

Chapter 3 - Matrices

Exercise 3.1

Question 1:

In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write:

- (i) The order of the matrix
- (ii) The number of elements,
- (iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$

Solution 1:

(i) In the given matrix, the number of rows is 3 and the number of columns is 4. Therefore, the order of the matrix is 3×4 .

(ii) Since the order of the matrix is 3×4 , there are $3 \times 4 = 12$ elements in it.

(iii) $a_{13} = 19, a_{21} = 35, a_{33} = -5, a_{24} = 12, a_{23} = \frac{5}{2}$

Question 2:

If a matrix has 24 elements, what are the possible order it can have? What, if it has 13 elements?

Solution 2:

We know that if a matrix is of the order $m \times n$, it has mn elements.

Thus, to find all the possible orders of a matrix having 24 elements, we have to find all the ordered pairs of natural numbers whose product is 24.

The ordered pairs are: $(1, 24), (24, 1), (2, 12), (12, 2), (3, 8), (8, 3), (4, 6)$, and $(6, 4)$

Hence, the possible orders of a matrix having 24 elements are:

$1 \times 24, 24 \times 1, 2 \times 12, 12 \times 2,$
 $3 \times 8, 8 \times 3, 4 \times 6$ and 6×4

$(1, 13)$ and $(13, 1)$ are the ordered pairs of natural numbers whose product is 13.

Hence the possible orders of matrix having 13 elements are 1×13 and 13×1 .

Question 3:

If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Solution 3:

We know that if a matrix is of the order $m \times n$, it has mn elements.

Thus, to find all the possible orders of a matrix having 18 elements, we have to find all the ordered pairs of natural numbers whose products is 18.

The ordered pairs are: $(1,18), (18,1), (2,9), (9,2), (3,6)$, and $(6,3)$

Hence, the possible orders of a matrix having 18 elements are :

$1 \times 18, 18 \times 1, 2 \times 9, 9 \times 2, 3 \times 6$ and 6×3

$(1,5)$ and $(5,1)$ are the ordered pairs of natural numbers whose product is 5.

Hence, the possible orders of a matrix having 5 elements are 1×5 and 5×1 .

Question 4:

Construct a 3×4 matrix, whose elements are given by

$$(i) a_{ij} = \frac{1}{2}|-3i + j| \quad (ii) a_{ij} = 2i - j$$

Solution 4:

In general, a 3×4 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

(i) Given : $a_{ij} = \frac{1}{2}|-3i + j|, i = 1, 2, 3$ and $j = 1, 2, 3, 4$

Thus, we have

$$\bullet a_{11} = \frac{1}{2}|-3 \times 1 + 1| = \frac{1}{2}|-3 + 1| = \frac{1}{2}|-2| = \frac{2}{2} = 1$$

$$\bullet a_{21} = \frac{1}{2}|-3 \times 2 + 1| = \frac{1}{2}|-6 + 1| = \frac{1}{2}|-5| = \frac{5}{2}$$

$$\bullet a_{31} = \frac{1}{2}|-3 \times 3 + 1| = \frac{1}{2}|-9 + 1| = \frac{1}{2}|-8| = 4$$

$$\bullet a_{12} = \frac{1}{2}|-3 \times 1 + 2| = \frac{1}{2}|-3 + 2| = \frac{1}{2}|-1| = \frac{1}{2}$$

$$\bullet \quad a_{22} = \frac{1}{2}|-3 \times 2 + 2| = \frac{1}{2}|-6 + 2| = \frac{1}{2}|-4| = \frac{4}{2} = 2$$

$$\bullet \quad a_{32} = \frac{1}{2}|-3 \times 3 + 2| = \frac{1}{2}|-9 + 2| = \frac{1}{2}|-7| = \frac{7}{2}$$

$$\bullet \quad a_{13} = \frac{1}{2}|-3 \times 1 + 3| = \frac{1}{2}|-3 + 3| = 0$$

$$\bullet \quad a_{23} = \frac{1}{2}|-3 \times 2 + 3| = \frac{1}{2}|-6 + 3| = \frac{1}{2}|-3| = \frac{3}{2}$$

$$\bullet \quad a_{33} = \frac{1}{2}|-3 \times 3 + 3| = \frac{1}{2}|-9 + 3| = \frac{1}{2}|-6| = \frac{6}{2} = 3$$

$$\bullet \quad a_{14} = \frac{1}{2}|-3 \times 1 + 4| = \frac{1}{2}|-3 + 4| = \frac{1}{2}|1| = \frac{1}{2}$$

$$\bullet \quad a_{24} = \frac{1}{2}|-3 \times 2 + 4| = \frac{1}{2}|-6 + 4| = \frac{1}{2}|-2| = \frac{2}{2} = 1$$

$$\bullet \quad a_{34} = \frac{1}{2}|-3 \times 3 + 4| = \frac{1}{2}|-9 + 4| = \frac{1}{2}|-5| = \frac{5}{2}$$

Therefore, the required matrix is $A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

(ii) Given : $a_{ij} = 2i - j$, $i = 1, 2, 3$ and $j = 1, 2, 3, 4$

Thus, we have

$$\bullet \quad a_{11} = 2 \times 1 - 1 = 2 - 1 = 1$$

- $a_{21} = 2 \times 2 - 1 = 4 - 1 = 3$
- $a_{31} = 2 \times 3 - 1 = 6 - 1 = 5$
- $a_{12} = 2 \times 1 - 2 = 2 - 2 = 0$
- $a_{22} = 2 \times 2 - 2 = 4 - 2 = 2$
- $a_{32} = 2 \times 3 - 2 = 6 - 2 = 4$
- $a_{13} = 2 \times 1 - 3 = 2 - 3 = -1$
- $a_{23} = 2 \times 2 - 3 = 4 - 3 = 1$
- $a_{33} = 2 \times 3 - 3 = 6 - 3 = 3$
- $a_{14} = 2 \times 1 - 4 = 2 - 4 = -2$
- $a_{24} = 2 \times 2 - 4 = 4 - 4 = 0$
- $a_{34} = 2 \times 3 - 4 = 6 - 4 = 2$

Therefore, the required matrix is $A = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$

Question 5:

Find the value of x, y and z from the following equation:

$$(i) \begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$$

$$(ii) \begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

$$(iii) \begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

Solution 5:

$$(i) \text{ Given : } \begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$$

As the given matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$$x=1, y=4 \text{ and } z=3$$

$$(ii) \begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

As the given matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$$x + y = 6, xy = 8, 5 + z = 5$$

$$\text{Now, } 5 + z = 5 \Rightarrow z = 0$$

Using $(x - y)^2 = (x + y)^2 - 4xy$, we get

$$\Rightarrow (x - y)^2 = 36 - 32 = 4$$

$$\Rightarrow x - y = \pm 2$$

- When $x - y = 2$ and $x + y = 6$, we get $x = 4$ and $y = 2$
- When $x - y = -2$ and $x + y = 6$ we get $x = 2$ and $y = 4$

$$\therefore x = 4, y = 2, \text{ and } z = 0 \text{ or } x = 2, y = 4, \text{ and } z = 0$$

$$(iii) \begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

As the two matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$$x + y + z = 9 \quad \dots(1)$$

$$x + z = 5 \quad \dots(2)$$

$$y + z = 7 \quad \dots(3)$$

From (1) and (2), we have:

$$y + 5 = 9 \Rightarrow y = 4$$

From (3), we have:

$$4 + z = 7 \Rightarrow z = 3$$

$$\therefore x + z = 5 \Rightarrow x = 2$$

$$\therefore x = 2, y = 4, \text{ and } z = 3$$

Question 6:

