DCAM classes

Dynamic Classes for Academic Mastery

limits,continuity and differentiability-CBSE

1 Mark Questions

1. Determine the value of '*k*' for which the following function is continuous at $x = 3$:

$$f(x) = \begin{cases} \dfrac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$

2. Determine the value of the constant '*k*' so that the function $f(x) = \begin{cases} \dfrac{kx}{|x|}, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases}$ is continuous at $x = 0$.

4 Marks Questions

3. Find the values of p and q for which

$$f(x) = \begin{cases} \dfrac{1 - \sin^3 x}{3\cos^2 x}, & \text{if } x < \dfrac{\pi}{2} \\ p, & \text{if } x = \dfrac{\pi}{2} \\ \dfrac{q(1 - \sin x)}{(\pi - 2x)^2}, & \text{if } x > \dfrac{\pi}{2} \end{cases}$$

is continuous at $x = \dfrac{\pi}{2}$.

4. If $f(x) = \begin{cases} \dfrac{\sin(a+1)x + 2\sin x}{x}, & x < 0 \\ 2, & x = 0 \\ \dfrac{\sqrt{1+bx} - 1}{x}, & x > 0 \end{cases}$

is continuous at $x = 0$, then find the values of a and b.

5. Find the value of k, so that the function

$$f(x) = \begin{cases} \left(\dfrac{1 - \cos 4x}{8x^2}\right), & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$

is continuous at $x = 0$

6. If $f(x)=\begin{cases}\dfrac{1-\cos 4x}{x^2}, & \text{when } x<0\\ a, & \text{when } x=0\\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & \text{when } x>0\end{cases}$

and f is continuous at $x=0$, then find the value of a.

7. Find the value of k, for which

$$f(x)=\begin{cases}\dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1\le x<0\\ \dfrac{2x+1}{x-1}, & \text{if } 0\le x<1\end{cases}$$

is continuous at $x=0$

8. Find the value of k, so that the following function is continuous at $x=2$.

$$f(x)=\begin{cases}\dfrac{x^3+x^2-16x+20}{(x-2)^2}, & x\ne 2\\ k, & x=2\end{cases}$$

9. Find the value of k, so that the function f defined by

$$f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & \text{if } x\ne\dfrac{\pi}{2}\\ 3, & \text{if } x=\dfrac{\pi}{2}\end{cases}$$

is continuous at $x=\dfrac{\pi}{2}$.

10. Find the value of a for which the function f is defined as

$$f(x)=\begin{cases}a\sin\dfrac{\pi}{2}(x+1), & x\le 0\\ \dfrac{\tan x-\sin x}{x^3}, & x>0\end{cases}$$

is continuous at $x=0$

11. If the function $f(x)$ given by

$$f(x)=\begin{cases}3ax+b, & \text{if } x>1\\ 11, & \text{if } x=1\\ 5ax-2b, & \text{if } x<1\end{cases}$$

is continuous at $x=1$, then find the values of a and b.

12. Find the values of a and b such that the following function $f(x)$ is a continuous function.

$$f(x)=\begin{cases}5, & x\le 2\\ ax+b, & 2<x<10\\ 21, & x\ge 10\end{cases}$$

13. Find the relationship between a and b, so that the function f defined by

$$f(x)=\begin{cases}ax+1, & \text{if } x\le 3\\ bx+3, & \text{if } x>3\end{cases}$$

is continuous at $x=3$.

14. Find the value of k, so that the function f defined by $f(x)=\begin{cases}kx+1, & \text{if } x\le\pi\\ \cos x, & \text{if } x>\pi\end{cases}$

is continuous at $x=\pi$.

15. For what values of λ, is the function

$$f(x)=\begin{cases}\lambda(x^2-2x), & \text{if } x\le 0\\ 4x+1, & \text{if } x>0\end{cases}$$

is continuous at $x=0$?

16. Discuss the continuity of the function $f(x)$ at $x=1/2$, when $f(x)$ is defined as follows.

$$f(x)=\begin{cases}1/2+x, & 0\le x<1/2\\ 1, & x=1/2\\ 3/2+x, & 1/2<x\le 1\end{cases}$$

17. Find the value of a, if the function $f(x)$ defined by

$$f(x)=\begin{cases}2x-1, & x<2\\ a, & x=2\\ x+1, & x>2\end{cases}$$

is continuous at $x=2$. Also, discuss the continuity of $f(x)$ at $x=3$.

