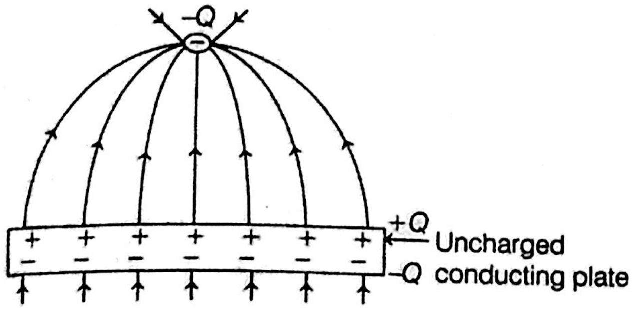
electrostatics-1(solutions)

DCAM classes

Dynamic Classes for Academic Mastery

Explanations

1.



(1)

2. Refer to diagram on page 4 (Electric Field Lines). (1)

3. Refer to diagram on page 4 (Electric Field Lines). (1)

4. The electrostatic field lines do not form closed loop because no electric field lines exist inside the charged body. (1)

5. According to the question, both the balls have same charge q. Let the balls be separated by a distance r. Hence, according to Coulomb's law, if F and F' are the force of attraction between balls in air and in medium respectively.

Then, $F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$

In medium, $F' = \frac{1}{4\pi K\varepsilon_0}\frac{q^2}{r^2}$ $\therefore F' = F/K$

where, K is dielectric constant of material and $K > 1$ for insulators, hence the force is reduced, when a plastic sheet is inserted. (1)

6. At the intersection point, if electric field lines cross each other, then there would be two directions of electric field which is not possible, so lines of forces never cross each other. (1)

7. As, electric field inside a conductor is always zero. The electric lines of forces exert lateral pressure on each other which leads to repulsion between like charges. Thus, in order to stabilize spacing, the electric field lines are normal to the surface. (1)

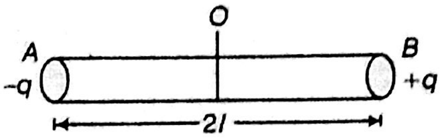
8. As per the condition given in question, *two conclusions that can be drawn are as follows*

(i) The two point charges (q_1 and q_2) should be of opposite nature. (1/2)

(ii) Magnitude of charge q_1 must be greater than magnitude of charge q_2. (1/2)

9. Electric dipole moment of an electric dipole is equal to the product of its either charge and the length of the electric dipole.

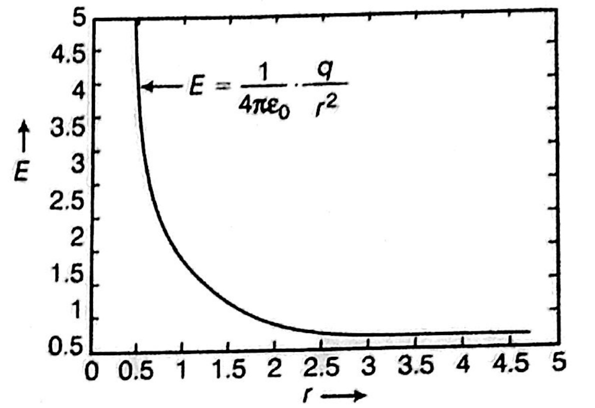
It is denoted by **p**. Its SI unit is coulomb-metre.



$$|\mathbf{p}| = q \times 2l$$

It is a vector quantity and its direction is from negative charge to positive charge. (1)

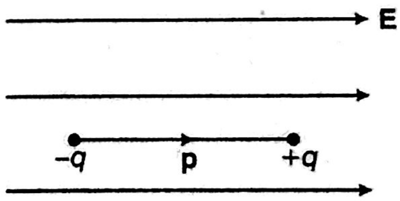
10. *The plot showing the variation of electric field with distance r due to a point charge q is shown as below*



(1)

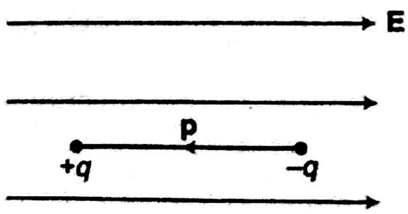
11. Force on positive charge is always in the direction of electric field. So, proton being positive will tend to move along the X-axis in the direction of a uniform electric field. (1)

12. (i) For stable equilibrium, the angle between **p** and **E** is 0° i.e. it should be placed parallel to electric field.



(1/2)

(ii) For unstable equilibrium, the angle between **p** and **E** is 180° i.e. it should be placed antiparallel to electric field.



(1/2)

13. Two point charges system is taken from air to water keeping other variables (e.g. distance, magnitude of charge) unchanged. So, only factor which may affect the interacting force is dielectric constant of medium.

Force acting between two point charges

$$F = \frac{1}{4\pi \varepsilon_0 K}\frac{q_1 q_2}{r^2}$$

or $$F \propto \frac{1}{K}$$

$\Rightarrow$ $$\frac{F_{air}}{F_{medium}} = K$$

$\Rightarrow$ $$\frac{8}{F_{water}} = 80$$

$\Rightarrow$ $$F_{water} = \frac{8}{80}$$

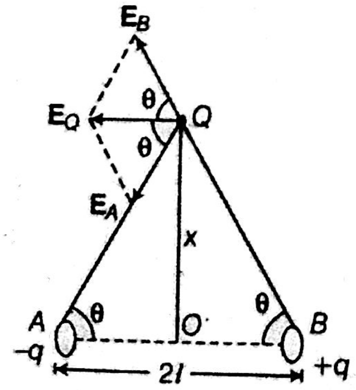
$\Rightarrow$ $$F_{water} = \frac{1}{10}\text{ N}$$ **(1)**

14. Path d is followed by electric field lines because electric field intensity inside the metallic sphere will be zero, therefore, no electric lines of force exist inside the sphere. Also electric field lines are always perpendicular to the surface of the conductor. **(1)**

15. Right, because mutual force acting between two point charges is proportional to the product of magnitude of charges and inversely proportional to the square of the distance between them i.e. independent of the other charges. **(1)**

16. (i) **Electric field at a point on the equatorial line of an electric dipole.**

Consider an electric dipole consisting of two point charges $+q$ and $-q$ separated by a small distance $AB = 2l$ with centre at O and dipole moment, $p = q(2l)$ as shown in the figure.



Resultant electric field intensity at the point Q,

$$\mathbf{E}_Q = \mathbf{E}_A + \mathbf{E}_B$$

Here, $$E_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(x^2 + l^2)}$$

and $$E_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(x^2 + l^2)}$$ **(1)**

On resolving $\mathbf{E}_A$ and $\mathbf{E}_B$ into two rectangular components, the vectors $E_A \sin\theta$ and $E_B \sin\theta$ are equal in magnitude and opposite to each other and hence, cancel out.

The vectors $E_A \cos\theta$ and $E_B \cos\theta$ are acting along the same direction and hence, add up.

$$\therefore E_Q = E_A\cos\theta + E_B\cos\theta$$

$$= 2E_A\cos\theta \qquad [\because E_A = E_B]$$

$$= \frac{2}{4\pi\varepsilon_0}\cdot\frac{q}{(x^2 + l^2)}\cdot\frac{l}{(x^2 + l^2)^{1/2}}$$

$$\left[\because \cos\theta = \frac{l}{(x^2 + l^2)^{1/2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2ql}{(x^2 + l^2)^{3/2}}$$

But, the dipole moment $|\mathbf{p}| = q \times 2l$

$$\therefore \quad E_Q = \frac{1}{4\pi\varepsilon_0}\cdot\frac{|\mathbf{p}|}{(x^2 + l^2)^{3/2}}$$

The direction of $\mathbf{E}$ is along E_Q that is parallel to BA, i.e. opposite to AB. In vector form, we can rewrite as, $$\mathbf{E}_Q = \frac{-\mathbf{p}}{4\pi\varepsilon_0(x^2 + l^2)^{3/2}}$$ **(1)**

17. Given,

Length $(2a) = 4\text{ cm} = 4 \times 10^{-2}\text{ m}$

Angle, $\theta = 60°$

Torque, $\tau = 4\sqrt{3}\text{ Nm}$

Charge, $Q = 8 \times 10^{-9}\text{C}$

We know that, $\tau = Q(2a)\,E\sin\theta$

Electric field, $$E = \frac{\tau}{Q(2a)\sin\theta}$$

$$= \frac{4\sqrt{3}}{8 \times 10^{-9} \times 4 \times 10^{-2} \times \sin 60°}$$ **(1)**

$$E = 2.5 \times 10^{10}\text{NC}^{-1}$$

$\therefore$ Potential energy,

$$U = -pE\cos\theta$$

$$= -Q(2a)\,E\cos\theta$$

$$U = -8 \times 10^{-9} \times 4 \times 10^{-2} \times 2.5 \times 10^{10}\cos 60°$$

$$= -4\text{ J}$$ **(1)**

18. 16 J, Refer to Sol. 17 **(2)**

19. 6 J, Refer to Sol. 17 **(2)**

20. (i) Work done in rotating the dipole, $W = \int_{\theta_1}^{\theta_2}\tau\,d\theta$

