probability(CBSE Solutions)

Solutions

1. Given, $P(A') = 0.7$, $P(B) = 0.7$ and $P\left(\frac{B}{A}\right) = 0.5$

Clearly, $P(A) = 1 - P(A') = 1 - 0.7 = 0.3$

Now, $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$ **(1)**

$\Rightarrow \quad 0.5 = \frac{P(A \cap B)}{0.3}$

$\Rightarrow \quad P(A \cap B) = 0.15$

$\therefore \quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.7} \Rightarrow P\left(\frac{A}{B}\right) = \frac{3}{14}$ **(1)**

2. When a die is thrown, the sample space is

$S = \{1, 2, 3, 4, 5, 6\}$

$\Rightarrow \quad n(S) = 6$

Also, A : number is even and B : number is red.

$\therefore \; A = \{2, 4, 6\}$ and $B = \{1, 2, 3\}$ and $A \cap B = \{2\}$

$\Rightarrow n(A) = 3, n(B) = 3$ and $n(A \cap B) = 1$

Now, $\quad P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

$P(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$ **(1)**

and $\quad P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{6}$

Now, $\quad P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq \frac{1}{6} = P(A \cap B)$

$\therefore \quad P(A \cap B) \neq P(A) \times P(B)$

Thus, A and B are not independent events. **(1)**

3. Let us denote the numbers on black die by $B_1, B_2, \ldots, B_6$ and the numbers on red die by $R_1, R_2, \ldots, R_6$.

Then, we get the following sample space.

$$S = \begin{Bmatrix} (B_1, R_1), (B_1, R_2), \ldots, (B_1, R_6), (B_2, R_1), (B_2, R_2) \\ \ldots, (B_2, R_6), \ldots, (B_6, R_1), (B_6, R_2), \ldots, (B_6, R_6) \end{Bmatrix}$$

Clearly, $n(S) = 36$ **(1/2)**

Now, let A be the event that sum of number obtained on the die is 8 and B be the event that red die shows a number less than 4.

Then, $A = \{(B_2, R_6), (B_6, R_2), (B_3, R_5), (B_5, R_3), (B_4, R_4)\}$

and $\quad B = \begin{Bmatrix} (B_1, R_1), (B_1, R_2), (B_1, R_3), (B_2, R_1), (B_2, R_2), \\ (B_2, R_3), \ldots\ldots, (B_6, R_1), (B_6, R_2), (B_6, R_3) \end{Bmatrix}$

$\Rightarrow \quad A \cap B = \{(B_6, R_2), (B_5, R_3)\}$ **(1/2)**

Now, required probability,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{2}{18} = \frac{1}{9}$$ **(1)**

4. We have, $2P(A) = P(B) = \frac{5}{13}$

$\Rightarrow P(A) = \frac{5}{26}$ and $P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$

$\because P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$\therefore \quad \frac{2}{5} = \frac{P(A \cap B)}{5/13}$

$\Rightarrow \quad P(A \cap B) = \frac{2}{5} \times \frac{5}{13} = \frac{2}{13}$ **(1)**

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$

$= \frac{5 + 10 - 4}{26} = \frac{11}{26}$ **(1)**

5. Given, E and F are independent events, therefore

$\Rightarrow \quad P(E \cap F) = P(E)\,P(F) \quad \ldots(i)$

Now, we have,

$P(E \cap F') + P(E \cap F) = P(E)$

$\Rightarrow \quad P(E \cap F') = P(E) - P(E \cap F)$ **(1)**

$\Rightarrow \quad P(E \cap F') = P(E) - P(E)\,P(F)$ [using Eq. (i)]

$\Rightarrow \quad P(E \cap F') = P(E)[1 - P(F)]$

$\Rightarrow \quad P(E \cap F') = P(E)\,P(F')$

$\therefore$ E and F' are also independent events.

Hence proved. **(1)**

6. Here, $n(S) = 6 \times 6 = 36$

Let A = Event of getting a sum of 7 in pair of dice
$= \{(1, 6), (2, 5), (3, 4), (6, 1), (5, 2), (4, 3)\}$

$\Rightarrow \quad n(A) = 6$ **(1/2)**

and B = Event of getting a sum of 10 in pair of dice $= \{(4, 6), (5, 5), (6, 4)\} \Rightarrow n(B) = 3$

$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

and $\quad P(B) = \frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$ **(1)**

$\Rightarrow \quad P(\bar{A}) = 1 - \frac{1}{6} = \frac{5}{6}$

and $\quad P(\bar{B}) = 1 - \frac{1}{12} = \frac{11}{12}$ **(1/2)**

Now, the probability that if A start the game, then B wins

$$P(B \text{ wins}) = P(\bar{A} \cap B) + P(\bar{A} \cap \bar{B} \cap \bar{A} \cap B) + P(\bar{A} \cap \bar{B} \cap \bar{A} \cap \bar{B} \cap \bar{A} \cap B) + \dots$$

$$= P(\bar{A})\,P(B) + P(\bar{A})P(\bar{B})P(\bar{A})P(B) + P(\bar{A})P(\bar{B})P(\bar{A})P(\bar{B})P(\bar{A})\,P(B) + \dots$$ **(1)**

[$\because$ events are independent]

$$= \frac{5}{6} \times \frac{1}{12} + \frac{5}{6} \times \frac{11}{12} \times \frac{5}{6} \times \frac{1}{12} + \frac{5}{6} \times \frac{11}{12} \times \frac{5}{6} \times \frac{11}{12} \times \frac{5}{6} \times \frac{1}{12} + \dots$$

$$= \frac{5}{72} + \frac{5}{72} \times \frac{55}{72} + \frac{5}{72} \times \left(\frac{55}{72}\right)^2 + \dots$$

$$= \frac{5}{72}\left[1 + \frac{55}{72} + \left(\frac{55}{72}\right)^2 + \dots\right] = \frac{5}{72}\left(\frac{1}{1 - \frac{55}{72}}\right)$$

$\left[\because \text{sum of infinite GP series is } \frac{a}{1-r}\right]$

$$= \frac{5}{72}\left(\frac{1}{\frac{17}{72}}\right) = \frac{5}{17}$$ **(1)**

7. Here, $n(S) = 6 \times 6 = 36$

Let E = Event of getting a total 10
$= \{(4, 6), (5, 5), (6, 4)\}$

$\therefore n(E) = 3$

$\therefore P(\text{getting a total of 10}) = P(E) = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$ **(1/2)**

and $P(\text{not getting a total of 10}) = P(\bar{E})$
$= 1 - P(E) = 1 - \frac{1}{12} = \frac{11}{12}$

Thus, $P(A \text{ getting } 10) = P(B \text{ getting } 10) = \frac{1}{12}$

and $P(A \text{ is not getting } 10) = P(B \text{ is not getting } 10) = \frac{11}{12}$ **(1/2)**

Now, $P(A \text{ winning}) = P(A) + P(\bar{A} \cap \bar{B} \cap A) + P(\bar{A} \cap \bar{B} \cap \bar{A} \cap \bar{B} \cap A) + \dots$

$$= P(A) + P(\bar{A})P(\bar{B})P(A) + P(\bar{A})P(\bar{B})P(\bar{A})P(\bar{B})P(A) + \dots$$

$$= \frac{1}{12} + \frac{11}{12} \times \frac{11}{12} \times \frac{1}{12} + \frac{11}{12} \times \frac{11}{12} \times \frac{11}{12} \times \frac{11}{12} \times \frac{1}{12} + \dots$$ **(1)**

$$= \frac{1}{12}\left[1 + \left(\frac{11}{12}\right)^2 + \left(\frac{11}{12}\right)^4 + \dots\right] = \frac{1}{12}\left[\frac{1}{1 - \left(\frac{11}{12}\right)^2}\right]$$

$\left[\because \text{the sum of an infinite GP is } S_\infty = \frac{a}{1-r}\right]$

$$= \frac{1}{12}\left[\frac{1}{\frac{144 - 121}{144}}\right] = \frac{12}{23}$$ **(1)**

Now, $P(B \text{ winning}) = 1 - P(A \text{ winning})$
$= 1 - \frac{12}{23} = \frac{11}{23}$ **(1/2)**

Hence, the probabilities of winning A and B are respectively $\frac{12}{23}$ and $\frac{11}{23}$. **(1/2)**

8.

> The problem is solved means atleast one of them solve it. Also, use the concept A and B are independent events, then their complements are also independent.

Let $P(A)$ = Probability that A solves the problem
$P(B)$ = Probability that B solves the problem
$P(\bar{A})$ = Probability that A does not solve the problem
and $P(\bar{B})$ = Probability that B does not solve the problem **(1)**

According to the question, we have

$P(A) = \frac{1}{2}$

then $\quad P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$ $\quad [\because P(A) + P(\bar{A}) = 1]$

and $\quad P(B) = \frac{1}{3}$

then $\quad P(\bar{B}) = 1 - P(B)$
$= 1 - \frac{1}{3} = \frac{2}{3}$

(i) P (problem is solved)

$= P(A\cap\bar{B}) + P(\bar{A}\cap B) + P(A\cap B)$

$= P(A)\cdot P(\bar{B}) + P(\bar{A})\cdot P(B) + P(A)\cdot P(B)$ **(1/2)**

[$\because$ A and B are independent events]

$$=\left(\frac{1}{2}\times\frac{2}{3}\right)+\left(\frac{1}{2}\times\frac{1}{3}\right)+\left(\frac{1}{2}\times\frac{1}{3}\right)$$

$$=\frac{2}{6}+\frac{1}{6}+\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$$

Hence, probability that the problem is solved, is $\frac{2}{3}$. **(1)**

(ii) P (exactly one of them solve the problem)

$= P$ (A solve but B do not solve)

$+ P$ (A do not solve but B solve)

$= P(A\cap\bar{B}) + P(\bar{A}\cap B)$

$= P(A)\cdot P(\bar{B}) + P(\bar{A})\cdot P(B)$

$$=\left(\frac{1}{2}\times\frac{2}{3}\right)+\left(\frac{1}{2}\times\frac{1}{3}\right)=\frac{2}{6}+\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$$ **(1½)**

Alternate Method

P (problem is solved)

$= 1 - P$ (none of them solve the problem)

$= 1 - P(\bar{A}\cap\bar{B})$

$$=1-P(\bar{A})\cdot P(\bar{B})=1-\left(\frac{1}{2}\times\frac{2}{3}\right)$$ **(2)**

$$\left[\because P(\bar{A})=\frac{1}{2}\text{ and }P(\bar{B})=\frac{2}{3}\right]$$

$$=1-\frac{1}{3}=\frac{2}{3}$$

(ii) P (exactly one of them solve the problem)

$= P(A) + P(B) - 2P(A\cap B)$

$= P(A) + P(B) - 2P(A)\times P(A)$

$$=\frac{1}{2}+\frac{1}{3}-2\times\frac{1}{2}\times\frac{1}{3}=\frac{1}{2}$$ **(2)**

9. Firstly, write the sample space of given data. Then, use concept of conditional probability $P(A/B)=\frac{P(A\cap B)}{P(B)}$ to get the desired result.

Let B and b represent older and younger boy child. Also, let G and g represent older and younger girl child. The sample space of the given question is

$$S = \{Bb, Bg, Gg, Gb\}$$

$\therefore$ $n(S) = 4$

Let A be the event that both children are boys.

Then, $A = \{Bb\}$

$\therefore$ $n(A) = 1$ **(1)**

(i) Let B : Atleast one of the children is a boy

$\therefore$ $B = \{Bb, Bg, Gb\}$ and $n(B) = 3$

Now, $P(B)=\frac{n(B)}{n(S)}=\frac{3}{4}$ ...(i)

Here, $A\cap B = \{Bb\}$, then $n(A\cap B) = 1$

$\therefore$ $P(A\cap B)=\frac{n(A\cap B)}{n(S)}=\frac{1}{4}$ ...(ii)

We have to find $P(A/B)$, we know that,

$P(A/B)=\frac{P(A\cap B)}{P(B)}$, $P(B)\neq 0$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{1/4}{3/4}=\frac{1}{3}$$

Hence, the required probability is $\frac{1}{3}$. **(1½)**

(ii) Let C : The older child is a boy.

