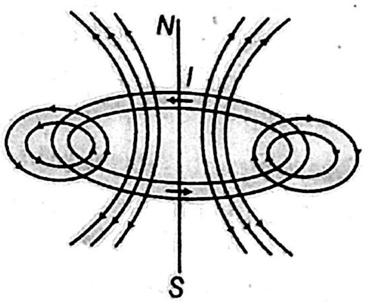
Magnetism (solution)

Explanations

1. Magnetic field lines due to a current carrying loop are given by



(1)

2. To calculate net magnetic field at point O, first of all, calculate the magnetic field at point O due to both coils separately, with direction. By vector addition of these two magnetic fields, net magnetic field can be obtained.

Magnetic field at O due to two rings will be in same direction ($Q \to P$, along the axis) and of equal magnitude.

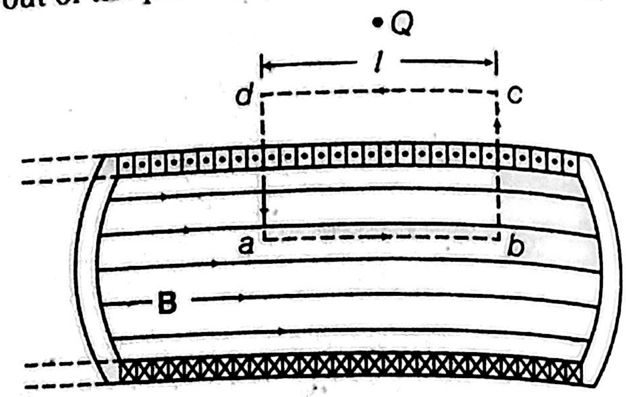
$$B = B_1 + B_2 \text{ but } B_2 = B_1$$ **(1/2)**

$$\Rightarrow B = 2B_1 = 2\left[\frac{\mu_0 Ir^2}{2(r^2+r^2)^{3/2}}\right]$$ **(1/2)**

$$B = \frac{\mu_0 Ir^2}{(2r^2)^{3/2}} = \frac{\mu_0 Ir^2}{2^{3/2} r^3}$$ **(1/2)**

$$B = \frac{\mu_0 I}{2^{3/2} r}$$ **(1/2)**

3. Figure shows the longitudinal sectional view of long current carrying solenoid. The current comes out of the plane of paper at points marked.



Let **B** be the magnetic field at any point inside the solenoid.

Considering the rectangular closed path $abcda$.

Applying Ampere's circuital law over loop $abcda$.

$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 \times$ (Total current passing through loop $abcda$)

$$\int_a^b \mathbf{B}\cdot d\mathbf{l} + \int_b^c \mathbf{B}\cdot d\mathbf{l} + \int_c^d \mathbf{B}\cdot d\mathbf{l} + \int_d^a \mathbf{B}\cdot d\mathbf{l} = \mu_0 \left(\frac{N}{L} lI\right)$$ **(1)**

where, $\frac{N}{L}$ = number of turns per unit length and $ab = cd = l$ = length of rectangle.

$$\int_a^b Bdl \cos 0° + \int_b^c Bdl \cos 90° + 0 + \int_d^a Bdl \cos 90° = \mu_0 \left(\frac{N}{L}\right) lI$$

$$B\int_a^b dl = \mu_0 \left(\frac{N}{L}\right) lI \Rightarrow Bl = \mu_0 \left(\frac{N}{L}\right) lI$$

$$\Rightarrow B = \mu_0 (N/L) I \text{ or } B = \mu_0 nI$$

where, n = number of turns per unit length.

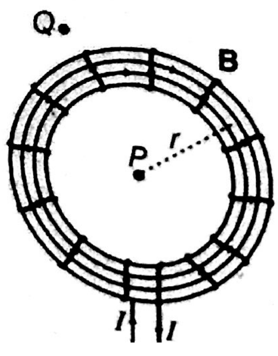
This is the required expression for magnetic field inside the long current carrying solenoid. **(1)**

4. Toroid is an endless solenoid in the form of ring. To calculate the magnetic field in the interior of it, Ampere's circuital law can be used.

Let I be the current, r be the mean radius and B be the magnetic field inside the toroid.

Then, the line integral of magnetic field around the closed path of circle of radius r is **(1)**

$$\oint \mathbf{B}\cdot d\mathbf{l} = \int Bdl \cos 0° = B\, 2\pi r \quad \text{...(i)}$$



Now, from Ampere's circuital law,

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ current enclosed by closed path

$\times$ number of turns ....(ii)

If n be the number of turns/unit length, then total number of turns $= n \times 2\pi r$

So, $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 n \times 2\pi r I$ ....(iii)

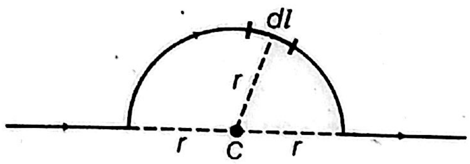
From Eqs. (i) and (iii), we get

$$B\, 2\pi r = \mu_0 n \times 2\pi r I$$

$$\Rightarrow \boxed{B = \mu_0 n I}$$

This is the required expression. **(1)**

5. When a straight wire is bent into semi-circular loop, then there are two parts which can produce the magnetic field at the centre, one is circular part and other is straight part due to which field is zero.



$\because$ Length L is bent into semi-circular loop.
Length of wire = Circumference of semi-circular wire

$$\Rightarrow \quad L = \pi r \Rightarrow r = \frac{L}{\pi} \quad ...(i)$$

Considering a small element dl on current loop. The magnetic field dB due to small current element Idl at centre C. Using Biot-Savart's law, we have

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90^\circ}{r^2} \quad [\because Idl \perp \mathbf{r}, \therefore \theta = 90^\circ]$$ **(1/2)**

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl}{r^2}$$

$\therefore$ Net magnetic field at C due to semi-circular loop,

$$B = \int_{\text{semicircle}} \frac{\mu_0}{4\pi} \frac{Idl}{r^2} \Rightarrow B = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int_{\text{semicircle}} dl$$ **(1/2)**

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{r^2} L$$

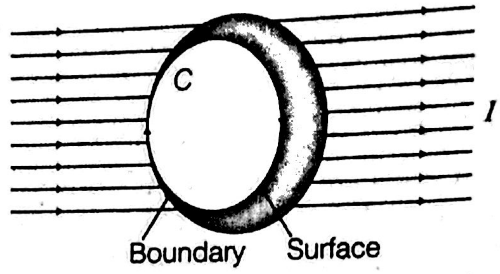
But, $r = L/\pi$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{IL}{(L/\pi)^2} = \frac{\mu_0}{4\pi} \times \frac{IL}{L^2} \times \pi^2 \Rightarrow B = \frac{\mu_0 I\pi}{4L}$$

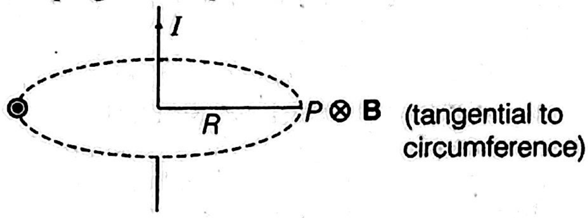
This is the required expression. **(1)**

6. Ampere's circuital law As, Ampere's circuital law states that the line integral of magnetic field **B** around any closed loop is equal to μ_0 times the total current threading through the loop. **(1)**

i.e. $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$



Derivation To explain the Ampere's circuital law consider an infinitely long conductor wire carrying a steady current I as shown in the figure.



In order to determine the magnetic field at point P which is situated at a distance R from the centre of the circular loop around the conductor wire, **B** (magnetic field) is tangential to circumference of the loop.

Now, $\oint \mathbf{B} \cdot d\mathbf{l} = \int B\, dl = B\, 2\pi R = \mu_0 I$

$\Rightarrow \quad B = \mu_0 I / 2\pi R$ [From Ampere's circuital law]

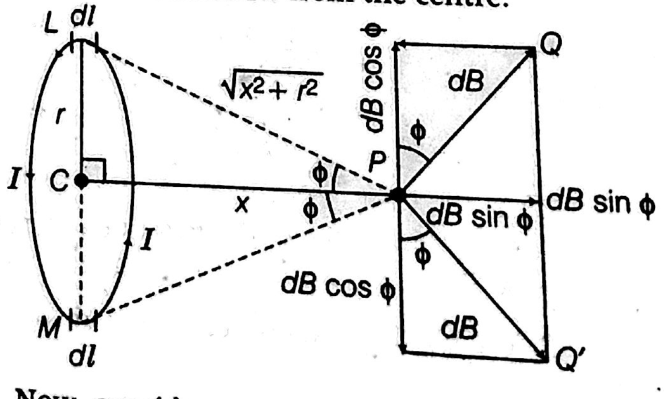
The direction of magnetic field will be determined by right hand rule. **(1)**

7. (i) Refer to Sol. 3
(Magnetic Field of a Solenoid) **(2)**

(ii) An endless solenoid in the form of a ring is called a toroid. **(1)**

8. (i) Refer to text (Biot-Savart's law). **(1)**

(ii) Let us consider a circular loop of radius r with centre C. Let the plane of the coil be perpendicular to the plane of the paper and current I be flowing in the direction as shown in the figure. Suppose P is any point on the axis at a direction x from the centre.



Now, consider a current element Idl on top (L) where current comes out of paper normally, whereas at bottom (M) enters into the plane of paper normally.

(along horizontal towards left)

$= \mu_0 I_2/2R$

Net magnitude of magnetic field at the common centre of the two coils,

$$B = \sqrt{B_P^2 + B_Q^2}$$

$$\Rightarrow B = \sqrt{\left(\frac{\mu_0 I_1}{2R}\right)^2 + \left(\frac{\mu_0 I_2}{2R}\right)^2}$$

$$B = \sqrt{\left(\frac{\mu_0}{2R}\right)^2 (I_1^2 + I_2^2)} \Rightarrow B = \frac{\mu_0}{2R}\sqrt{I_1^2 + I_2^2}$$

$$B = \frac{4\pi \times 10^{-7}}{2 \times R}\sqrt{(1)^2 + (\sqrt{3})^2}$$

$$B = 4\pi \times 10^{-7}/R \text{ Tesla}$$

Resultant magnetic field makes an angle θ with direction of B_Q, which is given by

$$\tan\theta = B_P/B_Q = 1/\sqrt{3} \Rightarrow \theta = 30°$$

$\because$ $LP \perp Idl$

Also, $MP \perp Idl$

$\because$ $LP = MP = \sqrt{x^2 + r^2}$

The magnetic field at point P due to current element Idl. According to Biot-Savart's law,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90°}{(x^2 + r^2)}$$

where, r = radius of circular loop,

x = distance of point P from centre along the axis.

The direction of dB is perpendicular to LP and along PQ, where $PQ \perp LP$. Similarly, the same magnitude of magnetic field is obtained due to current element Idl at the bottom and direction is along PQ', where $PQ' \perp MP$.

Now, resolving dB due to current element at L and M, $dB \cos\phi$ components balance each other and net magnetic field is given by

$$B = \oint dB \sin\phi = \oint \frac{\mu_0}{4\pi}\left(\frac{Idl}{x^2 + r^2}\right) \cdot \frac{a}{\sqrt{x^2 + r^2}}$$

$$\left[\because \text{In } \Delta PCL, \sin\phi = \frac{a}{\sqrt{x^2 + r^2}}\right]$$

$$= \frac{\mu_0}{4\pi}\frac{Ia}{(x^2 + r^2)^{3/2}}\oint dl = \frac{\mu_0}{4\pi}\frac{Ia}{(x^2 + r^2)^{3/2}}(2\pi r)$$

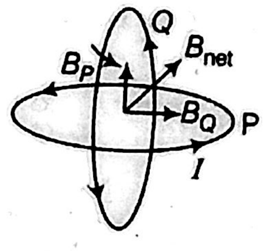
or $$B = \frac{\mu_0 I a^2}{2(x^2 + r^2)^{3/2}}$$

For N turns, $$B = \frac{\mu_0 N I r^2}{2(x^2 + r^2)^{3/2}} \text{ T}$$ (1)

For magnetic field lines Refer to Sol. 1 (1)

9. (i) Refer to the text (Biot-Savart law) (1)

(ii) (1)



Magnitude of magnetic field due to circular wire P,

$$B_P = \frac{\mu_0}{4\pi} \times \frac{2\pi I_1}{R} \text{ (along vertically upwards)}$$

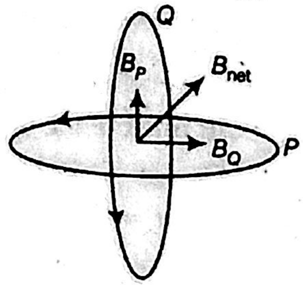
$= \mu_0 I_1/2R$

Magnitude of magnetic field due to circular wire Q,

$$B_Q = \mu_0/4\pi \times 2\pi I_2/R$$

10. Magnetic field due to circular loop P,

$$B_P = \frac{\mu_0 I_P}{2r}$$



Magnetic field due to circular loop Q,

$$B_Q = \mu_0 I_Q/2r$$

So, net magnetic field at the common centre of the loop is,

$$B_{net} = \sqrt{B_P^2 + B_Q^2}$$

$$= \sqrt{(\mu_0 I_P/2r)^2 + (\mu_0 I_Q/2r)^2}$$

$$= \frac{\mu_0}{2r}\sqrt{I_P^2 + I_Q^2} = \frac{4\pi \times 10^{-7}}{2 \times 5 \times 10^{-2}} \times 5$$

$$= 2\pi \times 10^{-5} \text{ T}$$ (1)

Resultant magnetic field makes an angle θ with B_Q which is given by,

$$\tan\theta = \frac{B_P}{B_Q} = \frac{I_P}{I_Q} = \frac{3}{4} \Rightarrow \theta = \tan^{-1}\left(\frac{3}{4}\right)$$ (1)

11. Refer to Sol. 8 (ii) (3)

12. (i) Refer to Sol. 6 (1)

(ii) According to Ampere's circuital law, the magnetic field is given by $B = \mu_0 n \hat{i}$

(a) The net magnetic field is given by

$B_{net} = B_2 - B_1$

$= \mu_0 n_2 I_2 - \mu_0 n_1 I_1 \qquad [\because I_2 = I_1 = I]$

$= \mu_0 I (n_2 - n_1)$

The direction is from B to A. **(1)**

(b) As the magnetic field due to S_1 is confined solely inside S_1 and the solenoids are assumed to be very long so, there is no magnetic field outside S_1 due to current in S_1. Similarly there is no field outside S_2.