If the dipole is turned from direction parallel to electric field to direction opposite to electric field, then angle θ will change from 0 to π.

$$\therefore \quad W = \int_0^{\pi} pE\sin\theta\,d\theta = pE\,[-\cos\theta]_0^{\pi} = 2pE$$ **(1)**

(ii) We know that, $\tau = pE\sin\theta$

If $\theta = \pi/2$, then τ is maximum

i.e. $\tau = pE\sin\frac{\pi}{2}$

$\Rightarrow \quad \tau = pE$ (maximum) **(1)**

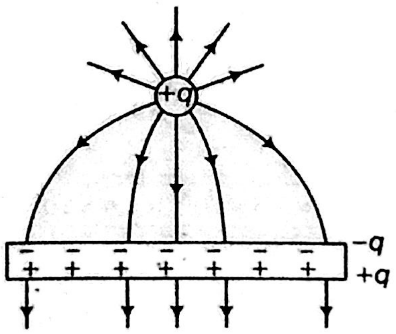
21. According to question, the charge on inner surface $= -Q$

According to question, the charge on outer surface $= +Q$

Electric field at point P_1 is given by

$$E = Q/4\pi\varepsilon_0 r_1^2$$ **(2)**

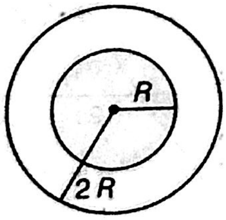
22. Equal charge of opposite nature induces in the surface of conductor nearer to the source charge.



Electric lines of forces should fall normally, i.e. at 90° on the conducting plate. **(2)**

23. Surface charge density, $\sigma = \frac{Q}{4\pi R^2}$

According to the question, surface charge density, σ = constant



Let Q_1 and Q_2 are two charges

Hence,

Charge, $Q_1 = 4\pi R^2\sigma$ ...(i)

Charge, $Q_2 = 4\pi(2R)^2\sigma$ ...(ii)

On dividing Eq. (i) with Eq. (ii), we get

$$\therefore \quad \frac{Q_1}{Q_2} = \frac{4\pi R^2\sigma}{4\pi(2R)^2\sigma} = \frac{1}{4}$$ **(2)**

24. According to question, for unstable equilibrium, the angle between **p** and **E** is $\theta_1 = 180°$

Finally, for stable equilibrium, $\theta_2 = 0°$ **(1/2)**

Required work done

$W = pE(\cos\theta_1 - \cos\theta_2)$ **(1/2)**

$= 3\times10^{-8}\times10^{3}(\cos180° - \cos0°)$

$[\because \cos180° = -1$ and $\cos0° = +1]$

$W = -6\times10^{-5}$ J **(1)**

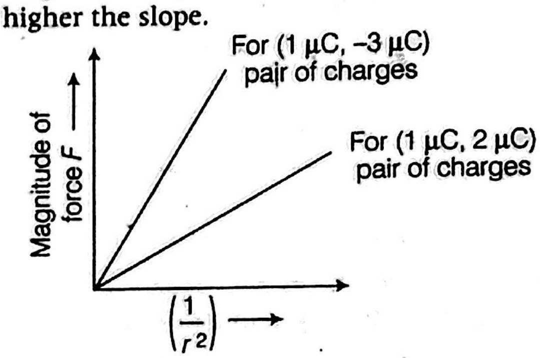
25. According to Coulomb's law, the magnitude of force acting between two stationary point charges is given by $F = \left(\frac{q_1 q_2}{4\pi\varepsilon_0}\right)\left(\frac{1}{r^2}\right)$

For given $q_1 q_2$, $\quad F \propto \left(\frac{1}{r^2}\right)$

The slope of F *versus* $\frac{1}{r^2}$, graph depends on $q_1 q_2$.

Magnitude of $q_1 q_2$ is higher for second pair.

$\therefore$ Slope of F *versus* $\frac{1}{r^2}$ graph corresponding to second pair (1μC, − 3μC) is greater. Higher the magnitude of product of charges q_1 and q_2, higher the slope. **(1)**



(1)

26. When two identical conducting charged spheres are brought in contact, then redistribution of charge takes place, i.e. the charge is equally divided on both the spheres.

When C and A are placed in contact, charge of A equally divides in two spheres. Therefore, charge on each A and $C = +2Q$. **(1)**

Now, C is placed in contact with B, then charge on each A and C becomes

$$\frac{2Q + (-10Q)}{2} = -4Q$$

When A and B are placed in contact, then charge on each A and B becomes

$$\frac{2Q + (-4Q)}{2} = -Q$$ **(1)**

27. For stable equilibrium, the angle between **p** and **E** $\theta_1 = 0°$.

For unstable equilibrium, $\theta_2 = 180°$. **(1)**

Work done in rotating the dipole from angle θ_1 to θ_2

$W = pE(\cos\theta_1 - \cos\theta_2) = pE(\cos0° - \cos180°)$

$W = 2pE$ **(1)**

28. Electric field intensity, $E = 10^6$ NC^{-1}

Work done, $W = 2\times10^{-23}$ J

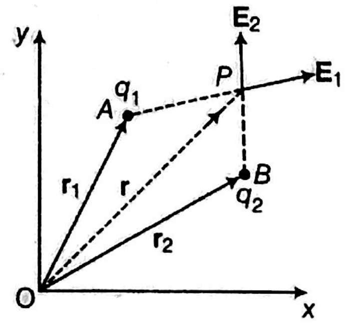
Work done in rotating the dipole from stable equilibrium position to unstable equilibrium position.

$$W = pE(\cos\theta_1 - \cos\theta_2)$$
$$W = pE\,(\cos 0° - \cos 180°) = 2pE \quad \textbf{(1)}$$

Magnitude of dipole moment is

$$\therefore \quad p = \frac{W}{2E}$$
$$= \frac{2\times10^{-23}}{2\times10^{6}} = 10^{-29}\ \text{C-m} \quad \textbf{(1)}$$

29. Let two point charges q_1 and q_2 are situated at points A and B have position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$.



$$\because \quad \mathbf{AP} = \mathbf{r} - \mathbf{r}_1$$
and $$\mathbf{BP} = \mathbf{r} - \mathbf{r}_2$$

Electric field intensity at point P due to q_1,

$$\mathbf{E}_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{|\mathbf{AP}|^3}\,\mathbf{AP}$$

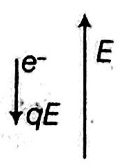
Similarly, $$\mathbf{E}_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{|\mathbf{BP}|^3}\,\mathbf{BP} \quad \textbf{(1)}$$

$\therefore$ Net electric field intensity at point P,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{|\mathbf{r}-\mathbf{r}_1|^3}(\mathbf{r}-\mathbf{r}_1) + \frac{q_2}{|\mathbf{r}-\mathbf{r}_2|^3}(\mathbf{r}-\mathbf{r}_2)\right] \quad \textbf{(1)}$$

30. (a) An electron falls through distance 1.5 cm, if electric field is 2×10^4 N/C.

So, net force on electron



$$F = q_e E$$
$$m_e a = q_e E$$
$$a = \frac{q_e E}{m_e} \quad [\because F = ma]$$

where, $q_e = 1.6\times10^{-19}$ C,

$m_e = 9.1\times10^{-31}$ kg

and $E = 2\times10^4$ N/C

So, $$a = \frac{1.6\times10^{-19}\times2\times10^4}{9.1\times10^{-31}}$$
$$= 3.5\times10^{15}\ \text{m/s}^2$$

As we know, $s = ut + \frac{1}{2}at^2$

So, $$1.5\times10^{-2} = \frac{0\times t + 3.5\times10^{15}t^2}{2}$$
$$t = \sqrt{\frac{2\times1.5\times10^{-2}}{3.5\times10^{15}}}$$
$$= \sqrt{8.57\times10^{-8}} = 292\,\text{ns} \quad \textbf{(1½)}$$

(b) Similarly time of fall of proton if direction of field is reversed

$$t_p = \sqrt{\frac{2s}{a_p}} = \sqrt{\frac{2sm_p}{q_pE}} \quad \left(\because a_p = \frac{q_pE}{m_p}\right)$$

where, $m_p = 1.6\times10^{-27}$ kg, $q_p = 1.6\times10^{-19}$ C

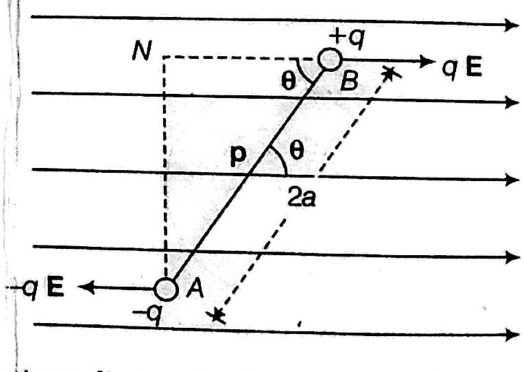
and $s = 1.5$ cm

$$t_p = \sqrt{\frac{2\times1.5\times10^{-2}\times1.6\times10^{-27}}{1.6\times10^{-19}\times2\times10^4}}$$
$$= \sqrt{1.5\times10^{-14}}$$
$$t_p = 1.22\times10^{-7}\ \text{s} \quad \textbf{(1½)}$$

31. (i) Refer to Sol. 16 **(1½)**

(ii) Refer to Sol. 12 (i) and (ii) **(1½)**

32. (i) Dipole in a uniform electric field



According to the figure, if we consider an electric dipole consisting of charges $-q$ and $+q$ and of length $2a$ placed in a uniform electric field $\mathbf{E}$ making an angle θ with electric field, then force exerted on charge $-q$ at $A = -q\,\mathbf{E}$ (opposite to $\mathbf{E}$) **(1)**

Force exerted on charge $+q$ at $A = q\,\mathbf{E}$ (along $\mathbf{E}$)

Hence, the net translating force on a dipole in a uniform electric field is zero. But the two equal and opposite forces act at different points and form couple which exerts a torque τ.

