☑ 1 Mark Questions

1. Write the value of
$\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$.

2. Find the principal value of
$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

3. If $\sin\left(\sin^{-1}\dfrac{1}{5} + \cos^{-1}x\right) = 1$, then find the value of x

4. If $\tan^{-1}x + \tan^{-1}y = \dfrac{\pi}{4}$; $xy < 1$, then write the value of $x + y + xy$.

5. Write the value of $\cos^{-1}\left(-\dfrac{1}{2}\right) + 2\sin^{-1}\left(\dfrac{1}{2}\right)$.

6. Write the principal value of
$\cos^{-1}[\cos(680)°]$.

7. Write the principal value of
$\tan^{-1}\left[\sin\left(\dfrac{-\pi}{2}\right)\right]$.

8. Find the value of the following.
$\cot\left(\dfrac{\pi}{2} - 2\cot^{-1}\sqrt{3}\right)$

9. Write the principal value of the following.
$\left[\cos^{-1}\dfrac{\sqrt{3}}{2} + \cos^{-1}\left(-\dfrac{1}{2}\right)\right]$

10. Write the principal value of
$\tan^{-1}(1) + \cos^{-1}\left(-\dfrac{1}{2}\right)$.

11. Write the value of $\tan\left(2\tan^{-1}\dfrac{1}{5}\right)$.

12. Write the value of
$\tan^{-1}\left[2\sin\left(2\cos^{-1}\dfrac{\sqrt{3}}{2}\right)\right]$.

13. Write the value of $\cos^{-1}\left(\dfrac{1}{2}\right) - 2\sin^{-1}\left(-\dfrac{1}{2}\right)$.

14. Using the principal values, write the value of $\cos^{-1}\left(\dfrac{1}{2}\right) + 2\sin^{-1}\left(\dfrac{1}{2}\right)$.

15. Write the value of $\sin\left[\dfrac{\pi}{3} - \sin^{-1}\left(-\dfrac{1}{2}\right)\right]$.

16. Write the value of $\tan^{-1}\left(\tan\dfrac{3\pi}{4}\right)$.

17. Write the value of $\cos^{-1}\left(\cos\dfrac{7\pi}{6}\right)$.

18. What is the principal value of
$\cos^{-1}\left(\cos\dfrac{2\pi}{3}\right) + \sin^{-1}\left(\sin\dfrac{2\pi}{3}\right)$?

19. What is the principal value of $\tan^{-1}(-1)$?

20. Using the principal values, write the value of $\sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$.

21. Write the principal value of $\sin^{-1}\left(-\dfrac{1}{2}\right)$.

22. What is the principal value of $\sec^{-1}(-2)$?

23. What is the domain of the function $\sin^{-1}x$?

24. Using the principal values, find the value of $\cos^{-1}\left(\cos\dfrac{13\pi}{6}\right)$.

25. If $\tan^{-1}(\sqrt{3}) + \cot^{-1}x = \dfrac{\pi}{2}$, then find the value of x.

2 Marks Questions

26. Prove that
$$3\sin^{-1} x = \sin^{-1}(3x - 4x^3), x \in \left[-\frac{1}{2}, \frac{1}{2}\right].$$

27. Prove that
$$3\cos^{-1} x = \cos^{-1}(4x^3 - 3x), x \in \left[\frac{1}{2}, 1\right].$$

4 Marks Questions

28. Solve for x,
$$\tan^{-1}(x + 1) + \tan^{-1}(x - 1) = \tan^{-1}\frac{8}{31}.$$

29. Find the value of $\sin\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right).$

30. Solve for x, $\tan^{-1} 3x + \tan^{-1} 2x = \frac{\pi}{4}.$

31. Solve $\tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{4}.$

32. If $\tan^{-1}\frac{x-3}{x-4} + \tan^{-1}\frac{x+3}{x+4} = \frac{\pi}{4}$, then find the value of x.

33. Prove that $\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\}$
$$+ \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\} = \frac{2b}{a}.$$

34. Solve the following equation for x.
$$\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$

35. Prove that
$$\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$$

36. Solve for x,
$$2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x).$$

37. Solve for x,
$$\tan^{-1}(x - 1) + \tan^{-1} x + \tan^{-1}(x + 1)$$
$$= \tan^{-1} 3x$$

38. Prove that
$$\tan^{-1}\left(\frac{6x - 8x^3}{1 - 12x^2}\right) - \tan^{-1}\left(\frac{4x}{1 - 4x^2}\right)$$
$$= \tan^{-1} 2x; |2x| < \frac{1}{\sqrt{3}}.$$

