

# Atoms and nuclei:solutions

## Explanations

1. **Key idea** In case of hydrogen atom, the kinetic energy is equal to the negative of total energy and potential energy is equal to twice of the total energy.

Given, total ground state energy

$$(TE) = (-13.6 \text{ eV})$$

$\therefore$  Kinetic energy = - Total Energy

$$= - (-13.6 \text{ eV}) = 13.6 \text{ eV}$$

Potential energy = 2 (TE) =  $2 \times (-13.6)$

$$= - 27.2 \text{ eV} \quad (1)$$

2.  $H_{\alpha}$ -line of the Balmer series in the emission spectrum of hydrogen atom is obtained when an electron makes a transition from third lowest energy level to second lowest energy level. (1)
3. Number of spectral lines obtained due to transition of electron from  $n=4$  ( $3^{\text{rd}}$  excited state) to  $n=1$  (ground state) is according to formula

$$N = \frac{n(n-1)}{2}$$

$$N = \frac{(4)(4-1)}{2} = 6 \quad (1)$$

4. The classical model could not explain the stability of an atom as in this atomic structure the electron revolving around the nucleus are accelerated and emits energy. As the result, the radius of the circular paths goes on decreasing. Ultimately electrons fall in to the nucleus, which is not practical possible. (1)

5. **Ionisation energy** The minimum amount of energy required to remove an electron from the ground state of the atom is known as ionisation energy.

$$\text{Ionisation energy for hydrogen atom} = E_{\infty} - E_1 \\ = -(-13.6\text{eV}) = +13.6\text{ eV} \quad (1)$$

6. (i) Since, the second permitted energy level to the first level  $= E_2 - E_1$   
 $=$  energy of photon released  
 $= (-3.4\text{ eV}) - (-13.6\text{ eV}) = 10.2\text{ eV} \quad (1/2)$

(ii) The highest permitted energy level to the first permitted level  
 $= E_{\infty} - E_1 = 0 - (-13.6) = 13.6\text{ eV}$   
 $\therefore$  Ratio of energies of photon  $= \frac{10.2}{13.6} = \frac{3}{4} = 3 : 4 \quad (1/2)$

7. For first excited state  $n = 2$ .

Ground state occurs for  $n = 1$ ,

$$\therefore r_n = r_0 n^2$$

$$\therefore r \propto n^2 \quad [\because r_0 = 0.53]$$

$$\Rightarrow r_1/r_2 = (n_1/n_2)^2 = (2/1)^2$$

$$\therefore r_1 : r_2 = 4 : 1$$

where,  $r_1$  and  $r_2$  are radii corresponding to first excited state and ground state of the atom, respectively. (1)

8. The radius of atom whose principal quantum number is  $n$  is given by  $r = n^2 r_0$

where,  $r_0 =$  radius of innermost electron orbit for hydrogen atom and  $r_0 = 5.3 \times 10^{-11}\text{ m}$

For second excited state,  $n = 3$

$$\therefore r = 3^2 \times r_0 = 9 \times 5.3 \times 10^{-11} \\ r = 4.77 \times 10^{-10}\text{ m} \quad (1)$$

9. Expression for Bohr's radius in hydrogen atom

$$r_n = \frac{n^2 h^2}{4\pi^2 m k Z e^2}$$

where,  $n =$  principal quantum number,

$m =$  mass of electron,

$k = 1/4\pi\epsilon_0 = 9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2$ ,

$Z =$  atomic number of atom  $= 1$  and

$h =$  Planck's constant. (1)

$$\Rightarrow r_1 = \frac{h^2}{4\pi^2 m k e^2}$$

10. According to Bohr's quantisation condition, electrons are permitted to revolve in only those orbits in which the angular momentum of electron is an integral multiple of  $\frac{h}{2\pi}$  i.e.

$$mvr = nh/2\pi \quad \text{where, } n = 1, 2, 3, \dots$$

$m, v, r$  are mass, speed and radius of electron and  $h$  being Planck's constant. (1)

11. According to Rutherford's experiment, following observations were made.

- (i) Most of the  $\alpha$ -particles passed through the gold foil without any appreciable deflection.
- (ii) Only 0.14% of incident  $\alpha$ -particles scattered by more than  $1^\circ$ . But about 1  $\alpha$ -particle in every 8000 particles deflected by more than  $90^\circ$ . (1)

Thus, all these leads to the conclusion that atom has a lot of empty space and practically the entire mass of the atom is confined to an extremely small centered core called nucleus, whose size is of the order from  $10^{-15}\text{ m}$  to  $10^{-14}\text{ m}$ . (1)

12. We know that,  $\lambda = \frac{h}{p} = \frac{h}{mv}$

$$\Rightarrow mv = \frac{h}{\lambda}$$

$$\Rightarrow mvr = \frac{hr}{\lambda} = \frac{nh}{\lambda}$$

$$\Rightarrow \lambda = \frac{2\pi r}{nh} \times hr = \frac{2\pi r^2}{nh}$$

$$\text{As, } r \propto n^2$$

$$\Rightarrow \lambda \propto \frac{1}{n} (n^2) = n \quad (1)$$

$$\text{Thus, we can say that, } \frac{\lambda_3}{\lambda_1} = \frac{3}{1} \Rightarrow \lambda_1 = \frac{\lambda_3}{3}$$

Thus, wavelength decreases 3 times as an electron jumps from third excited state to the ground state. (1)

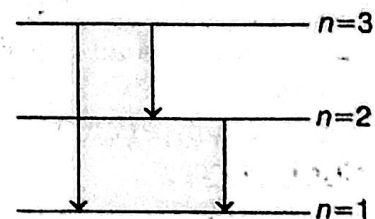
13. The energy absorbed by it is  $E_n$

$$-136 + 125 = -1.1\text{ eV}$$

$$\text{Energy, } E_n = -\frac{136}{n^2} \Rightarrow n^2 = \frac{-136}{-1.1} = 1236$$

$$\Rightarrow n = 3 \quad (1)$$

Thus, number of transitions  $= 3$



(1)

14. For Bohr's quantisation condition Refer to Sol. 10 on page 377. (1)

For Brackett-series,  $\frac{1}{\lambda} = R \left( \frac{1}{4^2} - \frac{1}{n^2} \right)$  where

$$n = 5, 6, 7, \dots$$

For shortest wavelength,  $n = 5$

$$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left( \frac{1}{16} - \frac{1}{25} \right)$$

$$= 1.097 \times 10^7 \times \frac{9}{16 \times 25} = 0.0246 \times 10^7$$

$$\Rightarrow \lambda = 40.514 \times 10^{-7} \approx 4051 \text{ nm}$$

It lies in infrared region of electromagnetic spectrum. (1)

15. The velocity of electron,  $v_n = \frac{1}{n} \frac{Ze^2}{2h\epsilon_0}$

Here,  $Z = 1, e = 1.6 \times 10^{-19} \text{ C}$ ,

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ NC}^2\text{m}^{-2},$$

$h = 6.62 \times 10^{-34} \text{ J-s}$  and  $n = 2$  (in 1st excited state)

$$\Rightarrow v_2 = \frac{1 \times (1.6 \times 10^{-19})^2}{2 \times 2 \times (6.62 \times 10^{-34}) \times (8.85 \times 10^{-12})}$$

$$= 1.09 \times 10^6 \text{ m/s} \quad (1)$$

Radius of orbit,  $r_2 = \frac{n^2 h^2 \epsilon_0}{\pi m e^2}$

Here,  $m = 9.1 \times 10^{-31} \text{ kg}$

$$\Rightarrow r_2 = \frac{(2)^2 \times (6.62 \times 10^{-34})^2 \times (8.85 \times 10^{-12})}{3.14 \times (9.1 \times 10^{-31}) \times (1.6 \times 10^{-19})^2}$$

$$= 212 \times 10^{-10} \text{ m}$$

Time period or orbital period,

$$T = \frac{2\pi r_2}{v_2} = \frac{2 \times 3.14 \times 212 \times 10^{-10}}{1.09 \times 10^6} = 1.22 \times 10^{-15} \text{ s} \quad (1)$$

16. Refer to Sol. 6 on page 377. (1)

∴ Ratio of energies of photon,

$$\frac{E'}{E''} = \frac{h\nu_1}{h\nu_2} = \frac{3}{4}$$

⇒ Ratio of the frequencies of photon is  $\nu_1 : \nu_2 = 3 : 4$  (1)

17. For an electron revolving in  $n$ th orbit of radius  $r_n$  then, we have  $n\lambda = 2\pi r_n$

where,  $\lambda$  is the wavelength of electron.

