DCAM classes
Dynamic Classes for Academic Mastery

vector algebra-CBSE

1 Mark Questions

1. Find the position vector of a point which divides the join of points with position vectors $\vec{a} - 2\vec{b}$ and $2\vec{a} + \vec{b}$ externally in the ratio 2:1.

2. If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find a unit vector parallel to the vector $\vec{a} + \vec{b}$.

3. The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represent the two sides AB and AC respectively of triangle ABC. Find the length of the median through A.

4. Write the direction ratios of the vector $3\vec{a} + 2\vec{b}$, where $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$.

5. Find the unit vector in the direction of the sum of the vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $4\hat{i} - 3\hat{j} + 2\hat{k}$.

6. Find a vector in the direction of vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ which has magnitude 21 units.

7. Find a vector $\vec{a}$ of magnitude $5\sqrt{2}$, making an angle of $\frac{\pi}{4}$ with X-axis, $\frac{\pi}{2}$ with Y-axis and an acute angle θ with Z-axis.

8. Write a unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 7\hat{k}$.

9. Find the value of p for which the vectors $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} - 2p\hat{j} + 3\hat{k}$ are parallel.

10. Write the value of cosine of the angle which the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ makes with Y-axis.

11. Find the angle between X-axis and the vector $\hat{i} + \hat{j} + \hat{k}$.

12. Write a vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 units.

13. Write a unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points (1, 3, 0) and (4, 5, 6), respectively.

14. If a unit vector $\vec{a}$ makes angle $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find the value of θ.

15. Write a unit vector in the direction of the sum of vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$.

16. If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vectors, then write the value of $x + y + z$.

17. P and Q are two points with position vectors $3\vec{a} - 2\vec{b}$ and $\vec{a} + \vec{b}$, respectively. Write the position vector of a point R which divides the line segment PQ in the ratio 2 : 1 externally.

18. L and M are two points with position vectors $2\vec{a} - \vec{b}$ and $\vec{a} + 2\vec{b}$, respectively. Write the position vector of a point N which divides the line segment LM in the ratio 2 : 1 externally.

19. A and B are two points with position vectors $2\vec{a} - 3\vec{b}$ and $6\vec{b} - \vec{a}$, respectively. Write the position vector of a point P which divides the line segment AB internally in the ratio 1 : 2.

20. Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

21. Find the sum of the following vectors. $\vec{a} = \hat{i} - 3\hat{k}$, $\vec{b} = 2\hat{j} - \hat{k}$, $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$

22. Find the sum of the following vectors. $\vec{a} = \hat{i} - 2\hat{j}$, $\vec{b} = 2\hat{i} - 3\hat{j}$, $\vec{c} = 2\hat{i} + 3\hat{k}$

23. Find the scalar components of $\overrightarrow{AB}$ with initial point $A(2, 1)$ and terminal point $B(-5, 7)$.

24. For what values of $\vec{a}$, the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear?

25. Write the direction cosines of vector $-2\hat{i} + \hat{j} - 5\hat{k}$.

26. Write the position vector of mid-point of the vector joining points $P(2, 3, 4)$ and $Q(4, 1, -2)$.

27. Write a unit vector in the direction of vector $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$.

28. Find the magnitude of the vector $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$.

29. Find a unit vector in the direction of vector $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$.

30. If A, B and C are the vertices of a ΔABC, then what is the value of $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$?

31. Find a unit vector in the direction of $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$.

32. Find a vector in the direction of $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, which has magnitude 6 units.

33. Find the position vector of mid-point of the line segment AB, where A is point $(3, 4, -2)$ and B is point $(1, 2, 4)$.

34. Write a vector of magnitude 9 units in the direction of vector $-2\hat{i} + \hat{j} + 2\hat{k}$.

35. Write a vector of magnitude 15 units in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$. Delhi 2010

36. What is the cosine of angle which the vector $\sqrt{2}\,\hat{i} + \hat{j} + \hat{k}$ makes with Y-axis?

4 Marks Questions

37. Find a vector of magnitude 5 units an parallel to the resultant of $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

38. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$. Find a vector of magnitude 6 units, which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

39. Find the position vector of a point R, which divides the line joining two points P and Q whose position vector are $2\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ respectively, externally in the ratio 1 : 2. Also, show that P is the mid-point of line segmet RQ.