Then, $C = \{Bb, Bg\}$

$\therefore$ $n(C) = 2$

and $P(C)=\frac{n(C)}{n(S)}=\frac{2}{4}=\frac{1}{2}$ ...(iv)

Here, $A\cap C = \{Bb\}$, then $n(A\cap C) = 1$

and $P(A\cap C)=\frac{n(A\cap C)}{n(C)}=\frac{1}{4}$ ...(v)

Now, $P(A/C)=\frac{P(A\cap C)}{P(C)}=\frac{1/4}{1/2}=\frac{1}{2}$

Hence, the required probability is $\frac{1}{2}$. **(1½)**

10. Let B and b represent elder and younger boy child. Also, G and g represent elder and younger girl child. If a family has two children, then all possible cases are

$$S = \{Bb, Bg, Gg, Gb\}$$ **(1)**

$\therefore$ $n(S) = 4$

Let us define event A : Both children are girls, then $A = \{Gg\} \Rightarrow n(A) = 1$ **(1)**

(i) Let E_1 : The event that youngest child is a girl.

Then, $E_1 = \{Bg, Gg\}$ and $n(E_1) = 2$

so $P(E_1)=\frac{n(E_1)}{n(S)}=\frac{2}{4}=\frac{1}{2}$

and $A\cap E_1 = \{Gg\} \Rightarrow n(A\cap E_1) = 1$

so $P(A\cap E_1)=\frac{n(A\cap E_1)}{n(S)}=\frac{1}{4}$

Now, $P\left(\frac{A}{E_1}\right)=\frac{P(A\cap E_1)}{P(E_1)}=\frac{1/4}{1/2}=\frac{1}{2}$

$\therefore$ Required probability $=\frac{1}{2}$ **(1)**

(ii) Let E_2 : The event that atleast one is girl.

Then, $E_2 = \{Bg, Gg, Gb\} \Rightarrow n(E_2) = 3,$

so $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{4}$

and $(A \cap E_2) = \{Gg\}$

$\Rightarrow n(A \cap E_2) = 1$

so $P(A \cap E_2) = \frac{n(A \cap E_2)}{n(S)} = \frac{1}{4}$

Now, $P\left(\frac{A}{E_2}\right) = \frac{P(A \cap E_2)}{P(E_2)} = \frac{1/4}{3/4} = \frac{1}{3}$

$\therefore$ Required probability $= \frac{1}{3}$ **(1)**

11. Let A_T : Event that A speaks truth

and B_T : Event that B speaks truth.

Given, $P(A_T) = \frac{75}{100}$, then $P(\bar{A}_T) = 1 - \frac{75}{100}$

$[\because P(\bar{A}) = 1 - P(A)]$

$= \frac{25}{100}$

and $P(B_T) = \frac{90}{100}$,

Then, $P(\bar{B}_T) = 1 - \frac{90}{100} = \frac{10}{100}$ **(1)**

Now, P (A and B are contradict to each other)

$= P(A_T \cap \bar{B}_T) + P(\bar{A}_T \cap B_T)$

$= P(A_T) \cdot P(\bar{B}_T) + P(\bar{A}_T) \cdot P(B_T)$ **(1)**

[$\because$ events A_T and B_T are independent events]

$= \frac{75}{100} \times \frac{10}{100} + \frac{25}{100} \times \frac{90}{100}$

$= \frac{750 + 2250}{10000} = \frac{3000}{10000} = \frac{3}{10}$

$\therefore$ Percentage of P (A and B are contradict to each other) $= \frac{3}{10} \times 100 = 30\%$ **(1)**

Since, B speaks truth in only 90% (i.e. not 100%) of the cases, therefore we think, the statement of B may be false. **(1)**

12. Let P_T : Event that P speaks truth

and Q_T : Event that Q speaks truth.

Given, $P(P_T) = \frac{70}{100}$, then $P(\bar{P}_T) = 1 - \frac{70}{100} = \frac{30}{100}$

and $P(Q_T) = \frac{80}{100}$, then $P(\bar{Q}_T) = 1 - \frac{80}{100} = \frac{20}{100}$ **(1)**

P (A and B are agree to each other)

$= P(P_T \cap Q_T) + P(\bar{P}_T \cap \bar{Q}_T)$

$= P(P_T) \cdot P(Q_T) + P(\bar{P}_T) \cdot P(\bar{Q}_T)$ **(1)**

[$\because$ events P_T and Q_T are independent events]

$= \frac{70}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{20}{100}$

$= \frac{5600}{10000} + \frac{600}{10000} = \frac{6200}{10000} = \frac{62}{100}$

$\therefore$ Percentage of P(A and B are agree to each other)

$= \frac{62}{100} \times 100 = 62\%$ **(1)**

No, agree does not mean that they are speaking truth. **(1)**

13. Do same as Q. No. 11. **[Ans. 42%, Yes]**

14. Given, A and B are two independent events with $P(\bar{A} \cap B) = \frac{2}{15}$ and $P(A \cap \bar{B}) = \frac{1}{6}$.

We know that, if A and B are independent, then $\bar{A}, B$ and $A, \bar{B}$ are independent events. **(1)**

Now, $P(\bar{A} \cap B) = \frac{2}{15} \Rightarrow P(B)P(\bar{A}) = \frac{2}{15}$

$\Rightarrow P(B)(1 - P(A)) = \frac{2}{15}$ $[\because P(A) + P(\bar{A}) = 1]$

$\Rightarrow P(B) - P(A) \cdot P(B) = \frac{2}{15}$ ...(i) **(1)**

and $P(A \cap \bar{B}) = \frac{1}{6} \Rightarrow P(A)P(\bar{B}) = \frac{1}{6}$

$\Rightarrow P(A)[1 - P(B)] = \frac{1}{6}$ $[\because P(B) + P(\bar{B}) = 1]$

$\Rightarrow P(A) - P(A)P(B) = \frac{1}{6}$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$P(A) - P(B) = \frac{1}{6} - \frac{2}{15} = \frac{5-4}{30} = \frac{1}{30}$

$\Rightarrow P(A) = \frac{1}{30} + P(B)$ ...(iii) **(1)**

Now, on substituting this value in Eq. (i), we get

$P(B) - \left(\frac{1}{30} + P(B)\right)P(B) = \frac{2}{15}$

Let $P(B) = x$, then

$\Rightarrow x - \left(\frac{1}{30} + x\right) \cdot x = \frac{2}{15}$ **(1)**

$\Rightarrow 30x - (1 + 30x)x = 4$

$\Rightarrow 30x - x - 30x^2 = 4$

$\Rightarrow 30x^2 - 29x + 4 = 0$

$$\Rightarrow \quad (6x-1)(5x-4)=0 \Rightarrow x=\frac{1}{6} \text{ or } x=\frac{4}{5}$$

$$\Rightarrow \quad P(B)=\frac{1}{6} \text{ or } P(B)=\frac{4}{5} \quad [\text{put } x=P(B)]$$ (1)

Now, if $P(B)=\frac{1}{6}$, then

$$P(A)=\frac{1}{30}+\frac{1}{6}=\frac{1+5}{30}=\frac{6}{30}=\frac{1}{5}$$

[using Eq. (iii)]

and if $P(B)=\frac{4}{5}$, then $P(A)=\frac{1}{30}+\frac{4}{5}$

$$=\frac{1+24}{30}=\frac{25}{30}=\frac{5}{6}$$ (1)

15. The sample space S of the experiment is given as

$S=\{(H,H),(H,T),(T,1),(T,2),(T,3),$
$(T,4),(T,5),(T,6)\}$ (1)

The probabilities of these elementary events are

$P\{(H,H)\}=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$, $\quad P\{(H,T)\}=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$,

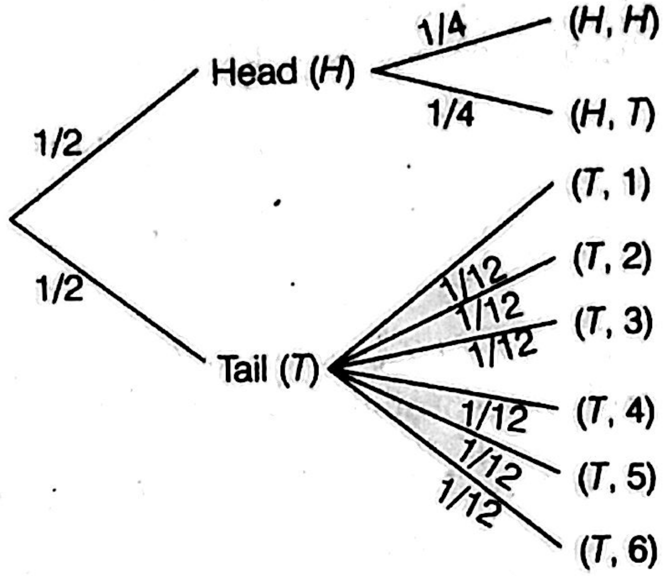
$P\{(T,1)\}=\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$, $\quad P\{(T,2)\}=\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$,

$P\{(T,3)\}=\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$, $\quad P\{(T,4)\}=\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$,

$P\{(T,5)\}=\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$ and $P\{(T,6)\}=\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$

(1)

The outcomes of the experiment can be represented in the following tree diagram.



(1)

Consider the following events:

A = the die shows a number greater than 4 and

B = there is atleast one tail.

We have, $A=\{(T,5),(T,6)\}$,

$B=\{(H,T),(T,1),(T,2),(T,3),$
$(T,4),(T,5),(T,6)\}$

and $A\cap B=\{(T,5),(T,6)\}$ (1)

$\therefore P(B)=P((H,T))+P((T,1))+P((T,2))$
$+P((T,3))+P((T,4))+P((T,5))+P((T,6))$

$\Rightarrow P(B)=\frac{1}{4}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}=\frac{3}{4}$

and $P(A\cap B)=P((T,5))+P((T,6))=\frac{1}{12}+\frac{1}{12}=\frac{1}{6}$

(1)

$\therefore$ Required probability

$$=P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{1/6}{3/4}=\frac{4}{18}=\frac{2}{9}$$ (1)

Solutions

1. When two coins are tossed, there may be 1 head, 2 heads or no head at all. Thus, the possible values of X are 0, 1, 2.

Now, $P(X=0) = P$ (Getting no head) $= P(TT) = \frac{1}{4}$

$P(X=1) = P$ (Getting one head) $= P(HT \text{ or } TH) = \frac{2}{4} = \frac{1}{2}$

$P(X=2) = P$ (getting two heads) $= P(HH) = \frac{1}{4}$ **(1)**

Thus, the required probability distribution of X is

X	0	1	2
$P(X)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(1)

2. Given, $P(X=x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$

Making it in tabular format, we get the following

X	0	1	2	otherwise
$P(X)$	k	$2k$	$3k$	0

Since, sum of all probabilities is equal to 1. **(1/2)**

$$\Sigma P(X=x) = 1$$

$\Rightarrow \quad P(X=0) + P(X=1) + P(X=2) + 0 + 0 + \ldots = 1$ **(1/2)**

$\Rightarrow \quad k + 2k + 3k = 1$

$\Rightarrow \quad 6k = 1 \Rightarrow k = \frac{1}{6}$ **(1)**

3. Let E_1 be the event that the girl gets 1 or 2,

E_2 be the event that the girl gets 3, 4, 5 or 6

and A be the event that the girl gets exactly a 'tail'.

Then, $\quad P(E_1) = \frac{2}{6} = \frac{1}{3}$

and $\quad P(E_2) = \frac{4}{6} = \frac{2}{3}$ **(1)**

$P\left(\frac{A}{E_1}\right) = P$ (getting exactly one tail when a coin is tossed three times) $= \frac{3}{8}$

$P\left(\frac{A}{E_2}\right) = P$ (getting exactly a tail when a coin is tossed once) $= \frac{1}{2}$ **(1)**

Now, required probability

$$= P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\cdot P\left(\frac{A}{E_2}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{2}{3}\cdot\frac{1}{2}}{\frac{1}{3}\cdot\frac{3}{8}+\frac{2}{3}\cdot\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{8}+\frac{1}{3}} = \frac{8}{11}$$ (2)

4. Total number of possible outcomes

$$= {}^5P_2 = \frac{5!}{3!} = 5\times 4 = 20$$

Here, X denotes the larger of two numbers obtained.

