linear programming solutions-CBSE

Solutions

1. Let number of necklaces and bracelets produced by firm per day be x and y, respectively.

Clearly, $x \geq 0,\ y \geq 0$

$\because$ Total number of necklaces and bracelets that the firm can handle per day is atmost 24. **(1/2)**

$\therefore \quad x + y \leq 24$

Since it takes one hour to make a bracelet and half an hour to make a necklace and maximum number of hours available per day is 16.

$$\therefore \quad \frac{1}{2}x + y \leq 16$$

$$\Rightarrow \quad x + 2y \leq 32$$ **(1/2)**

Let Z be the profit function.

Then, $\quad Z = 100x + 300y$ **(1/2)**

$\therefore$ The given LPP reduces to

Maximise $Z = 100x + 300y$

subject to,

$$x + y \leq 24$$

$$x + 2y \leq 32$$

and $\quad x, y \geq 0$ **(1/2)**

2. The given data can be summarised as follows

	Tailor A	Tailor B	Minimum requirement
No. of shirts	6	10	60
No. of pairs of trousers	4	4	32
Labour cost	₹ 300/day	₹ 400/day	

Let tailor A and tailor B works for x days and y days, respectively.

Then, $x \geq 0,\ y \geq 0$ **(1/2)**

$\because$ Minimum number of shirts = 60

$\therefore\ 6x + 10y \geq 60 \Rightarrow 3x + 5y \geq 30$ **(1/2)**

Minimum number of trousers = 32

$\therefore\ 4x + 4y \geq 32 \Rightarrow x + y \geq 8$ **(1/2)**

Let Z be the total labour cost.

Then, $Z = 300x + 400y$

So, the given LPP reduces to

$$Z = 300x + 400y$$

$x \geq 0,\ y \geq 0,\ 3x + 5y \geq 30$ and $x + y \geq 8$ **(1/2)**

3. Our problem is to minimise

$$Z = 5x + 10y \quad \text{... (i)}$$

Subject to constraints

$$x + 2y \leq 120 \quad \text{...(ii)}$$

$$x + y \geq 60 \quad \text{...(iii)}$$

$$x - 2y \geq 0 \quad \text{...(iv)}$$

and $x \geq 0, y \geq 0$

Table for line $x + 2y = 120$ is

x	0	120
y	60	0

Put (0, 0) in the inequality $x + 2y \leq 120$, we get

$$0 + 2 \times 0 \leq 120$$

$\Rightarrow \quad 0 \leq 120$ (which is true)

So, the half plane is towards the origin. Secondly, draw the graph of the line $x + y = 60$

x	0	60
y	60	0

(1)

On putting (0, 0) in the inequality $x + y \geq 60$, we get

$0 + 0 \geq 60 \Rightarrow 0 \geq 60$ (which is false)

So, the half plane is away from the origin.

Thirdly, draw the graph of the line $x - 2y = 0$.

x	0	10
y	0	5

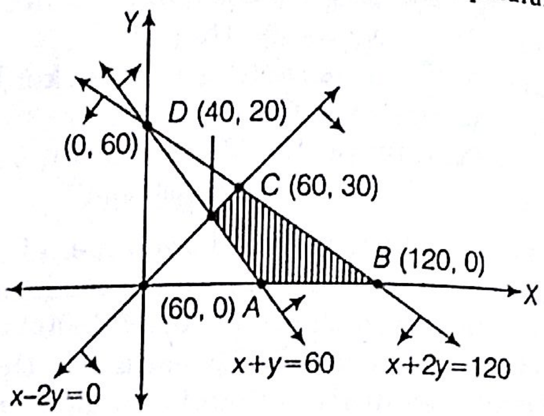
On putting (5, 0) in the inequality $x - 2y \geq 0$, we get

$5 - 2 \times 0 \geq 0 \Rightarrow 5 \geq 0$ (which is true)

Thus, the half plane is towards the X-axis.

Since, $x, y \geq 0$

$\therefore$ The feasible region lies in the first quadrant.



