optical instruments(solutions)

Explanations

1. Refer to class notes
[(Refracting) Astronomical Telescope] **(2)**

2. Refer to class notes
(Compound Microscope)

Resolving power of a microscope $= \dfrac{2\mu \sin\beta}{1.22\lambda}$. **(2)**

3. Angular magnification or magnifying power of compound microscope is defined as ratio of angle made at eye by image formed at infinity to the angle made by object, if placed at distance of distinct vision from an unaided eye. **(1)**

Magnification $= LD/f_o \cdot f_e$

where, L is length of the tube of microscope.

As, $m \propto \dfrac{1}{f_o}$ and $m \propto \dfrac{1}{f_e}$

$\therefore$ Both eyepiece and objective must be of smaller focal lengths, so that magnification is higher. **(1)**

4. When final image is at D,

then magnifying power, $M = \dfrac{-f_o}{f_e}\left(1 + \dfrac{f_e}{D}\right)$

In normal adjustment, $M = -f_o/f_e$

For telescope,

Focal length of objective lens, $f_o = 150\,\text{cm}$

Focal length of eye lens, $f_e = 5\,\text{cm}$

When final image forms at $D = 25\,\text{cm}$

$\therefore$ Magnification, $M = \dfrac{-f_o}{f_e}\left(1 + \dfrac{f_e}{D}\right)$

$$= \frac{-150}{5}\left(1 + \frac{5}{25}\right) = \frac{-150}{5} \times \frac{6}{5}$$

$\Rightarrow$ $M = -36$ **(1)**

Let height of final image be h cm.

$\Rightarrow$ $\tan\beta = \dfrac{h}{25}$

β = visual angle formed by final image at eye.

α = visual angle subtended by object at objective.

$$\tan\alpha = \frac{100\,\text{m}}{3000\,\text{m}} = \frac{1}{30}$$

But, $M = \dfrac{\tan\beta}{\tan\alpha}$

$$\Rightarrow \quad -36 = \frac{\left(\frac{h}{25}\right)}{\left(\frac{1}{30}\right)} \Rightarrow -36 = \frac{h}{25} \times 30$$

$$\Rightarrow \quad -36 = \frac{6h}{5}$$

$$\Rightarrow \quad h = \frac{-36 \times 5}{6}$$

$$h = -30\,\text{cm}$$

(1

5. Maximum magnification of a compound microscope is

$$m = \frac{v_0}{u_0}\left[1 + \frac{D}{f_e}\right]$$

So, for m to be 30,

$$30 = \frac{v_0}{u_0}\left[1 + \frac{25}{5}\right] \text{ or } 30 = \frac{v_0}{u_0}[6]$$

$$v_0 = 5u_0 \qquad \text{...(i)}$$ (1

For objective of focal length 1.25 cm,

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \Rightarrow \frac{1}{5u_0} - \frac{1}{-u_0} = \frac{1}{1.25}$$

$$1 + \frac{5}{5u_0} = \frac{1}{1.25}$$

$5u_0 = +7.5\,\text{cm}$ or $u_0 = 1.5\,\text{cm}$. So, $v_0 = +7.5\,\text{cm}$

Now, u_e for required magnification

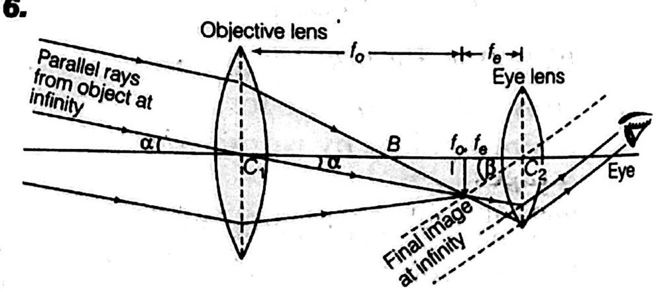
$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e} \text{ or } \frac{1}{-25} - \frac{1}{-u_e} = \frac{1}{5}$$

$$\frac{1}{u_e} = \frac{1}{5} + \frac{1}{25} = \frac{5+1}{25} \text{ or } u_e = \frac{25}{6}\,\text{cm}$$

Hence, separation between two lenses should be

$v_0 + u_e = 7.5\,\text{cm} + 25/6\,\text{cm} = 11.67\,\text{cm}$ (1

6.