39. Prove that
$$\cot^{-1}\left(\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}\right) = \frac{x}{2},$$
$$0 < x < \frac{\pi}{2}, \text{ or } x \in \left(0, \frac{\pi}{4}\right).$$

40. Solve for x,
$$\tan^{-1}\left(\frac{x - 2}{x - 1}\right) + \tan^{-1}\left(\frac{x + 2}{x + 1}\right) = \frac{\pi}{4}$$

41. If $\sin[\cot^{-1}(x + 1)] = \cos(\tan^{-1} x)$, then find x.

42. If $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$, then find x.

43. Prove the following.
$$\cot^{-1}\left(\frac{xy + 1}{x - y}\right) + \cot^{-1}\left(\frac{yz + 1}{y - z}\right)$$
$$+ \cot^{-1}\left(\frac{zx + 1}{z - x}\right) = 0,$$
$$(0 < xy, yz, zx < 1)$$

44. If $\tan^{-1}\left(\frac{1}{1 + 1 \cdot 2}\right) + \tan^{-1}\left(\frac{1}{1 + 2 \cdot 3}\right)$
$$+ \ldots + \tan^{-1}\left(\frac{1}{1 + n(n + 1)}\right) = \tan^{-1}\theta,$$
then find the value of θ.

45. Prove that
$$2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \sin^{-1}\left(\frac{31}{25\sqrt{2}}\right)$$

46. Solve the following equation for x.
$$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x, \; x > 0$$

47. Solve for x, $\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$.

48. Prove that
$$2\tan^{-1}\left(\frac{1}{5}\right) + \sec^{-1}\left(\frac{5\sqrt{2}}{7}\right) + 2\tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}.$$

49. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right]$
$$= \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x, \; -\frac{1}{\sqrt{2}} \le x \le 1.$$

50. If $\tan^{-1}\left(\frac{x-2}{x-4}\right) + \tan^{-1}\left(\frac{x+2}{x+4}\right) = \frac{\pi}{4}$, then find the value of x.

51. Prove that
$$\cos^{-1}(x) + \cos^{-1}\left\{\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right\} = \frac{\pi}{3}.$$

52. Prove that
$$\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \cot^{-1}3.$$

53. Prove that
$$\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \cos^{-1}\left(\frac{36}{85}\right).$$

54. Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$.

55. Solve for x, $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$.

56. Prove that $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$.

57. Find the value of the following.
$$\tan\frac{1}{2}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right],$$
if $|x| < 1, y > 0$ and $xy < 1$

58. Prove that
$$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}.$$

59. Prove that
$$\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right) = \frac{\pi}{4} - \frac{x}{2}, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right).$$