τ = Force × Perpendicular distance between the two forces

$$\tau = qE(AN) = qE(2a\sin\theta)$$

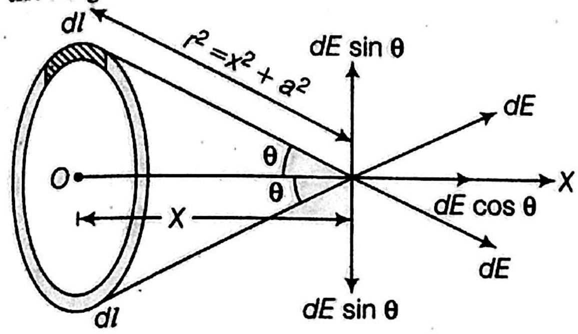
$$\tau = q(2a)E\sin\theta$$

$$\Rightarrow \quad \tau = pE\sin\theta$$

$$\Rightarrow \quad \tau = \mathbf{p} \times \mathbf{E} \quad \textbf{(1)}$$

(ii) When the dipole is placed in a non-uniform electric field, it experiences a net force and torque. **(1)**

33. According to question, suppose that the ring is placed with its plane perpendicular to the X-axis as shown in figure. Consider small element dl of the ring.



(1)

As the total charge q is uniformly distributed so, the charge dq on element dl is $dq = \frac{q}{2\pi a} \cdot dl$

$$= \frac{kq}{2\pi a}\frac{dl}{r^2}\cos\theta = dE\cos\theta \quad \left(\text{where, } \cos\theta = \frac{x}{r}\right)$$

Since, only the axial component gives the net **E** at point P due to charge on ring.

$$\text{So, } \int_0^E dE = \int_0^{2\pi a} dE\cos\theta$$

$$= \int_0^{2\pi a} \frac{kq}{2\pi a} \cdot \frac{dl}{r^2} \times \frac{x}{r}$$

$$= \frac{kqx}{2\pi a}\frac{1}{r^3}\int_0^{2\pi a} dl$$

$$= \frac{kq}{2\pi a} \cdot \frac{1}{r^3}[l]_0^{2\pi a}$$

$$= \frac{kqx}{2\pi a} \cdot \frac{1}{(x^2 + a^2)^{3/2}} \cdot 2\pi a \quad [\because r^2 = x^2 + a^2]$$

$$\text{So, } \quad E = \frac{kqx}{(x^2 + a^2)^{3/2}} \quad \textbf{(1)}$$

Now, for points at large distances from the ring $x >> a$,

$$\therefore \quad E = \frac{kq}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2} \quad \textbf{(1)}$$

This is same as the field due to a point charge indicating that for far-off axial point, the charged ring behaves as a point charge. **(2)**

34. Refer to Sol. 32 (i)

Pairs of perpendicular vectors

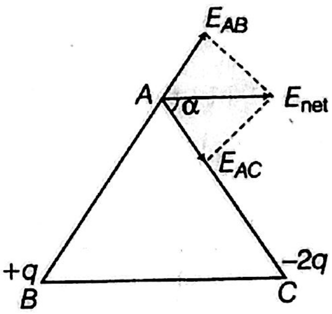
(a) $(\tau, \mathbf{p})$

(b) $(\tau, \mathbf{E})$ **(1)**

35. (i) The magnitude,

$$|\mathbf{E}_{AB}| = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{a^2} = E$$

$$|\mathbf{E}_{AC}| = \frac{1}{4\pi\varepsilon_0} \times \frac{2q}{a^2} = 2E \quad \textbf{(1)}$$



$$E_{net} = \sqrt{E_{AB}^2 + E_{AC}^2 + 2E_{AB}E_{AC}\cos\theta}$$

$$= \sqrt{(2E)^2 + E^2 + 2 \times 2E \times E \times \left(-\frac{1}{2}\right)}$$

$$= \sqrt{4E^2 + E^2 - 2E^2} = E\sqrt{3} \quad \text{...(i)}$$

We know that,

$$E = q/4\pi\varepsilon_0 a^2$$

$$\text{So, } \quad E_{net} = q\sqrt{3}/4\pi\varepsilon_0 a^2 \quad \textbf{(1)}$$

(ii) Direction of resultant electric field at vertex,

$$\tan\alpha = \frac{E_{AB}\sin 120°}{E_{AC} + E_{AB}\cos 120°} = \frac{E \times \sqrt{3}/2}{2E + E \times (-1/2)}$$

$$\tan\alpha = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\alpha = 30° \quad \text{(with side } AC) \quad \textbf{(1)}$$

36. For electric dipole moment Refer to Sol. 9. **(1)**

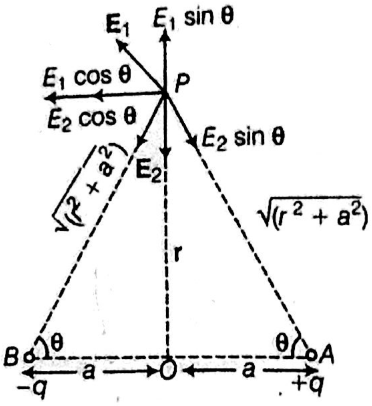
For derivation of E

Consider an electric dipole AB consists of two charges $+q$ and $-q$ separated by a distance $2a$. We have to find electric field at point P on equipotential line separated by a distance r. **(1)**

Electric field at point P due to charge $+q$

$$\mathbf{E}_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{[\sqrt{(r^2+a^2)}]^2} = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{(r^2+a^2)}$$

Along AP,



Electric field at point P due to charge $-q$

$$\mathbf{E}_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2+a^2} \text{ along } PB$$

On resolving $\mathbf{E}_1$ and $\mathbf{E}_2$ into rectangular components, we get resultant electric field at point P.

$$E = E_1 \cos\theta + E_2 \cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{q}{(r^2+a^2)} \cos\theta + \frac{1}{4\pi\varepsilon_0} \times \frac{q}{(r^2+a^2)} \cos\theta$$

$$= 2 \times \frac{1}{4\pi\varepsilon_0} \times \frac{q}{(r^2+a^2)} \times \frac{a}{\sqrt{(r^2+a^2)}}$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{q2a}{(r^2+a^2)^{3/2}} \qquad [\text{But } q \times 2a = p]$$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \times \frac{p}{(r^2+a^2)^{3/2}}$$

If $r >> a$, then $E = \frac{1}{4\pi\varepsilon_0} \times \frac{p}{r^3}$ **(1)**

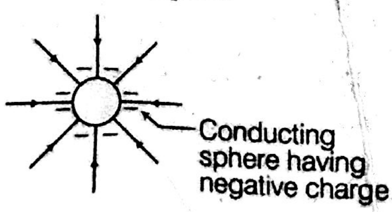
37. Refer to Sol. 32 (i) **(2)**

Conditions

(i) When $\theta = 0$; $\tau = 0$, then $\mathbf{p}$ and $\mathbf{E}$ are parallel and the dipole is in a position of stable equilibrium. **(1/2)**

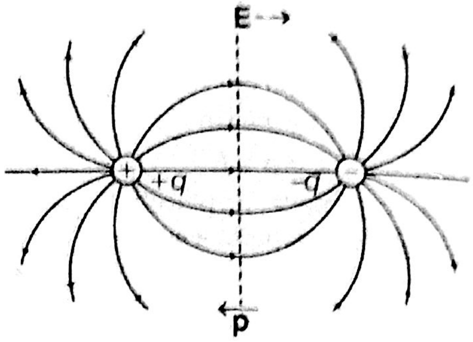
(ii) When $\theta = 180°$, $\tau = 0$, then $\mathbf{p}$ and $\mathbf{E}$ are anti-parallel and the dipole is in a position of unstable equilibrium. **(1/2)**

38. (i) Electric field lines due to a conducting sphere are shown in figure.