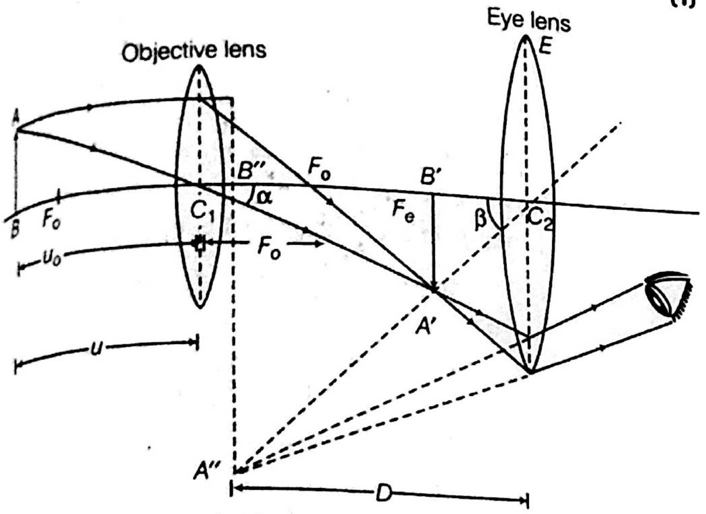
The final image is magnified and inverted (1

Limitations of refracting telescope over a reflecting type telescope.

(i) Refracting due to telescope suffers from chromatic aberration, due to uses of large size lenses. (1/2

(ii) It is difficult and expensive to make such large sized lenses. (1/2

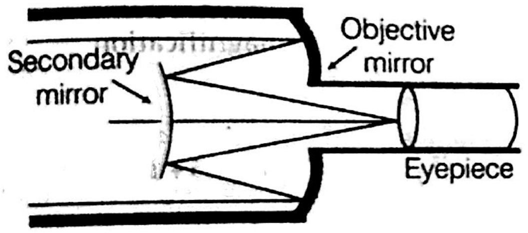
7. A compound microscope consists of two convex lenses coaxially separated by some distance. The lens nearer to the object is called the objective. The lens through which the final image is viewed is called the eyepiece. The focal length of objective lens is smaller than eyepiece. **(1)**



Compound microscope, final image at D. **(1)**

8. Ray diagram of a reflecting type telescope Refer to Sol. 9 **(2)**

9. Diagram of a reflecting telescope (Cassegrain) is shown as below:



(1)

Cassegrain reflecting telescope It consists of a large concave paraboloidal (primary) mirror having a hole at its centre. There is a small convex (secondary) mirror near the focus of the primary mirror. The eyepiece is placed on the axis of the telescope near the hole of the primary mirror.

Advantages

(i) Reflecting telescopes have high resolving power due to a large aperture of mirrors.

(ii) Due to availability of paraboloidal mirror, the image is free from chromatic and spherical aberration. **(1)**

10. Resolving power of telescope, $R_p = A/1.22\lambda$

where, A = aperture or diameter of the objective telescope

and λ = wavelength of the objective.

$\Rightarrow R \propto A$

$\therefore$ Ratio of resolving powers of two telescopes

$$R_1/R_2 = A_1/A_2$$

$\because \quad A_2 > A_1$ **(1)**

$\therefore \quad R_2 > R_1$

The larger the aperture of objective, higher the resolving power of telescope. As well more gathering of light to form the image and hence, brighter image would be obtained. **(1)**

11. Resolving power of a microscope is defined as the reciprocal of its limit of resolution (d) i.e.

RP of microscope $= 1/d$ where, limit of resolution is equal to the smallest distance between two closest objects whose vivid or clean image can be seen through the microscope and given by $d = \lambda/2\mu \sin\theta$

$\therefore$ Resolving power of microscope $= 2\mu \sin\theta/\lambda$

where, λ = wavelength of light used, **(1)**

θ = semivertical angle of the cone formed by object at objective and μ = refractive index of molecule between object and lens.