Answers

1) $3\bar{a} + 4\bar{b}$

2) $\frac{6\hat{i} - 3\hat{j} + 2\hat{k}}{7}$

3) $\sqrt{\frac{17}{2}}$

4) 7, −5, 4

5) $\frac{6}{\sqrt{37}}\hat{i} + \frac{k}{\sqrt{37}}$

6) $6\hat{i} - 9\hat{j} + 18k$

7) $5\hat{i} + 5k$

8) $\frac{1}{13}(4\hat{i} + 3\hat{j} - 12k)$

9) $P = \frac{-1}{3}$

10) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + k)$

11) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

12) $3\hat{i} - 6\hat{j} + 6k$

13) $\frac{1}{7}(3\hat{i} + 2\hat{j} + 6k)$

14) $\theta = \pi/3$

15) $\frac{1}{\sqrt{26}}(\hat{i} + 5k)$

16) $= 0$

17) $-\bar{a} + 4\bar{b}$

18) $5\bar{b}$

19) $\bar{a}$

20) $-4\hat{j} - 1k$

21) $3\hat{i} - \hat{j} - 2k$

22) $5\hat{i} - 5\hat{j} + 31k$

23) 6

24) $a = -4$

25) $\frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$

26) $3\hat{i} + 2\hat{j} + 1k$

27) $\frac{1}{3}(2\hat{i} + \hat{j} + 2k)$

28) 7

29) $\frac{1}{7}(2\hat{i} + 3\hat{j} + 6k)$

30) 0

31) $\frac{1}{7}(2\hat{i} - 3\hat{j} + 6k)$

32) $4\hat{i} - 2\hat{j} + 4k$

33) $2\hat{i} + 3\hat{j} + 1k$

34) $-6\hat{i} + 3\hat{j} + 6k$

35) $5\hat{i} - 10\hat{j} + 10k$

36) $1/2$

37) $\frac{1}{\sqrt{10}}(15\hat{i} + 5\hat{j})$

38) $2\hat{i} - 4\hat{j} + 4k$

1 Mark Questions

1. Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$.

2. Find the value of $[\hat{i}, \hat{k}, \hat{j}]$.

3. Find λ and μ, if $(\hat{i} + 3\hat{j} + 9\hat{k}) \times (3\hat{i} - \lambda\hat{j} + \mu\hat{k}) = \vec{0}$.

4. Write the number of vectors of unit length perpendicular to both the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$

5. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then write the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

6. If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 400$ and $|\vec{a}| = 5$, then write the value of $|\vec{b}|$.

7. Find λ, if the vectors $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{j} + 3\hat{k}$ are coplanar.

8. If $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$, then find the projection of $\vec{a}$ on $\vec{b}$.

9. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then find the value of $|2\hat{a} + \hat{b} + \hat{c}|$.

10. Write a unit vector perpendicular to both the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$.

11. Find the area of a parallelogram whose adjacent sides are represented by the vectors $2\hat{i} - 3\hat{k}$ and $4\hat{j} + 2\hat{k}$.

12. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a} + \vec{b}| = 13$ and $|\vec{a}| = 5$, then find the value of $|\vec{b}|$.

13. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + \vec{b}$ is also a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

14. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $2\hat{i} - 3\hat{j} + 6\hat{k}$.

15. Write the projection of vector $\hat{i} + \hat{j} + \hat{k}$ along the vector $\hat{j}$.

16. Write the value of the following.
$\hat{i} \times (\hat{j} + \hat{k}) + \hat{j} \times (\hat{k} + \hat{i}) + \hat{k} \times (\hat{i} + \hat{j})$

17. If vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$, $|\vec{b}| = 2/3$ and $\vec{a} \times \vec{b}$ is a unit vector, then write the angle between $\vec{a}$ and $\vec{b}$.

18. Find $\vec{a} \cdot (\vec{b} \times \vec{c})$, if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$.

19. If $\vec{a}$ and $\vec{b}$ are unit vectors, then find the angle between $\vec{a}$ and $\vec{b}$, given that $(\sqrt{3}\,\vec{a} - \vec{b})$ is a unit vector.

20. If $|\vec{a}| = 8, |\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, find the angle between $\vec{a}$ and $\vec{b}$.