(2)

Clearly, feasible region is $ABCDA$.

On solving equations $x - 2y = 0$ and $x + y = 60$, we get $D(40, 20)$ and on solving equations $x - 2y = 0$ and $x + 2y = 120$, we get C (60, 30). The corner points of the feasible region are A (60, 0), B (120, 0), C (60, 30) and D (40, 20). The values of Z at these points are as follows

Corner point	$Z = 5x + 10y$
A (60, 0)	300 (minimum)
B (120, 0)	600
C (60, 30)	600
D (40, 20)	400

Clearly, the minimum value of Z is 300 at the point (60, 0). **(1)**

4. Our problem is to minimise and maximise

$$Z = x + 2y \quad \text{...(i)}$$

Subject to constraints,

$$x + 2y \geq 100 \quad \text{...(ii)}$$

$$2x - y \leq 0 \quad \text{...(iii)}$$

$$2x + y \leq 200 \quad \text{...(iv)}$$

and $x \geq 0, y \geq 0$ ...(v)

Table for line $x + 2y = 100$ is

x	0	100
y	50	0

So, the line $x + 2y = 100$ is passing through the points (0, 50) and (100, 0).

On putting (0, 0) in the inequality $x + 2y \geq 100$, we get

$$0 + 2 \times 0 \geq 100$$

$\Rightarrow$ $0 \geq 100$ (which is false)

So, the half plane is away from the origin.

Table for line $2x - y = 0$ is

x	0	10
y	0	20

So, the line $2x - y = 0$ is passing through the points (0, 0) and (10, 20).

On pu... ng (5, 0) in the inequality $2x - y \leq 0$, we get

$$2 \times 5 - 0 \leq 0$$

$\Rightarrow$ $10 \leq 0$ (which is false)

So, the half plane is towards Y-axis.

Table for line $2x + y = 200$ is

x	0	100
y	200	0

(1)

So, the line $2x + y = 200$ is passing through the points (0, 200) and (100, 0).

On putting (0, 0) in the inequality $2x + y \leq 200$, we get

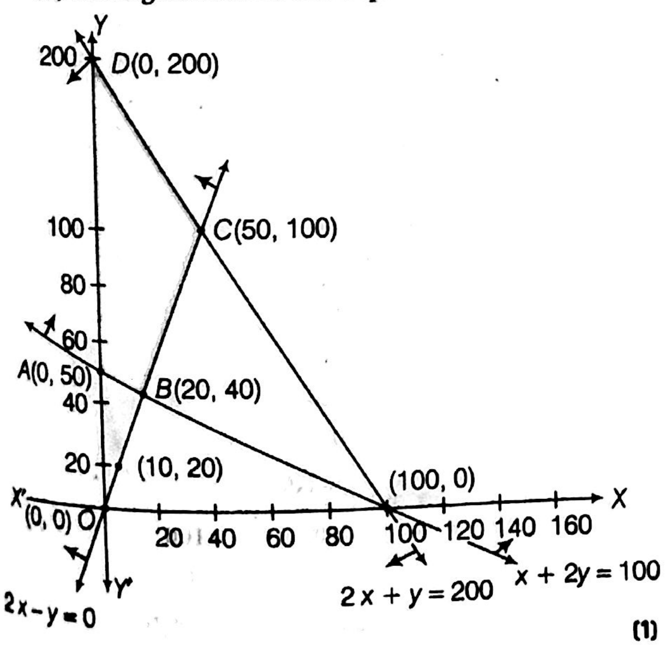
$$2 \times 0 + 0 \leq 200$$

$\Rightarrow$ $0 \leq 200$ (which is true)

So, the half plane is towards the origin.

Also, $x, y \geq 0$.

So, the region lies in the I quadrant.



(1)

Clearly, feasible region is $ABCDA$.

On solving equations $2x - y = 0$ and $x + 2y = 100$, we get $B(20, 40)$.

Again, solving the equations $2x - y = 0$ and $2x + y = 200$, we get $C(50, 100)$.

