

## Chapter - 07 - Integrals

## Exercise 7.1

**Question 1:**

Find an anti-derivative (or integral) of the function  $\sin 2x$  by the method of inspection.

**Solution 1:**

The anti-derivative of  $\sin 2x$  is a function of  $x$  whose derivative is  $\sin 2x$ . It is known that,

$$\frac{d}{dx}(\cos 2x) = -2 \sin 2x$$

$$\Rightarrow \sin 2x = -\frac{1}{2} \frac{d}{dx}(\cos 2x)$$

$$\therefore \sin 2x = \frac{d}{dx} \left( -\frac{1}{2} \cos 2x \right)$$

Therefore, the anti-derivative of  $\sin 2x$  is  $-\frac{1}{2} \cos 2x$ .

**Question 2:**

Find an anti-derivative (or integral) of the function  $\cos 3x$  by the method of inspection.

**Solution 2:**

The anti-derivative of  $\cos 3x$  is a function of  $x$  whose derivative is  $\cos 3x$ .

It is known that,

$$\frac{d}{dx}(\sin 3x) = 3 \cos 3x$$

$$\Rightarrow \cos 3x = \frac{1}{3} \frac{d}{dx}(\sin 3x)$$

$$\therefore \cos 3x = \frac{d}{dx} \left( \frac{1}{3} \sin 3x \right)$$

Therefore, the anti-derivative of  $\cos 3x$  is  $\frac{1}{3} \sin 3x$ .

**Question 3:**

Find an anti-derivative (or integral) of the function  $e^{2x}$  by the method of inspection.

**Solution 3:**

The anti-derivative of  $e^{2x}$  is the function of  $x$  whose derivative is  $e^{2x}$

It is known that,

$$\frac{d}{dx}(e^{2x}) = 2e^{2x}$$

$$\Rightarrow e^{2x} = \frac{1}{2} \frac{d}{dx}(e^{2x})$$

$$\therefore e^{2x} = \frac{d}{dx} \left( \frac{1}{2} e^{2x} \right)$$

Therefore, the anti-derivative of  $e^{2x}$  is  $\frac{1}{2} e^{2x}$ .

**Question 4:**

Find an anti-derivative (or integral) of the function  $(ax + b)^2$  by the method of inspection.

**Solution 4:**

The anti-derivative of  $(ax + b)^2$  is the function of  $x$  whose derivative is  $(ax + b)^2$ .

It is known that,

$$\frac{d}{dx} (ax+b)^3 = 3a(ax+b)^2$$

$$\Rightarrow (ax+b)^2 = \frac{1}{3a} \frac{d}{dx} (ax+b)^3$$

$$\therefore (ax+b)^2 = \frac{d}{dx} \left( \frac{1}{3a} (ax+b)^3 \right)$$

Therefore, the anti-derivative of  $(ax+b)^2$  is  $\frac{1}{3a} (ax+b)^3$ .

**Question 5:**

Find an anti-derivative (or integral) of the function  $\sin 2x - 4e^{3x}$  by the method of inspection.

**Solution 5:**

The anti-derivative of  $\sin 2x - 4e^{3x}$  is the function of  $x$  whose derivative is  $\sin 2x - 4e^{3x}$

It is known that,

$$\frac{d}{dx} \left( -\frac{1}{2} \cos 2x - \frac{4}{3} e^{3x} \right) = \sin 2x - 4e^{3x}$$

Therefore, the anti derivative of  $(\sin 2x - 4e^{3x})$  is  $\left( -\frac{1}{2} \cos 2x - \frac{4}{3} e^{3x} \right)$ .

**Question 6:**

$$\int (4e^{3x} + 1) dx$$

**Solution 6:**

$$\int (4e^{3x} + 1) dx$$

$$= 4 \int e^{3x} dx + \int 1 dx$$

$$= 4 \left( \frac{e^{3x}}{3} \right) + x + C$$

$$= \frac{4}{3} e^{3x} + x + C$$

where C is an arbitrary constant.

**Question 7:**

$$\int x^2 \left( 1 - \frac{1}{x^2} \right) dx$$

**Solution 7:**

$$\int x^2 \left( 1 - \frac{1}{x^2} \right) dx$$

$$= \int (x^2 - 1) dx$$

$$= \int x^2 dx - \int 1 dx$$

$$= \frac{x^3}{3} - x + C$$

where C is an arbitrary constant.

**Question 8:**

$$\int (ax^2 + bx + c) dx$$

**Solution 8:**

$$\int (ax^2 + bx + c) dx$$

$$= a \int x^2 dx + b \int x dx + c \int 1 dx$$

$$= a \left( \frac{x^3}{3} \right) + b \left( \frac{x^2}{2} \right) + cx + C$$

$$= \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C$$

where C is an arbitrary constant.

**Question 9:**

$$\int (2x^2 + e^x) dx$$

**Solution 9:**

$$\int (2x^2 + e^x) dx$$

$$= 2 \int x^2 dx + \int e^x dx$$

$$= 2 \left( \frac{x^3}{3} \right) + e^x + C$$

$$= \frac{2}{3} x^3 + e^x + C$$

where C is an arbitrary constant.

**Question 10:**

$$\int \left( \sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 dx$$

**Solution 10:**

$$\int \left( \sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 dx$$

$$= \int \left( x + \frac{1}{x} - 2 \right) dx$$

$$= \int x dx + \int \frac{1}{x} dx - 2 \int 1 dx$$

$$= \frac{x^2}{2} + \log|x| - 2x + C$$

where C is an arbitrary constant.

**Question 11:**

$$\int \frac{x^3 + 5x^2 - 4}{x^2} dx$$

**Solution 11:**

$$\int \frac{x^3 + 5x^2 - 4}{x^2} dx$$

$$= \int (x + 5 - 4x^{-2}) dx$$

$$= \int x dx + 5 \int 1 dx - 4 \int x^{-2} dx$$

$$= \frac{x^2}{2} + 5x - 4 \left( \frac{x^{-1}}{-1} \right) + C$$

$$= \frac{x^2}{2} + 5x + \frac{4}{x} + C$$

where C is an arbitrary constant.

**Question 12:**

$$\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$$

**Solution 12:**

$$\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$$

$$= \int \left( x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 4x^{-\frac{1}{2}} \right) dx$$

$$\frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{3 \left( x^{\frac{3}{2}} \right)}{\frac{3}{2}} + \frac{4 \left( x^{\frac{1}{2}} \right)}{\frac{1}{2}} + C$$

$$= \frac{2}{7} x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + C$$

$$= \frac{2}{7} x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8\sqrt{x} + C$$

where C is an arbitrary constant.

**Question 13:**

$$\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$$

**Solution 13:**

$$\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$$

On factorising, we obtain

$$\int \frac{(x^2 + 1)(x - 1)}{x - 1} dx$$

$$= \int (x^2 + 1) dx$$

$$= \int x^2 dx + \int 1 dx$$

$$= \frac{x^3}{3} + x + C$$

where C is an arbitrary constant.

**Question 14:**

$$\int (1-x)\sqrt{x} dx$$

**Solution 14:**

$$\begin{aligned} & \int (1-x)\sqrt{x} dx \\ &= \int \left( \sqrt{x} - x^{\frac{3}{2}} \right) dx \\ &= \int x^{\frac{1}{2}} dx - \int x^{\frac{3}{2}} dx \\ &= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C \\ &= \frac{2}{3} x^{\frac{3}{2}} - \frac{2}{5} x^{\frac{5}{2}} + C \end{aligned}$$

where C is an arbitrary constant.

**Question 15:**

$$\int \sqrt{x}(3x^2 + 2x + 3) dx$$

**Solution 15:**

$$\begin{aligned} & \int \sqrt{x}(3x^2 + 2x + 3) dx \\ &= 3 \int x^{\frac{5}{2}} dx + 2 \int x^{\frac{3}{2}} dx + 3 \int x^{\frac{1}{2}} dx \\ &= 3 \left( \frac{x^{\frac{7}{2}}}{\frac{7}{2}} \right) + 2 \left( \frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right) + 3 \left( \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right) + C \\ &= \frac{6}{7} x^{\frac{7}{2}} + \frac{4}{5} x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C \end{aligned}$$

where C is an arbitrary constant.

**Question 16:**

$$\int (2x - 3 \cos x + e^x) dx$$

**Solution 16:**

$$\begin{aligned} & \int (2x - 3 \cos x + e^x) dx \\ &= 2 \int x dx - 3 \int \cos x dx + \int e^x dx \\ &= \frac{2x^2}{2} - 3(\sin x) + e^x + C \\ &= x^2 - 3 \sin x + e^x + C \end{aligned}$$

where C is an arbitrary constant.

**Question 17:**

$$\int (2x^2 - 3\sin x + 5\sqrt{x}) dx$$

**Solution 17:**

$$\begin{aligned} & \int (2x^2 - 3\sin x + 5\sqrt{x}) dx \\ &= 2 \int x^2 dx - 3 \int \sin x dx + 5 \int x^{\frac{1}{2}} dx \\ &= \frac{2x^3}{3} - 3(-\cos x) + 5 \left( \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right) + C \\ &= \frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{\frac{3}{2}} + C \end{aligned}$$

where C is an arbitrary constant.

**Question 18:**

$$\int \sec x (\sec x + \tan x) dx$$

**Solution 18:**

$$\begin{aligned} & \int \sec x (\sec x + \tan x) dx \\ &= \int (\sec^2 x + \sec x \tan x) dx \\ &= \int \sec^2 x dx + \int \sec x \tan x dx \\ &= \tan x + \sec x + C \end{aligned}$$

where C is an arbitrary constant.

**Question 19:**

$$\int \frac{\sec^2 x}{\cos^2 x} dx$$

**Solution 19:**

$$\begin{aligned} & \int \frac{\sec^2 x}{\cos^2 x} dx \\ &= \int \frac{1}{\cos^2 x} dx \\ &= \int \frac{\sin^2 x}{\cos^2 x} dx \end{aligned}$$

$$\begin{aligned}
 &= \int \tan^2 x dx \\
 &= \int (\sec^2 x - 1) dx \\
 &= \int \sec^2 x dx - \int 1 dx \\
 &= \tan x - x + C \\
 &\text{where } C \text{ is an arbitrary constant.}
 \end{aligned}$$

**Question 20:**

$$\int \frac{2 - 3 \sin x}{\cos^2 x} dx$$

**Solution 20:**

$$\begin{aligned}
 &\int \frac{2 - 3 \sin x}{\cos^2 x} dx \\
 &= \int \left( \frac{2}{\cos^2 x} - \frac{3 \sin x}{\cos^2 x} \right) dx \\
 &= \int 2 \sec^2 x dx - 3 \int \tan x \sec x dx \\
 &= 2 \tan x - 3 \sec x + C \\
 &\text{where } C \text{ is an arbitrary constant.}
 \end{aligned}$$

**Question 21:**

The anti-derivative of  $\left( \sqrt{x} + \frac{1}{\sqrt{x}} \right)$  equals

$$\begin{aligned}
 (A) & \frac{1}{3} x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + C & (B) & \frac{2}{3} x^{\frac{2}{3}} + \frac{1}{2} x^2 + C \\
 (C) & \frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C & (D) & \frac{3}{2} x^{\frac{3}{2}} + \frac{1}{2} x^{\frac{1}{2}} + C
 \end{aligned}$$

**Solution 21:**

$$\begin{aligned}
 &\int \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right) dx \\
 &= \int x^{\frac{1}{2}} dx + \int x^{-\frac{1}{2}} dx \\
 &= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C \\
 &= \frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C, \text{ where } C \text{ is an arbitrary constant.}
 \end{aligned}$$

Hence, the correct Answer is C.



**Question 22:**

If  $\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$  such that  $f(2) = 0$ , then  $f(x)$  is

- (A)  $x^4 + \frac{1}{x^3} - \frac{129}{8}$       (B)  $x^3 + \frac{1}{x^4} + \frac{129}{8}$   
 (C)  $x^4 + \frac{1}{x^3} + \frac{129}{8}$       (D)  $x^3 + \frac{1}{x^4} - \frac{129}{8}$

**Solution 22:**

It is given that,  $\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$

Anti-derivative of  $4x^3 - \frac{3}{x^4} = f(x)$

$$\therefore f(x) = \int 4x^3 - \frac{3}{x^4} dx$$

$$f(x) = 4 \int x^3 dx - 3 \int (x^{-4}) dx$$

$$f(x) = 4 \left( \frac{x^4}{4} \right) - 3 \left( \frac{x^{-3}}{-3} \right) + C$$

$$f(x) = x^4 + \frac{1}{x^3} + C$$

Also,

$$f(2) = 0$$

$$\therefore f(2) = (2)^4 + \frac{1}{(2)^3} + C = 0$$

$$\Rightarrow 16 + \frac{1}{8} + C = 0$$

$$\Rightarrow C = -\left(16 + \frac{1}{8}\right)$$

$$\Rightarrow C = -\frac{129}{8}$$

$$\therefore f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$$

Hence, the correct Answer is A.

**Exercise 7.2****Question 1:**

Integrate  $\frac{2x}{1+x^2}$

**Solution 1:**

$$\text{Let } 1+x^2 = t$$

$$\therefore 2x \, dx = dt$$

$$\Rightarrow \int \frac{2x}{1+x^2} dx = \int \frac{1}{t} dt$$

$$= \log|t| + C$$

$$= \log|1+x^2| + C$$

$$= \log(1+x^2) + C$$

where C is an arbitrary constant.

**Question 2:**

$$\text{Integrate } \frac{(\log x)^2}{x}$$

**Solution 2:**

$$\text{Let } \log x = t$$

$$\therefore \frac{1}{x} dx = dt$$

$$\Rightarrow \int \frac{(\log|x|)^2}{x} dx = \int t^2 dt$$

$$= \frac{t^3}{3} + C$$

$$= \frac{(\log|x|)^3}{3} + C$$

where C is an arbitrary constant.

**Question 3:**

$$\text{Integrate } \frac{1}{x+x \log x}$$

**Solution 3:**

The given function can be rewritten as

$$\frac{1}{x+x \log x} = \frac{1}{x(1+\log x)}$$

$$\text{Let } 1 + \log x = t$$

$$\therefore \frac{1}{x} dx = dt$$

$$\Rightarrow \int \frac{1}{x(1+\log x)} dx = \int \frac{1}{t} dt$$

$$= \log|t| + C$$

$$= \log|1+\log x| + C$$

where C is an arbitrary constant.

#### Question 4:

Integrate  $\sin x \cdot \sin(\cos x)$

#### Solution 4:

Let  $\cos x = t$

$$\therefore -\sin x dx = dt$$

$$\Rightarrow \int \sin x \cdot \sin(\cos x) dx = -\int \sin t dt$$

$$= -[-\cos t] + C$$

$$= \cos t + C$$

$$= \cos(\cos x) + C$$

where C is an arbitrary constant.

#### Question 5:

Integrate  $\sin(ax+b) \cos(ax+b)$

#### Solution 5:

The given function can be rewritten as

$$\sin(ax+b) \cos(ax+b) = \frac{2 \sin(ax+b) \cos(ax+b)}{2} = \frac{\sin 2(ax+b)}{2}$$

Let  $2(ax+b) = t$

$$\therefore 2adx = dt$$

$$\Rightarrow \int \frac{\sin 2(ax+b)}{2} dx = \frac{1}{2} \int \frac{\sin t dt}{2a}$$

$$= \frac{1}{4a} [-\cos t] + C$$

$$= \frac{-1}{4a} \cos 2(ax+b) + C$$

where C is an arbitrary constant.

#### Question 6:

Integrate  $\sqrt{ax+b}$

#### Solution 6:

Let  $ax + b = t$

$$\Rightarrow adx = dt$$

$$\therefore dx = \frac{1}{a} dt$$

$$\Rightarrow \int (ax+b)^{\frac{1}{2}} dx = \frac{1}{a} \int t^{\frac{1}{2}} dt$$

$$= \frac{1}{a} \left( \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right) + C$$

$$= \frac{2}{3a} (ax+b)^{\frac{3}{2}} + C$$

where C is an arbitrary constant.

### Question 7:

Integrate  $x\sqrt{x+2}$

### Solution 7:

Let  $x + 2 = t$

$$\therefore dx = dt$$

$$\Rightarrow \int x\sqrt{x+2} dx = \int (t-2)\sqrt{t} dt$$

$$= \int \left( t^{\frac{3}{2}} - 2t^{\frac{1}{2}} \right) dt$$

$$= \int t^{\frac{3}{2}} dt - 2 \int t^{\frac{1}{2}} dt$$

$$= \frac{t^{\frac{3}{2}+1}}{\frac{3}{2}+1} - 2 \left( \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right) + C$$

$$= \frac{2}{5} t^{\frac{5}{2}} - \frac{4}{3} t^{\frac{3}{2}} + C$$

$$= \frac{2}{5} (x+2)^{\frac{5}{2}} - \frac{4}{3} (x+2)^{\frac{3}{2}} + C$$

where C is an arbitrary constant.

### Question 8:

$x\sqrt{1+2x^2}$

### Solution 8:

Let  $1 + 2x^2 = t$

$$\therefore 4x \, dx = dt$$

$$\Rightarrow \int x\sqrt{1+2x^2} \, dx = \int \frac{\sqrt{t}}{4} \, dt$$

$$= \frac{1}{4} \int t^{\frac{1}{2}} \, dt$$

$$= \frac{1}{4} \left( \frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$$

$$= \frac{1}{6} (1+2x^2)^{\frac{3}{2}} + C$$

where C is an arbitrary constant.

### Question 9:

Integrate  $(4x+2)\sqrt{x^2+x+1}$

### Solution 9:

Let  $x^2 + x + 1 = t$

$$\therefore (2x+1)dx = dt$$

$$\int (4x+2)\sqrt{x^2+x+1} \, dx$$

$$= \int 2\sqrt{t} \, dt$$

$$= 2 \int \sqrt{t} \, dt$$

$$= 2 \left( \frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$$

$$= \frac{4}{3} (x^2+x+1)^{\frac{3}{2}} + C$$

where C is an arbitrary constant.

### Question 10:

Integrate  $\frac{1}{x-\sqrt{x}}$

### Solution 10:

The given function can be rewritten as

$$\frac{1}{x-\sqrt{x}} = \frac{1}{\sqrt{x}(\sqrt{x}-1)}$$

Let  $(\sqrt{x}-1) = t$

$$\therefore \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow \int \frac{1}{\sqrt{x}(\sqrt{x}-1)} dx = \int \frac{2}{t} dt$$

$$= 2 \log|t| + C$$

$$= 2 \log|\sqrt{x}-1| + C$$

where C is an arbitrary constant.

### Question 11:

Integrate  $\frac{x}{\sqrt{x+4}}, x > 0$

### Solution 11:

Let  $x + 4 = t$

$$\therefore dx = dt$$

$$\int \frac{x}{\sqrt{x+4}} dx = \int \frac{(t-4)}{\sqrt{t}} dt$$

$$= \int \left( \sqrt{t} - \frac{4}{\sqrt{t}} \right) dt$$

$$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 4 \left( \frac{t^{\frac{1}{2}}}{\frac{1}{2}} \right) + C$$

$$= \frac{2}{3}(t)^{\frac{3}{2}} - 8(t)^{\frac{1}{2}} + C$$

$$= \frac{2}{3}t \cdot t^{\frac{1}{2}} - 8t^{\frac{1}{2}} + C$$

$$= \frac{2}{3}t^{\frac{1}{2}}(t-12) + C$$

$$= \frac{2}{3}(x+4)^{\frac{1}{2}}(x+4-12) + C$$

$$= \frac{2}{3}\sqrt{x+4}(x-8) + C$$

where C is an arbitrary constant.

### Question 12:

Integrate  $(x^3 - 1)^{\frac{1}{3}} x^5$

**Solution 12:**

Let  $x^3 - 1 = t$

$$\therefore 3x^2 dx = dt$$

$$\Rightarrow \int (x^3 - 1)^{\frac{1}{3}} x^5 dx = \int (x^3 - 1)^{\frac{1}{3}} x^3 \cdot x^2 dx$$

$$= \int t^{\frac{1}{3}} (t+1) \frac{dt}{3}$$

$$= \frac{1}{3} \int \left( t^{\frac{4}{3}} + t^{\frac{1}{3}} \right) dt$$

$$= \frac{1}{3} \left[ \frac{t^{\frac{7}{3}}}{\frac{7}{3}} + \frac{t^{\frac{4}{3}}}{\frac{4}{3}} \right] + C$$

$$= \frac{1}{3} \left[ \frac{3}{7} t^{\frac{7}{3}} + \frac{3}{4} t^{\frac{4}{3}} \right] + C$$

$$= \frac{1}{7} (x^3 - 1)^{\frac{7}{3}} + \frac{1}{4} (x^3 - 1)^{\frac{4}{3}} + C$$

where C is an arbitrary constant.

**Question 13:**

Integrate  $\frac{x^2}{(2+3x^3)^3}$

**Solution 13:**

Let  $2 + 3x^3 = t$

$$\therefore 9x^2 dx = dt$$

$$\Rightarrow \int \frac{x^2}{(2+3x^3)^3} dx = \frac{1}{9} \int \frac{dt}{(t)^3}$$

$$= \frac{1}{9} \left[ \frac{t^{-2}}{-2} \right] + C$$

$$= \frac{-1}{18} \left( \frac{1}{t^2} \right) + C$$

$$= \frac{-1}{18(2+3x^3)^2} + C$$

where C is an arbitrary constant.

**Question 14:**

Integrate  $\frac{1}{x(\log x)^m}, x > 0$

**Solution 14:**

Let  $\log x = t$

$$\frac{1}{x} dx = dt$$

$$\Rightarrow \int \frac{1}{x(\log x)^m} dx = \int \frac{dt}{(t)^m}$$

$$= \frac{t^{-m+1}}{-m+1} + C$$

$$= \frac{(\log x)^{1-m}}{(1-m)} + C$$

where C is an arbitrary constant.

**Question 15:**

Integrate  $\frac{x}{9-4x^2}$

**Solution 15:**

Let  $9 - 4x^2 = t$

$$\therefore -8x dx = dt$$

$$\Rightarrow \int \frac{x}{9-4x^2} dx = \frac{-1}{8} \int \frac{1}{t} dt$$

$$= \frac{-1}{8} \log|t| + C$$

$$= \frac{-1}{8} \log|9 - 4x^2| + C$$

where C is an arbitrary constant.

**Question 16:**

Integrate  $e^{2x+3}$

**Solution 16:**

Let  $2x + 3 = t$

$$\therefore 2dx = dt$$

$$\Rightarrow \int e^{2x+3} dx = \frac{1}{2} \int e^t dt$$



$$= \frac{1}{2}(e^t) + C$$

$$= \frac{1}{2}e^{(2x+3)} + C$$

where C is an arbitrary constant.

### Question 17:

Integrate  $\frac{x}{e^{x^2}}$

### Solution 17:

Let  $x^2 = t$

$$\therefore 2x \, dx = dt$$

$$\Rightarrow \int \frac{x}{e^{x^2}} dx = \frac{1}{2} \int \frac{1}{e^t} dt$$

$$= \frac{1}{2} \int e^{-t} dt$$

$$= \frac{1}{2} \left( \frac{e^{-t}}{-1} \right) + C$$

$$= -\frac{1}{2} e^{-x^2} + C$$

$$= \frac{-1}{2e^{x^2}} + C$$

where C is an arbitrary constant.

### Question 18:

Integrate  $\frac{e^{\tan^{-1}x}}{1+x^2}$

### Solution 18:

Let  $\tan^{-1}x = t$

$$\therefore \frac{1}{1+x^2} dx = dt$$

$$\Rightarrow \int \frac{e^{\tan^{-1}x}}{1+x^2} dx = \int e^t dt$$

$$= e^t + C$$

$$= e^{\tan^{-1}x} + C$$

where C is an arbitrary constant.

**Question 19:**

Integrate  $\frac{e^{2x} - 1}{e^{2x} + 1}$

**Solution 19:**

Dividing the given function's numerator and denominator by  $e^x$ , we obtain,

$$\frac{\left(\frac{e^{2x} - 1}{e^x}\right)}{\left(\frac{e^{2x} + 1}{e^x}\right)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Let  $e^x + e^{-x} = t$

$$(e^x - e^{-x})dx = dt$$

$$\Rightarrow \int \frac{e^{2x} - 1}{e^{2x} + 1} dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + C$$

$$= \log|e^x + e^{-x}| + C$$

where C is an arbitrary constant.

**Question 20:**

Integrate  $\frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$

**Solution 20:**

Let  $e^{2x} + e^{-2x} = t$

$$\Rightarrow 2e^{2x} - 2e^{-2x} dx = dt$$

$$\Rightarrow 2(e^{2x} - e^{-2x}) dx = dt$$

$$\Rightarrow \int \left( \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} \right) dx = \int \frac{dt}{2t}$$

$$= \frac{1}{2} \int \frac{1}{t} dt$$

$$= \frac{1}{2} \log|t| + C$$

$$= \frac{1}{2} \log|e^{2x} + e^{-2x}| + C$$

where C is an arbitrary constant.

**Question 21:**Integrate  $\tan^2(2x-3)$ **Solution 21:**

$$\tan^2(2x-3) = \sec^2(2x-3) - 1$$

Let  $2x - 3 = t$ 

$$\therefore 2dx = dt$$

$$\Rightarrow \int \tan^2(2x-3)dx = \int [\sec^2(2x-3) - 1]dx$$

$$= \frac{1}{2} \int (\sec^2 t) dt - \int 1 dx$$

$$= \frac{1}{2} \int \sec^2 t dt - \int 1 dx$$

$$= \frac{1}{2} \tan t - x + C$$

$$= \frac{1}{2} \tan(2x-3) - x + C$$

where C is an arbitrary constant.

**Question 22:**Integrate  $\sec^2(7-4x)$ **Solution 22:**Let  $7 - 4x = t$ 

$$\therefore -4dx = dt$$

$$\therefore \int \sec^2(7-4x) dx = \frac{-1}{4} \int \sec^2 t dt$$

$$= \frac{-1}{4} (\tan t) + C$$

$$= \frac{-1}{4} \tan(7-4x) + C$$

where C is an arbitrary constant.

**Question 23:**Integrate  $\frac{\sin^{-1} x}{\sqrt{1-x^2}}$ **Solution 23:**Let  $\sin^{-1} x = t$ 

$$\frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\begin{aligned} \Rightarrow \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx &= \int t dt \\ &= \frac{t^2}{2} + C \\ &= \frac{(\sin^{-1} x)^2}{2} + C \end{aligned}$$

where C is an arbitrary constant.

### Question 24:

Integrate  $\frac{2 \cos x - 3 \sin x}{6 \cos x + 4 \sin x}$

### Solution 24:

The given function is,

$$\frac{2 \cos x - 3 \sin x}{6 \cos x + 4 \sin x} = \frac{2 \cos x - 3 \sin x}{2(3 \cos x + 2 \sin x)}$$

Let  $3 \cos x + 2 \sin x = t$

$$(-3 \sin x + 2 \cos x) dx = dt$$

$$\int \frac{2 \cos x - 3 \sin x}{6 \cos x + 4 \sin x} dx = \int \frac{dt}{2t}$$

$$= \frac{1}{2} \int \frac{1}{t} dt$$

$$= \frac{1}{2} \log |t| + C$$

$$= \frac{1}{2} \log |2 \sin x + 3 \cos x| + C$$

where C is an arbitrary constant.

### Question 25:

Integrate  $\frac{1}{\cos^2 x (1 - \tan x)^2}$

### Solution 25:

The given function is

$$\frac{1}{\cos^2 x (1 - \tan x)^2} = \frac{\sec^2}{(1 - \tan x)^2}$$

Let  $(1 - \tan x) = t$

$$-\sec^2 x dx = dt$$

$$\begin{aligned} \Rightarrow \int \frac{\sec^2}{(1-\tan x)^2} dx &= \int \frac{-dt}{t^2} \\ &= -\int t^{-2} dt \\ &= +\frac{1}{t} + C \\ &= \frac{1}{(1-\tan x)} + C \end{aligned}$$

where C is an arbitrary constant.

### Question 26:

Integrate  $\frac{\cos \sqrt{x}}{\sqrt{x}}$

### Solution 26:

Let  $\sqrt{x} = t$

$$\frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow \int \frac{\cos \sqrt{x}}{\sqrt{x}} dx = 2 \int \cos t dt$$

$$= 2 \sin t + C$$

$$= 2 \sin \sqrt{x} + C$$

where C is an arbitrary constant.

### Question 27:

Integrate  $\sqrt{\sin 2x} \cos 2x$

### Solution 27:

Let  $\sin 2x = t$

So,  $2 \cos 2x dx = dt$

$$\Rightarrow \int \sqrt{\sin 2x} \cos 2x dx = \frac{1}{2} \int \sqrt{t} dt$$

$$= \frac{1}{2} \left( \frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$$

$$= \frac{1}{3} t^{\frac{3}{2}} + C$$

$$= \frac{1}{3} (\sin 2x)^{\frac{3}{2}} + C$$

where C is an arbitrary constant.

**Question 28:**

Integrate  $\frac{\cos x}{\sqrt{1+\sin x}}$

**Solution 28:**

Let  $1 + \sin x = t$

$\therefore \cos x \, dx = dt$

$$\Rightarrow \int \frac{\cos x}{\sqrt{1+\sin x}} dx = \int \frac{dt}{\sqrt{t}}$$

$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= 2\sqrt{t} + C$$

$$= 2\sqrt{1+\sin x} + C$$

where C is an arbitrary constant.

**Question 29:**

Integrate  $\cot x \log \sin x$

**Solution 29:**

Let  $\log \sin x = t$

$$\Rightarrow \frac{1}{\sin x} \cdot \cos x \, dx = dt$$

$\therefore \cot x \, dx = dt$

$$\Rightarrow \int \cot x \log \sin x \, dx = \int t \, dt$$

$$= \frac{t^2}{2} + C$$

$$= \frac{1}{2}(\log \sin x)^2 + C$$

where C is an arbitrary constant.

**Question 30:**

Integrate  $\frac{\sin x}{1+\cos x}$

**Solution 30:**

Let  $1 + \cos x = t$

$\therefore -\sin x \, dx = dt$

$$\Rightarrow \int \frac{\sin x}{1 + \cos x} dx = \int -\frac{dt}{t}$$

$$= -\log|t| + C$$

$$= -\log|1 + \cos x| + C$$

where C is an arbitrary constant.

### Question 31:

Integrate  $\frac{\sin x}{(1 + \cos x)^2}$

### Solution 31:

Let  $1 + \cos x = t$

$\therefore -\sin x \, dx = dt$

$$\Rightarrow \int \frac{\sin x}{(1 + \cos x)^2} dx = \int -\frac{dt}{t^2}$$

$$= -\int t^{-2} dt$$

$$= \frac{1}{t} + C$$

$$= \frac{1}{1 + \cos x} + C$$

where C is an arbitrary constant.

### Question 32:

Integrate  $\frac{1}{1 + \cot x}$

### Solution 32:

Let  $I = \int \frac{1}{1 + \cot x} dx$

$$= \int \frac{1}{1 + \frac{\cos x}{\sin x}} dx$$

$$= \int \frac{\sin x}{\sin x + \cos x} dx$$

$$= \frac{1}{2} \int \frac{2 \sin x}{\sin x + \cos x} dx$$

$$= \frac{1}{2} \int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{(\sin x + \cos x)} dx$$

$$= \frac{1}{2} \int 1 dx + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

$$= \frac{1}{2}(x) + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

Let  $\sin x + \cos x = t \Rightarrow (\cos x - \sin x) dx = dt$

$$\therefore I = \frac{x}{2} + \frac{1}{2} \int \frac{-(dt)}{t}$$

$$= \frac{x}{2} - \frac{1}{2} \log|t| + C$$

$$= \frac{x}{2} - \frac{1}{2} \log|\sin x + \cos x| + C$$

where C is an arbitrary constant.

### Question 33:

Integrate  $\frac{1}{1 - \tan x}$

### Solution 33:

$$\text{Let } I = \int \frac{1}{1 - \tan x} dx$$

$$= \int \frac{1}{1 - \frac{\sin x}{\cos x}} dx$$

$$= \int \frac{\cos x}{\cos x - \sin x} dx$$

$$= \frac{1}{2} \int \frac{2 \cos x}{\cos x - \sin x} dx$$

$$= \frac{1}{2} \int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{(\cos x - \sin x)} dx$$

$$= \frac{1}{2} \int 1 dx + \frac{1}{2} \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

$$= \frac{x}{2} + \frac{1}{2} \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Put  $\cos x - \sin x = t \Rightarrow (-\sin x - \cos x) dx = dt$

$$\therefore I = \frac{x}{2} + \frac{1}{2} \int \frac{-(dt)}{t}$$

$$= \frac{x}{2} - \frac{1}{2} \log|t| + C$$

$$= \frac{x}{2} - \frac{1}{2} \log|\cos x - \sin x| + C$$

where C is an arbitrary constant.



**Question 34:**

Integrate  $\frac{\sqrt{\tan x}}{\sin x \cos x}$

**Solution 34:**

$$\begin{aligned} \text{Let } I &= \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx \\ &= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x} dx \\ &= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x} dx \\ &= \int \frac{\sec^2 x dx}{\sqrt{\tan x}} \end{aligned}$$

Let  $\tan x = t \Rightarrow \sec^2 x dx = dt$

$$\therefore I = \int \frac{dt}{\sqrt{t}}$$

$$= 2\sqrt{t} + C$$

$$= 2\sqrt{\tan x} + C$$

where C is an arbitrary constant.

**Question 35:**

Integrate  $\frac{(1 + \log x)^2}{x}$

**Solution 35:**

Let  $1 + \log x = t$

$$\therefore \frac{1}{x} dx = dt$$

$$\begin{aligned} \Rightarrow \int \frac{(1 + \log x)^2}{x} dx &= \int t^2 dt \\ &= \frac{t^3}{3} + C \\ &= \frac{(1 + \log x)^3}{3} + C \end{aligned}$$

where C is an arbitrary constant.

**Question 36:**

Integrate  $\frac{(x+1)(x+\log x)^2}{x}$

**Solution 36:**

The given function can be rewritten as

$$\frac{(x+1)(x+\log x)^2}{x}$$

$$= \left(1 + \frac{1}{x}\right)(x + \log x)^2$$

Let  $(x + \log x) = t$

$$\therefore \left(1 + \frac{1}{x}\right) dx = dt$$

$$\Rightarrow \int \left(1 + \frac{1}{x}\right)(x + \log x)^2 dx = \int t^2 dt$$

$$= \frac{t^3}{3} + C$$

$$= \frac{1}{3}(x + \log x)^3 + C$$

where C is an arbitrary constant.