21. Write the projection of the vector $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$.

22. Write the value of λ, so that the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular to each other.

23. Write the projection of $(\vec{b} + \vec{c})$ on $\vec{a}$, where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

24. Write the projection of the vector $7\hat{i} + \hat{j} - 4\hat{k}$ on the vector $2\hat{i} + 6\hat{j} + 3\hat{k}$.

25. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} + \vec{b}| = |\vec{a}|$, then prove that vector $2\vec{a} + \vec{b}$ is perpendicular to vector $\vec{b}$.

26. Find $|\vec{x}|$, if for a unit vector $\hat{a}$, $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 15$.

27. Find λ, when projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units.

28. Write the value of $(\hat{k} \times \hat{j}) \cdot \hat{i} + \hat{j} \cdot \hat{k}$.

29. If $\vec{a} \cdot \vec{a} = 0$ and $\vec{a} \cdot \vec{b} = 0$, then what can be concluded about the vector $\vec{b}$?

30. Write the projection of vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$.

31. Write the angle between vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively, having $\vec{a} \cdot \vec{b} = \sqrt{6}$.

32. For what value of λ are the vectors $\hat{i} + 2\lambda\hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} - 3\hat{k}$ perpendicular?

33. If $|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$ and angle between $\vec{a}$ and $\vec{b}$ is 60°, then find $\vec{a} \cdot \vec{b}$.

34. Find the value of λ, if the vectors $2\hat{i} + \lambda\hat{j} + 3\hat{k}$ and $3\hat{i} + 2\hat{j} - 4\hat{k}$ are perpendicular to each other.

35. If $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 3$, then find the projection of $\vec{b}$ on $\vec{a}$.

36. If $\vec{a}$ and $\vec{b}$ are two vectors, such that $|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

37. Find λ, if $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$.

2 Marks Questions

38. If the sum of two unit vectors $\hat{a}$ and $\hat{b}$ is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

39. If $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$, find $[\vec{a}\ \vec{b}\ \vec{c}]$.

40. If $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a}\times\vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

41. Find the volume of cuboid whose edges are given by $-3\hat{i} + 7\hat{j} + 5\hat{k}$, $-5\hat{i} + 7\hat{j} - 3\hat{k}$ and $7\hat{i} - 5\hat{j} - 3\hat{k}$.

42. Show that the points $A(-2\hat{i} + 3\hat{j} + 5\hat{k})$, $B(\hat{i} + 2\hat{j} + 3\hat{k})$ and $C(7\hat{i} - \hat{k})$ are collinear.

43. Find $|\vec{a}\times\vec{b}|$, if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$.

44. If θ is the angle between two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$, find $\sin\theta$.

45. If $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 5$, $|\vec{b}| = 6$ and $|\vec{c}| = 9$, then find the angle between $\vec{a}$ and $\vec{b}$.

4 Marks Questions

46. If $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 5\hat{j}$, $3\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{i} - 6\hat{j} - \hat{k}$ respectively, are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear or not.

47. The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

48. Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d}\cdot\vec{a} = 21$.

49. Find x such that the four points $A(4,4,4)$, $B(5,x,8)$, $C(5,4,1)$ and $D(7,7,2)$ are coplanar.

50. Find the value of x such that the points $A(3,2,1)$, $B(4,x,5)$, $C(4,2,-2)$ and $D(6,5,-1)$ are coplanar.

51. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of the same magnitude, then prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$.

52. Using vectors, find the area of the ΔABC, whose vertices are $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$.

53. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$, then
    (a) Let $c_1 = 1$ and $c_2 = 2$, find c_3 which makes $\vec{a}$, $\vec{b}$ and $\vec{c}$ coplanar.
    (b) If $c_2 = -1$ and $c_3 = 1$, show that no value of c_1 can make $\vec{a}$, $\vec{b}$ and $\vec{c}$ coplanar.

54. Show that the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle.

55. Show that the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar, if $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar.

Or

Prove that, for any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$, $[\vec{a}+\vec{b} \;\; \vec{b}+\vec{c} \;\; \vec{c}+\vec{a}] = 2[\vec{a} \;\; \vec{b} \;\; \vec{c}]$.

56. Show that the four points $A(4,5,1)$, $B(0,-1,-1)$, $C(3,9,4)$ and $D(-4,4,4)$ are coplanar.

Or

Show that the four points A, B, C and D with position vectors $4\hat{i}+5\hat{j}+\hat{k}$, $-\hat{j}-\hat{k}$, $3\hat{i}+9\hat{j}+4\hat{k}$ and $4(-\hat{i}+\hat{j}+\hat{k})$, respectively are coplanar.