The corner points of the feasible region are $A(0, 50)$, $B(20, 40)$, $C(50, 100)$ and $D(0, 200)$. **(1)**

The values of Z at corner points are given below

Corner points	$Z = x + 2y$
$A(0, 50)$	$Z = 0 + 2 \times 50 = 100$
$B(20, 40)$	$Z = 20 + 2 \times 40 = 100$
$C(50, 100)$	$Z = 50 + 2 \times 100 = 250$
$D(0, 200)$	$Z = 0 + 2 \times 200 = 400$

The maximum value of Z is 400 at $D(0, 200)$ and the minimum value of Z is 100 at all the points on the line segment joining $A(0, 50)$ and $B(20, 40)$. **(1)**

5. Let the company produce x items of A model and y items of B model.

Maximize profit is $P = 15x + 10y$

Now, total time spent by 5 skilled men = $2x + y$ and it should be less than 40.

$\therefore$ $2x + y \leq 40$ ...(i)

Also, the total time spent by 10 semi-skilled men = $2x + 3y$ and it should be less than 80.

$\therefore$ $2x + 3y \leq 80$ ...(ii) **(1)**

Also, $x \geq 0$ and $y \geq 0$

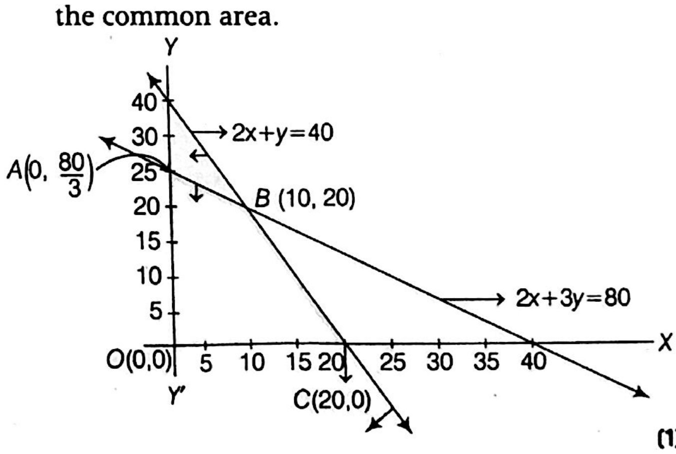
Now, our problem is to maximise $Z = 15x + 10y$

Subject to the constraints

$$2x + y \leq 40$$
$$2x + 3y \leq 80$$

and $x, y \geq 0$ **(1)**

Let us draw all the 4 lines on the graph and find the common area.



(1)

From above we get the region $OABC$ is the feasible region with $A\left(0, \frac{80}{3}\right)$, $B(10, 20)$, $C(20, 0)$, $O(0, 0)$. **(1)**

Since, the feasible is a bounded region, we can check the profit function at all the vertices to find the maxima.

At point $A : Z = 15(0) + 10\left(\frac{80}{3}\right) = \frac{800}{3}$

At point $B : Z = 15(10) + 10(20) = 350$

At point $C : Z = 15(20) + 10(0) = 300$

At point $O : Z = 15(0) + 10(0) = 0$ **(1)**

Thus, the maxima lies at point B having coordinates $x = 10$ and $y = 20$ and the maximum profit = ₹ 350.

Hence, the manufacturer should makes 10 items of A model and 20 items of B model for maximum profit of ₹ 350. **(1)**

6. Let number of goods $A = x$ units,

and number of goods $B = y$ units

Now, the given LPP is to maximise profit :

$$P = 40x + 50y$$

Subject to following constraints

$$3x + y \leq 9$$

$$x + 2y \leq 8$$

and $x \geq 0, y \geq 0$ **(1)**

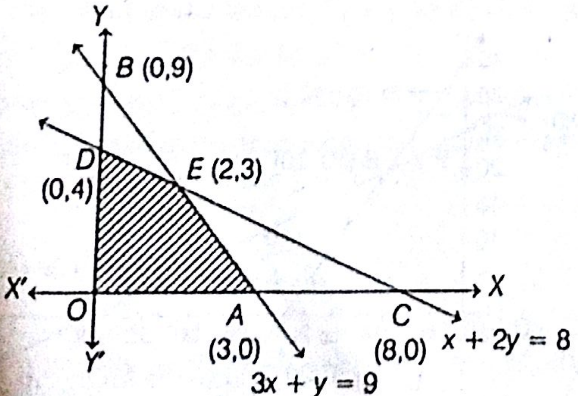
To solve this LPP, we first draw the following lines