**Question 37:**

Integrate  $\frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}$

**Solution 37:**

Let  $x^4 = t$

$$\therefore 4x^3 dx = dt$$

$$\Rightarrow \int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8} dx = \frac{1}{4} \int \frac{\sin(\tan^{-1} t)}{1+t^2} dt \quad \dots(1)$$

Let  $\tan^{-1} t = u$

$$\therefore \frac{1}{1+t^2} dt = du$$

From (1), we obtain

$$\int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8} dx = \frac{1}{4} \int \sin u du$$

$$= \frac{1}{4}(-\cos u) + C$$

$$= -\frac{1}{4} \cos(\tan^{-1} t) + C$$

$$= -\frac{1}{4} \cos(\tan^{-1} x^4) + C$$

where C is an arbitrary constant.

**Question 38:**

$$\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx \text{ equals}$$

- (A)  $10^x - x^{10} + C$                       (B)  $10^x + x^{10} + C$   
 (C)  $(10^x - x^{10})^{-1} + C$               (D)  $\log(10^x + x^{10}) + C$

**Solution 38:**

$$\text{Let } x^{10} + 10^x = t$$

$$\therefore (10x^9 + 10^x \log_e 10) dx = \int \frac{dt}{t}$$

$$\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$$

$$= \log t + C$$

$$= \log(10^x + x^{10}) + C$$

Hence, the correct Answer is D.

**Question 39:**

$$\int \frac{dx}{\sin^2 x \cos^2 x} \text{ equals}$$

- (A)  $\tan x + \cot x + C$                       (B)  $\tan x - \cot x + C$   
 (C)  $\tan x \cot x + C$                       (D)  $\tan x - \cot 2x + C$

**Solution 39:**

$$\text{Let } I = \int \frac{dx}{\sin^2 x \cos^2 x}$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$$

$$= \tan x - \cot x + C$$

Hence, the correct Answer is B.

**Exercise 7.3**

**Question 1:**Integrate  $\sin^2(2x + 5)$ **Solution 1:**

The given function can be rewritten as

$$\sin^2(2x+5) = \frac{1 - \cos 2(2x+5)}{2} = \frac{1 - \cos(4x+10)}{2}$$

$$\begin{aligned} \Rightarrow \int \sin^2(2x+5) dx &= \int \frac{1 - \cos(4x+10)}{2} dx \\ &= \frac{1}{2} \int 1 dx - \frac{1}{2} \int \cos(4x+10) dx \\ &= \frac{1}{2} x - \frac{1}{2} \left( \frac{\sin(4x+10)}{4} \right) + C \\ &= \frac{1}{2} x - \frac{1}{8} \sin(4x+10) + C \end{aligned}$$

**Question 2:**Integrate  $\sin 3x \cdot \cos 4x$ **Solution 2:**It is known that,  $\sin A \cos B = \frac{1}{2} \{ \sin(A+B) + \sin(A-B) \}$ 

$$\therefore \int \sin 3x \cos 4x dx = \frac{1}{2} \int \{ \sin(3x+4x) + \sin(3x-4x) \} dx$$

$$= \frac{1}{2} \int \{ \sin 7x + \sin(-x) \} dx$$

$$= \frac{1}{2} \int \{ \sin 7x - \sin x \} dx$$

$$= \frac{1}{2} \int \sin 7x dx - \frac{1}{2} \int \sin x dx$$

$$= \frac{1}{2} \left( \frac{-\cos 7x}{7} \right) - \frac{1}{2} (-\cos x) + C$$

$$= \frac{-\cos 7x}{14} + \frac{\cos x}{2} + C$$

where C is an arbitrary constant.

**Question 3:**Integrate  $\cos 2x \cos 4x \cos 6x$ **Solution 3:**

It is known that,  $\cos A \cos B = \frac{1}{2} \{ \cos(A+B) + \cos(A-B) \}$

$$\begin{aligned} \therefore \int \cos 2x (\cos 4x \cos 6x) dx &= \int \cos 2x \left[ \frac{1}{2} \{ \cos(4x+6x) + \cos(4x-6x) \} \right] dx \\ &= \frac{1}{2} \int \{ \cos 2x \cos 10x + \cos 2x \cos(-2x) \} dx \\ &= \frac{1}{2} \int \{ \cos 2x \cos 10x + \cos^2 2x \} dx \\ &= \frac{1}{2} \int \left[ \left\{ \frac{1}{2} \cos(2x+10x) + \cos(2x-10x) \right\} + \left( \frac{1+\cos 4x}{2} \right) \right] dx \\ &= \frac{1}{4} \int (\cos 12x + \cos 8x + 1 + \cos 4x) dx \\ &= \frac{1}{4} \left[ \frac{\sin 12x}{12} + \frac{\sin 8x}{8} + x + \frac{\sin 4x}{4} + C \right] \end{aligned}$$

where C is an arbitrary constant.

#### Question 4:

Integrate  $\sin^3(2x+1)$

#### Solution 4:

Let  $I = \int \sin^3(2x+1) dx$

$$\begin{aligned} \Rightarrow \int \sin^3(2x+1) dx &= \int \sin^2(2x+1) \cdot \sin(2x+1) dx \\ &= \int (1 - \cos^2(2x+1)) \sin(2x+1) dx \end{aligned}$$

Let  $\cos(2x+1) = t$

$$\Rightarrow -2 \sin(2x+1) dx = dt$$

$$\Rightarrow \sin(2x+1) dx = \frac{-dt}{2}$$

$$\Rightarrow I = \frac{-1}{2} \int (1-t^2) dt$$

$$= \frac{-1}{2} \left\{ t - \frac{t^3}{3} \right\} + C$$

$$= \frac{-1}{2} \left\{ \cos(2x+1) - \frac{\cos^3(2x+1)}{3} \right\} + C$$

$$= \frac{-\cos(2x+1)}{2} + \frac{\cos^3(2x+1)}{6} + C$$

where C is an arbitrary constant.

**Question 5:**Integrate  $\sin^3 x \cos^3 x$ **Solution 5:**

$$\begin{aligned} \text{Let } I &= \int \sin^3 x \cos^3 x dx \\ &= \int \cos^3 x \cdot \sin^2 x \cdot \sin x dx \\ &= \int \cos^3 x (1 - \cos^2 x) \sin x dx \end{aligned}$$

Let  $\cos x = t$ 

$$\Rightarrow -\sin x dx = dt$$

$$\Rightarrow I = -\int t^3 (1 - t^2) dt$$

$$= -\int (t^3 - t^5) dt$$

$$= -\left\{ \frac{t^4}{4} - \frac{t^6}{6} \right\} + C$$

$$= -\left\{ \frac{\cos^4 x}{4} - \frac{\cos^6 x}{6} \right\} + C$$

$$= \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + C$$

where C is an arbitrary constant.

**Question 6:**Integrate  $\sin x \sin 2x \sin 3x$ **Solution 6:**It is known that,  $\sin A \sin B = \frac{1}{2} \{ \cos(A - B) - \cos(A + B) \}$ 

$$\begin{aligned} \therefore \int \sin x \sin 2x \sin 3x dx &= \int \left[ \sin x \cdot \frac{1}{2} \{ \cos(2x - 3x) - \cos(2x + 3x) \} \right] dx \\ &= \frac{1}{2} \int (\sin x \cos(-x) - \sin x \cos 5x) dx \\ &= \frac{1}{2} \int (\sin x \cos x - \sin x \cos 5x) dx \\ &= \frac{1}{2} \int \frac{\sin 2x}{2} dx - \frac{1}{2} \int \sin x \cos 5x dx \\ &= \frac{1}{4} \left[ \frac{-\cos 2x}{2} \right] - \frac{1}{2} \int \frac{1}{2} \{ \sin(x + 5x) + \sin(x - 5x) \} dx \\ &= \frac{-\cos 2x}{8} - \frac{1}{4} \int (\sin 6x + \sin(-4x)) dx \end{aligned}$$

$$\begin{aligned}
 &= \frac{-\cos 2x}{8} - \frac{1}{4} \left[ \frac{-\cos 6x}{6} + \frac{\cos 4x}{4} \right] + C \\
 &= \frac{-\cos 2x}{8} - \frac{1}{8} \left[ \frac{-\cos 6x}{3} + \frac{\cos 4x}{2} \right] + C \\
 &= \frac{1}{8} \left[ \frac{\cos 6x}{3} + \frac{\cos 4x}{2} - \cos 2x \right] + C
 \end{aligned}$$

where C is an arbitrary constant.

### Question 7:

Integrate  $\sin 4x \sin 8x$

### Solution 7:

It is known that,  $\sin A \sin B = \frac{1}{2} \cos(A - B) - \cos(A + B)$

$$\begin{aligned}
 \therefore \int \sin 4x \sin 8x dx &= \int \left\{ \frac{1}{2} \cos(4x - 8x) - \cos(4x + 8x) \right\} dx \\
 &= \frac{1}{2} \int (\cos(-4x) - \cos 12x) dx \\
 &= \frac{1}{2} \int (\cos 4x - \cos 12x) dx \\
 &= \frac{1}{2} \left[ \frac{\sin 4x}{4} - \frac{\sin 12x}{12} \right] + C
 \end{aligned}$$

where C is an arbitrary constant.

### Question 8:

Integrate  $\frac{1 - \cos x}{1 + \cos x}$

### Solution 8:

Consider,

$$\begin{aligned}
 \frac{1 - \cos x}{1 + \cos x} &= \frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} && \left[ 2 \sin^2 \frac{x}{2} = 1 - \cos x \text{ and } 2 \cos^2 \frac{x}{2} = 1 + \cos x \right] \\
 &= \tan^2 \frac{x}{2} \\
 &= \left( \sec^2 \frac{x}{2} - 1 \right)
 \end{aligned}$$

$$\begin{aligned}\therefore \int \frac{1-\cos x}{1+\cos x} dx &= \int \left( \sec^2 \frac{x}{2} - 1 \right) dx \\ &= \left[ \frac{\tan \frac{x}{2}}{\frac{1}{2}} - x \right] + C \\ &= 2 \tan \frac{x}{2} - x + C\end{aligned}$$

where C is an arbitrary constant.

### Question 9:

Integrate  $\frac{\cos x}{1+\cos x}$

### Solution 9:

$$\begin{aligned}\frac{\cos x}{1+\cos x} &= \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} \quad \left[ \cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \text{ and } \cos x = 2\cos^2 \frac{x}{2} - 1 \right] \\ &= \frac{1}{2} \left[ 1 - \tan^2 \frac{x}{2} \right] \\ \therefore \int \frac{\cos x}{1+\cos x} dx &= \frac{1}{2} \int \left( 1 - \tan^2 \frac{x}{2} \right) dx \\ &= \frac{1}{2} \int \left( 1 - \sec^2 \frac{x}{2} + 1 \right) dx \\ &= \frac{1}{2} \int \left( 2 - \sec^2 \frac{x}{2} \right) dx \\ &= \frac{1}{2} \left[ 2x - \frac{\tan \frac{x}{2}}{\frac{1}{2}} \right] + C \\ &= x - \tan \frac{x}{2} + C\end{aligned}$$

where C is an arbitrary constant.

### Question 10:

Integrate  $\sin^4 x$

### Solution 10:

Consider  $\sin^4 x = \sin^2 x \sin^2 x$



$$\begin{aligned}
&= \left( \frac{1 - \cos 2x}{2} \right) \left( \frac{1 - \cos 2x}{2} \right) \\
&= \frac{1}{4} (1 - \cos 2x)^2 \\
&= \frac{1}{4} [1 + \cos^2 2x - 2 \cos 2x] \\
&= \frac{1}{4} \left[ 1 + \left( \frac{1 + \cos 4x}{2} \right) - 2 \cos 2x \right] \\
&= \frac{1}{4} \left[ 1 + \frac{1}{2} + \frac{1}{2} \cos 4x - 2 \cos 2x \right] \\
&= \frac{1}{4} \left[ \frac{3}{2} + \frac{1}{2} \cos 4x - 2 \cos 2x \right] \\
\therefore \int \sin^4 x dx &= \frac{1}{4} \int \left[ \frac{3}{2} + \frac{1}{2} \cos 4x - 2 \cos 2x \right] dx \\
&= \frac{1}{4} \left[ \frac{3}{2} x + \frac{1}{2} \left( \frac{\sin 4x}{4} \right) - \frac{2 \sin 2x}{2} \right] + C \\
&= \frac{1}{8} \left[ 3x + \frac{\sin 4x}{4} - 2 \sin 2x \right] + C \\
&= \frac{3x}{8} - \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + C
\end{aligned}$$

where C is an arbitrary constant.

**Question 11:**Integrate  $\cos^4 2x$ **Solution 11:**

$$\begin{aligned}
\cos^4 2x &= (\cos^2 2x)^2 \\
&= \left( \frac{1 + \cos 4x}{2} \right)^2 \\
&= \frac{1}{4} [1 + \cos^2 4x + 2 \cos 4x] \\
&= \frac{1}{4} \left[ 1 + \left( \frac{1 + \cos 8x}{2} \right) + 2 \cos 4x \right] \\
&= \frac{1}{4} \left[ 1 + \frac{1}{2} + \frac{\cos 8x}{2} + 2 \cos 4x \right] \\
&= \frac{1}{4} \left[ \frac{3}{2} + \frac{\cos 8x}{2} + 2 \cos 4x \right] \\
\therefore \int \cos^4 2x dx &= \int \left( \frac{3}{8} + \frac{\cos 8x}{8} + \frac{\cos 4x}{2} \right) dx
\end{aligned}$$

$$= \frac{3}{8}x + \frac{\sin 8x}{64} + \frac{\sin 4x}{8} + C$$

where C is an arbitrary constant.

### Question 12:

Integrate  $\frac{\sin^2 x}{1 + \cos x}$

### Solution 12:

$$\frac{\sin^2 x}{1 + \cos x} = \frac{\left(2 \sin \frac{x}{2} \cos \frac{x}{2}\right)^2}{2 \cos^2 \frac{x}{2}} \quad \left[ \sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}; \cos x = 2 \cos^2 \frac{x}{2} - 1 \right]$$

$$= \frac{4 \sin^2 \frac{x}{2} \cos^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}}$$

$$= 2 \sin^2 \frac{x}{2}$$

$$= 1 - \cos x$$

$$\therefore \int \frac{\sin^2 x}{1 + \cos x} dx = \int (1 - \cos x) dx$$

$$= x - \sin x + C$$

where C is an arbitrary constant.

### Question 13:

Integrate  $\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$

### Solution 13:

Consider,

$$\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} = \frac{-2 \sin \frac{2x+2\alpha}{2} \sin \frac{2x-2\alpha}{2}}{-2 \sin \frac{x+\alpha}{2} \sin \frac{x-\alpha}{2}} \quad \left[ \cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2} \right]$$

$$= \frac{\sin(x+\alpha) \sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right) \sin\left(\frac{x-\alpha}{2}\right)}$$

$$\begin{aligned}
&= \frac{\left[ 2 \sin\left(\frac{x+\alpha}{2}\right) \cos\left(\frac{x+\alpha}{2}\right) \right] \left[ 2 \sin\left(\frac{x-\alpha}{2}\right) \cos\left(\frac{x-\alpha}{2}\right) \right]}{\sin\left(\frac{x+\alpha}{2}\right) \sin\left(\frac{x-\alpha}{2}\right)} \\
&= 4 \cos\left(\frac{x+\alpha}{2}\right) \cos\left(\frac{x-\alpha}{2}\right) \\
&= 2 \left[ \cos\left(\frac{x+\alpha}{2} + \frac{x-\alpha}{2}\right) + \cos\frac{x+\alpha}{2} - \frac{x-\alpha}{2} \right] \\
&= 2 [\cos(x) + \cos \alpha] \\
&= 2 \cos x + 2 \cos \alpha \\
\therefore \int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx &= \int 2 \cos x + 2 \cos \alpha \\
&= 2 [\sin x + x \cos \alpha] + C
\end{aligned}$$

where C is an arbitrary constant.

#### Question 14:

Integrate  $\frac{\cos x - \sin x}{1 + \sin 2x}$

#### Solution 14:

$$\frac{\cos x - \sin x}{1 + \sin 2x} = \frac{\cos x - \sin x}{(\sin^2 x + \cos^2 x) + 2 \sin x \cos x} \quad \left[ \sin^2 x + \cos^2 x = 1; \sin 2x = 2 \sin x \cos x \right]$$

$$= \frac{\cos x - \sin x}{(\sin x + \cos x)^2}$$

Let  $\sin x + \cos x = t$

$$\therefore (\cos x - \sin x) dx = dt$$

$$\Rightarrow \int \frac{\cos x - \sin x}{1 + \sin 2x} dx = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx$$

$$= \int \frac{dt}{t^2}$$

$$= \int t^{-2} dt$$

$$= -t^{-1} + C$$

$$= -\frac{1}{t} + C$$

$$= \frac{-1}{\sin x + \cos x} + C$$

where C is an arbitrary constant.

**Question 15:**Integrate  $\tan^3 2x \sec 2x$ **Solution 15:**

$$\begin{aligned}\tan^3 2x \sec 2x &= \tan^2 2x \tan 2x \sec 2x \\ &= (\sec^2 2x - 1) \tan 2x \sec 2x \\ &= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x \\ \therefore \int \tan^3 2x \sec 2x dx &= \int \sec^2 2x \cdot \tan 2x \sec 2x dx - \int \tan 2x \sec 2x dx \\ &= \int \sec^2 2x \cdot \tan 2x \sec 2x dx - \frac{\sec 2x}{2} + C\end{aligned}$$

Let  $\sec 2x = t$ 

$$\therefore 2 \sec 2x \tan 2x dx = dt$$

$$\begin{aligned}\therefore \int \tan^3 2x \sec 2x dx &= \frac{1}{2} \int t^2 dt - \frac{\sec 2x}{2} + C \\ &= \frac{t^3}{6} - \frac{\sec 2x}{2} + C \\ &= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C\end{aligned}$$

where C is an arbitrary constant.

**Question 16:**Integrate  $\tan^4 x$ **Solution 16:**

$$\begin{aligned}\tan^4 x &= \tan^2 x \cdot \tan^2 x \\ &= (\sec^2 x - 1) \tan^2 x \\ &= \sec^2 x \tan^2 x - \tan^2 x \\ &= \sec^2 x \tan^2 x - (\sec^2 x - 1) \\ &= \sec^2 x \tan^2 x - \sec^2 x + 1 \\ \therefore \int \tan^4 x dx &= \int \sec^2 x \tan^2 x dx - \int \sec^2 x dx + \int 1 dx \\ &= \int \sec^2 x \tan^2 x dx - \tan x + x + C \quad \dots(1)\end{aligned}$$

Consider  $\int \sec^2 x \tan^2 x dx$ 

$$\text{Let } \tan x = t \Rightarrow \sec^2 x dx = dt$$

$$\Rightarrow \int \sec^2 x \tan^2 x dx = \int t^2 dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}$$

From equation (1), we obtain

$$\int \tan^4 x dx = \frac{1}{3} \tan^3 x - \tan x + x + C$$

where C is an arbitrary constant.

### Question 17:

Integrate  $\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$

### Solution 17:

$$\begin{aligned} \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} &= \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x} \\ &= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x} \\ &= \tan x \sec x + \cot x \operatorname{cosec} x \end{aligned}$$

$$\begin{aligned} \therefore \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx &= \int (\tan x \sec x + \cot x \operatorname{cosec} x) dx \\ &= \sec x - \operatorname{cosec} x + C \end{aligned}$$

where C is an arbitrary constant.

### Question 18:

Integrate  $\frac{\cos 2x + 2 \sin^2 x}{\cos^2 x}$

### Solution 18:

$$\begin{aligned} \frac{\cos 2x + 2 \sin^2 x}{\cos^2 x} &= \frac{\cos 2x + (1 - \cos 2x)}{\cos^2 x} \quad [\cos 2x = 1 - 2 \sin^2 x] \\ &= \frac{1}{\cos^2 x} \\ &= \sec^2 x \end{aligned}$$

$$\therefore \int \frac{\cos 2x + 2 \sin^2 x}{\cos^2 x} dx = \int \sec^2 x dx = \tan x + C$$

where C is an arbitrary constant.

### Question 19:

Integrate  $\frac{1}{\sin x \cos^3 x}$

### Solution 19:

$$\begin{aligned} \frac{1}{\sin x \cos^3 x} &= \frac{\sin^2 x + \cos^2 x}{\sin x \cos^3 x} \\ &= \frac{\sin x}{\cos^3 x} + \frac{1}{\sin x \cos x} \\ &= \tan x \sec^2 x + \frac{1/\cos^2 x}{\sin x \cos x} \\ &= \tan x \sec^2 x + \frac{\sec^2 x}{\tan x} \end{aligned}$$

$$\therefore \int \frac{1}{\sin x \cos^3 x} dx = \int \tan x \sec^2 x dx + \int \frac{\sec^2 x}{\tan x} dx$$

Let  $\tan x = t \Rightarrow \sec^2 x dx = dt$

$$\begin{aligned} \Rightarrow \int \frac{1}{\sin x \cos^3 x} dx &= \int t dt + \int \frac{1}{t} dt \\ &= \frac{t^2}{2} + \log|t| + C \\ &= \frac{1}{2} \tan^2 x + \log|\tan x| + C \end{aligned}$$

where C is an arbitrary constant.

### Question 20:

Integrate  $\frac{\cos 2x}{(\cos x + \sin x)^2}$

### Solution 20:

$$\frac{\cos 2x}{(\cos x + \sin x)^2} = \frac{\cos 2x}{\cos^2 x + \sin^2 x + 2 \sin x \cos x} = \frac{\cos 2x}{1 + \sin 2x}$$

$$\therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx = \int \frac{\cos 2x}{1 + \sin 2x} dx$$

Let  $1 + \sin 2x = t$

$$\Rightarrow 2 \cos 2x dx = dt$$

$$\begin{aligned} \therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx &= \frac{1}{2} \int \frac{1}{t} dt \\ &= \frac{1}{2} \log|t| + C \\ &= \frac{1}{2} \log|1 + \sin 2x| + C \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} \log |(\sin x + \cos x)^2| + C \\
 &= \log |\sin x + \cos x| + C
 \end{aligned}$$

where C is an arbitrary constant.

### Question 21:

Integrate  $\sin^{-1}(\cos x)$

### Solution 21:

$$\sin^{-1}(\cos x)$$

$$\text{Let } \cos x = t$$

$$\text{Then, } \sin x = \sqrt{1-t^2}$$

$$\Rightarrow (-\sin x) dx = dt$$

$$dx = \frac{-dt}{\sin x}$$

$$dx = \frac{-dt}{\sqrt{1-t^2}}$$

$$\begin{aligned}
 \therefore \int \sin^{-1}(\cos x) dx &= \int \sin^{-1} t \left( \frac{-dt}{\sqrt{1-t^2}} \right) \\
 &= - \int \frac{\sin^{-1} t}{\sqrt{1-t^2}} dt
 \end{aligned}$$

$$\text{Let } \sin^{-1} t = u$$

$$\Rightarrow \frac{1}{\sqrt{1-t^2}} dt = du$$

$$\begin{aligned}
 \therefore \int \sin^{-1}(\cos x) dx &= - \int 4du \\
 &= -\frac{u^2}{2} + C \\
 &= -\frac{(\sin^{-1} t)^2}{2} + C \\
 &= -\frac{[\sin^{-1}(\cos x)]^2}{2} + C \quad \dots(1)
 \end{aligned}$$

It is known that,

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\therefore \sin^{-1}(\cos x) = \frac{\pi}{2} - \cos^{-1}(\cos x) = \left( \frac{\pi}{2} - x \right)$$

Substituting in equation (1), we obtain

$$\begin{aligned}
 \int \sin^{-1}(\cos x) dx &= -\frac{\left[\frac{\pi}{2} - x\right]^2}{2} + C \\
 &= -\frac{1}{2} \left( \frac{\pi^2}{2} + x^2 - \pi x \right) + C \\
 &= -\frac{\pi^2}{8} - \frac{x^2}{2} + \frac{1}{2} \pi x + C \\
 &= \frac{\pi x}{2} - \frac{x^2}{2} + \left( C - \frac{\pi^2}{8} \right) \\
 &= \frac{\pi x}{2} - \frac{x^2}{2} + C_1
 \end{aligned}$$

**Question 22:**

Integrate  $\frac{1}{\cos(x-a)\cos(x-b)}$

**Solution 22:**

$$\begin{aligned}
 \frac{1}{\cos(x-a)\cos(x-b)} &= \frac{1}{\sin(a-b)} \left[ \frac{\sin(a-b)}{\cos(x-a)\cos(x-b)} \right] \\
 &= \frac{1}{\sin(a-b)} \left[ \frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)} \right] \\
 &= \frac{1}{\sin(a-b)} \left[ \frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)} \right] \\
 &= \frac{1}{\sin(a-b)} \left[ \tan(x-b) - \tan(x-a) \right] \\
 \Rightarrow \int \frac{1}{\cos(x-a)\cos(x-b)} dx &= \frac{1}{\sin(a-b)} \int \left[ \tan(x-b) - \tan(x-a) \right] dx \\
 &= \frac{1}{\sin(a-b)} \left[ -\log|\cos(x-b)| + \log|\cos(x-a)| \right] \\
 &= \frac{1}{\sin(a-b)} \left[ \log \left| \frac{\cos(x-a)}{\cos(x-b)} \right| \right] + C
 \end{aligned}$$

where C is an arbitrary constant.

**Question 23:**

$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$  is equal to



- (A)  $\tan x + \cot x + C$   
 (C)  $-\tan x + \cot x + C$

- (B)  $\tan x + \operatorname{cosec} x + C$   
 (D)  $\tan x + \sec x + C$

**Solution 23:**

$$\begin{aligned}\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx &= \int \left( \frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx \\ &= \int (\sec^2 x - \operatorname{cosec}^2 x) dx \\ &= \tan x + \cot x + C\end{aligned}$$

Hence, the correct Answer is A.

**Question 24:**

$$\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx \text{ equals}$$

- (A)  $-\cot(e^x) + C$                       (B)  $\tan(xe^x) + C$   
 (C)  $\tan(e^x) + C$                       (D)  $\cot(e^x) + C$

**Solution 24:**

$$\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$$

Let  $e^x = t$

$$\Rightarrow (e^x \cdot x + e^x \cdot 1) dx = dt$$

$$e^x(x+1) dx = dt$$

$$\begin{aligned}\therefore \int \frac{e^x(1+x)}{\cos^2(e^x x)} dx &= \int \frac{dt}{\cos^2 t} \\ &= \int \sec^2 t dt \\ &= \tan t + C \\ &= \tan(e^x \cdot x) + C\end{aligned}$$

Hence, the correct Answer is B.

**Exercise 7.4****Question 1:**

$$\text{Integrate } \frac{3x^2}{x^6 + 1}$$

**Solution 1:**

Let  $x^3 = t$

$\therefore 3x^2 dx = dt$

$$\Rightarrow \int \frac{3x^2}{x^6+1} dx = \int \frac{dt}{t^2+1}$$

$= \tan^{-1} t + C$

$= \tan^{-1} (x^3) + C$

where C is an arbitrary constant.

**Question 2:**

Integrate  $\frac{1}{\sqrt{1+4x^2}}$

**Solution 2:**

Let  $2x = t$

$\therefore 2dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{1+4x^2}} dx = \frac{1}{2} \int \frac{dt}{\sqrt{1+t^2}}$$

$$= \frac{1}{2} \left[ \log \left| t + \sqrt{t^2+1} \right| \right] + C$$

$$\left[ \int \frac{1}{\sqrt{x^2+a^2}} dt = \log \left| x + \sqrt{x^2+a^2} \right| \right]$$

$$= \frac{1}{2} \log \left| 2x + \sqrt{4x^2+1} \right| + C$$

where C is an arbitrary constant.

**Question 3:**

Integrate  $\frac{1}{\sqrt{(2-x)^2+1}}$

**Solution 3:**

Let  $2-x = t$

$\Rightarrow -dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{(2-x)^2+1}} dx = - \int \frac{1}{\sqrt{t^2+1}} dt$$

$$= -\log \left| t + \sqrt{t^2+1} \right| + C$$

$$\left[ \int \frac{1}{\sqrt{x^2+a^2}} dt = \log \left| x + \sqrt{x^2+a^2} \right| \right]$$

$$= -\log \left| 2-x + \sqrt{(2-x)^2+1} \right| + C$$

$$= \log \left| \frac{1}{(2-x) + \sqrt{x^2-4x+5}} \right| + C$$

where C is an arbitrary constant.

**Question 4:**

Integrate  $\frac{1}{\sqrt{9-25x^2}}$

**Solution 4:**

Let  $5x = t$

$\therefore 5dx = dt$

$$\begin{aligned}\Rightarrow \int \frac{1}{\sqrt{9-25x^2}} dx &= \frac{1}{5} \int \frac{1}{\sqrt{3^2-t^2}} dt \\ &= \frac{1}{5} \sin^{-1} \left( \frac{t}{3} \right) + C \\ &= \frac{1}{5} \sin^{-1} \left( \frac{5x}{3} \right) + C\end{aligned}$$

where C is an arbitrary constant.

**Question 5:**

Integrate  $\frac{3x}{1+2x^4}$

**Solution 5:**

Let  $\sqrt{2}x^2 = t$

$\therefore 2\sqrt{2}x dx = dt$

$$\begin{aligned}\Rightarrow \int \frac{3x}{1+2x^4} dx &= \frac{3}{2\sqrt{2}} \int \frac{dt}{1+t^2} \\ &= \frac{3}{2\sqrt{2}} [\tan^{-1} t] + C \\ &= \frac{3}{2\sqrt{2}} \tan^{-1} (\sqrt{2}x^2) + C\end{aligned}$$

where C is an arbitrary constant.

**Question 6:**

Integrate  $\frac{x^2}{1-x^6}$

**Solution 6:**

Let  $x^3 = t$

$\therefore 3x^2 dx = dt$

$$\begin{aligned}\Rightarrow \int \frac{x^2}{1-x^6} dx &= \frac{1}{3} \int \frac{dt}{1-t^2} \\ &= \frac{1}{3} \left[ \frac{1}{2} \log \left| \frac{1+t}{1-t} \right| \right] + C \\ &= \frac{1}{6} \log \left| \frac{1+x^3}{1-x^3} \right| + C\end{aligned}$$

where C is an arbitrary constant.

### Question 7:

Integrate  $\frac{x-1}{\sqrt{x^2-1}}$

### Solution 7:

$$\int \frac{x-1}{\sqrt{x^2-1}} dx = \int \frac{x}{\sqrt{x^2-1}} dx - \int \frac{1}{\sqrt{x^2-1}} dx \quad \dots(1)$$

For  $\int \frac{x}{\sqrt{x^2-1}} dx$ , let  $x^2-1=t \Rightarrow 2x dx = dt$

$$\begin{aligned}\therefore \int \frac{x}{\sqrt{x^2-1}} dx &= \frac{1}{2} \int \frac{dt}{\sqrt{t}} \\ &= \frac{1}{2} \int t^{-\frac{1}{2}} dt \\ &= \frac{1}{2} \left[ 2t^{\frac{1}{2}} \right] \\ &= \sqrt{t} \\ &= \sqrt{x^2-1}\end{aligned}$$

From (1), we obtain

$$\begin{aligned}\int \frac{x-1}{\sqrt{x^2-1}} dx &= \int \frac{x}{\sqrt{x^2-1}} dx - \int \frac{1}{\sqrt{x^2-1}} dx \quad \left[ \int \frac{x}{\sqrt{x^2-a^2}} dt = \log |x + \sqrt{x^2-a^2}| \right] \\ &= \sqrt{x^2-1} - \log |x + \sqrt{x^2-1}| + C\end{aligned}$$

where C is an arbitrary constant.

### Question 8:

Integrate  $\frac{x^2}{\sqrt{x^6+a^6}}$

### Solution 8:

Let  $x^3 = t$

$$\Rightarrow 3x^2 dx = dt$$

$$\begin{aligned}\therefore \int \frac{x^2}{\sqrt{x^6 + a^6}} dx &= \frac{1}{3} \int \frac{dt}{\sqrt{t^2 + (a^3)^2}} \\ &= \frac{1}{3} \log \left| t + \sqrt{t^2 + a^6} \right| + C \\ &= \frac{1}{3} \log \left| x^3 + \sqrt{x^6 + a^6} \right| + C\end{aligned}$$

where C is an arbitrary constant.

### Question 9:

Integrate  $\frac{\sec^2 x}{\sqrt{\tan^2 x + 4}}$

### Solution 9:

Let  $\tan x = t$

$$\therefore \sec^2 x dx = dt$$

$$\begin{aligned}\Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx &= \int \frac{dt}{\sqrt{t^2 + 2^2}} \\ &= \log \left| t + \sqrt{t^2 + 4} \right| + C \\ &= \log \left| \tan x + \sqrt{\tan^2 x + 4} \right| + C\end{aligned}$$

where C is an arbitrary constant.

### Question 10:

Integrate  $\frac{1}{\sqrt{x^2 + 2x + 2}}$

### Solution 10:

$$\int \frac{1}{\sqrt{x^2 + 2x + 2}} dx = \int \frac{1}{\sqrt{(x+1)^2 + (1)^2}} dx$$

Let  $x+1=t$

$$\therefore dx=dt$$

$$\begin{aligned}\Rightarrow \int \frac{1}{\sqrt{x^2 + 2x + 2}} dx &= \int \frac{1}{\sqrt{t^2 + 1}} dt \\ &= \log \left| t + \sqrt{t^2 + 1} \right| + C \\ &= \log \left| (x+1) + \sqrt{(x+1)^2 + 1} \right| + C\end{aligned}$$

$$= \log \left| (x+1) + \sqrt{x^2 + 2x + 2} \right| + C$$

where C is an arbitrary constant.

### Question 11:

Integrate  $\frac{1}{\sqrt{9x^2 + 6x + 5}}$

### Solution 11:

$$\int \frac{1}{\sqrt{9x^2 + 6x + 5}} dx = \int \frac{1}{\sqrt{(3x+1)^2 + 2^2}} dx$$

$$\text{Let } (3x+1) = t$$

$$\therefore 3dx = dt$$

$$\int \frac{1}{\sqrt{(3x+1)^2 + 2^2}} dx = \frac{1}{3} \int \frac{1}{\sqrt{t^2 + 2^2}} dt$$

$$= \frac{1}{3} \log | t + \sqrt{t^2 + 2^2} | + C$$

$$= \frac{1}{3} \log | (3x+1) + \sqrt{(3x+1)^2 + 2^2} | + C$$

where C is an arbitrary constant.

### Question 12:

Integrate  $\frac{1}{\sqrt{7 - 6x - x^2}}$

### Solution 12:

$7 - 6x - x^2$  can be written as  $7 - (x^2 + 6x + 9 - 9)$

Therefore,

$$7 - (x^2 + 6x + 9 - 9)$$

$$= 16 - (x^2 + 6x + 9)$$

$$= 16 - (x+3)^2$$

$$= (4)^2 - (x+3)^2$$

$$\therefore \int \frac{1}{\sqrt{7 - 6x - x^2}} dx = \int \frac{1}{\sqrt{(4)^2 - (x+3)^2}} dx$$

$$\text{Let } x+3 = t$$

$$\Rightarrow dx = dt$$

$$\begin{aligned}\Rightarrow \int \frac{1}{\sqrt{(4)^2 - (x+3)^2}} dx &= \int \frac{1}{\sqrt{(4)^2 - (t)^2}} dt \\ &= \sin^{-1}\left(\frac{t}{4}\right) + C \\ &= \sin^{-1}\left(\frac{x+3}{4}\right) + C\end{aligned}$$

where C is an arbitrary constant.