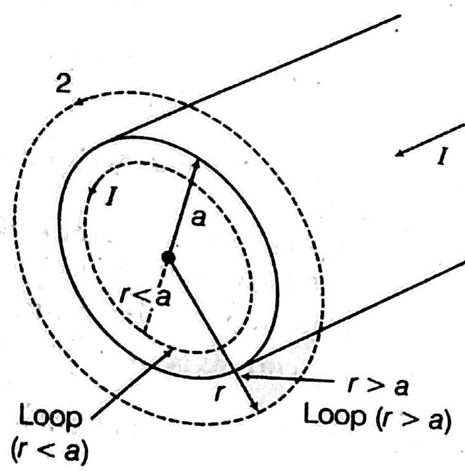
$\therefore \; B_{net} = 0$ **(1)**

13. (i) A toroid can be viewed as a solenoid which has been bent into circular shape to close on itself. **(1)**

(ii) Refer to Sol. 4 **(2)**

14. In these types of questions, first of all we have to calculate the current per unit area of cross-section, so that we can calculate the current in each loop, then only we can find the magnetic field.

As, the current is distributed uniformly across the cross-section of radius a.



$\therefore$ Current passes per unit cross-section $= I/\pi a^2$

$\therefore$ Current passes through the cross-section of radius r is,

$$I' = \left(\frac{I}{\pi a^2} \times \pi r^2\right) = \frac{Ir^2}{a^2} \qquad \text{...(i)}$$

(1/2)

(i) Consider a loop of radius r whose centre lies at the axis of wire where, $r < a$ as shown in figure inside the wire.

Applying Ampere's circuital law,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I'$$

(1/2)

$$\oint Bdl \cos 0^\circ = \mu_0 (Ir^2/a^2) \qquad \text{[From Eq. (i)]}$$

$$B \oint dl = \mu_0 \frac{Ir^2}{a^2} \Rightarrow B \times 2\pi r = \frac{\mu_0 Ir^2}{a^2}$$

$$\Rightarrow \quad B = \frac{\mu_0 Ir}{2\pi a^2} \Rightarrow B \propto r$$

(1)

(ii) Considering a loop of radius r whose centre lies at the axis of wire and $(r > a)$ as shown in outer dotted line.

$\therefore$ Current I threads the loops.

Applying Ampere's circuital law,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

$$\oint Bdl \cos 0^\circ = \mu_0 I \Rightarrow B \oint dl = \mu_0 I$$

$$B \times 2\pi r = \mu_0 I \Rightarrow B = \mu_0 I / 2\pi r \Rightarrow B \propto 1/r$$

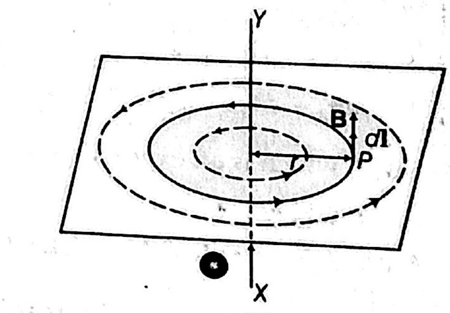
Thus, the field **B** is proportional to r as we move from the axis of cylinder towards its surface and then decreases as $1/r$. **(1)**

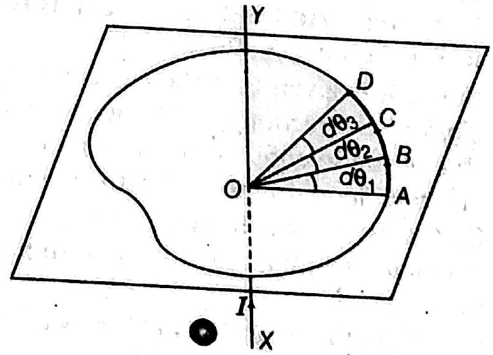
15. Refer to Sol. 14 **(3)**

16. (i) **For Biot-Savart's law** Refer to text

For magnetic field due to a current carrying loop on its axis Refer to Sol. 8 (ii) **(1+1 ½)**

(ii) When current in the coil is in anti-clockwise direction.





Consider any arbitrary closed path perpendicular to the plane of paper around a long straight conductor XY carrying current from X to Y, lying in the plane of paper.

Let, the closed path be made of large number of small elements, where

$$AB = d\mathbf{l}_1, \; BC = d\mathbf{l}_2, \; CD = d\mathbf{l}_3$$

Let $d\theta_1, d\theta_2, d\theta_3$, be the angles subtended by the various elements at point O through which conductor is passing. Then

$$d\theta_1 + d\theta_2 + d\theta_3 + \ldots = 2\pi$$

Suppose these small elements AB, BC, CD,... are small circular arcs of radii $r_1, r_2, r_3, \ldots$ respectively.

Then, $d\theta_1 = \dfrac{dl_1}{r_1}, d\theta_2 = \dfrac{dl_2}{r_2}, d\theta_3 = \dfrac{dl_3}{r_3}$

If $\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3$ are the magnetic field induction at a point along the small elements dl_1, dl_2, dl_3 ...... then from Biot-Savart's law we know that for the conductor of infinite length, magnetic field is given by

$$B_1 = \frac{\mu_0}{4\pi}\frac{2I}{r_1}; B_2 = \frac{\mu_0}{4\pi}\frac{2I}{r_2}; B_3 = \frac{\mu_0}{4\pi}\frac{2I}{r_3} \ldots$$

In case of each elements, the magnetic field induction $\mathbf{B}$ and current element vector $d\mathbf{l}$ are in the same direction. Line integral of $\mathbf{B}$ around closed path is

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mathbf{B}_1\cdot d\mathbf{l}_1 + \mathbf{B}_2\cdot d\mathbf{l}_2 + \mathbf{B}_3\cdot d\mathbf{l}_3 + \ldots$$

$$= B_1(dl_1) + B_2(dl_2) + B_3(dl_3) + \ldots$$

$$= \frac{\mu_0}{4\pi}\frac{2I}{r_1}dl_1 + \frac{\mu_0}{4\pi}\frac{2I}{r_2}dl_2 + \frac{\mu_0}{4\pi}\frac{2I}{r_3}dl_3 + \ldots$$

$$= \frac{\mu_0 2I}{4\pi}\left[\frac{dl_1}{r_1} + \frac{dl_2}{r_2} + \frac{dl_3}{r_3} + \ldots\right]$$

$$= \frac{\mu_0 2I}{4\pi}[d\theta_1 + d\theta_2 + d\theta_3 + \ldots] = \frac{\mu_0}{4\pi}2I \times 2\pi = \mu_0 I$$

$\Rightarrow \oint B\cdot dl = \mu_0 I$, which is an expression of Ampere's circuital law. **(2½)**

17. (i) **For Ampere's circuital law** Refer to Sol. 6

For magnetic field inside the toroid Refer to Sol. 4 **(3)**

(ii) Since, it is given that the current flows in the clockwise direction for an observer on the left side of the solenoid. It means that the left face of the solenoid acts as South pole and right face acts as North pole. Inside a bar, the magnetic field lines are directed from South to North. **(1)**

Therefore, the magnetic field lines are directed from left to right in the solenoid.

Magnetic moment of a single current carrying loop is given by, $m' = IA$.

So, magnetic moment of the whole solenoid is given by $m = Nm' = N(IA)$. **(1)**

18. The magnetic field at a point due to a circular loop is given by, $B = \dfrac{\mu_0}{4\pi}\cdot\dfrac{2\pi I a^2}{(a^2 + r^2)^{3/2}}$ **(1)**

where, I = current through the loop

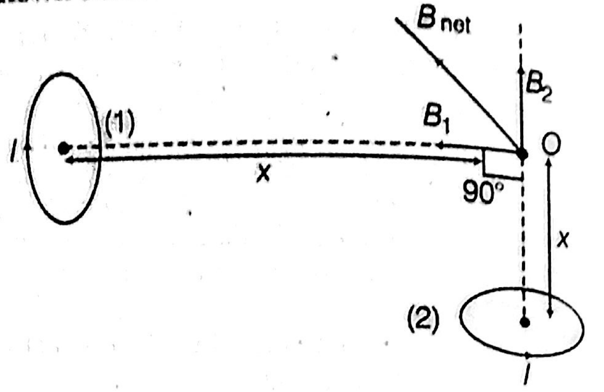
a = radius of the loop

and r = distance of O from the centre of the loop.

Since I, a and $r = x$ are the same for both the loops, the magnitude of B will be the same and is given by

$$B_1 = B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2\pi I a^2}{(a^2 + x^2)^{3/2}}$$ (1)

The direction of magnetic field due to loop (1) will be away from O and that of the magnetic field due to loop (2) will be towards O as shown. The direction of the net magnetic field will be as shown below: (1)



The magnitude of the net magnetic field is given by (1)

$$B_{net} = \sqrt{B_1^2 + B_2^2} \Rightarrow B_{net} = \frac{\mu_0}{4\pi}\frac{2\sqrt{2}\pi I a^2}{(a^2 + x^2)^{3/2}}$$ (1)

19. **For the statement of Biot-Savart's law** Refer to text (1)

For field at axial point of a circular coil Refer to Sol. 8 (ii) (1)

Magnetic field induction at the centre of the circular coil carrying current is, $B_1 = \dfrac{\mu_0}{4\pi}\cdot\dfrac{2\pi I}{a}$

and magnetic field induction at an axial point is

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2\pi a^2 I}{(a^2 + d^2)^{3/2}}$$

$$\frac{B_2}{B_1} = \frac{a^2 \times a}{(a^2 + d^2)^{3/2}} = \frac{a^3}{(a^2 + d^2)^{3/2}}$$

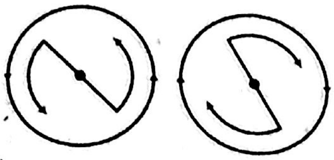
$$\frac{B_2}{B_1} = \frac{a^3}{(a^2 + 3a^2)^{3/2}} \quad [\because d = a\sqrt{3}]$$

$$= \frac{a^3}{(4a^2)^{3/2}} = \frac{a^3}{8a^3} \Rightarrow \frac{B_2}{B_1} = \frac{1}{8}$$

20. **For Biot-Savart's law** Refer to text

For the magnetic field due to a circular coil carrying current at a point along its axis Refer to Sol. 8(ii)

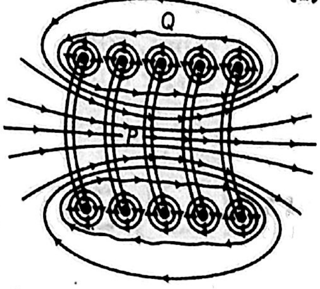
As current carrying loop has the magnetic field lines around it which exerts a force on a moving charge. Thus, it behaves as a magnet with two mutually opposite poles. **(1)**



The anti-clockwise flow of current behaves like a North pole, whereas clockwise flow as South pole. Hence, loop behaves as a magnet. **(1)**

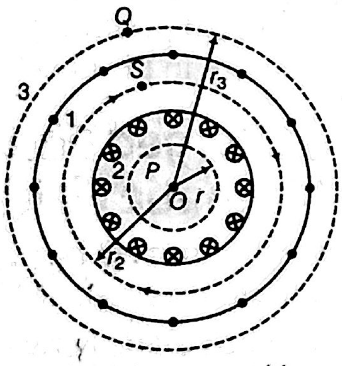
21. (i) Refer to Sol. 3 **(2)**

(ii) Solenoid is a hollow circular ring having large number of turns of insulated copper wire on it. Therefore, we can assume that toroid is a bent solenoid to close on itself.



Field due to solenoid **(1/2)**

The magnetic fields due to solenoid and toroid is shown in figures



Field due to toroid

Magnetic field inside the solenoid is uniform, strong and along its axis. Also, all field lines are mostly parallel while inside the toroid field line makes closed path. **(1½)**

(iii) The magnetic field in the solenoid can be increased by inserting a soft iron core inside it. **(1)**

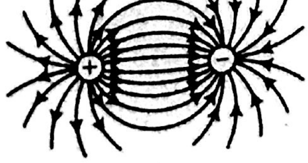
22. (i) **For statement of Ampere's circuital law**

Refer to Sol. 6 **(1)**

(ii) Refer to Sol. 3 **(2)**

(iii) Refer to Sol. 21 (ii) for magnetic field lines due to a finite solenoid

Electric field lines from an electric dipole has been shown below:



All the magnetic field lines are necessarily closed loops, whereas electric lines of force are not. **(2)**

(2)

23. (i) Refer to Sol. 8 (ii) **(1)**

(ii) Refer to Sol. 1

(iii) Magnetic field due to straight part

$$B = \int \frac{\mu_0}{4\pi} \frac{I d\mathbf{l} \times \mathbf{r}}{r^3}$$ **(1/2)**

For point O, $d\mathbf{l}$ and r for each element of the straight segments AB and DE are parallel. Therefore, $d\mathbf{l} \times \mathbf{r} = 0$. Hence, magnetic field due to straight segments is zero.