Find the value of a , b , c , and d from the equation:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

Solution 6:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

As the two matrices are equal, their corresponding elements are also equal. Comparing the corresponding elements, we get:

$$a - b = -1 \quad \dots(1)$$

$$2a - b = 0 \quad \dots(2)$$

$$2a + c = 5 \quad \dots(3)$$

$$3c + d = 13 \quad \dots(4)$$

From (2), we have:

$$b = 2a$$

Then, from (1), we have:

$$a - 2a = -1$$

$$\Rightarrow a = 1$$

$$\Rightarrow b = 2$$

Now, from (3), we have:

$$2 \times 1 + c = 5$$

$$\Rightarrow c = 3$$

From (4) we have:

$$3 \times 3 + d = 13$$

$$\Rightarrow 9 + d = 13 \Rightarrow d = 4$$

$$\therefore a = 1, b = 2, c = 3, \text{ and } d = 4$$

Question 7:

$A = [a_{ij}]_{m \times n}$ is a square matrix, if

- A. $m < n$
- B. $m > n$
- C. $m = n$
- D. None of these

Solution 7:

The correct answer is C.

It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

Therefore, $A = [a_{ij}]_{m \times n}$ is a square matrix, if $m = n$.

Question 8:

Which of the given values of x and y make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

- A. $x = \frac{-1}{3}, y = 7$
- B. Not possible to find
- C. $y = 7, x = \frac{-2}{3}$
- D. $x = \frac{-1}{3}, y = \frac{-2}{3}$

Solution 8:

The Correct answer is B.

It is given that $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

Equating the corresponding elements, we get:

$$3x + 7 = 0 \Rightarrow x = -\frac{7}{3}$$

$$5 = y - 2 \Rightarrow y = 7$$

$$y + 1 = 8 \Rightarrow y = 7$$

$$2 - 3x = 4 \Rightarrow x = -\frac{2}{3}$$

We find that on comparing the corresponding elements of the two matrices, we get two different values of x , which is not possible.

Hence, it is not possible to find the values of x and y for which the given matrices are equal.

Question 9:

The number of all possible matrices of order 3×3 with each entry 0 or 1 is:

- A. 27
- B. 18
- C. 81
- D. 512

Solution 9:

The correct answer is D.

The given matrix of the order 3×3 has 9 elements and each of these elements can be either 0 or 1.

Now, each of the 9 elements can be filled in two possible ways.

Therefore, by the multiplication principle, the required number of possible matrices is $2^9 = 512$

Exercise 3.2

Question 1:

Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

Find each of the following

- (i) $A + B$ (ii) $A - B$
 (iii) $3A - C$ (iv) AB
 (v) BA

Solution 1:

$$(i) \quad A + B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2+1 & 4+3 \\ 3-2 & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$$

$$(ii) \quad A - B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2-1 & 4-3 \\ 3-(-2) & 2-5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$$

$$(iii) \quad 3A - C = 3 \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \times 2 & 3 \times 4 \\ 3 \times 3 & 3 \times 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

- (iv) Matrix A has 2 columns. This number is equal to the number of rows in matrix B.
 Therefore, AB is defined as:

$$\begin{aligned}
 AB &= \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \\
 &= \begin{bmatrix} 2(1)+4(-2) & 2(3)+4(5) \\ 3(1)+2(-2) & 3(3)+2(5) \end{bmatrix} \\
 &= \begin{bmatrix} 2-8 & 6+20 \\ 3-4 & 9+10 \end{bmatrix} \\
 &= \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}
 \end{aligned}$$

- (v) matrix B has 2 columns. This number is equal to the number of rows in matrix A. Therefore, BA is defined as:

$$\begin{aligned}
 BA &= \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1(2)+3(3) & 1(4)+3(2) \\ -2(2)+5(3) & -2(4)+5(2) \end{bmatrix} \\
 &= \begin{bmatrix} 2+9 & 4+6 \\ -4+15 & -8+10 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}
 \end{aligned}$$

Question 2:

Compute the following:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

(ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

(iv) $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

Solution 2:

$$(i) \begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a+a & b+b \\ -b+b & a+a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$$

$$(ii) \begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix}$$

$$= \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

$$(iii) \begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

$$(iv) \begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad (\because \cos^2 x = 1)$$

Question 3:

Compute the indicated products

$$(i) \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$(ii) \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [2 \ 3 \ 4]$$

$$(iii) \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$(iv) \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$(v) \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$$

$$(vi) \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$$

Solution 3:

$$(i) \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$= \begin{bmatrix} a(a)+b(b) & a(-b)+b(a) \\ -b(a)+a(b) & -b(-b)+a(a) \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & -ab+ab \\ -ab+ab & b^2+a^2 \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1(2) & 1(3) & 1(4) \\ 2(2) & 2(3) & 2(4) \\ 3(2) & 3(3) & 3(4) \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$$

$$(iii) \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)-2(2) & 1(2)-2(3) & 1(3)-2(1) \\ 2(1)+3(2) & 2(2)+3(3) & 2(3)+3(1) \end{bmatrix}$$

$$= \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix} = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

$$(iv) \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2(1)+3(0)+4(3) & 2(-3)+3(2)+4(0) & 2(5)+3(4)+4(5) \\ 3(1)+4(0)+5(3) & 3(-3)+4(2)+5(0) & 3(5)+4(4)+5(5) \\ 4(1)+5(0)+6(3) & 4(-3)+5(2)+6(0) & 4(5)+5(4)+6(5) \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & 15+16+25 \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix} = \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

$$(v) \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2(1)+1(-1) & 2(0)+1(2) & 2(1)+1(1) \\ 3(1)+2(-1) & 3(0)+2(2) & 3(1)+2(1) \\ -1(1)+1(-1) & -1(0)+1(2) & -1(1)+1(1) \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 0+2 & 2+1 \\ 3-2 & 0+4 & 3+2 \\ -1-1 & 0+2 & -1+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

$$\text{(vi)} \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3(2)-1(1)+3(3) & 3(-3)-1(0)+3(1) \\ -1(2)+0(1)+2(3) & -1(-3)+0(0)+2(1) \end{bmatrix}$$

$$= \begin{bmatrix} 6-1+9 & -9-0+3 \\ -2+0+6 & 3+0+2 \end{bmatrix} = \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$$

Question 4:

If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$, and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$, then

Compute $(A+B)$ and $(B-C)$. Also, verify that $A+(B-C) = (A+B)-C$

Solution 4:

$$A+B = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3 & 2-1 & -3+2 \\ 5+4 & 0+2 & 2+5 \\ 1+2 & -1+0 & 1+3 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$$

$$B - C = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\begin{aligned} A + (B - C) &= \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1+(-1) & 2+(-2) & -3+0 \\ 5+4 & 0+(-1) & 2+3 \\ 1+1 & -1+2 & 1+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix} \end{aligned}$$

$$(A + B) - C = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4-4 & 1-1 & -1-2 \\ 9-0 & 2-3 & 7-2 \\ 3-1 & -1-(-2) & 4-3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

Hence, we have verified that $A + (B - C) = (A + B) - C$.