For electron orbiting in ground state  $n = 1$ .

$$1 \cdot \lambda = 2\pi r_1 = 2\pi \times n^2 r_0$$

$$= 2\pi \times 0.5 \text{ \AA} = \pi \text{ \AA} \quad [\because r_0 = 0.5 \text{ \AA}]$$

$$\Rightarrow \lambda = 3.14 \text{ \AA} \quad (2)$$

18. For electron in first excited state i.e.  $n = 2$ .

So, if  $\lambda$  be its wavelength (de-Broglie), then we have

$$n\lambda = 2\pi r_n$$

where,  $r_n$  is the radius of second orbit.

$$r_n = 0.5 \times n^2 \text{ (in \AA)} = 0.5 \times 4 = 2 \text{ \AA}$$

$$\therefore 2 \times \lambda = 2 \times \pi \times 2 \text{ \AA}$$

$$\Rightarrow \lambda = 2\pi \text{ \AA} = 6.28 \text{ \AA} \quad (2)$$

19. Given, energy of electron beam,  $E = 12.5 \text{ eV}$

Comparing with  $E = hc/\lambda$ , we get

$$\Rightarrow E = \frac{(1240 \text{ eV-nm})}{\lambda} = 12.5 \text{ eV} \quad [\because hc = 1240 \text{ eV nm}]$$

$$\therefore \text{Wavelength, } \lambda = 1240/12.5 \text{ nm}$$

$$= 99.2 \text{ nm} = 992 \text{ \AA}$$

This wavelength corresponds to Lyman series of hydrogen atom. (1)

20. Lyman series,  $n = 2, 3, 4 \dots$  to  $n = \infty$

For short wavelength,  $n = \infty$  to  $n = 1$

$$\text{Energy, } E = \frac{12375}{\lambda(\text{\AA})} = \frac{12375}{9134} \text{ eV} = 13.54 \text{ eV} \quad (1)$$

Also, energy of  $n$ th orbit,  $E = 13.54/n^2$

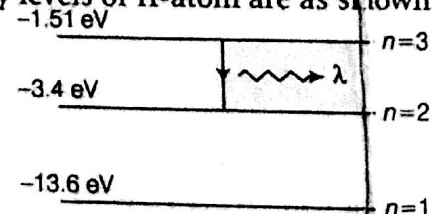
So, energy of  $n = 1$ , energy level = 13.54 eV

Energy of  $n = 2$ , energy level =  $13.54/2^2 = 3.387 \text{ eV}$

So, short wavelength of Balmer series =  $\frac{12375}{3.387}$

$$= 3653 \text{ \AA} \quad (1)$$

21. Energy levels of H-atom are as shown below



Wavelength of spectral line emitted

$$\Rightarrow \lambda = hc / \Delta E$$

Taking,  $hc = 1240 \text{ eV-nm}$ ,

We have,  $\Delta E = -1.51 - (-3.4) = 1.89 \text{ eV}$

$$\therefore \lambda = \frac{1240}{1.89} \approx 656 \text{ nm}$$

This belongs to Balmer spectral series. (2)

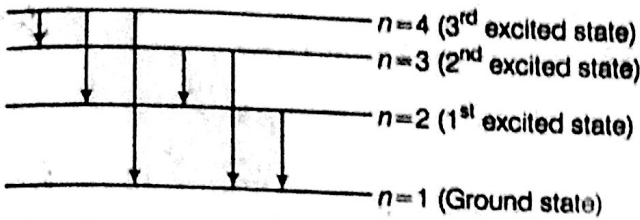
22. Frequency condition An atom can emit or absorb radiation in the form of discrete energy photons only when an electron jumps from a higher to a lower orbit or from a lower to a higher orbit, respectively. (1)

$$h\nu = E_i - E_f$$

where,  $\nu$  is frequency of radiation emitted,  $E_i$  and  $E_f$  are the energies associated with stationary orbits of principal quantum numbers  $n_i$  and  $n_f$  respectively (where  $n_i > n_f$ ).

23. Number of spectral lines obtained due to transition of electron from  $n = 4$  (3<sup>rd</sup> excited state) to  $n = 1$  (ground state) is

$$N = (4)(4-1)/2 = 6 \quad (1)$$



These lines correspond to Lyman series. (1)

24. Energy of electron at  $n = 2$  states is

$$E = \frac{-136}{n^2} = \frac{-136}{(2)^2} = -34 \text{ eV} \quad (1)$$

Now, de-Broglie wavelength of electron is given by

$$\begin{aligned} \lambda &= \frac{h}{p} = \frac{h}{\sqrt{2mK}} \\ &= h / \sqrt{2mE} \quad [\because |K| = |E|] \\ &= \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 34 \times 1.6 \times 10^{-19}}} \\ &= \frac{6.62 \times 10^{-34}}{10 \times 10^{-25}} = 0.662 \times 10^{-9} \\ &= 6.62 \times 10^{-10} = 6.62 \text{ \AA} \quad (1) \end{aligned}$$

25. According to de-Broglie's hypothesis, for any permissible orbit

$$2\pi r = n\lambda$$

As,  $\lambda = \frac{h}{mv}$

$$\therefore \dots(1) \quad mvr = \frac{nh}{2\pi} \quad \dots(1)$$

(where,  $mvr$  = angular momentum of an electron and  $n$  is an integer).

Thus, the centripetal force,  $mv^2/r$  (experienced by the electron) is due to the electrostatic attraction,  $\frac{kZe^2}{r^2}$

where,  $Z$  = atomic number of the atom. (1)

$$\text{Therefore, } \frac{mv^2}{r} = \frac{kZe^2}{r^2}$$

Substituting the value of  $v^2$  from Eq. (i),

$$\text{we obtain, } \frac{m}{r} \cdot \frac{n^2 h^4}{4\pi^2 m^2 r^2} = \frac{kZe^2}{r^2}, \therefore r = \frac{n^2 h^2}{4\pi^2 m k Z e^2}$$

The relation for the  $n$ th radius of Bohr orbit in terms of Bohr's quantization condition of orbital

$$\text{angular momentum} = \frac{n^2 h^2}{4\pi^2 m k Z e^2} \quad (1)$$

26. Kinetic energy of  $\alpha$ -particle is given as

$$K = \frac{1}{2} \frac{Ze \cdot Ze}{4\pi\epsilon_0 d^2}$$

where,  $d$  is the distance of closest approach.

$$d^2 = \frac{2Ze^2}{4\pi\epsilon_0 K} \Rightarrow d = \sqrt{\frac{2Ze^2}{4\pi\epsilon_0 K}}$$

This is the required expression for the distance of closest approach  $d$  in terms of kinetic energy  $K$ . (2)

27. For Balmer series,  $\frac{1}{\lambda_B} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right)$

For highest energy  $n \rightarrow \infty$

$$\Rightarrow \frac{1}{\lambda_B} = \frac{R}{2^2} = \frac{R}{4} \Rightarrow \lambda_B = \frac{4}{R} \quad (1)$$

$$\text{For Paschen series, } \frac{1}{\lambda_P} = R \left( \frac{1}{3^2} - \frac{1}{n^2} \right)$$