$3x + y = 9$ and $x + 2y = 8$

The line $3x + y = 9$ meets the coordinate axes at $A(3, 0)$ and $B(0, 9)$. Join these points to obtain the line represented by $3x + y = 9$.

The line $x + 2y = 8$ meets the coordinate axes at $C(8, 0)$ and $D(0, 4)$. Join these points to obtain the line represented by $x + 2y = 8$. **(1)**

Clearly, $x \geq 0$ and $y \geq 0$ represents the first quadrant.



(1)

Thus, the shaded region in figure is the feasible region of the LPP. The coordinates of the corner points of this region are $O(0, 0)$, $A(3, 0)$, $E(2, 3)$ and $D(0, 4)$, where the point $E(2, 3)$ is obtained by solving $3x + y = 9$ and $x + 2y = 8$ simultaneously. **(1)**

The value of the objective function $P = 40x + 50y$ at the corner points of the feasible region are given in the following table:

Points (x, y)	Value of the objective function $P = 40x + 50y$
$O(0, 0)$	$P = 40 \times 0 + 50 \times$ [illegible]
$A(3, 0)$	$P = 40 \times 3 + 50 \times 0 =$ ₹ 120
$E(2, 3)$	$P = 40 \times 2 + 50 \times 3 =$ ₹ 230
$D(0, 4)$	$P = 40 \times 0 + 50 \times 4 =$ ₹ 200

(1)

Clearly, P is maximum at $x = 2$ and $y = 3$

$\therefore$ The maximum value of P is ₹ 230. **(1)**

7. Let x be the number of packets of screw 'A' and y be the number of packets of screw 'B'.

Then, we have the following table from the given data.

Machines	Packet of screws		Availability
	$A(x)$	$B(y)$	
Automatic machine	4 min	6 min	240 min
Hand-operated machine	6 min	3 min	240 min

(1)

Now, the mathematical model of the given problem is

Maximize $Z = 0.7x + y$

Subject to the constraints,

$$4x + 6y \leq 240$$

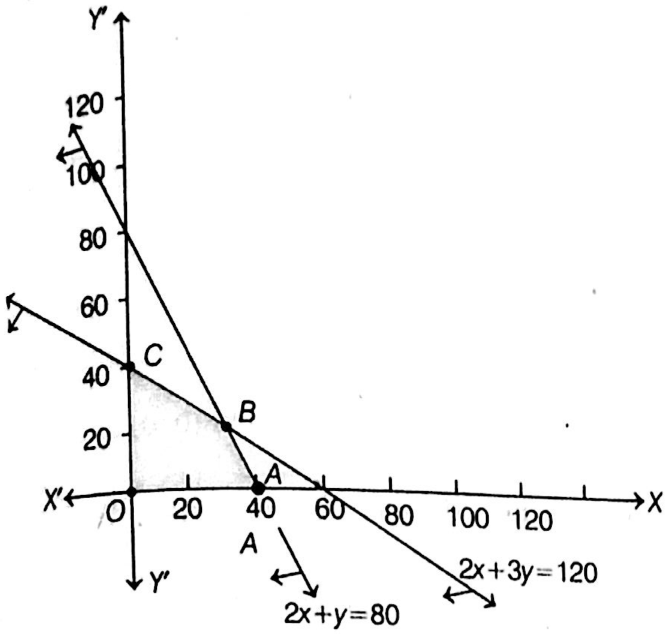
or $2x + 3y \leq 120$ ...(i)

$$6x + 3y \leq 240$$

or $2x + y \leq 80$ ...(ii)

and $x, y \geq 0$ ...(iii) **(1)**

Now let us draw the graph of inequalities (i) to (iii).