$\therefore$ X can take values 2, 3, 4 and 5.

Now, $P(X=2) = P$ (getting (1, 2) or (2, 1))

$$= \frac{2}{20} = \frac{1}{10}$$

$P(X=3) = P$ (getting (1, 3) or (3, 1) or (2, 3) or (3, 2)) $= \frac{4}{20} = \frac{2}{10}$ (1)

$P(X=4) = P$ (getting (1, 4) or (4, 1) or (2, 4) or (4, 2) or (3, 4) or (4, 3)) $= \frac{6}{20} = \frac{3}{10}$

and $P(X=5) = P$ (getting (1, 5) or (5, 1) or (2, 5) or (5, 2) or (3, 5) or (5, 3) or (4, 5) or (5, 4))

$$= \frac{8}{20} = \frac{4}{10}$$ (1)

Thus, the probability distribution of X is

X	2	3	4	5
$P(X)$	1/10	2/10	3/10	4/10

Now, mean of $X = E(X) = \Sigma X\cdot P(X)$

$$= 2\cdot\frac{1}{10} + 3\cdot\frac{2}{10} + 4\cdot\frac{3}{10} + 5\cdot\frac{4}{10}$$

$$= \frac{1}{10}(2+6+12+20) = \frac{40}{10} = 4$$ (1)

and variance of $X = E(X^2) - (E(X))^2$

$$= \left(2^2\cdot\frac{1}{10} + 3^2\cdot\frac{2}{10} + 4^2\cdot\frac{3}{10} + 5^2\cdot\frac{4}{10}\right) - 4^2$$

$$= \frac{1}{10}(4+18+48+100) - 16$$

$$= \frac{1}{10}\times 170 - 16 = 17 - 16 = 1$$ (1)

5. Let E_1 and E_2 denote the events that first and second group will win. Then,

$$P(E_1) = 0.6 \text{ and } P(E_2) = 0.4$$ (1)

Let E be the event of introducing the new product.

Then, $P\left(\frac{E}{E_1}\right) = 0.7$ and $P\left(\frac{E}{E_2}\right) = 0.3$ (1)

Now, we have to find the probability that new product is introduced by second event.

$$\therefore P\left(\frac{E_2}{E}\right) = \frac{P(E_2)P\left(\frac{E}{E_2}\right)}{P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right)}$$ (1)

$$= \frac{0.4\times 0.3}{0.6\times 0.7 + 0.4\times 0.3} = \frac{0.12}{0.42 + 0.12}$$

$$= \frac{0.12}{0.54} = 0.22$$ (1)

6. Given, $X = 0, 1, 2, 3$ and

$P(X=0) = P(X=1) = p, P(X=2) = P(X=3)$

such that $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$

Now, $\Sigma p_i = 1$

$\Rightarrow p_0 + p_1 + p_2 + p_3 = 1$

$\Rightarrow p + p + x + x = 1$

[let $P(X=2) = P(X=3) = x$]

$\Rightarrow 2p + 2x = 1$

$\Rightarrow 2x = 1 - 2p$

$\Rightarrow x = \frac{1-2p}{2}$ (1)

The probability distribution of X is given by

$X = x_i$	0	1	2	3
p_i	p	p	$\frac{1-2p}{2}$	$\frac{1-2p}{2}$

Now, $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$ (1)

$\Rightarrow p_0 x_0^2 + p_1 x_1^2 + p_2 x_2^2 + p_3 x_3^2$

$= 2[p_0 x_0 + p_1 x_1 + p_2 x_2 + p_3 x_3]$

$\Rightarrow p\times 0 + p\times 1^2 + \frac{1-2p}{2}\times(2)^2 + \frac{1-2p}{2}\times(3)^2$

$= 2\left[p\times 0 + p\times 1 + \frac{1-2p}{2}\times 2 + \frac{1-2p}{2}\times 3\right]$

$\Rightarrow p + (1-2p)\times 2 + (1-2p)\times\frac{9}{2}$

$= 2\left[p + (1-2p) + (1-2p)\times\frac{3}{2}\right]$

$$\Rightarrow p + 2 - 4p + \frac{9}{2} - 9p = 2\left[p + 1 - 2p + \frac{3}{2} - 3p\right] \quad (1)$$

$$\Rightarrow \quad -12p + 2 + \frac{9}{2} = 2\left[-4p + 1 + \frac{3}{2}\right]$$

$$\Rightarrow \quad -12p + \frac{13}{2} = 2\left[-4p + \frac{5}{2}\right]$$

$$\Rightarrow -12p + 8p = 5 - \frac{13}{2} \Rightarrow -4p = -\frac{3}{2} \Rightarrow 4p = \frac{3}{2}$$

$$\therefore \quad p = \frac{3}{8} \quad (1)$$

7. Here, $S = \{(1, 3), (1, 5), (1, 7), (3, 1)$

$(3, 5), (3, 7), (5, 1), (5, 3), (5, 7), (7, 1), (7, 3), (7, 5)\}$

$\Rightarrow n(S) = 12$

Let random variable X denotes the sum of the numbers on two cards drawn. So, the random variables X may have values 4, 6, 8, 10 and 12.

At $X = 4$, $P(X) = \frac{2}{12} = \frac{1}{6}$

At $X = 6$, $P(X) = \frac{2}{12} = \frac{1}{6}$

At $X = 8$, $P(X) = \frac{4}{12} = \frac{1}{3}$

At $X = 10$, $P(X) = \frac{2}{12} = \frac{1}{6}$

At $X = 12$, $P(X) = \frac{2}{12} = \frac{1}{6}$ (1)

Therefore, the required probability distribution is as follows

X	4	6	8	10	12
$P(X)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

$\therefore$ Mean, $E(X) = \Sigma X\, P(X)$

$$= 4 \times \frac{1}{6} + 6 \times \frac{1}{6} + 8 \times \frac{1}{3} + 10 \times \frac{1}{6} + 12 \times \frac{1}{6} \quad (1)$$

$$= \frac{2}{3} + 1 + \frac{8}{3} + \frac{5}{3} + 2 = \frac{2 + 8 + 5}{3} + 3 = \frac{15}{3} + 3 = 5 + 3$$

$= 8$

Also, $\Sigma X^2 P(X) = 16 \times \frac{1}{6} + 36 \times \frac{1}{6}$

$$+ 64 \times \frac{1}{3} + 100 \times \frac{1}{6} + 144 \times \frac{1}{6}$$

$$= \frac{8}{3} + 6 + \frac{64}{3} + \frac{50}{3} + 24 \quad (1)$$

$$= \frac{8 + 64 + 50}{3} + 30$$

$$= \frac{122}{3} + 30 = \frac{122 + 90}{3} = \frac{212}{3}$$

$$\therefore \text{Var}(X) = \Sigma X^2 P(X) - [\Sigma X\, P(X)]^2 = \frac{212}{3} - (8)^2$$

$$= \frac{212}{3} - 64 = \frac{212 - 192}{3} = \frac{20}{3} \quad (1)$$

8. Let us define the following events

A = selecting person A

B = selecting person B

C = selecting person C

$$P(A) = \frac{1}{1 + 2 + 4}, \; P(B) = \frac{2}{1 + 2 + 4}$$

and $$P(C) = \frac{4}{1 + 2 + 4}$$

$$\Rightarrow \quad P(A) = \frac{1}{7}, \; P(B) = \frac{2}{7}$$

and $$P(C) = \frac{4}{7}$$

Let E = Event to introduce the changes in their profit.

Also, given $P\left(\frac{E}{A}\right) = 0.8$, $P\left(\frac{E}{B}\right) = 0.5$ and $P\left(\frac{E}{C}\right) = 0.3$ (1)

$$\Rightarrow \quad P\left(\frac{\overline{E}}{A}\right) = 1 - 0.8 = 0.2, \; P\left(\frac{\overline{E}}{B}\right) = 1 - 0.5 = 0.5$$

and $P\left(\frac{\overline{E}}{C}\right) = 1 - 0.3 = 0.7$ (1)

The probability that change does not take place by the appointment of C,

$$P\left(\frac{C}{\overline{E}}\right) = \frac{P(C) \cdot P\left(\frac{\overline{E}}{C}\right)}{P(A) \times P\left(\frac{\overline{E}}{A}\right) + P(B) \times P\left(\frac{\overline{E}}{B}\right) + P(C) \times P\left(\frac{\overline{E}}{C}\right)} \quad (1)$$

$$= \frac{\frac{4}{7} \times 0.7}{\frac{1}{7} \times 0.2 + \frac{2}{7} \times 0.5 + \frac{4}{7} \times 0.7}$$

$$= \frac{2.8}{0.2 + 1.0 + 2.8} = \frac{2.8}{4} = 0.7 \quad (1)$$

9. Let us define the following events:

E_1 : Bag X is selected

E_2 : Bag Y is selected

and E : Getting one white and one black ball in a draw of two balls.

Here, $P(E_1) = P(E_2) = \frac{1}{2}$ (1/2)

[$\because$ probability of selecting each bag is equal]

Now, $P\left(\frac{E}{E_1}\right)$ = Probability of drawing one white and one black ball from bag X

$$= \frac{{}^4C_1 \times {}^2C_1}{{}^6C_2} = \frac{4\times 2}{\frac{6\times 5}{2\times 1}} = \frac{16}{6\times 5} = \frac{8}{15}$$ **(1)**

and $P\left(\frac{E}{E_2}\right)$ = Probability of drawing one white and one black ball from bag Y

$$= \frac{{}^3C_1 \times {}^3C_1}{{}^6C_2} = \frac{3\times 3}{\frac{6\times 5}{2\times 1}} = \frac{3}{5}$$ **(1)**

$\therefore$ The probability that the one white and one black balls are drawn from bag Y,

$$P\left(\frac{E_2}{E}\right) = \frac{P(E_2)\cdot P\left(\frac{E}{E_2}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right) + P(E_2)\cdot P\left(\frac{E}{E_2}\right)}$$

[by using Baye's theorem] **(1/2)**

$$= \frac{\frac{1}{2}\times\frac{3}{5}}{\frac{1}{2}\times\frac{8}{15}+\frac{1}{2}\times\frac{3}{5}} = \frac{\frac{3}{5}}{\frac{8}{15}+\frac{3}{5}} = \frac{\frac{3}{5}}{\frac{8+9}{15}} = \frac{3}{5\times\frac{17}{15}} = \frac{9}{17}$$ **(1)**

10. Let X be a random variable that denotes the amount received by the man. Then, X can take values 5, 4, 3 and -3. **(1)**

Now, $P(X=5) = P$ (getting a number greater than 4 in the first throw) $= \frac{2}{6} = \frac{1}{3}$ **(1/2)**

$P(X=4) = P$ (getting a number less than or equal to 4 in the first throw and getting a number greater than 4 in the second throw) $= \frac{4}{6}\times\frac{2}{6} = \frac{2}{9}$ **(1/2)**

$P(X=3) = P$ (getting a number less than or equal to 4 in first two throws and getting a number greater than 4 in the third throw)

$$= \frac{4}{6}\times\frac{4}{6}\times\frac{2}{6} = \frac{4}{27}$$ **(1/2)**

and $P(X=-3) = P$ (getting a number less than or equal to 4 in all three throws)

$$= \frac{4}{6}\times\frac{4}{6}\times\frac{4}{6} = \frac{8}{27}$$ **(1/2)**

Thus, the probability distribution of X is

X	5	4	3	-3
$P(X=x)$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{4}{27}$	$\frac{8}{27}$

Now, expected value of the amount

$$= E(X) = \sum x_i p_i = 5\cdot\frac{1}{3} + 4\cdot\frac{2}{9} + 3\cdot\frac{4}{27} - 3\cdot\frac{8}{27}$$

$$= \frac{45+24+12-24}{27} = \frac{57}{27} = \frac{19}{9}$$ **(1)**

11. Let A : Two drawn balls are white

E_1 : All the balls are white

E_2 : Three balls are white

E_3 : Two balls are white

Since, E_1, E_2 and E_3 are mutually exclusive and exhaustive events.

$$\therefore \quad P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$ **(1)**