Question 5:

If $A = \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix}$ and $B = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$ then compute $3A - 5B$

Solution 5:

$$\begin{aligned}
 3A - 5B &= 3 \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix} - 5 \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix} \\
 &= \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
 \end{aligned}$$

Question 6:

Simplify $\cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$

Solution 6:

$$\begin{aligned}
 &\cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix} \\
 &= \begin{bmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ -\sin \theta \cos \theta & \cos^2 \theta \end{bmatrix} + \begin{bmatrix} \sin^2 \theta & -\sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix} \\
 &= \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & \cos \theta \sin \theta - \sin \theta \cos \theta \\ -\sin \theta \cos \theta + \sin \theta \cos \theta & \cos^2 \theta + \sin^2 \theta \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (\because \sin^2 \theta = 1)
 \end{aligned}$$

Question 7:

Find X and Y , if

(i) $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$$\text{(ii)} \quad 2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \text{ and } 3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

Solution 7:

$$\text{(i)} \quad X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \quad \dots(1)$$

$$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad \dots(2)$$

Adding equations (1) and (2), we get:

$$2X = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 7+3 & 0+0 \\ 2+0 & 5+3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

$$\therefore X = \frac{1}{2} \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

$$\text{Now, } X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 7-5 & 0-0 \\ 2-1 & 5-4 \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\text{(ii)} \quad 2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \quad \dots(3)$$

$$3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} \quad \dots(4)$$

Multiplying equation (3) with (2), we get

$$2(2X + 3Y) = 2 \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 4X + 6Y = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} \quad \dots(5)$$

Multiplying equation (4) with (3), we get

$$3(3X + 2Y) = 3 \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$\Rightarrow 9X + 6Y = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix} \quad \dots(6)$$

From (5) and (6), we have

$$(4X + 6Y) - (9X + 6Y) = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} - \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix}$$

$$\Rightarrow -5X = \begin{bmatrix} 4-6 & 6-(-6) \\ 8-(-3) & 0-15 \end{bmatrix} = \begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix}$$

$$\therefore X = -\frac{1}{5} \begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$$

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

Now,

$$\Rightarrow 2 \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix}$$

$$\Rightarrow 3Y = \begin{bmatrix} 2 - \frac{4}{5} & 3 + \frac{24}{5} \\ 4 + \frac{22}{5} & 0 - 6 \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

$$\therefore Y = \frac{1}{3} \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

Question 8:

Find X , if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Solution 8:

$$2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\therefore X = \frac{1}{2} \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

Question 9:

Find X and Y , if $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Solution 9:

$$2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we have:

$$2+y=5 \Rightarrow y=3$$

$$2x+2=8 \Rightarrow x=3$$

$$\therefore x=3 \text{ and } y=3$$

Question 10:

Solve the equation for x, y, z and t if

$$2 \begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3 \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

Solution 10:

$$2 \begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3 \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$$2x + 3 = 9$$

$$\Rightarrow 2x = 6$$

$$\Rightarrow x = 3$$

$$2y = 12$$

$$\Rightarrow y = 6$$

$$2z - 3 = 15$$

$$\Rightarrow 2z = 18$$

$$\Rightarrow z = 9$$

$$2t + 6 = 18$$

$$\Rightarrow 2t = 12$$

$$\Rightarrow t = 6$$

$$\therefore x = 3, y = 6, z = 9, \text{ and } t = 6$$

Question 11:

If $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find values of x and y.

Solution 11:

$$x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x - y \\ 3x + y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$$2x - y = 10 \quad \text{and} \quad 3x + y = 5$$

Adding these two equations, we have:

$$5x = 15 \Rightarrow x = 3$$

Now, $3x + y = 5$

$$\Rightarrow y = 5 - 3x$$

$$\Rightarrow y = 5 - 9 = -4$$

$$\therefore x = 3 \text{ and } y = -4$$

Question 12:

Given $3 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the values of x , y , z and w .

Solution 12:

$$3 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$$3x = x + 4$$

$$\Rightarrow 2x = 4$$

$$\Rightarrow x = 2$$

$$3x = 6 + x + y$$

$$\Rightarrow 2y = 6 + x = 6 + 2 = 8$$

$$\Rightarrow y = 4$$

$$3w = 2w + 3$$

$$\Rightarrow w = 3$$

$$3z = -1 + z + w$$

$$\Rightarrow 2z = -1 + w = -1 + 3 = 2$$

$$\Rightarrow z = 1$$

$$\therefore x = 2, y = 4, z = 1, \text{ and } w = 3$$

Question 13:

If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)F(y) = F(x+y)$.

Solution 13:

$$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}, F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{R.H.S : } F(x+y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{L.H.S : } F(x)F(y)$$

$$= \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
 &= \begin{bmatrix} \cos x \cos y - \sin x \sin y + 0 & -\cos x \sin y - \sin x \cos y + 0 & 0 \\ \sin x \cos y + \cos x \sin y + 0 & -\sin x \sin y + \cos x \cos y + 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= F(x+y)
 \end{aligned}$$

$$\therefore F(x)F(y) = F(x+y)$$

Question 14:

Show that

$$(i) \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Solution 14:

$$(i) \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5(2) - 1(3) & 5(1) - 1(4) \\ 6(2) + 7(3) & 6(1) + 7(4) \end{bmatrix}$$

$$= \begin{bmatrix} 10 - 3 & 5 - 4 \\ 12 + 21 & 6 + 28 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2(5) + 1(6) & 2(-1) + 1(7) \\ 3(5) + 4(6) & 3(-1) + 4(7) \end{bmatrix}$$

$$= \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$\text{(ii)} \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4) \\ 0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4) \\ 1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) & 1(0)+1(1)+0(4) \end{bmatrix}$$

$$\begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1(1)+1(0)+0(1) & -1(2)+1(-1)+0(1) & -1(3)+1(0)+0(0) \\ 0(1)+(-1)(0)+1(1) & 0(2)+(-1)(1)+1(1) & 0(3)+(-1)(0)+1(0) \\ 2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Question 15:

Find $A^2 - 5A + 6I$ if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Solution 15:

We have $A^2 = A \times A$

$$A^2 = AA \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2(2)+0(2)+1(1) & 2(0)+0(1)+1(-1) & 2(1)+0(3)+1(0) \\ 2(2)+1(2)+3(1) & 2(0)+1(1)+3(-1) & 2(1)+1(3)+3(0) \\ 1(2)+(-1)(2)+0(1) & 1(0)+(-1)(1)+0(-1) & 1(1)+(-1)(3)+0(0) \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

Substituting the matrices in the given equation : $A^2 - 5A + 6I$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5 \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\begin{aligned}
 &= \begin{bmatrix} 5-10 & -1-0 & 2-5 \\ 9-10 & -2-5 & 5-15 \\ 0-5 & -1+5 & -2-0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \\
 &= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \\
 &= \begin{bmatrix} -5+6 & -1+0 & -3+0 \\ -1+0 & -7+6 & -10+0 \\ -5+0 & 4+0 & -2+6 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}
 \end{aligned}$$

Question 16:

If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$

Solution 16:

$$A^2 = AA = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

Now, $A^3 = A^2 \cdot A$

$$= \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

Substituting the matrices in the given equation $A^3 - 6A^2 + 7A + 2I$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6 \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 21+7+2 & 0+0+0 & 34+14+0 \\ 12+0+0 & 8+14+2 & 23+7+0 \\ 34+14+0 & 0+0+0 & 55+21+2 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}$$

$$= \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$$\therefore A^3 - 6A^2 + 7A + 2I = 0$$

Question 17:

If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$

Solution 17:

$$A^2 = A \cdot A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 3(3)+(-2)(4) & 3(-2)+(-2)(-2) \\ 4(3)+(-2)(4) & 4(-2)+(-2)(-2) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

Now $A^2 = kA - 2I$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

Comparing the corresponding elements, we have:

$$3k - 2 = 1$$

$$\Rightarrow 3k = 3$$

$$\Rightarrow k = 1$$

Thus, the value of k is 1.