(1½)

(ii) Electric field lines due to an electric dipole are shown in figure.

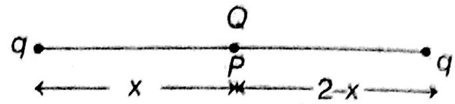


(1½)

39. (a) Refer to Sol. 16 **(2)**

(b) Let P be the point at which the system of charges as shown in the figure below is in equilibrium, then **(1)**

$$F(x) = F(2-x)$$



$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ}{x^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ}{(2-x)^2}$$

(from figure)

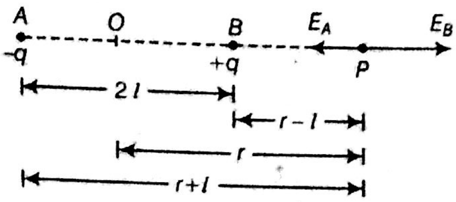
$$\Rightarrow \quad \frac{1}{x^2} = \frac{1}{(2-x)^2}$$

$$\Rightarrow \quad x = (2-x) \Rightarrow x = 1$$

Thus, the charge Q should be placed at the centre of line joining two given charges. Also the two given charges are identical, i.e. having same nature, so the third charge could be of any nature (positive or negative), as the forces on it at the centre are equal and opposite. **(2)**

40. (i) **Electric field due to dipole at axial point**

We have to calculate the field intensity E at a point P on the axial line of the dipole at distance $OP = r$ from the centre O of the dipole.



Resultant electric field intensity at the point P is

$$E_P = E_A + E_B$$

The vectors E_A and E_B are collinear and opposite.

$$\therefore \quad E_P = E_B - E_A$$

Here, $E_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(r+l)^2}$

$$E_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(r-l)^2}$$

$$\therefore \quad E_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(r-l)^2} - \frac{q}{(r+l)^2}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{4q \times l}{(r^2 - l^2)^2}$$

$$\therefore \quad E_P = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{(r^2 - l^2)^2}$$

If the length of dipole is short i.e. $2l << r$, then

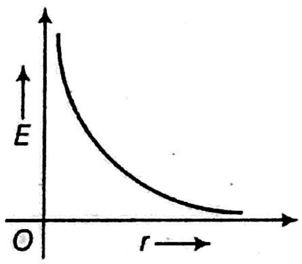
$$E_P = \frac{2p}{4\pi\varepsilon_0 \cdot r^3}$$

The direction of E_P is along BP produced.

So, $E_P \propto \frac{1}{r^3}$ **(2)**

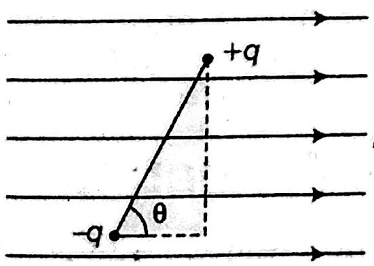
(ii) $E \propto \frac{1}{r^3}$. As r increases, E will sharply decrease.

The shape of the graph will be as given in the figure. **(1)**



(iii) When the dipole were kept in a uniform electric field E_0. The torque acting on dipole,

$$\tau = \mathbf{p} \times \mathbf{E} = pE \sin\theta$$



(a) If $\theta = 0°$, then $\tau = 0$, $\mathbf{p} \parallel \mathbf{E}$ **(1)**

For diagram Refer to Sol. 12 on page 9.

(b) If $\theta = 180°$, then $\tau = 0$, $\mathbf{p} \parallel -\mathbf{E}$ **(1)**

For diagrams Refer to Sol. 12 on page 9.

41. (i) $\tau = pE\sin\theta$

In vector notation, $\tau = \mathbf{p} \times \mathbf{E}$

SI unit of torque is newton-metre (N-m) and its dimensional formula is $[ML^2T^{-2}]$. Torque is always directed in plane perpendicular to the plane of dipole movement and electric field.

Case 1 If $\theta = 0°$, then $\tau = 0$

The dipole is in **stable equilibrium.**

Case 2 If $\theta = 90°$, then $\tau = pE$ (maximum value)

The torque acting on dipole will be **maximum.**

Case 3 If $\theta = 180°$, then $\tau = 0$ **(2)**

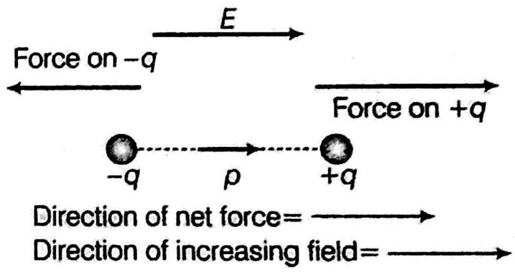
The dipole is in **unstable equilibrium.**

(ii) If the field is non-uniform, there would be a net force on the dipole in addition to the torque and the resulting motion would be a combination of translation and rotation.

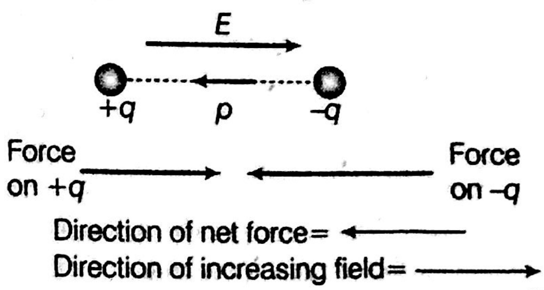
$$\tau = \mathbf{p} \times \mathbf{E}(\mathbf{r})$$

Net torque acts on the dipole depending on the location, where $\mathbf{r}$ is the position vector of the centre of the dipole. **(1)**

(iii) (a) $\mathbf{E}$ is increasing parallel to $\mathbf{p}$, then $\theta = 0°$. So, torque becomes zero but the net force on the dipole will be in the direction of increasing electric field and hence it will have linear motion along the dipole moment.



(b) $\mathbf{E}$ is increasing anti-parallel to $\mathbf{p}$. So, the torque still remains zero but the net force on the dipole will be in the direction of increasing electric field which is opposite to the dipole moment, hence it will have linear motion opposite to the dipole moment.

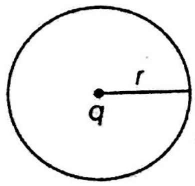


(2)

Electrostatics(solutions)

Explanations

1. According to question, electric flux (ϕ) due to a point charge enclosed by a spherical Gaussian surface is given by



$$\phi = \mathbf{E} \cdot \mathbf{A}$$

$$\phi = \frac{kq}{r^2} \cdot 4\pi r^2 = kq \cdot 4\pi$$

$$\left(\because E = \frac{kq}{r^2} \text{ and } A = 4\pi r^2\right)$$

So, there is no effect of change in radius on the electric flux. **(1)**

2. Since, according to the Gauss' law of electrostatics, electric flux through any closed surface is given by,

$$\phi_E = \oint \mathbf{E} \cdot d\mathbf{S} = q/\varepsilon_0 \quad \text{...(i)}$$

where, $\mathbf{E}$ = electrostatic field

q = total charge enclosed by the surface

ε_0 = absolute electric permittivity of free space

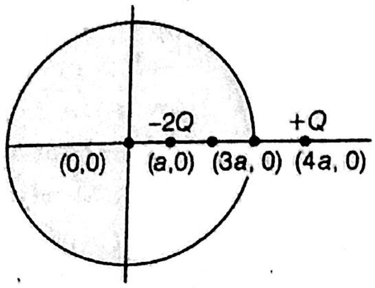
So, in the given case, cube encloses an electric dipole. Therefore, the total charge enclosed by the cube is zero. i.e. $q = 0$.

Therefore, from Eq. (i), we have

$$\phi_E = q/\varepsilon_0 = 0$$

i.e. electric flux is zero. **(1)**

3. Gauss' theorem states that the total electric flux linked with closed surface S is



$$\phi_E = \oint \mathbf{E} \cdot d\mathbf{S} = q/\varepsilon_0$$

where, q is the total charge enclosed by the closed Gaussian (imaginary) surface.

Charge enclosed by the sphere $= -2Q$

Therefore, $\phi = 2Q/\varepsilon_0$ (inwards) **(1)**

4. By Gauss' theorem, total electric flux linked with a closed surface is given by $\phi = q/\varepsilon_0$

where, q is the total charge enclosed by the closed surface.

$\therefore$ Total electric flux linked with cube, $\phi = q/\varepsilon_0$

As charge is at centre, therefore, electric flux is symmetrically distributed through all 6 faces.

