Solution to very important questions

Explanations

1. As the needle is displaced from the equilibrium position, the torque will try to bring it back in equilibrium position, hence acceleration will be related with negative of angular displacement.

When compass needle of magnetic moment M and moment of inertia I is slightly disturbed by an angle θ from the mean position of equilibrium.

Then, restoring torque begin to act on the needle which try to bring the needle back to its mean position which is given by

$$\tau = -MB\sin\theta$$

Since, θ is small

So, $\sin\theta \approx \theta$

$\therefore \quad \tau = -MB\theta \quad \ldots(\text{i})$

But $\quad \tau = I\alpha \quad \ldots(\text{ii})$

where, α = angular acceleration

and M = magnetic moment of dipole.

On comparing Eqs. (i) and (ii), we get

$\Rightarrow \quad I\alpha = -MB\theta$

$\Rightarrow \quad \alpha = -(MB/I)\theta$

$\therefore \quad \alpha \propto -\theta$ (1)

$\Rightarrow$ Angular acceleration $\propto$ – Angular displacement

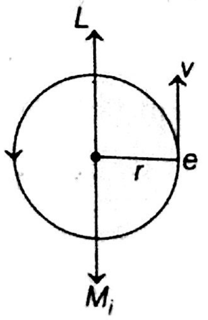
$\Rightarrow$ Therefore, needle execute SHM.

Hence, time period,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{MB/I}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{I}{MB}}$$ (1)

This is the required expression.

2. According to Bohr's model of atom, negatively charged electron revolves around the positively charged nucleus. This is same as that of a current loop of dipole moment = IA. Let the electron is moving in a circle with speed v in anti-clockwise direction of radius r and time period is T.



Current, $I = \frac{e}{T} = \frac{e}{2\pi r / v} = \frac{ev}{2\pi r}$ (1)

Area of loop $= \pi r^2$

$\therefore$ Orbital magnetic moment of electron is

$$M_l = IA = \frac{ev}{2\pi r} \times \pi r^2 = \frac{evr}{2}$$

The angular momentum of electron due to orbital motion is

$$L = m_e vr$$

It is directed upward in perpendicular direction to the plane.

$$\frac{M_l}{L} = \frac{evr/2}{m_e vr} = \frac{e}{2m_e}$$

This ratio is constant called gyromagnetic ratio. Its value is $8.8 \times 10^{10} \text{C kg}^{-1}$, so

$$M_l = \frac{e}{2m_e} L$$

The vector form

$$M_l = -\frac{e}{2m_e}\mathbf{L}$$ (1)

The negative sign shows that the direction of **L** is opposite to M_l. According to Bohr's quantisation condition, the angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$.

$$\therefore \quad L = \frac{nh}{2\pi}$$

$$\Rightarrow \quad M_l = n\left(\frac{eh}{4\pi m_e}\right)$$

This is the equation of magnetic moment of an electron revolving in nth orbit. (1)

3. (a) Refer to text on page 163 (Magnetism and Gauss' Law). (1/2)

(b) Refer to text on page 161 (Properties of Magnetic Field Lines). (1/2)

4. (a) Given, magnetic moment, $M = 6$ J/T

Aligned angle, $\theta_1 = 60°$

External magnetic field,

$$B = 0.44 \text{ T}$$

(i) When the bar magnet is align normal to the magnetic field, i.e. $\theta_2 = 90°$

$\therefore$ Amount of work done in turning the magnet,

$$W = -MB(\cos\theta_2 - \cos\theta_1)$$
$$= -6 \times 0.44(\cos 90° - \cos 60°)$$
$$= +6 \times 0.44 \times \frac{1}{2} \quad \left(\begin{array}{l}\because \cos 90° = 0 \\ \text{and } \cos 60° = 1/2\end{array}\right)$$
$$= 1.32 \text{ J}$$ (1)

(ii) When the bar magnet align opposite to the magnetic field, i.e. $\theta_2 = 180°$

$$\therefore W = -MB(\cos 180° - \cos 60°)$$
$$= -6 \times 0.44\left(-1 - \frac{1}{2}\right) \quad (\because \cos 180° = -1)$$
$$= 6 \times 0.44 \times \frac{3}{2}$$
$$= 3.96 \text{ J}$$ (1)

(b) We know that, torque,

$$\tau = \mathbf{M} \times \mathbf{B} = MB\sin\theta$$

For case (ii), $\theta = 180°$

$$\therefore \quad \tau = MB\sin 180° \quad (\because \sin 180° = 0)$$
$$= 0$$

$\therefore$ Amount of torque is zero for case (ii). (1)

Explanations

1. The susceptibility of magnetic material is inversely proportional to temperature, i.e.

$$\chi_m \propto \frac{1}{T}$$

$$\therefore \qquad \frac{\chi_m(T)}{\chi_m\,(300\,\text{K})} = \frac{300}{T}$$

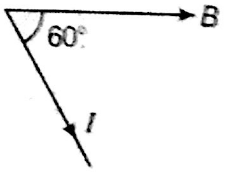
$$\Rightarrow \qquad T = \frac{300 \times 1.2 \times 10^5}{1.44 \times 10^5}$$

$$= 250\ \text{K}$$

(1)

2. Substance having (small) negative value (−0.5) of magnetic susceptibility χ_m are diamagnetic. (1)

3.