Question 18:

If $A = \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that

$$I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

Solution 18:

L.H.S.

$$\begin{aligned}
 & I + A \\
 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix} \\
 & \begin{bmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{bmatrix} \quad \dots(1)
 \end{aligned}$$

R.H.S :

$$\begin{aligned}
 & (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \\
 &= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix} \right) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \\
 &= \begin{bmatrix} 1 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \\
 &= \begin{bmatrix} \cos \alpha + \sin \alpha \tan \frac{\alpha}{2} & -\sin \alpha + \cos \alpha \tan \frac{\alpha}{2} \\ -\cos \alpha \tan \frac{\alpha}{2} + \sin \alpha & \sin \alpha \tan \frac{\alpha}{2} + \cos \alpha \end{bmatrix} \quad \dots (2)
 \end{aligned}$$

$$\begin{aligned}
&= \begin{bmatrix} 1 - 2\sin^2 \frac{\alpha}{2} + 2\sin \frac{\alpha}{2} - \cos \frac{\alpha}{2} \tan \frac{\alpha}{2} & -2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} + \left(2\cos^2 \frac{\alpha}{2} - 1\right) \tan \frac{\alpha}{2} \\ -\left(2\cos^2 \frac{\alpha}{2} - 1\right) \tan \frac{\alpha}{2} + 2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} & 2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \tan \frac{\alpha}{2} + 1 - 2\sin^2 \frac{\alpha}{2} \end{bmatrix} \\
&= \begin{bmatrix} 1 - 2\sin^2 \frac{\alpha}{2} + 2\sin^2 \frac{\alpha}{2} & -2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} + 2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} - \tan \frac{\alpha}{2} \\ -2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} + \tan \frac{\alpha}{2} + 2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} & 2\sin^2 \frac{\alpha}{2} + 1 - 2\sin^2 \frac{\alpha}{2} \end{bmatrix} \\
&= \begin{bmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{bmatrix}
\end{aligned}$$

Thus, from (1) and (2), we get **L.H.S. = R.H.S.**

Question 19:

A trust fund has Rs 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of :

- (a) Rs 1,800 (b) Rs 2,000

Solution 19:

- (a) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond pays
Rs $(30000 - x)$.

It is given that the first bond pays 5% interest per year and the second bond pay 7% interest per year.

Therefore, in order to obtain an annual total interest of Rs 1800, we have:

$$\left[x \quad (30000 - x) \right] \begin{bmatrix} 5 \\ 100 \\ 7 \\ 100 \end{bmatrix} = 1800 \quad \left[S.I \text{ for 1 year} = \frac{\text{Principal} \times \text{Rate}}{100} \right]$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000 - x)}{100} = 1800$$

$$\Rightarrow 5x + 210000 - 7x = 180000$$

$$\Rightarrow 210000 - 2x = 180000$$

$$\Rightarrow 2x = 210000 - 180000$$

$$\Rightarrow 2x = 30000$$

$$\Rightarrow x = 15000$$

Thus, in order to obtain an annual total interest of Rs 1800, the trust fund should invest Rs 15000 in the first bond and the remaining Rs 15000 in the second bond.

(b) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond will be Rs $(30000 - x)$.

Therefore, in order to obtain an annual total interest of Rs 2000, we have :

$$\left[x \quad (30000 - x) \right] \begin{bmatrix} 5 \\ 100 \\ 7 \\ 100 \end{bmatrix} = 2000$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000 - x)}{100} = 2000$$

$$\Rightarrow 5x + 210000 - 7x = 200000$$

$$\Rightarrow 210000 - 2x = 200000$$

$$\Rightarrow 2x = 210000 - 200000$$

$$\Rightarrow 2x = 10000$$

$$\Rightarrow x = 5000$$

Thus, in order to obtain an annual total interest of Rs 2000, the trust fund should invest Rs 5000 in the first bond and the remaining Rs 25000 in the second bond.

Question 20:

The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs 80, Rs 60 and Rs 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Solution 20:

The bookshop has 10 dozen chemistry books, 8 dozen physics books, and 10 dozen economics books.

The selling prices of a chemistry book, a physics book, and an economics book are respectively given as Rs 80, Rs 60 and Rs 40.

The total amount of money that will be received from the sale of all these books can be represented in the form of a matrix as :

$$\begin{aligned} & 12[10 \quad 8 \quad 10] \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix} \\ &= 12[10 \times 80 + 8 \times 60 + 10 \times 40] \\ &= 12(800 + 480 + 400) \\ &= 12(1680) \\ &= 20160 \end{aligned}$$

Thus, the bookshop will receive Rs 20160 from the sale of all these books.

Question 21:

Assume X, Y, Z, W and P are the matrices of order $2 \times n, 3 \times k, 2 \times p, n \times 3$ and $p \times k$ respectively. The restriction on n, k and p so that $PY + WY$ will be defined are:

- A. $k = 3, p = n$
- B. k is arbitrary, $p = 2$
- C. p is arbitrary, $k = 3$
- D. $k = 2, p = 3$

Solution 21:

Matrices P and Y are of the orders $p \times k$ and $3 \times k$ respectively.

Therefore, matrix PY will be defined if $k = 3$.

Consequently, PY will be of the order $p \times k$.

Matrices W and Y are of the orders $n \times 3$ and $3 \times k$ respectively.

Since the number of columns in W is equal to the number of rows in Y , matrix WY is well-defined and is of the order $n \times k$.

Matrices PY and WY can be added only when their orders are the same.

However, PY is of the order $p \times k$ and WY is of the order $n \times k$. Therefore, we must have $p = n$.

Thus, $k = 3$ and $p = n$ are the restrictions on n, k , and p so that $PY + WY$ will be defined.

Question 22:

Assume X, Y, Z, W and P are matrices of order $2 \times n, 3 \times k, 2 \times p, n \times 3$ and $p \times k$ respectively. If $n = p$, then the order of the matrix $7X - 5Z$ is

- A. $p \times 2$
- B. $2 \times n$
- C. $n \times 3$
- D. $p \times n$

Solution 22:

The correct answer is B.

Matrix X is of the order $2 \times n$.

Therefore, matrix $7X$ is also of the same order.

Matrix Z is of the order $2 \times p$, i.e, $2 \times n$ [since $n = p$]

Therefore, matrix $5Z$ is also of the same order.

Now, both the matrices $7X$ and $5Z$ are of the order $2 \times n$.