18. Find the values of a and b such that the function defined as follows is continuous.

$$f(x)=\begin{cases}x+2, & x\le 2\\ ax+b, & 2<x<5\\ 3x-2, & x\ge 5\end{cases}$$

19. For what value of k, is the function defined by $f(x) = \begin{cases} k(x^2+2), & \text{if } x \le 0 \\ 3x+1, & \text{if } x > 0 \end{cases}$, continuous at $x = 0$?
Also, find whether the function is continuous at $x = 1$.

20. Find all points of discontinuity of f, where f is defined as follows.

$$f(x) = \begin{cases} |x| + 3, & x \le -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \ge 3 \end{cases}$$

1 Mark Questions

1. Differentiate $e^{\sqrt{3x}}$, with respect to x.

2. If $y = \cos(\sqrt{3x})$, then find $\frac{dy}{dx}$.

3. If $f(x) = x + 1$, find $\frac{d}{dx}(fof)(x)$.

4. If $f(x) = x + 7$ and $g(x) = x - 7$, $x \in R$, then find the values of $\frac{d}{dx}(fog)\,x$.

5. If $y = x|x|$, find $\frac{dy}{dx}$ for $x < 0$.

2 Marks Questions

6. Differentiate $\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ with respect to x.

7. Differentiate $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$ with respect to x.

8. Find the value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in $[-\sqrt{3}, 0]$.

9. Find $\frac{dy}{dx}$ at $x = 1$, $y = \frac{\pi}{4}$ if $\sin^2 y + \cos xy = K$.

10. If $y = \sin^{-1}(6x\sqrt{1-9x^2})$, $-\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$, then find $\frac{dy}{dx}$.

4 Marks Questions

11. If $(\cos x)^y = (\cos y)^x$, then find $\frac{dy}{dx}$.

12. If $x\sqrt{1+y} + y\sqrt{1+x} = 0, (x \neq y)$, then prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$.

13. If $y = (\sin^{-1} x)^2$, prove that
$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$$

14. If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

15. If $x = ae^t(\sin t + \cos t)$ and $y = ae^t(\sin t - \cos t)$, then prove that
$$\frac{dy}{dx} = \frac{x+y}{x-y}$$

16. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x.

17. If $\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

18. If $x^y - y^x = a^b$, find $\frac{dy}{dx}$.

19. If $x = \cos t + \log\tan\left(\frac{t}{2}\right)$, $y = \sin t$, then find the values of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

20. If $y = \sin(\sin x)$, prove that
$$\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + y\cos^2 x = 0.$$

21. If $(x^2 + y^2)^2 = xy$, find $\frac{dy}{dx}$.

22. If $x = a(2\theta - \sin 2\theta)$ and $y = a(1 - \cos 2\theta)$, find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$.

23. If $\sin y = x\cos(a+y)$, then show that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\cos a}$.

Also, show that $\frac{dy}{dx} = \cos a$, when $x = 0$.

24. If $x = a\sec^3\theta$ and $y = a\tan^3\theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{3}$.

25. If $y = e^{\tan^{-1}x}$, prove that

$$(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0.$$

26. If $x^y + y^x = a^b$, then find $\frac{dy}{dx}$.

27. If $e^y(x+1) = 1$, then show that

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2.$$

28. If $y = x^x$, then prove that

$$\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0.$$

29. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, when $x \neq 0$.

30. If $x = a\sin 2t(1+\cos 2t)$ and $y = b\cos 2t(1-\cos 2t)$, then find the values of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ and $t = \frac{\pi}{3}$.

Or If $x = a\sin 2t(1+\cos 2t)$ and $y = b\cos 2t(1-\cos 2t)$, then show that at $t = \frac{\pi}{4}$, $\frac{dy}{dx} = \frac{b}{a}$

31. If $x\cos(a+y) = \cos y$, then prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$. Hence, show that

$$\sin a\frac{d^2y}{dx^2} + \sin 2(a+y)\frac{dy}{dx} = 0.$$

Or If $\cos y = x\cos(a+y)$, where $\cos a \neq \pm 1$, prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

32. Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left[\frac{6x - 4\sqrt{1-4x^2}}{5}\right]$.

33. Find the values of a and b, if the function f defined by

$$f(x) = \begin{cases} x^2 + 3x + a, & x \le 1 \\ bx + 2, & x > 1 \end{cases}$$

is differentiable at $x = 1$.