For highest energy  $n \rightarrow \infty$

$$\Rightarrow \lambda_P = \frac{9}{R} \Rightarrow \lambda_B : \lambda_P = \frac{4}{R} : \frac{9}{R} \Rightarrow 4:9 \quad (1)$$

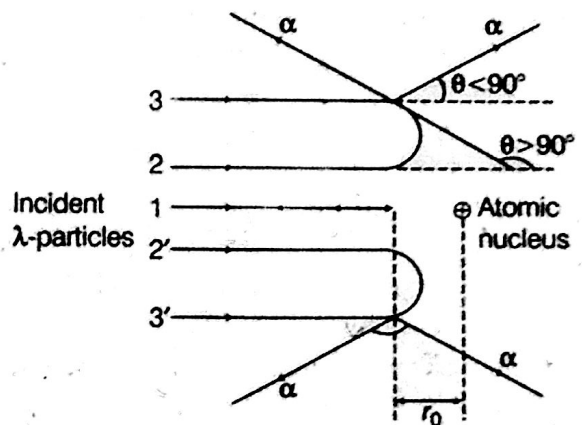
28. According to question, shortest wavelength of the spectral lines emitted in Balmer series is given by

$$\frac{1}{\lambda} = R \left[ \frac{1}{2^2} - \frac{1}{\infty^2} \right] \Rightarrow \frac{1}{\lambda} = \frac{10^7}{4} \quad [\because R = 10^7]$$

$$\Rightarrow \lambda = \frac{4}{10^7} = 4 \times 10^{-7}$$

$$\lambda = 4000 \text{ \AA} \quad (2)$$

29. Trajectory of an  $\alpha$ -particles in the Coulomb field of the target nucleus is given below as



(1)

From this experiment, the following is observed

- (i) Most of the  $\alpha$ -particles pass straight through the gold foil. It means that they do not suffer any collision with gold atoms.
- (ii) About one  $\alpha$ -particle in every 8000  $\alpha$ -particles deflects by more than  $90^\circ$ . As most of the  $\alpha$ -particles go undeflected and only a few get deflected, this shows that most of the space in an atom is empty and at the centre of the atom, there is a heavy mass, which is most commonly known as nucleus. Thus, with the help of these observations regarding the deflection of  $\alpha$ -particles, the size of the nucleus was predicted.

**30.** According to the Bohr's theory of hydrogen atom, the angular momentum of revolving electron is given by

$$mvr = nh/2\pi \quad \dots(i)$$

where,  $m$  = mass of the electron,  $v$  = velocity of the electron,  $r$  = radius of the orbit,  $h$  = Planck's constant

and  $n$  = principal quantum number of the atom.

If an electron of mass  $m$  and velocity  $v$  is moving in a circular orbit of radius  $r$ , then the centripetal force is given by

$$F_c = mv^2/r \quad \dots(ii)$$

(1)

Also, if the charge on the nucleus is  $Ze$ , then the force of electrostatic attraction between the nucleus and the electron will provide the necessary centripetal force.

$$\Rightarrow F_c = F_e$$

$$\Rightarrow \frac{mv^2}{r} = \frac{ke^2}{r^2} \quad [\because Z = 1]$$

$$\Rightarrow r = \frac{e^2 \cdot k}{mv^2} \quad \dots(iii)$$

From Eq. (i), we get  $v = nh/2\pi mr$

Putting this value in Eq. (iii), we get

$$r = \frac{ke^2 4\pi^2 m^2 r^2}{m \cdot n^2 h^2}$$

$$\Rightarrow r = \frac{n^2 h^2}{ke^2 4\pi^2 m} \Rightarrow r \propto n^2 \quad (1)$$

**31.** Since, we know that for Balmer series,

$$\frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n_2^2} \right), \quad n_2 = 3, 4, 5, \dots \quad (1)$$

For shortest wavelength in Balmer series, the spectral series is given by

$$n_1 = 2, n_2 = \infty$$

$$\Rightarrow \frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{\infty^2} \right)$$

$$\Rightarrow \frac{1}{\lambda} = R \times \frac{1}{4} \Rightarrow \frac{1}{\lambda} = \frac{R}{4} \Rightarrow \lambda = \frac{4}{R}$$

$$\lambda = \frac{4}{1.097 \times 10^7} \quad [\because R = 1.097 \times 10^7 \text{ m}^{-1}]$$

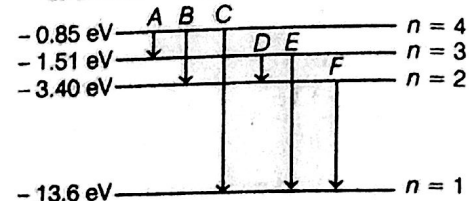
$$\Rightarrow \lambda = 364 \times 10^{-7} \text{ m.}$$

The lines of Balmer series are found in the visible part of the spectrum. (1)

**32.** (i) Given,  $\lambda = 496 \text{ nm} = 496 \times 10^{-9} \text{ m}$

$$\therefore E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{496 \times 10^{-9}} \text{ J}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{496 \times 10^{-9} \times 1.6 \times 10^{-19}} = 2.5 \text{ eV}$$



This energy corresponds to the transition B ( $n = 4$  to  $n = 2$ ) for which the energy change = 2 eV (1)

(ii) Energy of emitted photon is given by,

$$E = \frac{hc}{\lambda}$$

$$\therefore \lambda_{\max} \propto \frac{1}{E_{\min}}$$

Transition A, for which the energy emission is minimum, corresponds to the emission of radiation of maximum wavelength. (1)

**33.** The Rutherford nuclear model of the atom describes the atom as an electrically neutral sphere consisting of a very small, massive and positively charged nucleus at the centre surrounded by the revolving electrons in their respective dynamically stable orbits. The electrostatic force of attraction  $F_e$  between the revolving electrons and the nucleus provides the requisite centripetal force ( $F_c$ ) to keep them in their orbits. Thus, for a dynamically stable orbit in a hydrogen atom

$$F_c = F_e$$

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r^2} \quad [\because Z = 1] \quad (1/2)$$

Thus, the relation between the orbit radius and the electron velocity is

$$r = \frac{e^2}{4\pi\epsilon_0 mv^2}$$

The kinetic energy ( $K$ ) and electrostatic potential energy ( $U$ ) of the electron in hydrogen atom are

$$K = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\epsilon_0 r}$$

and 
$$U = -\frac{e^2}{4\pi\epsilon_0 r} \quad (1/2)$$

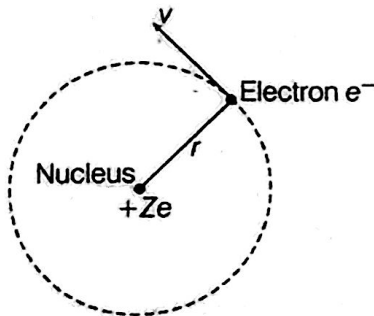
(The negative sign in  $U$  signifies that the electrostatic force is attractive in nature.)

Thus, the total mechanical energy  $E$  of the electron in a hydrogen atom is

$$E = K + U = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} = -\frac{e^2}{8\pi\epsilon_0 r} \quad (1/2)$$

The total energy of the electron is negative. This implies the fact that the electron is bound to the nucleus. If  $E$  were positive, an electron will not follow a closed orbit around the nucleus and it would leave the atom. (1/2)

34. A hydrogen like atom consists of a tiny positively charged nucleus and an electron revolving in a stable circular orbit around the nucleus.



(1/2)

Let  $e$ ,  $m$  and  $v$  be respectively the charge, mass and velocity of the electron and  $r$  the radius of the orbit.