(1)

Clearly, the feasible region is the shaded region, whose corner points are O, A, B and C. The coordinates of O, A and C are (0, 0), (40,0) and (0, 40) respectively. (1)

Let us find the coordinates of B which is the intersection point of $2x + y = 80$ and $2x + 3y = 120$.

On solving these two equations, we get

$$x = 30 \text{ and } y = 20$$

Thus, the coordinates of B are (30, 20).

Now, let us find the value of Z at corner points, as shown in the following table. (1)

Corner Points	Value of $Z = 0.7x + y$
O (0, 0)	0
A (40,0)	28
B (30, 20)	41 ← maximum
C (0, 40)	40

We find that maximum value of Z is 41 at B (30, 20).

Hence, the manufacturer should produce 30 packets of screw A and 20 packets of screw B to get a maximum profit of ₹41. (1)

8. Let the manufacturer produces x units of product A and y units of product B.

Let us construct the following table.

Products	No. of Product (in units)	Time on Machine I (in h)	Time on Machine II (in h)	Profit
A	x	$3x$	$3x$	$7x$
B	y	$2y$	$1y$	$4y$
Total	$x + y$	$3x + 2y$	$3x + y$	$7x + 4y$
Availability		12	9	

(1)

Here, total profit $Z = 7x + 4y$

So, our problem is to maximise $Z = 7x + 4y$ ...(i)

Subject to the constraints,

$$3x + 2y \le 12$$

$$3x + y \le 9$$

and $x \ge 0, y \ge 0$

Now, consider the given inequations as equations

$$3x + 2y = 12 \quad \text{... (ii)}$$

$$3x + y = 9 \quad \text{...(iii)}$$ (1)

Table for line $3x + 2y = 12$ or $y = \dfrac{12 - 3x}{2}$ is

x	0	4
y	6	0

It passes through the points (0, 6) and (4, 0).

On putting (0, 0) in the inequality $3x + 2y \le 12$, we get

$$0 + 0 \le 12$$

$\Rightarrow$ $0 \le 12$ [true]

So, the half plane is towards the origin.

Table for line $3x + y = 9$ or $y = 9 - 3x$ is

x	0	3
y	9	0

It passes through the points (0, 9) and (3, 0).

On putting (0, 0) in the inequality $3x + y \le 9$, we get

$$0 + 0 \le 9$$

$\Rightarrow$ $0 \le 9$ [true]

So, the half plane is towards the origin.

Also, $x \ge 0$ and $y \ge 0$, so the region lies in Ist quadrant. (1)

Now, on subtracting Eq. (iii) from Eq. (ii), we get

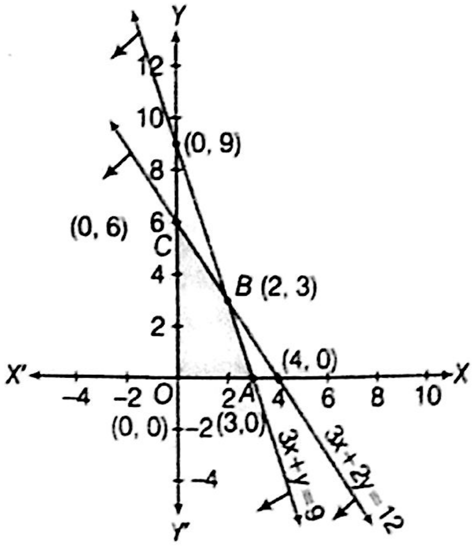
$(3x + 2y) - (3x + y) = 12 - 9$

$\Rightarrow \quad y = 3$

Now, $3x = 12 - 2y = 12 - 2 \times 3 = 6 \Rightarrow x = 2$

Thus, the point of intersection is $B(2, 3)$ **(1)**

The graph of above inequations is shown below



Here, we see that $OABC$ is the required feasible region, whose corner points are $O(0,0)$, $A(3, 0)$, $B(2, 3)$ and $C(0, 6)$. **(1)**