I is the total magnetic field.

Now, $I \cos 60^\circ = B$

$$\Rightarrow \quad I = \frac{B}{\cos 60^\circ} = \frac{B}{1/2} = 2B$$

At equator, dip angle is 0°.

$$\therefore \quad B_H = I\cos 0^\circ = I = 2B.$$ **(1)**

4. When paramagnetic materials are placed in external magnetic field, these are feebly magnetised in the direction of the applied external magnetic field whereas in case of diamagnetic materials, these are feebly magnetised opposite to that of applied external magnetic field. **(1)**

5. The nature of magnetic material is a diamagnetic. The relation between relative permeability and magnetic susceptibility is

$$\mu_r = 1 + \chi_m$$ **(1)**

6. Permanent magnets are those magnets which have high retentivity and coercivity. The magnetisation of permanent magnet is not easily destroyed even if it is handled roughly or exposed in stray reverse magnetic field, e.g. steel. **(1)**

7. At equator, vertical component of earth's magnetic field will be zero. **(1)**

8. Horizontal component of earth's magnetic field,

$$H = B_e \cos 60^\circ = B \qquad \text{(given)}$$

$$\Rightarrow \quad B_e \times \frac{1}{2} = B \quad \text{or} \quad B_e = 2B$$

Vertical component of earth's magnetic field,

$$V = B_e \sin 60^\circ \Rightarrow V = 2B \times (\sqrt{3}/2)$$

$$\Rightarrow \quad V = \sqrt{3}B$$ **(1)**

9. The angle of dip is given by

$$\delta = \tan^{-1}(B_V/B_H)$$

B_V = vertical component of the earth's magnetic field.

B_H = horizontal component of the earth's magnetic field.

So, as $B_V = B_H$

Then, $\delta = \tan^{-1}(1) = 45^\circ$

∴ The angle of dip will be $\delta = 45^\circ$. **(1)**

10. (i) The needle is free to move in vertical plane, it means that there is no component of the earth's magnetic field in horizontal direction, so the horizontal component of the earth's magnetic field is zero. **(1/2)**

(ii) The angle of dip is 0°. **(1/2)**

11. At poles, the angle of dip is 90°. **(1)**

12. The magnetic material is diamagnetic substance for which $\mu_r < 1$. **(1)**

13. The small and positive susceptibility of 1.9×10^{-5} represents paramagnetic substance. **(1)**

14. Negative susceptibility represents diamagnetic substance. **(1)**

15. Diamagnetic material acquires feeble magnetisation in the opposite direction of the magnetic field when they are placed in an external magnetic field. **(1)**

16. (i) The magnetic susceptibility of a magnetic material is defined as the ratio of the intensity of magnetisation (I) to the magnetic intensity (H).

i.e., $$\chi_m = \frac{I}{H}$$

Relation between magnetic susceptibility (χ_m) and relative magnetic permeability (μ_r) is given as **(1)**

$$\mu_r = 1 + \chi_\mu$$

(ii) For material $A, \mu_r = 0.96 < 1$

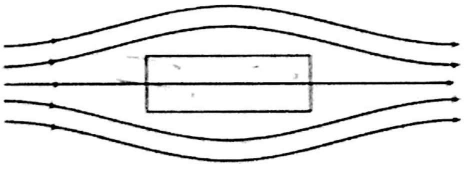
Hence, magnetic materia lA is diamagnetic.

For material $B, \mu_r = 500$

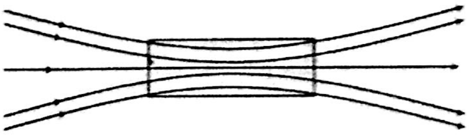
Since, μ_r is much greater than 1 for material B, therefore B is ferromagnetic material. **(1)**

17. Magnetic permeability of paramagnetic is more than air, so it allows more lines to pass through it while permeability of diamagnetic is less than air, so it does not allow lines to pass through it.

(i) Behaviour of magnetic field lines when diamagnetic substance is placed in an external field.



(ii) Behaviour of magnetic field lines when paramagnetic substance is placed in a external field.