The positive charge on the nucleus is  $Ze$ , where  $Z$  is the atomic number (in case of hydrogen atom,  $Z = 1$ ). As, the centripetal force is provided by the electrostatic force of attraction, we have

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \cdot \frac{(Ze) \times e}{r^2}$$

or 
$$mv^2 = \frac{Ze^2}{4\pi\epsilon_0 r} \quad \dots(i)$$

From the first postulate of Bohr's atomic model, the angular momentum of the electron is

$$mvr = n \frac{h}{2\pi} \quad \dots(ii)$$

where,  $n$  ( $= 1, 2, 3, \dots$ ) is **principal quantum number**. (1/2)

From Eqs. (i) and (ii), we get

$$r = n^2 \frac{h^2 \epsilon_0}{\pi m Z e^2} \quad \dots(iii)$$

This is the equation for the radii of the permitted orbits. According to this equation,  $r_n \propto n^2$ . Since,  $n = 1, 2, 3, \dots$  it follows that the radii of the permitted orbits increase in the ratio  $1 : 4 : 9 : 16 : \dots$  from the first orbit. (1/2)

The radius of the first orbit ( $n = 1$ ) of hydrogen atom ( $Z = 1$ ) will be,  $r_1 = h^2 \epsilon_0 / \pi m e^2$

This is called **Bohr's radius** and its value is  $0.53 \text{ \AA}$ . Since  $r \propto n^2$ , the radius of the second orbit of hydrogen atom will be  $(4 \times 0.53) \text{ \AA}$  and that of the third orbit  $(9 \times 0.53) \text{ \AA}$ . (1/2)

35. The ionisation energy (IE) is qualitatively defined as the amount of energy required to remove the most loosely bound electron, the valence electron of an isolated gaseous atom to form a cation. Since, total energy is directly proportional to the mass of electron. (1)

$$E_0 = \frac{me^4}{8\epsilon_0^2 h^2} \text{ i.e. } E_0 \propto m$$

So, the ionisation energy becomes 200 times on replacing an electron by a particle of mass 200 times of the electron and of same charge. (1)

36. (i) Given,  $r_1 = 5.3 \times 10^{-11} \text{ m} \Rightarrow r_2 = 21.2 \times 10^{-11} \text{ m}$   
 $n_1 = 1$

We know that,  $r \propto n^2$

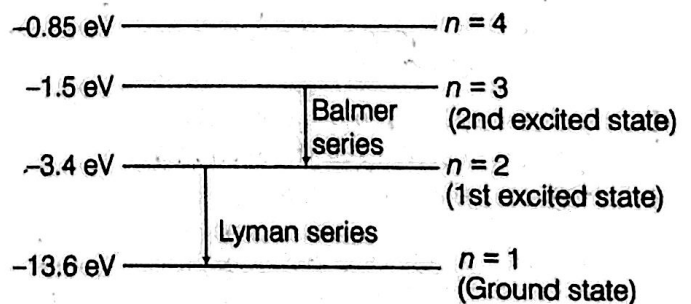
$$\frac{r_1}{r_2} = \frac{n_1^2}{n_2^2} \Rightarrow \frac{1}{21.2 \times 10^{-11}} = \frac{5.3 \times 10^{-11}}{n_2^2}$$

$$\Rightarrow n_2^2 = 4 \Rightarrow n_2 = 2 \quad (1)$$

- (ii) We know that,  $E = \frac{-136}{n^2} = \frac{-136}{4} = -34 \text{ eV}$  (1)

37. (i) An electron undergoes transition from second excited state to the first excited state belong to Balmer series and then to the ground state belongs to Lyman series. (1/2)

- (ii) The wavelength of the emitted radiations in the two cases.



For  $n_2 \xrightarrow{\lambda_L} n_1$

$$\Delta E = (-3.40 + 13.6) = 10.20 \text{ eV}$$

$$\lambda_L = \frac{12.43 \times 10^{-7}}{10.2} = 1.218 \times 10^{-7} \text{ m}$$

$$\lambda_L = 1218 \text{ \AA} \quad (1)$$

For  $n_3 \xrightarrow{\lambda_B} n_2$

$$\Rightarrow \Delta E = (-1.5 + 3.4) = 1.9 \text{ eV}$$

$$\Rightarrow \lambda_B = \frac{12.43 \times 10^{-7}}{1.9} = 6.54 \times 10^{-7} \text{ m} = 6540 \text{ \AA}$$

$$\therefore \frac{\lambda_B}{\lambda_L} = \frac{6540 \text{ \AA}}{1281 \text{ \AA}} = 5.10 \quad (1/2)$$

**38.** The following are the drawbacks of Rutherford's model

(i) A moving charge in a circular path must radiate energy because it is continuously subjected to a force. Due to this, continuous loss of energy of electrons, the radii of the orbits should be decreasing continuously. So, the concept of fixed orbit is erroneous. (1)

(ii) Due to this continuous decrease in radii of orbit, a charge moves over a spiral path must radiates a continuous spectrum. Hence, line emission spectrum of H-atom is not explained by Rutherford's model. (1)

**39.** Refer to Sol. 34 on page 381. (2)

**40.** Here,  $\phi = 2 \text{ eV}$ ,  $\frac{1}{\lambda} = R \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$

$$E = \frac{hc}{\lambda} = hcR \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = \phi + \text{KE}$$

Also,  $\text{KE} = eV_0$

$$n_1 = 2, n_2 = n$$

$$hcR \left( \frac{1}{4} - \frac{1}{n^2} \right) = 2 \times 1.6 \times 10^{-19} + 1.6 \times 10^{-19} \times 0.55$$

$$\Rightarrow 6.62 \times 10^{-34} \times 3 \times 10^8 \times 1.097 \times 10^7 \left( \frac{1}{4} - \frac{1}{n^2} \right)$$

$$= (3.2 + 0.88) \times 10^{-19}$$

$$\Rightarrow 21.786 \times 10^{-19} \left( \frac{1}{4} - \frac{1}{n^2} \right) = 4.08 \times 10^{-19}$$

$$\frac{1}{4} - \frac{1}{n^2} = 0.187 \Rightarrow n \approx 4 \quad (3)$$

**41.** (i) Refer to diagram

(Line Spectrum of the H-atom)

(ii) For largest wavelength,  $n = \infty$

$$\frac{1}{\lambda_m} = R \left( \frac{1}{2^2} - \frac{1}{\infty} \right)$$

$$\frac{1}{\lambda_m} = \frac{1.1 \times 10^7}{4}$$

$$\Rightarrow \lambda_m = \frac{4}{1.1} \times 10^{-7} = 3636 \times 10^{-7} \text{ m}$$

$$= 3636 \text{ \AA} \quad (1)$$

For shortest wavelength,

$$n = 3$$

$$\frac{1}{\lambda_s} = R \left( \frac{1}{2^2} - \frac{1}{3^2} \right)$$

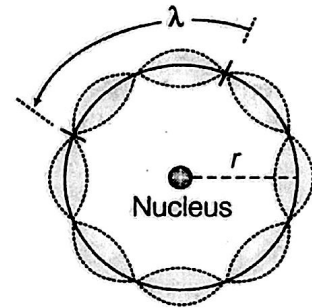
$$= 1.1 \times 10^7 \times \frac{5}{36}$$

$$\Rightarrow \lambda_s = \frac{36}{55} \times 10^{-7}$$

$$= 6.545 \times 10^{-7} \text{ m}$$

$$= 6545 \text{ \AA} \quad (1)$$

**42.** (i) Bohr's second postulate defines the stable orbits. This postulate states that the electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of  $h/2\pi$ , where  $h$  is the Planck's constant ( $= 6.63 \times 10^{-34} \text{ J-s}$ ).



According to de-Broglie hypothesis, wavelength of moving electron,  $\lambda = \frac{h}{mv_n}$

where,  $v_n$  is speed of electron revolving in  $n$ th orbit.