Magnetic field at the centre due to circular point

$$= \frac{\text{Magnetic field at the centre of circular coil}}{2}$$

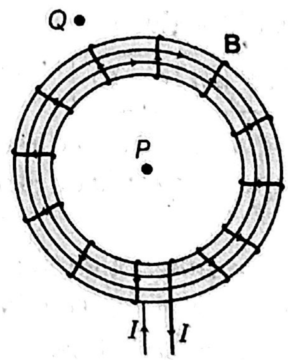
[$\because$ Here, coil is half]

$$= \frac{1}{2}\left(\frac{\mu_0 I}{2r}\right) = \frac{\mu_0 I}{4r}$$

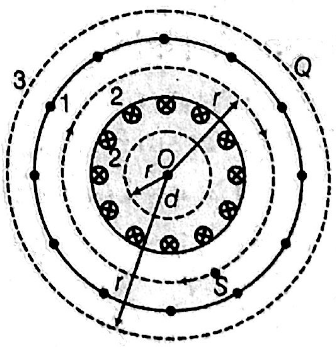
$$B = \frac{\mu_0 I}{4r} = \frac{(4\pi \times 10^{-7}) \times 12}{4 \times 2 \times 10^{-2}} = 6\pi \times 10^{-5}\ \text{T}$$ **(1½)**

24. (i) Refer to Sol. 6 **(1½)**

(ii) A solenoid bent into the form of closed loop is called **toroid.** The magnetic field **B** has a constant magnitude everywhere inside the toroid. **(1/2)**



(a) Let magnetic field inside the toroid is B along the considered loop (1) as shown in figure.



Applying Ampere's circuital law,

$$\oint_{\text{loop 1}} \mathbf{B} \cdot d\mathbf{l} = \mu_0\,(NI)$$

Since, toroid of N turns, threads the loop 1, N times, each carrying current I inside the loop. Therefore, total current threading the loop 1 is NI.

$$\Rightarrow \oint_{\text{loop 1}} \mathbf{B}\cdot d\mathbf{l} = \mu_0 NI$$

$$\Rightarrow B\oint_{\text{loop 1}} d\mathbf{l} = \mu_0 NI$$

$$B \times 2\pi r = \mu_0 NI \text{ or } B = \frac{\mu_0 NI}{2\pi r}$$ **(1)**

(b) **Magnetic field inside the open space interior the toroid.** Let the loop (2) is shown in figure experience magnetic field B.

No current threads the loop 2 which lie in the open space inside the toroid.

$\therefore$ Ampere's circuital law

$$\oint_{\text{loop 2}} \mathbf{B}\cdot d\mathbf{l} = \mu_0 (0) = 0 \Rightarrow B = 0$$ (1)

Magnetic field in the open space exterior of toroid Let us consider a coplanar loop (3) in the open space of exterior of toroid. Here, each turn of toroid threads the loop two times in opposite directions.

Therefore, net current threading the loop

$$= NI - NI = 0$$

$\therefore$ By Ampere's circuital law,

$$\oint_{\text{loop 3}} \mathbf{B}\cdot d\mathbf{l} = \mu_0 (NI - NI) = 0 \Rightarrow B = 0$$

Thus, there is no magnetic field in the open space interior and exterior of toroid.

(1)

Explanations

1. When a charged particle q moves with velocity $\mathbf{v}$ in a uniform magnetic field $\mathbf{B}$, then the force acting on it is given by

$$\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$$

(1)

2. $$F_{\text{lorentz}} = F_{\text{electric}} + F_{\text{magnetic}}$$ (1)

$$= qE + q\,(v \times B) = q[E + (v \times B)]$$ (1)

3. First proton has circular trajectories and second has helical. (1)

4. As we know that in a circular path, frequency of a charged particle is given by

$$\nu = \frac{qB}{2\pi m} \quad \text{or} \quad \nu \propto \frac{1}{m}$$

Since, $m_p > m_e$, therefore electron will move in circular path with higher frequency. (1)

5. The expression in vector form is given by

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}).$$

where, q is the charged particle. (1/2)

The direction of the magnetic force is in the direction of $\mathbf{v} \times \mathbf{B}$, i.e. perpendicular to the plane containing $\mathbf{v}$ and $\mathbf{B}$. (1/2)

6. For the given momentum of charge particle, radius of circular paths depends on charge and magnetic field as

$$r = mv/qB \qquad [\because qvB = mv^2/r]$$

For constant momentum, $r \propto (1/qB)$

$$\therefore\ r_{\text{proton}} : r_{\text{deuteron}} = q_{\text{deuteron}} : q_{\text{proton}} = 1:1$$ (1)

7. Ratio of forces acting on the two particles,

$$\frac{F_A}{F_B} = \frac{qv_1 B \sin 90^\circ}{(2q)\, v_2 B \sin 90^\circ} = \frac{1}{2}$$

[Given, $B_1 = B_2 = B$]

$$\Rightarrow \quad v_1/v_2 = 1 \Rightarrow v_1 : v_2 = 1:1$$ (1)

8. Velocity of α-particles

$$\mathbf{v} = v\hat{\mathbf{i}} \qquad \text{[Projected along } X\text{-axis]}$$

Magnetic force on α-particles,

$$\mathbf{F}_m = q\,(\mathbf{v} \times \mathbf{B}) = q\,(v\hat{\mathbf{i}} \times \mathbf{B})$$

As, $\mathbf{F}_m = F_m\,\hat{\mathbf{j}}$ [Oriented along Y-axis]

$$\Rightarrow \quad F_m\,\hat{\mathbf{j}} = q\,(v\hat{\mathbf{i}} \times \mathbf{B}) \Rightarrow \mathbf{B} = -B\hat{\mathbf{k}} = B(-\hat{\mathbf{k}})$$

The direction of magnetic field must be along $-Z$-axis. (1)

9. Given, $\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B}) \Rightarrow F = qvB \sin\theta$

where, θ is the angle between $\mathbf{v}$ and $\mathbf{B}$.

$$\Rightarrow \quad B = F/qv \sin\theta$$

If $q = 1$ C, $v = 1\ \text{ms}^{-1}$, $\theta = 90^\circ$

The magnetic field at any point can be given by

$$B = \frac{1\ \text{N}}{[(1\ \text{C})\,(1\ \text{ms}^{-1}) \sin 90^\circ]} = 1\ \text{N/A-m} = 1\ \text{T}$$

$\therefore$ SI unit of magnetic field = 1 T

Thus, the magnetic field induction at a point is said to be one tesla if a charge of one coulomb while moving at right angle to a magnetic field with a velocity of $1\ \text{ms}^{-1}$ experiences a force of 1 N at that point. (1)

10. The kinetic energy of proton due to potential V is given by

$$K = eV$$

where, e = charge on proton.

The radius of circular path of proton in a magnetic field is

$$r = \frac{\sqrt{2mK}}{qB} = \frac{\sqrt{2meV}}{qB}$$

If potential is doubled, i.e. **(1)**

$V' = 2V$, then

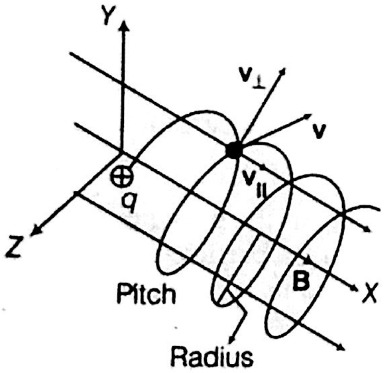
$$r' = \frac{\sqrt{2me \times 2V}}{qB} = \sqrt{2}r$$

Thus, radius becomes $\sqrt{2}$ times of previous value. **(1)**

11. When an charged particle q enters a uniform magnetic field at an angle of 30°, then its path becomes helix of radius

$$r = \frac{mv \sin 30°}{eB} = \frac{mv}{2eB}$$ **(1)**

For diagram and discription If a charged particle has a velocity not perpendicular to **B**, then component of velocity along **B** remains unchanged as the motion along the magnetic field will not be affected by the magnetic field. Then, the motion of the particle in a plane perpendicular to **B** is as before a circular one, thereby producing a helical motion.



(1)

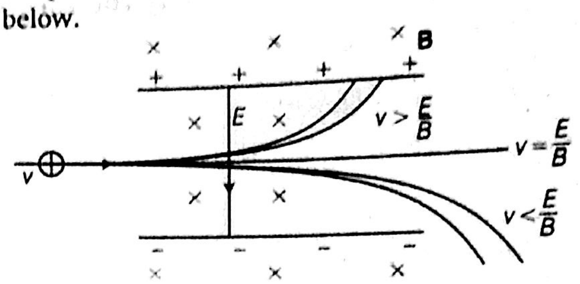
12. If we adjust the value of electric field (E) and magnetic field (B), such that the magnitude of two forces are equal, then the total force on the charge is zero. Also, the charge will move in the fields undeflected. This happens, when

Electric force, (F_e) = Magnetic force, (F_m)

$\Rightarrow \quad qE = qvB$ or $v = E/B$

The above condition can be used to select a charged particle of a particular velocity from the charges moving with different speeds.

A diagram in which particle has being deflected in the presence of magnetic and electric field is shown below. **(1)**



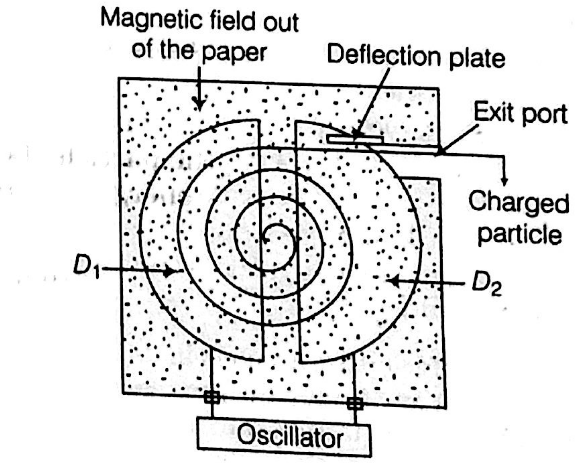
(1)

13. One tesla is defined as the field which produces a force of 1 newton when a charge of 1 coulomb moves perpendicularly in the region of the magnetic field at a velocity of $1\ \text{ms}^{-1}$.

As, $\quad F = qvB \Rightarrow B = F/qv$

$\Rightarrow \quad 1\ \text{T} = 1\ \text{N} / (1\text{C})\ (1\ \text{ms}^{-1})$ **(2)**

14. Principle A charged particle can be accelerated to very high energies by making it pass through a moderate electric field a number of times. This can be done with the help of a perpendicular magnetic field which throws the charged particle into a circular motion, the frequency of which does not depend on the speed of the particle and the radius of the circular orbit.



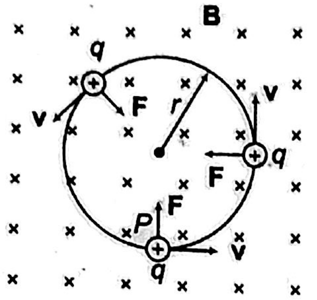
(1)

Working Inside the two semi-circular dees of the cyclotron, the particle is shielded from the electric field and normal magnetic field acts on the particle and makes it to go round in a circular path.

Every time, the particle moves from one dee to the other it comes under the influence of electric field which ensures to increase the energy of the particle as the sign of the electric field changed alternately.

The increased energy increases the radius of the circular path, so the accelerated particle moves in a spiral path. **(1)**

15. A charge q projected perpendicular to the uniform magnetic field B with velocity v. The perpendicular force, $F = qvB$, acts like a centripetal force perpendicular to the magnetic field. Then, the path followed by charge is circular as shown in the figure. **(1)**



$$qvB = \frac{mv^2}{r} \quad \text{or} \quad r = \frac{mv}{qB}$$

where, r = radius of the circular path followed by charge projected perpendicular to a uniform magnetic field. **(1)**

16. Lorentz force always acts along the direction perpendicular to the direction of velocity of the particle.

Magnetic Lorentz force,

$$\mathbf{F}_m = q\,(\mathbf{v} \times \mathbf{B})$$ **(1)**

$\because$ $\quad \mathbf{F} \perp \mathbf{v}$

$\Rightarrow$ Force is perpendicular to displacement made by charged particle.

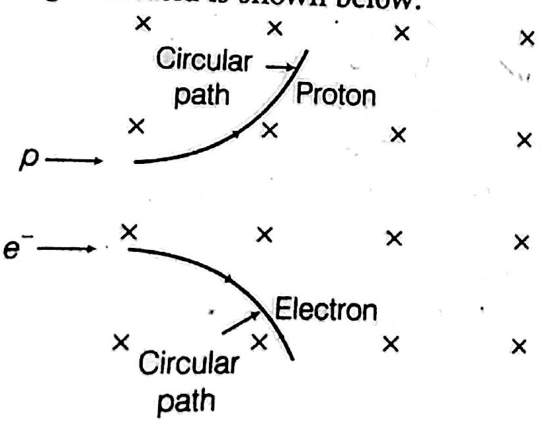
$\therefore$ $W = Fd \cos 90° = 0$

[$\because$ Force F and displacement d are perpendicular to each other]

$\Rightarrow$ $\quad W = 0$

No work is done by magnetic Lorentz force on the charged particle. **(1)**

17. When a charged particle enters in the magnetic field at right angle, then the particle follows a circular path. The trajectory of the two particles in the magnetic field is shown below.