Now, $P\left(\frac{A}{E_1}\right) = \frac{{}^4C_2}{{}^4C_2} = 1$, $P\left(\frac{A}{E_2}\right) = \frac{{}^3C_2}{{}^4C_2} = \frac{3}{6} = \frac{1}{2}$

and $P\left(\frac{A}{E_3}\right) = \frac{{}^2C_2}{{}^4C_2} = \frac{1}{6}$ **(1)**

$\therefore$ Probability that all balls in the bag are white

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{\left[P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right) + P(E_3)\cdot P\left(\frac{A}{E_3}\right)\right]}$$ **(1)**

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{6}} = \frac{1}{1+\frac{1}{2}+\frac{1}{6}} = \frac{6}{10} = 0.6$$ **(1)**

12. We have, $P(X=x) = \begin{cases} kx, & \text{if } x = 0 \text{ or } 1 \\ 2kx, & \text{if } x = 2 \\ k(5-x), & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{if } x > 4 \end{cases}$

We know that, sum of all the probabilities of a distribution is always 1.

$$\therefore P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + \ldots = 1$$

$$\Rightarrow 0 + k + 4k + 2k + k + 0 + 0 + \ldots = 1$$

$$\Rightarrow \quad 8k = 1 \Rightarrow k = \frac{1}{8}$$ **(1)**

$$\text{Now, } P(X=x) = \begin{cases} \frac{x}{8}, & \text{if } x = 0 \text{ or } 1 \\ \frac{x}{4}, & \text{if } x = 2 \\ \frac{(5-x)}{8}, & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{if } x > 4 \end{cases} \quad \ldots \text{(i)}$$

$\therefore$ (i) P (getting admission in exactly one college)

$$= P(X = 1) = \frac{1}{8} \quad \text{[using (i)]}$$ (1)

(ii) P (getting admission in at most 2 colleges)

$$= P(X \le 2)$$
$$= P(X = 0) + P(X = 1) + P(X = 2)$$
$$= 0 + \frac{1}{8} + \frac{2}{4} = \frac{5}{8}$$ (1)

(iii) P (getting admission in at least 2 colleges)

$$= P(X \ge 2) = 1 - P(X < 2)$$
$$= 1 - [P(X = 0) + P(X = 1)]$$
$$= 1 - \left[0 + \frac{1}{8}\right] = 1 - \frac{1}{8} = \frac{7}{8}$$ (1)

13. Given, bag A = 4 black and 6 red balls

bag B = 7 black and 3 red balls.

Let E_1 = The event that die show 1 or 2

E_2 = The event that die show 3 or 4 or 5 or 6

E = The event that among two drawn balls, one of them is red and other is black

Here, $P(E_1) = \frac{2}{6}$, $P(E_2) = \frac{4}{6}$

[$\because$ total number in a die is six] (1)

$\therefore P\left(\frac{E}{E_1}\right) = P$ (getting one red and one black from bag A) $= \frac{{}^4C_1 \times {}^6C_1}{{}^{10}C_2} = \frac{4 \times 6 \times 2}{10 \times 9}$

$\Rightarrow P\left(\frac{E}{E_2}\right) = P$ (getting one red and one black from bag B)

$$= \frac{{}^7C_1 \times {}^3C_1}{{}^{10}C_2} = \frac{7 \times 3 \times 2}{10 \times 9}$$ (2)

Now, by theorem of total probability

$$P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)$$
$$= \frac{2}{6} \cdot \left(\frac{4 \times 6 \times 2}{10 \times 9}\right) + \frac{4}{6} \cdot \left(\frac{7 \times 3 \times 2}{10 \times 9}\right)$$
$$= \frac{4 \times 6}{6 \times 10 \times 9}(4 + 7) = \frac{4 \times 6 \times 11}{6 \times 10 \times 9} = \frac{22}{45}$$ (1)

14. Let A_1 : Event that the bulb is produced by machine E_1

A_2 : Event that the bulb is produced by machine E_2

A_3 : Event that the bulb is produced by machine E_3

A : Event that the picked up bulb is defective (1)

Here, $P(A_1) = 50\% = \frac{50}{100} = \frac{1}{2}$,

$$P(A_2) = 25\% = \frac{25}{100} = \frac{1}{4},$$
$$P(A_3) = 25\% = \frac{25}{100} = \frac{1}{4}.$$ (1)

Also, $P\left(\frac{A}{A_1}\right) = 4\% = \frac{4}{100} = \frac{1}{25}$

$$P\left(\frac{A}{A_2}\right) = 4\% = \frac{4}{100} = \frac{1}{25}$$

and $P\left(\frac{A}{A_3}\right) = 5\% = \frac{5}{100} = \frac{1}{20}$ (1)

$\therefore$ The probability that the picked bulb is defective,

$$P(A) = P(A_1) \times P\left(\frac{A}{A_1}\right) + P(A_2) \times P\left(\frac{A}{A_2}\right) + P(A_3) \times P\left(\frac{A}{A_3}\right)$$
$$= \frac{1}{2} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{20}$$
$$= \frac{1}{50} + \frac{1}{100} + \frac{1}{80}$$
$$= \frac{8 + 4 + 5}{400} = \frac{17}{400} = 0.0425$$ (1)

15. Given, X denotes larger of the two numbers be obtained.

Obviously X can values 3, 4, 5, 6 and 7.

Now,

$P(X = 3) = P$ [getting 3 and a number is less than 3]

$$= \frac{{}^1C_1 \times {}^1C_1}{{}^6C_2} = \frac{1}{15}$$

$P(X = 4) = P$ [getting 4 and a number less than 4]

$$= \frac{{}^1C_1 \times {}^2C_1}{{}^6C_2} = \frac{2}{15}$$

$P(X = 5) = P$ [getting 5 and a number less than 5]

$$= \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{3}{15}$$ (1)

$P(X = 6) = P$ [getting 6 and a number less than 6]

$$= \frac{{}^1C_1 \times {}^4C_1}{{}^6C_2} = \frac{4}{15}$$

$P(X = 7) = P$ [getting 7 and a number less than 7]

$$= \frac{{}^1C_1 \times {}^5C_1}{{}^6C_2} = \frac{5}{15}$$

Therefore, required probability distribution is as follows

X	3	4	5	6	7
$P(X)$	1/15	2/15	3/15	4/15	5/15

(1)

$\therefore$ Required mean $= \mu = \sum X_i P_i$

$= 3 \times \frac{1}{15} + 4 \times \frac{2}{15} + 5 \times \frac{3}{15} + 6 \times \frac{4}{15} + 7 \times \frac{5}{15}$

$= \frac{3}{15} + \frac{8}{15} + \frac{15}{15} + \frac{24}{15} + \frac{35}{15} = \frac{85}{15} = \frac{17}{3}$ **(1)**

Variance $= \sum x_i^2 p_i - \mu^2$, where μ = mean

$= \left(9 \times \frac{1}{15} + 16 \times \frac{2}{15} + 25 \times \frac{3}{15} + 36 \times \frac{4}{15} + 49 \times \frac{5}{15}\right) - \left(\frac{17}{3}\right)^2$

$= \left(\frac{9}{15} + \frac{32}{15} + \frac{75}{15} + \frac{144}{15} + \frac{245}{15}\right) - \frac{289}{9}$

$= \frac{505}{15} - \frac{289}{9} = \frac{101}{3} - \frac{289}{9}$

$= \frac{303 - 289}{9} = \frac{14}{9}$ **(1)**

16. It is given that out of 15 bulbs, 5 are defective.

$\therefore$ Number of non-defective bulbs $= 15 - 5 = 10$

Let X be the random variable which denotes the defective bulbs. So, X may take values 0, 1, 2

$P(X = 0) = P$ (No defective bulb)

$= \frac{10}{15} \times \frac{9}{14} = \frac{90}{210} = \frac{9}{21}$ **(1)**

[$\because$ bulb is drawn without replacement]

$P(X = 1) = P$ (One defective bulb and one non-defective bulb)

$= \frac{5}{15} \times \frac{10}{14} + \frac{10}{15} \times \frac{5}{14} = 2\left(\frac{50}{210}\right) = \frac{100}{210} = \frac{10}{21}$ **(1)**

$P(X = 2) = P$ (both are defective bulbs)

$= \frac{5}{15} \times \frac{4}{14} = \frac{20}{210} = \frac{2}{21}$ **(1)**

$\therefore$ The probability distribution is as follows

X	0	1	2
$P(X)$	$\frac{9}{21}$	$\frac{10}{21}$	$\frac{2}{21}$

(1)

17. Let X be a random variable that denotes number of red cards in three draws.

Here, X can take values 0, 1, 2, 3.

Now, $P(X = 0) = P$ (getting all black cards)

$= \frac{^{26}C_3}{^{52}C_3} = \frac{26}{52} \times \frac{25}{51} \times \frac{24}{50} = \frac{2}{17}$

$P(X = 1) = P$ (getting one red card and two black cards)

$= \frac{^{26}C_1 \times {^{26}C_2}}{^{52}C_3} = 3 \times \frac{26}{52} \times \frac{26}{51} \times \frac{25}{50} = \frac{13}{34}$ **(1)**

$P(X = 2) = P$ (getting two red cards and one black card)

$= \frac{^{26}C_2 \times {^{26}C_1}}{^{52}C_3} = 3 \times \frac{26}{52} \times \frac{25}{51} \times \frac{26}{50} = \frac{13}{34}$

$P(X = 3) = P$ (getting all red cards)

$= \frac{^{26}C_3}{^{52}C_3} = \frac{26}{52} \times \frac{25}{51} \times \frac{24}{50} = \frac{2}{17}$ **(1)**

So, the probability distribution of X is as follows

X	0	1	2	3
$P(X)$	$\frac{2}{17}$	$\frac{13}{34}$	$\frac{13}{34}$	$\frac{2}{17}$

Now, mean **(1)**

$E(X) = \sum X_i \cdot P_i = 0 \times \frac{2}{17} + \frac{13}{34} \times 1 + \frac{13}{34} \times 2 + 3 \times \frac{2}{17}$

$= 0 + \frac{13}{34} + \frac{26}{34} + \frac{6}{17} = \frac{51}{34} = 1.5$ **(1)**

18. Firstly, find the probability of respective ages and make a probability distribution table then using the formula
Mean $(X) = \Sigma X \cdot P(X)$, calculate mean.

Here, total students $= 15$

The ages of students in ascending order are 14, 14, 15, 16, 16, 17, 17, 17, 18, 19, 19, 20, 20, 20 and 21.

Now, $P(X = 14) = \frac{2}{15}$, $P(X = 15) = \frac{1}{15}$,

$P(X = 16) = \frac{2}{15}$, $P(X = 17) = \frac{3}{15}$,

$P(X = 18) = \frac{1}{15}$, $P(X = 19) = \frac{2}{15}$,

$P(X = 20) = \frac{3}{15}$, $P(X = 21) = \frac{1}{15}$ **(1)**

Therefore, the probability distribution of random variable X is as follows

X	14	15	16	17	18	19	20	21
Number of students	2	1	2	3	1	2	3	1
$P(X)$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

(1)

$\therefore$ Mean $(X) = \sum X \cdot P(X)$ **(1)**

$$= 14\times\frac{2}{15} + 15\times\frac{1}{15} + \frac{16\times 2}{15} + 17\times\frac{3}{15} + 18\times\frac{1}{15} + 19\times\frac{2}{15} + 20\times\frac{3}{15} + 21\times\frac{1}{15}$$

$$= \frac{1}{15}[28 + 15 + 32 + 51 + 18 + 38 + 60 + 21]$$

$$= \frac{263}{15} = 17.53$$ **(1)**

19. Let E_1 : First ball is red, E_2 : First ball is black

A : Second ball is red **(1)**

Total number of balls is 10.

