

Areas by integration-CBSE

4. Using integration, prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0$, $x = 4$, $y = 4$ and $y = 0$ into three equal parts.
5. Using method of integration, find the area of the triangle whose vertices are $(1, 0)$, $(2, 2)$ and $(3, 1)$.
6. Using integration, find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.
7. Using integration, find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.
8. Find the area of the region in the first quadrant enclosed by the X-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.
9. Using integration, find the area of the region : $\{(x, y) : 0 \leq 2y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 3\}$.
10. Using integration, find the area of region bounded by the triangle whose vertices are $(-2, 1)$, $(0, 4)$ and $(2, 3)$.
11. Find the area bounded by the circle $x^2 + y^2 = 16$ and the line $\sqrt{3}y = x$ in the first quadrant, using integration.
12. Using the method of integration, find the area of the ΔABC , coordinates of whose vertices are $A(4, 1)$, $B(6, 6)$ and $C(8, 4)$.
13. Find the area enclosed between the parabola $4y = 3x^2$ and the straight line $3x - 2y + 12 = 0$.
14. Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \leq 2ax, y^2 \geq ax; x, y \geq 0\}$.
15. Using integration, find the area of the triangular region whose vertices are $(2, -2)$, $(4, 3)$ and $(1, 2)$.

26 Marks Questions

1. Using integration, find the area of ΔABC , the coordinates of whose vertices are $A(2, 5)$, $B(4, 7)$ and $C(6, 2)$.
2. Using integration, find the area of triangle whose vertices are $(2, 3)$, $(3, 5)$ and $(4, 4)$.
3. Find the area of the region lying above X-axis and included between the circle $x^2 + y^2 = 8x$ and inside the parabola $y^2 = 4x$.

Test 1

16. Using integration, find the area of the region bounded by the curves $y = \sqrt{4 - x^2}$, $x^2 + y^2 - 4x = 0$ and the X-axis.

17. Using integration, find the area of the region in the first quadrant enclosed by the Y-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

18. Using integration, find the area of the triangle formed by positive X-axis and tangent and normal to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$.

19. Using integration, find the area of the region bounded by the line $x - y + 2 = 0$, the curve $x = \sqrt{y}$ and Y-axis.

20. Find the area of the region $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$, using method of integration.

21. Using integration, find the area of the region in the first quadrant enclosed by the X-axis, the line $y = x$ and the circle $x^2 + y^2 = 18$.

22. Using integration, find the area of the region bounded by the curves $y = |x + 1| + 1$, $x = -3$, $x = 3$ and $y = 0$.

23. Using integration, find the area of ΔPQR , coordinates of whose vertices are $P(2, 0)$, $Q(4, 5)$ and $R(6, 3)$.

24. Using integration, find the area of the region bounded by the triangle whose vertices are $(-1, 2)$, $(1, 5)$ and $(3, 4)$.

25. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

Or Using integration, find the area of the following region.

$$\left\{ (x, y) : \frac{x^2}{9} + \frac{y^2}{4} \leq 1 \leq \frac{x}{3} + \frac{y}{2} \right\}$$

26. Using integration, find the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.

27. Using integration, find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

28. Using integration, find the area of the region bounded by the curves $y = x^2$ and $y = x$.

29. Find the area of the region $\{(x, y) : y^2 \leq 6ax$ and $x^2 + y^2 \leq 16a^2\}$, using method of integration.

30. Find the area of the region bounded by the parabola $y = x^2$ and the line $y = |x|$.

Or Find the area of the region given by $\{(x, y) : x^2 \leq y \leq |x|\}$.

31. Using integration, find the area of the circle $x^2 + y^2 = 16$, which is exterior to the parabola $y^2 = 6x$.

32. Using method of integration, find the area of region bounded by lines $3x - 2y + 1 = 0$, $2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$.

33. Using integration, find the area of the region bounded by the lines $3x - y - 3 = 0$, $2x + y - 12 = 0$, $x - 2y - 1 = 0$.

34. Using integration, find the area of the region bounded by the lines $5x - 2y - 10 = 0$, $x + y - 9 = 0$ and $2x - 5y - 4 = 0$.

35. Find the area of the region $\{(x, y) : x^2 + y^2 \leq 4, x + y \geq 2\}$.

36. Find the area of the region $\{(x, y) : (x^2 + y^2) \leq 1 \leq x + y\}$.

37. Sketch the graph of $y = |x + 3|$ and evaluate the area under the curve $y = |x + 3|$ above X-axis and between $x = -6$ to $x = 0$.

Test 1

Test-1
38. Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \leq 16, x^2 \leq 6y\}$

39. Find the area of circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.

Test-1
40. Using integration, find the area of the following region.

$$\{(x, y) : |x - 1| \leq y \leq \sqrt{5 - x^2}\}$$

Answers

1) 7 sq. unit.

2) $3\frac{1}{2} \text{ sq. unit.}$

3) $\frac{4}{3} (8+3\pi) \text{ sq. unit.}$

4) $\frac{16}{3} \text{ sq. unit}$

5) $3\frac{1}{2} \text{ sq. unit}$

6) $\frac{8\pi}{3} - 2\sqrt{3}$

7) 8

8) 4π

9) $\frac{23}{6}$

10) 4

11) $\frac{4\pi}{3}$

12) 7

13) 27

14) $a^2 \left(\frac{\pi}{4} - \frac{2}{3} \right)$

15) $\frac{13}{2}$

16) $\frac{4\pi}{3} - \sqrt{3}$

17) 4π

18) $2\sqrt{3}$

19) $10\frac{1}{3}$

20) $\frac{1}{3\sqrt{2}} + \frac{9\pi}{8} - \frac{9}{4} \sin^{-1}\left(\frac{1}{3}\right)$

21) $\frac{9\pi}{4}$

22) 16

23) 7

24) 4

25) $\frac{3}{2} (\pi - 2)$

26) $\frac{7}{2}$

27) $9/8$

28) $1/6$

29) $\frac{4\sqrt{3}}{3} a^2 + \frac{16a^2\pi}{3}$

30) $1/3$

31) $\frac{4}{3} (8\pi - \sqrt{3})$

32) $13\frac{1}{2}$

33) 10

34) $2\frac{1}{2}$

35) $\pi - 2$

36) $\frac{\pi}{4} - \frac{1}{2}$

37) 9

$$38.) \frac{4\sqrt{3}}{3} + \frac{16\pi}{3}$$

$$39.) \frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

$$40.) \left[\frac{5}{2} \left(\sin^{-1} \frac{2}{\sqrt{5}} + \sin^{-1} \frac{1}{\sqrt{5}} \right) - \frac{1}{2} \right]$$