Radius of the circular path, $r = \dfrac{mv}{qB}$

For same speed v, magnitude of charge and magnetic field

$$r \propto m \Rightarrow \frac{r_e}{r_p} = \frac{m_e}{m_p}$$ **(1)**

where, m_e and m_p are masses of electron and proton, respectively

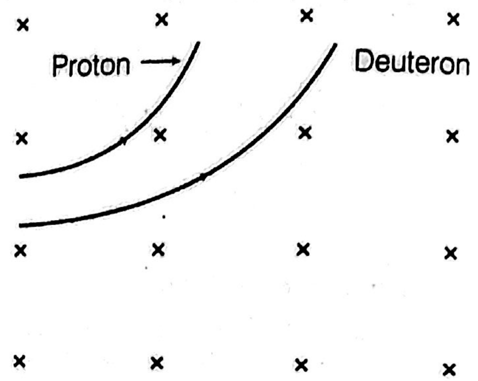
$\because$ $\quad m_e < m_p$

(Proton is much heavier than electron)

$\Rightarrow$ $\quad r_e < r_p$

The curvature of path of electron is much more than curvature of path of proton. **(1)**

18. The trajectory of the two particles in the magnetic field is shown below. **(1)**



$\because$ $\quad \dfrac{r_d}{r_p} = \dfrac{m_d}{m_p},$

$\because$ $\quad m_d = 2m_p$

$\Rightarrow$ $\quad r_d = 2r_p$

or $\quad r_d : r_p = 2:1$ **(1)**

NOTE Smaller the radius, greater the curvature and vice-versa. This is why, proton's path has got greater curvature.

19. In a cyclotron, in the presence of the magnetic field, the charged particle moves in the circular path whose radius is r, which depends on its speed. **(1)**

Hence, magnetic force on q = centripetal force on q.

$\Rightarrow$ $qvB \sin 90° = \dfrac{mv^2}{r} \Rightarrow qvB = \dfrac{mv^2}{r}$ $\quad$ [$\because \sin 90° = 1$]

$\Rightarrow$ $\quad r = \dfrac{mv}{qB}$ **(1)**

Time period of charged particle is, $T = \dfrac{2\pi r}{v} = \dfrac{2\pi m}{qB}$

Frequency, $f = \dfrac{1}{T} = \dfrac{qB}{2\pi m}$ ...(i)

$\therefore$ The frequency of oscillation of the charged particle from the above expression is

$$f_{osc} = qB/2\pi m$$

It is also known as **cyclotron frequency.** **(1)**

20. Refer to Sol. 14 and 19 **(3)**

21. (i) Refer to Sol. 19 on page 140. **(1½)**

(ii) Let the mass of proton $= m$

Charge of proton $= q$, Mass of deuteron $= 2m$

Charge of deuteron $= q$

Cyclotron frequency, $f = \dfrac{Bq}{2\pi m} \Rightarrow f \propto \dfrac{q}{m}$

For proton frequency, $f_p \propto \dfrac{q}{m}$ ...(i)

For deuteron frequency, $f_d \propto \dfrac{q}{2m}$ ...(ii)

From Eqs. (i) and (ii), we get, $f_p = 2f_d$

Thus, frequency of proton is twice that of deuteron.

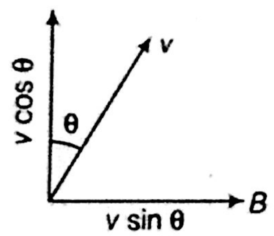
No, both cannot be accelerated with same oscillator frequency as they have different mass. **(1½)**

22. (i) Force acting on the particle, $F = Bqv$

In vector form, $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$

where, B is uniform magnetic field and v is velocity with particle which is moving.

From this equation, it is clear that direction of force is perpendicular to the plane containing both **v** and **B**. In other words, force acts perpendicular to both **v** and **B**. When velocity becomes perpendicular to force, the path of the object becomes circular. **(1)**



In this case, B is assumed to act perpendicular to **v**.

In case, B is not perpendicular to **v**, a component of **v** remains perpendicular to **v**. It creates circular path. The component of **v** parallel to **B** will create linear path.

Here, the particle will have circular path due to $v\cos\theta$ and linear path due to $v\sin\theta$. Both when combined gives helical path. **(1)**

(ii) Since, force always adjusts itself in a direction which becomes perpendicular to velocity, so only direction of velocity is changed not the magnitude. Hence, the kinetic energy of the particle always remains constant. **(1)**

23. (i) When a charged particle (q) moves with velocity ($\mathbf{v}$) inside a uniform magnetic field $\mathbf{B}$, then force acting on it is, $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$ **(1)**

(ii) The direction of force on the charged particle is given by $(\mathbf{v} \times \mathbf{B})$ with the sign of charged particle, i.e. for α-particle, charge is positive and direction of **v** is $+\hat{\mathbf{i}}$ and direction of **B** is $-\hat{\mathbf{k}}$.

So, direction of force is $+(\hat{\mathbf{i}} \times -\hat{\mathbf{k}})$, i.e. $+\hat{\mathbf{j}}$.

For α-particle

It describes a circle with anti-clockwise motion.

For neutron

It is a neutral particle so, it goes undeviated.

As $\quad \mathbf{F} = q(\mathbf{v} \times \mathbf{B}) = 0$

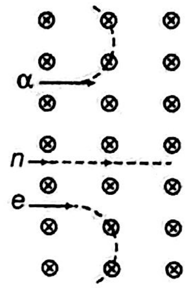
For electron

Force is given by $\mathbf{F} = -e(\mathbf{v} \times \mathbf{B})$

So, direction $= -(\hat{\mathbf{i}} \times -\hat{\mathbf{k}}) \Rightarrow -\hat{\mathbf{j}}$

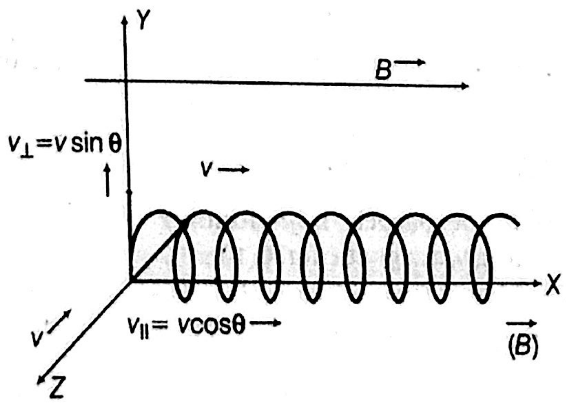
e^- describes a circle with clockwise motion.

Path of the particles in the presence of magnetic field is shown below.



(2)

24. The path of the charged particle will be helix. As, the charge moves linearly in the direction of the magnetic field with velocity $v\cos\theta$ and also describe the circular path due to velocity $v\sin\theta$.



(1)

Time taken by the charge to complete one circular rotation, $\quad T = 2\pi r / v_{\perp}$

$\Rightarrow \quad f = qv_{\perp}B$ ...(i)

and centripetal force = magnetic force

$\Rightarrow \quad mv_\perp^2/r = qv_\perp B \quad \ldots(ii)$

From Eqs. (i) and (ii), we get

$\Rightarrow \quad v_\perp m/qB = r \Rightarrow T = 2\pi v_\perp m/qB \cdot v_\perp$

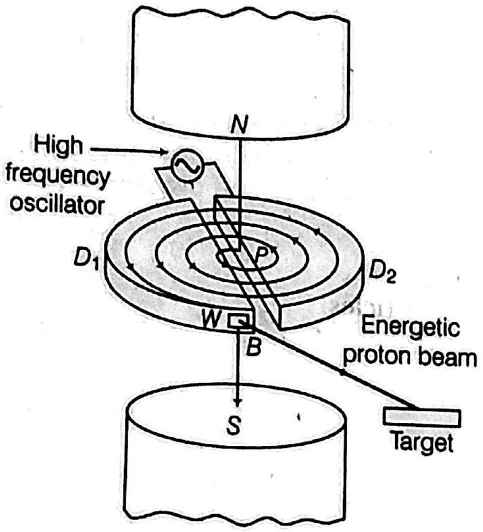
$\Rightarrow \quad T = 2\pi m/Bq$ **(1)**

Distance moved by the particle along the magnetic field in one rotation (pitch of the helix path)

$= v_{||} \times T \quad [\because v_{||} = v_{\text{parallel}}]$

$= v\cos\theta \times 2\pi m/Bq \Rightarrow P = 2\pi mv\cos\theta/qB$ **(1)**

25. Schematic diagram of cyclotron is shown as below



(1)

For Principle of Cyclotron Refer to Sol. 14 on page 139. **(1)**

$\because$ Radius of a charged particle, $r = mv/qB$

$\Rightarrow$ Time period of charged particle

$$T = 2\pi r/v = 2\pi m/qB$$

$\Rightarrow$ Frequency, $f = qB/2\pi m = 1/T$

From the above formula, it is clear that frequency of charged particle does not depend on the velocity of charged particle. **(1)**

26. (i) If the particle is performing circular motion due to magnetic force, then

Centripetal force = Magnetic force

$mv^2/r = qvB\sin 90° \Rightarrow r = mv/qB$ **(1)**

Time period is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\cdot\frac{mv}{qB} = \frac{2\pi m}{qB} \quad \therefore \quad T = \frac{2\pi m}{qB}$$

Thus, frequency, $f = \dfrac{1}{T} = \dfrac{qB}{2\pi m}$

From the above formula, it is clear that frequency of charged particle does not depend on the velocity of charged particle. **(1)**

(ii) **Essential details of construction of cyclotron** Cyclotron consists of

(a) two semi-circular, hollow metallic D shaped half cylinders known as dees.

(b) high frequency oscillating electric field is produced by AC oscillator (having high frequency of AC voltage of few kVs)

(c) strong normal magnetic field is produced in dees using field magnets.

(d) whole system is enclosed in high vacuum chamber. **(1)**

For working and schematic sketch Refer to Sol. 14 **(2)**

27. (i) **For schematic sketch** Refer to Sol. 25 on page 142.

The combination of crossed electric and magnetic fields is used to increase the energy of the charged particle. Cyclotron uses the fact that the frequency of revolution of the charged particle in a magnetic field is independent of its energy. Since, radius of trajectory

$$r = \frac{vm}{qB} \quad \left[\because \frac{mv^2}{r} = Bqv\right]$$

$$\therefore \quad v = rqB/m$$

Hence, the kinetic energy of ions

$$= \frac{1}{2}mv^2 = \frac{1}{2}m\frac{r^2q^2B^2}{m^2} \Rightarrow \text{KE} = \frac{1}{2}\frac{r^2q^2B^2}{m}$$ **(2)**

(ii) (a) Let the mass of proton $= m$, charge of proton $= q$, mass of α-particle $= 4m$

Charge of α-particle $= 2q$

Cyclotron frequency, $\nu = \dfrac{Bq}{2\pi m} \Rightarrow \nu \propto \dfrac{q}{m}$

For proton frequency, $\nu_p \propto \dfrac{q}{m}$

For α-particle, frequency, $\nu_\alpha \propto \dfrac{2q}{4m}$ or $\nu_\alpha \propto \dfrac{q}{2m}$

Thus, particles will not accelerate with same cyclotron frequency. The frequency of proton is twice than the frequency of α-particle. **(1½)**

(b) Velocity, $v = \dfrac{Bqr}{m} \Rightarrow v \propto \dfrac{q}{m}$

For proton velocity, $v_p \propto \dfrac{q}{m}$

For α-particle, velocity, $v_\alpha \propto \dfrac{2q}{4m}$ or $v_\alpha \propto \dfrac{q}{2m}$

Thus, particles will not exit the dees with same velocity. The velocity of proton is twice than the velocity of α-particles. **(1½)**

28. Force experienced by the charged particle in the presence of electric and magnetic field is the sum of electric force and magnetic force on it,

As, Electric force, $\mathbf{F}_e = q\mathbf{E}$

Magnetic force, $\mathbf{E}_m = q(\mathbf{v} \times \mathbf{B})$

Thus Lorentz force, $\mathbf{E} = \mathbf{F}_e + \mathbf{F}_m = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$

$$= q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$$ **(2)**

For a charged prticle moves undeflected through these field,

$$\mathbf{F} = 0$$

$$q(\mathbf{E} + (\mathbf{v} \times \mathbf{B})] = 0$$ **(1)**

or $$\mathbf{E} + (\mathbf{v} \times \mathbf{B}) = 0$$

$$\mathbf{E} = -(\mathbf{v} \times \mathbf{B}) = (\mathbf{B} \times \mathbf{v})$$

$\Rightarrow$ $$|\mathbf{E}| = |\mathbf{B}|\,|\mathbf{v}|\sin\theta$$ **(1)**

It is maximum, when $\theta = 90°$

$\Rightarrow$ $$|\mathbf{v}| = |\mathbf{B}|/|\mathbf{E}|$$

The above condition can be used to select a charged particle of a particular velocity from charges moving with diffeent speeds. **(1)**

29. For schematic digaram and working, Refer to Sol. 14 and 25 **(3)**

For cyclotron frequency Refer to Sol. 19. This means, that the frequency is independent of the energy of the particle.

Maximum KE of charged partcile $= q^2B^2r_0^2/2m$ **(2)**

where, r_0 = radius of dees

As, radius of dees is limited, therefore, KE_{max} will also have limited value.