Then, $P(E_1) = \frac{3}{10}$, $P(E_2) = \frac{7}{10}$

$\therefore \quad P\left(\frac{A}{E_1}\right) = \frac{2}{9}$ and $P\left(\frac{A}{E_2}\right) = \frac{3}{9}$ **(1)**

Then, by Baye's theorem, probability of second selected ball is red when first selected ball is also red is given by

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$ **(1)**

$$= \frac{\frac{3}{10}\times\frac{2}{9}}{\frac{3}{10}\times\frac{2}{9} + \frac{7}{10}\times\frac{3}{9}} = \frac{6}{6+21} = \frac{6}{27} = \frac{2}{9}$$

Hence, the probability that the first selected ball is red, is $\frac{2}{9}$ **(1)**

20. Let X be a random variable that denotes the number of white balls in a draw of 4 balls. Then, X can take values of 0, 1, 2, 3 and 4. **(1)**

Clearly, $P(X = 0) = P$ (getting no white ball)

$$= \frac{{}^6C_4}{{}^{10}C_4} = \frac{\frac{6\times5\times4\times3}{4\times3\times2\times1}}{\frac{10\times9\times8\times7}{4\times3\times2\times1}} = \frac{15}{210} = \frac{1}{14}$$ **(1/2)**

$P(X = 1) = P$ (getting one white ball)

$$= \frac{{}^4C_1\times{}^6C_3}{{}^{10}C_4} = \frac{4\times\frac{6\times5\times4}{3\times2\times1}}{\frac{10\times9\times8\times7}{4\times3\times2\times1}} = \frac{4\times20}{210} = \frac{80}{210} = \frac{8}{21}$$ **(1/2)**

$P(X = 2) = P$ (getting two white balls)

$$= \frac{{}^4C_2\times{}^6C_2}{{}^{10}C_4} = \frac{\frac{4\times3}{2\times1}\times\frac{6\times5}{2\times1}}{\frac{10\times9\times8\times7}{4\times3\times2\times1}} = \frac{6\times15}{210} = \frac{90}{210} = \frac{3}{7}$$ **(1/2)**

$P(X = 3) = P$ (getting three white balls)

$$= \frac{{}^4C_3\times{}^6C_1}{{}^{10}C_4} = \frac{4\times6}{210} = \frac{24}{210} = \frac{4}{35}$$ **(1/2)**

and $P(X = 4) = P$ (getting four white balls)

$$= \frac{{}^4C_4}{{}^{10}C_4} = \frac{1}{210}$$ **(1/2)**

$\therefore$ The probability distribution of X is

X	0	1	2	3	4
$P(X = x)$	$\frac{1}{14}$	$\frac{8}{21}$	$\frac{3}{7}$	$\frac{4}{35}$	$\frac{1}{210}$

(1/2)

21. Firstly, find the probability distribution table of number of red cards. Then, using this table, find the mean and variance.

Let X be the number of red cards. Then, X can take values 0, 1 and 2. **(1/2)**

Now, $P(X = 0) = P$ (having no red card)

$$= \frac{{}^{26}C_2}{{}^{52}C_2} = \frac{26\times25/(2\times1)}{52\times51/(2\times1)} = \frac{1}{2}\times\frac{25}{51} = \frac{25}{102}$$

$P(X = 1) = P$ (having one red card)

$$= \frac{{}^{26}C_1\times{}^{26}C_1}{{}^{52}C_2} = \frac{26\times26\times2}{52\times51} = \frac{26}{51}$$

$P(X = 2) = P$ (having two red cards)

$$= \frac{{}^{26}C_2}{{}^{52}C_2} = \frac{26\times25/2\times1}{52\times51/2\times1} = \frac{25}{102}$$ **(1)**

$\therefore$ The probability distribution of number of red cards is given below

X	0	1	2
$P(X)$	$\frac{25}{102}$	$\frac{26}{51}$	$\frac{25}{102}$

(1/2)

Now, we know that, mean $= \sum X \cdot P(X)$

and variance $= \sum X^2 \cdot P(X) - [\sum X \cdot P(X)]^2$

X	$P(X)$	$X\cdot P(X)$	$X^2\cdot P(X)$
0	$\frac{25}{102}$	0	0
1	$\frac{26}{51}$	$\frac{26}{51}$	$\frac{26}{51}$
2	$\frac{25}{102}$	$\frac{25}{51}$	$\frac{50}{51}$

(1/2)

$\therefore$ Mean $= \sum X \cdot P(X)$

$= 0 + \frac{26}{51} + \frac{25}{51} = \frac{51}{51} = 1$

Variance $= \sum X^2 \cdot P(X) - [\sum X \cdot P(X)]^2$

$= \left[0 + \frac{26}{51} + \frac{50}{51}\right] - (1)^2$

$= \frac{76}{51} - 1 = \frac{76-51}{51} = \frac{25}{51}$ **(1½)**

22. Firstly, we write the probability distribution table for the given experiment. Then, we find mean by using formula

Mean $= \sum X_i P(x_i)$.

Let X = Number of heads when a coin is tossed three times.

Sample space of given experiment is

$S = \{HHH, TTT, HHT, HTH, THH, TTH, THT, HTT\}$ **(1)**

X can take values 0, 1, 2 and 3.

Now, $P(X=0) = P$ (no head occur) $= \frac{1}{8}$

$P(X=1) = P$ (one head occur) $= \frac{3}{8}$

$P(X=2) = P$ (two heads occur) $= \frac{3}{8}$

$P(X=3) = P$ (three heads occur) $= \frac{1}{8}$ **(1½)**

$\therefore$ The probability distribution is as follows

X	0	1	2	3
P (X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Now, for finding mean

X	**P (X)**	X_iP_i
0	1/8	0
1	3/8	3/8
2	3/8	6/8
3	1/8	3/8
Total		12/8

$\therefore$ Mean $= \sum X_i \cdot P(X_i) = \frac{12}{8} = \frac{3}{2}$ **(1½)**

23. Firstly, use the result that sum of all the probabilities of an experiment is one. Find k and then find other values by using this value of k.

(i) We know that, the sum of a probability distribution of random variable is one, i.e. $\sum P(X) = 1$

$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow \quad 10k^2 + 9k - 1 = 0$

$\Rightarrow \quad 10k^2 + 10k - k - 1 = 0$

$\Rightarrow \quad 10k(k+1) - 1(k+1) = 0$

$\Rightarrow \quad (10k-1)(k+1) = 0$

$\Rightarrow \quad k = \frac{1}{10}$ or -1

But $k = -1$ is rejected because probability cannot be negative.

$\therefore \quad k = \frac{1}{10}$ **(1)**

(ii) $P(X < 3) = P(X=0) + P(X=1) + P(X=2)$

$= 0 + k + 2k = 3k$

$= 3\left(\frac{1}{10}\right) \quad \left[\because k = \frac{1}{10}\right]$

$= \frac{3}{10}$ **(1)**

(iii) $P(X > 6) = P(X=7) = 7k^2 + k$

$= 7\left(\frac{1}{10}\right)^2 + \frac{1}{10} \quad \left[\because k = \frac{1}{10}\right]$

$= \frac{7}{100} + \frac{1}{10} = \frac{7+10}{100} = \frac{17}{100}$ **(1)**

(iv) $P(0 < X < 3) = P(X=1) + P(X=2)$

$= k + 2k = 3k$

$= 3\left(\frac{1}{10}\right) \quad \left[\because k = \frac{1}{10}\right]$

$= \frac{3}{10}$. **(1)**

24. We know that, when a pair of dice is thrown, then total number of outcomes = 36

Also, probability of getting a doublet in one throw $= \frac{6}{36} = \frac{1}{6}$

[$\because$ doublets in pair of dice are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6)]

$\therefore$ Probability of not getting a doublet

$= 1 - \frac{1}{6} = \frac{5}{6}$

Let X = Number of doublets in three tosses of pair of dice

So, X can take values 0, 1, 2 and 3. **(1)**

Now, $P(X=0) = P$ (not getting a doublet)

$$= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$$

$P(X=1) = P$ (getting a doublet once only)

$= P$ (getting a doublet in 1st throw)

$+ P$ (getting a doublet in 2nd throw)

$+ P$ (getting a doublet in 3rd throw)

$$= \left(\frac{1}{6} \times \frac{5}{6} \times \frac{5}{6}\right) + \left(\frac{5}{6} \times \frac{1}{6} \times \frac{5}{6}\right) + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right)$$

$$= \frac{25}{216} + \frac{25}{216} + \frac{25}{216} = \frac{75}{216}$$ **(1)**

$P(X=2) = P$ (getting a doublet two times)

$= P$ (doublet in 1st and 2nd throw)

$+ P$ (doublet in 2nd and 3rd throw)

$+ P$ (doublet in 1st and 3rd throw)

$$= \left(\frac{1}{6} \times \frac{1}{6} \times \frac{5}{6}\right) + \left(\frac{5}{6} \times \frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times \frac{5}{6} \times \frac{1}{6}\right)$$

$$= \frac{5}{216} + \frac{5}{216} + \frac{5}{216} = \frac{15}{216}$$ **(1)**

$P(X=3) = P$ (getting a doublet in all the three throws)

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$$

∴ The probability distribution is as follows

X	0	1	2	3
$P(X)$	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

(1)

25. Let X = Number of aces

Since, two cards are drawn, so X can take values 0, 1 and 2.

Now, probability of getting an ace $= \frac{4}{52} = \frac{1}{13}$

and probability of not getting an ace **(1)**

$$= 1 - \frac{1}{13} = \frac{12}{13}$$

$P(X=0) = P$ (not getting an ace card)

$$= \frac{12}{13} \times \frac{12}{13} = \frac{144}{169}$$

$P(X=1) = P$ (getting one ace card)

$$= \frac{1}{13} \times \frac{12}{13} + \frac{12}{13} \times \frac{1}{13} = \frac{24}{169}$$ **(1)**

$P(X=2) = P$ (getting two ace cards)

$$= \frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$$ **(1)**

∴ The probability distribution is as follows

X	0	1	2
$P(X)$	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

(1)

26. Let X denotes the number of ace cards. Since, two cards are drawn, so X can take values 0, 1 and 2. We know that, probability of getting an ace card

$$= \frac{{}^4C_1}{{}^{52}C_1} = \frac{4}{52} = \frac{1}{13}$$ **(1/2)**

Now, $P(X=0) = P$ (getting no ace card)

$$= \frac{{}^4C_0 \times {}^{48}C_2}{{}^{52}C_2} = \frac{\frac{1 \times 48 \times 47}{2 \times 1}}{\frac{52 \times 51}{2 \times 1}}$$

$$= \frac{48 \times 47}{52 \times 51} = \frac{188}{221}$$ **(1/2)**

$P(X=1) = P$ (getting one ace card)

$$= \frac{{}^4C_1 \times {}^{48}C_1}{{}^{52}C_2} = \frac{4 \times 48}{\frac{52 \times 51}{2 \times 1}}$$

$$= \frac{4 \times 48 \times 2 \times 1}{52 \times 51} = \frac{32}{221}$$ **(1/2)**

$P(X=2) = P$ (getting both ace cards)

$$= \frac{{}^4C_2}{{}^{52}C_2} = \frac{\frac{4 \times 3}{2 \times 1}}{\frac{52 \times 51}{2 \times 1}} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}$$ **(1/2)**

The probability distribution is as follows

X	0	1	2
$P(X)$	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

(1)

Now, required mean $= \sum X_i \cdot P_i$

$$= 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + 2 \times \frac{1}{221}$$

$$= 0 + \frac{32}{221} + \frac{2}{221} = \frac{34}{221}$$ **(1)**

27. Let E_1 be the event that Bag I is chosen, E_2 be the event that Bag II is chosen, and A be the event that the drawn ball is white. **(1/2)**

Then, $P(E_1) = \frac{1}{2}$, **(1/2)**

$P(E_2) = \frac{1}{2}$, **(1/2)**

$$P(A/E_1)=\frac{4}{7},\quad \textbf{(1/2)}$$

and $$P(A/E_2)=\frac{3}{10},\quad \textbf{(1/2)}$$

Now, (required probability $= P(E_1/A)$

$$=\frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1)+P(E_2).P(A/E_2)}\quad \textbf{(1/2)}$$

$$=\frac{\frac{1}{2}.\frac{4}{7}}{\frac{1}{2}.\frac{4}{7}+\frac{1}{2}.\frac{3}{10}}=\frac{\frac{4}{7}}{\frac{4}{7}+\frac{3}{10}}=\frac{\frac{4}{7}}{\frac{40+21}{70}}$$

$$=\frac{4}{7}\times\frac{70}{61}=\frac{40}{61}\quad \textbf{(1)}$$

28. Let E_1 = event of selecting two headed coin

E_2 = event of selecting biased coin

E_3 = event of selecting unbiased coin

A = event of getting head **(2)**

Then, $P(E_1)=P(E_2)=P(E_3)=\frac{1}{3}$

$$P\left(\frac{A}{E_1}\right)=1,\ P\left(\frac{A}{E_2}\right)=\frac{75}{100}=\frac{3}{4},\ P\left(\frac{A}{E_3}\right)=\frac{1}{2}\quad \textbf{(2)}$$

By Baye's theorem

$$P\left(\frac{E_1}{A}\right)=\frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)+P(E_3)\cdot P\left(\frac{A}{E_3}\right)}$$

$$=\frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times\frac{3}{4}+\frac{1}{3}\times\frac{1}{2}}=\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{4}+\frac{1}{6}}$$

$$=\frac{1}{3}\times\frac{12}{9}=\frac{4}{9}\quad \textbf{(2)}$$

29. Let E_1, E_2 and A denote the following events

E_1 = first bag is chosen, E_2 = second bag is chosen and A = two balls drawn at random are red.