57. The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}-5\hat{k}$ and $2\hat{i}+2\hat{j}+3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram.

58. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, then show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, where $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.

59. If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, find $(\vec{r}\times\hat{i})\cdot(\vec{r}\times\hat{j})+xy$.

60. If $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}$ and $\vec{c}=3\hat{i}-4\hat{j}-5\hat{k}$, then find a unit vector perpendicular to both of the vectors $(\vec{a}-\vec{b})$ and $(\vec{c}-\vec{b})$.

61. Find the value of λ so that the four points A, B, C and D with position vectors $4\hat{i}+5\hat{j}+\hat{k}$, $-\hat{j}-\hat{k}$, $3\hat{i}+\lambda\hat{j}+4\hat{k}$ and $-4\hat{i}+4\hat{j}+4\hat{k}$, respectively are coplanar.

62. Prove that $[\vec{a} \;\; \vec{b}+\vec{c} \;\; \vec{d}] = [\vec{a} \;\; \vec{b} \;\; \vec{d}] + [\vec{a} \;\; \vec{c} \;\; \vec{d}]$.

63. If $\vec{a}=2\hat{i}-3\hat{j}+\hat{k}$, $\vec{b}=-\hat{i}+\hat{k}$, $\vec{c}=2\hat{j}-\hat{k}$ are three vectors, find the area of the parallelogram having diagonals $(\vec{a}+\vec{b})$ and $(\vec{b}+\vec{c})$.

64. Vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3$, $|\vec{b}|=5$ and $|\vec{c}|=7$. Find the angle between $\vec{a}$ and $\vec{b}$.

65. The scalar product of the vector $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ and hence, find the unit vector along $\vec{b}+\vec{c}$.

Or

The scalar product of vector $\hat{i}+\hat{j}+\hat{k}$ with the unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

66. Find the vector $\vec{p}$ which is perpendicular to both $\vec{\alpha}=4\hat{i}+5\hat{j}-\hat{k}$ and $\vec{\beta}=\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{p}\cdot\vec{q}=21$, where $\vec{q}=3\hat{i}+\hat{j}-\hat{k}$.

67. Find the unit vector perpendicular to both of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ where, $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$.

68. Find the unit vector perpendicular to the plane ABC where the position vectors of A, B and C are $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+\hat{j}+2\hat{k}$ and $2\hat{i}+3\hat{k}$, respectively.

69. Dot product of a vector with vectors $\hat{i}-\hat{j}+\hat{k}$, $2\hat{i}+\hat{j}-3\hat{k}$ and $\hat{i}+\hat{j}+\hat{k}$ are respectively 4, 0 and 2. Find the vector.

70. Find the values of λ for which the angle between the vectors $\vec{a} = 2\lambda^2\hat{i} + 4\lambda\hat{j} + \hat{k}$ and $\vec{b} = 7\hat{i} - 2\hat{j} + \lambda\hat{k}$ is obtuse.

71. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that each one is perpendicular to the vector obtained by sum of the other two and $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 5$, then prove that $|\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$.

Or

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors, such that $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 5$ and each one of these is perpendicular to the sum of other two, then find $|\vec{a} + \vec{b} + \vec{c}|$.

72. If $\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\vec{b}$ in the form $\vec{b} = \vec{b_1} + \vec{b_2}$, where $\vec{b_1} \parallel \vec{a}$ and $\vec{b_2} \perp \vec{a}$.

73. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a vector $\vec{c}$, such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.

74. If $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$, then find the value of λ, so that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors.

75. If $\vec{p} = 5\hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{q} = \hat{i} + 3\hat{j} - 5\hat{k}$, then find the value of λ, so that $\vec{p} + \vec{q}$ and $\vec{p} - \vec{q}$ are perpendicular vectors.

76. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors, such that $|\vec{a}| = 5, |\vec{b}| = 12, |\vec{c}| = 13$ and $\vec{a} + \vec{b} + \vec{c} = 0$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

77. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{p}$, which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p} \cdot \vec{c} = 18$.

78. Find a unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$.

79. If $\vec{a}$ and $\vec{b}$ are two vectors, such that $|\vec{a}| = 2, |\vec{b}| = 1$ and $\vec{a} \cdot \vec{b} = 1$, then find $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$.