### Question 13:

Integrate  $\frac{1}{\sqrt{(x-1)(x-2)}}$

### Solution 13:

$(x-1)(x-2)$  can be written as  $x^2 - 3x + 2$ .

Therefore,

$$x^2 - 3x + 2$$

$$= x^2 - 3x + \frac{9}{4} - \frac{9}{4} + 2$$

$$= \left(x - \frac{3}{2}\right)^2 - \frac{1}{4}$$

$$= \left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$\therefore \int \frac{1}{\sqrt{(x-1)(x-2)}} dx = \int \frac{1}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} dx$$

Let  $x - \frac{3}{2} = t$

$\therefore dx = dt$

$$\begin{aligned}\Rightarrow \int \frac{1}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} dx &= \int \frac{1}{\sqrt{t^2 - \left(\frac{1}{2}\right)^2}} dt \\ &= \log \left| t + \sqrt{t^2 - \left(\frac{1}{2}\right)^2} \right| + C \\ &= \log \left| \left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + 2} \right| + C\end{aligned}$$

where C is an arbitrary constant.

**Question 14:**

Integrate  $\frac{1}{\sqrt{8+3x-x^2}}$

**Solution 14:**

$8+3x-x^2$  can be written as  $8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$

Therefore,

$$8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$$

$$= \frac{41}{4}-\left(x-\frac{3}{2}\right)^2$$

$$\Rightarrow \int \frac{1}{\sqrt{8+3x-x^2}} dx = \int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}} dx$$

$$\text{Let } x-\frac{3}{2}=t$$

$$\therefore dx = dt$$

$$\Rightarrow \int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}} dx = \int \frac{1}{\sqrt{\left(\frac{41}{4}\right)-t^2}} dt$$

$$= \sin^{-1} \left( \frac{t}{\frac{\sqrt{41}}{2}} \right) + C$$

$$= \sin^{-1} \left( \frac{x-\frac{3}{2}}{\frac{\sqrt{41}}{2}} \right) + C$$

$$= \sin^{-1} \left( \frac{2x-3}{\sqrt{41}} \right) + C$$

where C is an arbitrary constant.

**Question 15:**

Integrate  $\frac{1}{\sqrt{(x-a)(x-b)}}$

**Solution 15:**

$(x-a)(x-b)$  can be written as  $x^2-(a+b)x+ab$ .



Therefore,

$$\begin{aligned} & x^2 - (a+b)x + ab \\ &= x^2 - (a+b)x + \frac{(a+b)^2}{4} - \frac{(a+b)^2}{4} + ab \\ &= \left[ x - \left( \frac{a+b}{2} \right) \right]^2 - \frac{(a-b)^2}{4} \\ & \int \frac{1}{\sqrt{(x-a)(x-b)}} dx = \int \frac{1}{\sqrt{\left[ x - \left( \frac{a+b}{2} \right) \right]^2 - \frac{(a-b)^2}{4}}} dx \end{aligned}$$

$$\text{Let } x - \left( \frac{a+b}{2} \right) = t$$

$$\therefore dx = dt$$

$$\begin{aligned} & \int \frac{1}{\sqrt{\left[ x - \left( \frac{a+b}{2} \right) \right]^2 - \frac{(a-b)^2}{4}}} dx = \int \frac{1}{\sqrt{t^2 - \left( \frac{a-b}{2} \right)^2}} dt \\ &= \log \left| t + \sqrt{t^2 - \left( \frac{a-b}{2} \right)^2} \right| + C \\ &= \log \left| \left\{ x - \left( \frac{a+b}{2} \right) \right\} + \sqrt{(x-a)(x-b)} \right| + C \end{aligned}$$

### Question 16:

$$\text{Integrate } \frac{4x+1}{\sqrt{2x^2+x-3}}$$

### Solution 16:

$$\text{Let } 2x^2 + x - 3 = t$$

$$\therefore (4x + 1) dx = dt$$

$$\begin{aligned} \Rightarrow \int \frac{4x+1}{\sqrt{2x^2+x-3}} dx &= \int \frac{1}{\sqrt{t}} dt \\ &= 2\sqrt{t} + C \\ &= 2\sqrt{2x^2+x-3} + C \end{aligned}$$

where C is an arbitrary constant.

### Question 17:

$$\text{Integrate } \frac{x+2}{\sqrt{x^2-1}}$$

**Solution 17:**

$$\text{Let } x + 2 = A \frac{d}{dx}(x^2 - 1) + B \quad \dots(1)$$

$$\Rightarrow x + 2 = A(2x) + B$$

Equating the coefficients of x and constant terms on both sides, we obtain

$$2A = 1 \Rightarrow A = \frac{1}{2}$$

$$B = 2$$

From (1), we obtain

$$(x + 2) = \frac{1}{2}(2x) + 2$$

$$\begin{aligned} \text{Then, } \int \frac{x+2}{\sqrt{x^2-1}} dx &= \int \frac{\frac{1}{2}(2x) + 2}{\sqrt{x^2-1}} dx \\ &= \frac{1}{2} \int \frac{2x}{\sqrt{x^2-1}} dx + \int \frac{2}{\sqrt{x^2-1}} dx \quad \dots(2) \end{aligned}$$

$$\text{In } \frac{1}{2} \int \frac{2x}{\sqrt{x^2-1}} dx \text{ let } x^2 - 1 = t \Rightarrow 2xdx = dt$$

$$\begin{aligned} \frac{1}{2} \int \frac{2x}{\sqrt{x^2-1}} dx &= \frac{1}{2} \int \frac{dt}{\sqrt{t}} \\ &= \frac{1}{2} [2\sqrt{t}] \\ &= \sqrt{t} \\ &= \sqrt{x^2-1} \end{aligned}$$

$$\text{Then, } \int \frac{2}{\sqrt{x^2-1}} dx = 2 \int \frac{1}{\sqrt{x^2-1}} dx = 2 \log |x + \sqrt{x^2-1}|$$

From equation (2), we obtain

$$\int \frac{x+2}{\sqrt{x^2-1}} dx = \sqrt{x^2-1} + 2 \log |x + \sqrt{x^2-1}| + C$$

where C is an arbitrary constant.

**Question 18:**

$$\text{Integrate } \frac{5x-2}{1+2x+3x^2}$$

**Solution 18:**

$$\text{Let } 5x - 2 = A \frac{d}{dx}(1 + 2x + 3x^2) + B$$

$$\Rightarrow 5x - 2 = A(2 + 6x) + B$$

Equating the coefficient of x and constant term on both sides, we obtain

$$5 = 6A \Rightarrow A = \frac{5}{6}$$

$$2A + B = -2 \Rightarrow B = -\frac{11}{3}$$

$$\therefore 5x - 2 = \frac{5}{6}(2 + 6x) + \left(-\frac{11}{3}\right)$$

$$\Rightarrow \int \frac{5x-2}{1+2x+3x^2} dx = \int \frac{\frac{5}{6}(2+6x) - \frac{11}{3}}{1+2x+3x^2} dx$$

$$= \frac{5}{6} \int \frac{2+6x}{1+2x+3x^2} dx - \frac{11}{3} \int \frac{1}{1+2x+3x^2} dx$$

$$\text{Let } I_1 = \int \frac{2+6x}{1+2x+3x^2} dx \text{ and } I_2 = \int \frac{1}{1+2x+3x^2} dx$$

$$\therefore \int \frac{5x-2}{1+2x+3x^2} dx = \frac{5}{6} I_1 - \frac{11}{3} I_2 \quad \dots(1)$$

$$I_1 = \int \frac{2+6x}{1+2x+3x^2} dx$$

$$\text{Let } 1+2x+3x^2 = t$$

$$\Rightarrow (2+6x) dx = dt$$

$$\therefore I_1 = \int \frac{dt}{t}$$

$$I_1 = \log|t|$$

$$I_1 = \log|1+2x+3x^2| \quad \dots(2)$$

$$I_2 = \int \frac{1}{1+2x+3x^2} dx$$

$$1+2x+3x^2 \text{ can be written as } 1+3\left(x^2 + \frac{2}{3}x\right)$$

Therefore,

$$1+3\left(x^2 + \frac{2}{3}x\right)$$

$$= 1+3\left(x^2 + \frac{2}{3}x + \frac{1}{9} - \frac{1}{9}\right)$$

$$= 1+3\left(x + \frac{1}{3}\right)^2 - \frac{1}{3}$$

$$= \frac{2}{3} + 3\left(x + \frac{1}{3}\right)^2$$

$$= 3\left[\left(x + \frac{1}{3}\right)^2 + \frac{2}{9}\right]$$

$$\begin{aligned}
 &= 3 \left[ \left( x + \frac{1}{3} \right)^2 + \left( \frac{\sqrt{2}}{3} \right)^2 \right] \\
 I_2 &= \frac{1}{3} \int \frac{1}{\left[ \left( x + \frac{1}{3} \right)^2 + \left( \frac{\sqrt{2}}{3} \right)^2 \right]} dx \\
 &= \frac{1}{3} \left[ \frac{1}{\frac{\sqrt{2}}{3}} \tan^{-1} \left( \frac{x + \frac{1}{3}}{\frac{\sqrt{2}}{3}} \right) \right] \\
 &= \frac{1}{3} \left[ \frac{3}{\sqrt{2}} \tan^{-1} \left( \frac{3x+1}{\sqrt{2}} \right) \right] \\
 &= \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{3x+1}{\sqrt{2}} \right) \quad \dots(3)
 \end{aligned}$$

Substituting equations (2) and (3) in equation (1), we obtain

$$\begin{aligned}
 \int \frac{5x-2}{1+2x+3x^2} dx &= \frac{5}{6} \left[ \log |1+2x+3x^2| \right] - \frac{11}{3} \left[ \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{3x+1}{\sqrt{2}} \right) \right] + C \\
 &= \frac{5}{6} \log |1+2x+3x^2| - \frac{11}{3\sqrt{2}} \tan^{-1} \left( \frac{3x+1}{\sqrt{2}} \right) + C
 \end{aligned}$$

where C is an arbitrary constant.

### Question 19:

Integrate  $\frac{6x+7}{\sqrt{(x-5)(x-4)}}$

### Solution 19:

$$\frac{6x+7}{\sqrt{(x-5)(x-4)}} = \frac{6x+7}{\sqrt{x^2-9x+20}}$$

Let  $6x+7 = A \frac{d}{dx}(x^2-9x+20) + B$

$$\Rightarrow 6x+7 = A(2x-9) + B$$

Equating the coefficients of x and constant term, we obtain

$$2A = 6 \Rightarrow A = 3$$

$$-9A + B = 7 \Rightarrow B = 34$$

$$\therefore 6x+7 = 3(2x-9) + 34$$

$$\int \frac{6x+7}{\sqrt{x^2-9x+20}} = \int \frac{3(2x-9)+34}{\sqrt{x^2-9x+20}} dx$$

$$= 3 \int \frac{2x-9}{\sqrt{x^2-9x+20}} dx + 34 \int \frac{1}{\sqrt{x^2-9x+20}} dx$$

$$\text{Let } I_1 = \int \frac{2x-9}{\sqrt{x^2-9x+20}} dx \text{ and } I_2 = \int \frac{1}{\sqrt{x^2-9x+20}} dx$$

$$\therefore \int \frac{6x+7}{\sqrt{x^2-9x+20}} = 3I_1 + 34I_2 \quad (1)$$

Then,

$$I_1 = \int \frac{2x-9}{\sqrt{x^2-9x+20}} dx$$

$$\text{Let } x^2 - 9x + 20 = t$$

$$\Rightarrow (2x-9) dx = dt$$

$$\Rightarrow I_1 = \frac{dt}{\sqrt{t}}$$

$$I_1 = 2\sqrt{t}$$

$$I_1 = 2\sqrt{x^2-9x+20} \quad \dots(2)$$

$$\text{and } I_2 = \int \frac{1}{\sqrt{x^2-9x+20}} dx$$

$$x^2 - 9x + 20 \text{ can be written as } x^2 - 9x + 20 + \frac{81}{4} - \frac{81}{4}.$$

Therefore,

$$x^2 - 9x + 20 + \frac{81}{4} - \frac{81}{4}$$

$$= \left(x - \frac{9}{2}\right)^2 - \frac{1}{4}$$

$$= \left(x - \frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$\Rightarrow I_2 = \int \frac{1}{\left(x - \frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$$

$$I_2 = \log \left| \left(x - \frac{9}{2}\right) + \sqrt{x^2 - 9x + 20} \right| \quad \dots(3)$$

Substituting equations (2) and (3) in (1), we obtain

$$\int \frac{6x+7}{\sqrt{x^2-9x+20}} dx = 3 \left[ 2\sqrt{x^2-9x+20} \right] + 34 \log \left[ \left(x - \frac{9}{2}\right) + \sqrt{x^2-9x+20} \right] + C$$

$$= 6\sqrt{x^2-9x+20} + 34 \log \left[ \left(x - \frac{9}{2}\right) + \sqrt{x^2-9x+20} \right] + C$$

where C is an arbitrary constant.

**Question 20:**

Integrate  $\frac{x+2}{\sqrt{4x-x^2}}$

**Solution 20:**

Let  $x+2 = A \frac{d}{dx}(4x-x^2) + B$

$$\Rightarrow x+2 = A(4-2x) + B$$

Equating the coefficients of  $x$  and constant term on both sides, we obtain

$$-2A = 1 \Rightarrow A = -\frac{1}{2}$$

$$4A + B = 2 \Rightarrow B = 4$$

$$\Rightarrow (x+2) = -\frac{1}{2}(4-2x) + 4$$

$$\begin{aligned} \therefore \int \frac{x+2}{\sqrt{4x-x^2}} dx &= \int \frac{-\frac{1}{2}(4-2x) + 4}{\sqrt{4x-x^2}} dx \\ &= -\frac{1}{2} \int \frac{4-2x}{\sqrt{4x-x^2}} dx + 4 \int \frac{1}{\sqrt{4x-x^2}} dx \end{aligned}$$

$$\text{Let } I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}} dx \text{ and } I_2 = \int \frac{1}{\sqrt{4x-x^2}} dx$$

$$\therefore \int \frac{x+2}{\sqrt{4x-x^2}} dx = -\frac{1}{2} I_1 + 4 I_2 \quad \dots (1)$$

$$\text{Then, } I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}} dx$$

$$\text{Let } 4x-x^2 = t$$

$$\Rightarrow (4-2x) dx = dt$$

$$\Rightarrow I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{4x-x^2} \quad \dots(2)$$

$$I_2 = \int \frac{1}{\sqrt{4x-x^2}} dx$$

$$\Rightarrow 4x-x^2 = -(-4x+x^2)$$

$$= (-4x+x^2+4-4)$$

$$= 4-(x-2)^2$$

$$= (2)^2 - (x-2)^2$$

$$\therefore I_2 = \int \frac{1}{\sqrt{(2)^2 - (x-2)^2}} dx = \sin^{-1} \left( \frac{x-2}{2} \right) \quad \dots(3)$$

Using equations (2) and (3) in (1), we obtain

$$\int \frac{x+2}{\sqrt{4x-x^2}} dx = -\frac{1}{2} \left( 2\sqrt{4x-x^2} \right) + 4 \sin^{-1} \left( \frac{x-2}{2} \right) + C$$

$$= -\sqrt{4x-x^2} + 4 \sin^{-1} \left( \frac{x-2}{2} \right) + C$$

Where C is an arbitrary constant

### Question 21:

Integrate  $\frac{x+2}{\sqrt{x^2+2x+3}}$

### Solution 21:

$$\int \frac{x+2}{\sqrt{x^2+2x+3}} dx = \frac{1}{2} \int \frac{2(x+2)}{\sqrt{x^2+2x+3}} dx$$

$$= \frac{1}{2} \int \frac{2x+4}{\sqrt{x^2+2x+3}} dx$$

$$= \frac{1}{2} \int \frac{2x+2}{\sqrt{x^2+2x+3}} dx + \frac{1}{2} \int \frac{2}{\sqrt{x^2+2x+3}} dx$$

$$= \frac{1}{2} \int \frac{2x+2}{\sqrt{x^2+2x+3}} dx + \int \frac{1}{\sqrt{x^2+2x+3}} dx$$

Let  $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}} dx$  and  $I_2 = \int \frac{1}{\sqrt{x^2+2x+3}} dx$

$$\therefore \int \frac{x+2}{\sqrt{x^2+2x+3}} dx = \frac{1}{2} I_1 + I_2 \quad \dots(1)$$

Then,  $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}} dx$

Let  $x^2 + 2x + 3 = t$

$$\Rightarrow (2x + 2) dx = dt$$

$$I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{x^2+2x+3} \quad \dots(2)$$

$$I_2 = \int \frac{1}{\sqrt{x^2+2x+3}} dx$$

$$\Rightarrow x^2 + 2x + 3 = x^2 + 2x + 1 + 2 = (x+1)^2 + (\sqrt{2})^2$$

$$\therefore I_2 = \int \frac{1}{\sqrt{(x+1)^2 + (\sqrt{2})^2}} dx = \log \left| (x+1) + \sqrt{x^2+2x+3} \right| \quad \dots(3)$$

Using equations (2) and (3) in (1), we obtain

$$\int \frac{x+2}{\sqrt{x^2+2x+3}} dx = \frac{1}{2} \left[ 2\sqrt{x^2+2x+3} \right] + \log \left| (x+1) + \sqrt{x^2+2x+3} \right| + C$$

$$= \sqrt{x^2+2x+3} + \log \left| (x+1) + \sqrt{x^2+2x+3} \right| + C$$

Where C is an arbitrary constant

**Question 22:**

Integrate  $\frac{x+3}{x^2-2x-5}$

**Solution 22:**

Let  $(x+3) = A \frac{d}{dx}(x^2-2x-5) + B$

$$(x+3) = A(2x-2) + B$$

Equating the coefficients of x and constant term on both sides, we obtain

$$2A = 1 \Rightarrow A = \frac{1}{2}$$

$$-2A + B = 3 \Rightarrow B = 4$$

$$\therefore (x+3) = \frac{1}{2}(2x-2) + 4$$

$$\Rightarrow \int \frac{x+3}{x^2-2x-5} dx = \int \frac{\frac{1}{2}(2x-2) + 4}{x^2-2x-5} dx$$

$$= \frac{1}{2} \int \frac{2x-2}{x^2-2x-5} dx + 4 \int \frac{1}{x^2-2x-5} dx$$

Let  $I_1 = \int \frac{2x-2}{x^2-2x-5} dx$  and  $I_2 = \int \frac{1}{x^2-2x-5} dx$

$$\therefore \int \frac{x+3}{x^2-2x-5} dx = \frac{1}{2} I_1 + 4 I_2 \quad \dots(1)$$

Then,  $I_1 = \int \frac{2x-2}{x^2-2x-5} dx$

Let  $x^2-2x-5 = t$

$$\Rightarrow (2x-2) dx = dt$$

$$\Rightarrow I_1 = \int \frac{dt}{t} = \log|t| = \log|x^2-2x-5| \quad \dots(2)$$

$$I_2 = \int \frac{1}{x^2-2x-5} dx$$

$$= \int \frac{1}{(x^2-2x+1)-6} dx$$

$$= \int \frac{1}{(x-1)^2 - (\sqrt{6})^2} dx$$

$$= \frac{1}{2\sqrt{6}} \log \left( \frac{x-1-\sqrt{6}}{x-1+\sqrt{6}} \right) \quad \dots(3)$$

Substituting (2) and (3) in (1), we obtain



$$\int \frac{x+3}{x^2-2x-5} dx = \frac{1}{2} \log|x^2-2x-5| + \frac{4}{2\sqrt{6}} \log \left| \frac{x-1-\sqrt{6}}{x-1+\sqrt{6}} \right| + C$$

$$= \frac{1}{2} \log|x^2-2x-5| + \frac{2}{\sqrt{6}} \log \left| \frac{x-1-\sqrt{6}}{x-1+\sqrt{6}} \right| + C$$

Where C is an arbitrary constant.

### Question 23:

Integrate  $\frac{5x+3}{\sqrt{x^2+4x+10}}$

### Solution 23:

Let  $5x+3 = A \frac{d}{dx}(x^2+4x+10) + B$

$$\Rightarrow 5x+3 = A(2x+4) + B$$

Equating the coefficients of x and constant term, we obtain

$$2A = 5 \Rightarrow A = \frac{5}{2}$$

$$4A + B = 3 \Rightarrow B = -7$$

$$\therefore 5x+3 = \frac{5}{2}(2x+4) - 7$$

$$\Rightarrow \int \frac{5x+3}{\sqrt{x^2+4x+10}} dx = \int \frac{\frac{5}{2}(2x+4) - 7}{\sqrt{x^2+4x+10}} dx$$

$$= \frac{5}{2} \int \frac{2x+4}{\sqrt{x^2+4x+10}} dx - 7 \int \frac{1}{\sqrt{x^2+4x+10}} dx$$

Let  $I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}} dx$  and  $I_2 = \int \frac{1}{\sqrt{x^2+4x+10}} dx$

$$\therefore \int \frac{5x+3}{\sqrt{x^2+4x+10}} dx = \frac{5}{2} I_1 - 7 I_2 \quad \dots(1)$$

Then,  $I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}} dx$

Let  $x^2+4x+10 = t$

$$\therefore (2x+4) dx = dt$$

$$\Rightarrow I_1 = \int \frac{dt}{t} = 2\sqrt{t} = 2\sqrt{x^2+4x+10} \quad \dots(2)$$

$$I_2 = \int \frac{1}{\sqrt{x^2+4x+10}} dx$$

$$\begin{aligned}
 &= \int \frac{1}{\sqrt{(x^2 + 4x + 4) + 6}} dx \\
 &= \int \frac{1}{(x+2)^2 + (\sqrt{6})^2} dx \\
 &= \log |(x+2) + \sqrt{x^2 + 4x + 10}| \quad \dots (3)
 \end{aligned}$$

Using equations (2) and (3) in (1), we obtain

$$\begin{aligned}
 \int \frac{5x+3}{\sqrt{x^2+4x+10}} dx &= \frac{5}{2} \left[ 2\sqrt{x^2+4x+10} \right] - 7 \log |(x+2)\sqrt{x^2+4x+10}| + C \\
 &= 5\sqrt{x^2+4x+10} - 7 \log |(x+2)\sqrt{x^2+4x+10}| + C
 \end{aligned}$$

Where C is an arbitrary constant.

### Question 24:

$$\int \frac{dx}{x^2 + 2x + 2} \text{ equals}$$

- (A)  $x \tan^{-1}(x+1) + C$                       (B)  $\tan^{-1}(x+1) + C$   
 (C)  $(x+1) \tan^{-1} x + C$                       (D)  $\tan^{-1} x + C$

### Solution 24:

$$\begin{aligned}
 \int \frac{dx}{x^2 + 2x + 2} &= \int \frac{dx}{(x^2 + 2x + 1) + 1} \\
 &= \int \frac{1}{(x+1)^2 + (1)^2} dx \\
 &= [\tan^{-1}(x+1)] + C
 \end{aligned}$$

Hence, the correct Answer is B.

### Question 25:

$$\int \frac{dx}{\sqrt{9x-4x^2}} \text{ equals}$$

- (A)  $\frac{1}{9} \sin^{-1} \left( \frac{9x-8}{8} \right) + C$                       (B)  $\frac{1}{2} \sin^{-1} \left( \frac{8x-9}{9} \right) + C$   
 (C)  $\frac{1}{3} \sin^{-1} \left( \frac{9x-8}{8} \right) + C$                       (D)  $\frac{1}{2} \sin^{-1} \left( \frac{9x-8}{9} \right) + C$

### Solution 25:

$$\int \frac{dx}{\sqrt{9x-4x^2}}$$

$$\begin{aligned}
 &= \int \frac{1}{\sqrt{-4\left(x^2 - \frac{9}{4}x\right)}} dx \\
 &= \int \frac{1}{-4\left(x^2 - \frac{9}{4}x + \frac{81}{64} - \frac{81}{64}\right)} dx \\
 &= \int \frac{1}{\sqrt{-4\left[\left(x - \frac{9}{8}\right)^2 - \left(\frac{9}{8}\right)^2\right]}} dx \\
 &= \frac{1}{2} \left[ \sin^{-1} \left( \frac{x - \frac{9}{8}}{\frac{9}{8}} \right) \right] + C \qquad \left( \int \frac{dy}{\sqrt{a^2 - y^2}} = \sin^{-1} \frac{y}{a} + C \right) \\
 &= \frac{1}{2} \sin^{-1} \left( \frac{8x - 9}{9} \right) + C
 \end{aligned}$$

Hence, the correct Answer is B.

### Exercise 7.5

#### Question 1:

Integrate  $\frac{x}{(x+1)(x+2)}$

#### Solution 1:

$$\text{Let } \frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$\Rightarrow x = A(x+2) + B(x+1)$$

Equating the coefficients of x and constant term, we obtain

$$A + B = 1$$

$$2A + B = 0$$

On solving, we obtain

$$A = -1 \text{ and } B = 2$$

$$\therefore \frac{x}{(x+1)(x+2)} = \frac{-1}{x+1} + \frac{2}{x+2}$$

$$\Rightarrow \int \frac{x}{(x+1)(x+2)} dx = \int \frac{-1}{x+1} + \frac{2}{x+2} dx$$

$$\begin{aligned}
 &= -\log|x+1| + 2\log|x+2| + C \\
 &= \log(x+2)^2 - \log|x+1| + C \\
 &= \log \frac{(x+2)^2}{(x+1)} + C
 \end{aligned}$$

Where C is an arbitrary constant

### Question 2:

Integrate  $\frac{1}{x^2-9}$

### Solution 2:

Let  $\frac{1}{(x+3)(x-3)} = \frac{A}{(x+3)} + \frac{B}{(x-3)}$

$$1 = A(x-3) + B(x+3)$$

Equating the coefficients of x and constant term, we obtain

$$A + B = 0$$

$$-3A + 3B = 1$$

On solving, we obtain

$$A = -\frac{1}{6} \text{ and } B = \frac{1}{6}$$

$$\therefore \frac{1}{(x+3)(x-3)} = \frac{-1}{6(x+3)} + \frac{1}{6(x-3)}$$

$$\Rightarrow \int \frac{1}{(x^2-9)} dx = \int \left( \frac{-1}{6(x+3)} + \frac{1}{6(x-3)} \right) dx$$

$$= -\frac{1}{6} \log|x+3| + \frac{1}{6} \log|x-3| + C$$

$$= \frac{1}{6} \log \left| \frac{(x-3)}{(x+3)} \right| + C$$

Where C is an arbitrary constant

### Question 3:

Integrate  $\frac{3x-1}{(x-1)(x-2)(x-3)}$

### Solution 3:

Let  $\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) \quad \dots(1)$$

Equating the coefficients of  $x^2$ ,  $x$  and constant term, we obtain

$$A + B + C = 0$$

$$-5A - 4B - 3C = 3$$

$$6A + 3B + 2C = -1$$

Solving these equations, we obtain

$$A = 1, B = -5, \text{ and } C = 4$$

$$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)}$$

$$\Rightarrow \int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \int \left\{ \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)} \right\} dx$$

$$= \log|x-1| - 5\log|x-2| + 4\log|x-3| + C$$

Where  $C$  is an arbitrary constant.

#### Question 4:

Integrate  $\frac{x}{(x-1)(x-2)(x-3)}$

#### Solution 4:

$$\text{Let } \frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

$$x = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) \quad \dots(1)$$

Equating the coefficients of  $x^2$ ,  $x$  and constant term, we obtain

$$A + B + C = 0$$

$$-5A - 4B - 3C = 1$$

$$6A + 4B + 2C = 0$$

Solving these equations, we obtain

$$A = \frac{1}{2}, B = 2 \text{ and } C = \frac{3}{2}$$

$$\therefore \frac{x}{(x-1)(x-2)(x-3)} = \frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)}$$

$$\Rightarrow \int \frac{x}{(x-1)(x-2)(x-3)} dx = \int \left\{ \frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)} \right\} dx$$

$$= \frac{1}{2} \log|x-1| - 2\log|x-2| + \frac{3}{2} \log|x-3| + C$$

#### Question 5:

Integrate  $\frac{2x}{x^2 + 3x + 2}$

**Solution 5:**

$$\text{Let } \frac{2x}{x^2 + 3x + 2} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$$

$$2x = A(x+2) + B(x+1) \quad \dots(1)$$

Equating the coefficients of  $x^2$ ,  $x$  and constant term, we obtain

$$A + B = 2$$

$$2A + B = 0$$

Solving these equations, we obtain

$$A = -2 \text{ and } B = 4$$

$$\therefore \frac{2x}{(x+1)(x+2)} = \frac{-2}{(x+1)} + \frac{4}{(x+2)}$$

$$\Rightarrow \int \frac{2x}{(x+1)(x+2)} dx = \int \left\{ \frac{4}{(x+2)} - \frac{2}{(x+1)} \right\} dx$$

$$= 4 \log|x+2| - 2 \log|x+1| + C$$

Where  $C$  is an arbitrary constant.

**Question 6:**

$$\text{Integrate } \frac{1-x^2}{x(1-2x)}$$

**Solution 6:**

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing  $(1-x^2)$  by  $x(1-2x)$ , we obtain

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2} \left( \frac{2-x}{x(1-2x)} \right)$$

$$\text{Let } \frac{2-x}{x(1-2x)} = \frac{A}{x} + \frac{B}{(1-2x)}$$

$$\Rightarrow (2-x) = A(1-2x) + Bx \quad \dots(1)$$

Equating the coefficients of  $x^2$ ,  $x$  and constant term, we obtain

$$-2A + B = -1$$

$$\text{And } A = 2$$

Solving these equations, we obtain

$$A = 2 \text{ and } B = 3$$

$$\therefore \frac{2-x}{x(1-2x)} = \frac{2}{x} + \frac{3}{1-2x}$$

Substituting in equation (1), we obtain

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2} \left\{ \frac{2}{x} + \frac{3}{(1-2x)} \right\}$$

$$\int \frac{1-x^2}{x(1-2x)} dx = \int \left\{ \frac{1}{2} + \frac{1}{2} \left( \frac{2}{x} + \frac{3}{1-2x} \right) \right\} dx$$

$$= \frac{x}{2} + \log|x| + \frac{3}{2(-2)} \log|1-2x| + C$$

$$= \frac{x}{2} + \log|x| - \frac{3}{4} \log|1-2x| + C$$

Where C is an arbitrary constant.

### Question 7:

Integrate  $\frac{x}{(x^2+1)(x-1)}$

### Solution 7:

Let  $\frac{x}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1}$

$$x = (Ax+B)(x-1) + C(x^2+1)$$

$$x = Ax^2 - Ax + Bx - B + Cx^2 + C$$

Equating the coefficients of  $x^2$ ,  $x$ , and constant term, we obtain

$$A + C = 0$$

$$-A + B = 1$$

$$-B + C = 0$$

On solving these equations, we obtain

$$A = -\frac{1}{2}, B = \frac{1}{2}, \text{ and } C = \frac{1}{2}$$

From equation (1), we obtain

$$\therefore \frac{x}{(x^2+1)(x-1)} = \frac{\left(-\frac{1}{2}x + \frac{1}{2}\right)}{x^2+1} + \frac{\frac{1}{2}}{x-1}$$

$$\Rightarrow \int \frac{x}{(x^2+1)(x-1)} = -\frac{1}{2} \int \frac{x}{x^2+1} dx + \frac{1}{2} \int \frac{1}{x^2+1} dx + \frac{1}{2} \int \frac{1}{x-1} dx$$

$$= -\frac{1}{4} \int \frac{2x}{x^2+1} dx + \frac{1}{2} \tan^{-1} x + \frac{1}{2} \log|x-1| + C$$

Consider  $\int \frac{2x}{x^2+1} dx$ , let  $(x^2+1) = t \Rightarrow 2x dx = dt$

$$\Rightarrow \int \frac{2x}{x^2+1} dx = \int \frac{dt}{t} = \log|t| = \log|x^2+1|$$

$$\therefore \int \frac{x}{(x^2+1)(x-1)} = -\frac{1}{4} \log|x^2+1| + \frac{1}{2} \tan^{-1} x + \frac{1}{2} \log|x-1| + C$$

$$= \frac{1}{2} \log|x-1| - \frac{1}{4} \log|x^2+1| + \frac{1}{2} \tan^{-1} x + C$$

Where C is an arbitrary constant.

### Question 8:

Integrate  $\frac{x}{(x-1)^2(x+2)}$

### Solution 8:

$$\text{Let } \frac{x}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$$

$$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

Equating the coefficients of  $x^2$ ,  $x$  and constant term, we obtain

$$A + C = 0$$

$$A + B - 2C = 1$$

$$-2A + 2B + C = 0$$

On solving, we obtain

$$A = \frac{2}{9} \text{ and } C = \frac{-2}{9}$$

$$B = \frac{1}{3}$$

$$\therefore \frac{x}{(x-1)^2(x+2)} = \frac{2}{9(x-1)} + \frac{1}{3(x-1)^2} - \frac{2}{9(x+2)}$$

$$\Rightarrow \int \frac{x}{(x-1)^2(x+2)} dx = \frac{2}{9} \int \frac{1}{x-1} dx + \frac{1}{3} \int \frac{1}{(x-1)^2} dx - \frac{2}{9} \int \frac{1}{x+2} dx$$

$$= \frac{2}{9} \log|x-1| + \frac{1}{3} \left( \frac{-1}{x-1} \right) - \frac{2}{9} \log|x+2| + C$$

$$= \frac{2}{9} \log \left| \frac{x-1}{x+2} \right| - \frac{1}{3(x-1)} + C$$

Where C is an arbitrary constant.

### Question 9:

Integrate  $\frac{3x+5}{x^3-x^2-x+1}$

### Solution 9:

$$\frac{3x+5}{x^3-x^2-x+1} = \frac{3x+5}{(x-1)^2(x+1)}$$



$$\text{Let } \frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$$

$$3x+5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

$$3x+5 = A(x^2-1) + B(x+1) + C(x^2+1-2x) \quad \dots(1)$$

Equating the coefficients of  $x^2$ ,  $x$  and constant term, we obtain

$$A + C = 0$$

$$B - 2C = 3$$

$$-A + B + C = 5$$

On solving, we obtain

$$B = 4$$

$$A = -\frac{1}{2} \text{ and } C = \frac{1}{2}$$

$$\therefore \frac{3x+5}{(x-1)^2(x+1)} = \frac{-1}{2(x-1)} + \frac{4}{(x-1)^2} + \frac{1}{2(x+1)}$$

$$\Rightarrow \int \frac{3x+5}{(x-1)^2(x+1)} dx = -\frac{1}{2} \int \frac{1}{x-1} dx + 4 \int \frac{1}{(x-1)^2} dx + \frac{1}{2} \int \frac{1}{(x+1)} dx$$

$$= -\frac{1}{2} \log|x-1| + 4 \left( \frac{-1}{x-1} \right) + \frac{1}{2} \log|x+1| + C$$

$$= \frac{1}{2} \log \left| \frac{x+1}{x-1} \right| - \frac{4}{(x-1)} + C$$

Where C is an arbitrary constant.