Magnetic susceptibility distinguishes the behaviour of the field lines due to diamagnetic and paramagnetic substance. **(1)**

This difference can be explained as diamagnetic substances repel or expel the magnetic field lines while paramagnetic substance attract the magnetic field lines. **(1)**

18. The nature of the material A is paramagnetic and its susceptibility χ_m is positive.

The nature of the material B is diamagnetic and its susceptibility χ_m is negative. **(2)**

19.

Paramagnetic substance	Diamagnetic substance
A paramagnetic substance is feebly attracted by magnet.	A diamagnetic substance is feebly repelled by a magnet.
For a paramagnetic substance, the intensity of magnetisation has a small positive value.	For a diamagnetic substance, the intensity of magnetism has a small negative value. **(2)**

20. (i) An electromagnet consist of a core made of a ferromagnetic material placed inside a solenoid. It behaves like a strong magnet when current flows through the solenoid and effectively loses its magnetism when the current is switched off.

A permanent magnet is also made up of a ferromagnetic material but it retains its magnetism at room temperature for a long time after being magnetised one. **(1)**

(ii) *Properties of material are as below:*

(a) High permeability (c) Low retentivity

(b) Low coercivity **(1)**

21. Ferromagnetic substance are those substances which have very high magnetic permeability. **(1)**

Properties (i) High retentivity

(ii) High susceptibility ($\chi_m > 1000$)

(2 × 1/2)

22. (i) For diamagnetic substances, the variation of susceptibility is very small ($0 < \chi_m < \varepsilon$), i.e. diamagnetic materials are unaffected by the change in temperature (except bismuth). **(1)**

(ii) Paramagnetic materials when cooled due to thermal agitation tendency alignment of magnetic dipoles decreases. Hence, they shows greater magnetisation. **(1)**

23. (i) Magnetic lines of force come out from North pole and enter into the South pole outside the magnet and travels from South pole to North pole inside the magnet. So, magnetic lines of force form closed loop, magnetic monopoles do not exist. **(1)**

(ii) The diamagnetic material gets slightly magnetised in a direction opposite to external field, therefore lines of force are repelled by diamagnetic material. **(1)**

NOTE When South pole of the magnet is viewed from the frame of reference, then inside the magnet, it appears as North pole and *vice-versa*. Due to this reason, magnetic field lines are traversed from South pole to North pole inside the magnet.

24. Angle of dip, $\delta = 60° = \pi/3$

Horizontal component of the earth's magnetic field, $H = 0.4\text{ G}$

Earth magnetic field $(B_e) = ?$

$\because$ Horizontal component of the earth's magnetic field, $H = B_e \cos\delta$ **(1)**

$$\Rightarrow \quad B_e = \frac{H}{\cos\delta} = \frac{0.4\text{ G}}{\cos 60°} = \frac{0.4\text{ G}}{(1/2)} = 0.8\text{ G}$$

$$\therefore \quad B_e \doteq 0.8\text{ G}$$ **(1)**

25. (i) The earth's magnetic field at a place can be completely described by three parameters which are called elements of earth's magnetic field. They are as follows **(1)**

(a) Angle of declination (θ)

(b) Angle of dip (δ) or magnetic inclination

(c) Horizontal component of earth's magnetic field (H_e).

(ii) At the magnetic equator, the dip needle rests horizontally, so that the angle of dip is zero at the magnetic equator. **(1)**

26. (i) **Susceptibility for diamagnetic material**

It is independent of magnetic field and temperature (except for bismuth at low temperature). **(1)**

Susceptibility for ferromagnetic material

The susceptibility of ferromagnetic materials decreases steadily with increase in temperature. At the Curie temperature, the ferromagnetic materials become paramagnetic.

(ii) **Behaviour in non-uniform magnetic field**

Diamagnets are feebly repelled, whereas ferromagnets are strongly attracted by non-uniform field, i.e. diamagnets move in the direction of decreasing field, whereas ferromagnet feels force in the direction of increasing field intensity. **(1)**

27. (i) Two characteristics of material used for making permanent magnets are

(a) high coercivity

(b) high retentivity and high hysteresis loss.

(2 × 1/2)

(ii) Core of an electromagnet made of ferromagnetic material because of its

(a) low coercivity

(b) low hysteresis loss **(2 × 1/2)**

28. According to the question, $H = \sqrt{3}V$
where, H and V are the horizontal and vertical components of the earth's magnetic field.
If angle of dip at that place is δ, then

$$\tan\delta = (V/H) = (V/\sqrt{3}V) \quad [\because H = \sqrt{3}V]$$

$$\tan\delta = \frac{1}{\sqrt{3}} \Rightarrow \delta = \frac{\pi}{6}$$ **(1)**

$\because$ Horizontal component of the earth's magnetic field, $H = B_e \cos\delta$

where, B_e = Earth's magnetic field

$$\frac{H}{B_e} = \cos\delta = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad H : B_e = \sqrt{3} : 2$$ **(1)**