Thus, matrix $7X - 5Z$ is well-defined and is of the order $2 \times n$.

Exercise 3.3**Question 1:**

Find the transpose of each of the following matrices:

(i) $\begin{bmatrix} 5 \\ 1/2 \\ -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

Solution 1:

$$\text{(i) Let } A = \begin{bmatrix} 5 \\ 1/2 \\ -1 \end{bmatrix}, \text{ then } A^T = \begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$$

$$\text{(ii) Let } A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, \text{ then } A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$\text{(iii) Let } A = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}, \text{ then } A^T = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$$

Question 2:

$$\text{If } A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}, \text{ then verify that}$$

$$\text{(i) } (A + B)' = A' + B'$$

$$\text{(ii) } (A - B)' = A' - B'$$

Solution 2:

We have:

$$A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}, B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

$$\text{(i) } A + B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$$

$$\therefore (A + B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

$$A' + B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

Hence, we have verified that $(A + B)' = A' + B'$

$$(ii) A - B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$$

$$\therefore (A - B)' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

$$A' - B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

Hence, we have verified that $(A - B)' = A' - B'$.

Question 3:

If $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that

(i) $(A+B)' = A' + B'$

(ii) $(A-B)' = A' - B'$

Solution 3:

(i) It is known that $A = (A')'$

Therefore, we have:

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$B' = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}$$

$$\therefore (A+B)' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

$$A' + B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

Thus, we verified that $(A+B)' = A'+B'$

$$(ii) A - B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$$

$$\therefore (A - B)' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

$$A' - B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

Thus, we have verified that $(A - B)' = A' - B'$.

Question 4:

If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)'$

Solution 4:

We know that $A = (A')'$

$$\therefore A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$\therefore A + 2B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2 \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$$

$$\therefore (A + 2B)' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$$

Question 5:

For the matrices A and B , verify that $(AB)' = B'A'$ where

$$(i) A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = [-1 \quad 2 \quad 1]$$

$$(ii) A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = [1 \quad 5 \quad 7]$$

Solution 5:

$$(i) AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} [-1 \quad 2 \quad 1] = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

$$\text{Now, } A' = [1 \quad -4 \quad 3], B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} [1 \quad -4 \quad 3] = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

Hence, we have verified $(AB)' = B'A'$.

$$(ii) \quad AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} [1 \ 5 \ 7] = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

$$\text{Now, } A' = [0 \ 1 \ 2], B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} [0 \ 1 \ 2] = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Hence, we have verified that $(AB)' = B'A'$.

Question 6:

If (i) $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, then verify that $A'A = 1$

(ii) $A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$, then verify that $A'A = 1$

Solution 6:

$$A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$\begin{aligned}
 A' A &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \\
 &= \begin{bmatrix} (\cos \alpha)(\cos \alpha) + (-\sin \alpha)(-\sin \alpha) & (\cos \alpha)(\sin \alpha) + (-\sin \alpha)(\cos \alpha) \\ (\sin \alpha)(\cos \alpha) + (\cos \alpha)(-\sin \alpha) & (\sin \alpha)(\sin \alpha) + (\cos \alpha)(\cos \alpha) \end{bmatrix} \\
 &= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I
 \end{aligned}$$

Hence, we have verified that $A' A = 1$.

$$(ii) \quad A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix}$$

$$\begin{aligned}
 A' A &= \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} \\
 &= \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} \\
 &= \begin{bmatrix} (\sin \alpha)(\sin \alpha) + (-\cos \alpha)(-\cos \alpha) & (\sin \alpha)(\cos \alpha) + (-\cos \alpha)(\sin \alpha) \\ (\cos \alpha)(\sin \alpha) + (\sin \alpha)(-\cos \alpha) & (\cos \alpha)(\cos \alpha) + (\sin \alpha)(\sin \alpha) \end{bmatrix} \\
 &= \begin{bmatrix} \sin^2 \alpha + \cos^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \cos^2 \alpha + \sin^2 \alpha \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I
 \end{aligned}$$

Hence, we have verified that $A' A = 1$.

Question 7:

(i) Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix

(ii) Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix

Solution 7:

(i) We have:

$$A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = A$$

$$\therefore A' = A$$

Hence, A is a symmetric matrix.

(ii) We have:

$$A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$$

$$\therefore A' = -A$$

Hence, A is a skew – symmetric matrix.

Question 8:

For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(i) $(A + A')$ is a symmetric matrix

(ii) $(A - A')$ is a skew symmetric matrix

Solution 8:

Given : $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, then $A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

(i) $A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

$\therefore (A + A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = A + A'$

Hence, $(A + A')$ is a symmetric matrix.

(ii) $A - A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$(A - A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -(A - A')$

Hence, $(A - A')$ is a skew-symmetric matrix.

Question 9:

Find $\frac{1}{2}(A + A')$ and $\frac{1}{2}(A - A')$, when $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

Solution 9:

The given matrix is $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$, then

$$A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$A + A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A + A') = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Now, } A - A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} - \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A - A') = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

Question 10:

Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

(i) $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

Solution 10:

Let $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$, then $A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

$$\text{Now, } A + A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(A + A') = \frac{1}{2} \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$$

$$\text{Now, } P' = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} = P$$

Thus, $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

$$\text{Now, } A - A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

$$\text{Let } Q = \frac{1}{2}(A - A') = \frac{1}{2} \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\text{Now, } Q' = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = -Q$$

Thus, $Q = \frac{1}{2}(A - A')$ is a skew-symmetric matrix.

Representing A as the sum of P and Q :

$$P + Q = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = A$$

$$\text{(ii) Let } A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}, \text{ then } A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\text{Now, } A + A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(A + A') = \frac{1}{2} \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\text{Now, } P' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = P$$

Thus, $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

$$\text{Now, } A - A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Let } Q = \frac{1}{2}(A - A') = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Now, } Q' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -Q$$

Thus $Q = \frac{1}{2}(A - A')$ is a skew-symmetric matrix.

Representing A as the sum of P and Q

$$P + Q = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = A$$

$$\text{(iii) Let } A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}, \text{ then } A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\text{Now, } A + A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$P = \frac{1}{2}(A + A') = \frac{1}{2} \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}$$

$$\text{Now, } P' = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} = P$$

Thus, $P = \frac{1}{2}(A + A')$ is symmetric matrix.

$$\text{Now, } A - A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

$$\text{Let } Q = \frac{1}{2}(A - A') = \frac{1}{2} \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$$

$$\text{Now, } Q' = \begin{bmatrix} 0 & -\frac{5}{2} & -\frac{3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = -Q$$

Thus, $Q = \frac{1}{2}(A - A')$ is a skew-symmetric matrix.

Representing A as the sum of P and Q :

$$P+Q = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = A$$

(iv) Let $A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$, then $A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

Now $A + A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$

Let $P = \frac{1}{2}(A + A') = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$

Now $P' = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = P$

Thus $P = \frac{1}{2}(A + A')$ is symmetric matrix.

Now, $A - A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$

Let $Q = \frac{1}{2}(A - A') = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

Now $Q' = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} = -Q$

Thus $Q = \frac{1}{2}(A - A')$ is a skew symmetric matrix.

Representing A as the sum of P and Q.

$$P+Q = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} = A$$

Question 11:

If A, B are symmetric matrices of same order, then $AB - BA$ is a

- A. Skew symmetric matrix
- B. Symmetric matrix
- C. Zero matrix
- D. Identity matrix

Solution 11:

The correct answer is A.