80. If vectors $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

81. Using vectors, find the area of triangle with vertices $A(1, 1, 2)$, $B(2, 3, 5)$ and $C(1, 5, 5)$.

82. Using vectors, find the area of triangle with vertices $A(2, 3, 5)$, $B(3, 5, 8)$ and $C(2, 7, 8)$.

Answers

1) 3

2) −1

3) $\mu = 27$, $\lambda = -9$

4) two

5) $-3/2$

6) 4

7) $\lambda = 7$

8) $8/7$

9) $\sqrt{6}$

10) $\frac{-\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}$

11) $4\sqrt{14}$

12) 12

13) $\theta = \frac{2\pi}{3}$

14) 5

15) 1

16) 0

17) $\theta = \pi/6$

18) -10

19) $\pi/6$

20) $\pi/6$

21) $2/3$

22) $\lambda = 5/2$

23) 2

24) $8/7$

25) $|\bar{a}+\bar{b}| = |\bar{a}|$ ⇒ squaring both sides

$|\bar{a}+\bar{b}|^2 = |\bar{a}|^2$

$2|a|.|b| + |b|.|b| = 0$

$(2|a| + |b|).b = 0$

$(2\bar{a}+\bar{b})\,\bar{b} = 0$ i.e $(2\bar{a}+\bar{b}) \perp$ to $\bar{b}$ proved.

26) 4

27) $\lambda = 5$

28) -1

29) 0

30) 0

31) $\pi/4$

32) $1/2$

33) $\sqrt{3}$

34) 3

35) $3/2$

36) $\pi/4$

37) -3

38) $\sqrt{3}$

39) -30

40) $\pi/6$

41) 264

42) can be Proved easily.

43) $-17\hat{i} + 13\hat{j} + 7k$

44) $\sin\theta = \frac{2\sqrt{6}}{7}$

45) $\theta = \cos^{-1}(1/3)$

46)

47) $\frac{1}{7}(3\hat{i} + 6\hat{j} - 2k)$

48) $-\frac{1}{3}(i - 16j - 13k)$

49) $x = 7$

50) 5

51)

51. If three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular to each other, then $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$ and if all three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are equally inclined with the vector $(\vec{a}+\vec{b}+\vec{c})$, that means each vector $\vec{a}$, $\vec{b}$ and $\vec{c}$ makes equal angle with $(\vec{a}+\vec{b}+\vec{c})$ by using formula

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}.$$

Given, $|\vec{a}| = |\vec{b}| = |\vec{c}| = \lambda$ (say) ...(i)

and $\vec{a}\cdot\vec{b} = 0, \vec{b}\cdot\vec{c} = 0$ and $\vec{c}\cdot\vec{a} = 0$ ...(ii) **(1/2)**

Now, $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$

$= \lambda^2 + \lambda^2 + \lambda^2 + 2(0+0+0) = 3\lambda^2$

$\Rightarrow \quad |\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}\lambda$

[length cannot be negative] **(1)**

Suppose $(\vec{a}+\vec{b}+\vec{c})$ is inclined at angles θ_1, θ_2 and θ_3 respectively with vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, then

$(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a} = |\vec{a}+\vec{b}+\vec{c}|\ |\vec{a}|\cos\theta_1$

$[\because \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta]$

$\Rightarrow \quad |\vec{a}|^2 + \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} = \sqrt{3}\lambda \times \lambda\cos\theta_1$

$\Rightarrow \quad \lambda^2 + 0 + 0 = \sqrt{3}\lambda^2\cos\theta_1$

[from Eqs. (i) and (ii)]

$\therefore \quad \cos\theta_1 = \frac{1}{\sqrt{3}}$

$(\vec{a}+\vec{b}+\vec{c})\cdot\vec{b} = |\vec{a}+\vec{b}+\vec{c}||\vec{b}|\cos\theta_2$ **(1)**

$\Rightarrow \quad \vec{a}\cdot\vec{b} + |\vec{b}|^2 + \vec{c}\cdot\vec{b} = \sqrt{3}\lambda\cdot\lambda\cos\theta_2$

$\Rightarrow \quad 0 + \lambda^2 + 0 = \sqrt{3}\lambda^2\cos\theta_2$

[from Eqs. (i) and (ii)]