Flux linked with each face $= \frac{1}{6}\phi = \frac{1}{6} \times \frac{q}{\varepsilon_0} = \frac{q}{6\varepsilon_0}$ **(1)**

5. Electric flux through the closed surface S is

$$\phi_S = \frac{\Sigma q}{\varepsilon_0} = \frac{+2q - q}{\varepsilon_0}$$

$$= \frac{q}{\varepsilon_0} \Rightarrow \phi_S = \frac{q}{\varepsilon_0}$$

Charge $+3q$ is outside the closed surface S, therefore, it would not be taken into consideration in applying Gauss' theorem. **(1)**

6. According to the question, $\mathbf{E} = 3 \times 10^3\, \hat{\mathbf{i}}\, \text{NC}^{-1}$.

Side of square $(S) = 10\,\text{cm} = 0.1\,\text{m}$.

Area of square $(A) = (\text{side})^2 = (0.1)^2 = 1 \times 10^{-2}\,\text{m}^2$

Hence, electric flux through the square,

$$\phi = E.A = (3 \times 10^3) \cdot 10^{-2} = 30\,\text{Nm}^2\text{C}^{-1}$$ **(1)**

7. Given, electric field intensity

$$\mathbf{E} = 5 \times 10^3 \hat{\mathbf{i}}\, \text{NC}^{-1}$$

Magnitude of electric field intensity

$$|\mathbf{E}| = 5 \times 10^3\, \text{NC}^{-1}$$

Side of square, $S = 10\,\text{cm} = 0.1\,\text{m}$

Area of square, $A = (0.1)^2 = 0.01\,\text{m}^2$

The plane of the square is parallel to the YZ-plane. Hence, the angle between the unit vector normal to the plane and electric field is zero.

i.e., $\theta = 0°$ **(1)**

$\therefore$ Flux through the plane,

$$\phi = |\mathbf{E}| \times A \cos\theta$$

$$\Rightarrow \phi = 5 \times 10^3 \times 0.01 \cos 0°$$

$$\phi = 50\,\text{Nm}^2\text{C}^{-1}$$

If the plane makes an angle of $30°$ with the X-axis, then $\theta = 60°$

$\therefore$ Flux through the plane,

$$\phi = |\mathbf{E}| \times A \times \cos 60°$$

$$= 5 \times 10^3 \times 0.01 \times \cos 60°$$

$$= 25\,\text{Nm}^2\text{C}^{-1}$$

8. $40\,\text{Nm}^2\text{C}^{-1}$ and $20\,\text{Nm}^2\text{C}^{-1}$ **(1)**

Refer to Sol. 7

9. $200\,\text{Nm}^2\text{C}^{-1}$ and $100\,\text{Nm}^2\text{C}^{-1}$ **(2)**

Refer to Sol. 7

10. According to Gauss' law, **(2)**

Flux through S_1, $\phi_1 = \frac{Q}{\varepsilon_0}$ ...(i)

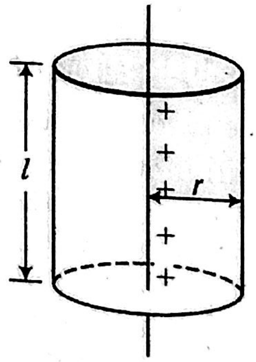
Flux through S_2, $\phi_2 = \frac{Q + 2Q}{\varepsilon_0} = \frac{3Q}{\varepsilon_0}$ ...(ii)

On dividing Eq. (i) by Eq. (ii), we get

$$\phi_1 / \phi_2 = 1/3$$ **(1)**

There is no change in the flux through S_1 with dielectric medium inside the sphere S_2. **(1)**

11. A thin straight conducting wire will be a uniform linear charge distribution.



Let q charge be enclosed by the cylindrical surface.

$\therefore$ Linear charge density,

$$\lambda = \frac{q}{l}$$

$\therefore$ $q = \lambda l$ ...(i) **(1)**

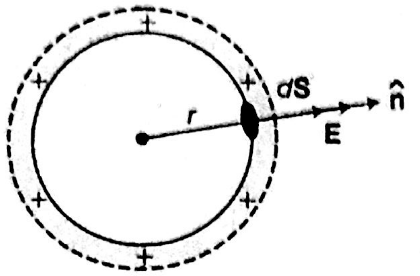
By Gauss' theorem,

$\therefore$ Total electric flux through the surface of cylinder

$$\phi = \frac{q}{\varepsilon_0}$$ [Gauss' theorem]

$\therefore$ $$\phi = \frac{\lambda l}{\varepsilon_0}$$ [from Eq. (i)] **(1)**

12. Let q charge be uniformly distributed over the spherical shell of radius r.



$\therefore$ Surface charge density on spherical shell

$$\sigma = \frac{q}{4\pi r^2} \quad ...(i)$$

(1)

$\because$ Electric field intensity on the surface of spherical shell

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\,\hat{\mathbf{n}}$$

(1/2)

[$\because$ **E** acts along radially outward and along $\hat{\mathbf{n}}$]

$$\mathbf{E} = \frac{(q/4\pi r^2)}{\varepsilon_0}\,\hat{\mathbf{n}} \Rightarrow \mathbf{E} = \frac{\sigma}{\varepsilon_0}\,\hat{\mathbf{n}} \quad ...(ii)$$

(1/2)

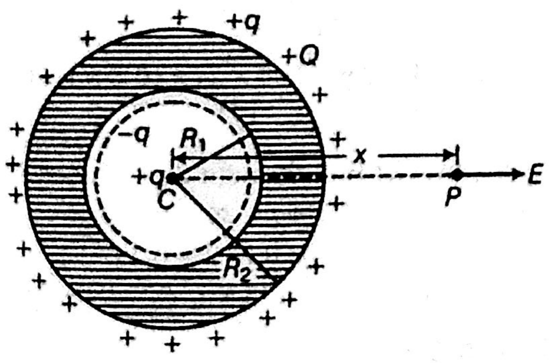
13.

> Here, *two points are important*
> 1. Charge resides on the outer surface of spherical conductor (skin effect).
> 2. Equal charge of opposite nature induces in the surface of conductor nearer to source charge.

(i) (a) Charge produced on inner surface due to induction $= -q$

$\therefore$ Surface charge density of inner surface

$$= \frac{-q}{4\pi R_1^2}$$



(b) When charge $-q$ is induced on inner walls, then equal charge $+q$ is produced at outer surface.

Charge on outer surface $= q + Q$

$\therefore$ Surface charge density of outer surface

$$= \frac{q + Q}{4\pi R_2^2}$$

(1)

(ii) Electric field intensity at point P at a distance $x\ (x > R_2)$

$$E = \frac{1}{4\pi\varepsilon_0}\times\frac{(q+Q)}{x^2}$$

[along CP and away from spherical shell]

Whole charge is assumed to be concentrated at the centre. **(1)**

14. (i) According to Gauss' theorem,

$$\phi = \frac{\Sigma q}{\varepsilon_0} \propto \Sigma q \Rightarrow \frac{\phi_{S_1}}{\phi_{S_2}} = \frac{2Q}{2Q + 4Q} = \frac{1}{3}$$

(1)

(ii) If the medium is filled in S_1, then

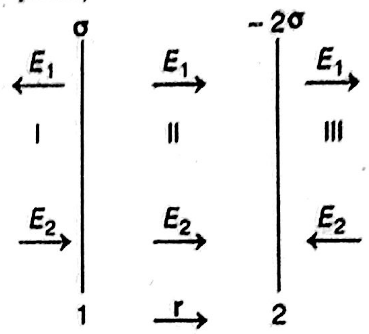
$$\phi_{S_1} = \frac{\Sigma q}{\varepsilon_0\varepsilon_r} = \frac{2Q}{\varepsilon_0\varepsilon_r}$$

(1)

15. (i) Electric field to the left of plate 1 (region I)

$$E_I = E_1 + E_2 = \frac{2\sigma}{2\varepsilon_0}\mathbf{r} - \frac{\sigma}{2\varepsilon_0}\mathbf{r}$$

where **r** be the unit vector in the direction from plate 1 (+ ve plate) to plate 2 (− ve plate)



(1)

(ii) Electric field to the right of plate 2 (region III)

$$E_{III} = \frac{\sigma}{2\varepsilon_0}\mathbf{r} - \frac{2\sigma}{2\varepsilon_0}\mathbf{r}$$

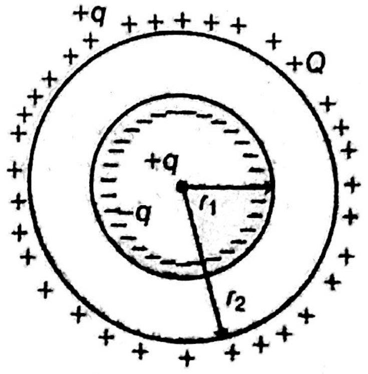
(1)

(iii) Electric field between two plates (region II)

$$E_{II} = \frac{\sigma}{2\varepsilon_0}\mathbf{r} + \frac{2\sigma}{2\varepsilon_0}\mathbf{r}$$

(1)

16. (a) When a charge $+q$ is placed at the centre of spherical cavity, as shown in the figure, Then, charge induced one the inner surface of a shell is $-q$ and charge induced on the outer surface of shell is $+q$. So,



(i) outer surface charge density $= \dfrac{Q+q}{4\pi r_2^2}$

(ii) Inner surface charge density $= \dfrac{-q}{4\pi r_1^2}$ **(2)**

(b) Yes, the electric field inside a cavity is zero irrespective of shape because the cavity has zero net charge. **(1)**

17. Refer to text on page 16 [Electric flux].

Since the electric field only an x component, for face perpendicular to x direction.