### Question 10:

$$\text{Integrate } \frac{2x-3}{(x^2-1)(2x+3)}$$

### Solution 10:

$$\frac{2x-3}{(x^2-1)(2x+3)} = \frac{2x-3}{(x+1)(x-1)(2x+3)}$$

$$\text{Let } \frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(2x+3)}$$

$$\Rightarrow (2x-3) = A(x-1)(2x+3) + B(x+1)(2x+3) + C(x+1)(x-1)$$

$$\Rightarrow (2x-3) = A(2x^2+x-3) + B(2x^2+5x+3) + C(x^2-1)$$

$$\Rightarrow (2x-3) = (2A+2B+C)x^2 + (A+5B)x + (-3A+3B-C)$$

Equating the coefficients of  $x^2$ ,  $x$  and constant, we obtain

$$2A + 2B + C = 0$$

$$A + 5B = 2$$

$$-3A + 3B - C = -3$$

On solving, we obtain

$$B = -\frac{1}{10}, A = \frac{5}{2}, \text{ and } C = -\frac{24}{5}$$

$$\begin{aligned} \therefore \frac{2x-3}{(x+1)(x-1)(2x+3)} &= \frac{5}{2(x+1)} - \frac{1}{10(x-1)} - \frac{24}{5(2x+3)} \\ \Rightarrow \int \frac{2x-3}{(x^2-1)(2x+3)} dx &= \frac{5}{2} \int \frac{1}{(x+1)} dx - \frac{1}{10} \int \frac{1}{x-1} dx - \frac{24}{5} \int \frac{1}{(2x+3)} dx \\ &= \frac{5}{2} \log|x+1| - \frac{1}{10} \log|x-1| - \frac{24}{5 \times 2} \log|2x+3| \\ &= \frac{5}{2} \log|x+1| - \frac{1}{10} \log|x-1| - \frac{12}{5} \log|2x+3| + C \end{aligned}$$

Where C is an arbitrary constant.

### Question 11:

Integrate  $\frac{5x}{(x+1)(x^2-4)}$

### Solution 11:

$$\frac{5x}{(x+1)(x^2-4)} = \frac{5x}{(x+1)(x+2)(x-2)}$$

Let  $\frac{5x}{(x+1)(x+2)(x-2)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-2}$

$$5x = A(x+2)(x-2) + B(x+1)(x-2) + C(x+1)(x+2) \quad \dots(1)$$

Equating the coefficients of  $x^2$ ,  $x$  and constant, we obtain

$$A + B + C = 0$$

$$-B + 3C = 5 \quad \text{and}$$

$$-4A - 2B + 2C = 0$$

On solving, we obtain

$$A = \frac{5}{3}, B = -\frac{5}{2}, \text{ and } C = \frac{5}{6}$$

$$\therefore \frac{5x}{(x+1)(x+2)(x-2)} = \frac{5}{3(x+1)} + \frac{5}{2(x+2)} + \frac{5}{6(x-2)}$$

$$\Rightarrow \int \frac{5x}{(x+1)(x^2-4)} dx = \frac{5}{3} \int \frac{1}{(x+1)} dx - \frac{5}{2} \int \frac{1}{(x+2)} dx + \frac{5}{6} \int \frac{1}{(x-2)} dx$$

$$= \frac{5}{3} \log|x+1| - \frac{5}{2} \log|x+2| + \frac{5}{6} \log|x-2| + C$$

Where C is an arbitrary constant.

### Question 12:

Integrate  $\frac{x^3+x+1}{x^2-1}$

### Solution 12:

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing  $(x^3 + x + 1)$  by  $x^2 - 1$ , we obtain

$$\frac{x^3 + x + 1}{x^2 - 1} = x + \frac{2x + 1}{x^2 - 1}$$

$$\text{Let } \frac{2x + 1}{x^2 - 1} = \frac{A}{(x + 1)} + \frac{B}{(x - 1)}$$

$$2x + 1 = A(x - 1) + B(x + 1) \quad \dots(1)$$

Equating the coefficients of  $x$  and constant, we obtain

$$A + B = 2$$

$$-A + B = 1$$

On solving, we obtain

$$A = \frac{1}{2} \text{ and } B = \frac{3}{2}$$

$$\therefore \frac{x^3 + x + 1}{x^2 - 1} = x + \frac{1}{2(x + 1)} + \frac{3}{2(x - 1)}$$

$$\Rightarrow \int \frac{x^3 + x + 1}{x^2 - 1} dx = \int x dx + \frac{1}{2} \int \frac{1}{(x + 1)} dx + \frac{3}{2} \int \frac{1}{(x - 1)} dx$$

$$= \frac{x^2}{2} + \log|x + 1| + \frac{3}{2} \log|x - 1| + C$$

Where  $C$  is an arbitrary constant.

### Question 13:

$$\text{Integrate } \frac{2}{(1 - x)(1 + x^2)}$$

### Solution 13:

$$\text{Let } \frac{2}{(1 - x)(1 + x^2)} = \frac{A}{(1 - x)} + \frac{Bx + C}{(1 + x^2)}$$

$$2 = A(1 + x^2) + (Bx + C)(1 - x)$$

$$2 = A + Ax^2 + Bx - Bx^2 + C - Cx$$

Equating the coefficient of  $x^2$ ,  $x$ , and constant term, we obtain

$$A - B = 0$$

$$B - C = 0$$

$$A + C = 2$$

On solving these equations, we obtain

$$A = 1, B = 1, \text{ and } C = 1$$

$$\therefore \frac{2}{(1 - x)(1 + x^2)} = \frac{1}{1 - x} + \frac{x + 1}{1 + x^2}$$

$$\begin{aligned} \Rightarrow \int \frac{2}{(1-x)(1+x^2)} dx &= \int \frac{1}{1-x} dx + \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx \\ &= -\int \frac{1}{1-x} dx + \frac{1}{2} \int \frac{2x}{1+x^2} dx + \int \frac{1}{1+x^2} dx \\ &= -\log|x-1| + \frac{1}{2} \log|1+x^2| + \tan^{-1} x + C \end{aligned}$$

Where C is an arbitrary constant.

#### Question 14:

Integrate  $\frac{3x-1}{(x+2)^2}$

#### Solution 14:

Let  $\frac{3x-1}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}$

$$\Rightarrow 3x-1 = A(x+2) + B$$

Equating the coefficient of x and constant term, we obtain

$$A = 3$$

$$2A + B = -1 \Rightarrow B = -7$$

$$\therefore \frac{3x-1}{(x+2)^2} = \frac{3}{x+2} - \frac{7}{(x+2)^2}$$

$$\Rightarrow \int \frac{3x-1}{(x+2)^2} dx = 3 \int \frac{1}{x+2} dx - 7 \int \frac{1}{(x+2)^2} dx$$

$$= 3 \log|x+2| - 7 \left( \frac{-1}{x+2} \right) + C$$

$$= 3 \log|x+2| + \frac{7}{x+2} + C$$

Where C is an arbitrary constant.

#### Question 15:

Integrate  $\frac{1}{x^4-1}$

#### Solution 15:

$$\frac{1}{(x^4-1)} = \frac{1}{(x^2-1)(x^2+1)} = \frac{1}{(x+1)(x-1)(1+x^2)}$$

Let  $\frac{1}{(x+1)(x-1)(1+x^2)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{Cx+D}{x^2+1}$

$$1 = A(x-1)(1+x^2) + B(x+1)(1+x^2) + (Cx+D)(x^2-1)$$

$$1 = A(x^3 + x - x^2 - 1) + B(x^3 + x + x^2 + 1) + Cx^3 + Dx^2 - Cx - D$$

$$1 = (A+B+C)x^3 + (-A+B+D)x^2 + (A+B-C)x + (-A+B-D)$$

Equating the coefficient of  $x^3$ ,  $x^2$ ,  $x$ , and constant term, we obtain

$$A+B+C=0$$

$$-A+B+D=0$$

$$A+B-C=0$$

$$-A+B-D=1$$

$$A = -\frac{1}{4}, B = \frac{1}{4}, C = 0, \text{ and } D = -\frac{1}{2}$$

$$\therefore \frac{1}{x^4-1} = \frac{-1}{4(x+1)} + \frac{1}{4(x-1)} + \frac{1}{2(x^2+1)}$$

$$\Rightarrow \int \frac{1}{x^4-1} dx = -\frac{1}{4} \log|x-1| + \frac{1}{4} \log|x+1| - \frac{1}{2} \tan^{-1} x + C$$

$$= \frac{1}{4} \log \left| \frac{x-1}{x+1} \right| - \frac{1}{2} \tan^{-1} x + C$$

Where C is an arbitrary constant.

### Question 16:

Integrate  $\frac{1}{x(x^n+1)}$

[Hint: multiply numerator and denominator by  $x^{n-1}$  and put  $x^n = t$ ]

### Solution 16:

$$\frac{1}{x(x^n+1)}$$

Multiplying numerator and denominator by  $x^{n-1}$ , we obtain

$$\frac{1}{x(x^n+1)} = \frac{x^{n-1}}{x^{n-1}x(x^n+1)} = \frac{x^{n-1}}{x^n(x^n+1)}$$

$$\text{Let } x^n = t \Rightarrow n x^{n-1} dx = dt$$

$$\therefore \int \frac{1}{x(x^n+1)} dx = \int \frac{x^{n-1}}{x^n(x^n+1)} dx = \frac{1}{n} \int \frac{1}{t(t+1)} dt$$

$$\text{Let } \frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{(t+1)}$$

$$1 = A(1+t) + Bt \quad \dots(1)$$

Equating the coefficients of t and constant, we obtain

$$A = 1 \text{ and } B = -1$$

$$\begin{aligned} \therefore \frac{1}{t(t+1)} &= \frac{1}{t} - \frac{1}{(1+t)} \\ \Rightarrow \int \frac{1}{x(x^n+1)} dx &= \frac{1}{n} \int \left\{ \frac{1}{t} - \frac{1}{(1+t)} \right\} dx \\ &= \frac{1}{n} [\log|t| - \log|t+1|] + C \\ &= \frac{1}{n} [\log|x^n| - \log|x^n+1|] + C \\ &= \frac{1}{n} \log \left| \frac{x^n}{x^n+1} \right| + C \end{aligned}$$

Where C is an arbitrary constant.

### Question 17:

Integrate  $\frac{\cos x}{(1-\sin x)(2-\sin x)}$

[Hint: Put  $\sin x = t$ ]

### Solution 17:

$$\frac{\cos x}{(1-\sin x)(2-\sin x)}$$

Let  $\sin x = t \Rightarrow \cos x dx = dt$

$$\therefore \int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx = \int \frac{dt}{(1-t)(2-t)}$$

$$\text{Let } \frac{1}{(1-t)(2-t)} = \frac{A}{(1-t)} + \frac{B}{(2-t)}$$

$$1 = A(2-t) + B(1-t) \quad \dots(1)$$

Equating the coefficients of t and constant, we obtain

$$-A - B = 0 \quad \text{and}$$

$$2A + B = 1$$

On solving, we obtain

$$A = 1 \quad \text{and} \quad B = -1$$

$$\begin{aligned} \therefore \frac{1}{(1-t)(2-t)} &= \frac{1}{(1-t)} - \frac{1}{(2-t)} \\ \Rightarrow \int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx &= \int \left\{ \frac{1}{1-t} - \frac{1}{(2-t)} \right\} dt \\ &= -\log|1-t| + \log|2-t| + C \\ &= \log \left| \frac{2-t}{1-t} \right| + C \end{aligned}$$

$$= \log \left| \frac{2 - \sin x}{1 - \sin x} \right| + C$$

Where C is an arbitrary constant.

### Question 18:

Integrate  $\frac{(x^2 + 1)(x^2 + 2)}{(x^2 + 3)(x^2 + 4)}$

### Solution 18:

$$\frac{(x^2 + 1)(x^2 + 2)}{(x^2 + 3)(x^2 + 4)} = 1 - \frac{4x^2 + 10}{(x^2 + 3)(x^2 + 4)}$$

$$\text{Let } \frac{(4x^2 + 10)}{(x^2 + 3)(x^2 + 4)} = \frac{Ax + B}{x^2 + 3} + \frac{Cx + D}{x^2 + 4}$$

$$4x^2 + 10 = (Ax + B)(x^2 + 4) + (Cx + D)(x^2 + 3)$$

$$4x^2 + 10 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + 3Cx + Dx^2 + 3D$$

$$4x^2 + 10 = (A + C)x^3 + (B + D)x^2 + (4A + 3C)x + (4B + 3D)$$

Equating the coefficients of  $x^3$ ,  $x^2$ ,  $x$  and constant term, we obtain

$$A + C = 0$$

$$B + D = 4$$

$$4A + 3C = 0$$

$$4B + 3D = 10$$

On solving these equations, we obtain

$$A = 0, B = -2, C = 0, \text{ and } D = 6$$

$$\therefore \frac{(4x^2 + 10)}{(x^2 + 3)(x^2 + 4)} = \frac{-2}{x^2 + 3} + \frac{6}{x^2 + 4}$$

$$\frac{(x^2 + 1)(x^2 + 2)}{(x^2 + 3)(x^2 + 4)} = 1 - \left( \frac{-2}{x^2 + 3} + \frac{6}{x^2 + 4} \right)$$

$$\Rightarrow \int \frac{(x^2 + 1)(x^2 + 2)}{(x^2 + 3)(x^2 + 4)} dx = \int \left\{ 1 + \frac{2}{x^2 + 3} - \frac{6}{x^2 + 4} \right\} dx$$

$$= \int \left\{ 1 + \frac{2}{x^2 + (\sqrt{3})^2} - \frac{6}{x^2 + 2^2} \right\}$$

$$= x + 2 \left( \frac{1}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} \right) - 6 \left( \frac{1}{2} \tan^{-1} \frac{x}{2} \right) + C$$

$$= x + \frac{2}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} - 3 \tan^{-1} \frac{x}{2} + C$$

**Question 19:**

Integrate  $\frac{2x}{(x^2+1)(x^2+3)}$

**Solution 19:**

$$\frac{2x}{(x^2+1)(x^2+3)}$$

Let  $x^2 = t \Rightarrow 2x dx = dt$

$$\therefore \int \frac{2x}{(x^2+1)(x^2+3)} dx = \int \frac{dt}{(t+1)(t+3)} \quad \dots(1)$$

$$\text{Let } \frac{1}{(t+1)(t+3)} = \frac{A}{(t+1)} + \frac{B}{(t+3)}$$

$$1 = A(t+3) + B(t+1) \quad \dots(2)$$

Equating the coefficients of  $t$  and constant, we obtain

$$A + B = 0 \text{ and } 3A + B = 1$$

On solving, we obtain

$$A = \frac{1}{2} \text{ and } B = -\frac{1}{2}$$

$$\therefore \frac{1}{(t+1)(t+3)} = \frac{1}{2(t+1)} - \frac{1}{2(t+3)}$$

$$\Rightarrow \int \frac{2x}{(x^2+1)(x^2+3)} dx = \int \left\{ \frac{1}{2(t+1)} - \frac{1}{2(t+3)} \right\} dt$$

$$= \frac{1}{2} \log|(t+1)| - \frac{1}{2} \log|t+3| + C$$

$$= \frac{1}{2} \log \left| \frac{t+1}{t+3} \right| + C$$

$$= \frac{1}{2} \log \left| \frac{x^2+1}{x^2+3} \right| + C$$

Where  $C$  is an arbitrary constant.

**Question 20:**

Integrate  $\frac{1}{x(x^4-1)}$

**Solution 20:**

$$\frac{1}{x(x^4-1)}$$

Multiplying numerator and denominator by  $x^3$ , we obtain



$$\frac{1}{x(x^4-1)} = \frac{x^3}{x^4(x^4-1)}$$

$$\therefore \int \frac{1}{x(x^4-1)} dx = \int \frac{x^3}{x^4(x^4-1)} dx$$

$$\text{Let } x^4 = t \Rightarrow 4x^3 dx = dt$$

$$\therefore \int \frac{1}{x(x^4-1)} dx = \frac{1}{4} \int \frac{dt}{t(t-1)}$$

$$\text{Let } \frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{(t-1)}$$

$$1 = A(t-1) + Bt \quad \dots(1)$$

Equating the coefficients of t and constant, we obtain

$$A + B = 0 \text{ and } -A = 1$$

$$A = -1 \text{ and } B = 1$$

$$\Rightarrow \frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\Rightarrow \int \frac{1}{x(x^4-1)} dx = \frac{1}{4} \int \left\{ \frac{-1}{t} + \frac{1}{t-1} \right\} dt$$

$$= \frac{1}{4} [-\log|t| + \log|t-1|] + C$$

$$= \frac{1}{4} \log \left| \frac{t-1}{t} \right| + C$$

$$= \frac{1}{4} \log \left| \frac{x^4-1}{x^4} \right| + C$$

Where C is an arbitrary constant.

### Question 21:

$$\text{Integrate } \frac{1}{(e^x-1)}$$

[Hint: Put  $e^x = t$ ]

### Solution 21:

$$\text{Let } e^x = t \Rightarrow e^x dx = dt$$

$$\Rightarrow \int \frac{1}{(e^x-1)} dx = \int \frac{1}{t-1} \times \frac{dt}{t} = \int \frac{1}{t(t-1)} dt$$

$$\text{Let } \frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$$

$$1 = A(t-1) + Bt \quad \dots(1)$$

Equating the coefficients of t and constant, we obtain

$$A + B = 0 \text{ and } -A = 1$$

$$A = -1 \text{ and } B = 1$$

$$\therefore \frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\Rightarrow \int \frac{1}{t(t-1)} dt = \log \left| \frac{t-1}{t} \right| + C$$

$$= \log \left| \frac{e^x - 1}{e^x} \right| + C$$

Where C is an arbitrary constant.

### Question 22:

$$\int \frac{x dx}{(x-1)(x-2)} \text{ equals}$$

A.  $\log \left| \frac{(x-1)^2}{x-2} \right| + C$

B.  $\log \left| \frac{(x-2)^2}{x-1} \right| + C$

C.  $\log \left| \left( \frac{x-1}{x-2} \right)^2 \right| + C$

D.  $\log |(x-1)(x-2)| + C$

### Solution 22:

$$\text{Let } \frac{x}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

$$x = A(x-2) + B(x-1) \quad \dots(1)$$

Equating the coefficients of x and constant, we obtain

$$A + B = 1 \text{ and } -2A - B = 0$$

$$A = -1 \text{ and } B = 2$$

$$\therefore \frac{x}{(x-1)(x-2)} = -\frac{1}{x-1} + \frac{2}{x-2}$$

$$\Rightarrow \int \frac{x}{(x-1)(x-2)} dx = \int \left\{ \frac{-1}{x-1} + \frac{2}{x-2} \right\} dx$$

$$= -\log|x-1| + 2\log|x-2| + C$$

$$= \log \left| \frac{(x-2)^2}{x-1} \right| + C$$

Hence, the correct Answer is B.

**Question 23:**

$\int \frac{dx}{x(x^2+1)}$  equals

- A.  $\log|x| - \frac{1}{2}\log(x^2+1) + C$   
 B.  $\log|x| + \frac{1}{2}\log(x^2+1) + C$   
 C.  $-\log|x| + \frac{1}{2}\log(x^2+1) + C$   
 D.  $\frac{1}{2}\log|x| + \log(x^2+1) + C$

**Solution 23:**

$$\text{Let } \frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$1 = A(x^2+1) + (Bx+C)x$$

Equating the coefficients of  $x^2$ ,  $x$ , and constant term, we obtain

$$A + B = 0$$

$$C = 0$$

$$A = 1$$

On solving these equations, we obtain

$$A = 1, B = -1, \text{ and } C = 0$$

$$\therefore \frac{1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{x^2+1}$$

$$\Rightarrow \int \frac{1}{x(x^2+1)} dx = \int \left\{ \frac{1}{x} - \frac{x}{x^2+1} \right\} dx$$

$$= \log|x| - \frac{1}{2}\log|x^2+1| + C$$

Hence, the correct Answer is A.

**Alter:**

$$\Rightarrow \int \frac{1}{x(x^2+1)} dx = \int \left\{ \frac{x}{x^2(x^2+1)} \right\} dx$$

Let  $x^2 = t$ , therefore,  $2x dx = dt$

$$\therefore \int \frac{x}{x^2(x^2+1)} dx = \frac{1}{2} \int \frac{dt}{t(t+1)} = \frac{1}{2} \int \frac{(t+1)-t}{t(t+1)} dt = \frac{1}{2} \int \frac{1}{t} - \frac{1}{t+1} dt$$

$$= \frac{1}{2} [\log t - \log(t+1)] + C$$

$$= \log|x| - \frac{1}{2}\log|x^2+1| + C$$

## Exercise 7.6

**Question 1:**

Integrate  $x \sin x$

**Solution 1:**

$$\text{Let } I = \int x \sin x dx$$

Taking  $x$  as first function and  $\sin x$  as second function and integrating by parts, we obtain,

$$\begin{aligned} I &= x \int \sin x dx - \int \left\{ \left( \frac{d}{dx} x \right) \int \sin x dx \right\} dx \\ &= x(-\cos x) - \int 1.(-\cos x) dx \\ &= -x \cos x + \sin x + C \end{aligned}$$

Where  $C$  is an arbitrary constant.

**Question 2:**

Integrate  $x \sin 3x$

**Solution 2:**

$$\text{Let } I = \int x \sin 3x dx$$

Taking  $x$  as first function and  $\sin 3x$  as second function and integrating by parts, we obtain

$$\begin{aligned} I &= x \int \sin 3x dx - \int \left\{ \left( \frac{d}{dx} x \right) \int \sin 3x dx \right\} \\ &= x \left( \frac{-\cos 3x}{3} \right) - \int 1. \left( \frac{-\cos 3x}{3} \right) dx \\ &= \frac{-x \cos 3x}{3} + \frac{1}{3} \int \cos 3x dx \\ &= \frac{-x \cos 3x}{3} + \frac{1}{9} \sin 3x + C \end{aligned}$$

Where  $C$  is an arbitrary constant.

**Question 3:**

Integrate  $x^2 e^x$

**Solution 3:**

$$\text{Let } I = \int x^2 e^x dx$$

Taking  $x^2$  as first function and  $e^x$  as second function and integrating by parts, we obtain

$$I = x^2 \int e^x dx - \int \left\{ \left( \frac{d}{dx} x^2 \right) \int e^x dx \right\} dx$$

$$= x^2 e^x - \int 2x e^x dx$$

$$= x^2 e^x - 2 \int x \cdot e^x dx$$

Again integrating by parts, we obtain

$$= x^2 e^x - 2 \left[ x \cdot \int e^x dx - \int \left\{ \left( \frac{d}{dx} x \right) \int e^x dx \right\} dx \right]$$

$$= x^2 e^x - 2 \left[ x e^x - \int e^x dx \right]$$

$$= x^2 e^x - 2 \left[ x e^x - e^x \right]$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

$$= e^x (x^2 - 2x + 2) + C$$

Where C is an arbitrary constant.

#### Question 4:

Integrate  $x \log x$

#### Solution 4:

Let  $I = \int x \log x dx$

Taking  $\log x$  as first function and  $x$  as second function and integrating by parts, we obtain

$$I = \log x \int x dx - \int \left\{ \left( \frac{d}{dx} \log x \right) \int x dx \right\} dx$$

$$= \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \log x}{2} - \int \frac{x}{2} dx$$

$$= \frac{x^2 \log x}{2} - \frac{x^2}{4} + C$$

Where C is an arbitrary constant.

#### Question 5:

Integrate  $x \log 2x$

#### Solution 5:

Let  $I = \int x \log 2x dx$

Taking  $\log 2x$  as first function and  $x$  as second function and integrating by parts, we obtain

$$I = \log 2x \int x dx - \int \left\{ \left( \frac{d}{dx} \log 2x \right) \int x dx \right\} dx$$

$$= \log 2x \cdot \frac{x^2}{2} - \int \frac{2}{2x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \log 2x}{2} - \int \frac{x}{2} dx$$

$$= \frac{x^2 \log 2x}{2} - \frac{x^2}{4} + C$$

Where C is an arbitrary constant.

### Question 6:

Integrate  $x^2 \log x$

### Solution 6:

Let  $I = \int x^2 \log x dx$

Taking  $\log x$  as first function and  $x^2$  as second function and integrating by parts, we obtain

$$I = \log x \int x^2 dx - \int \left\{ \left( \frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$$

$$= \log x \cdot \left( \frac{x^3}{3} \right) - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \frac{x^3 \log x}{3} - \int \frac{x^2}{3} dx$$

$$= \frac{x^3 \log x}{3} - \frac{x^3}{9} + C$$

Where C is an arbitrary constant.

### Question 7:

Integrate  $x \sin^{-1} x$

### Solution 7:

Let  $I = \int x \sin^{-1} x dx$

Taking  $\sin^{-1} x$  as first function and  $x$  as second function and integrating by parts, we obtain

$$I = \sin^{-1} x \int x dx - \int \left\{ \left( \frac{d}{dx} \sin^{-1} x \right) \int x dx \right\} dx$$

$$= \sin^{-1} x \left( \frac{x^2}{2} \right) - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \right\} dx$$

$$\begin{aligned}
&= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}} \right\} dx \\
&= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \int \sqrt{1-x^2} dx - \int \frac{1}{\sqrt{1-x^2}} dx \right\} \\
&= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x - \sin^{-1} x \right\} + C \\
&= \frac{x^2 \sin^{-1} x}{2} + \frac{x}{4} \sqrt{1-x^2} + \frac{1}{4} \sin^{-1} x - \frac{1}{2} \sin^{-1} x + C \\
&= \frac{1}{4} (2x^2 - 1) \sin^{-1} x + \frac{x}{4} \sqrt{1-x^2} + C
\end{aligned}$$

Where C is an arbitrary constant.

### Question 8:

Integrate  $x \tan^{-1} x$

### Solution 8:

Let  $I = \int x \tan^{-1} x dx$

Taking  $\tan^{-1} x$  as first function and  $x$  as second function and integrating by parts, we obtain

$$\begin{aligned}
I &= \tan^{-1} x \int x dx - \int \left\{ \left( \frac{d}{dx} \tan^{-1} x \right) \int x dx \right\} dx \\
&= \tan^{-1} x \left( \frac{x^2}{2} \right) - \int \frac{1}{1+x^2} \cdot \frac{x^2}{2} dx \\
&= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \frac{x^2}{1+x^2} dx \\
&= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \left( \frac{x^2+1}{1+x^2} - \frac{1}{1+x^2} \right) dx \\
&= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \left( 1 - \frac{1}{1+x^2} \right) dx \\
&= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} (x - \tan^{-1} x) + C \\
&= \frac{x^2}{2} \tan^{-1} x - \frac{x}{2} + \frac{1}{2} \tan^{-1} x + C
\end{aligned}$$

Where C is an arbitrary constant.

### Question 9:

Integrate  $x \cos^{-1} x$

### Solution 9:

$$\text{Let } I = \int x \cos^{-1} x dx$$

Taking  $\cos^{-1} x$  as first function and  $x$  as second function and integrating by parts, we obtain

$$\begin{aligned} I &= \cos^{-1} x \int x dx - \int \left\{ \left( \frac{d}{dx} \cos^{-1} x \right) \int x dx \right\} dx \\ &= \cos^{-1} x \frac{x^2}{2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx \\ &= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2} \int \frac{1-x^2-1}{\sqrt{1-x^2}} dx \\ &= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2} \int \left\{ \sqrt{1-x^2} + \left( \frac{-1}{\sqrt{1-x^2}} \right) \right\} dx \\ &= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2} \int \sqrt{1-x^2} dx - \frac{1}{2} \int \left( \frac{-1}{\sqrt{1-x^2}} \right) dx \\ I &= \frac{x \cos^{-1} x}{2} - \frac{1}{2} \left( \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x \right) - \frac{1}{2} \cos^{-1} x \end{aligned}$$

Where  $C$  is an arbitrary constant.

### Question 10:

Integrate  $(\sin^{-1} x)^2$

### Solution 10:

$$\text{Let } I = \int (\sin^{-1} x)^2 \cdot 1 dx$$

Taking  $(\sin^{-1} x)^2$  as first function and 1 as second function and integrating by parts, we obtain

$$\begin{aligned} I &= \int (\sin^{-1} x) \cdot \int 1 dx - \int \left\{ \frac{d}{dx} (\sin^{-1} x)^2 \cdot \int 1 dx \right\} dx \\ &= (\sin^{-1} x)^2 \cdot x - \int \frac{2 \sin^{-1} x}{\sqrt{1-x^2}} \cdot x dx \\ &= x (\sin^{-1} x)^2 + \int \sin^{-1} x \cdot \left( \frac{-2x}{\sqrt{1-x^2}} \right) dx \\ &= x (\sin^{-1} x)^2 + \left[ \sin^{-1} x \int \frac{-2x}{\sqrt{1-x^2}} dx - \int \left\{ \left( \frac{d}{dx} \sin^{-1} x \right) \int \frac{-2x}{\sqrt{1-x^2}} dx \right\} dx \right] \\ &= x (\sin^{-1} x)^2 + \left[ \sin^{-1} x \cdot 2\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2} dx \right] \\ &= x (\sin^{-1} x)^2 + 2\sqrt{1-x^2} \sin^{-1} x - \int 2 dx \\ &= x (\sin^{-1} x)^2 + 2\sqrt{1-x^2} \sin^{-1} x - 2x + C \end{aligned}$$



Where C is an arbitrary constant.

**Question 11:**

Integrate  $\frac{x \cos^{-1} x}{\sqrt{1-x^2}}$

**Solution 11:**

$$\text{Let } I = \int \frac{x \cos^{-1} x}{\sqrt{1-x^2}} dx$$

$$I = \frac{-1}{2} \int \frac{-2x}{\sqrt{1-x^2}} \cdot \cos^{-1} x dx$$

Taking  $\cos^{-1} x$  as first function and  $\left(\frac{-2x}{\sqrt{1-x^2}}\right)$  as second function and integrating by parts, we obtain

$$I = \frac{-1}{2} \left[ \cos^{-1} x \int \frac{-2x}{\sqrt{1-x^2}} dx - \int \left\{ \left( \frac{d}{dx} \cos^{-1} x \right) \int \frac{-2x}{\sqrt{1-x^2}} dx \right\} dx \right]$$

$$= \frac{-1}{2} \left[ \cos^{-1} x \cdot 2\sqrt{1-x^2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2} dx \right]$$

$$= \frac{-1}{2} \left[ 2\sqrt{1-x^2} \cos^{-1} x + \int 2 dx \right]$$

$$= \frac{-1}{2} \left[ 2\sqrt{1-x^2} \cos^{-1} x + 2x \right] + C$$

$$= - \left[ \sqrt{1-x^2} \cos^{-1} x + x \right] + C$$

Where C is an arbitrary constant.

**Question 12:**

Integrate  $x \sec^2 x$

**Solution 12:**

$$\text{Let } I = \int x \sec^2 x dx$$

Taking x as first function and  $\sec^2 x$  as second function and integrating by parts, we obtain

$$I = x \int \sec^2 x dx - \int \left\{ \left( \frac{d}{dx} x \right) \int \sec^2 x dx \right\} dx$$

$$= x \tan x - \int 1 \cdot \tan x dx$$

$$= x \tan x + \log |\cos x| + C$$

Where C is an arbitrary constant.

**Question 13:**Integrate  $\tan^{-1} x$ **Solution 13:**

$$\text{Let } I = \int 1 \cdot \tan^{-1} x dx$$

Taking  $\tan^{-1} x$  as first function and 1 as second function and integrating by parts, we obtain

$$I = \tan^{-1} x \int 1 dx - \int \left\{ \left( \frac{d}{dx} \tan^{-1} x \right) \int 1 dx \right\} dx$$

$$= \tan^{-1} x \cdot x - \int \frac{1}{1+x^2} \cdot x dx$$

$$= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx$$

$$= x \tan^{-1} x - \frac{1}{2} \log |1+x^2| + C$$

$$= x \tan^{-1} x - \frac{1}{2} \log(1+x^2) + C$$

Where C is an arbitrary constant.

**Question 14:**Integrate  $x(\log x)^2 dx$ **Solution 14:**

$$I = \int x(\log x)^2 dx$$

Taking  $(\log x)^2$  as first function and 1 as second function and integrating by parts, we obtain

$$I = (\log x)^2 \int x dx - \int \left\{ \left( \frac{d}{dx} \log x \right)^2 \int x dx \right\} dx$$

$$= \frac{x^2}{2} (\log x)^2 - \left[ \int 2 \log x \cdot \frac{1}{x} \cdot \frac{x^2}{2} dx \right]$$

$$= \frac{x^2}{2} (\log x)^2 - \int x \log x dx$$

Again integrating by parts, we obtain

$$I = \frac{x^2}{2} (\log x)^2 - \left[ \log x \int x dx - \int \left\{ \left( \frac{d}{dx} \log x \right) \int x dx \right\} dx \right]$$

$$= \frac{x^2}{2} (\log x)^2 - \left[ \frac{x^2}{2} \log x - \int \frac{1}{x} \cdot \frac{x^2}{2} dx \right]$$

$$= \frac{x^2}{2} (\log x)^2 - \frac{x^2}{2} \log x + \frac{1}{2} \int x dx$$

$$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{x^2}{4} + C$$

Where C is an arbitrary constant.

### Question 15:

Integrate  $(x^2 + 1)\log x$

### Solution 15:

$$\text{Let } I = \int (x^2 + 1)\log x dx = \int x^2 \log x dx + \int \log x dx$$

$$\text{Let } I = I_1 + I_2 \dots (1)$$

$$\text{Where, } I_1 = \int x^2 \log x dx \text{ and } I_2 = \int \log x dx$$

$$I_1 = \int x^2 \log x dx$$

Taking  $\log x$  as first function and  $x^2$  as second function and integrating by parts, we obtain

$$I_1 = \log x \int x^2 dx - \int \left\{ \left( \frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$$

$$= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \frac{x^3}{3} \log x - \frac{1}{3} \left( \int x^2 dx \right)$$

$$= \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 \quad \dots(2)$$

$$I_2 = \int \log x dx$$

Taking  $\log x$  as first function and 1 as second function and integrating by parts, we obtain

$$I_2 = \log x \int 1 dx - \int \left\{ \left( \frac{d}{dx} \log x \right) \int 1 dx \right\}$$

$$= \log x \cdot x - \int \frac{1}{x} \cdot x dx$$

$$= x \log x - \int 1 dx$$

$$= x \log x - x + C_2 \quad \dots(3)$$

Using equations (2) and (3) in (1), we obtain

$$I = \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 + x \log x - x + C_2$$

$$= \frac{x^3}{3} \log x - \frac{x^3}{9} + x \log x - x + (C_1 + C_2)$$

$$= \left( \frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + C$$

Where C is an arbitrary constant.