Since, one of the bag is chosen at random.

$$\therefore\quad P(E_1)=P(E_2)=\frac{1}{2}\quad \textbf{(1)}$$

If E_1 has already occurred, i.e. first bag is chosen. Therefore, the probability of drawing two red balls in this case

$$=P\left(\frac{A}{E_1}\right)=\frac{{}^5C_2}{{}^9C_2}=\frac{10}{36}\quad \textbf{(1)}$$

Similarly, $P\left(\frac{A}{E_2}\right)=\frac{{}^3C_2}{{}^9C_2}=\frac{3}{36}$ **(1)**

We are required to find $P\left(\frac{E_2}{A}\right)$. **(1)**

By Baye's theorem,

$$P\left(\frac{E_2}{A}\right)=\frac{P(E_2)\cdot P\left(\frac{A}{E_2}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)}$$

$$=\frac{\frac{1}{2}\times\frac{3}{36}}{\frac{1}{2}\times\frac{10}{36}+\frac{1}{2}\times\frac{3}{36}}$$

$$=\frac{\frac{3}{72}}{\frac{10}{72}+\frac{3}{72}}=\frac{\frac{3}{72}}{\frac{13}{72}}=\frac{3}{13}\quad \textbf{(2)}$$

30. Let A : Event that item produced by operator A

B : Event that item produced by operator B

C : Event that item produced by operator C

D : Event that item produced is defective **(1)**

We need to find out the probability that item is produced by operator A if it is defective, i.e. $P(A/D)$

so,

$$P(A/D)=\frac{P(A)\cdot P(D/A)}{P(A)\cdot P(D/A)+P(B)\cdot P(D/B)+P(C)\cdot P(D/C)}\quad ...(i)$$

[by Baye's theorem] **(1)**

$P(A)$ = Probability of item is produced by operator A

$$=50\%=\frac{50}{100}=0.5$$

$P(B)$ = Probability of item is produced by operator B

$$=30\%=\frac{30}{100}=0.3\quad \textbf{(1)}$$

$P(C)$ = Probability of item is produced by operator C

$$=20\%=\frac{20}{100}=0.2$$

$P(D/A)$ = Probability of a defective item produced by operator A

$$=1\%=\frac{1}{100}=0.01\quad \textbf{(1)}$$

$P(D/B)$ = Probability of a defective item produced by operator B

$$=5\%=\frac{5}{100}=0.05$$

$P(D/C)$ = Probability of a defective item produced by operator C

$$=7\%=\frac{7}{100}=0.07\quad \textbf{(1)}$$

Putting these values in the Eq. (i), we get

$$P(A/D) = \frac{0.5 \times 0.01}{0.5 \times 0.01 + 0.3 \times 0.05 + 0.2 \times 0.07}$$

$$= \frac{0.005}{0.005 + 0.015 + 0.014} = \frac{0.005}{0.034} = \frac{5}{34}$$

Therefore, required probability $= \frac{5}{34}$. **(1)**

31. Let X be the number of kings obtained.

We can get 0, 1 or 2 kings.

So, value of X is 0, 1 or 2.

Total number of ways to draw 2 cards out of 52

i.e. Total ways $= {}^{52}C_2 = 1326$

$P(X = 0)$

i.e. Probability of getting 0 king **(1)**

Number of ways to get 0 king

= Number of ways to select 2 cards out of non-king cards

= Number of ways to select 2 cards out of $(52 - 4)$ = 48 cards $= {}^{48}C_2 = 1128$

$$P(X = 0) = \frac{\text{Number of ways to get 0 king}}{\text{Total number of ways}} = \frac{1128}{1326}$$

$P(X = 1)$ i.e. Probability of getting 1 king **(1)**

Number of ways to get 1 king

= Number of ways to select 1 king out of 4 king cards × Number of ways to select 1 card from 48 non-king cards

$= {}^4C_1 \times {}^{48}C_1 = 4 \times 48 = 192$

$$P(X = 1) = \frac{\text{Number of ways to get 1 king}}{\text{Total number of ways}} = \frac{192}{1326}$$

$P(X = 2)$, i.e. Probability of geeting 2 kings **(1)**

Number of ways to get 2 king

= Number of ways of selecting 2 kings out of 4 king cards

$= {}^4C_2 = 6$

$$P(X = 2) = \frac{6}{1326}$$

Now, $\mu = E(X) = \sum_{i=1}^{n} X_i p_i$

$$= 0 \times \frac{1128}{1326} + 1 \times \frac{192}{1326} + 2 \times \frac{6}{1326}$$

$$= \frac{192 + 12}{1326} = \frac{204}{1326} = \frac{34}{221}$$ **(1)**

The variance of X is given by

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$ **(1)**

Now, $E(X^2) = \sum_{i=1}^{n} X_i^2 p_i$

$$= 0^2 + \frac{1128}{1326} + 1^2 \times \frac{192}{1326} + 2^2 \times \frac{6}{1326}$$

$$= \frac{36}{221}$$

So, $\text{Var}(X) = \frac{36}{221} - \left(\frac{34}{221}\right)^2 = \frac{1}{221}\left[36 - \frac{34^2}{221}\right]$

$$= \frac{6800}{(221)^2}$$ **(1)**

32. We have, first six positive integers as 1, 2, 3, 4, 5 and 6 and let X denotes the largest of the three positive integers.

So, the random variable X may have values 3, 4, 5 or 6.

$P(X = 3) = P$ [getting 3 and two numbers less than 3]

$$= \frac{{}^1C_1 \times {}^2C_2}{{}^6C_3} = \frac{1}{20}$$ **(1)**

$P(X = 4) = P$ [getting 4 and two numbers less than 4]

$$= \frac{{}^1C_1 \times {}^3C_2}{{}^6C_3} = \frac{3}{20}$$ **(1)**

$P(X = 5) = P$ [getting 5 and two numbers less than 5]

$$= \frac{{}^1C_1 \times {}^4C_2}{{}^6C_3} = \frac{6}{20}$$ **(1)**

$P(X = 6) = P$ [getting 6 and two numbers less than 6]

$$= \frac{{}^1C_1 \times {}^5C_2}{{}^6C_3} = \frac{10}{20}$$ **(1)**

∴ The probability distribution is shown below

X	3	4	5	6
$P(X)$	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{6}{20}$	$\frac{10}{20}$

Now, mean of distribution, $E(X) = \sum X \cdot P(X)$

$$= 3 \times \frac{1}{20} + 4 \times \frac{3}{20} + 5 \times \frac{6}{20} + 6 \times \frac{10}{20}$$

$$= \frac{3 + 12 + 30 + 60}{20} = \frac{105}{20} = \frac{21}{4}$$ **(1)**

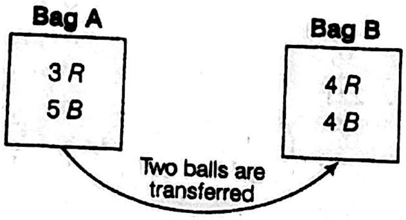
and variance of distribution $= E(X^2) - [E(X)]^2$

$= \sum X^2 P(X) - [E(X)]^2$

$$= 3^2 \times \frac{1}{20} + 4^2 \times \frac{3}{20} + 5^2 \times \frac{6}{20} + 6^2 \times \frac{10}{20} - \left(\frac{21}{4}\right)^2$$

$$= \frac{9}{20} + \frac{48}{20} + \frac{150}{20} + \frac{360}{20} - \frac{441}{16} = \frac{567}{20} - \frac{441}{16} = \frac{63}{80}$$ **(1)**

33.



Let us define the following events :

E_1 = one red and one black ball is transferred

E_2 = two red balls are transferred

E_3 = two black balls are transferred

E = drawn ball is red.

Then, **(1)**

$$P(E_1) = \frac{{}^3C_1 \times {}^5C_1}{{}^8C_2} = \frac{3\times 5}{28} = \frac{15}{28}$$ **(1/2)**

$$P(E_2) = \frac{{}^3C_2}{{}^8C_2} = \frac{3}{28},$$ **(1/2)**

$$P(E_3) = \frac{{}^5C_2}{{}^8C_2} = \frac{10}{28}$$ **(1/2)**

$$P(E/E_1) = \frac{5}{10},$$ **(1/2)**

$$P(E/E_2) = \frac{6}{10},$$ **(1/2)**

$$P(E/E_3) = \frac{4}{10}$$ **(1/2)**

Now, required probability $= P(E_2/E)$

$$= \frac{P(E_2)\cdot P(E/E_2)}{P(E_1)\cdot P(E/E_1) + P(E_2)\cdot P(E/E_2) + P(E_3)\cdot P(E/E_3)}$$

$$= \frac{\frac{3}{28}\cdot\frac{6}{10}}{\frac{15}{28}\cdot\frac{5}{10} + \frac{3}{28}\cdot\frac{6}{10} + \frac{10}{28}\cdot\frac{4}{10}}$$ **(1)**

$$= \frac{18}{75+18+40} = \frac{18}{133}$$ **(1)**

34. Let E_1 : Event that the selected bolt is manufactured by machine A,

E_2 : Event that the selected bolt is manufactured by machine B,

E_3 : Event that the selected bolt is manufactured by machine C,

and E : Event that the selected bolt is defective.

Then, we have $P(E_1) = 30\% = \frac{30}{100}$ **(1)**

$$P(E_2) = 50\% = \frac{50}{100}$$

and $P(E_3) = 20\% = \frac{20}{100}$ **(1)**

Also, given that 3%, 4% and 1% bolts manufactured by machines A, B and C respectively are defective. So,

$$P\left(\frac{E}{E_1}\right) = 3\% = \frac{3}{100}$$

$$P\left(\frac{E}{E_2}\right) = 4\% = \frac{4}{100}$$

$$P\left(\frac{E}{E_3}\right) = 1\% = \frac{1}{100}$$ **(1)**

Now, the probability that selected bolt which is defective, is manufactured by machine B

$$= P\left(\frac{E_2}{E}\right) = \frac{P(E_2)\cdot P\left(\frac{E}{E_2}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right) + P(E_2)\cdot P\left(\frac{E}{E_2}\right) + P(E_3)\cdot P\left(\frac{E}{E_3}\right)}$$ **(1)**

$$= \frac{\frac{50}{100}\times\frac{4}{100}}{\frac{30}{100}\times\frac{3}{100} + \frac{50}{100}\times\frac{4}{100} + \frac{20}{100}\times\frac{1}{100}}$$

$$= \frac{200}{90+200+20} = \frac{200}{310}$$ **(1)**

$\therefore$ The probability that selected bolt which is defective, is not manufactured by machine B

$$= 1 - P\left(\frac{E_2}{E}\right)$$

$$= 1 - \frac{200}{310} = \frac{110}{310} = \frac{11}{31}$$ **(1)**

35. Let E_1 : Event that the student knows the answer

E_2 : Event that the student guesses the answer

E : Event that the answer is correct **(1)**

Here, E_1 and E_2 are mutually exclusive and exhaustive events.