The flux, $\phi = E\Delta s$ is separately zero for each face of the cube except the two front and its opposite ones.

Now, the magnitude of electric field at the left face is

$$E_L = \alpha x = \alpha a \qquad (\because x = a \text{ at the left face})$$

So, $\quad \phi = E_L \Delta s = \alpha a[a^2 \cos 180°] = -\alpha a^3$

Field at right face, i.e $x = 2a$

$$E_R = \alpha x = \alpha 2a$$

$\Rightarrow \quad \phi = E_R \Delta s = 2a\alpha[a^2 \cos 0°] = 2\alpha a^3$

$\therefore \quad \phi_{net} = 2\alpha a^3 - \alpha a^3 = \alpha a^3$

According to Gauss' law

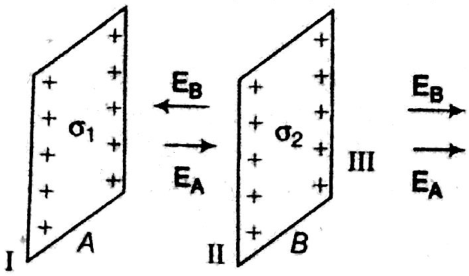
$$\phi_{net} = \frac{Q}{\varepsilon_0} \Rightarrow Q = \varepsilon_0 \phi_{net}$$

$$= 8.85 \times 10^{-12} \times 100 \times (0.1)^3 = 8.85 \times 10^{-13}\ \text{C}$$

18. According to the figure, A and B are two thin plane parallel sheets of charge having uniform densities σ_1 and σ_2 with $\sigma_1 > \sigma_2$

$\phi = E \times$ area of the end faces of the cylinder

$$E \times 2A = \frac{\sigma A}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$



(1)

In region II

The electric field due to the sheet of charge A will be from left to right (along the positive direction) and that due to the sheet of charge B will be from right to left (along the negative direction). Therefore, in region II, we have

$$E = \frac{\sigma_1}{2\varepsilon_0} + \left(-\frac{\sigma_2}{2\varepsilon_0}\right)$$

$$\Rightarrow \mathbf{E} = \frac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2) \quad \text{(along positive direction)}$$

In region III **(1)**

The electric fields due to both the charged sheets will be from left to right, i.e. along the positive direction. Therefore, in region III, we have

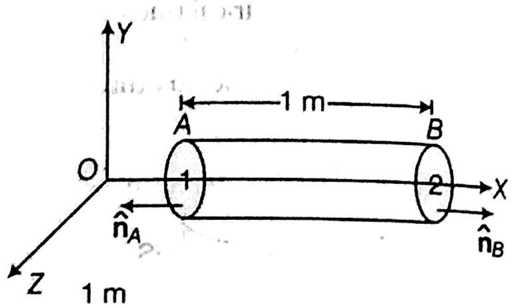
$$E = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0}$$

$$\mathbf{E} = \frac{1}{\varepsilon_0}(\sigma_1 + \sigma_2) \quad \text{(along positive direction)}$$

(1)

19. (i) Given, $E = 50\,x\,\hat{\mathbf{i}}$

and $\quad \Delta S = 25\ \text{cm}^2 = 25 \times 10^{-4}\ \text{m}^2$



As the electric field is only along the X-axis, so flux will pass only through the cross-section of the cylinder.

Magnitude of electric field at cross-section A,

$$E_A = 50 \times 1 = 50\ \text{NC}^{-1}$$

Magnitude of electric field at cross-section B,

$$E_B = 50 \times 2 = 100\ \text{NC}^{-1}$$ **(1/2)**

The corresponding electric fluxes are

$$\phi_A = \mathbf{E}_A \cdot \Delta\mathbf{S} = 50 \times 25 \times 10^{-4} \times \cos 180°$$
$$= -0.125\ \text{Nm}^2\text{C}^{-1}$$

$$\phi_B = \mathbf{E}_B \cdot \Delta\mathbf{S} = 100 \times 25 \times 10^{-4} \times \cos 0°$$
$$= 0.25\ \text{Nm}^2\text{C}^{-1}$$

So, the net flux through the cylinder,

$$\phi = \phi_A + \phi_B$$
$$= -0.125 + 0.25$$
$$= 0.125\ \text{Nm}^2\text{C}^{-1}$$ **(1)**

(ii) Using Gauss' law,

$$\phi = \oint \mathbf{E} \cdot d\mathbf{l} = \frac{q}{\varepsilon_0}$$

$\Rightarrow \quad 0.125 = \dfrac{q}{8.85 \times 10^{-12}}$

$\Rightarrow \quad q = 8.85 \times 0.125 \times 10^{-12}$

$\Rightarrow \quad q = 1.1 \times 10^{-12}\ \text{C}$ **(1/2)**

So, the charge enclosed by the cylinder is 1.1×10^{-12} C. **(1)**

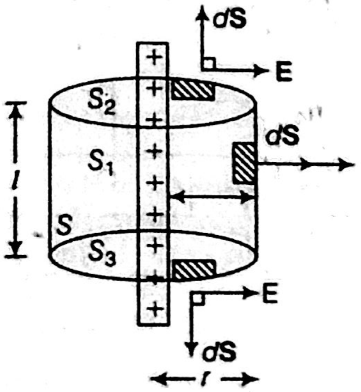
20. (i) **Gauss' law** states that the total flux through a closed surface is $\frac{1}{\varepsilon_0}$ times to the net charge enclosed by the closed surface.

Mathematically, $\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$.

Here, ε_0 is the absolute permittivity of the free space, q is the total charge enclosed and $\mathbf{E}$ is the electric field at the area element $d\mathbf{S}$. **(1)**

(ii) Electric field intensity due to an infinitely long uniformly charged wire at point P at distance r from it is obtained as follows:

Consider a thin cylindrical Gaussian surface S with charged wire on its axis and point P on its surface, then net electric flux through surface S is



$$\phi = \oint_S \mathbf{E} \cdot d\mathbf{S}$$

$$= \underbrace{\int EdS \cos 90°}_{\text{Upper plane face}} + \underbrace{\int EdS \cos 0°}_{\text{Curved surface}} + \underbrace{\int EdS \cos 90°}_{\text{Lower plane face}}$$

$$\phi = 0 + EA + 0 \text{ or } \phi = E \cdot 2\pi r l$$ **(1)**

But by Gauss's theorem, $\phi = q/\varepsilon_0 = \lambda l/\varepsilon_0$

where, q is the charge on length l of wire enclosed by cylindrical surface S and λ is uniform linear charge density of wire.

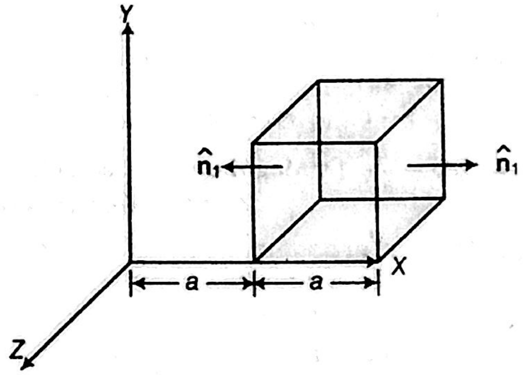
$$\therefore \quad E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Thus, electric field of a line charge is inversely proportional to distance directed normal to the surface of charged wire. **(1)**

21. **Gauss' law** Refer to text on page 16.

Now, the electric field $\mathbf{E} = C x \hat{\mathbf{i}}$ is in X-direction only. So, faces with surface normal vector perpendicular to this field would give zero electric flux, i.e. $\phi = E\, dS \cos 90° = 0$ through it. **(1)**



So, flux would be across only two surfaces.

Magnitude of E at left face,

$$E_L = Cx = Ca \qquad [x = a \text{ at left face}]$$

Magnitude of E at right face

$$E_R = Cx = C2a = 2aC \; [x = 2a \text{ at right face}]$$

Thus, corresponding fluxes are

$$\phi_L = E_L \cdot dS = E_L \, dS \cos\theta$$
$$= -aC \times a^2 \qquad [\text{As, } \theta = 180°]$$

$$\phi_R = E_R \cdot dS = 2aC \, dS \cos\theta \qquad [\because \theta = 0°]$$
$$= 2aCa^2 = 2a^3C$$ **(1/2)**

(i) Now, net flux through the cube is

$$= \phi_L + \phi_R = -a^3C + 2a^3C$$
$$= a^3C \text{ Nm}^2\text{C}^{-1}$$ **(1/2)**

(ii) Net charge inside the cube

Again, we can use Gauss' law to find total charge q inside the cube.