**Question 16:**

Integrate  $e^x (\sin x + \cos x)$

**Solution 16:**

$$\text{Let } I = \int e^x (\sin x + \cos x) dx$$

$$\text{Let } f(x) = \sin x$$

$$f'(x) = \cos x$$

$$I = \int e^x \{f(x) + f'(x)\} dx$$

$$\text{It is known that, } \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$$

$$\therefore I = e^x \sin x + C$$

Where C is an arbitrary constant.

**Question 17:**

Integrate  $\frac{xe^x}{(1+x)^2}$

**Solution 17:**

$$\text{Let } I = \int \frac{xe^x}{(1+x)^2} dx = \int e^x \left\{ \frac{x}{(1+x)^2} \right\} dx$$

$$= \int e^x \left\{ \frac{1+x-1}{(1+x)^2} \right\} dx$$

$$= \int e^x \left\{ \frac{1}{1+x} - \frac{1}{(1+x)^2} \right\} dx$$

$$\text{Let } f(x) = \frac{1}{1+x} \quad f'(x) = \frac{-1}{(1+x)^2}$$

$$\Rightarrow \int \frac{xe^x}{(1+x)^2} dx = \int e^x \{f(x) + f'(x)\} dx$$

$$\text{It is known that, } \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$$

$$\therefore \int \frac{xe^x}{(1+x)^2} dx = \frac{e^x}{1+x} + C$$

Where C is an arbitrary constant.

**Question 18:**

Integrate  $e^x \left( \frac{1 + \sin x}{1 + \cos x} \right)$

**Solution 18:**

$$\begin{aligned}
& e^x \left( \frac{1 + \sin x}{1 + \cos x} \right) \\
&= e^x \left( \frac{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right) \\
&= \frac{e^x \left( \sin \frac{x}{2} + \cos \frac{x}{2} \right)^2}{2 \cos^2 \frac{x}{2}} \\
&= \frac{1}{2} e^x \cdot \left( \frac{\sin \frac{x}{2} + \cos \frac{x}{2}}{\cos \frac{x}{2}} \right)^2 \\
&= \frac{1}{2} e^x \left[ \tan \frac{x}{2} + 1 \right]^2 \\
&= \frac{1}{2} e^x \left( 1 + \tan \frac{x}{2} \right)^2 \\
&= \frac{1}{2} e^x \left[ 1 + \tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right] \\
&= \frac{1}{2} e^x \left[ \sec^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right] \\
&\frac{e^x (1 + \sin x) dx}{(1 + \cos x)} = e^x \left[ \frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right] \quad \dots(1)
\end{aligned}$$

Let  $\tan \frac{x}{2} = f(x)$  so  $f'(x) = \frac{1}{2} \sec^2 \frac{x}{2}$

It is known that,  $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

From equation (1), we obtain

$$\int \frac{e^x (1 + \sin x)}{(1 + \cos x)} dx = e^x \tan \frac{x}{2} + C$$

Where C is an arbitrary constant.

**Question 19:**

Integrate  $e^x \left( \frac{1}{x} - \frac{1}{x^2} \right)$

**Solution 19:**

$$\text{Let } I = \int e^x \left[ \frac{1}{x} - \frac{1}{x^2} \right] dx$$

$$\text{Also, let } \frac{1}{x} = f(x) \quad f'(x) = \frac{-1}{x^2}$$

$$\text{It is known that, } \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$$

$$\therefore I = \frac{e^x}{x} + C$$

Where C is an arbitrary constant.

### Question 20:

$$\text{Integrate } \frac{(x-3)e^x}{(x-1)^3}$$

### Solution 20:

$$\int e^x \left\{ \frac{x-3}{(x-1)^3} \right\} dx = \int e^x \left\{ \frac{x-1-2}{(x-1)^3} \right\} dx$$

$$= \int e^x \left\{ \frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right\} dx$$

$$\text{Let } f(x) = \frac{1}{(x-1)^2} \quad f'(x) = \frac{-2}{(x-1)^3}$$

$$\text{It is known that, } \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$$

$$\therefore \int e^x \left\{ \frac{(x-3)}{(x-1)^2} \right\} dx = \frac{e^x}{(x-1)^2} + C$$

Where C is an arbitrary constant.

### Question 21:

$$\text{Integrate } e^{2x} \sin x$$

### Solution 21:

$$\text{Let } I = \int e^{2x} \sin x dx \quad \dots(1)$$

Integrating by parts, we obtain

$$I = \sin x \int e^{2x} dx - \int \left\{ \left( \frac{d}{dx} \sin x \right) \int e^{2x} dx \right\} dx$$

$$\Rightarrow I = \sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2} dx$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \int e^{2x} \cos x dx$$

Again integrating by parts, we obtain

$$I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \left[ \cos x \int e^{2x} dx - \int \left\{ \left( \frac{d}{dx} \cos x \right) \int e^{2x} dx \right\} dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \left[ \cos x \cdot \frac{e^{2x}}{2} - \int (-\sin x) \frac{e^{2x}}{2} dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \left[ \frac{e^{2x} \cos x}{2} + \frac{1}{2} \int e^{2x} \sin x dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} - \frac{1}{4} I \quad [\text{From (1)}]$$

$$\Rightarrow I + \frac{1}{4} I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow \frac{5}{4} I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow I = \frac{4}{5} \left[ \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} \right] + C$$

$$\Rightarrow I = \frac{e^{2x}}{5} [2 \sin x - \cos x] + C$$

Where C is an arbitrary constant.

### Question 22:

Integrate  $\sin^{-1} \left( \frac{2x}{1+x^2} \right)$

### Solution 22:

Let  $x = \tan \theta$                        $dx = \sec^2 \theta d\theta$

$$\therefore \sin^{-1} \left( \frac{2x}{1+x^2} \right) = \sin^{-1} \left( \frac{2 \tan \theta}{1 + \tan^2 \theta} \right) = \sin^{-1} (\sin 2\theta) = 2\theta$$

$$\int \sin^{-1} \left( \frac{2x}{1+x^2} \right) dx = \int 2\theta \cdot \sec^2 \theta d\theta = 2 \int \theta \cdot \sec^2 \theta d\theta$$

Integrating by parts, we obtain

$$2 \left[ \theta \cdot \int \sec^2 \theta d\theta - \int \left\{ \left( \frac{d}{d\theta} \theta \right) \int \sec^2 \theta d\theta \right\} d\theta \right]$$

$$= 2 \left[ \theta \cdot \tan \theta - \int \tan \theta d\theta \right]$$

$$= 2 \left[ \theta \tan \theta + \log |\cos \theta| \right] + C$$

$$\begin{aligned}
 &= 2 \left[ x \tan^{-1} x + \log \left| \frac{1}{\sqrt{1+x^2}} \right| \right] + C \\
 &= 2x \tan^{-1} x + 2 \left[ -\frac{1}{2} \log(1+x^2) \right] + C \\
 &= 2x \tan^{-1} x - \log(1+x^2) + C
 \end{aligned}$$

Where C is an arbitrary constant.

**Question 23:**

$\int x^2 e^{x^3} dx$  equals

- (A)  $\frac{1}{3} e^{x^3} + C$       (B)  $\frac{1}{3} e^{x^2} + C$   
 (C)  $\frac{1}{2} e^{x^3} + C$       (D)  $\frac{1}{3} e^{x^2} + C$

**Solution 23:**

Let  $I = \int x^2 e^{x^3} dx$

Also, let  $x^3 = t$  so  $3x^2 dx = dt$

$$\Rightarrow I = \frac{1}{3} \int e^t dt$$

$$= \frac{1}{3} (e^t) + C$$

$$= \frac{1}{3} e^{x^3} + C$$

Hence, the correct Answer is A.

**Question 24:**

$\int e^x \sec x (1 + \tan x) dx$  equals

- (A)  $e^x \cos x + C$       (B)  $e^x \sec x + C$   
 (C)  $e^x \sin x + C$       (D)  $e^x \tan x + C$

**Solution 24:**

$\int e^x \sec x (1 + \tan x) dx$

Let  $I = \int e^x \sec x (1 + \tan x) dx = \int e^x (\sec x + \sec x \tan x) dx$

Also, let  $\sec x = f(x)$        $\sec x \tan x = f'(x)$

It is known that,  $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$$\therefore I = e^x \sec x + C$$

Hence, the correct Answer is B.



## Exercise 7.7

**Question 1:**Integrate  $\sqrt{4-x^2}$ **Solution 1:**

$$\text{Let } I = \int \sqrt{4-x^2} dx = \int \sqrt{(2)^2 - (x)^2} dx$$

It is known that,

$$\sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$\therefore I = \frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} + C$$

$$= \frac{x}{2} \sqrt{4-x^2} + 2 \sin^{-1} \frac{x}{2} + C$$

Where C is an arbitrary constant.

**Question 2:**Integrate  $\sqrt{1-4x^2}$ **Solution 2:**

$$\text{Let } I = \int \sqrt{1-4x^2} dx = \int \sqrt{(1)^2 - (2x)^2} dx$$

$$\text{Let } 2x = t \Rightarrow 2dx = dt$$

$$\therefore I = \frac{1}{2} \int \sqrt{(1)^2 - (t)^2}$$

It is known that,

$$\sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$\Rightarrow I = \frac{1}{2} \left[ \frac{t}{2} \sqrt{1-t^2} + \frac{1}{2} \sin^{-1} t \right] + C$$

$$= \frac{t}{4} \sqrt{1-t^2} + \frac{1}{4} \sin^{-1} t + C$$

$$= \frac{2x}{4} \sqrt{1-4x^2} + \frac{1}{4} \sin^{-1} 2x + C$$

$$= \frac{x}{2} \sqrt{1-4x^2} + \frac{1}{4} \sin^{-1} 2x + C$$

Where C is an arbitrary constant.

**Question 3:**Integrate  $\sqrt{x^2 + 4x + 6}$ **Solution 3:**

Let  $I = \int \sqrt{x^2 + 4x + 6} dx$

$$= \int \sqrt{x^2 + 4x + 4 + 2} dx$$

$$= \int \sqrt{(x^2 + 4x + 4) + 2} dx$$

$$= \int \sqrt{(x+2)^2 + (\sqrt{2})^2} dx$$

It is known that,

$$\sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$$

$$\therefore I = \frac{(x+2)}{2} \sqrt{x^2 + 4x + 6} + \frac{2}{2} \log |(x+2) + \sqrt{x^2 + 4x + 6}| + C$$

$$= \frac{(x+2)}{2} \sqrt{x^2 + 4x + 6} + \log |(x+2) + \sqrt{x^2 + 4x + 6}| + C$$

Where C is an arbitrary constant.

**Question 4:**Integrate  $\sqrt{x^2 + 4x + 1}$ **Solution 4:**

Let  $I = \int \sqrt{x^2 + 4x + 1} dx$

$$= \int \sqrt{(x^2 + 4x + 4) - 3} dx$$

$$= \int \sqrt{(x+2)^2 - (\sqrt{3})^2} dx$$

It is known that,

$$\sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$$

$$\therefore I = \frac{(x+2)}{2} \sqrt{x^2 + 4x + 1} - \frac{3}{2} \log |(x-2) + \sqrt{x^2 + 4x + 1}| + C$$

Where C is an arbitrary constant.

**Question 5:**Integrate  $\sqrt{1 - 4x - x^2}$

**Solution 5:**

$$\begin{aligned} \text{Let } I &= \int \sqrt{1-4x-x^2} dx \\ &= \int \sqrt{1-(x^2+4x+4-4)} dx \\ &= \int \sqrt{1+4-(x+2)^2} dx \\ &= \int \sqrt{(\sqrt{5})^2-(x+2)^2} dx \end{aligned}$$

It is known that,

$$\begin{aligned} \sqrt{a^2-x^2} dx &= \frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C \\ \therefore I &= \frac{(x+2)}{2} \sqrt{1-4x-x^2} + \frac{5}{2} \sin^{-1} \left( \frac{x+2}{\sqrt{5}} \right) + C \end{aligned}$$

Where C is an arbitrary constant.

**Question 6:**

Integrate  $\sqrt{x^2+4x-5}$

**Solution 6:**

$$\begin{aligned} \text{Let } I &= \int \sqrt{x^2+4x-5} dx \\ &= \int \sqrt{(x^2+4x+4)-9} dx \\ &= \int \sqrt{(x+2)^2-(3)^2} dx \end{aligned}$$

It is known that,  $\int \sqrt{x^2-a^2} dx = \frac{x}{2} \sqrt{x^2-a^2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2-a^2} \right| + C$

$$\therefore I = \frac{(x+2)}{2} \sqrt{x^2+4x-5} - \frac{9}{2} \log \left| (x+2) + \sqrt{x^2+4x-5} \right| + C$$

Where C is an arbitrary constant.

**Question 7:**

Integrate  $\sqrt{1+3x-x^2}$

**Solution 7:**

$$\begin{aligned} \text{Let } I &= \int \sqrt{1+3x-x^2} dx \\ &= \int \sqrt{1-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)} dx \\ &= \int \sqrt{\left(1+\frac{9}{4}\right)-\left(x-\frac{3}{2}\right)^2} dx \end{aligned}$$

$$= \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2} dx$$

It is known that,

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$\therefore I = \frac{x - \frac{3}{2}}{2} \sqrt{1 + 3x - x^2} + \frac{13}{4 \times 2} \sin^{-1} \left( \frac{x - \frac{3}{2}}{\frac{\sqrt{13}}{2}} \right) + C$$

$$= \frac{2x - 3}{4} \sqrt{1 + 3x - x^2} + \frac{13}{8} \sin^{-1} \left( \frac{2x - 3}{\sqrt{13}} \right) + C$$

Where C is an arbitrary constant.

### Question 8:

Integrate  $\sqrt{x^2 + 3x}$

### Solution 8:

Let  $I = \int \sqrt{x^2 + 3x} dx$

$$= \int \sqrt{x^2 + 3x + \frac{9}{4} - \frac{9}{4}} dx$$

$$= \int \sqrt{\left(x + \frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2} dx$$

It is known that,

$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2 - a^2} \right| + C$$

$$\therefore I = \frac{\left(x + \frac{3}{4}\right)}{2} \sqrt{x^2 + 3x} - \frac{9}{2} \log \left| \left(x + \frac{3}{4}\right) + \sqrt{x^2 + 3x} \right| + C$$

$$= \frac{(2x + 3)}{4} \sqrt{x^2 + 3x} - \frac{9}{8} \log \left| \left(x + \frac{3}{4}\right) + \sqrt{x^2 + 3x} \right| + C$$

Where C is an arbitrary constant.

### Question 9:

Integrate  $\sqrt{1 + \frac{x^2}{9}}$

### Solution 9:

$$\text{Let } I = \int \sqrt{1 + \frac{x^2}{9}} dx = \frac{1}{3} \int \sqrt{9 + x^2} dx = \frac{1}{3} \int \sqrt{(3)^2 + x^2} dx$$

It is known that,

$$\sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$$

$$\therefore I = \frac{1}{3} \left[ \frac{x}{2} \sqrt{x^2 + 9} + \frac{9}{2} \log |x + \sqrt{x^2 + 9}| \right] + C$$

$$= \frac{x}{6} \sqrt{x^2 + 9} + \frac{3}{2} \log |x + \sqrt{x^2 + 9}| + C$$

Where C is an arbitrary constant.

### Question 10:

$\int \sqrt{1+x^2}$  is equal to

A.  $\frac{x}{2} \sqrt{1+x^2} + \frac{1}{2} \log |x + \sqrt{1+x^2}| + C$

B.  $\frac{2}{3} (1+x^2)^{\frac{2}{3}} + C$

C.  $\frac{2}{3} x (1+x^2)^{\frac{2}{3}} + C$

D.  $\frac{x^3}{2} \sqrt{1+x^2} + \frac{1}{2} x^2 \log |x + \sqrt{1+x^2}| + C$

### Solution 10:

It is known that,

$$\sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$$

$$\therefore \int \sqrt{1+x^2} dx = \frac{x}{2} \sqrt{1+x^2} + \frac{1}{2} \log |x + \sqrt{1+x^2}| + C$$

Hence, the correct Answer is A.

### Question 11:

$\int \sqrt{x^2 - 8x + 7} dx$  is equal to

A.  $\frac{1}{2} (x-4) \sqrt{x^2 - 8x + 7} + 9 \log |x-4 + \sqrt{x^2 - 8x + 7}| + C$

B.  $\frac{1}{2} (x+4) \sqrt{x^2 - 8x + 7} + 9 \log |x+4 + \sqrt{x^2 - 8x + 7}| + C$

$$C. \frac{1}{2}(x-4)\sqrt{x^2-8x+7} - 3\sqrt{2} \log|x-4+\sqrt{x^2-8x+7}| + C$$

$$D. \frac{1}{2}(x-4)\sqrt{x^2-8x+7} - \frac{9}{2} \log|x-4+\sqrt{x^2-8x+7}| + C$$

**Solution 11:**

$$\text{Let } I = \int \sqrt{x^2-8x+7} dx$$

$$= \int \sqrt{(x^2-8x+16)-9} dx$$

$$= \int \sqrt{(x-4)^2-(3)^2} dx$$

$$\text{It is known that, } \int \sqrt{x^2-a^2} dx = \frac{x}{2} \sqrt{x^2-a^2} - \frac{a^2}{2} \log|x+\sqrt{x^2-a^2}| + C$$

$$\therefore I = \frac{(x-4)}{2} \sqrt{x^2-8x+7} - \frac{9}{2} \log|(x-4)+\sqrt{x^2-8x+7}| + C$$

Hence, the correct Answer is D.

**Exercise 7.8****Question 1:**

$$\int_a^b x dx$$

**Solution 1:**

It is known that,

$$\int_a^b f(x) dx = (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} [f(a) + f(a+h) + \dots + f(a+(n-1)h)] \text{ where } h = \frac{b-a}{n}$$

Here,  $a = a, b = b$ , and  $f(x) = x$

$$\therefore \int_a^b x dx = (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} [a + (a+h) \dots (a+2h) \dots a + (n-1)h]$$

$$= (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} \left[ \left( a + \underset{\text{ntimes}}{a} + a + \dots + a \right) + (h + 2h + 3h + \dots + (n-1)h) \right]$$

$$= (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} [na + h(1+2+3+\dots+(n-1))]$$

$$= (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} \left[ na + h \left\{ \frac{(n-1)(n)}{2} \right\} \right]$$

$$= (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} \left[ na + \frac{n(n-1)h}{2} \right]$$

$$\begin{aligned}
&= (b-a) \lim_{n \rightarrow \infty} \frac{n}{n} \left[ a + \frac{(n-1)h}{2} \right] \\
&= (b-a) \lim_{n \rightarrow \infty} \left[ a + \frac{(n-1)h}{2} \right] \\
&= (b-a) \lim_{n \rightarrow \infty} \left[ a + \frac{(n-1)(b-a)}{2n} \right] \\
&= (b-a) \lim_{n \rightarrow \infty} \left[ a + \frac{\left(1 - \frac{1}{n}\right)(b-a)}{2} \right] \\
&= (b-a) \left[ a + \frac{(b-a)}{2} \right] \\
&= (b-a) \left[ \frac{2a+b-a}{2} \right] \\
&= \frac{(b-a)(b+a)}{2} \\
&= \frac{1}{2}(b^2 - a^2)
\end{aligned}$$

**Question 2:**

$$\int_0^b (x+1) dx$$

**Solution 2:**

$$\text{Let } I = \int_0^b (x+1) dx$$

It is known that,

$$\int_a^b f(x) dx = (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} \left[ f(a) + f(a+h) + \dots + f(a+(n-1)h) \right], \text{ where } h = \frac{b-a}{n}$$

Here,  $a = 0$ ,  $b = 5$ , and  $f(x) = (x+1)$

$$\Rightarrow h = \frac{5-0}{n} = \frac{5}{n}$$

$$\therefore \int_0^5 (x+1) dx = (5-0) \lim_{n \rightarrow \infty} \frac{1}{n} \left[ f(0) + f\left(\frac{5}{n}\right) + \dots + f\left((n-1)\frac{5}{n}\right) \right]$$

$$= 5 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ 1 + \left(\frac{5}{n} + 1\right) + \dots + \left\{ 1 + \left(\frac{5(n-1)}{n}\right) \right\} \right]$$

$$= 5 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ \left( \underset{n \text{ times}}{1+1+1 \dots 1} \right) + \left[ \frac{5}{n} + 2 \cdot \frac{5}{n} + 3 \cdot \frac{5}{n} + \dots + (n-1) \frac{5}{n} \right] \right]$$

$$\begin{aligned}
&= 5 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ n + \frac{5}{n} \{1+2+3 \dots (n-1)\} \right] \\
&= 5 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ n + \frac{5}{n} \cdot \frac{(n-1)n}{2} \right] \\
&= 5 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ n + \frac{5(n-1)}{2} \right] \\
&= 5 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ 1 + \frac{5}{2} \left( 1 - \frac{1}{n} \right) \right] \\
&= 5 \left[ 1 + \frac{5}{2} \right] \\
&= 5 \left[ \frac{7}{2} \right] \\
&= \frac{35}{2}
\end{aligned}$$

**Question 3:**

$$\int_2^3 x^2 dx$$

**Solution 3:**

It is known that,

$$\int_a^b f(x) dx = (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} [f(a) + f(a+h) + f(a+2h) \dots + f\{a+(n-1)h\}], \text{ where } h = \frac{b-a}{n}$$

Here,  $a=2, b=3$ , and  $f(x) = x^2$

$$\Rightarrow h = \frac{3-2}{n} = \frac{1}{n}$$

$$\therefore \int_2^3 x^2 dx = (3-2) \lim_{n \rightarrow \infty} \frac{1}{n} \left[ f(2) + f\left(2 + \frac{1}{n}\right) + f\left(2 + \frac{2}{n}\right) \dots f\left\{2 + (n-1)\frac{1}{n}\right\} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left[ (2)^2 + \left(2 + \frac{1}{n}\right)^2 + \left(2 + \frac{2}{n}\right)^2 + \dots \left(2 + \frac{(n-1)}{n}\right)^2 \right]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left[ 2^2 + \left\{ 2^2 + \left(\frac{1}{n}\right)^2 + 2 \cdot 2 \cdot \frac{1}{n} \right\} + \dots + \left\{ (2)^2 + \frac{(n-1)^2}{n^2} + 2 \cdot 2 \cdot \frac{(n-1)}{n} \right\} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left[ \left( 2^2 + \dots + 2^2 \right) + \left\{ \left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \dots + \left(\frac{n-1}{n}\right)^2 \right\} + 2 \cdot 2 \cdot \left\{ \frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \dots + \frac{(n-1)}{n} \right\} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left[ 4n + \frac{1}{n^2} \{1^2 + 2^2 + 3^2 \dots + (n-1)^2\} + \frac{4}{n} \{1+2+\dots+(n-1)\} \right]$$



$$\begin{aligned}
&= \lim_{n \rightarrow \infty} \frac{1}{n} \left[ 4n + \frac{1}{n^2} \left\{ \frac{n(n-1)(2n-1)}{6} \right\} + \frac{4}{n} \left\{ \frac{n(n-1)}{2} \right\} \right] \\
&= \lim_{n \rightarrow \infty} \frac{1}{n} \left[ 4n + \frac{n \left( 1 - \frac{1}{n} \right) \left( 2 - \frac{1}{n} \right)}{6} + \frac{4n-4}{2} \right] \\
&= \lim_{n \rightarrow \infty} \left[ 4 + \frac{1}{6} \left( 1 - \frac{1}{n} \right) \left( 2 - \frac{1}{n} \right) + 2 - \frac{2}{n} \right] \\
&= 4 + \frac{2}{6} + 2 \\
&= \frac{19}{3}
\end{aligned}$$

**Question 4:**

$$\int_1^4 (x^2 - x) dx$$

**Solution 4:**

$$\text{Let } I = \int_1^4 (x^2 - x) dx$$

$$= \int_1^4 x^2 dx - \int_1^4 x dx$$

$$\text{Let } I = I_1 - I_2, \text{ where } I_1 = \int_1^4 x^2 dx \text{ and } I_2 = \int_1^4 x dx \quad \dots(1)$$

It is known that,

$$\int_a^b f(x) dx = (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} [f(a) + f(a+h) + f(a+(n-1)h)], \text{ where } h = \frac{b-a}{n}$$

$$\text{For, } I_1 = \int_1^4 x^2 dx,$$

$$a = 1, b = 4, \text{ and } f(x) = x^2$$

$$\therefore h = \frac{4-1}{n} = \frac{3}{n}$$

$$I_1 = \int_1^4 x^2 dx = (4-1) \lim_{n \rightarrow \infty} \frac{1}{n} [f(1) + f(1+h) + \dots + f(1+(n-1)h)]$$

$$= 3 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ 1^2 + \left( 1 + \frac{3}{n} \right)^2 + \left( 1 + 2 \cdot \frac{3}{n} \right)^2 + \dots + \left( 1 + \frac{(n-1)3}{n} \right)^2 \right]$$

$$= 3 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ 1^2 + \left\{ 1^2 + \left( \frac{3}{n} \right)^2 + 2 \cdot \frac{3}{n} \right\} + \dots + \left\{ 1^2 + \left( \frac{(n-1)3}{n} \right)^2 + \frac{2 \cdot (n-1) \cdot 3}{2} \right\} \right]$$

$$= 3 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ \left( 1^2 + \dots + 1^2 \right) + \left( \frac{3}{n} \right)^2 \left\{ 1^2 + 2^2 + \dots + (n-1)^2 \right\} + 2 \cdot \frac{3}{n} \left\{ 1 + 2 + \dots + (n-1) \right\} \right]$$

$$\begin{aligned}
&= 3 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ n + \frac{9}{n^2} \left\{ \frac{(n-1)(n)(2n-1)}{6} \right\} + \frac{6}{n} \left\{ \frac{(n-1)(n)}{2} \right\} \right] \\
&= 3 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ n + \frac{9n}{6} \left( 1 - \frac{1}{n} \right) \left( 2 - \frac{1}{n} \right) + \frac{6n-6}{2} \right] \\
&= 3 \lim_{n \rightarrow \infty} \left[ 1 + \frac{9}{6} \left( 1 - \frac{1}{n} \right) \left( 2 - \frac{1}{n} \right) + 3 - \frac{3}{n} \right] \\
&= 3[1+3+3] \\
&= 3[7] \\
I_1 &= 21 \quad \dots(2)
\end{aligned}$$

For  $I_2 = \int_1^4 x dx$ ,

$a = 1, b = 4$ , and  $f(x) = x$

$$\Rightarrow h = \frac{4-1}{n} = \frac{3}{n}$$

$$\therefore I_2 = (4-1) \lim_{n \rightarrow \infty} \frac{1}{n} [f(1) + f(1+h) + \dots + f(a+(n-1)h)]$$

$$\begin{aligned}
&= 3 \lim_{n \rightarrow \infty} \frac{1}{n} [1 + (1+h) + \dots + (1+(n-1)h)] \\
&= 3 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ 1 + \left( 1 + \frac{3}{n} \right) + \dots + \left\{ 1 + (n-1) \frac{3}{n} \right\} \right] \\
&= 3 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ \left( 1+1+\dots+1 \right) + \frac{3}{n} (1+2+\dots+(n-1)) \right] \\
&= 3 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ n + \frac{3}{n} \left\{ \frac{(n-1)n}{2} \right\} \right] \\
&= 3 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ 1 + \frac{3}{2} \left( 1 - \frac{1}{n} \right) \right] \\
&= 3 \left[ 1 + \frac{3}{2} \right] \\
&= 3 \left[ \frac{5}{2} \right] \\
I_2 &= \frac{15}{2} \quad \dots(3)
\end{aligned}$$

From equations (2) and (3), we obtain

$$I = I_1 - I_2 = 21 - \frac{15}{2} = \frac{27}{2}$$

### Question 5:

$$\int_{-1}^1 e^x dx$$

**Solution 5:**

$$\text{Let } I = \int_{-1}^1 e^x dx \quad \dots(1)$$

It is known that,

$$\int_a^b f(x) dx = (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} [f(a) + f(a+h) + \dots + f(a+(n-1)h)] \text{ where } h = \frac{b-a}{n}$$

Here,  $a = -1, b = 1$ , and  $f(x) = e^x$

$$\therefore h = \frac{1+1}{n} = \frac{2}{n}$$

$$\therefore I = (1+1) \lim_{n \rightarrow \infty} \frac{1}{n} \left[ f(-1) + f\left(-1 + \frac{2}{n}\right) + f\left(-1 + 2 \cdot \frac{2}{n}\right) + \dots + f\left(-1 + \frac{(n-1)2}{n}\right) \right]$$

$$= 2 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ e^{-1} + e^{\left(-1 + \frac{2}{n}\right)} + e^{\left(-1 + 2 \cdot \frac{2}{n}\right)} + \dots + e^{\left(-1 + \frac{(n-1)2}{n}\right)} \right]$$

$$= 2 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ e^{-1} \left\{ 1 + e^{\frac{2}{n}} + e^{\frac{4}{n}} + e^{\frac{6}{n}} + e^{\frac{(n-1)2}{n}} \right\} \right]$$

$$= 2 \lim_{n \rightarrow \infty} \frac{e^{-1}}{n} \left[ \frac{e^{\frac{2n-1}{n}}}{e^{\frac{2-1}{n}}} \right]$$

$$= e^{-1} \times 2 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ \frac{e^2 - 1}{e^{\frac{2-1}{n}}} \right]$$

$$= \frac{e^{-1} \times 2(e^2 - 1)}{\lim_{\frac{2}{n} \rightarrow 0} \left( \frac{e^{\frac{2-1}{n}}}{\frac{2}{n}} \right) \times 2}$$

$$= e^{-1} \left[ \frac{2(e^2 - 1)}{2} \right] \quad \left[ \lim_{h \rightarrow 0} \left( \frac{e^h - 1}{h} \right) = 1 \right]$$

$$= \frac{e^2 - 1}{e}$$

$$= \left( e - \frac{1}{e} \right)$$

**Question 6:**

$$\int_0^4 (x + e^{2x}) dx$$

**Solution 6:**

It is known that,

$$\int_a^b f(x) dx = (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} [f(a) + f(a+h) + \dots + f(a+(n-1)h)] \text{ where } h = \frac{b-a}{n}$$

Here,  $a=0, b=4$ , and  $f(x) = x + e^{2x}$

$$\therefore h = \frac{4-0}{n} = \frac{4}{n}$$

$$\Rightarrow \int_0^4 (x + e^{2x}) dx = (4-0) \lim_{n \rightarrow \infty} \frac{1}{n} [f(0) + f(h) + f(2h) + \dots + f((n-1)h)]$$

$$= 4 \lim_{n \rightarrow \infty} \frac{1}{n} [(0 + e^0) + (h + e^{2h}) + (2h + e^{2 \cdot 2h}) + \dots + \{(n-1)h + e^{2(n-1)h}\}]$$

$$= 4 \lim_{n \rightarrow \infty} \frac{1}{n} [1 + (h + e^{2h}) + (2h + e^{4h}) + \dots + \{(n-1)h + e^{2(n-1)h}\}]$$

$$= 4 \lim_{n \rightarrow \infty} \frac{1}{n} [\{h + 2h + 3h + \dots + (n-1)h\} + (1 + e^{2h} + e^{4h} + \dots + e^{2(n-1)h})]$$

$$= 4 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ h\{1 + 2 + \dots + (n-1)\} + \left( \frac{e^{2hn} - 1}{e^{2h} - 1} \right) \right]$$

$$= 4 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ \frac{(h(n-1)n)}{2} + \left( \frac{e^{2hn} - 1}{e^{2h} - 1} \right) \right]$$

$$= 4 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ \frac{4}{n} \cdot \frac{(n-1)n}{2} + \left( \frac{e^8 - 1}{\frac{8}{n} - 1} \right) \right]$$

$$= 4(2) + 4 \lim_{n \rightarrow \infty} \frac{(e^8 - 1)}{\left( \frac{\frac{8}{n} - 1}{\frac{8}{n}} \right) 8}$$

$$= 8 + \frac{4 \cdot (e^8 - 1)}{8} \quad \left( \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 \right)$$

$$= 8 + \frac{e^8 - 1}{2}$$

$$= \frac{15 + e^8}{2}$$

### Exercise 7.9

#### Question 1:

$$\int_{-1}^1 (x+1) dx$$

**Solution 1:**

$$\text{Let } I = \int_{-1}^1 (x+1) dx$$

$$\int (x+1) dx = \frac{x^2}{2} + x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(-1)$$

$$= \left(\frac{1}{2} + 1\right) - \left(\frac{1}{2} - 1\right)$$

$$= \frac{1}{2} + 1 - \frac{1}{2} + 1$$

$$= 2$$

**Question 2:**

$$\int_2^3 \frac{1}{x} dx$$

**Solution 2:**

$$\text{Let } I = \int_2^3 \frac{1}{x} dx$$

$$\int \frac{1}{x} dx = \log|x| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(3) - F(2)$$

$$= \log|3| - \log|2| = \log \frac{3}{2}$$

**Question 3:**

$$\int_1^2 (4x^3 - 5x^2 + 6x + 9) dx$$

**Solution 3:**

$$\text{Let } I = \int_1^2 (4x^3 - 5x^2 + 6x + 9) dx$$

$$\int (4x^3 - 5x^2 + 6x + 9) dx = 4\left(\frac{x^4}{4}\right) - 5\left(\frac{x^3}{3}\right) + 6\left(\frac{x^2}{2}\right) + 9(x)$$

$$= x^4 - \frac{5x^3}{3} + 3x^2 + 9x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(2) - F(1)$$

$$\begin{aligned}
 I &= \left\{ 2^4 - \frac{5 \cdot (2)^3}{3} + 3(2)^2 + 9(2) \right\} - \left\{ (1)^4 - \frac{5(1)^3}{3} + 3(1)^2 + 9(1) \right\} \\
 &= \left( 16 - \frac{40}{3} + 12 + 18 \right) - \left( 1 - \frac{5}{3} + 3 + 9 \right) \\
 &= 16 - \frac{40}{3} + 12 + 18 - 1 + \frac{5}{3} - 3 - 9 \\
 &= 33 - \frac{35}{3} \\
 &= \frac{99 - 35}{3} \\
 &= \frac{64}{3}
 \end{aligned}$$