As,  $2\pi r_n = n\lambda$  [From figure]

$$\therefore 2\pi r_n = \frac{nh}{mv_n}$$

$$\text{or } mv_n r_n = \frac{nh}{2\pi} = n(h/2\pi)$$

i.e. Angular momentum of electron revolving in  $n$ th orbit must be an integral multiple of  $h/2\pi$ , which is the quantum condition proposed by Bohr in his second postulate. (1/2)

(ii) We know that, energy of electron in  $n$ th orbit is

$$E_n = -\frac{136}{n^2} \text{ eV}$$

$$\text{For } n = 1, E_1 = -136 \text{ eV}$$

$$\text{Similarly, for } n = 4, E_4 = -\frac{136}{(4)^2} \text{ eV}$$

$$\therefore \text{Energy difference, } \Delta E = E_4 - E_1$$

$$= \left[ -\frac{136}{16} - (-136) \right] \text{ eV} \quad \dots (1)$$

Also, energy of photon is

$$\Delta E = h\nu \Rightarrow \nu = \frac{\Delta E}{h} \quad \dots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\nu = \left( -\frac{136}{16} + 136 \right) \times \frac{1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$\therefore \nu = 31 \times 10^{15} \text{ Hz} \quad \dots \text{(1\frac{1}{2})}$$

43. When an  $\alpha$ -particle is bombarded over a gold nucleus, it is repelled by electrostatic repulsion. As a result KE of  $\alpha$ -particle is converted into electrostatic PE. At a certain distance between the  $\alpha$ -particle and nucleus at which the moving charge loses all its kinetic energy and becomes stationary momentarily. This distance ( $r$ ) is known as distance of closest approach. In this process, all the kinetic energy ( $K$ ) of moving particle is converted into potential energy. (1)

From the given data,

$$\text{Initially, } \frac{1}{4\pi\epsilon_0} \cdot \frac{2e \times Ze}{r} = K \quad \dots \text{(i)}$$

Let  $r_0$  be the new distance of closest approach for a twice energetic  $\alpha$ -particle. Then, we have,

$$\frac{1}{4\pi\epsilon_0} \times \frac{2e \times Ze}{r_0} = 2K \quad \dots \text{(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{r_0}{r} = \frac{1}{2} \Rightarrow r_0 = \frac{r}{2} \quad \dots \text{(1)}$$

**For Limitations of Rutherford Nuclear Model**  
Refer to Sol. 33 (1)

44. For longest wavelength of Lyman series,  $n_i = 2$

$$\frac{1}{\lambda_{\max}} = R \left( \frac{1}{1^2} - \frac{1}{2^2} \right) = \frac{3R}{4}$$

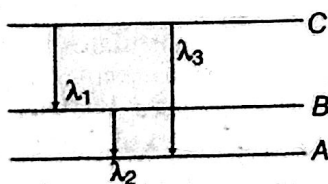
$$\lambda_{\max} = \frac{4}{3R} = \frac{4}{3 \times 1.097 \times 10^7}$$

$$= 1.215 \times 10^{-7} \text{ m}$$

$$\lambda_{\max} = 1215 \text{ \AA}$$

The lines of the Lyman series are found in ultraviolet region. (3)

45. (i) Refer to Sol. 42 (i) (1\frac{1}{2})  
(ii) According to question,



$$E_C - E_B = hc/\lambda_1 \quad \dots \text{(i)}$$

$$E_B - E_A = hc/\lambda_2 \quad \dots \text{(ii)}$$

$$E_C - E_A = hc/\lambda_3 \quad \dots \text{(iii)}$$

On adding Eqs. (i), (ii) and (iii), we get

$$E_C - E_B + E_B - E_A + E_C - E_A$$

$$2(E_C - E_A) = hc (1/\lambda_1 + 1/\lambda_2 + 1/\lambda_3)$$

$$2 \frac{hc}{\lambda_3} = hc \left[ \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} \right]$$

$$\Rightarrow \frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \Rightarrow \lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

This is the required expression. (1\frac{1}{2})

46. Here,  $\Delta E = 12.5 \text{ eV}$

Energy of an electron in  $n^{\text{th}}$  orbit of hydrogen atom is  $E_n = -\frac{136}{n^2} \text{ eV}$

In ground state,  $n = 1 \Rightarrow E_1 = -13.6 \text{ eV}$

Energy of an electron in the excited state after absorbing a photon of 12.5 eV energy will be

$$E_n = -13.6 + 12.5 = -1.1 \text{ eV}$$

$$\therefore n^2 = \frac{-136}{E_n} = \frac{-136}{-1.1} = 12.36$$

$$\Rightarrow n = 3.5 \quad \dots \text{(1)}$$

Here, state of electron cannot be fraction.

So,  $n = 3$  (2<sup>nd</sup> excited state).

$$\text{Lyman Series } \frac{1}{\lambda} = R \left[ \frac{1}{1^2} - \frac{1}{n^2} \right]$$

For first member,  $n = 2$

$$\therefore \frac{1}{\lambda_1} = R \left[ \frac{1}{1^2} - \frac{1}{2^2} \right] = 1.097 \times 10^7 \left[ \frac{4-1}{4} \right]$$

$$\Rightarrow \lambda_1 = 1.215 \times 10^{-7} \text{ m}$$

$$\text{Balmer Series } \frac{1}{\lambda} = R \left[ \frac{1}{2^2} - \frac{1}{n^2} \right]$$

For first member,  $n = 3$

$$\therefore \frac{1}{\lambda_1} = R \left[ \frac{1}{2^2} - \frac{1}{3^2} \right]$$

$$= 1.097 \times 10^7 \left[ \frac{1}{4} - \frac{1}{9} \right]$$

$$\Rightarrow \lambda_1 = 6.56 \times 10^{-7} \text{ m} \quad \dots \text{(2)}$$

47. (i) According to Bohr's postulates, in a hydrogen atom, as single electron revolves around a nucleus of charge  $+e$ . For an electron moving with a uniform speed in a circular orbit of a given radius, the centripetal force is provided by coulomb force of attraction between the electron and the nucleus. The gravitational attraction may be neglected as the mass of electron and proton is very small. (1/2)



So,  $mv^2/r = ke^2/r^2$  (where,  $k = 1/4\pi\epsilon_0$ )

or  $mv^2 = ke^2/r$  ... (i)

where,  $m$  = mass of electron  
 $r$  = radius of electronic orbit  
 $v$  = velocity of electron

Again, by Bohr's second postulates

$$mvr = nh/2\pi$$

where,  $n = 1, 2, 3, \dots$  or  $v = nh/2\pi mr$

Putting the value of  $v$  in Eq. (i),

$$m \left( \frac{nh}{2\pi mr} \right)^2 = \frac{ke^2}{r} \Rightarrow r = \frac{n^2 h^2}{4\pi^2 k m e^2} \dots (ii)$$

Kinetic energy of electron,

$$E_K = \frac{1}{2}mv^2 = \frac{ke^2}{2r} \quad \left( \because \frac{mv^2}{r} = \frac{ke^2}{r^2} \right)$$

Using Eq. (ii), we get

$$E_K = \frac{ke^2}{2} \frac{4\pi^2 k m e^2}{n^2 h^2} = \frac{2\pi^2 k^2 m e^4}{n^2 h^2} \quad (1/2)$$

Potential energy of electron,

$$E_P = -\frac{k(e) \times (e)}{r} = -\frac{ke^2}{r}$$

Using Eq. (ii), we get

$$E_P = -ke^2 \times \frac{4\pi^2 k m e^2}{n^2 h^2} = -\frac{4\pi^2 k^2 m e^4}{n^2 h^2}$$

Hence, total energy of the electron in the  $n$ th orbit

$$E = E_P + E_K = -\frac{4\pi^2 k^2 m e^4}{n^2 h^2} + \frac{2\pi^2 k^2 m e^4}{n^2 h^2} \\ = -\frac{2\pi^2 k^2 m e^4}{n^2 h^2} = -\frac{136}{n^2} \text{ eV} \quad (1/2)$$

This is the required expression.