For advantages of reflecting telescope Refer to Sol. 9 **(1)**

12. Magnifying power The magnifying power of a telescope is equal to the ratio of the visual angle subtended at the eye by final image formed at least distance of distance vision to the visual angle subtended at naked eye by the object at infinity. **(1)**

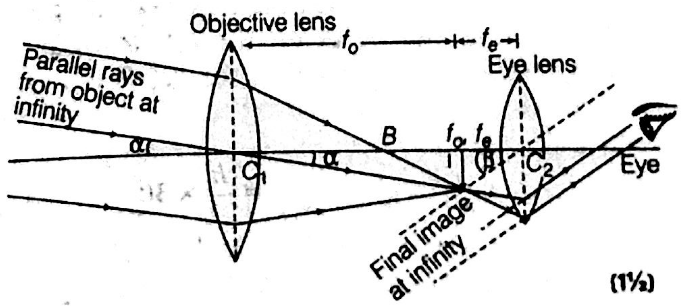
Telescope has objective of large aperture and large focal length whereas microscope have objective of small aperture and focal length.

The relative distance between objective and eye lens may change in telescope whereas the separation between objective and eye lens in compound microscope remains fixed. **(1)**

13. Astronomical telescope Refer to diagram (Refracting Astronomical Telescope) **(1½)**

For numerical Refer to Sol. 30 **(1½)**

14. (i) An astronomical telescope is an optical instrument which is used for observing distinct images of heavenly bodies like planets, stars, etc. It has two convex lens (objective and eye lens) placed coaxially and separated by some distance in normal adjustment. Final image is formed at infinity as depicted below.



(1½)

(ii) In the astronomical telescope, aperture of objective must be less than eyepiece. Therefore, possible combinations are (L_1 and L_3) or (L_1 and L_2). Also, focal length of the objective (f_o) must be greater than that of eyepiece (f_e).

$$\therefore \quad f_o > f_e \Rightarrow \frac{1}{f_o} < \frac{1}{f_e} \Rightarrow P_o < P_e$$

∴ Power of objective (P_o) must be less than power of eyepiece (P_e).

Now, for (L_1 and L_3) combination,

$$\left(\frac{f_o}{f_e}\right)_1 = \frac{P_e}{P_o} = \frac{10}{3}$$

For (L_1 and L_2) combination,

$$\left(\frac{f_o}{f_e}\right)_2 = \frac{P_e}{P_o} = \frac{6}{3} = 2 < \left(\frac{f_o}{f_e}\right)_1$$

Thus, the best combination of the lenses is (L_1 and L_3). **(1½)**

15. Refer to Sol. 9 **(3)**

16. (i) Let f_o = focal length of the objective lens $= 15\text{m} = 1500$ cm

f_e = focal length of the eye lens = 1 cm

∵ Angular magnification of giant refracting telescope is given by

$$m_0 = \left|\frac{f_o}{f_e}\right| = \left|\frac{1500}{1}\right| = 1500$$ **(1½)**

(ii) Diameter of the image of the moon formed by the objective lens, $d = \alpha f_0$

$$\Rightarrow \quad d = \frac{\text{Diameter of the moon}}{\text{Diameter of the lunar orbit}} = \frac{3.48 \times 10^6}{3.8 \times 10^8} \times 15$$

$$= 0.135\text{m} = 13.5\text{cm}$$ **(1½)**

17. An astronomical telescope should have an objective of larger aperture and longer focal length while an eyepiece of small aperture and small focal length. Therefore, we will use L_2 as an objective and L_3 as an eyepiece. For constructing microscope, L_3 should be used as objective and L_1 as eyepiece because both the lenses of microscope should have short focal lengths and the focal length of objective should be smaller than the eyepiece. **(3)**

18. (i) Refer to text in notes [Compound Microscope]. **(1)**

(ii) Given, magnification, $M = 20$

Magnification of eyepiece, $m_e = 5$

Least distance vision, $D = 20$ cm

Distance between objective and eyepiece, $L = 14$ cm

We know that,

Magnification, $M = m_e \times m_o$

$$m_o = \frac{m}{m_e} = \frac{20}{5} = 4 \Rightarrow m_e = 1 + \frac{D}{f_e}$$

where, f_e is focal length of eyepiece.