$\Rightarrow \quad \cos\theta_2 = \frac{1}{\sqrt{3}}$

Similarly, $(\vec{a}+\vec{b}+\vec{c})\cdot\vec{c} = |\vec{a}+\vec{b}+\vec{c}||\vec{c}|\cos\theta_3$

$\Rightarrow \quad \cos\theta_1 = \frac{1}{\sqrt{3}}$ **(1)**

Thus, $\cos\theta_1 = \cos\theta_2 = \cos\theta_3 = \frac{1}{\sqrt{3}}$

Hence, it is proved that $(\vec{a}+\vec{b}+\vec{c})$ is equally inclined with the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$. **(1/2)**

52) $\frac{1}{2}\sqrt{274}$

53) $c_3 = 2$

54) $\frac{1}{2}\sqrt{210}$

55) consider

$[(\bar{a}+\bar{b})\ (\bar{b}+\bar{c})\ (\bar{c}+\bar{a})]$

$= (\bar{a}+\bar{b})\cdot\{(\bar{b}+\bar{c})\times(\bar{c}+\bar{a})\}$

$= (\bar{a}+\bar{b})\cdot(\bar{b}\times\bar{c} + \bar{b}\times\bar{a} + \bar{c}\times\bar{c} + \bar{c}\times\bar{a})$

$= (\vec{a}+\vec{b})\cdot(\vec{b}\times\vec{c} + \vec{b}\times\vec{a} + \vec{c}\times\vec{a}) \quad [\because \vec{c}\times\vec{c} = \vec{0}]$

$= \vec{a}\cdot(\vec{b}\times\vec{c}) + \vec{a}\cdot(\vec{b}\times\vec{a}) + \vec{a}\cdot(\vec{c}\times\vec{a}) + \vec{b}\cdot(\vec{b}\times\vec{c}) + \vec{b}\cdot(\vec{b}\times\vec{a}) + \vec{b}\cdot(\vec{c}\times\vec{a})$ **(1)**

$= [\vec{a}\ \vec{b}\ \vec{c}] + [\vec{a}\ \vec{b}\ \vec{a}] + [\vec{a}\ \vec{c}\ \vec{a}] + [\vec{b}\ \vec{b}\ \vec{c}] + [\vec{b}\ \vec{b}\ \vec{a}] + [\vec{b}\ \vec{c}\ \vec{a}]$

$= [\vec{a}\ \vec{b}\ \vec{c}] + [\vec{b}\ \vec{c}\ \vec{a}]$

$[\because [\vec{a}\ \vec{b}\ \vec{a}] = [\vec{b}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{b}\ \vec{a}] = [\vec{a}\ \vec{c}\ \vec{a}] = 0]$

$= [\vec{a}\ \vec{b}\ \vec{c}] + [\vec{a}\ \vec{b}\ \vec{c}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$ ...(i)

$[\because [\vec{b}\ \vec{c}\ \vec{a}] = -[\vec{b}\ \vec{a}\ \vec{c}] = -(-[\vec{a}\ \vec{b}\ \vec{c}]) = [\vec{a}\ \vec{b}\ \vec{c}]]$ **(1)**

Now, if $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar $\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = 0$

$\Rightarrow \quad 2[\vec{a}\ \vec{b}\ \vec{c}] = 0$

$\Rightarrow [\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}] = 0$ [from Eq. (i)]

$\Rightarrow \vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar. **(1)**

57) $2\sqrt{101}$

58) & 59)

58. Use the result, if two vectors are parallel, then their cross-product will be a zero vector.

Given, $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ ...(i)

and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ ...(ii) (1)

On subtracting Eq. (ii) from Eq. (i), we get

$$(\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c}) = (\vec{c} \times \vec{d}) - (\vec{b} \times \vec{d})$$

$$\Rightarrow (\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c}) + (\vec{b} \times \vec{d}) - (\vec{c} \times \vec{d}) = \vec{0}$$

$$\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) + (\vec{b} - \vec{c}) \times \vec{d} = \vec{0}$$

$$\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) - \vec{d} \times (\vec{b} - \vec{c}) = \vec{0}$$

$[\because \vec{a} \times \vec{b} = -\vec{b} \times \vec{a}]$ (1)

$$\therefore (\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = \vec{0}$$

$[\because \vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$, given] (1/2)

Thus, we have that cross-product of vectors $\vec{a} - \vec{d}$ and $\vec{b} - \vec{c}$ is a zero vector, so $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$. (1½)