34. If $x = \sin t$ and $y = \sin pt$, then prove that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0.$$

35. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$, $x^2 \le 1$, then find dy/dx.

36. If $x = a\cos\theta + b\sin\theta$, $y = a\sin\theta - b\cos\theta$, then show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0.$

37. Show that the function $f(x) = |x+1| + |x-1|$, for all $x \in R$, is not differentiable at the points $x = -1$ and $x = 1$.

38. If $y = e^{m\sin^{-1}x}$, then show that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0.$$

39. If $f(x) = \sqrt{x^2+1}$; $g(x) = \frac{x+1}{x^2+1}$ and $h(x) = 2x - 3$, then find $f'[h'\{g'(x)\}]$.

40. If $y=(x+\sqrt{1+x^2})^n$, then show that $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=n^2y$.

41. Find whether the following function is differentiable at $x=1$ and $x=2$ or not.

$$f(x)=\begin{cases} x, & x<1 \\ 2-x, & 1\le x\le 2 \\ -2+3x-x^2, & x>2 \end{cases}$$

42. For what value of λ, the function defined by $f(x)=\begin{cases}\lambda(x^2+2), \text{ if } x\le 0 \\ 4x+6, \text{ if } x>0\end{cases}$ is continuous at $x=0$? Hence, check the differentiability of $f(x)$ at $x=0$.

43. If $y=(\sin x)^x+\sin^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$.

44. If $y=\frac{x\cos^{-1}x}{\sqrt{1-x^2}}-\log\sqrt{1-x^2}$, then prove that $\frac{dy}{dx}=\frac{\cos^{-1}x}{(1-x^2)^{3/2}}$.

45. Write the derivative of $\sin x$ with respect to $\cos x$.

46. If $y=\sin^{-1}\{x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\}$ and $0<x<1$, then find $\frac{dy}{dx}$.

47. If $e^x+e^y=e^{x+y}$, prove that $\frac{dy}{dx}+e^{y-x}=0$.

48. Find the value of $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$, if $x=ae^{\theta}(\sin\theta-\cos\theta)$ and $y=ae^{\theta}(\sin\theta+\cos\theta)$

49. If $x=a\left(\cos t+\log\tan\frac{t}{2}\right)$, $y=a\sin t$, then evaluate $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{3}$.

50. If $x^my^n=(x+y)^{m+n}$, prove that $\frac{dy}{dx}=\frac{y}{x}$.

51. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ w.r.t. $\cos^{-1}(2x\sqrt{1-x^2})$, when $x\neq 0$.

52. Differentiate $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.

53. If $y=Pe^{ax}+Qe^{bx}$, then show that $\frac{d^2y}{dx^2}-(a+b)\frac{dy}{dx}+aby=0$.

54. If $x=\cos t(3-2\cos^2 t)$ and $y=\sin t(3-2\sin^2 t)$, then find the value of $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$.

55. If $(x-y)e^{\frac{x}{x-y}}=a$, prove that $y\frac{dy}{dx}+x=2y$.

56. If $x=a(\cos t+t\sin t)$ and $y=a(\sin t-t\cos t)$, then find the value of $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{4}$.

57. If $y=\tan^{-1}\left(\frac{a}{x}\right)+\log\sqrt{\frac{x-a}{x+a}}$, prove that $\frac{dy}{dx}=\frac{2a^3}{x^4-a^4}$.

58. If $(\tan^{-1}x)^y+y^{\cot x}=1$, then find dy/dx.

59. If $x=2\cos\theta-\cos 2\theta$ and $y=2\sin\theta-\sin 2\theta$, then prove that $\frac{dy}{dx}=\tan\left(\frac{3\theta}{2}\right)$.

60. If $y=x\log\left(\frac{x}{a+bx}\right)$, then prove that $x^3\frac{d^2y}{dx^2}=\left(x\frac{dy}{dx}-y\right)^2$.

61. If $x=\cos\theta$ and $y=\sin^3\theta$, then prove that $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=3\sin^2\theta(5\cos^2\theta-1)$.