30. For principle, working and schematic sketch of cyclotron Refer to Sol. 14 and 25 **(1½)**

For the expression of time period Refer to Sol. 26 (i)

This is clear from the expression that time period of revolution of an ion is independent of its speed or radius of the orbit. **(1½)**

Uses of cyclotron

(i) It is used to accelerate the charged particles such as protons/ negatively charged ions.

(ii) It is used to bombard nuclei with energetic particles, ions into solids and in hospitals to produce radioactive substances. **(2)**

Explanations

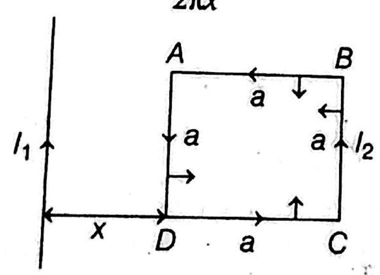
1. The principle of moving coil galvanometer is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a torque. (1)

2. One ampere is the current which flows through each of two parallel uniform long linear conductors, which are placed in free space at a distance of 1 m from each other and which attract or repel each other with a force of 2×10^{-7} N/m of length. (1)

3. No, steady current is not the only source of magnetic field. As, magnetic field can be produced by other sources also, for example, alternating current, moving charged particle, permanent magnets, etc. (1)

4. According to right-hand screw rule, force on AD is

$$F_1 = \frac{\mu_0 I_1 I_2 a}{2\pi x} \text{ (toward right)}$$



Force on BC is

$$F_2 = \frac{\mu_0 I_1 I_2 a}{2\pi(x+a)} \text{ (toward left)}$$ (1)

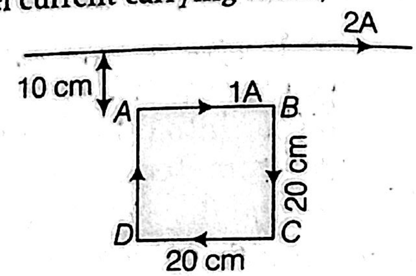
The forces on AB and DC are equal and opposite, so they will cancel each other.

Thus, net force on loop is

$$F_R = \frac{\mu_0 I_1 I_2 a}{2\pi}\left(\frac{1}{x} - \frac{1}{x+a}\right)$$

$$= \frac{\mu_0 I_1 I_2 a^2}{2\pi x(x+a)} \text{ (towards right)}$$ (1)

5. According to the question, as the loop is square, its sides are parallel. So, force between two parallel current carrying wires,



$$F = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

Force on arm AB, $F_{AB} = \dfrac{\mu_0 \times 2\times1\times20\times10^{-2}}{2\pi\times10\times10^{-2}}$ (1)

$$= \frac{2\mu_0}{\pi} \text{N (Attractive, towards the wire)}$$

Force on arm CD, $F_{CD} = \dfrac{\mu_0 \times 2\times1\times20\times10^{-2}}{2\pi\times30\times10^{-2}}$

$$= \frac{2\mu_0}{3\pi} \text{N (Repulsive, away from the wire)}$$

Force on arms BC and DA are equal and opposite. So, they cancel out each other.

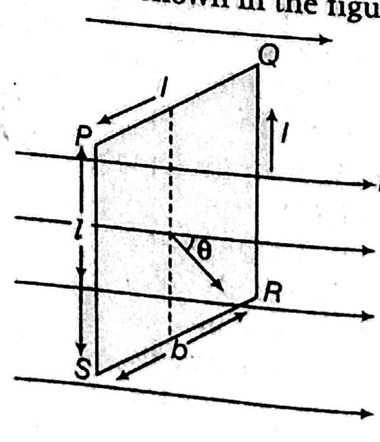
Net force on the loop is $F = F_{AB} - F_{CD} = \dfrac{\mu_0}{\pi}\left[2 - \dfrac{2}{3}\right]$

$$= \frac{4\mu_0}{3\pi} = \frac{4\times4\pi\times10^{-7}}{3\pi}$$

$$= 5.33\times10^{-7} \text{ N}$$

(Attractive, towards the wire) (1)

6. A rectangular coil $PQRS$ is placed in a uniform magnetic field as shown in the figure below.



Forces on arm PQ and RS are equal and opposite and they cancel each other as they are collinear.

Force on SP is F_1 and force on QR is F_2

and $F_1 = F_2 = IlB$ **(1)**

Thus, magnitude of torque due to these forces on the coil will be given as

$$\tau = IlbB\sin\theta = IAB\sin\theta$$

where, $A = lb$ (area of coil)

or in vector form, $\tau = MB\sin\theta\,\hat{n} = \mathbf{M}\times\mathbf{B}$

where, $\mathbf{M} = NIA$, is the magnetic moment of the coil.

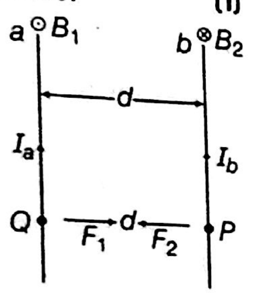
Since, according to the question,

$\theta = 0°$ (as plane of coil is perpendicular to the field).

Thus, $\tau = 0$

Torque acting on the coil will be zero. **(1)**

7. (i) Let a and b be two long straight parallel conductors, I_a and I_b are the current flowing through them and separated by a distance d.



Magnetic field induction at a point P on a conductor b due to current I_a passing through a is

$$B_1 = \frac{\mu_0 2I_a}{4\pi d}$$

Now, unit length of b will experience a force as $\quad F_2 = B_1 I_b \times 1 = B_1 I_b$

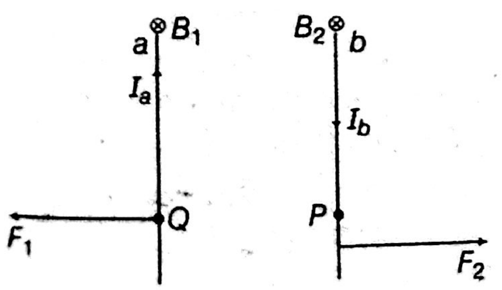
$$\therefore \quad F_2 = \frac{\mu_0}{4\pi}\frac{2I_a I_b}{d}$$

Conductor a also experiences the same amount of force directed towards b. Hence, a and b attract each other.

$\therefore$ The force between two current carrying parallel conductors per unit length is

$$F = \frac{\mu_0}{4\pi}\cdot\frac{2I_a I_b}{d}$$ **(1)**

(ii)



Now, let the direction of current in conductor b be reversed. The magnetic field B_2 at point P due to current I_a flowing through a will be downwards. Similarly, the magnetic field B_1 at point Q due to current I_b passing through b will also be downward as shown. The force on a will be, therefore towards the left. Also, the force on b will be towards the right. Hence, the two conductors will repel each other as shown. **(1)**

8. According to the question,

$$2\pi r = 4a \Rightarrow a = \frac{\pi r}{2} \quad ..(i)$$ **(1)**

Thus, the ratio of magnetic moment of square coil and circular coil is given as

$$\frac{M_s}{M_c} = \frac{NIA_s}{NIA_C} = \frac{NI(a)^2}{NI\pi r^2} = \frac{NI(\pi r/2)^2}{NI\pi r^2} \quad [\because \text{ from Eq (i)}]$$

$$= \pi/4$$

$\Rightarrow \quad M_s : M_c = \pi : 4$ **(1)**

9. (i) Magnetic field at centre due to circular current carrying coil, $B = \mu_0 NI/2r$ **(1)**

(ii) Magnetic moment, $M = NIA = NI(\pi r^2)$

$$M = \pi NIr^2$$

where, r is the radius of circular coil, μ_0 is permeability of free space and N is number of turns. **(1)**

10. The length of wire will be same in two cases as the same coil is unwound and rewound.

Length of the wire is same

$\therefore \quad N_1 \times (2\pi R) = N_2 \times 2\pi\,(R/2)$

[N_1 and N_2 = number of turns in two coils]

$N_2 = 2N_1$ **(1/2)**

Now, the ratio of magnetic moments is given by

$$\frac{M_1}{M_2} = \frac{N_1 I A_1}{N_2 I A_2} = \frac{N_1 \times \pi R_1^2}{N_2 \times \pi R_2^2}$$ **(1/2)**

$$\frac{M_1}{M_2} = \left(\frac{N_1}{2N_1}\right)\times\left(\frac{R}{R/2}\right)^2$$

$$= \frac{1}{2}\times 4 = 2$$ **(1/2)**

$M_1 : M_2 = 2:1$ **(1/2)**

11. The length of wire will be same in two cases as the same coil in unwound and rewound.

Length of wire of coil 1 = Length of wire of coil 2

$$N_1 \times \pi d_1 = N_2 \times \pi d_2$$

$$N_1 \times \pi d = N_2 \times \pi \times 2d$$

$$N_2 = \frac{N_1}{2} \quad [\text{where, } N_1 = N]$$

$\Rightarrow \quad N_2 : N_1 = 1 : 2$

$\Rightarrow \quad N_1 : N_2 = 2:1$ **(1)**

Magnetic moment, $M = NIA$

$$\therefore \quad \frac{M_1}{M_2} = \frac{N_1 I A_1}{N_2 I A_2} = \frac{N_1 \pi d^2}{N_2 \pi (2d)^2}$$

$$\frac{M_1}{M_2} = \left(\frac{N_1}{N_2}\right) \times \frac{1}{4}$$

$$= 2 \times \frac{1}{4} = \frac{1}{2}$$

$$\frac{M_1}{M_2} = \frac{1}{2}$$

$\Rightarrow \quad M_1 : M_2 = 1:2$ **(1)**

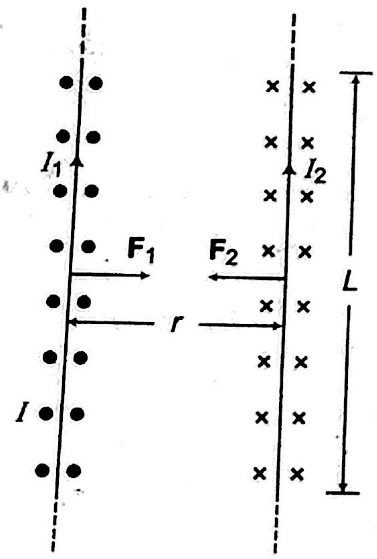
12. In these types of questions, we are calculating force on a wire in the field produced by the other current carrying wire.

To find expression of force between two parallel wires. Let two infinitely long straight current carrying conductor carry currents I_1 and I_2 in the same direction.

Magnetic field B_1 due to first wire on seconds, i.e. **(1/2)**

$$B_1 = \frac{\mu_0}{4\pi} \frac{2I_1}{r} = \frac{\mu_0 I_1}{2\pi r} \quad \ldots \text{(i)}$$ **(1/2)**

The magnetic field is perpendicular to the plane of paper and directed inwards.



Now, magnetic force on length L of second wire is given by **(1/2)**

$$F_2 = I_2 B_1 \, L \sin 90°$$

$$\Rightarrow \quad F_2 = I_2 \left(\frac{\mu_0}{4\pi} \cdot \frac{2I_1}{r}\right) L$$

$$\Rightarrow \quad \frac{F_2}{L} = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r} = \frac{\mu_0 I_1 I_2}{2\pi r} \quad \ldots \text{(ii)}$$

By Fleming's left hand rule, the direction of force F_2 is perpendicular to the second wire in the plane of paper towards the first wire.

Similarly, magnetic force on Ist wire is given by

$$\frac{F_1}{L} = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r} \quad \ldots \text{(iii)}$$

$$\mathbf{F}_1 = -\mathbf{F}_2$$

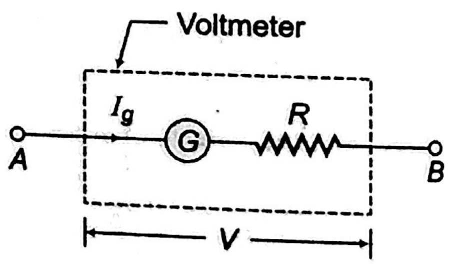
The force F_1 is directed towards the second wire.

Thus, two straight parallel current carrying conductor have the same direction of flow of currents attracting each other. **(1/2)**

13. The resistance of an ideal voltmeter is infinity or very high in practical condition. So, to convert a galvanometer into voltmeter, its resistance needs to be increased, which can be done by connecting a high resistance in series with it.

A galvanometer can be converted into a voltmeter by connecting a very high resistance R in series with it. This is done, so that there is no potential drop across it. **(1/2)**

Let R is so chosen that current I_g gives full deflection in the galvanometer where I_g is the range of galvanometer.



(1/2)

Let galvanometer of resistance G, range I_g is to be converted into voltmeter of range V (volt). Now,

$$V = I_g (G + R)$$

$$\Rightarrow \quad R + G = \frac{V}{I_g}$$

$$\Rightarrow \quad R = \frac{V}{I_g} - G$$

The appropriate scale need to be graduated to measure potential difference. **(1)**

14. Here, Area (A) of coil $= 10 \times 10 = 100 \text{ cm}^2 = 10^{-2} \text{ m}^2$

Number of turns, $N = 20$ turns

Current, $I = 12$ A

Coil makes an angle with magnetic field $= \theta = ?$

Magnetic field, $B = 0.8$T

Torque, $\tau = 0.96$ N-m **(1/2)**

$\because$ Torque (τ) experienced by current carrying coil, the magnetic field is

$$\tau = NIAB \sin\theta$$

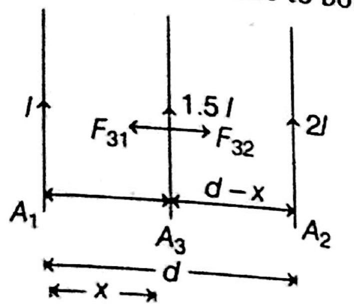
$$0.96 = 20 \times 12 \times 10^{-2} \times 0.8 \times \sin\theta \quad (1)$$

$$\Rightarrow \quad \sin\theta = \frac{0.96}{1.92} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{6} \text{ rad}$$

(1/2)

15. Let the current in the third wire A_3 be in same direction as that of A_1 and A_2. So, it will experience attractive force due to both.