29. Refer to Sol. 28 on page 173 (Ans. $1 : \sqrt{2}$). **(2)**

30. (i) Refer to Sol. 17 on pages 171 and 172.
(ii) Refer to Sol. 17 on pages 171 and 172. **(1)**
Magnetic susceptibility distinguishes the behaviour of the field lines due to diamagnetic and paramagnetic substance. **(1)**

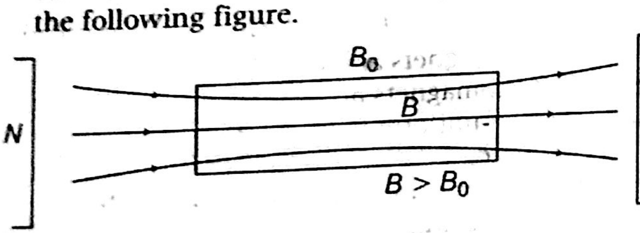
31. Difference between para-, dia- and ferro-magnetic materials
Refer to text on page 167.

32. Given, susceptibility, $\chi_m = 0.9853$

As the susceptibility of material is positive but small.

$\therefore$ The material is paramagnetic in nature. For paramagnetic material, magnetic lines of external magnetic field will passes through the material without much deviation, when it is placed in between magnetic poles.

The modification of the field pattern is shown in the following figure.



(3)

33. $\because$ Horizontal component,
$H = B\cos\theta = 0.4 \cos 60° = 0.4 \times (1/2) = 0.2\text{G}$
$H = 0.2 \times 10^{-4}$ T $\quad [\because \cos 60° = 1/2]$

This component is parallel to the plane of wheel. The wheel is rotating in a plane normal to the horizontal component, so it will cut the horizontal component only, vertical component of earth will contribute nothing in emf. **(1)**
Thus, the emf induced is given as

$$E = \frac{1}{2}Hl^2\omega$$

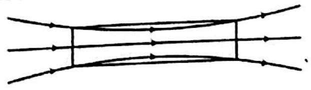
where, $\omega = 2\pi N/t$ and
l = length of the spoke = 50 cm = 0.5 m

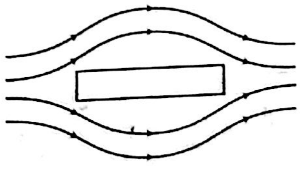
$$\therefore \quad E = \frac{1}{2} \times 0.2 \times 10^{-4} \times (0.5)^2 \times \frac{2 \times 3.14 \times 120}{60}$$

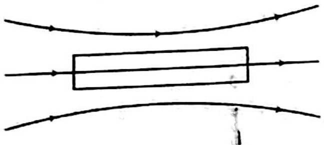
$$E = 3.14 \times 10^{-5}\ \text{V}$$ **(1)**

The value of emf induced is independent of the number of spokes as the emf's across the spokes are in parallel. So, the emf will be unaffected with the increase in spokes. **(1)**

34. The modifications are shown in the figure.







(½ × 3 = ½)

It happens because
(i) nickel is a ferromagnetic substance.
(ii) antimony is a diamagnetic substance.
(iii) aluminium is a paramagnetic substance.
(½ × 3 = 1 ½)

35. The torque always tries to bring back the needle in equilibrium position i.e. parallel to the existing field.

(i) The torque on the needle is $\tau = M \times B$
In magnitude, $\tau = MB\sin\theta$
Here, τ is restoring torque and θ is the angle between M and B.
Therefore, in equilibrium,
Restoring force = Deflecting torque

$$I\frac{d^2\theta}{dt^2} = -MB\sin\theta$$ **(1)**

Negative sign with $MB\sin\theta$ implies that restoring torque is in opposition to deflecting torque.
For small values of θ in radians, we approximate $\sin\theta = \theta$ and get

$$I\frac{d^2\theta}{dt^2} = -MB\theta \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{MB}{I}\theta$$

$$\Rightarrow \quad d^2\theta/dt^2 = -\omega^2\theta$$ **(1)**

This equation represents a simple harmonic motion. The square of the angular frequency is

$$\omega^2 = MB/I$$

i.e. $\omega^2 = MB/I$ or $\omega = \sqrt{MB/I}$

Time period, $T = 2\pi/\omega = 2\pi\sqrt{I/MB}$ **(1)**

(ii) (a) As, horizontal component of earth's magnetic field, $B_H = B\cos\delta$

Putting $\delta = 90°$ (as compass needle orients itself vertically)

$\therefore \quad B_H = 0$

(b) For a compass needle oriented itself with its axis vertical at a certain place, angle of dip, $\delta = 90°$. **(2)**