62. Differentiate the following function with respect to x.

$$(\log x)^x + x^{\log x}$$

63. If $y = \log[x + \sqrt{x^2 + a^2}]$, then show that

$$(x^2 + a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0.$$

64. Show that the function $f(x) = |x - 3|, x \in R$, is continuous but not differentiable at $x = 3$.

65. If $x = a \sin t$ and $y = a[\cos t + \log \tan (t/2)]$, then find $\frac{d^2y}{dx^2}$.

66. Differentiate the following with respect to x.

$$\sin^{-1}\left[\frac{2^{x+1} \cdot 3^x}{1 + (36)^x}\right]$$

67. If $x = a \cos^3 \theta$ and $y = a \sin^3 \theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$.

68. If $x \sin(a + y) + \sin a \cos(a + y) = 0$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$.

69. If $x^y = e^{x-y}$, then prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$

Or If $x^y = e^{x-y}$, then prove that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$.

70. If $y^x = e^{y-x}$, then prove that $\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}$.

71. If $\sin y = x \sin(a + y)$, then prove that

$$\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}.$$

72. If $y = \sin^{-1} x$, show that

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0.$$

73. If $x = \sqrt{a^{\sin^{-1} t}}$ and $y = \sqrt{a^{\cos^{-1} t}}$, then show that $\frac{dy}{dx} = \frac{-y}{x}$.

74. Differentiate $\tan^{-1}\left[\frac{\sqrt{1 + x^2} - 1}{x}\right]$ w.r.t. x.

75. If $y = (\tan^{-1} x)^2$, then show that

$$(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1)\frac{dy}{dx} = 2.$$

76. If $y = x^{\sin x - \cos x} + \frac{x^2 - 1}{x^2 + 1}$, then find $\frac{dy}{dx}$.

77. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, then find $\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$.

78. If $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$ and $y = a \sin t$, find $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$.

79. Find $\frac{dy}{dx}$, when $y = x^{\cot x} + \frac{2x^2 - 3}{x^2 + x + 2}$.

80. If $x = \tan\left(\frac{1}{a} \log y\right)$, then show that

$$(1 + x^2)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0.$$

81. Differentiate $x^{x \cos x} + \frac{x^2 + 1}{x^2 - 1}$ w.r.t. x.

82. If $x = a(\theta - \sin \theta)$, $y = a(1 + \cos \theta)$, then find $\frac{d^2y}{dx^2}$.

83. Prove that

$$\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right)\right] = \sqrt{a^2 - x^2}.$$

84. If $y = \log [x + \sqrt{x^2 + 1}]$, then prove that

$$(x^2 + 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0.$$

85. If $\log(\sqrt{1 + x^2} - x) = y\sqrt{1 + x^2}$, then show that $(1 + x^2)\frac{dy}{dx} + xy + 1 = 0$.

86. If $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$, then find $\frac{d^2y}{dx^2}$.

87. If $y = a\sin x + b\cos x$, then prove that

$$y^2 + \left(\frac{dy}{dx}\right)^2 = a^2 + b^2.$$

88. If $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$, then find $\frac{d^2y}{dx^2}$.

89. If $x = a(\theta - \sin\theta)$ and $y = a(1 + \cos\theta)$, then find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$.

90. If $y = (\sin x - \cos x)^{(\sin x - \cos x)}, \frac{\pi}{4} < x < \frac{3\pi}{4}$, then find $\frac{dy}{dx}$.

91. If $y = \cos^{-1}\left[\frac{2x - 3\sqrt{1 - x^2}}{\sqrt{13}}\right]$, then find $\frac{dy}{dx}$.

92. If $y = (\cot^{-1} x)^2$, then show that

$$(x^2 + 1)^2\frac{d^2y}{dx^2} + 2x(x^2 + 1)\frac{dy}{dx} = 2.$$

93. If $y = \operatorname{cosec}^{-1}x, x > 1$, then show that

$$x(x^2 - 1)\frac{d^2y}{dx^2} + (2x^2 - 1)\frac{dy}{dx} = 0.$$

94. If $y = \cos^{-1}\left(\frac{3x + 4\sqrt{1 - x^2}}{5}\right)$, then find $\frac{dy}{dx}$.

95. Show that the function defined as follows, is continuous at $x = 1, x = 2$ but not differentiable at $x = 2$.

$$f(x) = \begin{cases} 3x - 2, & 0 < x \le 1 \\ 2x^2 - x, & 1 < x \le 2 \\ 5x - 4, & x > 2 \end{cases}$$

96. If $y = e^{a\cos^{-1}x}, -1 \le x \le 1$, then show that

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0.$$

97. Find $\frac{dy}{dx}$, if $y = (\cos x)^x + (\sin x)^{1/x}$.

98. If $y = e^x\sin x$, then prove that

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$$

99. If $y = (x)^x + (\sin x)^x$, then find $\frac{dy}{dx}$.