**Question 4:**

$$\int_0^{\frac{\pi}{4}} \sin 2x dx$$

**Solution 4:**

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \sin 2x dx$$

$$\int \sin 2x dx = \left( \frac{-\cos 2x}{2} \right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F(0)$$

$$= -\frac{1}{2} \left[ \cos 2\left(\frac{\pi}{4}\right) - \cos 0 \right]$$

$$= -\frac{1}{2} [0 - 1]$$

$$= \frac{1}{2}$$

**Question 5:**

$$\int_0^{\frac{\pi}{2}} \cos 2x dx$$

**Solution 5:**

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \cos 2x dx$$

$$\int \cos 2x dx = \left( \frac{\sin 2x}{2} \right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F\left(\frac{\pi}{2}\right) - F(0) \\ &= \frac{1}{2} \left[ \sin 2\left(\frac{\pi}{2}\right) - \sin 0 \right] \\ &= \frac{1}{2} [\sin \pi - \sin 0] \\ &= \frac{1}{2} [0 - 0] = 0 \end{aligned}$$

**Question 6:**

$$\int_4^5 e^x dx$$

**Solution 6:**

$$\text{Let } I = \int_4^5 e^x dx$$

$$\int e^x dx = e^x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F(5) - F(4) \\ &= e^5 - e^4 \\ &= e^4(e - 1) \end{aligned}$$

**Question 7:**

$$\int_0^{\frac{\pi}{4}} \tan x dx$$

**Solution 7:**

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \tan x dx$$

$$\int \tan x dx = -\log |\cos x| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F\left(\frac{\pi}{4}\right) - F(0) \\ &= -\log \left| \cos \frac{\pi}{4} \right| + \log |\cos 0| \\ &= -\log \left| \frac{1}{\sqrt{2}} \right| + \log |1| \end{aligned}$$

$$= -\log(2)^{-\frac{1}{2}}$$

$$= \frac{1}{2} \log 2$$

**Question 8:**

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$$

**Solution 8:**

$$\text{Let } I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$$

$$\int \operatorname{cosec} x \, dx = \log |\operatorname{cosec} x - \cot x| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F\left(\frac{\pi}{6}\right)$$

$$= \log \left| \operatorname{cosec} \frac{\pi}{4} - \cot \frac{\pi}{4} \right| - \log \left| \operatorname{cosec} \frac{\pi}{6} - \cot \frac{\pi}{6} \right|$$

$$= \log |\sqrt{2} - 1| - \log |2 - \sqrt{3}|$$

$$= \log \left( \frac{\sqrt{2} - 1}{2 - \sqrt{3}} \right)$$

**Question 9:**

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

**Solution 9:**

$$\text{Let } I = \int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(0)$$

$$= \sin^{-1}(1) - \sin^{-1}(0)$$

$$= \frac{\pi}{2} - 0$$

$$= \frac{\pi}{2}$$



**Question 10:**

$$\int_0^1 \frac{dx}{1+x^2}$$

**Solution 10:**

$$\text{Let } I = \int_0^1 \frac{dx}{1+x^2}$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(0)$$

$$= \tan^{-1}(1) - \tan^{-1}(0)$$

$$= \frac{\pi}{4}$$

**Question 11:**

$$\int_2^3 \frac{dx}{x^2-1}$$

**Solution 11:**

$$\text{Let } I = \int_2^3 \frac{dx}{x^2-1}$$

$$\int \frac{dx}{x^2-1} = \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(3) - F(2)$$

$$= \frac{1}{2} \left[ \log \left| \frac{3-1}{3+1} \right| - \log \left| \frac{2-1}{2+1} \right| \right]$$

$$= \frac{1}{2} \left[ \log \left| \frac{2}{4} \right| - \log \left| \frac{1}{3} \right| \right]$$

$$= \frac{1}{2} \left[ \log \frac{1}{2} - \log \frac{1}{3} \right]$$

$$= \frac{1}{2} \left[ \log \frac{3}{2} \right]$$

**Question 12:**

$$\int_0^{\frac{\pi}{2}} \cos^2 x dx$$

**Solution 12:**

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \cos^2 x dx$$

$$\int \cos^2 x dx = \int \left( \frac{1 + \cos 2x}{2} \right) dx = \frac{x}{2} + \frac{\sin 2x}{4} = \frac{1}{2} \left( x + \frac{\sin 2x}{2} \right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= \left[ F\left(\frac{\pi}{2}\right) - F(0) \right] \\ &= \frac{1}{2} \left[ \left( \frac{\pi}{2} + \frac{\sin \pi}{2} \right) - \left( 0 + \frac{\sin 0}{2} \right) \right] \\ &= \frac{1}{2} \left[ \frac{\pi}{2} + 0 - 0 - 0 \right] \\ &= \frac{\pi}{4} \end{aligned}$$

### Question 13:

$$\int_2^3 \frac{x dx}{x^2 + 1}$$

### Solution 13:

$$\text{Let } I = \int_2^3 \frac{x}{x^2 + 1} dx$$

$$\int \frac{x}{x^2 + 1} dx = \frac{1}{2} \int \frac{2x}{x^2 + 1} dx = \frac{1}{2} \log(1 + x^2) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= F(3) - F(2) \\ &= \frac{1}{2} \left[ \log(1 + (3)^2) - \log(1 + (2)^2) \right] \\ &= \frac{1}{2} \left[ \log(10) - \log(5) \right] \\ &= \frac{1}{2} \log\left(\frac{10}{5}\right) = \frac{1}{2} \log 2 \end{aligned}$$

### Question 14:

$$\int_0^1 \frac{2x + 3}{5x^2 + 1} dx$$

### Solution 14:

$$\text{Let } I = \int_0^1 \frac{2x + 3}{5x^2 + 1} dx$$

$$\begin{aligned}
 \int \frac{2x+3}{5x^2+1} dx &= \frac{1}{5} \int \frac{5(2x+3)}{5x^2+1} dx \\
 &= \frac{1}{5} \int \frac{10x+15}{5x^2+1} dx \\
 &= \frac{1}{5} \int \frac{10x}{5x^2+1} dx + 3 \int \frac{1}{5x^2+1} dx \\
 &= \frac{1}{5} \int \frac{10x}{5x^2+1} dx + 3 \int \frac{1}{5\left(x^2+\frac{1}{5}\right)} dx \\
 &= \frac{1}{5} \log(5x^2+1) + \frac{3}{5} \cdot \frac{1}{\sqrt{5}} \tan^{-1} \frac{x}{\frac{1}{\sqrt{5}}} \\
 &= \frac{1}{5} \log(5x^2+1) + \frac{3}{\sqrt{5}} \tan^{-1}(\sqrt{5})x \\
 &= F(x)
 \end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(0)$$

$$\begin{aligned}
 &= \left\{ \frac{1}{5} \log(5+1) + \frac{3}{\sqrt{5}} \tan^{-1}(\sqrt{5}) \right\} - \left\{ \frac{1}{5} \log(1) + \frac{3}{\sqrt{5}} \tan^{-1}(0) \right\} \\
 &= \frac{1}{5} \log 6 + \frac{3}{\sqrt{5}} \tan^{-1} \sqrt{5}
 \end{aligned}$$

### Question 15:

$$\int_0^1 x e^{x^2} dx$$

### Solution 15:

$$\text{Let } I = \int_0^1 x e^{x^2} dx$$

$$\text{Put } x^2 = t \Rightarrow 2x dx = dt$$

As  $x \rightarrow 0, t \rightarrow 0$  and as  $x \rightarrow 1, t \rightarrow 1$ ,

$$\therefore I = \frac{1}{2} \int_0^1 e^t dt$$

$$\frac{1}{2} \int e^t dt = \frac{1}{2} e^t = F(t)$$

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(0)$$

$$= \frac{1}{2} e - \frac{1}{2} e^0$$

$$= \frac{1}{2} (e - 1)$$

**Question 16:**

$$\int_0^1 \frac{5x^2}{x^2 + 4x + 3} dx$$

**Solution 16:**

$$\text{Let } I = \int_1^2 \frac{5x^2}{x^2 + 4x + 3} dx$$

Dividing  $5x^2$  by  $x^2 + 4x + 3$ , we obtain

$$\begin{aligned} I &= \int_1^2 \left\{ 5 - \frac{20x + 15}{x^2 + 4x + 3} \right\} dx \\ &= \int_1^2 5 dx - \int_1^2 \frac{20x + 15}{x^2 + 4x + 3} dx \\ &= [5x]_1^2 - \int_1^2 \frac{20x + 15}{x^2 + 4x + 3} dx \end{aligned}$$

$$I = 5 - I_1, \text{ where } I_1 = \int_1^2 \frac{20x + 15}{x^2 + 4x + 3} dx \quad \dots (1)$$

Consider

$$\begin{aligned} \text{Let } 20x + 15 &= A \frac{d}{dx}(x^2 + 4x + 3) + B \\ &= 2Ax + (4A + B) \end{aligned}$$

Equating the coefficients of  $x$  and constant term, we obtain

$$A = 10 \text{ and } B = -25$$

$$\text{Let } x^2 + 4x + 3 = t$$

$$\Rightarrow (2x + 4) dx = dt$$

$$\Rightarrow I_1 = 10 \int \frac{dt}{t} - 25 \int \frac{dx}{(x+2)^2 - 1^2}$$

$$= 10 \log t - 25 \left[ \frac{1}{2} \log \left( \frac{x+2-1}{x+2+1} \right) \right]$$

$$= \left[ 10 \log(x^2 + 4x + 3) \right]_1^2 - 25 \left[ \frac{1}{2} \log \left( \frac{x+1}{x+3} \right) \right]_1^2$$

$$= [10 \log 15 - 10 \log 8] - 25 \left[ \frac{1}{2} \log \frac{3}{5} - \frac{1}{2} \log \frac{2}{4} \right]$$

$$= [10 \log(5 \times 3) - 10 \log(4 \times 2)] - \frac{25}{2} [\log 3 - \log 5 - \log 2 + \log 4]$$

$$= [10 \log 5 + 10 \log 3 - 10 \log 4 - 10 \log 2] - \frac{25}{2} [\log 3 - \log 5 - \log 2 + \log 4]$$

$$= \left[ 10 + \frac{25}{2} \right] \log 5 + \left[ -10 - \frac{25}{2} \right] \log 4 + \left[ 10 - \frac{25}{2} \right] \log 3 + \left[ -10 + \frac{25}{2} \right] \log 2$$

$$\begin{aligned}
 &= \frac{45}{2} \log 5 = \frac{45}{2} \log 4 - \frac{5}{2} \log 3 + \frac{5}{2} \log 2 \\
 &= \frac{45}{2} \log \frac{5}{4} - \frac{5}{2} \log \frac{3}{2}
 \end{aligned}$$

Substituting the value of  $I_1$  in (1), we obtain

$$\begin{aligned}
 I &= 5 - \left[ \frac{45}{2} \log \frac{5}{4} - \frac{5}{2} \log \frac{3}{2} \right] \\
 &= 5 - \frac{5}{2} \left[ 9 \log \frac{5}{4} - \log \frac{3}{2} \right]
 \end{aligned}$$

### Question 17:

$$\int_0^{\frac{\pi}{4}} (2 \sec^2 x + x^3 + 2) dx$$

### Solution 17:

$$\text{Let } I = \int_0^{\frac{\pi}{4}} (2 \sec^2 x + x^3 + 2) dx$$

$$\int (2 \sec^2 x + x^3 + 2) dx = 2 \tan x + \frac{x^4}{4} + 2x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}
 I &= F\left(\frac{\pi}{4}\right) - F(0) \\
 &= \left\{ \left( 2 \tan \frac{\pi}{4} + \frac{1}{4} \left( \frac{\pi}{4} \right)^4 + 2 \left( \frac{\pi}{4} \right) \right) - (2 \tan 0 + 0 + 0) \right\} \\
 &= 2 \tan \frac{\pi}{4} + \frac{\pi^4}{4^5} + \frac{\pi}{2} \\
 &= 2 + \frac{\pi}{2} + \frac{\pi^4}{1024}
 \end{aligned}$$

### Question 18:

$$\int_0^{\pi} \left( \sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} \right) dx$$

### Solution 18:

$$\text{Let } I = \int_0^{\pi} \left( \sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} \right) dx$$

$$= -\int_0^{\pi} \left( \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \right) dx$$

$$= -\int_0^{\pi} \cos x dx$$

$$-\int_0^{\pi} \cos x \, dx = -\sin x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(\pi) - F(0)$$

$$= -\sin \pi + \sin 0$$

$$= 0$$

### Question 19:

$$\int_0^2 \frac{6x+3}{x^2+4} \, dx$$

### Solution 19:

$$\text{Let } I = \int_0^2 \frac{6x+3}{x^2+4} \, dx$$

$$\int \frac{6x+3}{x^2+4} \, dx = 3 \int \frac{2x+1}{x^2+4} \, dx$$

$$= 3 \int \frac{2x}{x^2+4} \, dx + 3 \int \frac{1}{x^2+4} \, dx$$

$$= 3 \log(x^2+4) + \frac{3}{2} \tan^{-1} \frac{x}{2} = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(2) - F(0)$$

$$= \left\{ 3 \log(2^2+4) + \frac{3}{2} \tan^{-1} \left( \frac{2}{2} \right) \right\} - \left\{ 3 \log(0+4) + \frac{3}{2} \tan^{-1} \left( \frac{0}{2} \right) \right\}$$

$$= 3 \log 8 + \frac{3}{2} \tan^{-1} 1 - 3 \log 4 - \frac{3}{2} \tan^{-1} 0$$

$$= 3 \log 8 + \frac{3}{2} \left( \frac{\pi}{4} \right) - 3 \log 4 - 0$$

$$= 3 \log \left( \frac{8}{4} \right) + \frac{3\pi}{8}$$

$$= 3 \log 2 + \frac{3\pi}{8}$$

### Question 20:

$$\int_0^1 \left( xe^x + \sin \frac{\pi x}{4} \right) dx$$

### Solution 20:

$$\text{Let } I = \int_0^1 \left( xe^x + \sin \frac{\pi x}{4} \right) dx$$

$$\int_0^1 \left( xe^x + \sin \frac{\pi x}{4} \right) dx = x \int e^x dx - \int \left\{ \left( \frac{d}{dx} x \right) \int e^x dx \right\} dx + \left\{ \frac{-\cos \frac{\pi x}{4}}{\frac{\pi}{4}} \right\}$$

$$= xe^x - \int e^x dx - \frac{4}{\pi} \cos \pi \frac{x}{4}$$

$$= xe^x - e^x - \frac{4}{\pi} \cos \pi \frac{x}{4}$$

$$= F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(0)$$

$$= \left( 1.e^1 - e^1 - \frac{4}{\pi} \cos \frac{\pi}{4} \right) - \left( 0.e^0 - e^0 - \frac{4}{\pi} \cos 0 \right)$$

$$= e - e - \frac{4}{\pi} \left( \frac{1}{\sqrt{2}} \right) + 1 + \frac{4}{\pi}$$

$$= 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$$

### Question 21:

$$\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$$

A.  $\frac{\pi}{3}$

B.  $\frac{2\pi}{3}$

C.  $\frac{\pi}{6}$

D.  $\frac{\pi}{12}$  equals

### Solution 21:

$$\int \frac{dx}{1+x^2} = \tan^{-1} x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = F(\sqrt{3}) - F(1)$$

$$= \tan^{-1} \sqrt{3} - \tan^{-1} 1$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{\pi}{12}$$

Hence, the correct Answer is D.

**Question 22:**

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$$

A.  $\frac{\pi}{6}$

B.  $\frac{\pi}{12}$

C.  $\frac{\pi}{24}$

D.  $\frac{\pi}{4}$  equals

**Solution 22:**

$$\int \frac{dx}{4+9x^2} = \int \frac{dx}{(2)^2 + (3x)^2}$$

Put  $3x=t \Rightarrow 3dx=dt$

$$\therefore \int \frac{dx}{(2)^2 + (3x)^2} = \frac{1}{3} \int \frac{dt}{(2)^2 + t^2}$$

$$= \frac{1}{3} \left[ \frac{1}{2} \tan^{-1} \frac{t}{2} \right]$$

$$= \frac{1}{6} \tan^{-1} \left( \frac{3x}{2} \right)$$

$$= F(x)$$

By second fundamental theorem of calculus, we obtain

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} = F\left(\frac{2}{3}\right) - F(0)$$

$$= \frac{1}{6} \tan^{-1} \left( \frac{3}{2} \cdot \frac{2}{3} \right) - \frac{1}{6} \tan^{-1} 0$$

$$= \frac{1}{6} \tan^{-1} 1 - 0$$

$$= \frac{1}{6} \times \frac{\pi}{4}$$

$$= \frac{\pi}{24}$$

Hence, the correct Answer is C.

**Exercise 7.10**



**Question 1:**

$$\int_0^1 \frac{x}{x^2+1} dx$$

**Solution 1:**

$$\int_0^1 \frac{x}{x^2+1} dx$$

Let  $x^2 + 1 = t \Rightarrow 2x dx = dt$

When  $x = 0$ ,  $t = 1$  and when  $x = 1$ ,  $t = 2$

$$\therefore \int_0^1 \frac{x}{x^2+1} dx = \frac{1}{2} \int_1^2 \frac{dt}{t}$$

$$= \frac{1}{2} [\log |t|]_1^2$$

$$= \frac{1}{2} [\log 2 - \log 1]$$

$$= \frac{1}{2} \log 2$$

**Question 2:**

$$\int_0^{\frac{\pi}{2}} \sqrt{\sin \phi} \cos^5 \phi d\phi$$

**Solution 2:**

Let

$$I = \int_0^{\frac{\pi}{2}} \sqrt{\sin \phi} \cos^5 \phi d\phi$$

Also, let  $\sin \phi = t \Rightarrow \cos \phi d\phi = dt$

When  $\phi = 0$ ,  $t = 0$  and when  $\phi = \frac{\pi}{2}$ ,  $t = 1$

$$\therefore I = \int_0^1 \sqrt{t} (1-t^2)^2 dt$$

$$= \int_0^1 t^{\frac{1}{2}} (1+t^4 - 2t^2) dt$$

$$= \int_0^1 \left[ t^{\frac{1}{2}} + t^{\frac{9}{2}} - 2t^{\frac{5}{2}} \right] dt$$

$$= \left[ \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{11}{2}}}{\frac{11}{2}} - \frac{2t^{\frac{7}{2}}}{\frac{7}{2}} \right]_0^1$$

$$= \frac{2}{3} + \frac{2}{11} - \frac{4}{7}$$

$$= \frac{154 + 42 - 132}{231}$$

$$= \frac{64}{231}$$

**Question 3:**

$$\int_0^1 \sin^{-1} \left( \frac{2x}{1+x^2} \right) dx$$

**Solution 3:**

$$\text{Let } I = \int_0^1 \sin^{-1} \left( \frac{2x}{1+x^2} \right) dx$$

Also, let  $x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$

When  $x = 0$ ,  $\theta = 0$  and when  $x = 1$ ,  $\theta = \frac{\pi}{4}$

$$I = \int_0^{\frac{\pi}{4}} \sin^{-1} \left( \frac{2 \tan \theta}{1 + \tan^2 \theta} \right) \sec^2 \theta d\theta$$

$$= \int_0^{\frac{\pi}{4}} \sin^{-1} (\sin 2\theta) \sec^2 \theta d\theta$$

$$= \int_0^{\frac{\pi}{4}} 2\theta \cdot \sec^2 \theta d\theta$$

$$= 2 \int_0^{\frac{\pi}{4}} \theta \cdot \sec^2 \theta d\theta$$

Taking  $\theta$  as first function and  $\sec^2 \theta$  as second function and integrating by parts, we obtain

$$I = 2 \left[ \theta \int \sec^2 \theta d\theta - \int \left\{ \left( \frac{d}{d\theta} \theta \right) \int \sec^2 \theta d\theta \right\} d\theta \right]_0^{\frac{\pi}{4}}$$

$$= 2 \left[ \theta \tan \theta - \int \tan \theta d\theta \right]_0^{\frac{\pi}{4}}$$

$$= 2 \left[ \theta \tan \theta + \log |\cos \theta| \right]_0^{\frac{\pi}{4}}$$

$$= 2 \left[ \frac{\pi}{4} \tan \frac{\pi}{4} + \log \left| \cos \frac{\pi}{4} \right| - \log |\cos 0| \right]$$

$$= 2 \left[ \frac{\pi}{4} + \log \left( \frac{1}{\sqrt{2}} \right) - \log 1 \right]$$

$$= 2 \left[ \frac{\pi}{4} - \frac{1}{2} \log 2 \right]$$

$$= \frac{\pi}{2} - \log 2$$

**Question 4:**

$$\int_0^2 x\sqrt{x+2} \text{ (Put } x+2=t^2 \text{)}$$

**Solution 4:**

$$\int_0^2 x\sqrt{x+2} dx$$

$$\text{Let } x + 2 = t^2 \Rightarrow dx = 2t dt$$

$$\text{When } x = 0, t = \sqrt{2} \text{ and when } x = 2, t = 2$$

$$\therefore \int_0^2 x\sqrt{x+2} dx = \int_{\sqrt{2}}^2 (t^2 - 2)\sqrt{t^2} 2dt$$

$$= \int_{\sqrt{2}}^2 (t^2 - 2)t dt$$

$$= 2 \int_{\sqrt{2}}^2 (t^3 - 2t) dt = 2 \left[ \frac{t^4}{4} - t^2 \right]_{\sqrt{2}}^2 = 2[(4 - 4) - (1 - 2)] = 2$$

**Question 5:**

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos^2 x} dx$$

**Solution 5:**

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos^2 x} dx$$

$$\text{Let } \cos x = t \Rightarrow -\sin x dx = dt$$

$$\text{When } x = 0, t = 1 \text{ and when } x = \frac{\pi}{2}, t = 0$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos^2 x} dx = -\int_1^0 \frac{dt}{1 + t^2}$$

$$= -\left[ \tan^{-1} t \right]_1^0$$

$$= -\left[ \tan^{-1} 0 - \tan^{-1} 1 \right]$$

$$= -\left[ -\frac{\pi}{4} \right]$$

$$= \frac{\pi}{4}$$

**Question 6:**

$$\int_0^2 \frac{dx}{x+4-x^2}$$

**Solution 6:**

$$\begin{aligned} \int_0^2 \frac{dx}{x+4-x^2} &= \int_0^2 \frac{dx}{-(x^2-x-4)} \\ &= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)} \\ &= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]} \\ &= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2} \end{aligned}$$

Let  $x-\frac{1}{2}=t$  so  $dx=dt$

when  $x=0$ ,  $t=-\frac{1}{2}$  and when  $x=2$ ,  $t=\frac{3}{2}$

$$\begin{aligned} \therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2} &= \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2-t^2} \\ &= \left[ \frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\frac{\sqrt{17}}{2}+t}{\frac{\sqrt{17}}{2}-t} \right]_{-\frac{1}{2}}^{\frac{3}{2}} \\ &= \frac{1}{\sqrt{17}} \left[ \log \frac{\frac{\sqrt{17}}{2}+\frac{3}{2}}{\frac{\sqrt{17}}{2}-\frac{3}{2}} - \log \frac{\frac{\sqrt{17}}{2}-\frac{1}{2}}{\frac{\sqrt{17}}{2}+\frac{1}{2}} \right] \\ &= \frac{1}{\sqrt{17}} \left[ \log \frac{\sqrt{17}+3}{\sqrt{17}-3} - \log \frac{\sqrt{17}-1}{\sqrt{17}+1} \right] \\ &= \frac{1}{\sqrt{17}} \log \frac{\sqrt{17}+3}{\sqrt{17}-3} \times \frac{\sqrt{17}+1}{\sqrt{17}-1} \\ &= \frac{1}{\sqrt{17}} \log \left[ \frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}} \right] \\ &= \frac{1}{\sqrt{17}} \log \left[ \frac{20+4\sqrt{17}}{20-4\sqrt{17}} \right] \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{\sqrt{17}} \log \left( \frac{5 + \sqrt{17}}{5 - \sqrt{17}} \right) \\
 &= \frac{1}{\sqrt{17}} \log \left[ \frac{(5 + \sqrt{17})(5 + \sqrt{17})}{25 - 17} \right] \\
 &= \frac{1}{\sqrt{17}} \log \left[ \frac{25 + 17 + 10\sqrt{17}}{8} \right] \\
 &= \frac{1}{\sqrt{17}} \log \left( \frac{42 + 10\sqrt{17}}{8} \right) \\
 &= \frac{1}{\sqrt{17}} \log \left( \frac{21 + 5\sqrt{17}}{4} \right)
 \end{aligned}$$

**Question 7:**

$$\int_{-1}^1 \frac{dx}{x^2 + 2x + 5}$$

**Solution 7:**

$$\int_{-1}^1 \frac{dx}{x^2 + 2x + 5} = \int_{-1}^1 \frac{dx}{(x^2 + 2x + 1) + 4} = \int_{-1}^1 \frac{dx}{(x+1)^2 + (2)^2}$$

Let  $x + 1 = t \Rightarrow dx = dt$

When  $x = -1$ ,  $t = 0$  and when  $x = 1$ ,  $t = 2$

$$\int_{-1}^1 \frac{dx}{(x+1)^2 + (2)^2} = \int_0^2 \frac{dx}{t^2 + 2^2}$$

$$= \left[ \frac{1}{2} \tan^{-1} \frac{t}{2} \right]_0^2$$

$$= \frac{1}{2} \tan^{-1} 1 - \frac{1}{2} \tan^{-1} 0$$

$$= \frac{1}{2} \left( \frac{\pi}{4} \right) = \frac{\pi}{8}$$

**Question 8:**

$$\int_1^2 \left( \frac{1}{x} - \frac{1}{2x^2} \right) e^{2x} dx$$

**Solution 8:**

$$\int_1^2 \left( \frac{1}{x} - \frac{1}{2x^2} \right) e^{2x} dx$$

Let  $2x = t \Rightarrow 2dx = dt$

When  $x = 1$ ,  $t = 2$  and when  $x = 2$ ,  $t = 4$

$$\int_1^2 \left( \frac{1}{x} - \frac{1}{2x^2} \right) e^{2x} dx = \frac{1}{2} \int_2^4 \left( \frac{2}{t} - \frac{2}{t^2} \right) e^t dt$$

Let  $\frac{1}{t} = f(t)$

Then,  $f'(t) = -\frac{1}{t^2}$

$$= \int_2^4 \left( \frac{1}{t} - \frac{1}{t^2} \right) e^t dt = \int_2^4 (f(t) + f'(t)) e^t dt$$

$$= [e^t f(t)]_2^4$$

$$= \left[ e^t \cdot \frac{1}{t} \right]_2^4$$

$$= \left[ \frac{e^t}{t} \right]_2^4$$

$$= \frac{e^4}{4} - \frac{e^2}{2}$$

$$= \frac{e^2(e^2 - 2)}{4}$$

### Question 9:

The value of the integral  $\int_{\frac{1}{3}}^1 \frac{(x-x^3)^{\frac{1}{3}}}{x^4} dx$  is

- A. 6
- B. 0
- C. 3
- D. 4

### Solution 9:

Let  $I = \int_{\frac{1}{3}}^1 \frac{(x-x^3)^{\frac{1}{3}}}{x^4} dx$

Also, let  $x = \sin \theta \Rightarrow dx = \cos \theta d\theta$

When  $x = \frac{1}{3}$ ,  $\theta = \sin^{-1}\left(\frac{1}{3}\right)$  and when  $x=1$ ,  $\theta = \frac{\pi}{2}$

$$\Rightarrow I = \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin \theta - \sin^3 \theta)^{\frac{1}{3}}}{\sin^4 \theta} \cos \theta d\theta$$

$$\begin{aligned}
&= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin \theta)^{\frac{1}{3}} (1 - \sin^2 \theta)^{\frac{1}{3}}}{\sin^4 \theta} \cos \theta d\theta \\
&= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin \theta)^{\frac{1}{3}} (\cos \theta)^{\frac{2}{3}}}{\sin^4 \theta} \cos \theta d\theta \\
&= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin \theta)^{\frac{1}{3}} (\cos \theta)^{\frac{2}{3}}}{\sin^2 \theta \sin^2 \theta} \cos \theta d\theta \\
&= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\cos \theta)^{\frac{5}{3}}}{(\sin \theta)^{\frac{5}{3}}} \cos \theta d\theta \\
&= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} (\cot \theta)^{\frac{5}{3}} \cos \theta d\theta
\end{aligned}$$

Let  $\cot \theta = t \Rightarrow -\operatorname{cosec}^2 \theta d\theta = dt$

When  $\theta = \sin^{-1}\left(\frac{1}{3}\right)$ ,  $t = 2\sqrt{2}$  and when  $\theta = \frac{\pi}{2}$ ,  $t = 0$

$$\begin{aligned}
\therefore I &= -\int_{2\sqrt{2}}^0 (t)^{\frac{5}{3}} dt \\
&= -\left[ \frac{3}{8} (t)^{\frac{8}{3}} \right]_{2\sqrt{2}}^0 \\
&= -\frac{3}{8} \left[ -(2\sqrt{2})^{\frac{8}{3}} \right]_{2\sqrt{2}}^0 \\
&= \frac{3}{8} \left[ (\sqrt{8})^{\frac{8}{3}} \right] \\
&= \frac{3}{8} \left[ (8)^{\frac{4}{3}} \right] \\
&= \frac{3}{8} [16] \\
&= 3 \times 2 \\
&= 6
\end{aligned}$$

Hence, the correct Answer is A.

### Question 10:

If  $f(x) = \int_0^x t \sin t dt$ , then  $f'(x)$  is

- A.  $\cos x + x \sin x$
- B.  $x \sin x$
- C.  $x \cos x$

D.  $\sin x + x \cos x$

**Solution 10:**

$$f(x) = \int_0^x t \sin t dt$$

Integrating by parts, we obtain

$$f(x) = t \int_0^x \sin t dt - \int_0^x \left\{ \left( \frac{d}{dt} t \right) \int \sin t dt \right\} dt$$

$$= [t(-\cos t)]_0^x - \int_0^x (-\cot t) dt$$

$$= [-t \cos t + \sin t]_0^x$$

$$= -x \cos x + \sin x$$

$$\Rightarrow f'(x) = -[x(-\sin x)] + \cos x + \cos x$$

$$= x \sin x - \cos x + \cos x$$

$$= x \sin x$$

Hence, the correct Answer is B.

### Exercise 7.11

**Question 1:**

$$\int_0^{\frac{\pi}{2}} \cos^2 x dx$$

**Solution 1:**

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2 \left( \frac{\pi}{2} - x \right) dx \quad \left( \int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} (\sin^2 x + \cos^2 x) dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$



**Question 2:**

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx$$

**Solution 2:**

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right) + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} dx$$

$$\left(\int_0^a f(x) dx = \int_0^a f(a-x) dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x + \sqrt{\sin x}}} dx \quad \dots (2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

**Question 3:**

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$$

**Solution 3:**

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)} dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

#### Question 4:

$$\int_0^{\frac{\pi}{2}} \frac{\cos^5 x dx}{\sin^5 x + \cos^5 x}$$

#### Solution 4:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x dx}{\sin^5 x + \cos^5 x} \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^5\left(\frac{\pi}{2}-x\right)}{\sin^5\left(\frac{\pi}{2}-x\right) + \cos^5\left(\frac{\pi}{2}-x\right)} dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^5 x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

**Question 5:**

$$\int_{-5}^5 |x+2| dx$$

**Solution 5:**

$$\text{Let } I = \int_{-5}^5 |x+2| dx$$

It can be seen that  $(x+2) \leq 0$  on  $[-5, -2]$  and  $(x+2) \geq 0$  on  $[-2, 5]$ .

$$\therefore I = \int_{-5}^{-2} -(x+2) dx + \int_{-2}^5 (x+2) dx$$

$$\begin{aligned} I &= -\left[\frac{x^2}{2} + 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^5 \\ &= -\left[\frac{(-2)^2}{2} + 2(-2) - \frac{(-5)^2}{2} - 2(-5)\right] + \left[\frac{(5)^2}{2} + 2(5) - \frac{(-2)^2}{2} - 2(-2)\right] \\ &= -\left[2 - 4 - \frac{25}{2} + 10\right] + \left[\frac{25}{2} + 10 - 2 + 4\right] \\ &= -2 + 4 + \frac{25}{2} - 10 + \frac{25}{2} + 10 - 2 + 4 \\ &= 29 \end{aligned}$$

**Question 6:**

$$\int_2^8 |x-5| dx$$

**Solution 6:**

$$\text{Let } I = \int_2^8 |x-5| dx$$

It can be seen that  $(x-5) \leq 0$  on  $[2, 5]$  and  $(x-5) \geq 0$  on  $[5, 8]$ .

$$I = \int_2^5 -(x-5) dx + \int_5^8 (x-5) dx \quad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$\begin{aligned} &= -\left[\frac{x^2}{2} - 5x\right]_2^5 + \left[\frac{x^2}{2} - 5x\right]_5^8 \\ &= -\left[\frac{25}{2} - 25 - 2 + 10\right] + \left[32 - 40 - \frac{25}{2} + 25\right] \\ &= 9 \end{aligned}$$

**Question 7:**

$$\int_0^1 x(1-x)^n dx$$

**Solution 7:**

$$\text{Let } I = \int_0^1 x(1-x)^n dx$$

$$\therefore I = \int_0^1 (1-x)(1-(1-x))^n dx$$

$$= \int_0^1 (1-x)(x)^n dx$$

$$= \int_0^1 (x^n - x^{n+1}) dx$$

$$= \left[ \frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2} \right]_0^1$$

$$\left( \int_1^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$= \left[ \frac{1}{n+1} - \frac{1}{n+2} \right]$$

$$= \frac{(n+2) - (n+1)}{(n+1)(n+2)}$$

$$= \frac{1}{(n+1)(n+2)}$$

**Question 8:**

$$\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$$

**Solution 8:**

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx \quad \dots(1)$$

$$\therefore I = \int_0^{\frac{\pi}{4}} \log \left[ 1 + \tan \left( \frac{\pi}{4} - x \right) \right] dx \quad \left( \int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log \left\{ 1 + \frac{\tan \frac{\pi}{4} - \tan x}{1 + \tan \frac{\pi}{4} \tan x} \right\} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log \left\{ 1 + \frac{1 - \tan x}{1 + \tan x} \right\} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log \frac{2}{(1 + \tan x)} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2 dx - \int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2 dx - I \quad [From(1)]$$

$$\Rightarrow 2I = [x \log 2]_0^{\frac{\pi}{4}}$$

$$\Rightarrow 2I = \frac{\pi}{4} \log 2$$

$$\Rightarrow I = \frac{\pi}{8} \log 2$$

**Question 9:**

$$\int_0^2 x\sqrt{2-x} dx$$

**Solution 9:**

$$\text{Let } I = \int_0^2 x\sqrt{2-x} dx$$

$$I = \int_0^2 (2-x)\sqrt{x} dx \quad \left( \int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$= \int_0^2 \left\{ 2x^{\frac{1}{2}} - x^{\frac{3}{2}} \right\} dx$$

$$= \left[ 2 \left( \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right) - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right]_0^2$$

$$= \left[ \frac{4}{3} x^{\frac{3}{2}} - \frac{2}{5} x^{\frac{5}{2}} \right]_0^2$$

$$= \frac{4}{3} (2)^{\frac{3}{2}} - \frac{2}{5} (2)^{\frac{5}{2}}$$

$$= \frac{4 \times 2\sqrt{2}}{3} - \frac{2}{5} \times 4\sqrt{2}$$

$$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$$

$$= \frac{40\sqrt{2} - 24\sqrt{2}}{15}$$

$$= \frac{16\sqrt{2}}{15}$$

**Question 10:**

$$\int_0^{\frac{\pi}{2}} (2 \log \sin x - \log \sin 2x) dx$$

**Solution 10:**

$$\text{Let } I = \int_0^{\frac{\pi}{2}} (2 \log \sin x - \log \sin 2x) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \{2 \log \sin x - \log(2 \sin x \cos x)\} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \{2 \log \sin x - \log \sin x - \log \cos x - \log 2\} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \{\log \sin x - \log \cos x - \log 2\} dx \quad \dots(1)$$

It is known that,  $\left(\int_0^a f(x) dx = \int_0^a f(a-x) dx\right)$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \{\log \cos x - \log \sin x - \log 2\} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} (-\log 2 - \log 2) dx$$

$$\Rightarrow 2I = -2 \log 2 \int_0^{\frac{\pi}{2}} 1 dx$$

$$\Rightarrow I = -\log 2 \left[ \frac{\pi}{2} \right]$$

$$\Rightarrow I = \frac{\pi}{2} (-\log 2)$$

$$\Rightarrow I = \frac{\pi}{2} \left[ \log \frac{1}{2} \right]$$

$$\Rightarrow I = \frac{\pi}{2} \log \frac{1}{2}$$

**Question 11:**

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx$$

**Solution 11:**

$$\text{Let } I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx$$

As  $\sin^2(-x) = (\sin(-x))^2 = (-\sin x)^2 = \sin^2 x$ , therefore,  $\sin^2 x$  is an even function.