A and B are symmetric matrices, therefore, we have:

$$A' = A \text{ and } B' = B \quad \dots\dots(1)$$

$$\begin{aligned} \text{Consider } (AB - BA)' &= (AB)' - (BA)' && \left[\because \quad \quad \quad = A' - B' \right] \\ &= B' A' - A' B' && \left[\because \quad \quad \quad = B' A' \right] \\ &= BA - AB && \left[\text{by (1)} \right] \\ &= -(AB - BA) \end{aligned}$$

$$\therefore (AB - BA)' = -(AB - BA)$$

Thus, $(AB - BA)$ is a skew-symmetric matrix.

Question 12:

If $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$, then $A + A' = I$, if the value of α is

- A. $\frac{\pi}{6}$
- B. $\frac{\pi}{3}$
- C. π
- D. $\frac{3\pi}{2}$

Solution 12:

The correct answer is **B**.

$$A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$\Rightarrow A' = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

Now $A + A' = I$

$$\therefore \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} + \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos \alpha & 0 \\ 0 & 2\cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Comparing the corresponding elements of the two matrices, we have:

$$\cos \alpha = \frac{1}{2}$$

$$\alpha = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\therefore \alpha = \frac{\pi}{3}$$

Exercise 3.4

Question 1:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

Solution 1:

$$\text{Let } A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow \frac{1}{5} R_2$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$$

Post – multiplying both sides by, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A \cdot A^{-1}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} \cdot I$$

$$\text{Thus, } A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

Question 2:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

Solution 2:

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

We know that $AI = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - R_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

Question 3:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

Solution 3:

$$\text{Let } A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - 3R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

Question 4:

Find the inverse of each of the matrices, if it exists. $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

Solution 4:

$$\text{Let } A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{2}R_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 5R_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{2} & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + 3R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ -\frac{5}{2} & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow -2R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$$

Question 5:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

Solution 5:

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{2}R_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 7R_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{7}{2} & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -\frac{7}{2} & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow 2R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

Question 6:

Find the inverse of each of the matrices, if it exists. $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

Solution 6:

$$\text{Let } A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{2}R_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - R_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_2 - 5R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -\frac{1}{2} & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow 2R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

Question 7:

Find the inverse of each matrices, if exists. $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

Solution 7:

$$\text{Let } A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$$

We know that $AA^{-1} = IA$

$$\therefore \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \quad (C_1 \rightarrow C_2 - 2C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \quad (C_2 \rightarrow C_2 - C_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} \quad (C_1 \rightarrow C_1 - C_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

Question 8:

Find the inverse of each of the matrices, if it exists. $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

Solution 8:

$$\text{Let } A = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - 3R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$

Question 9:

Find the inverse of each of the matrices, if it exists. $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

Solution 9:

$$\text{Let } A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - 2R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A \quad (R_1 \rightarrow R_2 - 3R_1)$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$$

Question 10:

Find The inverse of each of the matrices, if it exists. $\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

Solution 10:

$$\text{Let } A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$$

We know that $A = AI$

$$\therefore \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \quad (C_1 \rightarrow C_1 + 2C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \quad (C_2 \rightarrow C_2 + C_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix} \quad \left(C_2 \rightarrow \frac{1}{2} C_2 \right)$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

Question 11:

Find the inverse of each of the matrices, if it exists. $\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

Solution 11:

$$\text{Let } A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$$

We know that $A = AI$

$$\therefore \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (C_2 \rightarrow C_2 + 3C_1)$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix} \quad (C_1 \rightarrow C_1 - C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} \quad \left(C_1 \rightarrow \frac{1}{2}C_1 \right)$$

$$\therefore A^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

Question 12:

Find the inverse of each of the matrices, if it exists. $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

Solution 12:

$$\text{Let } A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

We Know that $A = IA$

$$\therefore \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left(R_1 \rightarrow \frac{1}{6}R_1 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 + 2R_1)$$

Now, in the above equation, we can see all the zeros in the second row of the matrix on the L.H.S,

Therefore, A^{-1} does not exist.

Question 13:

Find the inverse of each of the matrices, if it exists. $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Solution 13:

$$\text{Let } A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 + R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} A \quad (R_2 \rightarrow R_2 + R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A \quad (R_1 \rightarrow R_1 + R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

Question 14:

Find the inverse of each of the matrices, if it exist.

$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

Solution 14:

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - \frac{1}{2}R_2$, we have:

$$\begin{bmatrix} 0 & 0 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 1 \end{bmatrix} A$$

Now, in the above equation, we can see all the zeros in the first row of the matrix on the L.H.S. Therefore, A^{-1} does not exist.

Question 15:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

Solution 15:

$$\text{Let } A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 + 3R_1$ and $R_3 \rightarrow R_3 - 2R_1$, we have:

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + 3R_3$ and $R_2 \rightarrow R_2 + 8R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow R_3 + R_2$, we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 25 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -15 & 1 & 9 \end{bmatrix} A$$

Applying $R_3 \rightarrow \frac{1}{25}R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - 10R_3$, and $R_2 \rightarrow R_2 - 21R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

Question 16:

Find the inverse of each of the matrices, if it exists. $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Solution 16:

$$\text{Let } A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{2}R_1$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 5R_1$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow R_3 - R_2$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow 2R_3$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + \frac{1}{2}R_3$, and $R_2 \rightarrow R_2 - \frac{5}{2}R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Question 17:

Matrices A and B will be inverse of each other only if

- A. $AB = BA$
- B. $AB = 0, BA = I$
- C. $AB = BA = 0$
- D. $AB = BA = I$

Solution 17:

We know that if A is a square matrix of order m , and if there exists another square matrix B of the same order m , such that $AB = BA = I$, then B is said to be the inverse of A . In this case, it is clear that A is the inverse of B .

Thus, matrices A and B will be inverses of each other only if $AB = BA = I$.

Miscellaneous Solutions**Question 1:**

Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bA)^n = a^n I + na^{n-1}bA$, where I is the identity matrix of order 2 and $n \in \mathbf{N}$

Solution 1:

It is given that $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

To show: $P(n) : (aI + bA)^n = a^n I + na^{n-1}bA, n \in \mathbf{N}$

We shall prove the result by using the principal of mathematical induction.

For $n = 1$, we have:

$$\mathbf{P(1): (aI + bA) = aI + ba^0 A = aI + bA}$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

That is,

$$\mathbf{P(k): (aI + bA)^k = a^k I + ka^{k-1} bA}$$

Now, we prove that the result is true for $n = k + 1$.

Consider

$$\begin{aligned} (aI + bA)^{k+1} &= (aI + bA)^k (aI + bA) \\ &= (a^k I + ka^{k-1} bA)(aI + bA) \\ &= a^{k+1} I + ka^k bAI + a^k bIA + ka^{k-1} b^2 A^2 \\ &= a^{k+1} I + (k+1)a^k bA + ka^{k-1} b^2 A^2 \quad \dots (1) \end{aligned}$$

$$\text{Now, } A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

From (1), we have:

$$\begin{aligned} (aI + bA)^{k+1} &= a^{k+1} I + (k+1)a^k bA + 0 \\ &= a^{k+1} I + (k+1)a^k bA \end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principal of mathematical induction, we have:

$$(aI + bA)^n = a^n I + na^{n-1} bA \text{ where } A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, n \in \mathbf{N}$$

Question 2:

$$\text{If } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \text{ prove that } A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in \mathbf{N}$$

Solution 2:

$$\text{It is given that } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

To show: $P(n) A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in \mathbf{N}$

We shall prove the result by using the principles of mathematical induction.

