r+1, 1-3r, 2-r)$ which lies on $3x + 2y + z = 6$.

$$\therefore \quad 3(2r+1) + 2(1-3r) + 2 - r = 6 \Rightarrow r = 1$$

So, the required point is (3, − 2, 1).

11. (*d*) The equation of plane containing the given line is

$$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0.$$

Since, normal to the plane is perpendicular to the line.

$$\therefore \quad al + bm + cn = 0$$

12. (*b*) Equation of plane is $a(x-1) + b(y+1) + cz = 0$.

[$\because$ plane is passing through (1, − 1, 0)]

Above plane also passing through (0, 2, − 1).

$$\therefore \quad -a + 3b - c = 0$$

Also, $2a - b + 3c = 0 \Rightarrow \frac{a}{8} = \frac{b}{1} = \frac{c}{-5}$

Hence, equation of plane is

$$8x + y - 5z - 7 = 0$$

13. (*b*) Given line can be rewritten as

$$\frac{x - \frac{1}{3}}{\frac{2b}{3}} = \frac{y-3}{-1} = \frac{z-1}{a}$$

Given plane $3x + y + 2z + 6 = 0$ is parallel to the above line.

$$\therefore \frac{2b}{3}\cdot 3 + 1\cdot(-1) + 2\cdot a = 0 \Rightarrow 2a + 2b = 1$$

$$\Rightarrow \quad 3a + 3b = \frac{3}{2}$$