The values of Z at these corner points are as follows

Corner points	$Z = 7x + 4y$
$O(0,0)$	$Z = 0 + 0 = 0$
$A(3,0)$	$Z = 7 \times 3 + 0 = 21$
$B(2,3)$	$Z = 7 \times 2 + 4 \times 3 = 26$ (maximum)
$C(0,6)$	$Z = 7 \times 0 + 4 \times 6 = 24$

Hence, for maximum profit, the manufacturer should produce 2 units of product A and 3 units of product B. **(1)**

9. Let the amounts invested by the person in bonds A and B are respectively, ₹ x and ₹ y.

Our problem is to maximise

$Z = 10\%$ of $x + 9\%$ of y or $Z = 0.1x + 0.09y$

Subject to constraints, $x + y = 50000$

$x \geq y$ or $x - y \geq 0$

$x \geq 20000$ and $y \geq 10000$ **(1)**

Now, consider the given inequations as equations

$x + y = 50000$ ... (i)

$x - y = 0$ ... (ii)

$x = 20000$ ... (iii)

and $y = 10000$ ... (iv)

The table for line $x + y = 50000$ is

x	0	50000
y	50000	0

∴ It passes through the points (0, 50000) and (50000, 0).

The table for line $x - y = 0$ is

x	0	20000
y	0	20000

∴ It passes through the points (0, 0) and (20000, 20000).

On putting (10000, 0) in the inequality $x \geq y$, we get

$10000 \geq 0$ (true)

So, the half plane is towards the X-axis. **(1)**

The line $x = 20000$ is perpendicular to the X-axis.

On putting (0, 0) in the inequality $x \geq 20000$, we get

$0 \geq 20000$ (false)

So, the half plane is away from the origin.

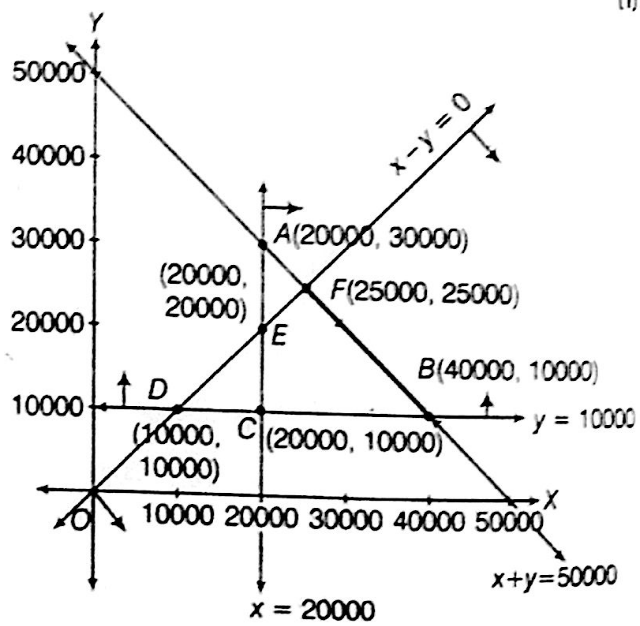
The line $y = 10000$ is perpendicular to the Y-axis.

On putting (0, 0) in the inequality $y \geq 10000$, we get

$0 \geq 10000$ (false)

So, the half plane is away from the origin. **(1)**

Now, draw the graph of given system of inequalities as shown below. **(1)**



(1)

From the graph, it is clear that feasible region lies on the line segment BF. The corner points of feasible region are B (40000, 10000) and F(25000, 25000).