$\therefore \quad P(E_1) = \frac{3}{5}$ and $P(E_2) = \frac{2}{5}$ **(1)**

Now, $P\left(\frac{E}{E_1}\right) = P$ (the student answered correctly, given he knows the answer) $= 1$ **(1)**

$P\left(\frac{E}{E_2}\right) = P$ (the student answered correctly, given he guesses) $= \frac{1}{3}$ (1)

The probability that the student knows the answer given that he answered it correctly is given by $P\left(\frac{E_1}{E}\right)$

By using Baye's theorem, we get

$$P\left(\frac{E_1}{E}\right) = \frac{P\left(\frac{E}{E_1}\right)\cdot P(E_1)}{P\left(\frac{E}{E_1}\right)P(E_1) + P\left(\frac{E}{E_2}\right)\cdot P(E_2)} \quad (1)$$

$$= \frac{1\times\frac{3}{5}}{1\times\frac{3}{5}+\frac{1}{3}\times\frac{2}{5}} = \frac{\frac{3}{5}}{\frac{3}{5}+\frac{2}{15}}$$

$$= \frac{\frac{3}{5}}{\frac{15+10}{15}} = \frac{3\times 3}{25} = \frac{9}{25} \quad (1)$$

36. Do same as Q. No. 29. $\left[\text{Ans. } \frac{6}{7}\right]$

37. Do same as Q. No. 15.

Ans.

X	2	3	4	5	6
$P(X)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

and mean $= \frac{14}{3}$

38. Do same as Q. No. 28. $\left[\text{Ans. } \frac{20}{47}\right]$

39. Do same as Q. No. 10. [Ans. 0]

40. Let us define the events as

E_1 : Insured person is a scooter driver

E_2 : Insured person is a car driver

E_3 : Insured person is a truck driver

A : Insured person meets with an accident

Then, $n(E_1) = 2000, n(E_2) = 4000$ and $n(E_3) = 6000$

Here, total insured person, $n(S) = 12000$

Now, $P(E_1)$ = Probability that the insured person is a scooter driver $= \frac{n(E_1)}{n(S)} = \frac{2000}{12000} = \frac{1}{6}$ (1)

$P(E_2)$ = Probability that the insured person is a car driver $= \frac{n(E_2)}{n(S)} = \frac{4000}{12000} = \frac{1}{3}$

and $P(E_3)$ = Probability that the insured person is a truck driver $= \frac{n(E_3)}{n(S)} = \frac{6000}{12000} = \frac{1}{2}$ (1)

Also, $P(A/E_1)$ = Probability that scooter driver meets with an accidnt = 0.01

$P(A/E_2)$ = Probability that car driver meets with an accident = 0.03

and $P(A/E_3)$ = Probability that truck driver meets with an accident = 0.15 (1)

The probability that the person met with an accident was a scooter driver,

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{\left[P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)\right]}$$

[by Baye's theorem] (1)

$$= \frac{\frac{1}{6}\times 0.01}{\left[\left(\frac{1}{6}\times 0.01\right)+\left(\frac{1}{3}\times 0.03\right)+\left(\frac{1}{2}\times 0.15\right)\right]} \quad (1)$$

$$= \frac{\frac{1}{6}}{\frac{1}{6}+1+\frac{15}{2}} = \frac{1}{6}\times\frac{6}{1+6+45} = \frac{1}{52}$$

The probability that the person met with an accident was a car driver, $P(E_2/A)$

$$= \frac{P(E_2)\cdot P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)}$$

$$= \frac{\frac{1}{3}\times 0.03}{\left(\frac{1}{6}\times 0.01\right)+\left(\frac{1}{3}\times 0.03\right)+\left(\frac{1}{2}\times 0.15\right)}$$

$$= \frac{1}{\frac{1}{6}+1+\frac{15}{2}} = \frac{6}{52} \quad (1)$$

Hence, the required probability is $P(E_1 \cup E_2/A)$

$$= P(E_1/A) + P(E_2/A) = \frac{1}{52}+\frac{6}{52} = \frac{7}{52}$$

41. Let E_1 = Event that 1 occurs in a die

E_2 = Event that 1 does not occur in a die

A = Event that the man reports that 1 occur in a die

Then, $P(E_1) = \frac{1}{6}$ and $P(E_2) = \frac{5}{6}$ (1)

$\therefore P$ (man reports that 1 occurs when 1 occur)

$$= P\left(\frac{A}{E_1}\right) = \frac{3}{5}$$ (1)

and P (man reports that 1 occur but 1 does not occur)

$$= P\left(\frac{A}{E_2}\right) = \frac{2}{5}$$ (1)

Thus, by Baye's theorem, we get

P(get actually 1 when he reports that 1 occur)

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right)}$$ (1)

$$= \frac{\frac{1}{6}\times\frac{3}{5}}{\frac{1}{6}\times\frac{3}{5}+\frac{5}{6}\times\frac{2}{5}} = \frac{3}{13}$$ (2)

42. Let us define the following events :

E_1 = lost card is a spade card

E_2 = lost card is not a spade card

A = drawn cards are spade cards (1)

Then,

$$P(E_1) = \frac{13}{52} = \frac{1}{4},$$ (1/2)

$$P(E_2) = \frac{39}{52} = \frac{3}{4},$$ (1/2)

$$P(A/E_1) = \frac{^{12}C_3}{^{51}C_3} = \frac{220}{20825}$$ (1)

and $$P(A/E_2) = \frac{^{13}C_3}{^{51}C_3} = \frac{286}{20825}$$ (1)

Now, required probability $= P(E_1/A)$

$$= \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1)+P(E_2).P(A/E_2)}$$ (1)

$$= \frac{\frac{1}{4}.\frac{220}{20825}}{\frac{1}{4}.\frac{220}{20825}+\frac{3}{4}.\frac{286}{20825}}$$

$$= \frac{220}{220+858} = \frac{220}{1078} = \frac{20}{98} = \frac{10}{49}$$ (1)

43. Let E_1 : The patient follows meditation and yoga.

E_2 : The patient uses drug.

Then, E_1 and E_2 are mutually exclusive

and $P(E_1) = P(E_2) = 1/2$

Also, let E : The selected patient suffers a heart attack. (1)

Then, $$P(E/E_1) = \frac{40}{100}\left(1-\frac{30}{100}\right) = \frac{40}{100}\times\frac{70}{100} = \frac{28}{100}$$ (1)

and $$P(E/E_2) = \frac{40}{100}\left(1-\frac{25}{100}\right) = \frac{40}{100}\times\frac{75}{100} = \frac{30}{100}$$ (1)

$\therefore P$ (patient who suffers heart attack follows meditation and yoga) $= P(E_1/E)$

$$= \frac{P(E/E_1)\cdot P(E_1)}{P(E/E_1)\cdot P(E_1) + P(E/E_2)\cdot P(E_2)}$$ (1)

[using Baye's theorem]

$$= \frac{\frac{28}{100}\times\frac{1}{2}}{\frac{28}{100}\times\frac{1}{2}+\frac{30}{100}\times\frac{1}{2}} = \frac{28}{58} = \frac{14}{29}$$ (1)

Yoga course and meditation are more beneficial for the heart patient. (1)

44. Do same as Q. No. 40. $\left[\text{Ans. } \frac{5}{26}\right]$

45. Do same as Q. No. 33. $\left[\text{Ans. } \frac{4}{17}\right]$

46. Let us define the events as

E_1 : Students reside in a hostel

E_2 : Students are day scholars

A : Students get A grade

Then,

$P(E_1)$ = Probability that student reside in a hostel

$$= 60\% = \frac{60}{100}$$ (1)

and $P(E_2)$ = Probability that students are day scholars $= 1 - \frac{60}{100} = \frac{40}{100}$ (1)

Also, $P(A/E_1)$ = Probability that hostelers get A grade $= 30\% = \frac{30}{100}$

and $P(A/E_2)$ = Probability that students having day scholars get A grade $= 20\% = \frac{20}{100}$ (1)

$\therefore$ The probability that the selecting student is a hosteler having A grade,

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$

[by Baye's theorem] (1)

$$= \frac{\frac{60}{100}\times\frac{30}{100}}{\left(\frac{60}{100}\times\frac{30}{100}\right)+\left(\frac{40}{100}\times\frac{20}{100}\right)}$$ (1)

$$= \frac{1800}{1800+800} = \frac{1800}{2600} = \frac{18}{26} = \frac{9}{13}$$ (1)

47. Let us define the events as

E_1 : Girl gets 5 or 6 on a die

E_2 : Girl gets 1, 2, 3 or 4 on a die

A : She gets exactly one head

Now, $P(E_1)$ = Probability of getting 5 or 6 on a die $= \frac{2}{6} = \frac{1}{3}$

and $P(E_2)$ = Probability of getting 1, 2, 3 or 4 on a die $= \frac{4}{6} = \frac{2}{3}$ **(1)**

Also, $P(A/E_1)$ = Probability that girl gets exactly one head when she throws coin thrice $= \frac{3}{8}$ **(1)**

[$\because$ {HHH, TTT, HHT, HTH, THH, TTH, THT, TTH}]

$P(A/E_2)$ = Probability that girl gets exactly one head when she throws coin once

$= \frac{1}{2}$ **(1)**

The probability that she throws 1, 2, 3 or 4 with the die for getting exactly one head,

$$P(E_2/A) = \frac{P(E_2)\cdot P(A/E_2)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$

[by Baye's theorem] **(1)**

$$= \frac{\frac{2}{3}\times\frac{1}{2}}{\left(\frac{1}{3}\times\frac{3}{8}\right)+\left(\frac{2}{3}\times\frac{1}{2}\right)}$$ **(1)**

$$= \frac{\frac{1}{3}}{\frac{1}{8}+\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{3+8}{24}} = \frac{\frac{1}{3}}{\frac{11}{24}} = \frac{1}{3}\times\frac{24}{11} = \frac{8}{11}$$ **(1)**

48. Do same as Q. No. 47. $\left[\textbf{Ans. } \frac{4}{7}\right]$

49. Let us define the events as

E_1 : Person selected is a male

E_2 : Person selected is a female

A : Person selected has grey hair

Now, $P(E_1) = P$ (person selected is a male) $= 1/2$

and $P(E_2) = P$ (person selected is a female) $= \frac{1}{2}$ **(1)**

[$\because$ there are equal number of males and females]

Now, $P(A/E_1)$ = Probability of selecting a person having grey hair is male $= \frac{5}{100}$ **(1)**

and $P(A/E_2)$ = Probability of selecting a person having grey hair is female $= \frac{0.25}{100}$ **(1)**

The probability of selecting person is a male having grey hair,

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$

[by Baye's theorem] **(1)**

$$\Rightarrow P(E_1/A) = \frac{\frac{1}{2}\times\frac{5}{100}}{\left(\frac{1}{2}\times\frac{5}{100}\right)+\left(\frac{1}{2}\times\frac{0.25}{100}\right)}$$ **(1)**

$$= \frac{\frac{5}{200}}{\frac{5}{200}+\frac{0.25}{200}} = \frac{5}{5+0.25}$$

$$= \frac{5}{5.25} = \frac{500}{525} = \frac{100}{105} = \frac{20}{21}$$

Hence, the required probability is $\frac{20}{21}$. **(1)**

50. Do same as Q. No. 27. $\left[\textbf{Ans. } \frac{35}{68}\right]$

51. Do same as Q. No. 41. $\left[\textbf{Ans. } \frac{3}{8}\right]$

52. Do same as Q. No. 34. $\left[\textbf{Ans. } \frac{1}{4}\right]$

53. Let us define the events as

E_1 : Box I is selected E_2 : Box II is selected

E_3 : Box III is selected

A : The drawn coin is a gold coin

Since events E_1, E_2 and E_3 are mutually exclusive and exhaustive events.

$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ **(1)**

Now, $P(A/E_1)$

= Probability that a gold coin is drawn from box I

$= \frac{2}{2} = 1$ [$\because$ box I contain both gold coins]

$P(A/E_2)$ = Probability that a gold coin is drawn from box II = 0 [$\because$ box II has both silver coins]

and $P(A/E_3)$ = Probability that a gold coin is drawn from box III $= \frac{1}{2}$ **(2)**

[$\because$ box III contains 1 gold and 1 silver coin]

The probability that other coin in box is also of gold = The probability that the drawing gold coin from bag I,

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{\left[P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)\right]}$$

[by Baye's theorem] (1)

$$= \frac{\frac{1}{3}\times 1}{\left(\frac{1}{3}\times 1\right) + \left(\frac{1}{3}\times 0\right) + \left(\frac{1}{3}\times\frac{1}{2}\right)}$$ (1)

$$= \frac{1}{1+0+\frac{1}{2}} = \frac{1}{3/2} = \frac{2}{3}$$

Hence, the required probability is $\frac{2}{3}$. (1)

54. Do same as Q. No. 28. $\left[\text{Ans. } \frac{10}{19}\right]$

55. Let us define the events as

E_1 : Boy is selected

E_2 : Girl is selected

A : The student has an IQ of more than 150 students.