For $n = 1$, we have:

$$P(1) = \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix} = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = A$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$$P(k): A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

That is Now, we prove that the result is true for $n = k + 1$.

Now, $A^{k+1} = A \cdot A^k$

$$\begin{aligned} &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix} \\ &= \begin{bmatrix} 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \\ 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \\ 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \end{bmatrix} \\ &= \begin{bmatrix} 3 \cdot 3^{(k+1)-1} & 3 \cdot 3^{(k+1)-1} & 3 \cdot 3^{(k+1)-1} \\ 3 \cdot 3^{(k+1)-1} & 3 \cdot 3^{(k+1)-1} & 3 \cdot 3^{(k+1)-1} \\ 3 \cdot 3^{(k+1)-1} & 3 \cdot 3^{(k+1)-1} & 3 \cdot 3^{(k+1)-1} \end{bmatrix} \end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principal of mathematical induction, we have:

$$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in \mathbf{N}$$

Question 3:

If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ where n is any positive integer

Solution 3:

It is given that $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

To prove: $P(n): A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}, n \in \mathbf{N}$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1): A^1 = \begin{bmatrix} 1+2 & -4 \\ 1 & 1-2 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} A$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

That is,

$$P(k): A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}, n \in \mathbf{N}$$

Now, we prove that the result is true for $n = k + 1$.

Consider

$$\begin{aligned} A^{k+1} &= A^k \cdot A \\ &= \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 3(1+2k) - 4k & -4(1+2k) + 4k \\ 3k + 1 - 2k & -4k - 1(1-2k) \end{bmatrix} \\ &= \begin{bmatrix} 3 + 6k - 4k & -4 - 8k + 4k \\ 3k + 1 - 2k & -4k - 1 + 2k \end{bmatrix} \\ &= \begin{bmatrix} 3 + 2k & -4 - 4k \\ 1 + k & -1 - 2k \end{bmatrix} \\ &= \begin{bmatrix} 1 + 2(k+1) & -4(k+1) \\ 1 + k & 1 - 2(k+1) \end{bmatrix} \end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principal of mathematical induction, we have:

$$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}, n \notin \mathbf{N}$$

Question 4:

If A and B are symmetric matrices, prove that $AB - BA$ is a skew symmetric matrix.

Solution 4:

It is given that A and B are symmetric matrices. Therefore, we have:

$$A' = A \text{ and } B' = B \quad \dots(1)$$

$$\begin{aligned} \text{Now, } (AB - BA)' &= (AB)' - (BA)' && [(A - B)' = A' - B'] \\ &= B'A' - A'B' && [(AB)' = B'A'] \\ &= BA - AB && [\text{Using (1)}] \\ &= -(AB - BA) \end{aligned}$$

$$\therefore (AB - BA)' = -(AB - BA)$$

Thus, $(AB - BA)$ is a skew – symmetric matrix.

Question 5:

Show that the matrix $B'AB$ is symmetric or skew symmetric according as A is symmetric or skew symmetric.

Solution 5:

We suppose that A is a symmetric matrix, then $A' = A \quad \dots (1)$

Consider

$$\begin{aligned} (B'AB)' &= \{B'(AB)\}' \\ &= (AB)'(B)' && [(AB)' = B'A'] \\ &= B'A'(B) && [(B')' = B] \end{aligned}$$

$$= B'(A'B)$$

$$= B'(AB) \quad [\text{Using (1)}]$$

$$\therefore (B'AB)' = B'AB$$

Thus, if A is a symmetric matrix, then $B'AB$ is a symmetric matrix.

Now, we suppose that A is a skew – symmetric matrix.

Then, $A' = -A$

Consider

$$(B'AB)' = [B'(AB)]' = (AB)'(B)'$$

$$= (B'A')B = B'(-A)B$$

$$= -B'AB$$

$$\therefore (B'AB)' = -B'AB$$

Thus, if A is a skew – symmetric matrix then $B'AB$ is a skew – symmetric matrix.

Hence, if A is a asymmetric or skew – symmetric matrix, then $B'AB$ is a symmetric or skew – symmetric matrix accordingly.

Question 6:

Solve system of linear equations, using matrix method.

$$2x - y = -2$$

$$3x + 4y = 3$$

Solution 6:

The given system of equation can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Now,

$$|A| = 8 + 3 = 11 \neq 0$$

Thus, A is non – singular. Therefore, its inverse exists.

$$A^{-1} = \frac{1}{|A|} \text{adj}A = \frac{1}{11} \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{11} \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -8+3 \\ 6+6 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -5 \\ 12 \end{bmatrix} = \begin{bmatrix} -\frac{5}{11} \\ \frac{12}{11} \end{bmatrix}$$

Hence, $x = \frac{-5}{11}$ and $y = \frac{12}{11}$

Question 7:

For what values of x , $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$?

Solution 7:

We have $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$

$$\Rightarrow [1+4+1 \quad 2+0+0 \quad 0+2+2] \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow [6 \quad 2 \quad 4] \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow [6(0)+2(2)+4(x)] = 0$$

$$\Rightarrow [4+4x] = [0]$$

$$\therefore 4 + 4x = 0$$

$$\Rightarrow x = -1$$

Thus, the required value of x is -1 .

Question 8:

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$

Solution 8:

It is given that $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$\begin{aligned} \therefore A^2 &= A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 3(3)+1(-1) & 3(1)+1(2) \\ -1(3)+2(-1) & -1(1)+2(2) \end{bmatrix} \\ &= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \end{aligned}$$

$$\text{L.H.S.} = A^2 - 5A + 7I$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0 = \text{R.H.S.}$$

$$\therefore A^2 - 5A + 7I = 0$$

Question 9:

Find X , if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

Solution 9:

We have:

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow [x+0-2 \quad 0-10+0 \quad 2x-5-3] \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow [x-2 \quad -10 \quad 2x-8] \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow [x(x-2) - 40 + 2x - 8] = 0$$

$$\Rightarrow [x^2 - 2x - 40 + 2x - 8] = [0]$$

$$\Rightarrow [x^2 - 48] = [0]$$

$$\therefore x^2 - 48 = 0$$

$$\Rightarrow x^2 = 48$$

$$\Rightarrow x = \pm 4\sqrt{3}$$

Question 10:

A manufacture produces three products X , Y , Z which he sells in two markets.

Annual sales are indicated below:

Market	Products		
	X	Y	Z
I	10000	2000	18000
II	6000	20000	8000

- If unit sale prices of X , Y and Z are Rs 2.50, Rs 1.50 and Rs 1.00, respectively, find the Total revenue in each market with the help of matrix algebra.
- If the unit costs of the above three commodities are Rs 2.00, Rs 1.00 and 50 paise respectively. Find the gross profit.