The values of Z at corner points are given below

Corner Points	$Z = 0.1x + 0.09y$
$B(40000, 10000)$	$Z = 0.1 \times 40000 + 0.09 \times 10000$ $= 4900$ (maximum)
$F(25000, 25000)$	$Z = 0.1 \times 25000 + 0.09 \times 25000$ $= 4750$

Hence, to get a maximum returns, he has to invest ₹ 40000 in bond A and ₹ 10000 in bond B. (1)

10. Let the farmer uses x kg of fertiliser A and y kg of fertiliser B. The given data can be summarised as follows

	Fertiliser A	Fertiliser B	Minimum requirement (in kg)
Nitrogen (in%)	12	4	12
Phosphoric acid (in %)	5	5	12
Cost (in ₹/kg)	10	8	

(1)

The inequations thus formed based on the given problem are as follows:

$$\frac{12x}{100} + \frac{4y}{100} \geq 12$$

$$\Rightarrow \quad 12x + 4y \geq 1200$$

$$\Rightarrow \quad 3x + y \geq 300$$

Also, $$\frac{5x}{100} + \frac{5y}{100} \geq 12$$

$$\Rightarrow \quad 5x + 5y \geq 1200$$

$$\Rightarrow \quad x + y \geq 240$$

and $$x \geq 0, y \geq 0$$

Let Z be the total cost of the fertilisers. Then,
$Z = 10x + 8y$

The LPP can be stated mathematically as

Minimise $Z = 10x + 8y$

Subject to constraints $3x + y \geq 300$, $x + y \geq 240$, $x \geq 0, y \geq 0$ (1)

To solve the LPP graphically, let us convert the inequations into equations as follows :

$$3x + y = 300 \quad \ldots(i)$$

$$x + y = 240 \quad \ldots(ii)$$

Table for line $3x + y = 300$ is

x	0	100
y	300	0

So, it passes through (0,300) and (100,0).

On putting (0,0) in the inequality $3x + y \geq 300$, we get $3(0) + 0 \geq 300 \Rightarrow 0 \geq 300$ (which is false)

So, the half plane is away from origin (1)

Table for line $x + y = 240$ is

x	0	240
y	240	0

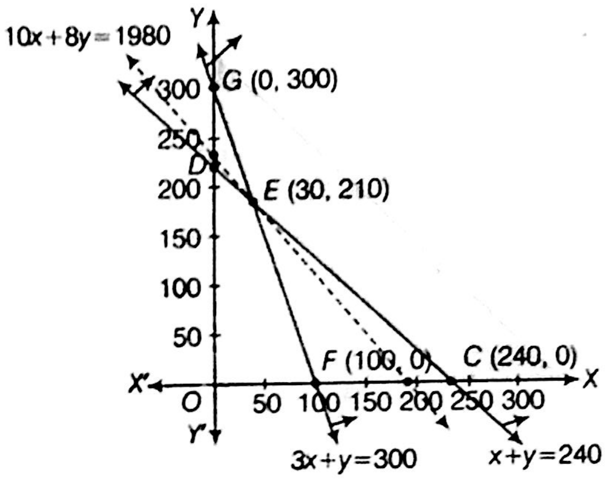
So, it passes through (240, 0) and (0, 240).

On putting (0, 0) in $x + y \geq 240$, we get $0 + 0 \geq 240$ (which is false)

So, the half plane is away from origin.

Also, $x \geq 0$ and $y \geq 0$, so the region lies in Ist quadrant. (1)

The graph of inequations is shown below



The shaded region GEC represents the feasible region of given LPP and it is unbounded.

On solving Eqs. (i) and (ii), we get

$x = 30$ and $y = 210$

So, the point of intersection is $E(30, 210)$.

Corner points	Value of $Z = 10x + 8y$
$G(0, 300)$	2400
$C(240, 0)$	2400
$E(30, 210)$	1980 (minimum)

(1)

From the table, we find that 1980 is the minimum value of Z at $E(30, 210)$. Since, the region is unbounded, therefore, 1980 may or may not be the minimum value of Z. For this we have to check that the open half plane $10x + 8y < 1980$ has any point common or not with the feasible region. Since, it has no point in common with the feasible region. So, the minimum value of Z is obtained at $E(30, 210)$ and the miniumum value of Z is 1980. So, the farmer should use 30 kg of fertiliser A and 210 kg of fertiliser B. (1)