$$\Rightarrow \quad 5 = 1 + \frac{20}{f_e} \quad \Rightarrow \quad f_e = 5\text{ cm}$$ **(1)**

Using lens formula for eyepiece,

$$\frac{1}{u_e} = \frac{-1}{20} - \frac{1}{5} = \frac{-5}{20} = \frac{-1}{4}$$

$u_e = -4$ cm

(objective distance for eyepiece)

$$L = v_o + |u_e|$$

$$\Rightarrow \quad v_o = L - |u_e| = 14 - 4 = 10\text{ cm}$$

Magnification produced by objective,

$$m_o = -v_o/u_o$$

Object distance for objective,

$$u_o = \frac{-v_o}{m_o} = \frac{-10}{4} = -2.5\text{ cm}$$

Using lens formula for objective,

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{10} - \frac{1}{-2.5} = \frac{1}{10} + \frac{1}{2.5} \Rightarrow f_o = 2\text{ cm}$$ **(1)**

19. Refer to Sol. 6

Magnifying power of a telescope is the ratio of the angle β subtended at the eye by the image to the angle α subtended at the eye by the object.

$m = \beta/\alpha = f_o/f_e$. **(3)**

20. Refer to Sol. 7 **(3)**

21. For compound microscope,

$f_o = 4$ cm, $f_e = 10$ cm

$u_o = -6$ cm, $v_e = -D = -25$ cm

For objective lens, $\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$

$$\Rightarrow \quad \frac{1}{4} = \frac{1}{v_o} + \left(\frac{1}{6}\right)$$

$$\Rightarrow \quad \frac{1}{v_o} = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$$

$v_o = 12$ cm

$$\therefore \text{ Magnifying power } M = -\left(\frac{v_o}{u_o}\right)\left(1 + \frac{D}{f_e}\right)$$

$$= -\left(\frac{12}{6}\right)\left(1 + \frac{25}{10}\right) = -2\left(\frac{7}{2}\right) = -7$$

Magnifying power, $M = -7$

Length of microscope $= |v_o| + |u_e|$ **(1)**

where, $v_o = 12$ cm

For eye lens, $v_e = -25$ cm, $f_e = 10$ cm, $u_e = ?$

$$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$

$$\Rightarrow \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{10}$$

$$\frac{1}{u_e} = \frac{-2-5}{50} = -\frac{7}{50}$$

$$u_e = -50/7 \text{ cm} = -7.14 \text{ cm}$$ **(1)**

$\therefore$ Length of microscope

$$= |v_o| + |u_e| = 12 + 7.14 = 19.14 \text{ cm}$$ **(1)**

NOTE

1. The separation between objective and eye lens is known as length of microscope.
2. The image formed by objective is an object for eye lens.

22. Given, $f_o = 20$ cm, $f_e = 1$ cm, $v_e = -25$ cm

For objective

$u_o = -100$ cm, $f_o = 20$ cm

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o} \Rightarrow \frac{1}{20} = \frac{1}{v_o} - \frac{1}{(-100)}$$

$$\frac{1}{v_o} = \frac{1}{20} - \frac{1}{100} = \frac{5-1}{100} = \frac{4}{100}$$

$$v_o = 25 \text{ cm}$$ **(1/2)**

For eye lens

$f_e = 1$ cm, $u_e = ?$, $v_e = -25$

$$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$

$$\Rightarrow \frac{1}{1} = \frac{1}{-25} - \frac{1}{u_e}$$

$$1 + \frac{1}{25} = -\frac{1}{u_e} \Rightarrow \frac{26}{25} = -\frac{1}{u_e}$$

$u_e = -25/26 \Rightarrow |u_e| = 0.96$ cm **(1/2)**

Magnification

$$m = -\frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right) = -\left(\frac{25}{100}\right)\left(1 + \frac{25}{1}\right)$$