The force on A_3 due to A_1 is $F_{31} = \frac{\mu_0}{2\pi} \cdot \frac{I \times 1.5 I \times l}{x}$

where, l = unit length of conductor wire A_2 and x = distance between A_1 and A_3. (1½)

Similarly, force on A_3 due to A_2 is

$$F_{32} = \frac{\mu_0}{2\pi} \cdot \frac{1.5I \times 2I \times l}{(d-x)}$$

According to question, $F_{31} = F_{32}$

$$\Rightarrow \quad \frac{\mu_0}{2\pi} \cdot \frac{1.5I^2 l}{x} = \frac{\mu_0}{2\pi} \frac{3I^2}{(d-x)} l$$

$$\Rightarrow \quad \frac{1.5}{x} = \frac{3}{d-x}$$

$$\Rightarrow \quad d - x = 2x \quad \text{or} \quad x = \frac{d}{3}$$

Yes, the net force acting on A_3 depends on the current flowing through it. (1½)

16. (i) Refer to Sol. 6 (2)

(ii) In a radial magnetic field, the magnetic torque remains maximum for all positions of the coils. (1)

17. (i) Refer to Sol. 1 (1)

(ii) The galvanometer cannot be used to measure the current because

(a) all the currents to be measured passes through coil and it gets damaged easily.

(b) its coil has considerable resistance because of length and it may affect original current. (1)

(iii) **Current sensitivity** of the galvanometer is the deflection per unit current flowing through it.

It is given by $I_S = \frac{\theta}{I} = \frac{NAB}{k}$

Its unit is rad/A or div/A.

Voltage sensitivity is the deflection per unit voltage.

It is given by

$$V_S = \frac{\theta}{V} = \left(\frac{NAB}{k}\right)\frac{I}{V}$$

or $$V_S = \frac{NAB}{k} \times \frac{I}{IR} = \frac{NAB}{kR}$$

[$\because$ according to Ohm's law, $V = IR$]

Its unit is rad/V or div/V. (1)

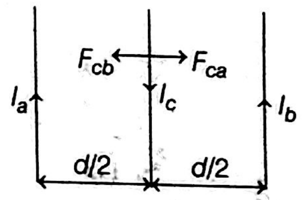
18. (i) Refer to Sol. 2 (1)

(ii) Refer to Sol. 7(i) (1)

When a third conductor of current I_C is placed in between, them having current in opposite direction then the forces will be

Force on C due to current I_a is

$$F_{ca} = \frac{\mu_0}{4\pi} \frac{2I_a I_c}{d/2} \text{ (toward right)}$$



Force on C due to current I_b is

$$F_{cb} = \frac{\mu_0}{4\pi} \frac{2I_b I_c}{d/2} \text{ (toward left)}$$

Then net force,

$$F = F_{ca} - F_{cb} = \frac{\mu_0}{4\pi} \frac{4I_c}{d}(I_a - I_b) \text{ (toward right)}$$

(1)

19. When electron revolves around a nucleus, it creates circular current around it. In this way, it is equivalent to a current carrying coil. So, it behaves as a tiny magnetic dipole. (1)

We know that a current carrying coil behaves like a magnetic dipole having dipole moment equal to I A, where I is current and A is area of the coil.

$$\therefore \quad I = \frac{e}{T} = \frac{e\omega}{2\pi} = \frac{eM_e r^2 \omega}{2\pi m r^2} \quad \text{...(i)}$$

where, e is charge of electron. (1)

Angular momentum of electron is $L = M_e r^2 \omega$

Substituting the above in Eq. (I), we get

$$I = \frac{eL}{2\pi M_e r^2}$$

Hence, $\mu = IA = \frac{eL\pi r^2}{2\pi M_e r^2} = \frac{eL}{2M_e}$

In vector form, $\mu = \frac{-e\mathbf{L}}{2M_e}$

Here, negative sign indicates μ is directs away from **L**.

(1)

20. **For the expression of force** Refer to Sol.12

(1½)

For the definition of one Ampere Refer to Sol. 2

(1½)

21. **For principle of galvanometer** Refer to Sol. 1

(1)

A high resistance is connected in series with the galvanometer to convert into voltmeter. The value of the resistance is given by $R = \frac{V}{I_g} - G$

(1)

where, V = potential difference across the terminals of the voltmeter, I_g = current through the galvanometer and

G = resistance of the galvanometer.

When resistance R_1 is connected in series with the galvanometer, then $R_1 = (V / I_g) - G$ ...(i)

When resistance R_2 is connected in series with the galvanometer, then $R_2 = \frac{V}{2I_g} - G$ ...(ii)

From Eqs. (i) and (ii), we get

$$R_1 - R_2 = V/2I_g \text{ and } G = R_1 - 2R_2$$

The resistance R_3 required to convert the given galvanometer into voltmeter of range 0 to 2V is given by $R_3 = (2V/I_g) - G$

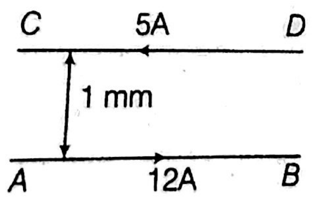
$\Rightarrow$ $R_3 = 4(R_1 - R_2) - (R_1 - 2R_2)$

$= 3R_1 - 2R_2$

G in terms of R_1 and R_2 is given by

$G = R_1 - 2R_2$ (1)

22. Force per unit length between two parallel current carrying wires separated by a distance r is given as $\frac{F}{l} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$



It is repulsive, if the current in the wires is in opposite direction; (otherwise attractive)

The magnetic force of repulsion on the upper wire (CD) should be balancing its own weight in order to remain suspended.

Thus, to remain suspended at its position in equilibrium,

magnetic force on CD due to AB = weight of CD

$\therefore$ $\frac{\mu_0 I_1 I_2}{2\pi r} l = mg$ $(g = 10\ \text{m/s}^2)$

$$m/l = 2\times10^{-7}\frac{I_1 I_2}{rg}$$

$$m/l = \frac{2\times10^{-7}\times12\times5}{10^{-3}\times10}$$

$$= 1.2\times10^{-3}\ \text{kgm}^{-1}$$

$\therefore$ Mass per unit length of wire CD is 1.2×10^{-3} kgm^{-1}

Current in CD should be in opposite direction to that in AB.

(2)

23. There will be force of attraction between the straight wire and 4 cm long arm of loop nearer to the straight conductor. Thus,

$$F_1 = \frac{\mu_0}{4\pi}\frac{2\times2\times1}{(1\times10^{-2})}\times(4\times10^{-2})$$

[towards straight wire]

$F_1 = 16\times10^{-7}$N ...(i)

Similarly, force on other 4 cm arm of loop, away from the straight conductor,

$$F_2 = \frac{\mu_0}{4\pi}\times\frac{2\times2\times1}{(3.5\times10^{-2})}\times(4\times10^{-2})$$

$F_2 = 4.57\times10^{-7}$N ...(ii)

[away from straight wire]

(i) Since, F_1 and F_2 are of different magnitudes therefore they do not form couple and hence Torque, $\tau = 0$

(1)

(ii) The magnitude of net force on loop,

$F = F_1 - F_2$

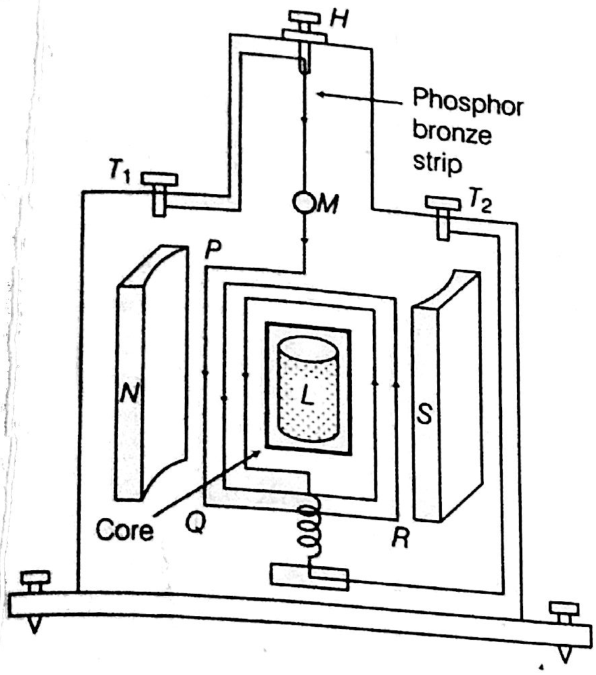
$F = 16\times10^{-7} - 4.57\times10^{-7}$

$\Rightarrow$ $F = 11.43\times10^{-7}$N

The direction of the force would be towards the straight wire.

(1)

24. The labelled diagram of moving coil galvanometer is shown below.



Working (1)

Suppose, the coil $PQRS$ is suspended freely in the magnetic field.

Let l = length PQ or RS of the coil

b = breadth QR or SP of the coil

n = number of turns in the coil

Area of each turns of the coil, $A = l \times b$

Let B = strength of the magnetic field in which coil is suspended and I = current passing through the coil in the direction of $PQRS$.

Let at any instant of time, α be the angle which the normal drawn on the plane of the coil makes with the direction of magnetic field. The rectangular current carrying coil when placed in the magnetic field experiences a torque whose magnitude is given by $\tau = NIBA \sin \alpha$ (1)

Due to this deflecting torque, the coil rotates and suspended wire gets twisted. A restoring torque is set up in the suspension wire.

Let θ be the twist produced in the phosphor bronze strip due to rotation of the coil and k be the restoring torque per unit twist of the phosphor bronze strip.

Then, total restoring torque produced = $k\theta$

In equilibrium position of the coil,

Deflecting torque = Restoring torque

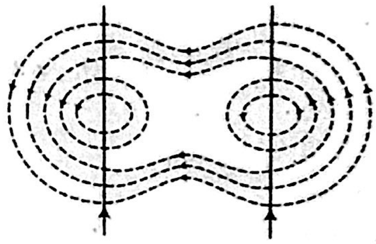
$$\therefore \quad NIBA = k\theta \text{ or } I = \frac{k}{NBA}\theta = G\theta$$

where, $\frac{k}{NBA} = G$ [constant for a galvanometer]

It is known as **galvanometer constant.**

The uniform radial magnetic field keeps the plane of the coil always parallel to the direction of the magnetic field, i.e. the angle between the plane of the coil and the magnetic field is zero for all the orientations of the coil. (1)

25. Magnetic field lines due to straight long parallel conductors carrying current I_1 and I_2 in the same direction is shown below.



Magnetic field lines due to both conductors

(1)

For deduction of the force acting between two long parallel conductors Refer to Sol. 7 (i) (1)

As, the current carrying conductors has same direction of flow of current, so the force between them will be attractive. (1)

26. As electric current associated with the revolving electron, $I = e/T = ev/2\pi r$

where, time period $T = 2\pi r/v$,

r = radius of orbit and

v = velocity of electron

The magnetic moment of revolving electron due to the current,

$$\mu = IA = \frac{ev}{2\pi r} \times \pi r^2$$

$$\Rightarrow \quad \mu = \frac{evr}{2}$$ (1)

If electron revolves in anti-clockwise sense, the current will be in clockwise sense. Hence, according to right hand rule, the direction of magnetic moment must will be perpendicular to the plane of orbit and directed inwards to the plane.

So, $\mu = evrm/2m = el/2m$

where, $mvr = l$ = angular momentum of electron and m is the mass of electron.

$$\Rightarrow \quad \mu = -e(l/2m)$$

Negative sign indicates μ and l are in mutually opposite directions. From Bohr's postulates, $l = mvr = \frac{nh}{2\pi}$, where $n = 1, 2, 3, \ldots$

$$\therefore \quad \mu = \frac{e}{2m} \cdot \frac{nh}{2\pi} = n\mu_{min}$$ (1)

where, $\mu_{min} = \frac{eh}{4\pi m}$ called Bohr's magneton.

Bohr's magneton is defined as the magnetic moment of revolving electron in its first orbit. Its value is 9.27×0^{-24} A-m^2. **(1)**

27. Principle The current carrying coil placed in normal magnetic field experiences a torque which is given by

$$\tau = NIAB$$

where, N = number of turns,
I = current, A = area of coil
and B = magnetic field **(1)**

The galvanometer cannot be used to measure the current because

(i) all the currents to be measured has to be, then passes through coil which would gets damaged as it is a hair line spring or

(ii) its coil has considerable resistance because of length and it may affect original current. **[½×2=1]**

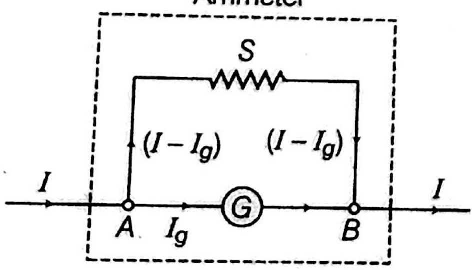
Current sensitivity of galvanometer can be increased by

(i) increasing the magnetic field and

(ii) decreasing the value of torsional constant. **(1)**

28. The resistance of an ideal ammeter is zero or very low in practical condition, so to convert a galvanometer into ammeter its resistance needs to be decreased which can be done by connecting a low resistance in its parallel order.