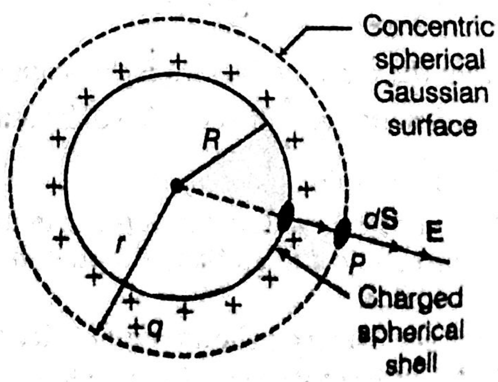
We have $\phi = \frac{q}{\varepsilon_0}$

or $\quad q = \phi\varepsilon_0 \quad q = a^3C\varepsilon_0$ coulomb **(1)**

22. Let us consider charge $+q$ be uniformly distributed over a spherical shell of radius R. Let E is to be obtained at P lying outside of spherical shell.

$\because$ $\mathbf{E}$ at any point is radially outward (if charge q is positive) and has same magnitude at all points which lie at the same distance r from centre of spherical shell such that $r > R$.

Therefore, Gaussian surface is concentric sphere of radius r such that $r > R$. **(1/2)**



By Gauss' theorem,

$$\oint \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0} \Rightarrow \oint E\, dS \cos 0° = \frac{q}{\varepsilon_0}$$

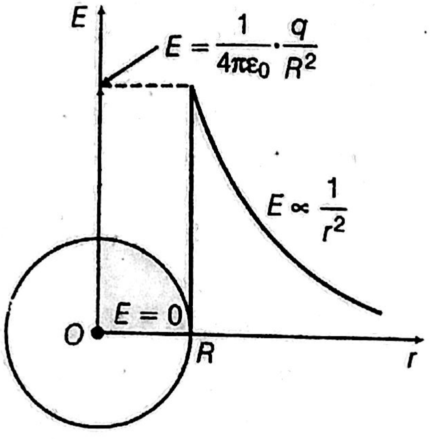
[$\because$ E and dS are along the same direction]

$$E \cdot \oint dS = \frac{q}{\varepsilon_0}$$ [$\because$ Magnitude of E is same at every point on Gaussian surface]

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$ **(1)**

Now, graph



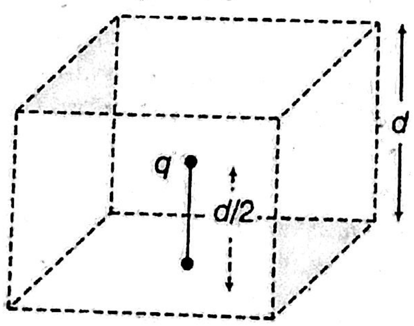
(1)

23. (a) **Electric flux** It is defined as the total number of electric field lines that are normally pass through that surface.

Total electric flux ϕ over the whole surface S due to an electric field E is given as

$$\phi = \oint_S \mathbf{E}\cdot d\mathbf{S} = \oint E dS \cos\theta$$

It is a scalar quantity. **(1)**



From the given problem, q is the point charge at a distance of $\frac{d}{2}$ directly above the centre of the square side.

Now, construct a Gaussian surface in form of a cube of side d to evaluate the amount of electric flux.

$\therefore$ We can calculate the amount of electric flux for six surfaces by using Gauss'S law,

$$\phi_E = \int_s \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

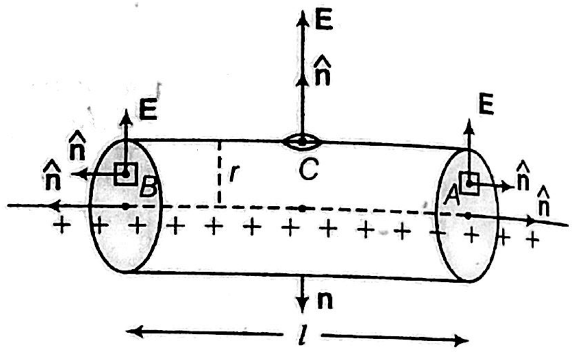
$\therefore$ For one surface of the cube, amount of electric flux is given as $\phi_E' = \frac{q}{6\varepsilon_0}$ **(2)**

(b) Even if the point charge is moved to a distance d from the centre of the square and side of the square is doubled, but amount of charge enclosed into the Gaussian surface does not changes.

$\therefore$ The amount of electric flux remains same. **(2)**

24. (a) **Field due to an infinitely long thin straight charged line**

Consider an infinitely long thin straight line with uniform linear charge density (λ).



From symmetry, the electric field is everywhere radial in the plane cutting the wire normally and its magnitude only depends on the radial distance (r).

From Gauss' law,

$$\phi_E = \oint_S \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

Now, $$\phi_E = \oint_S \mathbf{E}\cdot d\mathbf{S} = \oint_S \mathbf{E}\cdot\hat{\mathbf{n}}\, dS$$

$$= \oint_A \mathbf{E}\cdot\hat{\mathbf{n}}\, dS + \oint_B \mathbf{E}\cdot\hat{\mathbf{n}}\, dS + \oint_C \mathbf{E}\cdot\hat{\mathbf{n}}\, dS$$

$$\therefore \oint_S \mathbf{E}\cdot d\mathbf{S} = \oint_A \mathbf{E}\, dS \cos 90° + \oint_B \mathbf{E}\, dS \cos 90° + \oint_C \mathbf{E}\, dS \cos 0°$$

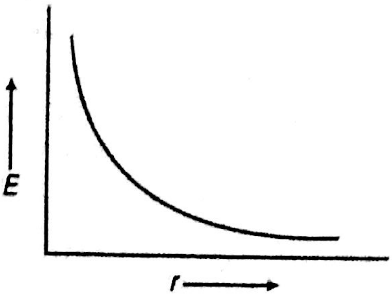
$$= \oint_C E\, dS = E(2\pi r l)$$

Charge enclosed in the cylinder, $q = \lambda l$

$$\therefore \quad E(2\pi r l) = \frac{\lambda l}{\varepsilon_0} \quad \text{or} \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

The direction of the electric field is radially outward from the positive line charge. For negative line charge, it will be radially inward.

(b) Electric field (E) due to the linear charge is inversely proportional to the distance (r) from the linear charge. The variation of electric field (E) with distance (r) is shown in figure. **(1)**



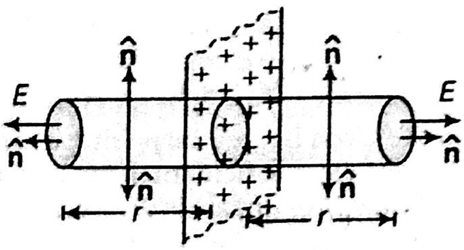
(c) $V = \int \mathbf{E} \cdot d\mathbf{r} = \int_{r_2}^{r_2} \frac{\lambda}{2\pi\varepsilon_0 r} dr = \frac{\lambda}{2\pi\varepsilon_0} \cdot \int_{r_1}^{r_2} \frac{1}{r} dr$

$$= \frac{\lambda}{2\pi\varepsilon_0}\left[\log \frac{r_2}{r_1}\right]$$

Work done $= qV = q\left[\frac{\lambda}{2\pi\varepsilon_0}\left(\log \frac{r_2}{r_1}\right)\right]$ **(2)**

25. (i) According to the question, σ is the surface charge density of the sheet. From symmetry, E on either side of the sheet must be perpendicular to the plane of the sheet, having same magnitude at all points equidistant from the sheet. We take a cylinder of cross-sectional area A and length $2r$ as the Gaussian surface. On the curved surface of the cylinder, E and $\hat{n}$ are perpendicular to each other.

Therefore, the flux through the curved surface of the cylinder = 0. **(1½)**



Flux through the flat surfaces = $EA + EA = 2EA$

The total electric flux over the entire surface of cylinder

$$\phi_E = 2EA$$

Total charge enclosed by the cylinder, $q = \sigma A$

According to Gauss's law,

$$\oint \mathbf{E} \cdot d\mathbf{A} = \phi_E = \frac{q}{\varepsilon_0} \Rightarrow 2AE = \frac{\sigma A}{\varepsilon_0} \text{ or } E = \frac{\sigma}{2\varepsilon_0}$$

E is independent of r, the distance of the point from the plane charged sheet. E at any point is directed away from the sheet for positive charge and directed towards the sheet in case of negative charge. **(1½)**

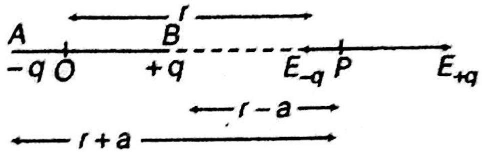
(ii) Surface charge density of the uniform plane sheet which is infinitely large $= +\sigma$. The electric potential (V) due to infinite sheet of uniform charge density $+\sigma$

$$V = -\frac{\sigma r}{2\varepsilon_0}$$

The amount of work done in bringing a point charge q from infinite to point, at distance r in front of the charged plane sheet.

$$W = q \times V = q \cdot \frac{-\sigma r}{2\varepsilon_0} = -\frac{\sigma r \cdot q}{2\varepsilon_0} \text{ J}$$ **(2)**

26. (i) Electric field on an axial line of an electric dipole



Let P be at distance r from the centre of the dipole on the side of charge $-q$.