59. Given, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Now, $\vec{r} \times \hat{i} = (x\hat{i} + y\hat{j} + z\hat{k}) \times \hat{i}$

$= x(\hat{i} \times \hat{i}) + y(\hat{j} \times \hat{i}) + z(\hat{k} \times \hat{i})$

$= x \cdot 0 + y(-\hat{k}) + z(\hat{j})$

$= -y\hat{k} + z\hat{j}$ (1)

$[\because \vec{a} \times \vec{a} = \vec{0};\ \hat{j} \times \hat{i} = -\hat{k},\ \hat{k} \times \hat{i} = \hat{j}]$

and $(\vec{r} \times \hat{j}) = (x\hat{i} + y\hat{j} + z\hat{k}) \times \hat{j}$

$= x(\hat{i} \times \hat{j}) + y(\hat{j} \times \hat{j}) + z(\hat{k} \times \hat{j})$

$= x\hat{k} + y \cdot 0 + z(-\hat{i})$

$= x\hat{k} - z\hat{i}$

$[\because \vec{a} \times \vec{a} = 0;\ \hat{i} \times \hat{j} = \hat{k}$ and $\hat{k} \times \hat{j} = -\hat{i}]$ (1)

$\therefore (\vec{r} \times \hat{i}) \cdot (\vec{r} \times \hat{j}) = (-y\hat{k} + z\hat{j}) \cdot (x\hat{k} - z\hat{i})$

$= -yx + yz \cdot 0 + 0 \cdot zx - z^2 \cdot 0$

$[\because \hat{k} \cdot \hat{k} = 1,\ \hat{k} \cdot \hat{i} = 0,\ \hat{j} \cdot \hat{k} = 0,\ \hat{j} \cdot \hat{i} = 0]$

$= -xy$

$\therefore (\vec{r} \times \hat{i}) \cdot (\vec{r} \times \hat{j}) + xy = -xy + xy = 0$ (2)

60) $-\frac{\hat j}{\sqrt{2}} + \frac{\hat k}{\sqrt{2}}$

61) $\lambda = 9$

62)

62. To prove $[\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{d}] = [\vec{a}\ \ \vec{b}\ \ \vec{d}] + [\vec{a}\ \ \vec{c}\ \ \vec{d}]$

Let LHS $= [\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{d}] = \vec{a} \cdot \{(\vec{b}+\vec{c}) \times \vec{d}\}$

[by definition of scalar triple product] (1)

$= \vec{a} \cdot (\vec{b} \times \vec{d} + \vec{c} \times \vec{d})$ (1)

$= \vec{a}(\vec{b} \times \vec{d}) + \vec{a}(\vec{c} \times \vec{d}) = [\vec{a}\ \ \vec{b}\ \ \vec{d}] + [\vec{a}\ \ \vec{c}\ \ \vec{d}]$ (2)

Hence proved.

63) $\frac{1}{2}\sqrt{21}$

64) $\pi/3$

65) $\frac{1}{7}(3\hat i + 6\hat j - 2\hat k)$

66) $7\hat i - 7\hat j - 7\hat k$

67) $\frac{1}{\sqrt{6}}(-\hat i + 2\hat j - \hat k)$

68) $\frac{1}{\sqrt{14}}(3\hat i + 2\hat j\ \hat k)$

69) $2\hat i - \hat j + \hat k$

70) $0 < \lambda < \frac{1}{2}$

71) $5\sqrt{2}$

72) $\frac{1}{2}(3\hat i - \hat j) + \frac{1}{2}(\hat i + 3\hat j - 3\hat k)$

73) $\frac{1}{3}(5\hat i + 2\hat j + 2\hat k)$

74) $\lambda = \pm 5$

75) $\lambda = \pm 1$

76) -169

77) $64\hat i - 2\hat j - 28\hat k$

78) $\frac{1}{3}(2\hat i - 2\hat j - \hat k)$

79) 0

80) $\lambda = 8$

81) $\frac{1}{2}\sqrt{61}$

82) $\frac{1}{2}\sqrt{61}$

Objectives Answers

1) a 2) c 3) b 4) d 5) a

6) b 7) a 8) d 9) c 10) b

11) b 12) c 13) c 14) c

15) b

Objective Questions

(For Complete Chapter)