A moving coil galvanometer of range I_g and resistance G can be converted into ammeter by connecting a very low shunt resistance (S) in parallel with galvanometer.



(1)

This is done, so that the potential difference across the combination is same.

∴ PD across galvanometer = PD across shunt S

$$I_g G = I_s S \quad \textbf{(1)}$$

But $I_s + I_g = I$

$\Rightarrow$ $I_s = I - I_g \Rightarrow I_g G = (I - I_g) S$

$\Rightarrow$ $S = \frac{I_g G}{I - I_g}$ **(1)**

29. The magnetic moment of a current carrying loop

$$\mathbf{m} = I\mathbf{A}$$

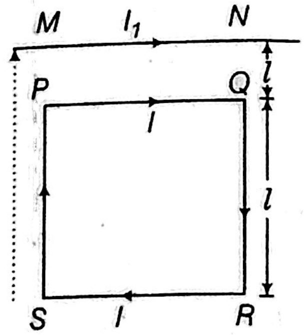
where, $\mathbf{A}$ = area of the loop (square)

$$\mathbf{A} = l^2 \hat{\mathbf{n}}$$

∴

Here, $\hat{\mathbf{n}}$ is a unit vector normal to the direction of area vector.

The forces acting on the arms QR and SP of given loop are equal, mutually opposite and collinear. Hence, they are balanced by one another. **(1)**



Force on arm PQ, $F_1 = B_1 \cdot Il = \frac{\mu_0 I_1}{2\pi l} Il = \frac{\mu_0 I_1 I}{2\pi}$

Since, the direction of the current in the arm PQ and the wire is same, so F_1 is of the attractive nature and directed towards MN.

Again, force on arm RS,

$$F_2 = B_2 I l = \frac{\mu_0 I_1 I l}{2\pi(2l)} = \frac{\mu_0 I_1 I}{4\pi}$$

(1)

F_2 is perpendicular to wire RS and directed away from the conductor MN.

∴ Net force on loop $PQRS$,

$\Rightarrow$ $\mathbf{F}_{net} = \mathbf{F}_1 - \mathbf{F}_2 = \frac{\mu_0 I_1 I}{2\pi} - \frac{\mu_0 I_1 I}{4\pi}$

or $F_{net} = \frac{\mu_0 I_1 I}{4\pi}$ [attractive]

As, $\mathbf{F}_1$ and $\mathbf{F}_2$ are collinear, hence they do not produce torque on the loop $PQRS$. **(1)**

30. For Principle Refer to Sol. 1. **(2)**

For figure and working Refer to Sol. 24

For current and voltage senstivity Refer to Sol. 17 (iii)

Current sensitivity, $I_s = \frac{NAB}{k}$

and voltage sensitivity, $V_s = \frac{NAB}{kR}$

Since, the resistance of the coil may vary, it implies an increase in current sensitivity may not necessarily increase voltage sensitivity. **(3)**

31. (i) Refer to Sol. 24 **(2)**

(ii) (a) It is necessary to introduce a cylindrical soft iron core inside the coil of a galvanometer because it increases its magnetic field. Thus, its sensitivity increases and magnetic field becomes radial. So, angle between the plane of coil and magnetic line of force is zero in all orientations of coil. **(1)**

(b) Refer to Sol. 30

32. (i) Refer to Sol. 30 **(2)**

(ii) **For converting galvanometer into ammeter** Refer to Sol. 28 **(3)**

In the case, when galvanometer is converted into voltmeter, its resistance needs to be increased, so that there is no potential drop across it because of high resistance no current passes through it. Hence, a high resistance is connected in series with the galvanometer. **(2)**.

33. (i) A galvanometer of range I_g and resistance G_1 can be converted into

(a) a voltmeter of range V by connecting a high resistance R in series with it whose value is given by $R = \frac{V}{I_s} - G$

(b) an ammeter of range I by connecting a very low resistance (shunt) in parallel with galvanometer whose value is given by

$$S = I_g G / (I - I_g)$$ **(4)**

(ii) Refer to Sol. 12 **(1)**

34. Refer to Sol. 6 **(1)**

35. (i) **For figure and principle of cyclotron** Refer to Sol. 14 on page 139 (Topic 2).

For working of cyclotron Refer to Sol. 14 on page 139 (Topic 2).

$\because$ In case of the cyclotron, the particle moves on a circular path, the centripetal force required is provided by magnetic force, so magnetic Lorentz force = centripetal force

$$qvB = \frac{mv^2}{r}$$

$$\Rightarrow \quad r = \frac{mv}{qB} \Rightarrow v = \frac{qBr}{m}$$

$$\therefore \quad \text{KE} = \frac{1}{2} mv^2$$

$$= \frac{1}{2} m \left(\frac{qBr}{m}\right)^2 = \frac{q^2B^2r^2}{2m}$$ **(2)**

For maximum KE, $r = r_0$ (radius of dees). **(1)**

(ii) Given, $I_1 = 2\text{A}, I_2 = 5\text{A}$,

$r = 1 \text{ cm} = 1 \times 10^{-2}$ m

Force per unit length between two wires.

$$\frac{F}{L} = \frac{\mu_0}{4\pi} \frac{2I_1 I_2}{r}$$

$$= \frac{10^{-7} \times 2 \times 2 \times 5}{1 \times 10^{-2}} = 20 \times 10^{-5}$$

$$\frac{F}{L} = 2 \times 10^{-4} \text{ Nm}^{-1}$$

Currents flowing in wires are in opposite directions, so the force will be of repulsive nature. **(1)**

36. (i) Torque on rectangular loop,

$$\tau = NIAB \sin\theta \quad \ldots\text{(i)}$$

Also, torque on the loop can be expressed in terms of magnetic moment of the coil and the magnetic field as

$$\tau = mB \sin\theta \quad \ldots\text{(ii)}$$

Comparing Eqs. (i) and (ii), we get

The magnetic dipole moment,

$$m = NIA$$

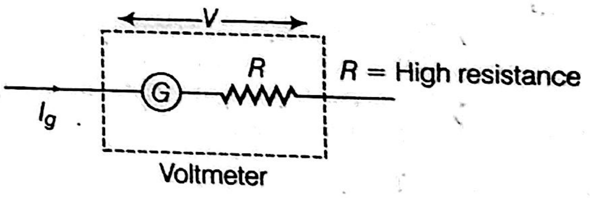
Also, **m** is along **A**.

$$\Rightarrow \quad \mathbf{m} = NI\mathbf{A}$$ **(1)**

(ii) Refer to Sol. 6 on pages 150 and 151. **(2)**

(iii) Given, $G = 50\ \Omega$,

$I_g = 5 \times 10^{-3}$ A, $V = 15$ V



$$\because \quad V = I_g (G + R)$$

$$\Rightarrow \quad \boxed{R = \frac{V}{I_g} - G}$$

$$\Rightarrow \quad \frac{15}{5 \times 10^{-3}} - 50$$

$$\Rightarrow \quad R = 2950\ \Omega$$

A resistance $R = 2950\Omega$ is to be connected in series with galvanometer to convert it into a desired voltmeter. **(2)**

37. (i) Refer to Sol. 24 **(2)**

(ii) Refer to Sol. 24 **(2)**

(iii) Refer to Sol. 32 (i). **(1)**

(2)

38. (i) Refer to Sol. 7 (i)

(ii) As, $\frac{F}{L} = \frac{\mu_0}{4\pi} \frac{2I_1 I_2}{r}$

$\Rightarrow$ $I_1 = I_2 = 1\text{ A}, r = 1\text{m}$

$\frac{F}{L} = 2\times10^{-7}\text{ Nm}^{-1}$

For definition Refer to Sol. 2 (1)

(iii) Here, magnetic field due to tne current carrying conductor at a distance d from it is given by

$$B = \frac{\mu_0}{4\pi} \frac{2I}{d}$$ (1/2)

$\therefore$ Force on proton,

$$F = (e)(v) B \sin 90^\circ$$

$\Rightarrow$ $F = evB$

$$F = ev\left(\frac{\mu_0}{4\pi} \frac{2I}{d}\right)$$

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2Iev}{d}$$ (1)

The proton is directed perpendicular to straight conductor and away from it. (1/2)

39. (i) Refer to Sol. 12 and 25

(3)

(ii) (a) The direction of the magnetic moment of the current loop is perpendicular to the plane of the paper and directed inward. (1)

(b) When angle between area vector of coil and magnetic field is 90°, then maximum torque experienced by the coil.

When $\theta = 0^\circ$ or 180°, then torque will be minimum, i.e. zero. (1)

Explanations

1. As the needle is displaced from the equilibrium position, the torque will try to bring it back in equilibrium position, hence acceleration will be related with negative of angular displacement

When compass needle of magnetic moment M and moment of inertia I is slightly disturbed by an angle θ from the mean position of equilibrium.

Then, restoring torque begin to act on the needle which try to bring the needle back to its mean position which is given by

$$\tau = -MB\sin\theta$$

Since, θ is small

So, $\sin\theta \approx \theta$

$\therefore \quad \tau = -MB\theta \quad \ldots(i)$

But $\quad \tau = I\alpha \quad \ldots(ii)$

where, α = angular acceleration

and M = magnetic moment of dipole.

On comparing Eqs. (i) and (ii), we get

$\Rightarrow \quad I\alpha = -MB\theta$

$\Rightarrow \quad \alpha = -(MB/I)\theta$

$\therefore \quad \alpha \propto -\theta$

$\Rightarrow$ Angular acceleration $\propto -$ Angular displacement

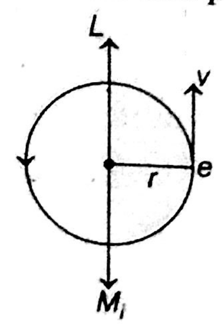
$\Rightarrow$ Therefore, needle execute SHM.

Hence, time period,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{MB/I}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{I}{MB}}$$

This is the required expression.

2. According to Bohr's model of atom, negatively charged electron revolves around the positively charged nucleus. This is same as that of a current loop of dipole moment = IA. Let the electron is moving in a circle with speed v in anti-clockwise direction of radius r and time period is T.



Current, $I = \frac{e}{T} = \frac{e}{2\pi r / v} = \frac{ev}{2\pi r}$

Area of loop $= \pi r^2$ **(1)**

$\therefore$ Orbital magnetic moment of electron is

$$M_i = IA = \frac{ev}{2\pi r} \times \pi r^2 = \frac{evr}{2}$$

The angular momentum of electron due to orbital motion is

$$L = m_e vr$$

It is directed upward in perpendicular direction to the plane.

$$\frac{M_i}{L} = \frac{evr/2}{m_e vr} = \frac{e}{2m_e}$$

This ratio is constant called gyromagnetic ratio. Its value is $8.8 \times 10^{10} \text{C kg}^{-1}$, so

$$M_i = \frac{e}{2m_e} L$$

The vector form

$$\mathbf{M}_i = -\frac{e}{2m_e} \mathbf{L}$$

(1)

The negative sign shows that the direction of **L** is opposite to M_i. According to Bohr's quantisation condition, the angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$.

$$\therefore \quad L = \frac{nh}{2\pi}$$

$$\Rightarrow \quad M_i = n\left(\frac{eh}{4\pi m_e}\right)$$

This is the equation of magnetic moment of an electron revolving in nth orbit. **(1)**

3. (a) Refer to text
(Magnetism and Gauss' Law). **(1/2)**

(b) Refer to text
(Properties of Magnetic Field Lines). **(1/2)**

4. (a) Given, magnetic moment, $M = 6$ J/T

Aligned angle, $\theta_1 = 60°$

External magnetic field,

$$B = 0.44 \text{ T}$$

(i) When the bar magnet is align normal to the magnetic field, i.e. $\theta_2 = 90°$

$\therefore$ Amount of work done in turning the magnet,

$$W = -MB(\cos\theta_2 - \cos\theta_1)$$

$$= -6 \times 0.44 (\cos 90° - \cos 60°)$$

$$= +6 \times 0.44 \times \frac{1}{2} \quad \left(\because \cos 90° = 0 \text{ and } \cos 60° = 1/2\right)$$

$$= 1.32 \text{J}$$ **(1)**

(ii) When the bar magnet align opposite to the magnetic field, i.e. $\theta_2 = 180°$

$$\therefore W = -MB(\cos 180° - \cos 60°)$$

$$= -6 \times 0.44\left(-1 - \frac{1}{2}\right) \quad (\because \cos 180° = -1)$$

$$= 6 \times 0.44 \times \frac{3}{2}$$

$$= 3.96 \text{ J}$$ **(1)**

(b) We know that, torque,

$$\tau = \mathbf{M} \times \mathbf{B} = MB\sin\theta$$

For case (ii), $\theta = 180°$

$$\therefore \quad \tau = MB\sin 180° \quad (\because \sin 180° = 0)$$

$$= 0$$

$\therefore$ Amount of torque is zero for case (ii). **(1)**

Explanations

1. The susceptibility of magnetic material is inverse proportional to temperature, i.e.

$$\chi_m \propto \frac{1}{T}$$

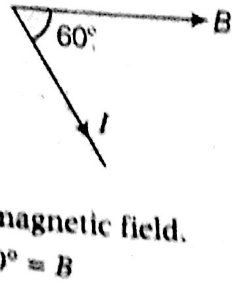
$$\therefore \qquad \frac{\chi_m(T)}{\chi_m\,(300\,\text{K})} = \frac{300}{T}$$

$$\Rightarrow \qquad T = \frac{300 \times 1.2 \times 10^5}{1.44 \times 10^5}$$

$$= 250\ \text{K}$$

2. Substance having (small) negative value (−0.9 magnetic susceptibility χ_m are diamagnetic.

3.