Then, the electric field at point P due to charge $-q$ of the dipole is given by, $\mathbf{E}_{-q} = -\frac{q}{4\pi\varepsilon_0 (r+a)^2}\hat{\mathbf{p}}$

where, $\hat{\mathbf{p}}$ is the unit vector along the dipole axis (from $-q$ to q). **(1)**

Also, the electric field at point P due to charge $+q$ of the dipole is given by, $\mathbf{E}_{+q} = \frac{q}{4\pi\varepsilon_0 (r-a)^2}\hat{\mathbf{p}}$

The total field at point P is

$$\mathbf{E} = \mathbf{E}_{+q} + \mathbf{E}_{-q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{\mathbf{p}}$$

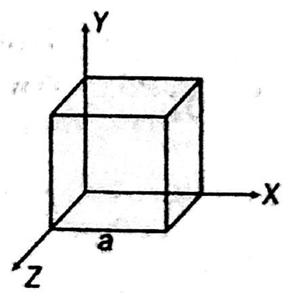
$$\Rightarrow \quad \mathbf{E} = \frac{q}{4\pi\varepsilon_0} \cdot \frac{4ar}{(r^2-a^2)^2}\hat{\mathbf{p}} \quad \because r = x \quad \text{(given)}$$

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0}\frac{4ax}{(x^2-a^2)^2}\hat{\mathbf{p}}$$

For $x >> a$, $\mathbf{E} = \frac{4qa}{4\pi\varepsilon_0 x^3}\hat{\mathbf{p}} \Rightarrow \mathbf{E} = \frac{2p}{4\pi\varepsilon_0 x^3}$

$[\because p = 2qa]$ **(2)**

(ii)



Since, the electric field has only x component, for faces normal to X - direction, the angle between E and ΔS is $\pm \pi/2$ Therefore, the flux is separately zero for each of the cube except the surface perpendicular to X-axis.

The magnitude of the electric field at the left face is

$E_l = 0$ (as, $x = 0$ at the left face).

The magnitude of the electric field at the right face is $E_R = 2a$ (as, $x = a$ at the right face). **(1)**

The corresponding fluxes are

$\phi_L = \mathbf{E}_L \cdot \Delta\mathbf{S} = 0$

$\phi_R = \mathbf{E}_R \cdot \Delta\mathbf{S} = E_R \Delta S \cos\theta = E_R \Delta S \quad (\because \theta = 0°)$

$\Rightarrow \quad \phi_R = E_R a^2$

Net flux (ϕ) through the cube

$= \phi_L + \phi_R = 0 + E_R a^2 = E_R a^2 \Rightarrow \phi = 2a\,(a)^2 = 2a^3$

We can use Gauss' law to find the total charge q inside the cube.

$\phi = q/\varepsilon_0 \quad \therefore \quad \phi = \phi E_0 = 2a^3 \varepsilon_0$ **(3)**

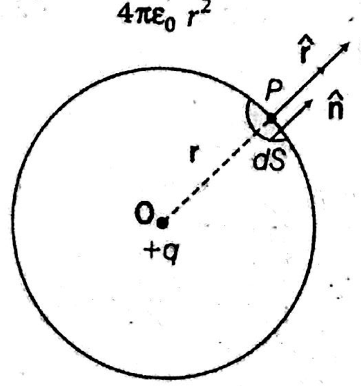
27. (i) **Electric Flux** Refer to notes

The SI unit of electric flux is Nm^2C^{-1}.

According to Gauss' law in electrostatics, the surface integral of electrostatic field **E** produced by any sources over any closed surface S enclosing a volume V in vacuum, i.e. total electric flux over the closed surface S in vacuum, is $1/\varepsilon_0$ times the total charge (q) contained inside S, i.e. $\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$

Gauss' law in electrostatics is true for an closed surface, no matter what its shape or size is. In order to justify the above statement, consider an isolated positive charge q situated at the centre O of a sphere of radius r.

According to Coulomb's law, electric field intensity at any point P on the surface of the sphere is $\mathbf{E} = \frac{q}{4\pi\varepsilon_0} \frac{\hat{\mathbf{r}}}{r^2}$



where, $\hat{\mathbf{r}}$ is unit vector directed from O to P.

Consider a small area element dS of the sphere around P. Let it be represented by the vector $d\mathbf{S} + \hat{\mathbf{n}} \cdot dS$.

where, $\hat{n}$ is unit vector along out drawn normal to the area element.

$\therefore$ Electric flux over the area element,

$$d\phi_E = \mathbf{E} \cdot d\mathbf{S} = (q/4\pi\varepsilon_0 \cdot \hat{\mathbf{r}}/r^2).(\hat{n} \cdot dS)$$

$$\mathbf{E} \cdot d\mathbf{S} = q/4\pi\varepsilon_0 \cdot dS/r^2 \cdot \hat{\mathbf{r}} \cdot \hat{\mathbf{n}}$$

As normal to a surface of every point is along the radius vector at that point, therefore, $\hat{\mathbf{r}} \cdot \hat{\mathbf{n}} = 1$

$$\mathbf{E} \cdot d\mathbf{S} = q/4\pi\varepsilon_0 \cdot dS/r^2$$

Integrating over the closed surface area of the sphere, we get total normal electric flux over the entire sphere,

$$\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{4\pi\varepsilon_0 r^2} \oint_S dS$$

$$= \frac{q}{4\pi\varepsilon_0 r^2} \times \text{total area of surface of sphere}$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}(4\pi r^2) = \frac{q}{\varepsilon_0}$$

Hence, $\oint_S \mathbf{E} d\mathbf{S} = q/\varepsilon_0$, which proves Gauss' theorem. **(2½)**

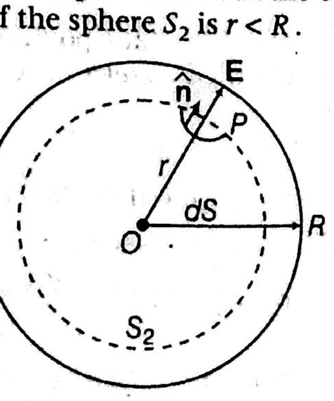
(ii) **Electric field inside a uniformly charged spherical shell**

According to Gauss' theorem

$$\oint_S \mathbf{E} \cdot d\mathbf{S} = \oint_S E\, \hat{\mathbf{n}} \cdot dS = \frac{q}{\varepsilon_0} \text{ or } E \oint_S dS = \frac{q}{\varepsilon_0}$$

$$\therefore E \cdot 2\pi r^2 = q/\varepsilon_0 \Rightarrow E = q/4\pi\varepsilon_0 r^2 \quad \text{...(i)}$$

In the given figure, the point P where we have to find the electric field intensity is inside the shell. The Gaussian surface is the surface of a sphere S_2 passing through P and with the centre at O. The radius of the sphere S_2 is $r < R$.



The electric flux through the Gaussian surface, as calculated in Eq. (i), i.e. $E \times 4\pi r^2$. As, charge inside a spherical shell is zero, the Gaussian surface encloses no charge. The Gauss' theorem gives

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0} = 0$$

$\therefore$ $E = 0$ for $r < R$.

Hence, the field due to a uniformly charged spherical shell is zero at all points inside the shell. **(2½)**

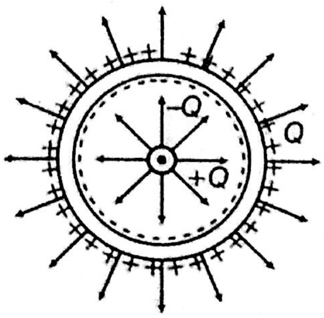
28. (a) Refer to Sol. 32(i) in Topic 1 **(5)**

(b) Refer to Sol. 10

29. Refer to Sol. 22 and 27(ii)

30. (i) **Electric flux** Refer to notes **(2½)**

(ii) Using Gauss' theorem,



Let point P_1 be at a distance r_1 from the centre.

$$E \times 4\pi r_1^2 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_1^2}$$

Field at point $P_2 = 0$, because the electric field inside the conductor is zero. **(2½)**

31. (i) Refer to text [Electric flux] **(2)**

(ii) Refer to Sol. 25 (i) on page 27.

$$E = \frac{\sigma}{2\varepsilon_0}$$

Hence, electric field at a point is independent of distance from the sheet.

(a) Normally away from the sheet when sheet is positively charged.

(b) Normally inward towards the sheet when plane sheet is negatively charged.

(3)

32. (i) Refer to Sol. 29 **(3)**

(ii) Refer to Sol. 26 of Topic 1 **(2)**