(ii) Refer to Sol. 46 (1/2)

48. (i) Energy of electron in  $n$ th orbit of hydrogen atom.

$$E_n = \frac{-136}{n^2} \text{ eV} \quad (1/2)$$

For  $n = 1 \Rightarrow E_1 = -136 \text{ eV}$

For  $n = 2 \Rightarrow E_2 = -136/4 = -34 \text{ eV} \quad (1/2)$

Energy required to move an electron

$$= E_2 - E_1 = -34 - (-136) \\ = -34 + 136 = 10.2 \text{ eV} \quad (1)$$

(ii) (a) Kinetic energy = - (Total energy of the electron in first excited state).

$$= -(-3.4) = 3.4 \text{ eV} \quad (1/2)$$

(b) Orbital radius in the excited state,  $r = r_0 n^2$

[For first excited state,  $n = 2$ ]

$$= 4 \times 0.53 = 2.12 \text{ \AA} \quad (1/2)$$

49. Given, Bohr's radius ( $r_0$ ) =  $53 \times 10^{-11} \text{ m}$

$$n = 3$$

Total energy of an electron in first excited state =  $-3.4 \text{ eV} \quad (1/2)$

(i) Radius of orbit is given by,  $r_n = n^2 r_0$

$$\text{For } n = 3 \quad r_3 = (3)^2 \times 53 \times 10^{-11}$$

$$\Rightarrow r_3 = 47.7 \times 10^{-11} \text{ or } r_3 = 4.77 \times 10^{-10} \text{ m} \quad (1)$$

(ii) (a) Kinetic energy = - Total energy

$$= -(-3.4 \text{ eV}) = 3.4 \text{ eV}$$

(b) Potential energy =  $-2 \times$  Kinetic energy

$$= -2 \times 3.4 = -6.8 \text{ eV} \quad (1/2)$$

50. Refer to Sol. 47(i) (1/2)

When the electron in a hydrogen atom jumps from higher energy level to the lower energy level, the difference of energies of the two energy levels is emitted as a radiation of particular wavelength. It is called a **spectral line**.

In H-atom, when an electron jumps from the orbit  $n_i$  to orbit  $n_f$ , the wavelength of the emitted radiation is given by

$$1/\lambda = R (1/n_f^2 - 1/n_i^2)$$

where,  $R$  = Rydberg's constant =  $1.09678 \times 10^7 \text{ m}^{-1} \quad (1/2)$

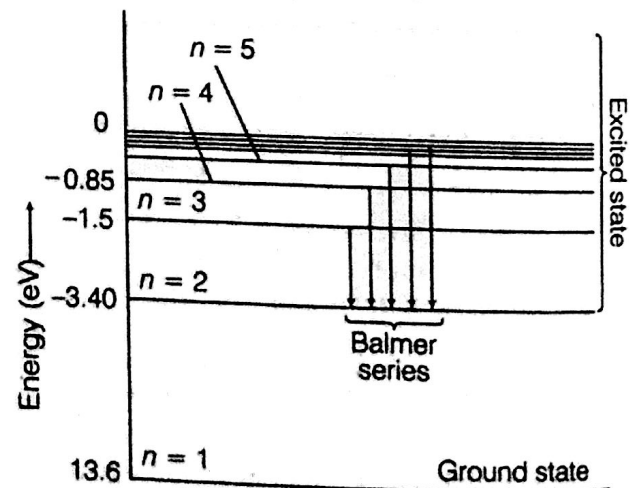
For Balmer series,  $n_f = 2$  and  $n_i = 3, 4, 5, \dots$

$$1/\lambda = R (1/2^2 - 1/n_i^2)$$

where,  $n_i = 3, 4, 5, \dots$

These spectral lines lie in the visible region.

Line spectra for the Balmer series is shown below



51. According to Bohr's postulates for hydrogen atom electron revolves in a circular orbit around the heavy positively charged nucleus. These are the stationary (orbits) states of the atom.

For a particular orbit, electron moves there, so it possess both kinetic energy and potential energy.

Hence, total energy ( $E$ ) of atom is sum of kinetic energy ( $K$ ) and potential energy ( $U$ ).

$$\text{i.e. } E = K + U \quad (1/2)$$

Let us assume that the nucleus has positive charge  $Ze$ . An electron moving with a constant speed  $v$  along a circle of radius  $r$  with centre at the nucleus.

Force acting on electron due to nucleus is given by,

$$F = Ze^2 / 4\pi\epsilon_0 r^2$$

The acceleration of electron  $v^2/r$  (towards the centre)

If  $m$  = mass of an electron, then from Newton's second law

$$F = m(v^2/r) \Rightarrow \frac{Ze^2}{4\pi\epsilon_0 r^2} = m \left( \frac{v^2}{r} \right)$$

$$\Rightarrow r = Ze^2 / 4\pi\epsilon_0 mv^2 \quad \dots(i)$$

From Bohr's quantisation rules,

$$mvr = n \frac{h}{2\pi} \quad \dots(ii)$$

where,  $n$  is a positive integer. (1/2)

Substituting the value of  $r$  from Eq. (i) in Eq. (ii), we get

$$mv \cdot \frac{Ze^2}{4\pi\epsilon_0 (mv^2)} = n \frac{h}{2\pi}$$

$$\Rightarrow v = Ze^2 / 2\epsilon_0 hn \quad \dots(iii)$$

$$\text{So, kinetic energy, } K = \frac{1}{2}mv^2 = \frac{Z^2e^4}{8\epsilon_0^2 h^2 n^2} \quad \dots(iv)$$

Potential energy of the atom,

$$U = - \frac{Ze^2}{4\pi\epsilon_0 r} \quad \dots(v)$$

Using Eq. (iii) in Eq. (i), we get

$$r = \frac{Ze^2}{4\pi\epsilon_0 m \frac{(Ze^2)^2}{(2\epsilon_0 hn)^2}} = \frac{4\epsilon_0^2 h^2 n^2}{(4\pi\epsilon_0) m Ze^2}$$

$$\Rightarrow r = \frac{\epsilon_0 h^2 n^2}{\pi m Ze^2} \quad (1)$$

Using value of  $r$  in Eq. (v), we get

$$U = \frac{-Ze^2}{4\pi\epsilon_0 (\epsilon_0 h^2 n^2 / \pi m Ze^2)} = \frac{-Z^2 e^4 m}{4\epsilon_0^2 h^2 n^2}$$

This implies,  $K = -2U$

So, the total energy in  $n^{\text{th}}$  energy level of hydrogen atom is

$$E = K + U = + \frac{mZ^2e^4}{8\epsilon_0^2 h^2 n^2} - \frac{mZ^2e^4}{4\epsilon_0^2 h^2 n^2} = - \frac{Z^2e^4 m}{8\epsilon_0^2 h^2 n^2} \quad (1)$$

52. (i)  $\therefore$  Energy of photon and associated wavelength  $\lambda$  are given by

$$E = \frac{1242 \text{ eV}}{\lambda \text{ (in nm)}}$$

$$\text{Here, } \lambda = 275 \text{ nm} \Rightarrow E = \frac{1242}{275} = 4.5 \text{ eV}$$

This energy of photon exist corresponding to the transition B.

(ii) The maximum wavelength occurs during the transition of energy level having minimum difference of their energies. It is possible for transition A. (1/2)

For details Refer to Sol. 32 (ii) (1/2)

53. (i) Bohr's second postulate (quantum condition) states that the electron revolves around the nucleus in certain privileged orbit which satisfy certain quantum condition that angular momentum of an electron is an integral multiple of  $h/2\pi$ , where,  $h$  is Planck's constant,

$$\text{i.e. } L = mvr = nh/2\pi \quad (1)$$

where,  $m$  = mass of electron,  $v$  = speed of electron and  $r$  = radius of orbit of electron

$$\Rightarrow 2\pi r = n(h/mv) \quad (1)$$

Since, de-Broglie wavelength associated with an electron is given as

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$\therefore$  Circumference of electron in  $n^{\text{th}}$  orbit =  $n \times$  de-Broglie wavelength associated with electron.