It is known that if  $f(x)$  is an even function, then  $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$

$$I = 2 \int_0^{\frac{\pi}{2}} \sin^2 x dx$$

$$= 2 \int_0^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2} dx$$

$$\begin{aligned}
 &= \int_0^{\frac{\pi}{2}} (1 - \cos 2x) dx \\
 &= \left[ x - \frac{\sin 2x}{2} \right]_0^{\frac{\pi}{2}} \\
 &= \frac{\pi}{2}
 \end{aligned}$$

**Question 12:**

$$\int_0^{\pi} \frac{x dx}{1 + \sin x}$$

**Solution 12:**

$$\text{Let } I = \int_0^{\pi} \frac{x dx}{1 + \sin x} \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1 + \sin(\pi - x)} dx \quad \left( \int_0^{\pi} f(x) dx = \int_0^{\pi} f(a - x) dx \right)$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1 + \sin x} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\pi} \frac{\pi}{1 + \sin x} dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{(1 - \sin x)}{(1 + \sin x)(1 - \sin x)} dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{1 - \sin x}{\cos^2 x} dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \{ \sec^2 x - \tan x \sec x \} dx$$

$$\Rightarrow 2I = \pi [2]$$

$$\Rightarrow I = \pi$$

**Question 13:**

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$$

**Solution 13:**

$$\text{Let } I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx \quad \dots(1)$$

As  $\sin^7(-x) = (\sin(-x))^7 = (-\sin x)^7 = -\sin^7 x$ , therefore,  $\sin^7 x$  is an odd function.

It is known that, if  $f(x)$  is an odd function, then  $\int_{-a}^a f(x) dx = 0$

$$\therefore I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx = 0$$

**Question 14:**

$$\int_0^{2\pi} \cos^5 x dx$$

**Solution 14:**

$$\text{Let } I = \int_0^{2\pi} \cos^5 x dx \quad \dots(1)$$

$$\cos^5(2\pi - x) = \cos^5 x$$

It is known that,

$$\begin{aligned} \int_0^{2a} f(x) dx &= 2 \int_0^a f(x) dx, \text{ if } f(2a - x) = f(x) \\ &= 0 \text{ if } f(2a - x) = -f(x) \end{aligned}$$

$$\therefore I = 2 \int_0^{\pi} \cos^5 x dx$$

$$\Rightarrow I = 2(0) = 0 \quad [\cos^5(\pi - x) = -\cos^5 x]$$

**Question 15:**

$$\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$$

**Solution 15:**

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - x\right)} dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{0}{1 + \sin x \cos x} dx$$

$$\Rightarrow I = 0$$

**Question 16:**

$$\int_0^{\pi} \log(1 + \cos x) dx$$



**Solution 16:**

$$\text{Let } I = \int_0^{\pi} \log(1 + \cos x) dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\pi} \log(1 + \cos(\pi - x)) dx \quad \left( \int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$\Rightarrow I = \int_0^{\pi} \log(1 - \cos x) dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\pi} \{ \log(1 - \cos x) + \log(1 - \cos x) \} dx$$

$$\Rightarrow 2I = \int_0^{\pi} \log(1 - \cos^2 x) dx$$

$$\Rightarrow 2I = \int_0^{\pi} \log \sin^2 x dx$$

$$\Rightarrow 2I = 2 \int_0^{\pi} \log \sin x dx$$

$$\Rightarrow I = \int_0^{\pi} \log \sin x dx \quad \dots(3)$$

$$\sin(\pi - x) = \sin x$$

$$\therefore I = 2 \int_0^{\frac{\pi}{2}} \log \sin x dx \quad \dots(4)$$

$$\Rightarrow I = 2 \int_0^{\frac{\pi}{2}} \log \sin \left( \frac{\pi}{2} - x \right) dx = 2 \int_0^{\frac{\pi}{2}} \log \cos x dx \quad \dots(5)$$

Adding (4) and (5), we obtain

$$2I = 2 \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x + \log 2 - \log 2) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log 2 \sin x \cos x - \log 2) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \sin 2x dx - \int_0^{\frac{\pi}{2}} \log 2 dx$$

$$\text{Let } 2x = t \Rightarrow 2dx = dt$$

$$\text{When } x = 0, t = 0 \text{ and when } x = \frac{\pi}{2}, t = \pi$$

$$\therefore I = \frac{1}{2} \int_0^{\pi} \log \sin t dt - \frac{\pi}{2} \log 2$$

$$\Rightarrow I = \frac{I}{2} - \frac{\pi}{2} \log 2$$

$$\Rightarrow \frac{I}{2} = -\frac{\pi}{2} \log 2$$

$$\Rightarrow I = -\pi \log 2$$

**Question 17:**

$$\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx$$

**Solution 17:**

$$\text{Let } I = \int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx \quad \dots(1)$$

It is known that,  $\left(\int_0^a f(x) dx = \int_0^a f(a-x) dx\right)$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \frac{\sqrt{x} + \sqrt{a-x}}{\sqrt{x} + \sqrt{a-x}} dx$$

$$\Rightarrow 2I = \int_0^a 1 dx$$

$$\Rightarrow 2I = [x]_0^a$$

$$\Rightarrow 2I = a$$

$$\Rightarrow I = \frac{a}{2}$$

**Question 18:**

$$\int_0^4 |x-1| dx$$

**Solution 18:**

$$I = \int_0^4 |x-1| dx$$

It can be seen that,  $(x-1) \leq 0$  when  $0 \leq x \leq 1$  and  $(x-1) \geq 0$  when  $1 \leq x \leq 4$

$$I = \int_0^1 |x-1| dx + \int_1^4 |x-1| dx \quad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$I = \int_0^1 -(x-1) dx + \int_1^4 (x-1) dx$$

$$= \left[ x - \frac{x^2}{2} \right]_0^1 + \left[ \frac{x^2}{2} - x \right]_1^4$$

$$= 1 - \frac{1}{2} + \frac{(4)^2}{2} - 4 - \frac{1}{2} + 1$$

$$= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1$$

$$= 5$$

**Question 19:**

Show that  $\int_0^a f(x)g(x)dx = 2\int_0^a f(x)dx$ , if  $f$  and  $g$  are defined as  $f(x) = f(a-x)$  and  $g(x) + g(a-x) = 4$

**Solution 19:**

$$\text{Let } \int_0^a f(x)g(x)dx \quad \dots(1)$$

$$\Rightarrow \int_0^a f(a-x)g(a-x)dx \quad \left( \int_0^a f(x)dx = \int_0^a f(a-x)dx \right)$$

$$\Rightarrow \int_0^a f(x)g(a-x)dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \{f(x)g(x) + f(x)g(a-x)\}dx$$

$$\Rightarrow 2I = \int_0^a f(x)\{g(x) + g(a-x)\}dx$$

$$\Rightarrow 2I = \int_0^a f(x) \times 4dx \quad [g(x) + g(a-x) = 4]$$

$$\Rightarrow I = 2\int_0^a f(x)dx$$

**Question 20:**

The value of  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x \cos x + \tan^5 x + 1)dx$  is

- A. 0
- B. 2
- C.  $\pi$
- D. 1

**Solution 20:**

$$\text{Let } \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x \cos x + \tan^5 x + 1)dx$$

$$\Rightarrow I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^3 dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan^5 x dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 dx$$

It is known that if  $f(x)$  is an even function, then  $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$

if  $f(x)$  is an odd function, then  $\int_{-a}^a f(x)dx = 0$

$$\text{and } I = 0 + 0 + 0 + 2\int_0^{\frac{\pi}{2}} 1 dx$$

$$\begin{aligned}
 &= 2[x]_0^{\frac{\pi}{2}} \\
 &= \frac{2\pi}{2} \\
 &= \pi
 \end{aligned}$$

Hence, the correct Answer is C.

### Question 21:

The value of  $\int_0^{\frac{\pi}{2}} \left( \frac{4+3\sin x}{4+3\cos x} \right) dx$  is

- A. 2
- B.  $\frac{3}{4}$
- C. 0
- D. -2

### Solution 21:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \left( \frac{4+3\sin x}{4+3\cos x} \right) dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \left[ \frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)} \right] dx \quad \left( \int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\cos x}{4+3\sin x}\right) dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left\{ \log\left(\frac{4+3\sin x}{4+3\cos x}\right) + \log\left(\frac{4+3\cos x}{4+3\sin x}\right) \right\} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \left( \frac{4+3\sin x}{4+3\cos x} \times \frac{4+3\cos x}{4+3\sin x} \right) dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log 1 dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 0 dx$$

$$\Rightarrow I = 0$$

Hence, the correct Answer is C.

### Miscellaneous Exercise

**Question 1:**

Integrate  $\frac{1}{x-x^3}$

**Solution 1:**

$$\frac{1}{x-x^3} = \frac{1}{x(1-x^2)} = \frac{1}{x(1-x)(1+x)}$$

$$\text{Let } \frac{1}{x(1-x)(1+x)} = \frac{A}{x} + \frac{B}{(1-x)} + \frac{C}{1+x} \quad \dots(1)$$

$$\Rightarrow 1 = A(1-x^2) + Bx(1+x) + Cx(1-x)$$

$$\Rightarrow 1 = A - Ax^2 + Bx + Bx^2 + Cx - Cx^2$$

Equating the coefficients of  $x^2$ ,  $x$ , and constant term, we obtain

$$-A + B - C = 0$$

$$B + C = 0$$

$$A = 1$$

On solving these equations, we obtain

$$A = 1, B = \frac{1}{2}, \text{ and } C = -\frac{1}{2}$$

From equation (1), we obtain

$$\frac{1}{x(1-x)(1+x)} = \frac{1}{x} + \frac{1}{2(1-x)} - \frac{1}{2(1+x)}$$

$$\Rightarrow \int \frac{1}{x(1-x)(1+x)} dx = \int \frac{1}{x} dx + \frac{1}{2} \int \frac{1}{(1-x)} dx - \frac{1}{2} \int \frac{1}{(1+x)} dx$$

$$= \log|x| - \frac{1}{2} \log|(1-x)| - \frac{1}{2} \log|(1+x)|$$

$$= \log|x| - \log\left|(1-x)^{\frac{1}{2}}\right| - \log\left|(1+x)^{\frac{1}{2}}\right|$$

$$= \log\left|\frac{x}{(1-x)^{\frac{1}{2}}(1+x)^{\frac{1}{2}}}\right| + C$$

$$= \log\left|\left(\frac{x^2}{1-x^2}\right)^{\frac{1}{2}}\right| + C$$

$$= \frac{1}{2} \log\left|\frac{x^2}{1-x^2}\right| + C$$

**Question 2:**

Integrate  $\frac{1}{\sqrt{x+a} + \sqrt{(x+b)}}$

**Solution 2:**

$$\begin{aligned}
\frac{1}{\sqrt{x+a}+\sqrt{(x+b)}} &= \frac{1}{\sqrt{x+a}+\sqrt{x+b}} \times \frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}} \\
&= \frac{\sqrt{x+a}-\sqrt{x+b}}{(x+a)-(x-b)} \\
&= \frac{(\sqrt{x+a}-\sqrt{x+b})}{a-b} \\
\Rightarrow \int \frac{1}{\sqrt{x+a}+\sqrt{(x+b)}} dx &= \frac{1}{a-b} \int (\sqrt{x+a}-\sqrt{x+b}) dx \\
&= \frac{1}{(a-b)} \left[ \frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}} \right] \\
&= \frac{2}{3(a-b)} \left[ (x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}} \right] + C
\end{aligned}$$

**Question 3:**

Integrate  $\frac{1}{x\sqrt{ax-x^2}}$  [Hint:  $x = \frac{a}{t}$ ]

**Solution 3:**

$$\begin{aligned}
&\frac{1}{x\sqrt{ax-x^2}} \\
\text{Let } x = \frac{a}{t} \Rightarrow dx &= -\frac{a}{t^2} dt \\
\Rightarrow \int \frac{1}{x\sqrt{ax-x^2}} dx &= \int \frac{1}{\frac{a}{t} \sqrt{a \cdot \frac{a}{t} - \left(\frac{a}{t}\right)^2}} \left(-\frac{a}{t^2} dt\right) \\
&= -\int \frac{1}{at} \cdot \frac{1}{\sqrt{\frac{1}{t} - \frac{1}{t^2}}} dt \\
&= -\frac{1}{a} \int \frac{1}{\sqrt{t-1}} dt \\
&= -\frac{1}{a} [2\sqrt{t-1}] + C \\
&= -\frac{1}{a} \left[ 2\sqrt{\frac{a}{x}-1} \right] + C
\end{aligned}$$

$$= -\frac{2}{a} \left( \frac{\sqrt{a-x}}{\sqrt{x}} \right) + C$$

$$= -\frac{2}{a} \left( \sqrt{\frac{a-x}{x}} \right) + C$$

**Question 4:**

Integrate  $\frac{1}{x^2(x^4+1)^{\frac{3}{4}}}$

**Solution 4:**

$$\frac{1}{x^2(x^4+1)^{\frac{3}{4}}}$$

Multiplying and dividing by  $x^{-3}$ , we obtain

$$\frac{x^{-3}}{x^2 x^{-3} (x^4+1)^{\frac{3}{4}}} = \frac{x^{-3} (x^4+1)^{-\frac{3}{4}}}{x^2 x^{-3}}$$

$$= \frac{(x^4+1)^{-\frac{3}{4}}}{x^5 \cdot (x^4)^{-\frac{3}{4}}}$$

$$= \frac{1}{x^5} \left( \frac{x^4+1}{x^4} \right)^{-\frac{3}{4}}$$

$$= \frac{1}{x^5} \left( 1 + \frac{1}{x^4} \right)^{-\frac{3}{4}}$$

Let  $\frac{1}{x^4} = t \Rightarrow -\frac{4}{x^5} dx = dt \Rightarrow \frac{1}{x^5} dx = -\frac{dt}{4}$

$$\therefore \int \frac{1}{x^2(x^4+1)^{\frac{3}{4}}} dx = \int \frac{1}{x^5} \left( 1 + \frac{1}{x^4} \right)^{-\frac{3}{4}} dx$$

$$= -\frac{1}{4} \int (1+t)^{-\frac{3}{4}} dt$$

$$= -\frac{1}{4} \left[ \frac{(1+t)^{\frac{1}{4}}}{\frac{1}{4}} \right] + C$$

$$= -\frac{1}{4} \frac{\left(1 + \frac{1}{x^4}\right)^{\frac{1}{4}}}{\frac{1}{4}} + C$$

$$= -\left(1 + \frac{1}{x^4}\right)^{\frac{1}{4}} + C$$

**Question 5:**

Integrate  $\frac{1}{x^{\frac{1}{2}} + x^{\frac{1}{3}}}$

$$\left[ \text{Hint: } \frac{1}{x^{\frac{1}{2}} + x^{\frac{1}{3}}} = \frac{1}{x^{\frac{1}{3}} \left(1 + x^{\frac{1}{6}}\right)} \text{ Put } x = t^6 \right]$$

**Solution 5:**

Let  $x = t^6 \Rightarrow dx = 6t^5 dt$

$$\int \frac{1}{x^{\frac{1}{2}} + x^{\frac{1}{3}}} dx = \int \frac{6t^5}{t^3 + t^2} dt$$

$$= \int \frac{6t^5}{t^2(1+t)} dt$$

$$= 6 \int \frac{t^3}{(1+t)} dt$$

On dividing, we obtain

$$\int \frac{1}{x^{\frac{1}{2}} + x^{\frac{1}{3}}} dx = 6 \int \left\{ (t^2 - t + 1) - \frac{1}{1+t} \right\} dt$$

$$= 6 \left[ \left( \frac{t^3}{3} \right) - \left( \frac{t^2}{2} \right) + t - \log|1+t| \right]$$

$$= 2x^{\frac{1}{2}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6 \log \left( 1 + x^{\frac{1}{6}} \right) + C$$

$$= 2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6 \log \left( 1 + x^{\frac{1}{6}} \right) + C$$

**Question 6:**



Integrate  $\frac{5x}{(x+1)(x^2+9)}$

**Solution 6:**

$$\text{Let } \frac{5x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9} \quad \dots(1)$$

$$\Rightarrow 5x = A(x^2+9) + (Bx+C)(x+1)$$

$$\Rightarrow 5x = Ax^2 + 9A + Bx^2 + Bx + Cx + C$$

Equating the coefficients of  $x^2$ ,  $x$ , and constant term, we obtain

$$A + B = 0$$

$$B + C = 5$$

$$9A + C = 0$$

On solving these equations, we obtain

$$A = -\frac{1}{2}, B = \frac{1}{2}, \text{ and } C = \frac{9}{2}$$

From equation (1), we obtain

$$\frac{5x}{(x+1)(x^2+9)} = \frac{-1}{2(x+1)} + \frac{\frac{x}{2} + \frac{9}{2}}{x^2+9}$$

$$\int \frac{5x}{(x+1)(x^2+9)} dx = \int \left\{ \frac{-1}{2(x+1)} + \frac{(x+9)}{2(x^2+9)} \right\} dx$$

$$= -\frac{1}{2} \log|x+1| + \frac{1}{2} \int \frac{x}{x^2+9} dx + \frac{9}{2} \int \frac{1}{x^2+9} dx$$

$$= -\frac{1}{2} \log|x+1| + \frac{1}{4} \int \frac{2x}{x^2+9} dx + \frac{9}{2} \int \frac{1}{x^2+9} dx$$

$$= -\frac{1}{2} \log|x+1| + \frac{1}{4} \log|x^2+9| + \frac{9}{2} \cdot \frac{1}{3} \tan^{-1} \frac{x}{3}$$

$$= -\frac{1}{2} \log|x+1| + \frac{1}{4} \log(x^2+9) + \frac{3}{2} \tan^{-1} \frac{x}{3} + C$$

**Question 7:**

Integrate  $\frac{\sin x}{\sin(x-a)}$

**Solution 7:**

$$\frac{\sin x}{\sin(x-a)}$$

Let  $x - a = t \Rightarrow dx = dt$

$$\begin{aligned}
\int \frac{\sin x}{\sin(x-a)} dx &= \int \frac{\sin(t+a)}{\sin t} dt \\
&= \int \frac{\sin t \cos a + \cos t \sin a}{\sin t} dt \\
&= \int (\cos a + \cot t \sin a) dt \\
&= t \cos a + \sin a \log |\sin t| + C_1 \\
&= (x-a) \cos a + \sin a \log |\sin(x-a)| + C_1 \\
&= x \cos a + \sin a \log |\sin(x-a)| - a \cos a + C_1 \\
&= \sin a \log |\sin(x-a)| + x \cos a + C
\end{aligned}$$

**Question 8:**

Integrate  $\frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}$

**Solution 8:**

$$\begin{aligned}
\frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} &= \frac{e^{4\log x} (e^{\log x} - 1)}{e^{2\log x} (e^{\log x} - 1)} \\
&= e^{2\log x} \\
&= e^{\log x^2} \\
&= x^2 \\
\therefore \int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} dx &= \int x^2 dx = \frac{x^3}{3} + C
\end{aligned}$$

**Question 9:**

Integrate  $\frac{\cos x}{\sqrt{4 - \sin^2 x}}$

**Solution 9:**

$$\begin{aligned}
&\frac{\cos x}{\sqrt{4 - \sin^2 x}} \\
\text{Let } \sin x = t &\Rightarrow \cos x \, dx = dt \\
\Rightarrow \int \frac{\cos x}{\sqrt{4 - \sin^2 x}} dx &= \int \frac{dt}{\sqrt{(2)^2 - (t)^2}} \\
&= \sin^{-1} \left( \frac{t}{2} \right) + C \\
&= \sin^{-1} \left( \frac{\sin x}{2} \right) + C
\end{aligned}$$

**Question 10:**

Integrate  $\frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}$

**Solution 10:**

$$\begin{aligned} \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} &= \frac{(\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x)}{1 - 2\sin^2 x + \cos^2 x} \\ &= \frac{(\sin^4 x + \cos^4 x)(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)}{1 - 2\sin^2 x + \cos^2 x} \\ &= \frac{(\sin^4 x + \cos^4 x)(\sin^2 x - \cos^2 x)}{1 - 2\sin^2 x + \cos^2 x} \\ &= \frac{-(\sin^4 x + \cos^4 x)(\cos^2 x - \sin^2 x)}{(\sin^4 x + \cos^4 x)} \\ &= -\cos 2x \\ \therefore \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx &= \int -\cos 2x dx = -\frac{\sin 2x}{2} + C \end{aligned}$$

**Question 11:**

Integrate  $\frac{1}{\cos(x+a)\cos(x+b)}$

**Solution 11:**

$$\begin{aligned} &\frac{1}{\cos(x+a)\cos(x+b)} \\ \text{Multiplying and dividing by } \sin(a-b), \text{ we obtain.} \\ &\frac{1}{\sin(a-b)} \left[ \frac{\sin(a-b)}{\cos(x+a)\cos(x+b)} \right] \\ &= \frac{1}{\sin(a-b)} \left[ \frac{\sin[(x+a)-(x+b)]}{\cos(x+a)\cos(x+b)} \right] \\ &= \frac{1}{\sin(a-b)} \left[ \frac{\sin(x+a)\cos(x+b) - \cos(x+a)\sin(x+b)}{\cos(x+a)\cos(x+b)} \right] \\ &= \frac{1}{\sin(a-b)} \left[ \frac{\sin(x+a)}{\cos(x+a)} - \frac{\sin(x+b)}{\cos(x+b)} \right] \\ &= \frac{1}{\sin(a-b)} [\tan(x+a) - \tan(x+b)] \end{aligned}$$

$$\begin{aligned} \int \frac{1}{\cos(x+a)\cos(x+b)} dx &= \frac{1}{\sin(a-b)} \int [\tan(x+a) - \tan(x+b)] dx \\ &= \frac{1}{\sin(a-b)} [-\log |\cos(x+a)| + \log |\cos(x+b)|] + C \\ &= \frac{1}{\sin(a-b)} \log \left| \frac{\cos(x+b)}{\cos(x+a)} \right| + C \end{aligned}$$

**Question 12:**

Integrate  $\frac{x^3}{\sqrt{1-x^8}}$

**Solution 12:**

$$\begin{aligned} &\frac{x^3}{\sqrt{1-x^8}} \\ \text{Let } x^4 &= t \Rightarrow 4x^3 dx = dt \\ \Rightarrow \int \frac{x^3}{\sqrt{1-x^8}} dx &= \frac{1}{4} \int \frac{dt}{\sqrt{1-t^2}} \\ &= \frac{1}{4} \sin^{-1} t + C \\ &= \frac{1}{4} \sin^{-1}(x^4) + C \end{aligned}$$

**Question 13:**

Integrate  $\frac{e^x}{(1+e^x)(2+e^x)}$

**Solution 13:**

$$\begin{aligned} &\frac{e^x}{(1+e^x)(2+e^x)} \\ \text{Let } e^x &= t \Rightarrow e^x dx = dt \\ \Rightarrow \int \frac{e^x}{(1+e^x)(2+e^x)} dx &= \int \frac{dt}{(t+1)(t+2)} \\ &= \int \left[ \frac{1}{(t+1)} - \frac{1}{(t+2)} \right] dt \\ &= \log |t+1| - \log |t+2| + C \\ &= \log \left| \frac{t+1}{t+2} \right| + C \end{aligned}$$

$$= \log \left| \frac{1+e^x}{2+e^x} \right| + C$$

**Question 14:**

Integrate  $\frac{1}{(x^2+1)(x^2+4)}$

**Solution 14:**

$$\therefore \frac{1}{(x^2+1)(x^2+4)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+4}$$

$$\Rightarrow 1 = (Ax+B)(x^2+4) + (Cx+D)(x^2+1)$$

$$\Rightarrow 1 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + Cx + Dx^2 + D$$

Equating the coefficients of  $x^3$ ,  $x^2$ ,  $x$ , and constant term, we obtain

$$A + C = 0$$

$$B + D = 0$$

$$4A + C = 0$$

$$4B + D = 1$$

On solving these equations, we obtain

$$A = 0, B = \frac{1}{3}, C = 0 \text{ and } D = -\frac{1}{3}$$

From equation (1), we obtain

$$\frac{1}{(x^2+1)(x^2+4)} = \frac{1}{3(x^2+1)} - \frac{1}{3(x^2+4)}$$

$$\int \frac{1}{(x^2+1)(x^2+4)} dx = \frac{1}{3} \int \frac{1}{x^2+1} dx - \frac{1}{3} \int \frac{1}{x^2+4} dx$$

$$= \frac{1}{3} \tan^{-1} x - \frac{1}{3} \cdot \frac{1}{2} \tan^{-1} \frac{x}{2} + C$$

$$= \frac{1}{3} \tan^{-1} x - \frac{1}{6} \tan^{-1} \frac{x}{2} + C$$

**Question 15:**

Integrate  $\cos^3 x e^{\log \sin x}$

**Solution 15:**

$$\cos^3 x e^{\log \sin x} = \cos^3 x \times \sin x$$

Let  $\cos x = t \Rightarrow -\sin x dx = dt$

$$\Rightarrow \int \cos^3 x e^{\log \sin x} dx = \int \cos^3 x \sin x dx$$

$$= -\int t^3 dx$$

$$= -\frac{t^4}{4} + C$$

$$= -\frac{\cos^4 x}{4} + C$$

**Question 16:**Integrate  $e^{3\log x} (x^4 + 1)^{-1}$ **Solution 16:**

$$e^{3\log x} (x^4 + 1)^{-1} = e^{\log x^3} (x^4 + 1)^{-1} = \frac{x^3}{(x^4 + 1)}$$

$$\text{Let } x^4 + 1 = t \Rightarrow 4x^3 dx = dt$$

$$\Rightarrow \int e^{3\log x} (x^4 + 1)^{-1} dx = \int \frac{x^3}{(x^4 + 1)} dx$$

$$= \frac{1}{4} \int \frac{dt}{t}$$

$$= \frac{1}{4} \log |t| + C$$

$$= \frac{1}{4} \log |x^4 + 1| + C$$

$$= \frac{1}{4} \log (x^4 + 1) + C$$

**Question 17:**Integrate  $f'(ax+b)[f(ax+b)]^n$ **Solution 17:**

$$f'(ax+b)[f(ax+b)]^n$$

$$\text{Let } f(ax+b) = t \Rightarrow a f'(ax+b) dx = dt$$

$$\Rightarrow \int f'(ax+b)[f(ax+b)]^n dx = \frac{1}{a} \int t^n dt$$

$$= \frac{1}{a} \left[ \frac{t^{n+1}}{n+1} \right]$$

$$= \frac{1}{a(n+1)} (f(ax+b))^{n+1} + C$$

**Question 18:**

Integrate  $\frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}}$

**Solution 18:**

$$\begin{aligned} \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}} &= \frac{1}{\sqrt{\sin^3 x (\sin x \cos \alpha + \cos x \sin \alpha)}} \\ &= \frac{1}{\sqrt{\sin^4 x \cos \alpha + \sin^3 x \cos x \sin \alpha}} \\ &= \frac{1}{\sin^2 x \sqrt{\cos \alpha + \cot x \sin \alpha}} \\ &= \frac{\operatorname{cosec}^2 x}{\sqrt{\cos \alpha + \cot x \sin \alpha}} \end{aligned}$$

Let  $\cos \alpha + \cot x \sin \alpha = t \Rightarrow -\operatorname{cosec}^2 x \sin \alpha dx = dt$

$$\begin{aligned} \therefore \int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}} dx &= \int \frac{\operatorname{cosec}^2 x}{\sqrt{\cos \alpha + \cot x \sin \alpha}} dx \\ &= \frac{-1}{\sin \alpha} \int \frac{dt}{\sqrt{t}} \\ &= \frac{-1}{\sin \alpha} [2\sqrt{t}] + C \\ &= \frac{-1}{\sin \alpha} [2\sqrt{\cos \alpha + \cot x \sin \alpha}] + C \\ &= \frac{-2}{\sin \alpha} \sqrt{\cos \alpha + \frac{\cos x \sin \alpha}{\sin x}} + C \\ &= \frac{-2}{\sin \alpha} \sqrt{\frac{\sin x \cos \alpha + \cos x \sin \alpha}{\sin x}} + C \\ &= \frac{-2}{\sin \alpha} \sqrt{\frac{\sin(x+\alpha)}{\sin x}} + C \end{aligned}$$

**Question 19:**

Integrate  $\frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}}, x \in [0, 1]$

**Solution 19:**

$$\text{Let } I = \int \frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} dx$$

It is known that,  $\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x} = \frac{\pi}{2}$

$$\begin{aligned} \Rightarrow I &= \int \frac{\left(\frac{\pi}{2} - \cos^{-1} \sqrt{x}\right) - \cos^{-1} \sqrt{x}}{\frac{\pi}{2}} dx \\ &= \frac{2}{\pi} \int \left(\frac{\pi}{2} - 2 \cos^{-1} \sqrt{x}\right) dx \\ &= \frac{2}{\pi} \cdot \frac{\pi}{2} \int 1 \cdot dx - \frac{4}{\pi} \int \cos^{-1} \sqrt{x} dx \\ &= x - \frac{4}{\pi} \int \cos^{-1} \sqrt{x} dx \quad \dots(1) \end{aligned}$$

Let  $I_1 = \int \cos^{-1} \sqrt{x} dx$

Also, let  $\sqrt{x} = t \Rightarrow dx = 2t dt$

$$\begin{aligned} \Rightarrow I_1 &= 2 \int \cos^{-1} t \cdot t dt \\ &= 2 \left[ \cos^{-1} t \cdot \frac{t^2}{2} - \int \frac{-1}{\sqrt{1-t^2}} \cdot \frac{t^2}{2} dt \right] \\ &= t^2 \cos^{-1} t + \int \frac{t^2}{\sqrt{1-t^2}} dt \\ &= t^2 \cos^{-1} t - \int \frac{1-t^2-1}{\sqrt{1-t^2}} dt \\ &= t^2 \cos^{-1} t - \int \sqrt{1-t^2} dt + \int \frac{1}{\sqrt{1-t^2}} dt \\ &= t^2 \cos^{-1} t - \frac{1}{2} \sqrt{1-t^2} - \frac{1}{2} \sin^{-1} t + \sin^{-1} t \\ &= t^2 \cos^{-1} t - \frac{1}{2} \sqrt{1-t^2} + \frac{1}{2} \sin^{-1} t \end{aligned}$$

From equation (1), we obtain

$$\begin{aligned} I &= x - \frac{4}{\pi} \left[ t^2 \cos^{-1} t - \frac{t}{2} \sqrt{1-t^2} + \frac{1}{2} \sin^{-1} t \right] \\ &= x - \frac{4}{\pi} \left[ x \cos^{-1} \sqrt{x} - \frac{\sqrt{x}}{2} \sqrt{1-x} + \frac{1}{2} \sin^{-1} \sqrt{x} \right] \\ &= x - \frac{4}{\pi} \left[ x \left( \frac{\pi}{2} - \sin^{-1} \sqrt{x} \right) - \frac{\sqrt{x-x^2}}{2} + \frac{\pi}{2} \sin^{-1} \sqrt{x} \right] \\ &= x - 2x + \frac{4x}{\pi} \sin^{-1} \sqrt{x} + \frac{2}{\pi} \sqrt{x-x^2} - \frac{2}{\pi} \sin^{-1} \sqrt{x} \\ &= -x + \frac{2}{\pi} \left[ (2x-1) \sin^{-1} \sqrt{x} \right] + \frac{2}{\pi} \sqrt{x-x^2} + C \\ &= \frac{2(2x-1)}{\pi} \sin^{-1} \sqrt{x} + \frac{2}{\pi} \sqrt{x-x^2} - x + C \end{aligned}$$



**Question 20:**

Integrate  $\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$

**Solution 20:**

$$I = \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$$

$$\text{Let } x = \cos^2 \theta \Rightarrow dx = -2 \sin \theta \cos \theta d\theta$$

$$\begin{aligned} I &= \int \sqrt{\frac{1-\cos \theta}{1+\cos \theta}} (-2 \sin \theta \cos \theta) d\theta \\ &= -\int \sqrt{\frac{2 \sin^2 \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}}} \sin 2\theta d\theta \\ &= -\int \tan \frac{\theta}{2} \cdot 2 \sin \theta \cos \theta d\theta \\ &= -2 \int \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} \left( 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right) \cos \theta d\theta \\ &= -4 \int \sin^2 \frac{\theta}{2} \cos \theta d\theta \\ &= -4 \int \sin^2 \frac{\theta}{2} \cdot \left( 2 \cos^2 \frac{\theta}{2} - 1 \right) d\theta \\ &= -4 \int \left( 2 \sin^2 \frac{\theta}{2} \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} \right) d\theta \\ &= -8 \int \sin^2 \frac{\theta}{2} \cdot \cos^2 \frac{\theta}{2} d\theta + 4 \int \sin^2 \frac{\theta}{2} d\theta \\ &= -2 \int \sin^2 \theta d\theta + 4 \int \sin^2 \frac{\theta}{2} d\theta \\ &= -2 \int \left( \frac{1 - \cos 2\theta}{2} \right) d\theta + 4 \int \frac{1 - \cos \theta}{2} d\theta \\ &= -2 \left[ \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right] + 4 \left[ \frac{\theta}{2} - \frac{\sin \theta}{2} \right] + C \\ &= -\theta + \frac{\sin 2\theta}{2} + 2\theta - 2 \sin \theta + C \\ &= \theta + \frac{\sin 2\theta}{2} + 2 \sin \theta + C \\ &= \theta + \frac{2 \sin \theta \cos \theta}{2} - 2 \sin \theta + C \end{aligned}$$

$$\begin{aligned}
 &= \theta + \sqrt{1 - \cos^2 \theta} \cdot \cos \theta - 2\sqrt{1 - \cos^2 \theta} + C \\
 &= \cos^{-1} \sqrt{x} + \sqrt{1-x} \cdot \sqrt{x} - 2\sqrt{1-x} + C \\
 &= -2\sqrt{1-x} + \cos^{-1} \sqrt{x} + \sqrt{x(1-x)} + C \\
 &= -2\sqrt{1-x} + \cos^{-1} \sqrt{x} + \sqrt{x-x^2} + C
 \end{aligned}$$