Now, $P(E_1) = 60\% = \frac{60}{100}$

Now, $P(E_2) = 40\% = \frac{40}{100}$ (1)

[$\because$ in the class 60% students are boys, so 40% are girls, given]

Now, $P(A/E_1)$ = Probability that boys has an IQ of more than 150

$$= 5\% = \frac{5}{100}$$ (1)

and $P(A/E_2)$ = Probability that girls has an IQ of more than 150 $= 10\% = \frac{10}{100}$ (1)

The probability that the selected boy having IQ more than 150 is

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$ (1)

[by Baye's theorem]

$$= \frac{\frac{60}{100}\times\frac{5}{100}}{\left(\frac{60}{100}\times\frac{5}{100}\right) + \left(\frac{40}{100}\times\frac{10}{100}\right)}$$ (1)

$$= \frac{300}{300+400} = \frac{300}{700} = \frac{3}{7}$$

Hence, the required probability is 3/7. (1)

56. Do same as Q. No. 34. $\left[\text{Ans. } \frac{28}{69}\right]$

57. Do same as Q. No. 42 $\left[\text{Ans. } \frac{11}{50}\right]$

58. Do same as Q. No. 16.

Ans.

X	0	1	2
$P(X)$	7/15	7/15	1/15

Objective Questions

(For Complete Chapter)

1 Mark Questions

1. Out of 50 tickets numbered 00, 01, 02, ..., 49, one ticket is drawn randomly, the probability of the ticket having the product of its digits 7, given that the sum of the digits is 8, is

(a) $\frac{1}{14}$ (b) $\frac{3}{14}$
(c) $\frac{1}{5}$ (d) None of these

2. If $P(A \cup B) = 0.83$, $P(A) = 0.3$ and $P(B) = 0.6$, then the events will be

(a) dependent
(b) independent
(c) cannot say anything
(d) None of the above

3. If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.3$, then $P\left(\frac{A'}{B'}\right)$ is equal to

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

4. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(A/B) = \frac{1}{2}$ and $P(B/A) = \frac{2}{3}$. Then, $P(B)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

5. For two events A and B, if $P(A) = P\left(\frac{A}{B}\right) = \frac{1}{4}$ and $P\left(\frac{B}{A}\right) = \frac{1}{2}$, then

(a) A and B are independent events
(b) $P\left(\frac{A'}{B}\right) = \frac{3}{4}$
(c) $P\left(\frac{B'}{A}\right) = \frac{1}{2}$
(d) All of the above

6. If $P(A) = \frac{1}{12}$, $P(B) = \frac{5}{12}$ and $P\left(\frac{B}{A}\right) = \frac{1}{15}$, then $P(A \cup B)$ is equal to

(a) $\frac{89}{180}$ (b) $\frac{90}{180}$ (c) $\frac{91}{180}$ (d) $\frac{92}{180}$

7. If A and B are mutually exclusive events with $P(B) \neq 1$, then $P(A/\overline{B})$ is equal to (here, $\overline{B}$ is the complement of the event B)

(a) $\frac{1}{P(B)}$ (b) $\frac{1}{1 - P(B)}$
(c) $\frac{P(A)}{P(B)}$ (d) $\frac{P(A)}{1 - P(B)}$

8. A bag A contains 4 green and 3 red balls and bag B contains 4 red and 3 green balls. One bag is taken at random and a ball is drawn and noted to be green. The probability that it comes from bag B is

(a) $\frac{2}{7}$ (b) $\frac{2}{3}$
(c) $\frac{3}{7}$ (d) $\frac{1}{3}$

9. A coin and six faced die, both unbiased, are thrown simultaneously. The probability of getting a head on the coin and an odd number on the die is

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{2}{3}$

10. If A and B are two independent events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$, then P (neither A nor B) is equal to

(a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) $\frac{5}{6}$ (d) $\frac{1}{3}$

11. A random variable X has the probability distribution given below

X	1	2	3	4	5
$P(X = x)$	K	$2K$	$3K$	$2K$	K

Its variance is

(a) $\frac{16}{3}$ (b) $\frac{4}{3}$ (c) $\frac{5}{3}$ (d) $\frac{10}{3}$

12. The probability distribution of a random variable X is given below

X	−5	−4	−3	−2	−1	0	1	2	3	4	5
$P(X)$	p	$2p$	$3p$	$4p$	$5p$	$7p$	$8p$	$9p$	$10p$	$11p$	$12p$

Then, the value of p is

(a) $\frac{1}{72}$ (b) $\frac{3}{73}$ (c) $\frac{5}{72}$ (d) $\frac{1}{74}$

13. Two dice are thrown n times in succession. The probability of obtaining a double six atleast once is

(a) $\left(\frac{1}{36}\right)^n$ (b) $1-\left(\frac{35}{36}\right)^n$

(c) $\left(\frac{1}{12}\right)^n$ (d) None of these

14. If m and σ^2 are the mean and variance of the random variable X, whose distribution is given by

X	0	1	2	3
$P(X)$	1/3	1/2	0	1/6

Then,

(a) $m=\sigma^2=2$ (b) $m=1, \sigma^2=2$

(c) $m=\sigma^2=1$ (d) $m=2, \sigma^2=1$

Solutions

1. (c) Total number of cases $= {}^{50}C_1 = 50$

Let A be the event of selecting ticket with sum of digits '8'.

Favourable cases to A are {08, 17, 26, 35, 44}.

Let B be the event of selecting ticket with product of its digits '7'.

Favourable cases to B is only {17}.

Now, $P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}=\frac{1/50}{5/50}=\frac{1}{5}$

2. (a) Given, $P(A\cup B)=0.83, P(A)=0.3$

and $P(B)=0.6$

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$\Rightarrow$ $0.83=0.3+0.6-P(A\cap B)$

$\Rightarrow$ $P(A\cap B)=0.9-0.83=0.07$

Now, $P(A)\cdot P(B)=0.3\times 0.6=0.18$

$\therefore$ $P(A)\cdot P(B)\neq P(A\cap B)$

Hence, events are dependent.

3. (c) $P(A'/B')=\frac{P(A'\cap B')}{P(B')}=\frac{P(A\cup B)'}{P(B')}$

$$=\frac{1-P(A\cup B)}{1-P(B)}$$

$$=\frac{1-[P(A)+P(B)-P(A\cap B)]}{1-P(B)}$$

$$=\frac{1-[0.5+0.4-0.3]}{1-0.4}=\frac{0.4}{0.6}=\frac{2}{3}$$

4. (c) We know that, $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$

and $P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}$

$\therefore$ $P(B)=\frac{P\left(\frac{B}{A}\right)\cdot P(A)}{P\left(\frac{A}{B}\right)}=\frac{\left(\frac{2}{3}\right)\cdot\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)}=\frac{1}{3}$

5. (d) Given, $P\left(\frac{B}{A}\right)=\frac{1}{2}$ $\Rightarrow$ $P(B\cap A)=\frac{1}{8}$

and $P\left(\frac{A}{B}\right)=\frac{1}{4}$ $\Rightarrow$ $P(B)=\frac{1}{2}$

$\therefore$ $P(A\cap B)=\frac{1}{8}=P(A)\cdot P(B)$

So, events are independent.

Now, $P\left(\frac{A'}{B}\right)=\frac{P(A'\cap B)}{P(B)}=\frac{P(B)-P(A\cap B)}{P(B)}=\frac{3}{4}$

and $P\left(\frac{B'}{A}\right)=\frac{P(A\cap B')}{P(A)}=\frac{P(A)-P(A\cap B)}{P(A)}=\frac{1}{2}$

6. (a) $P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}$

$\therefore$ $\frac{1}{15}=\frac{P(A\cap B)}{\frac{1}{12}}$ $\Rightarrow$ $P(A\cap B)=\frac{1}{180}$

Also, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow$ $P(A\cup B)=\frac{1}{12}+\frac{5}{12}-\frac{1}{180}=\frac{89}{180}$

7. (d) Given, A and B are mutually exclusive events.

$\therefore$ $P(A\cap B)=0$

Now, $P(A/\bar{B})=\frac{P(A\cap\bar{B})}{P(\bar{B})}=\frac{P(A)-P(A\cap B)}{P(\bar{B})}$

$$=\frac{P(A)}{1-P(B)}$$

8. (c) $P\left(\frac{G}{A}\right)=\frac{{}^4C_1}{{}^7C_1}=\frac{4}{7}$ and $P\left(\frac{G}{B}\right)=\frac{{}^3C_1}{{}^7C_1}=\frac{3}{7}$

Now, $P\left(\frac{B}{G}\right)=\dfrac{P(B)\cdot P\left(\frac{G}{B}\right)}{P(A)P\left(\frac{G}{A}\right)+P(B)\,P\left(\frac{G}{B}\right)}$

$$=\frac{\frac{1}{2}\cdot\frac{3}{7}}{\frac{1}{2}\cdot\frac{4}{7}+\frac{1}{2}\cdot\frac{3}{7}}=\frac{3}{7}$$

9. (c) Let E = Event of getting a head on a coin and F = Event of getting an odd number $\{1, 3, 5\}$, on a die

$$P(E)=\frac{1}{2},\quad P(F)=\frac{3}{6}=\frac{1}{2}$$

Since, E and F are independent events.

$$\therefore\quad P(E\cap F)=P(E)\cdot P(F)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$$

10. (d) Since, A and B are independent events.

$$\therefore\quad P(A\cap B)=P(A)\cdot P(B)=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$$

Now, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$=\frac{1}{2}+\frac{1}{3}-\frac{1}{6}=\frac{2}{3}$$

$$\therefore\quad P(\bar{A}\cap\bar{B})=1-P(A\cup B)$$

$$=1-\frac{2}{3}=\frac{1}{3}$$

11. (b) Given distribution is

X	1	2	3	4	5
$P(X=x)$	k	$2k$	$3k$	$2k$	k

$\therefore$ Variance $=\Sigma x_i^2 p-(\Sigma x_i p)^2$

$$=(1k+8k+27k+32k+25k)-(k+4k+9k+8k+5k)^2$$

$$=(93k)-(27k)^2=\left(93\times\frac{1}{9}\right)-\left(27\times\frac{1}{9}\right)^2$$

$$\left[\because \Sigma p=1,\text{ so } k=\frac{1}{9}\right]$$

$$=\frac{93}{9}-9$$

$$=\frac{93-81}{9}=\frac{12}{9}=\frac{4}{3}$$

12. (a) $\because$ Sum of probabilities distribution = 1

$$\Rightarrow\quad p+2p+3p+4p+5p+7p+8p+9p+10p+11p+12p=1$$

$$\Rightarrow\quad 72p=1$$

$$\therefore\quad p=\frac{1}{72}$$

13. (b) Here,

p = Probability of getting double six in two dice

$$=\frac{1}{6^2}=\frac{1}{36}\quad\text{and}\quad q=\frac{35}{36}$$

$\therefore$ Required probability

$$=1-(\text{Probability of not getting double six})^n$$

$$=1-\left(\frac{35}{36}\right)^n$$

14. (c) Given, distribution is

X	0	1	2	3
$P(X)$	$\frac{1}{3}$	$\frac{1}{2}$	0	$\frac{1}{6}$

$\therefore$ Mean, $m=\sum_{i=1}^{4} p_i x_i$

$$=0\times\frac{1}{3}+1\times\frac{1}{2}+2\times 0+3\times\frac{1}{6}$$

$$=0+\frac{1}{2}+0+\frac{1}{2}=1$$

Variance, $\sigma^2=\sum_{i=1}^{4} p_i (x_i-m)^2$

$$=\frac{1}{3}(0-1)^2+\frac{1}{2}(1-1)^2+0(2-1)^2+\frac{1}{6}(3-1)^2$$

$$=\frac{1}{3}+0+0+\frac{2}{3}=1$$

$$\therefore\quad m=\sigma^2=1$$