(ii) Refer to Sol. 3 (1)

54. **Key Idea** Photon is emitted when electron transits from higher energy state to lower energy state, the difference of energy of the state appear in form of energy of photon.

According to Bohr's theory of hydrogen atom, energy of photon released,  $E_2 - E_1 = h\nu$  (1/2)

$$\text{Given, } E_1 = -1.51 \text{ eV}$$

$$E_2 = -0.85 \text{ eV}$$

$$E_2 - E_1 = -0.85 - (-1.51) = 1.51 - 0.85$$

$$E_2 - E_1 = 0.66 \text{ eV} \quad (1/2 \times 2 = 1)$$

So, the wavelength of emitted spectral line,

$$\lambda = \frac{1242 \text{ eV} \cdot \text{nm}}{E \text{ (in eV)}} = \frac{1242 \text{ eV} \cdot \text{nm}}{0.66 \text{ eV}} = 1.88 \times 10^{-6} \text{ m} \quad (1/2)$$

As here,  $\lambda = 1.88 \times 10^{-6} \text{ m} = 18751 \times 10^{-10} \text{ m}$

The wavelength belongs to Paschen series of hydrogen spectrum. (1)

55. Given,  $Z = 80$ ,

$$KE = K = 8 \text{ MeV} = 8 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$\therefore \text{Energy, } K = \frac{(Ze)(2e)}{4\pi\epsilon_0 r_0} \quad (1)$$

where,  $r_0$  = distance of closest approach.

$$\Rightarrow r_0 = 2Ze^2 / 4\pi\epsilon_0(K) \quad (1/2)$$

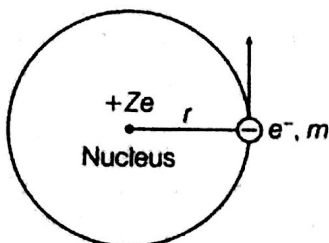
$$\Rightarrow r_0 = \frac{9 \times 10^9 \times 2 \times 80 \times (1.6 \times 10^{-19})^2}{8 \times 10^6 \times 1.6 \times 10^{-19}} \quad (1/2)$$

$$\Rightarrow r_0 = 2.88 \times 10^{-14} \text{ m} \quad (1/2)$$

As,  $r_0 \propto (1/K)$

If KE gets doubled, distance of closest approach reduces to half. (1/2)

56. Let an electron revolves around the nucleus of hydrogen atom. The necessary centripetal force is provided by electrostatic force of attraction.



$$\therefore \frac{mv^2}{r} = \frac{ke^2}{r^2} \Rightarrow r = \frac{ke^2}{mv^2} \quad \dots(i)$$

where,  $m$  is mass of electron and  $v$  is its speed of a circular path of radius  $r$ .

By Bohr's second postulates,

$$mvr = nh/2\pi \quad \text{where, } n = 1, 2, 3 \dots$$

$$r = nh/2\pi mv \quad \dots(ii)$$

On comparing Eqs. (i) and (ii), we get

$$\frac{ke^2}{mv^2} = \frac{nh}{2\pi mv} \Rightarrow v = \frac{2\pi ke^2}{nh}$$

Substituting in Eq. (ii), we get

$$r = \frac{n^2 h^2}{4\pi^2 m k e^2} \quad \dots(iii)$$

$$\Rightarrow r \propto n^2 \quad (1)$$

Now, kinetic energy of electron

$$KE = 1/2 mv^2 = ke^2/2r$$

Also, potential energy,  $PE = -ke^2/2r$

Energy of electron in  $n$ th orbit,

$$E_n = -\frac{ke^2}{2r} = -\frac{ke^2}{2} \cdot \frac{4\pi^2 m k e^2}{n^2 h^2}$$

$$\Rightarrow E_n = -\frac{2\pi^2 m k^2 e^4}{n^2 h^2} \quad \dots(iv)$$

where,  $R = \frac{2\pi^2 m k^2 e^4}{ch^3} \Rightarrow E_n = -\frac{Rhc}{n^2} \quad \dots(v)$   
(1)

where,  $n = 1, 2, 3 \dots$

For  $n = n_1 \Rightarrow E_n \propto \frac{1}{n^2}$

$$E_{n_1} = -\frac{Rhc}{n_1^2}$$

and  $E_{n_f} = -\frac{Rhc}{n_f^2}$

By Bohr's postulates,  $E_{n_f} - E_{n_1} = h\nu$

$$\Rightarrow Rhc [1/n_1^2 - 1/n_f^2] = h\nu$$

where,  $c$  = velocity of light

$$\nu = Rc [1/n_1^2 - 1/n_f^2]$$

This is required expression for frequency associated with photon. (1)

57. (i) The kinetic energy ( $E_K$ ) of the electron in an orbit is equal to negative of its total energy ( $E$ ).

$$E_K = -E = -(-1.5) = 1.5 \text{ eV} \quad (1)$$

(ii) The potential energy ( $E_P$ ) of the electron in an orbit is equal to twice its total energy ( $E$ ).

$$\text{i.e. } E_P = 2E = -1.5 \times 2 = -3 \text{ eV} \quad (1)$$

(iii) As, a result of transition of electron from excited state to ground state.

$$\text{Energy of radiation} = -1.5 - (-13.6)$$

( $\therefore$  Ground state energy of H-atom =  $-13.6 \text{ eV}$ )

$$E = h\nu = h \frac{c}{\lambda}$$

$$\Rightarrow hc/\lambda = 12.1 \text{ eV} = \text{energy of radiation}$$

$$\therefore \frac{1}{\lambda} = \frac{12.1 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34} \times 3 \times 10^8}$$

$$\Rightarrow \lambda = 1.025 \times 10^{-7} \text{ m} = 1025 \text{ \AA} \quad (1)$$

58. (i) Refer to Sol. 34

As, radius of electron's  $n$ th orbit in hydrogen atom

$$r_n = \frac{\epsilon_0 h^2}{\pi m e^2} n^2$$

$$\Rightarrow r_n \propto n^2 \quad (1/2)$$

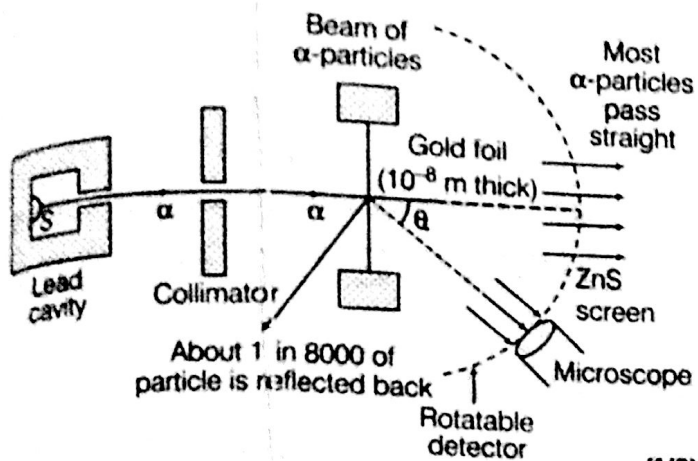
(ii) Refer to Sol. 51 on pages 384 and 385.