I is the total magnetic field.

Now, $I \cos 60^\circ = B$

$$\Rightarrow \quad I = \frac{B}{\cos 60^\circ} = \frac{B}{1/2} = 2B$$

At equator, dip angle is 0°.

$\therefore \quad B_H = I\cos 0^\circ = I = 2B.$ **(1)**

4. When paramagnetic materials are placed in external magnetic field, these are feebly magnetised in the direction of the applied external magnetic field whereas in case of diamagnetic materials, these are feebly magnetised opposite to that of applied external magnetic field. **(1)**

5. The nature of magnetic material is a diamagnetic. The relation between relative permeability and magnetic susceptibility is

$$\mu_r = 1 + \chi_m$$ **(1)**

6. Permanent magnets are those magnets which have high retentivity and coercivity. The magnetisation of permanent magnet is not easily destroyed even if it is handled roughly or exposed in stray reverse magnetic field, e.g. steel. **(1)**

7. At equator, vertical component of earth's magnetic field will be zero. **(1)**

8. Horizontal component of earth's magnetic field,

$$H = B_e \cos 60^\circ = B \quad \text{(given)}$$

$$\Rightarrow \quad B_e \times \frac{1}{2} = B \quad \text{or} \quad B_e = 2B$$

Vertical component of earth's magnetic field,

$$V = B_e \sin 60^\circ \Rightarrow V = 2B \times (\sqrt{3}/2)$$

$$\Rightarrow \quad V = \sqrt{3}B$$ **(1)**

9. The angle of dip is given by

$$\delta = \tan^{-1}(B_V/B_H)$$

B_V = vertical component of the earth's magnetic field.

B_H = horizontal component of the earth's magnetic field.

So, as $\quad B_V = B_H$

Then, $\quad \delta = \tan^{-1}(1) = 45^\circ$

$\therefore$ The angle of dip will be $\delta = 45^\circ$. **(1)**

10. (i) The needle is free to move in vertical plane, it means that there is no component of the earth's magnetic field in horizontal direction, so the horizontal component of the earth's magnetic field is zero. **(1/2)**

(ii) The angle of dip is 0°. **(1/2)**

11. At poles, the angle of dip is 90°. **(1)**

12. The magnetic material is diamagnetic substance for which $\mu_r < 1$. **(1)**

13. The small and positive susceptibility of 1.9×10^{-5} represents paramagnetic substance. **(1)**

14. Negative susceptibility represents diamagnetic substance. **(1)**

15. Diamagnetic material acquires feeble magnetisation in the opposite direction of the magnetic field when they are placed in an external magnetic field. **(1)**

16. (i) The magnetic susceptibility of a magnetic material is defined as the ratio of the intensity of magnetisation (I) to the magnetic intensity (H).

i.e., $\quad \chi_m = \dfrac{I}{H}$

Relation between magnetic susceptibility (χ_m) and relative magnetic permeability (μ_r) is given as **(1)**

$$\mu_r = 1 + \chi_\mu$$

(ii) For material $A, \mu_r = 0.96 < 1$

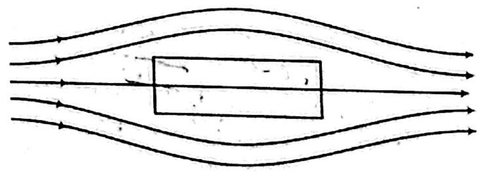
Hence, magnetic materia $1A$ is diamagnetic.

For material $B, \mu_r = 500$

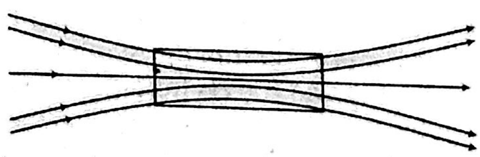
Since, μ_r is much greater than 1 for material B, therefore B is ferromagnetic material. **(1)**

17. Magnetic permeability of paramagnetic is more than air, so it allows more lines to pass through it while permeability of diamagnetic is less than air, so it does not allow lines to pass through it.

(i) Behaviour of magnetic field lines when diamagnetic substance is placed in an external field.



(ii) Behaviour of magnetic field lines when paramagnetic substance is placed in a external field.



Magnetic susceptibility distinguishes the behaviour of the field lines due to diamagnetic and paramagnetic substance. **(1)**

This difference can be explained as diamagnetic substances repel or expel the magnetic field lines while paramagnetic substance attract the magnetic field lines. **(1)**

18. The nature of the material A is paramagnetic and its susceptibility χ_m is positive.

The nature of the material B is diamagnetic and its susceptibility χ_m is negative. **(2)**

19.

Paramagnetic substance	Diamagnetic substance
A paramagnetic substance is feebly attracted by magnet.	A diamagnetic substance is feebly repelled by a magnet.
For a paramagnetic substance, the intensity of magnetisation has a small positive value.	For a diamagnetic substance, the intensity of magnetism has a small negative value. **(2)**

20. (i) An electromagnet consist of a core made of a ferromagnetic material placed inside a solenoid. It behaves like a strong magnet when current flows through the solenoid and effectively loses its magnetism when the current is switched off.

A permanent magnet is also made up of a ferromagnetic material but it retains its magnetism at room temperature for a long time after being magnetised one. **[1]**

(ii) *Properties of material are as below:*

(a) High permeability (c) Low retentivity

(b) Low coercivity **(1)**

21. Ferromagnetic substance are those substances which have very high magnetic permeability. **(1)**

Properties (i) High retentivity

(ii) High susceptibility $(\chi_m > 1000)$

(2 × 1/2)

22. (i) For diamagnetic substances, the variation of susceptibility is very small $(0 < \chi_m < \varepsilon)$, i.e. diamagnetic materials are unaffected by the change in temperature (except bismuth). **(1)**

(ii) Paramagnetic materials when cooled due to thermal agitation tendency alignment of magnetic dipoles decreases. Hence, they shows greater magnetisation. **(1)**

23. (i) Magnetic lines of force come out from North pole and enter into the South pole outside the magnet and travels from South pole to North pole inside the magnet. So, magnetic lines of force form closed loop, magnetic monopoles do not exist. **(1)**

(ii) The diamagnetic material gets slightly magnetised in a direction opposite to external field, therefore lines of force are repelled by diamagnetic material. **(1)**

NOTE When South pole of the magnet is viewed from the frame of reference, then inside the magnet, it appears as North pole and *vice-versa*. Due to this reason, magnetic field lines are traversed from South pole to North pole inside the magnet.

24. Angle of dip, $\delta = 60° = \pi/3$

Horizontal component of the earth's magnetic field, $H = 0.4$ G

Earth magnetic field $(B_e) = ?$

$\because$ Horizontal component of the earth's magnetic field, $H = B_e \cos\delta$ **(1)**

$$\Rightarrow \quad B_e = \frac{H}{\cos\delta} = \frac{0.4\text{ G}}{\cos 60°} = \frac{0.4\text{ G}}{(1/2)} = 0.8\text{ G}$$

$\therefore \quad B_e = 0.8$ G **(1)**

25. (i) The earth's magnetic field at a place can be completely described by three parameters which are called elements of earth's magnetic field. They are as follows **(1)**

(a) Angle of declination (θ)

(b) Angle of dip (δ) or magnetic inclination

(c) Horizontal component of earth's magnetic field (H_e).

(ii) At the magnetic equator, the dip needle rests horizontally, so that the angle of dip is zero at the magnetic equator. **(1)**

26. (i) **Susceptibility for diamagnetic material** It is independent of magnetic field and temperature (except for bismuth at low temperature). **(1)**

Susceptibility for ferromagnetic material The susceptibility of ferromagnetic materials decreases steadily with increase in temperature. At the Curie temperature, the ferromagnetic materials become paramagnetic.

(ii) **Behaviour in non-uniform magnetic field** Diamagnets are feebly repelled, whereas ferromagnets are strongly attracted by non-uniform field, i.e. diamagnets move in the direction of decreasing field, whereas ferromagnet feels force in the direction of increasing field intensity. **(1)**

27. (i) Two characteristics of material used for making permanent magnets are

(a) high coercivity

(b) high retentivity and high hysteresis loss **(2 × 1/2)**

(ii) Core of an electromagnet made of ferromagnetic material because of its

(a) low coercivity

(b) low hysteresis loss **(2 × 1/2)**

28. According to the question, $H = \sqrt{3}V$

where, H and V are the horizontal and vertical components of the earth's magnetic field.

If angle of dip at that place is δ, then

$$\tan\delta = (V/H) = (V/\sqrt{3}V) \quad [\because H = \sqrt{3}V]$$

$$\tan\delta = \frac{1}{\sqrt{3}} \Rightarrow \delta = \frac{\pi}{6}$$

(1)

$\because$ Horizontal component of the earth's magnetic field, $H = B_e \cos\delta$

where, B_e = Earth's magnetic field

$$\frac{H}{B_e} = \cos\delta = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad H : B_e = \sqrt{3} : 2$$

(1)

29. Refer to Sol. 28 (Ans. $1 : \sqrt{2}$). (2)

30. (i) Refer to Sol. 17

(ii) Refer to Sol. 17

(1)

Magnetic susceptibility distinguishes the behaviour of the field lines due to diamagnetic and paramagnetic substance. (1)

31. Difference between para-, dia- and ferro-magnetic materials

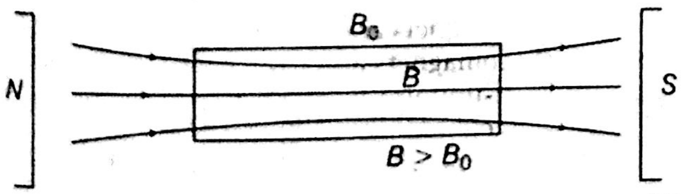
Refer to text on page 167.

32. Given, susceptibility, $\chi_m = 0.9853$

As the susceptibility of material is positive but small.

$\therefore$ The material is paramagnetic in nature. For paramagnetic material, magnetic lines of external magnetic field will passes through the material without much deviation, when it is placed in between magnetic poles.

The modification of the field pattern is shown in the following figure.



(3)

33. $\because$ Horizontal component,

$$H = B\cos\theta = 0.4\cos 60^\circ = 0.4 \times (1/2) = 0.2\,\text{G}$$

$$H = 0.2 \times 10^{-4}\ \text{T} \quad [\because \cos 60^\circ = 1/2]$$

This component is parallel to the plane of wheel. The wheel is rotating in a plane normal to the horizontal component, so it will cut the horizontal component only, vertical component of earth will contribute nothing in emf. (1)

Thus, the emf induced is given as

$$E = \frac{1}{2}Hl^2\omega$$

where, $\omega = 2\pi N/t$ and

l = length of the spoke = 50 cm = 0.5 m

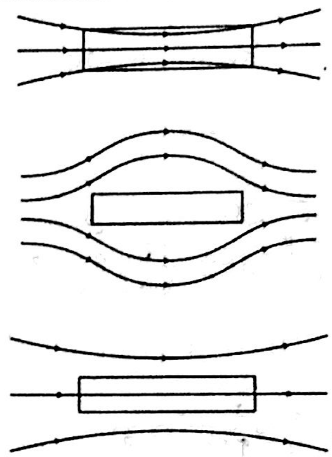
$$\therefore \quad E = \frac{1}{2} \times 0.2 \times 10^{-4} \times (0.5)^2 \times \frac{2 \times 3.14 \times 120}{60}$$

$$E = 3.14 \times 10^{-5}\ \text{V}$$

(1)

The value of emf induced is independent of the number of spokes as the emf's across the spokes are in parallel. So, the emf will be unaffected with the increase in spokes. (1)

34. The modifications are shown in the figure.



(½ × 3 = ½)

It happens because

(i) nickel is a ferromagnetic substance.

(ii) antimony is a diamagnetic substance.

(iii) aluminium is a paramagnetic substance.

(½ × 3 = 1 ½)

35. The torque always tries to bring back the needle in equilibrium position i.e. parallel to the existing field.

(i) The torque on the needle is $\tau = M \times B$

In magnitude, $\tau = MB\sin\theta$

Here, τ is restoring torque and θ is the angle between M and B.

Therefore, in equilibrium,

Restoring force = Deflecting torque

$$I\frac{d^2\theta}{dt^2} = -MB\sin\theta$$

(1)

Negative sign with $MB\sin\theta$ implies that restoring torque is in opposition to deflecting torque.

For small values of θ in radians, we approximate $\sin\theta = \theta$ and get

$$I\frac{d^2\theta}{dt^2} = -MB\theta \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{MB}{I}\theta$$

$$\Rightarrow \quad d^2\theta/dt^2 = -\omega^2\theta$$

(1)

This equation represents a simple harmonic motion. The square of the angular frequency is

$$\omega^2 = MB/I$$

i.e. $\omega^2 = MB/I$ or $\omega = \sqrt{MB/I}$

Time period, $T = 2\pi/\omega = 2\pi\sqrt{I/MB}$ **(1)**

(ii) (a) As, horizontal component of earth's magnetic field, $B_H = B\cos\delta$

Putting $\delta = 90°$ (as compass needle orients itself vertically)

$\therefore \quad B_H = 0$

(b) For a compass needle oriented itself with its axis vertical at a certain place, angle of dip, $\delta = 90°$. **(2)**