Solution 10:

- The unit sale prices of X , Y and Z are respectively given as Rs 2.50, Rs 1.50, and Rs 1.00

Consequently, the total revenue in market **I** can be represented in the form of a matrix as :

$$\begin{aligned}
 & [10000 \quad 2000 \quad 18000] \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix} \\
 &= 10000 \times 2.50 + 2000 \times 1.50 + 18000 \times 1.00 \\
 &= 25000 + 3000 + 18000 \\
 &= 46000
 \end{aligned}$$

The total revenue in market **II** can be represented in the form of a matrix as:

$$\begin{aligned}
 & [6000 \quad 20000 \quad 8000] \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix} \\
 &= 6000 \times 2.50 + 20000 \times 1.50 + 8000 \times 1.00 \\
 &= 15000 + 30000 + 8000 \\
 &= 53000
 \end{aligned}$$

Therefore, the total revenue in market **I** is Rs 46000 and the same in market **II** is Rs 53000

- b.** The unit cost price of X , Y and Z are respectively given as Rs 2.00, Rs 1.00 and 50paise.

Consequently, the total cost prices of all the products in the market **I** can be represented in the form of a matrix as:

$$\begin{aligned}
 & [10000 \quad 2000 \quad 18000] \begin{bmatrix} 2.50 \\ 1.00 \\ 0.50 \end{bmatrix} \\
 &= 10000 \times 2.00 + 2000 \times 1.00 + 18000 \times 0.50 \\
 &= 20000 + 2000 + 9000 = 31000
 \end{aligned}$$

Since the total revenue in market **I** is Rs 46000, the gross profit in this market is Rs 46000 – Rs 31000 = Rs 15000.

The total cost prices of all the products in market **II** can be represented in the form of a matrix as:

$$\begin{aligned}
 & [6000 \quad 20000 \quad 8000] \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix} \\
 &= 6000 \times 2.00 + 20000 \times 1.00 + 8000 \times 0.50 \\
 &= 12000 + 20000 + 4000 \\
 &= 36000
 \end{aligned}$$

Since the total revenue in market **II** is Rs 53000, the gross profit in this market is Rs 53000 – Rs 36000 = Rs 17000.

Question 11:

Find the matrix X so that $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Solution 11:

It is given that:

$$X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

The matrix given on the R.H.S. of the equation is a 2×3 matrix and the one given on the L.H.S. of the equation is 2×3 matrix.

Therefore, X has to be a 2×2 matrix.

$$\text{Now, let } X = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

Therefore, we have:

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Equating the corresponding elements of two matrices, we have:

$$a + 4c = -7, \quad 2a + 5c = -8, \quad 3a + 6c = -9$$

$$b + 4d = 2, \quad 2b + 5d = 4, \quad 3b + 6d = 6$$

$$\text{Now, } a + 4c = -7 \Rightarrow a = -7 - 4c \quad \dots (1)$$

$$2a + 5c = -8 \Rightarrow -14 - 8c + 5c = -8 \quad [\text{From (1)}]$$

$$\Rightarrow -3c = 6$$

$$\Rightarrow c = -2$$

$$\therefore a = -7 - 4(-2) = -7 + 8 = 1$$

Now, $b + 4d = 2 \Rightarrow b = 2 - 4d$

$$\therefore 2b + 5d = 4 \Rightarrow 4 - 8d + 5d = 4$$

$$\Rightarrow -3d = 0$$

$$\Rightarrow d = 0$$

$$\therefore b = 2 - 4(0) = 2$$

Thus, $a = 1, b = 2, c = -2, d = 0$

Hence, the required matrix X is $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$.

Question 12:

If A and B are square matrices of the same order such that $AB = BA$, then prove by induction that $AB^n = B^n A$. Further, prove that $(AB)^n = A^n B^n$ for all $n \in \mathbf{N}$

Solution 12:

A and B are square matrices of the same order such that $AB = BA$.

To prove: $P(n): AB^n = B^n A, n \in \mathbf{N}$

For $n = 1$, we have:

$$P(1): AB = BA \quad [\text{Given}]$$

$$\Rightarrow AB^1 = B^1 A$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$$P(k): AB^k = B^k A \quad \dots(1)$$

Now, we prove that the result is true for $n = k + 1$.

$$AB^{k+1} = AB^k \cdot B$$

$$= (B^k A) B \quad [\text{By (1)}]$$

$$= B^k (AB) \quad [\text{Associative law}]$$

$$= B^k (BA) \quad [AB = BA(\text{Given})]$$

$$= (B^k B) A \quad [\text{Associative law}]$$

$$= B^{k+1} A$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have $AB^n = B^n A$, $n \in \mathbf{N}$.

Now, we prove that $(AB)^n = A^n B^n$ for all $n \in \mathbf{N}$

For $n = 1$, we have:

$$(AB)^1 = A^1 B^1 = AB$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$$(AB)^k = A^k B^k \quad \dots\dots\dots(2)$$

Now, we prove that the result is true for $n = k + 1$.

$$\begin{aligned} (AB)^{k+1} &= (AB)^k \cdot (AB) \\ &= (A^k B^k) \cdot (AB) && \text{[By (2)]} \\ &= A^k (B^k A) B && \text{[Associative law]} \\ &= A^k (AB^k) B && \text{[} AB^n = B^n A \text{ for all } n \in \mathbf{N} \text{]} \\ &= (A^k A) \cdot (B^k B) && \text{[Associative law]} \\ &= A^{k+1} B^{k+1} \end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have $(AB)^n = A^n B^n$, for all natural numbers.

Question 13:

Choose the correct answer in the following questions:

If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$ then

- A. $1 + \alpha^2 + \beta\gamma = 0$
- B. $1 - \alpha^2 + \beta\gamma = 0$
- C. $1 - \alpha^2 - \beta\gamma = 0$
- D. $1 + \alpha^2 - \beta\gamma = 0$

Solution 13:

Given : $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

$$\begin{aligned} \text{Then, } A^2 = A.A &= \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \\ &= \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix} \\ &= \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} \end{aligned}$$

$$\text{Now, } A^2 = 1 \Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing the corresponding elements, we have:

$$\alpha^2 + \beta\gamma = 1$$

$$\Rightarrow \alpha^2 + \beta\gamma - 1 = 0$$

$$\Rightarrow 1 - \alpha^2 - \beta\gamma = 0$$

Hence, the correct answer is C.

Question 14:

If the matrix A is both symmetric and skew symmetric, then

- A. A is diagonal matrix
- B. A is a zero matrix
- C. A is a square matrix
- D. None of these

Solution 14:

If A is both symmetric and skew – symmetric matrix, then we should have

$$A' = A \text{ and } A' = -A$$

$$\Rightarrow A = -A$$

$$\Rightarrow A + A = 0$$

$$\Rightarrow 2A = 0$$

$$\Rightarrow A = 0$$

Therefore, A is a zero matrix.

Hence, the correct answer is B.

Question 15:

If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

- A. A
- B. $I - A$
- C. I
- D. $3A$

Solution 15:

$$\begin{aligned}(I + A)^3 - 7A &= I^3 + A^3 + 3I^2A + 3A^2I - 7A \\ &= I + A^3 + 3A + 3A^2 - 7A \\ &= I + A^2 \cdot A + 3A + 3A - 7A && [A^2 = A] \\ &= I + A \cdot A - A \\ &= I + A^2 - A \\ &= I + A - A \\ &= I \\ \therefore (I + A)^3 - 7A &= I\end{aligned}$$