Also, the total energy of an electron belonging to  $n$ th orbit,

$$E_n = -\frac{me^2}{8\epsilon_0 n^2 h^2} \Rightarrow |E_n| \propto \frac{1}{n^2}$$

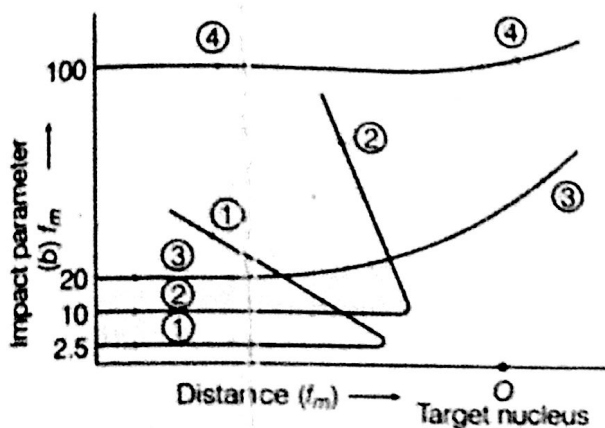
i.e. total energy of electron increases as  $\frac{1}{n^2}$  (1/2)

59. Given figure shows a schematic diagram of Geiger-Marsden experiment.



(1/2)

Trajectories of the scattered  $\alpha$ -particles



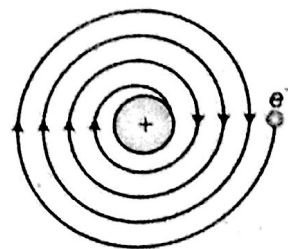
(1/2)  
(2)

Refer to Sol. 29

60. Basic assumptions of Rutherford atomic model are given below:

- (i) Atom consists of small central core, called atomic nucleus in which whole mass and positive charge is assumed to be concentrated.
- (ii) The size of the nucleus is much smaller than size of the atom.
- (iii) The nucleus is surrounded by electrons. Atoms are electrically neutral as total negative charge of electrons surrounding the nucleus is equal to total positive charge on the nucleus.
- (iv) Electrons revolve around the nucleus in various circular orbits and necessary centripetal force is provided by electrostatic force of attraction between positively charged nucleus and negatively charged electrons. (2)

**Stability of atom** When an electron revolves around the nucleus, then it radiates electromagnetic energy and hence, radius of orbit of electron decreases gradually. Thus, electron revolve on spiral path of decreasing radius and finally, it should fall into nucleus, but this does not happen. Thus, Rutherford atomic model cannot account for stability of atom.



Instability of Atom

(1)

61. (i) Speed of the electron in  $n$ th orbit.

$\therefore$  Centripetal force of revolution is provided by electrostatic force of attraction.

$$mv^2/r = ke^2/r^2$$

where,  $m$  = mass of electron

$v$  = speed of electron

$r$  = radius of orbit of the electron

$$\Rightarrow r = ke^2/mv^2 \quad \dots(i)$$

Also, from Bohr's postulates,

$$mvr = \frac{nh}{2\pi} \Rightarrow r = \frac{nh}{2\pi mv} \quad \dots(ii)$$

On comparing Eqs. (i) and (ii), we get

$$\frac{ke^2}{mv^2} = \frac{nh}{2\pi mv} \Rightarrow v = \frac{2\pi ke^2}{nh} \quad \dots(iii) \quad (1/2)$$

$$\text{or } v = \left( \frac{2\pi ke^2}{ch} \right) \frac{c}{n} \quad \left[ \because k = \frac{1}{4\pi\epsilon_0} \right] \quad (1/2)$$

where,  $c$  = velocity of light or  $v = \alpha \frac{c}{n} \quad \dots(iv)$

where,  $\alpha = 2\pi ke^2/ch$  and known as fine structure constant.

$$\text{Also, } \alpha = \frac{1}{137} \Rightarrow v = \frac{1}{137} \frac{c}{n} \quad \dots(v)$$

$$\text{For } n=1, v = \frac{1}{137} \times c$$

In K-shell of hydrogen atom, electron revolves with  $\frac{1}{137}$  times of speed of light. (1/2)

(ii) For radius of  $n$ th orbit of electron Refer to Sol. 34 (1/2)

**62. Key Idea** For minimum (i.e. maximum energy) wavelength, electron will transit from ground to highest energy level and for maximum wavelength (i.e. minimum energy) electron will transit to first immediate energy state.

**For Bohr's Postulates**

(Bohr's Model of Hydrogen Atom). (1/2)

In second excited state, i.e.  $n=3$  the number of spectral lines obtained would be

$$N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = \frac{3 \times 2}{2} = 3$$

∴ 3 spectral lines would be obtained.

For minimum wavelength, the transition would be from  $n = 3$  to  $n = 1$

$$\frac{1}{\lambda_{\min}} = R \left[ \frac{1}{1^2} - \frac{1}{3^2} \right] = \frac{8R}{9} \quad \dots(i)$$

For maximum wavelength the transition would be from

$$\begin{aligned} n &= 3 \text{ to } n = 2 \\ \Rightarrow \frac{1}{\lambda_{\max}} &= R \left[ \frac{1}{2^2} - \frac{1}{3^2} \right] \\ &= \left( \frac{9-4}{36} \right) R = \frac{5R}{36} \quad \dots(ii) \end{aligned}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{\lambda_{\max}}{\lambda_{\min}} = \frac{8R/9}{5R/36} = \frac{8R}{9} \times \frac{36}{5R} = \frac{32}{5}$$

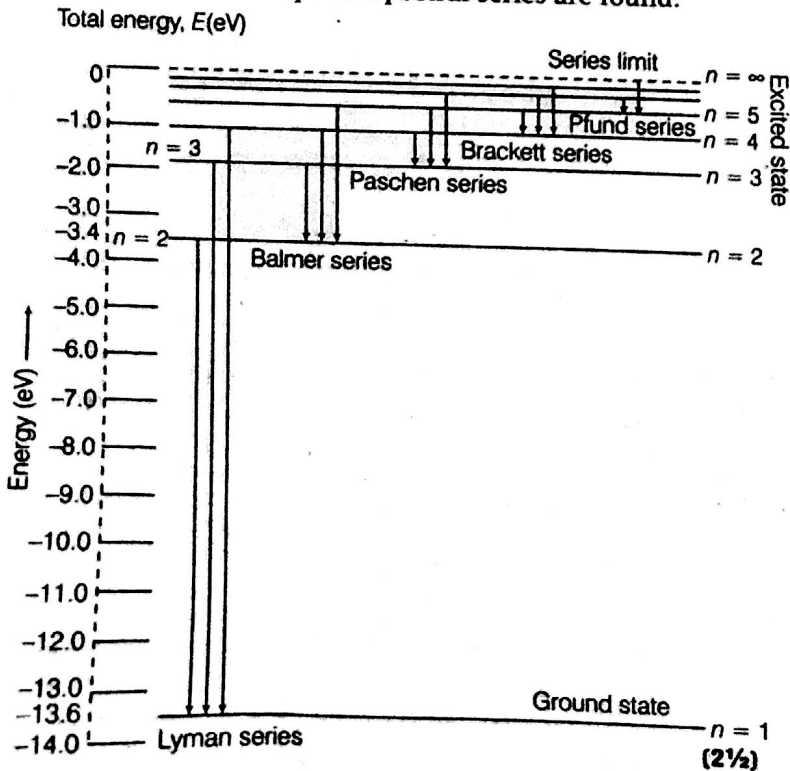
$$\Rightarrow \lambda_{\max} : \lambda_{\min} = 32 : 5 \quad (1\frac{1}{2})$$

63. Refer to Sol. 56 (1\frac{1}{2})

But  $\Delta E = h\nu$

$$\nu = Rc \left[ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right] \text{ or } \nu = \frac{me^4}{8\epsilon_0^2 h^3} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad (1)$$

When electron in hydrogen atom jumps from energy state  $n_i = 4$  to  $n_f = 3, 2, 1$ , the Paschen, Balmer and Lyman spectral series are found.



64. (i) (a) Refer to Sol. 58(i)  
 (b) Refer to Sol. 58(ii)  
 (ii) Refer to Sol. 28

(3)

(2)