**Question 21:**

Integrate  $\frac{2 + \sin 2x}{1 + \cos 2x} e^x$

**Solution 21:**

$$\begin{aligned}
 I &= \int \left( \frac{2 + \sin 2x}{1 + \cos 2x} \right) e^x \\
 &= \int \left( \frac{2 + 2 \sin x \cos x}{2 \cos^2 x} \right) e^x \\
 &= \int \left( \frac{1 + \sin x \cos x}{\cos^2 x} \right) e^x \\
 &= \int (\sec^2 x + \tan x) e^x
 \end{aligned}$$

Let  $f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$

$$\therefore I = \int (f(x) + f'(x)) e^x dx$$

$$= e^x f(x) + C$$

$$= e^x \tan x + C$$

**Question 22:**

Integrate  $\frac{x^2 + x + 1}{(x+1)^2(x+2)}$

**Solution 22:**

$$\text{Let } \frac{x^2 + x + 1}{(x+1)^2(x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2} \quad \dots(1)$$

$$\Rightarrow x^2 + x + 1 = A(x+1)(x+2) + B(x+2) + C(x^2 + 2x + 1)$$

$$\Rightarrow x^2 + x + 1 = A(x^2 + 3x + 2) + B(x+2) + C(x^2 + 2x + 1)$$

$$\Rightarrow x^2 + x + 1 = (A+C)x^2 + (3A+B+2C)x + (2A+2B+C)$$

Equating the coefficients of  $x^2$ ,  $x$  and constant term, we obtain

$$A + C = 1$$

$$3A + B + 2C = 1$$

$$2A + 2B + C = 1$$

On solving these equations, we obtain

$A = -2$ ,  $B = 1$ , and  $C = 3$

From equation (1), we obtain

$$\frac{x^2 + x + 1}{(x+1)^2(x+2)} = \frac{-2}{(x+1)} + \frac{3}{(x+2)} + \frac{1}{(x+1)^2}$$

$$\int \frac{x^2 + x + 1}{(x+1)^2(x+2)} dx = -2 \int \frac{1}{x+1} dx + 3 \int \frac{1}{(x+2)} dx + \int \frac{1}{(x+1)^2} dx$$

$$= -2 \log|x+1| + 3 \log|x+2| - \frac{1}{(x+1)} + C$$

### Question 23:

Integrate  $\tan^{-1} \sqrt{\frac{1-x}{1+x}}$

### Solution 23:

$$I = \tan^{-1} \sqrt{\frac{1-x}{1+x}}$$

Let  $x = \cos \theta \Rightarrow dx = -\sin \theta d\theta$

$$I = \int \tan^{-1} \sqrt{\frac{1-\cos \theta}{1+\cos \theta}} \cdot (-\sin \theta d\theta)$$

$$= - \int \tan^{-1} \sqrt{\frac{2 \sin^2 \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}}} \sin \theta d\theta$$

$$= - \int \tan^{-1} \tan \frac{\theta}{2} \cdot \sin \theta d\theta$$

$$= - \frac{1}{2} \int \theta \cdot \sin \theta d\theta$$

$$= - \frac{1}{2} \left[ \theta \cdot (-\cos \theta) - \int 1 \cdot (-\cos \theta) d\theta \right]$$

$$= - \frac{1}{2} \left[ -\theta \cos \theta + \sin \theta \right]$$

$$= + \frac{1}{2} \theta \cos \theta - \frac{1}{2} \sin \theta$$

$$= \frac{1}{2} \cos^{-1} x \cdot x - \frac{1}{2} \sqrt{1-x^2} + C$$

$$= \frac{x}{2} \cos^{-1} x - \frac{1}{2} \sqrt{1-x^2} + C$$

$$= \frac{1}{2} \left( x \cos^{-1} x - \sqrt{1-x^2} \right) + C$$

**Question 24:**

Integrate  $\frac{\sqrt{x^2+1}[\log(x^2+1)-2\log x]}{x^4}$

**Solution 24:**

$$\frac{\sqrt{x^2+1}[\log(x^2+1)-2\log x]}{x^4} = \frac{\sqrt{x^2+1}}{x^4}[\log(x^2+1)-\log x^2]$$

$$= \frac{\sqrt{x^2+1}}{x^4} \left[ \log\left(\frac{x^2+1}{x^2}\right) \right]$$

$$= \frac{\sqrt{x^2+1}}{x^4} \log\left(1 + \frac{1}{x^2}\right)$$

$$= \frac{1}{x^3} \sqrt{\frac{x^2+1}{x^2}} \log\left(1 + \frac{1}{x^2}\right)$$

$$= \frac{1}{x^3} \sqrt{1 + \frac{1}{x^2}} \log\left(1 + \frac{1}{x^2}\right)$$

$$\text{Let } 1 + \frac{1}{x^2} = t \Rightarrow \frac{-2}{x^3} dx = dt$$

$$\therefore I = \int \frac{1}{x^3} \sqrt{1 + \frac{1}{x^2}} \log\left(1 + \frac{1}{x^2}\right) dx$$

$$= -\frac{1}{2} \int \sqrt{t} \log t dt$$

$$= -\frac{1}{2} \int t^{\frac{1}{2}} \cdot \log t dt$$

Integrating by parts, we obtain

$$I = -\frac{1}{2} \left[ \log t \cdot \int t^{\frac{1}{2}} dt - \left\{ \left( \frac{d}{dt} \log t \right) \int t^{\frac{1}{2}} dt \right\} dt \right]$$

$$= -\frac{1}{2} \left[ \log t \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \int \frac{1}{t} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} dt \right]$$

$$= -\frac{1}{2} \left[ \frac{2}{3} t^{\frac{3}{2}} \log t - \frac{2}{3} \int t^{\frac{1}{2}} dt \right]$$

$$= -\frac{1}{2} \left[ \frac{2}{3} t^{\frac{3}{2}} \log t - \frac{4}{9} t^{\frac{3}{2}} \right]$$

$$= -\frac{1}{3} t^{\frac{3}{2}} \log t + \frac{2}{9} t^{\frac{3}{2}}$$

$$= -\frac{1}{3} t^{\frac{3}{2}} \left[ \log t - \frac{2}{3} \right]$$

$$= -\frac{1}{3} \left(1 + \frac{1}{x^2}\right)^{\frac{3}{2}} \left[ \log \left(1 + \frac{1}{x^2}\right) - \frac{2}{3} \right] + C$$

**Question 25:**

$$\int_{\frac{\pi}{2}}^{\pi} e^x \left( \frac{1 - \sin x}{1 - \cos x} \right) dx$$

**Solution 25:**

$$\begin{aligned} I &= \int_{\frac{\pi}{2}}^{\pi} e^x \left( \frac{1 - \sin x}{1 - \cos x} \right) dx \\ &= \int_{\frac{\pi}{2}}^{\pi} e^x \left( \frac{1 - 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \sin^2 \frac{x}{2}} \right) dx \\ &= \int_{\frac{\pi}{2}}^{\pi} \left( \frac{\operatorname{cosec}^2 \frac{x}{2}}{2} - \cot \frac{x}{2} \right) dx \end{aligned}$$

$$\text{Let } f(x) = -\cot \frac{x}{2}$$

$$\Rightarrow f'(x) = -\left( -\frac{1}{2} \operatorname{cosec}^2 \frac{x}{2} \right) = \frac{1}{2} \operatorname{cosec}^2 \frac{x}{2}$$

$$\therefore I = \int_{\frac{\pi}{2}}^{\pi} e^x (f(x) + f'(x)) dx$$

$$= \left[ e^x \cdot f(x) dx \right]_{\frac{\pi}{2}}^{\pi}$$

$$= - \left[ e^x \cdot \cot \frac{x}{2} \right]_{\frac{\pi}{2}}^{\pi}$$

$$= - \left[ e^{\pi} \cdot \cot \frac{\pi}{2} - e^{\frac{\pi}{2}} \cdot \cot \frac{\pi}{4} \right]$$

$$= - \left[ e^{\pi} \cdot 0 - e^{\frac{\pi}{2}} \cdot 1 \right]$$

$$= e^{\frac{\pi}{2}}$$

**Question 26:**

$$\int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$$

**Solution 26:**

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \frac{\frac{\sin x \cos x}{\cos^4 x}}{\frac{\cos^4 x + \sin^4 x}{\cos^4 x}} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \frac{\tan x \sec^2 x}{1 + \tan^4 x} dx$$

$$\text{Let } \tan^2 x = t \Rightarrow 2 \tan x \sec^2 x dx = dt$$

$$\text{When } x = 0, t = 0 \text{ and when } x = \frac{\pi}{4}, t = 1$$

$$\therefore I = \frac{1}{2} \int_0^1 \frac{dt}{1+t^2}$$

$$= \frac{1}{2} \left[ \tan^{-1} t \right]_0^1$$

$$= \frac{1}{2} \left[ \tan^{-1} 1 - \tan^{-1} 0 \right]$$

$$= \frac{1}{2} \left[ \frac{\pi}{4} \right]$$

$$= \frac{\pi}{8}$$

**Question 27:**

$$\int_0^{\frac{\pi}{2}} \frac{\cos^2 x dx}{\cos^2 x + 4 \sin^2 x}$$

**Solution 27:**

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4 \sin^2 x} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4(1 - \cos^2 x)} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4 - 4 \cos^2 x} dx$$

$$\Rightarrow I = \frac{-1}{3} \int_0^{\frac{\pi}{2}} \frac{4 - 3 \cos^2 x}{\cos^2 x + 4 - 4 \cos^2 x} dx$$

$$\Rightarrow I = \frac{-1}{3} \int_0^{\frac{\pi}{2}} \frac{4 - 3 \cos^2 x}{4 - 3 \cos^2 x} dx + \frac{1}{3} \int_0^{\frac{\pi}{2}} \frac{4}{4 - 3 \cos^2 x} dx$$

$$\Rightarrow I = \frac{-1}{3} \int_0^{\frac{\pi}{2}} 1 dx + \frac{1}{3} \int_0^{\frac{\pi}{2}} \frac{4 \sec^2 x}{4 \sec^2 x - 3} dx$$

$$\Rightarrow I = \frac{-1}{3} [x]_0^{\frac{\pi}{2}} + \frac{1}{3} \int_0^{\frac{\pi}{2}} \frac{4 \sec^2 x}{4(1 + \tan^2 x) - 3} dx$$

$$\Rightarrow I = -\frac{\pi}{6} + \frac{2}{3} \int_0^{\frac{\pi}{2}} \frac{2 \sec^2 x}{1 + 4 \tan^2 x} dx \quad \dots(1)$$

Consider,  $\int_0^{\frac{\pi}{2}} \frac{2 \sec^2 x}{1 + 4 \tan^2 x} dx$

Let  $2 \tan x = t \Rightarrow 2 \sec^2 x dx = dt$

When  $x = 0$ ,  $t = 0$  and when  $x = \frac{\pi}{2}$ ,  $t = \infty$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \frac{2 \sec^2 x}{1 + 4 \tan^2 x} dx = \int_0^{\infty} \frac{dt}{1 + t^2}$$

$$= [\tan^{-1} t]_0^{\infty}$$

$$= [\tan^{-1}(\infty) - \tan^{-1}(0)]$$

$$= \frac{\pi}{2}$$

Therefore, from (1), we obtain

$$I = -\frac{\pi}{6} + \frac{2}{3} \left[ \frac{\pi}{2} \right] = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

### Question 28:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$$

### Solution 28:

Let  $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

$$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\sin x + \cos x)}{\sqrt{-(-\sin 2x)}} dx$$

$$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{-(-1 + 1 - 2 \sin x \cos x)}} dx$$

$$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\sin x + \cos x)}{\sqrt{1 - (\sin^2 x \cos^2 x - 2 \sin x \cos x)}} dx$$

$$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\sin x + \cos x) dx}{\sqrt{1 - (\sin x - \cos x)^2}}$$

Let  $(\sin x - \cos x) = t = (\sin x + \cos x) dx = dt$

when  $x = \frac{\pi}{6}, t = \left(\frac{1-\sqrt{3}}{2}\right)$  and when  $x = \frac{\pi}{3}, t = \left(\frac{\sqrt{3}-1}{2}\right)$

$$I = \int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}}$$

$$\Rightarrow I = \int_{-\left(\frac{1-\sqrt{3}}{2}\right)}^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}}$$

As  $\frac{1}{\sqrt{1-(-t)^2}} = \frac{1}{\sqrt{1-t^2}}$ , therefore,  $\frac{1}{\sqrt{1-t^2}}$  is an even function.

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

It is known that if  $f(x)$  is an even function, then

$$\Rightarrow I = 2 \int_0^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}}$$

$$= \left[ 2 \sin^{-1} t \right]_0^{\frac{\sqrt{3}-1}{2}}$$

$$= 2 \sin^{-1} \left( \frac{\sqrt{3}-1}{2} \right) = 2(\pi/12) = \pi/6$$

### Question 29:

$$\int_0^1 \frac{dx}{\sqrt{1+x} - \sqrt{x}}$$

### Solution 29:

$$\text{Let } I = \int_0^1 \frac{dx}{\sqrt{1+x} - \sqrt{x}}$$

$$I = \int_0^1 \frac{1}{(\sqrt{1+x} - \sqrt{x})} \times \frac{(\sqrt{1+x} + \sqrt{x})}{(\sqrt{1+x} + \sqrt{x})} dx$$

$$= \int_0^1 \frac{(\sqrt{1+x} + \sqrt{x})}{1+x-x} dx$$

$$= \int_0^1 \sqrt{1+x} dx + \int_0^1 \sqrt{x} dx$$

$$= \left[ \frac{2}{3}(1+x)^{\frac{3}{2}} \right]_0^1 \left[ \frac{2}{3}(x)^{\frac{3}{2}} \right]_0^1$$

$$= \frac{2}{3} \left[ (2)^{\frac{3}{2}} - 1 \right] + \frac{2}{3} [1]$$

$$= \frac{2}{3} (2)^{\frac{3}{2}}$$



$$= \frac{2.2\sqrt{2}}{3}$$

$$= \frac{4\sqrt{2}}{3}$$

**Question 30:**

$$\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$

**Solution 30:**

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$

Also let  $\sin x - \cos x = t \Rightarrow (\cos x + \sin x) dx = dt$

When  $x = 0$ ,  $t = -1$  and when  $x = \frac{\pi}{4}$ ,  $t = 0$

$$\Rightarrow (\sin x - \cos x)^2 = t^2$$

$$\Rightarrow \sin^2 + \cos^2 - 2 \sin x \cos x = t^2$$

$$\Rightarrow 1 - \sin 2x = t^2$$

$$\Rightarrow \sin 2x = 1 - t^2$$

$$\therefore I = \int_{-1}^0 \frac{dt}{9 + 16(1 - t^2)}$$

$$= \int_{-1}^0 \frac{dt}{9 + 16 - 16t^2}$$

$$= \int_{-1}^0 \frac{dt}{25 - 16t^2} = \int_{-1}^0 \frac{dt}{(5)^2 - (4t)^2}$$

$$= \frac{1}{4} \left[ \frac{1}{2(5)} \log \left| \frac{5+4t}{5-4t} \right| \right]_{-1}^0$$

$$= \frac{1}{40} \left[ \log(1) - \log \left| \frac{1}{9} \right| \right]$$

$$= \frac{1}{40} \log 9$$

**Question 31:**

$$\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$$

**Solution 31:**

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx = \int_0^{\frac{\pi}{2}} 2 \sin x \cos x \tan^{-1}(\sin x) dx$$

Also, let  $\sin x = t \Rightarrow \cos x \, dx = dt$

When  $x = 0$ ,  $t = 0$  and when  $x = \frac{\pi}{2}$ ,  $t = 1$

$$\Rightarrow I = 2 \int_0^1 t \tan^{-1}(t) \, dt \quad \dots (1)$$

Consider  $\int t \cdot \tan^{-1} t \, dt = \tan^{-1} t \cdot \int t \, dt - \int \left\{ \frac{d}{dt} (\tan^{-1} t) \int t \, dt \right\} dt$

$$= \tan^{-1} t \cdot \frac{t^2}{2} - \int \frac{1}{1+t^2} \cdot \frac{t^2}{2} dt$$

$$= \frac{t^2 \tan^{-1} t}{2} - \frac{1}{2} \int \frac{t^2 + 1 - 1}{1+t^2} dt$$

$$= \frac{t^2 \tan^{-1} t}{2} - \frac{1}{2} \int 1 \cdot dt + \frac{1}{2} \int \frac{1}{1+t^2} dt$$

$$= \frac{t^2 \tan^{-1} t}{2} - \frac{1}{2} t + \frac{1}{2} \tan^{-1} t$$

$$\Rightarrow \int_0^1 t \cdot \tan^{-1} t \, dt = \left[ \frac{t^2 \tan^{-1} t}{2} - \frac{t}{2} + \frac{1}{2} \tan^{-1} t \right]_0^1$$

$$= \frac{1}{2} \left[ \frac{\pi}{4} - 1 + \frac{\pi}{4} \right]$$

$$= \frac{1}{2} \left[ \frac{\pi}{2} - 1 \right] = \frac{\pi}{4} - \frac{1}{2}$$

From equation (1), we obtain

$$I = 2 \left[ \frac{\pi}{4} - \frac{1}{2} \right] = \frac{\pi}{2} - 1$$

### Question 32:

$$\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$$

### Solution 32:

$$\text{Let } \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx \quad \dots(1)$$

$$I = \int_0^{\pi} \left\{ \frac{(\pi-x) \tan(\pi-x)}{\sec(\pi-x) + \tan(\pi-x)} \right\} dx \quad \left( \int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$\Rightarrow I = \int_0^{\pi} \left\{ \frac{-(\pi-x) \tan x}{-(\sec x + \tan x)} \right\} dx$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi-x) \tan x}{\sec x + \tan x} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$\begin{aligned}
 2I &= \int_0^{\pi} \frac{\pi \tan x}{\sec x + \tan x} dx \\
 &= \int_0^{\pi} \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} dx \\
 &= \int_0^{\pi} \frac{\sin x + 1 - 1}{1 + \sin x} dx \\
 &= \int_0^{\pi} 1 \cdot dx - \int_0^{\pi} \frac{1}{1 + \sin x} dx \\
 &= \pi [x]_0^{\pi} - \int_0^{\pi} \frac{1 - \sin x}{\cos^2 x} dx \\
 &= \pi^2 - \pi \int_0^{\pi} (\sec^2 x - \tan x \sec x) dx \\
 &= \pi^2 - \pi [\tan x - \sec x]_0^{\pi} \\
 &= \pi^2 - \pi [\tan \pi - \sec \pi - \tan 0 + \sec 0] \\
 &= \pi^2 - \pi [0 - (-1) - 0 + 1] \\
 &= \pi^2 - 2\pi \\
 &= \pi(\pi - 2) \\
 &\Rightarrow I = \frac{\pi}{2}(\pi - 2)
 \end{aligned}$$

**Question 33:**

$$\int_1^4 [|x-1| + |x-2| + |x-3|] dx$$

**Solution 33:**

$$\text{Let } I = \int_1^4 [|x-1| + |x-2| + |x-3|] dx$$

$$\Rightarrow I = \int_1^4 |x-1| dx + \int_1^4 |x-2| dx + \int_1^4 |x-3| dx$$

$$I = I_1 + I_2 + I_3 \quad \dots(1)$$

$$\text{where, } I_1 = \int_1^4 |x-1| dx, I_2 = \int_1^4 |x-2| dx, \text{ and } I_3 = \int_1^4 |x-3| dx$$

$$I_1 = \int_1^4 |x-1| dx$$

$$(x-1) \geq 0 \text{ for } 1 \leq x \leq 4$$

$$\therefore I_1 = \int_1^4 (x-1) dx$$

$$\Rightarrow I_1 = \left[ \frac{x^2}{2} - x \right]_1^4$$

$$\Rightarrow I_1 = \left[ 8 - 4 - \frac{1}{2} + 1 \right] = \frac{9}{2} \quad \dots(2)$$

$$I_2 = \int_1^4 |x-2| dx$$

$x-2 \geq 0$  for  $2 \leq x \leq 4$  and  $x-2 \leq 0$  for  $1 \leq x \leq 2$

$$\therefore I_2 = \int_1^2 (2-x) dx + \int_2^4 (x-2) dx$$

$$\Rightarrow I_2 = \left[ 2x - \frac{x^2}{2} \right]_1^2 + \left[ \frac{x^2}{2} - 2x \right]_2^4$$

$$\Rightarrow I_2 = \left[ 4 - 2 - 2 + \frac{1}{2} \right] + [8 - 8 - 2 + 4]$$

$$\Rightarrow I_2 = \frac{1}{2} + 2 = \frac{5}{2} \quad \dots(3)$$

$$I_3 = \int_1^4 |x-3| dx$$

$x-3 \geq 0$  for  $3 \leq x \leq 4$  and  $x-3 \leq 0$  for  $1 \leq x \leq 3$

$$\therefore I_3 = \int_1^3 (3-x) dx + \int_3^4 (x-3) dx$$

$$\Rightarrow I_3 = \left[ 3x - \frac{x^2}{2} \right]_1^3 + \left[ \frac{x^2}{2} - 3x \right]_3^4$$

$$\Rightarrow I_3 = \left[ 9 - \frac{9}{2} - 3 + \frac{1}{2} \right] + \left[ 8 - 12 - \frac{9}{2} + 9 \right]$$

$$\Rightarrow I_3 = [6 - 4] + \left[ \frac{1}{2} \right] = \frac{5}{2} \quad \dots(4)$$

From equations (1), (2), (3), and (4), we obtain

$$I = \frac{9}{2} + \frac{5}{2} + \frac{5}{2} = \frac{19}{2}$$

### Question 34:

Prove  $\int_1^3 \frac{dx}{x^2(x+1)} = \frac{2}{3} + \log \frac{2}{3}$

### Solution 34:

Let  $I = \int_1^3 \frac{dx}{x^2(x+1)}$

Also, let  $\frac{1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

$$\Rightarrow 1 = Ax(x+1) + B(x+1) + C(x^2)$$

$$\Rightarrow 1 = Ax^2 + Ax + Bx + B + Cx^2$$

Equating the coefficients of  $x^2$ ,  $x$ , and constant term, we obtain

$$A + C = 0$$

$$A + B = 0$$

$$B = 1$$

On solving these equations, we obtain

$$A = -1, C = 1, \text{ and } B = 1$$

$$\therefore \frac{1}{x^2(x+1)} = \frac{-1}{x} + \frac{1}{x^2} + \frac{1}{(x+1)}$$

$$\Rightarrow I = \int_1^3 \left\{ -\frac{1}{x} + \frac{1}{x^2} + \frac{1}{(x+1)} \right\} dx$$

$$= \left[ -\log x - \frac{1}{x} + \log(x+1) \right]_1^3$$

$$= \left[ \log\left(\frac{x+1}{x}\right) - \frac{1}{x} \right]_1^3$$

$$= \log\left(\frac{4}{3}\right) - \frac{1}{3} - \log\left(\frac{2}{1}\right) + 1$$

$$= \log 4 - \log 3 - \log 2 + \frac{2}{3}$$

$$= \log 2 - \log 3 + \frac{2}{3}$$

$$= \log\left(\frac{2}{3}\right) + \frac{2}{3}$$

Hence, the given result is proved.

### Question 35:

Prove  $\int_0^4 xe^x dx = 1$

### Solution 35:

$$\text{Let } I = \int_0^4 xe^x dx$$

Integrating by parts, we obtain

$$I = x \int_0^4 e^x dx - \int_0^4 \left\{ \left( \frac{d}{dx}(x) \right) \int e^x dx \right\} dx$$

$$= [xe^x]_0^4 - \int_0^4 e^x dx$$

$$= [xe^x]_0^4 - [e^x]_0^4$$

$$= e - e + 1$$

$$= 1$$

Hence, the given result is proved.

**Question 36:**

Prove  $\int_{-1}^1 x^{17} \cos^4 x dx = 0$

**Solution 36:**

Let  $I = \int_{-1}^1 x^{17} \cos^4 x dx$

Also, let  $f(x) = x^{17} \cos^4 x$

$$\Rightarrow f(-x) = (-x)^{17} \cos^4(-x) = -x^{17} \cos^4 x = -f(x)$$

Therefore,  $f(x)$  is an odd function.

It is known that if  $f(x)$  is an odd function, then  $\int_{-a}^a f(x) dx = 0$

$$\therefore I = \int_{-1}^1 x^{17} \cos^4 x dx = 0$$

Hence, the given result is proved.

**Question 37:**

Prove  $\int_0^{\frac{\pi}{2}} \sin^3 x dx = \frac{2}{3}$

**Solution 37:**

Let  $I = \int_0^{\frac{\pi}{2}} \sin^3 x dx$

$$I = \int_0^{\frac{\pi}{2}} \sin^2 x \cdot \sin x dx$$

$$= \int_0^{\frac{\pi}{2}} (1 - \cos^2 x) \sin x dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x dx - \int_0^{\frac{\pi}{2}} \cos^2 x \cdot \sin x dx$$

$$= [-\cos x]_0^{\frac{\pi}{2}} + \left[ \frac{\cos^3 x}{3} \right]_0^{\frac{\pi}{2}}$$

$$= 1 + \frac{1}{3}[-1] = 1 - \frac{1}{3} = \frac{2}{3}$$

Hence, the given result is proved.

**Question 38:**

Prove  $\int_0^{\frac{\pi}{4}} 2 \tan^3 x dx = 1 - \log 2$

**Solution 38:**

Let  $I = \int_0^{\frac{\pi}{4}} 2 \tan^3 x dx$

$$\begin{aligned}
 I &= \int_0^{\frac{\pi}{4}} 2 \tan^2 x \tan x dx = 2 \int_0^{\frac{\pi}{4}} (\sec^2 x - 1) \tan x dx \\
 &= 2 \int_0^{\frac{\pi}{4}} \sec^2 x \tan x dx - 2 \int_0^{\frac{\pi}{4}} \tan x dx \\
 &= 2 \left[ \frac{\tan^2 x}{2} \right]_0^{\frac{\pi}{4}} + 2 [\log \cos x]_0^{\frac{\pi}{4}} \\
 &= 1 + 2 \left[ \log \cos \frac{\pi}{4} - \log \cos 0 \right] \\
 &= 1 + 2 \left[ \log \frac{1}{\sqrt{2}} - \log 1 \right] \\
 &= 1 - \log 2 - \log 1 = 1 - \log 2
 \end{aligned}$$

Hence, the given result is proved.

**Question 39:**

Prove  $\int_0^1 \sin^{-1} x dx = \frac{\pi}{2} - 1$

**Solution 39:**

Let  $\int_0^1 \sin^{-1} x dx$

$$\Rightarrow I = \int_0^1 \sin^{-1} x \cdot 1 \cdot dx$$

Integrating by parts, we obtain

$$\begin{aligned}
 I &= \left[ \sin^{-1} x \cdot x \right]_0^1 - \int_0^1 \frac{1}{\sqrt{1-x^2}} \cdot x dx \\
 &= \left[ x \sin^{-1} x \right]_0^1 + \frac{1}{2} \int_0^1 \frac{(-2x)}{\sqrt{1-x^2}} dx
 \end{aligned}$$

Let  $1 - x^2 = t \Rightarrow -2x dx = dt$

When  $x = 0$ ,  $t = 1$  and when  $x = 1$ ,  $t = 0$

$$\begin{aligned}
 I &= \left[ x \sin^{-1} x \right]_0^1 + \frac{1}{2} \int_0^1 \frac{dt}{\sqrt{t}} \\
 &= \left[ x \sin^{-1} x \right]_0^1 + \frac{1}{2} \left[ 2\sqrt{t} \right]_1^0 \\
 &= \sin^{-1}(1) + \left[ -\sqrt{1} \right] \\
 &= \frac{\pi}{2} - 1
 \end{aligned}$$

Hence, the given result is proved.

**Question 40:**

Evaluate  $\int_0^1 e^{2-3x} dx$  as a limit of a sum.

**Solution 40:**

$$\text{Let } I = \int_0^1 e^{2-3x} dx$$

It is known that,

$$\int_a^b f(x) dx = (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} [f(a) + f(a+h) + \dots + f(a+(n-1)h)]$$

$$\text{where, } h = \frac{b-a}{n}$$

$$\text{Here, } a = 0, b = 1, \text{ and } f(x) = e^{2-3x}$$

$$\Rightarrow h = \frac{1-0}{n} = \frac{1}{n}$$

$$\therefore \int_0^1 e^{2-3x} dx = (1-0) \lim_{n \rightarrow \infty} \frac{1}{n} [f(0) + f(0+h) + \dots + f(0+(n-1)h)]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} [e^2 + e^{2-3x} + \dots + e^{2-3(n-1)h}]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} [e^2 \{1 + e^{-3h} + e^{-6h} + e^{-9h} + \dots + e^{-3(n-1)h}\}]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left[ e^2 \left\{ \frac{1 - (e^{-3h})^n}{1 - (e^{-3h})} \right\} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left[ e^2 \left\{ \frac{1 - e^{-\frac{3 \times n}{n}}}{1 - e^{-\frac{3}{n}}} \right\} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left[ \frac{e^2 (1 - e^{-3})}{1 - e^{-\frac{3}{n}}} \right]$$

$$= e^2 (e^{-3} - 1) \lim_{n \rightarrow \infty} \frac{1}{n} \left[ \frac{1}{e^{-\frac{3}{n}} - 1} \right]$$

$$= e^2 (e^{-3} - 1) \lim_{n \rightarrow \infty} \left( -\frac{1}{3} \right) \left[ \frac{-\frac{3}{n}}{e^{-\frac{3}{n}} - 1} \right]$$

$$= \frac{e^2 (e^{-3} - 1)}{3} \lim_{n \rightarrow \infty} \left[ \frac{-\frac{3}{n}}{e^{-\frac{3}{n}} - 1} \right]$$

$$= \frac{-e^2 (e^{-3} - 1)}{3} (1) \quad \left[ \lim_{n \rightarrow \infty} \frac{x}{e^x - 1} \right]$$



$$= \frac{-e^{-1} + e^2}{3}$$

$$= \frac{1}{3} \left( e^2 - \frac{1}{e} \right)$$

**Question 41:**

$\int \frac{dx}{e^x + e^{-x}}$  is equal to

- A.  $\tan^{-1}(e^x) + C$   
 B.  $\tan^{-1}(e^{-x}) + C$   
 C.  $\log(e^x - e^{-x}) + C$   
 D.  $\log(e^x + e^{-x}) + C$

**Solution 41:**

$$\text{Let } I = \int \frac{dx}{e^x + e^{-x}} = \int \frac{e^x}{e^{2x} + 1} dx$$

Also, let  $e^x = t \Rightarrow e^x dx = dt$

$$\therefore I = \int \frac{dt}{1+t^2}$$

$$= \tan^{-1} t + C$$

$$= \tan^{-1}(e^x) + C$$

Hence, the correct Answer is A.

**Question 42:**

$\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$  is

- A.  $\frac{-1}{\sin x + \cos x} + C$   
 B.  $\log|\sin x + \cos x| + C$   
 C.  $\log|\sin x - \cos x| + C$   
 D.  $\frac{1}{(\sin x + \cos x)^2} + C$       equal to

**Solution 42:**

$$\text{Let } I = \int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$$

$$\begin{aligned}
 I &= \int \frac{\cos^2 x - \sin^2 x}{(\sin x + \cos x)^2} dx \\
 &= \int \frac{(\cos x + \sin x)(\cos x - \sin x)}{(\sin x + \cos x)^2} dx \\
 &= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx
 \end{aligned}$$

$$\text{Let } \cos x + \sin x = t \Rightarrow (\cos x - \sin x) dx = dt$$

$$\therefore I = \int \frac{dt}{t}$$

$$= \log|t| + C$$

$$= \log|\cos x + \sin x| + C$$

Hence, the correct Answer is B.

### Question 43:

If  $f(a + b - x) = f(x)$ , then  $\int_a^b xf(x) dx$  is equal to

A.  $\frac{a+b}{2} \int_a^b f(b-x) dx$

B.  $\frac{a+b}{2} \int_a^b f(b+x) dx$

C.  $\frac{b-a}{2} \int_a^b f(x) dx$

D.  $\frac{a+b}{2} \int_a^b f(x) dx$

### Solution 43:

$$\text{Let } I = \int_a^b xf(x) dx \quad \dots (1)$$

$$I = \int_a^b (a+b-x)f(a+b-x) dx \quad \left( \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right)$$

$$\Rightarrow I = \int_a^b (a+b-x)f(x) dx$$

$$\Rightarrow I = (a+b) \int_a^b f(x) dx - I \quad [\text{using (1)}]$$

$$\Rightarrow I + I = (a+b) \int_a^b f(x) dx$$

$$\Rightarrow 2I = (a+b) \int_a^b f(x) dx$$

$$\Rightarrow I = \left( \frac{a+b}{2} \right) \int_a^b f(x) dx$$

Hence, the correct Answer is D.

**Question 44:**

The value of  $\int_0^1 \tan^{-1} \left( \frac{2x-1}{1+x-x^2} \right) dx$  is

- A. 1
- B. 0
- C. -1
- D.  $\frac{\pi}{4}$

**Solution 44:**

$$\text{Let } I = \int_0^1 \tan^{-1} \left( \frac{2x-1}{1+x-x^2} \right) dx$$

$$\Rightarrow I = \int_0^1 \tan^{-1} \left( \frac{x-(1-x)}{1+x(1-x)} \right) dx$$

$$\Rightarrow I = \int_0^1 [\tan^{-1} x - \tan^{-1} (1-x)] dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^1 [\tan^{-1} (1-x) - \tan^{-1} (1-1+x)] dx$$

$$\Rightarrow I = \int_0^1 [\tan^{-1} (1-x) - \tan^{-1} (x)] dx$$

$$\Rightarrow I = \int_0^1 [\tan^{-1} (1-x) - \tan^{-1} (x)] dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$\Rightarrow 2I = \int_0^1 (\tan^{-1} x - \tan^{-1} (1-x) - \tan^{-1} (1-x) - \tan^{-1} x) dx$$

$$\Rightarrow 2I = 0$$

$$\Rightarrow I = 0$$

Hence, the correct Answer is B.