60. Prove that
$$\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right).$$

61. Prove the following.
$$\cos\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right) = \frac{6}{5\sqrt{13}}$$

62. Prove that
$$\sin^{-1}\left(\frac{63}{65}\right) = \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right).$$

63. Solve for x,
$$2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x), x \ne \frac{\pi}{2}.$$

64. Find the value of
$$\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right).$$

65. Prove that
$$2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right).$$

66. Prove that
$$\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right).$$

67. Prove that $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\tan^{-1}\frac{4}{3}$.

68. Solve for x, $\cos(2\sin^{-1}x) = \dfrac{1}{9}$; $x > 0$.

69. Prove that $2\tan^{-1}\dfrac{3}{4} - \tan^{-1}\dfrac{17}{31} = \dfrac{\pi}{4}$.

70. Solve for x,
$$\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3},$$
$-1 < x < 1.$

71. Prove that
$$\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in (0,1).$$

72. Prove that
$$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right).$$

73. Prove that
$$\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi.$$

74. Prove that
$$\tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right).$$

75. Prove that
$$\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}.$$

76. Solve for x, $\cos^{-1}x + \sin^{-1}\left(\dfrac{x}{2}\right) = \dfrac{\pi}{6}$.

77. Prove that
$$2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}.$$

78. Solve for x,
$$\tan^{-1}\frac{x}{2} + \tan^{-1}\frac{x}{3} = \frac{\pi}{4}, \sqrt{6} > x > 0.$$

79. Solve for x, $\tan^{-1}(x+2) + \tan^{-1}(x-2)$
$= \tan^{-1}\left(\dfrac{8}{79}\right), x > 0$

2. If $\tan^{-1}\left(\dfrac{a}{x}\right) + \tan^{-1}\left(\dfrac{b}{x}\right) = \dfrac{\pi}{2}$, then x is equal to

(a) $\sqrt{ab}$ (b) $\sqrt{2\,ab}$
(c) $2\,ab$ (d) ab

3. The value of $\sin(2\sin^{-1}0.8)$ is

(a) 0.48 (b) $\sin 1.2°$
(c) $\sin 1.6°$ (d) 0.96

4. The principal value of $\sin^{-1}\left[\sin\left(\dfrac{2\pi}{3}\right)\right]$ is

(a) $\dfrac{-2\pi}{3}$ (b) $\dfrac{2\pi}{3}$

(c) $\dfrac{4\pi}{3}$ (d) None of these

5. The value of $\sin(2\tan^{-1}x), |x| \le 1$ is

(a) $1/x$ (b) x
(c) $1/x^2$ (d) $2x/(1+x^2)$

6. If $\tan^{-1}x = \dfrac{\pi}{4} - \tan^{-1}\left(\dfrac{1}{3}\right)$, then x is equal to

(a) $\dfrac{1}{3}$ (b) $\dfrac{1}{2}$

(c) $\dfrac{1}{4}$ (d) $\dfrac{1}{6}$

7. The value of
$$\sin\left[\tan^{-1}\left(\dfrac{1-x^2}{2x}\right) + \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)\right] \text{ is}$$

(a) 1 (b) 0
(c) -1 (d) $\dfrac{\pi}{2}$

8. If $\sin^{-1}\dfrac{1}{3} + \sin^{-1}\dfrac{2}{3} = \sin^{-1}x$, then the value of x is

(a) 0 (b) $\dfrac{(\sqrt{5}-4\sqrt{2})}{9}$

(c) $\dfrac{(\sqrt{5}+4\sqrt{2})}{9}$ (d) $\dfrac{\pi}{2}$

9. If x takes negative permissible value, then $\sin^{-1}x$ is equal to

(a) $-\cos^{-1}\sqrt{1-x^2}$ (b) $\cos^{-1}\sqrt{x^2-1}$
(c) $\pi - \cos^{-1}\sqrt{1-x^2}$ (d) $\cos^{-1}\sqrt{1-x^2}$

Objective Questions
(For Complete Chapter)
☑ 1 Mark Questions

1. If $\cos^{-1}x > \sin^{-1}x$, then

(a) $\dfrac{1}{\sqrt{2}} < x \le 1$ (b) $0 \le x < \dfrac{1}{\sqrt{2}}$

(c) $-1 \le x < \dfrac{1}{\sqrt{2}}$ (d) $x > 0$

10. If $-\dfrac{\pi}{2} < \sin^{-1} x < \dfrac{\pi}{2}$, then $\tan(\sin^{-1} x)$ is equal to

(a) $\dfrac{x}{1-x^2}$

(b) $\dfrac{x}{1+x^2}$

(c) $\dfrac{x}{\sqrt{1-x^2}}$

(d) $\dfrac{1}{\sqrt{1-x^2}}$

11. The value of $\cot\left(\operatorname{cosec}^{-1} \dfrac{5}{3} + \tan^{-1} \dfrac{2}{3}\right)$ is

(a) $\dfrac{5}{17}$

(b) $\dfrac{6}{17}$

(c) $\dfrac{3}{17}$

(d) $\dfrac{4}{17}$

12. If $\sin^{-1} x + \sin^{-1} y = \dfrac{\pi}{2}$, then $\cos^{-1} x + \cos^{-1} y$ is equal to

(a) $\dfrac{\pi}{2}$

(b) $\dfrac{\pi}{4}$

(c) π

(d) $\dfrac{3\pi}{4}$

13. If $\sin^{-1} x - \cos^{-1} x = \pi / 6$, then x is equal to

(a) $\dfrac{1}{2}$

(b) $\dfrac{\sqrt{3}}{2}$

(c) $\dfrac{-1}{2}$

(d) None of these

14. The value of $\tan\left\{\cos^{-1}\left(-\dfrac{2}{7}\right) - \dfrac{\pi}{2}\right\}$ is

(a) $\dfrac{2}{3\sqrt{5}}$

(b) $\dfrac{2}{3}$

(c) $\dfrac{1}{\sqrt{5}}$

(d) $\dfrac{4}{\sqrt{5}}$