1 Mark Questions

1. If $\lambda\,(3\hat{i} + 2\hat{j} - 6\,\hat{k})$ is a unit vector, then the value of λ is

(a) $\pm\dfrac{1}{7}$ (b) ± 7

(c) $\pm\sqrt{43}$ (d) $\pm\dfrac{1}{\sqrt{43}}$

2. The figure formed by four points $\hat{i} + \hat{j} + \hat{k}$, $2\,\hat{i} + 3\hat{j}$, $3\hat{i} + 5\hat{j} - 2\hat{k}$, $\hat{k} - \hat{j}$ is a

(a) parallelogram (b) rectangle

(c) trapezium (d) square

3. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle $\pi/3$, then the value of $|\vec{a} + \vec{b}|$ is

(a) equal to (b) greater than 1

(c) equal to 0 (d) less than 1

4. If $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} + \vec{b}$ makes an angle of $60°$ with $\vec{a}$, then

(a) $|\vec{a}| = 2|\vec{b}|$ (b) $2|\vec{a}| = |\vec{b}|$

(c) $|\vec{a}| = \sqrt{3}\,|\vec{b}|$ (d) $\sqrt{3}\,|\vec{a}| = |\vec{b}|$

5. If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b}$ is a vector such that $\vec{a} \cdot \vec{b} = |\vec{b}|^2$ and $|\vec{a} - \vec{b}| = \sqrt{7}$, then $|\vec{b}|$ is equal to

(a) $\sqrt{7}$ (b) $\sqrt{3}$ (c) 7 (d) 3

6. If $\vec{a} \cdot \vec{b} = -|\vec{a}||\vec{b}|$, then the angle between a and b is

(a) $45°$ (b) $180°$ (c) $90°$ (d) $60°$

7. Suppose $\vec{a} = \lambda\hat{i} - 7\hat{j} + 3\hat{k}$, $\vec{b} = \lambda\,\hat{i} + \hat{j} + 2\lambda\hat{k}$.

If the angle between $\vec{a}$ and $\vec{b}$ is greater than $90°$, then λ satisfies the inequality

(a) $-7 < \lambda < 1$ (b) $\lambda > 1$

(c) $1 < \lambda < 7$ (d) $-5 < \lambda < 1$

8. If $\vec{x}$ and $\vec{y}$ are unit vectors and $\vec{x} \cdot \vec{y} = 0$, then

(a) $|\vec{x} + \vec{y}| = 1$ (b) $|\vec{x} + \vec{y}| = \sqrt{3}$

(c) $|\vec{x} + \vec{y}| = 2$ (d) $|\vec{x} + \vec{y}| = \sqrt{2}$

9. The projection of $\vec{a} = 3\hat{i} - \hat{j} + 5\hat{k}$ on $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$ is

(a) $\dfrac{8}{\sqrt{35}}$ (b) $\dfrac{8}{\sqrt{39}}$ (c) $\dfrac{8}{\sqrt{14}}$ (d) $\sqrt{14}$

10. If $|\vec{a}| = 1, |\vec{b}| = 4$, $\vec{a} \cdot \vec{b} = 2$ and $\vec{c} = 2\vec{a} \times \vec{b} - 3\vec{b}$, then the angle between $\vec{b}$ and $\vec{c}$ is

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{5\pi}{6}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{2\pi}{3}$

11. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$. Then, which one of the following is correct?

(a) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} = 0$

(b) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq 0$

(c) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{a} \times \vec{c} = 0$

(d) $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}$ are mutually perpendicular

12. If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is equal to

(a) 16 (b) 8 (c) 3 (d) 12

13. If a and b represent the adjacent sides of a parallelogram whose area is 15 units, then the area of the parallelogram whose adjacent sides are $3\vec{a} + 2\vec{b}$ and $\vec{a} + 3\vec{b}$ is

(a) 45 units (b) 75 units

(c) 105 units (d) 165 units

14. If $\vec{r} \cdot \vec{a} = \vec{r} \cdot \vec{b} = \vec{r} \cdot \vec{c} = 0$ for some non-zero vector $\vec{r}$, then the value of $[\vec{a}\;\vec{b}\;\vec{c}]$ is

(a) 2 (b) 3

(c) 0 (d) None of these

15. If the volume of parallelopiped with coterminous edges $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} + \hat{k}$ and $3\hat{i} + 9\hat{j} + p\hat{k}$ is 34 cu units, then p is equal to

(a) 4 (b) -13

(c) 13 (d) 6