$$m = -\frac{1}{4} \times 26 = -6.5$$ **(1)**

Length of telescope, $L = v_o + u_e = 25 + 0.96$

$$L = 25.96 \text{ cm}$$ **(1)**

23. The diameter of objective of the telescope

$$= 150 \times 10^{-3} \text{ m}$$

$f_o = 4$ m

$f_e = 25 \times 10^{-3}$ m and $D = 0.25$ m

Magnifying power,

$$M = -\frac{f_o}{f_e}\left(1 + \frac{D}{f_e}\right)$$ **(1)**

$$M = -\frac{4}{25 \times 10^{-3}}\left(1 + \frac{0.25}{25 \times 10^{-3}}\right)$$

$$M = -\frac{4000}{25}(1 + 10)$$

$$= -\frac{4000 \times 11}{25}$$

$$M = -1760$$ **(1)**

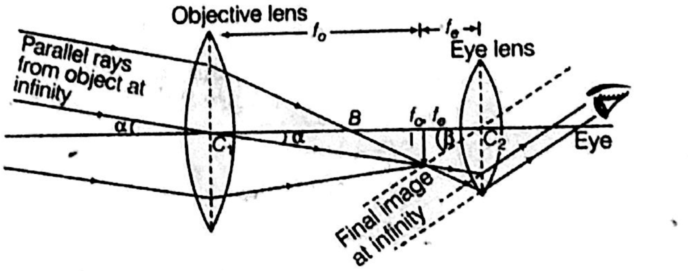
Resolving power $= \dfrac{1}{d\theta}$

$$d\theta = 1.22\lambda/D$$

$$= \frac{1.22 \times 6 \times 10^{-7}}{0.25} = 2.9 \times 10^{-6} \text{ rad}$$

$\therefore$ Resolving power $= \dfrac{1}{2.9 \times 10^{-6}} = 0.34 \times 10^{6}$ **(1)**

24. (i) Light from the distant object enters the objective and real image is formed at second focal point of objective. The eyepiece magnifies this image producing a final inverted image.



Angular magnification is given by

$$m = \frac{\beta}{\alpha}$$

Since, β and α are very small.

$\therefore \quad \beta \approx \tan\beta$ or $\alpha \approx \tan\alpha$

$$\Rightarrow m = \frac{\tan\beta}{\tan\alpha} \quad \text{...(i)}$$

Now, $\tan\alpha = \dfrac{I}{f_o}$ and $\tan\beta = \dfrac{I}{-f_e}$

where, I is the image formed by the objective, f_o and f_e are the focal lengths of objective and eyepiece, respectively.

Substituting the values of $\tan\alpha$ and $\tan\beta$ in Eq. (i), we get

$$m = \frac{-\frac{I}{f_e}}{\frac{I}{f_o}} \text{ or } m = -\frac{f_o}{f_e}$$ **(2)**

(ii) As, $f_e = \frac{1}{10} = 0.1\text{ m} = 10\text{ cm}$

$$f_e = \frac{1}{1} = 1\text{ m} = 100\text{ cm}$$

Magnifying power in normal adjustment,

$$m = -\frac{f_o}{f_e} = -\frac{100}{10}$$

$\therefore$ $m = -10$ **(1)**

25. (i) Refer to Sol. 7

The magnification by compound microscope is two step process.

Firstly, the objective gives a magnified image of the object and after that the eyepiece produces the angular magnification. **(1½)**

(ii) f_o and f_e of compound microscope must be small so as to have large magnifying power as

$$M = -\frac{L}{f_o}\left(1 + \frac{D}{f_e}\right)$$ **(1½)**

26. Refer to Sol. 9 **(3)**

27. Refer to Sol. 24 (i) **(1)**

In normal adjustment,

$$M = |f_o/f_e| = 20$$

$\Rightarrow$ $f_o = 20f_e$

Also, length of telescope,

$$f_o + f_e = 105$$
$$20f_e + f_e = 105$$
$\Rightarrow$ $21\,f_e = 105$
$\Rightarrow$ $f_e = 5\text{ cm}$
$$f_o = 20f_e$$
$$= 20 \times 5 = 100\text{ cm}$$ **(2)**

28. (i) Refer to text

When the final image is formed at infinity, angular magnification is given by $M = \frac{\beta}{\alpha}$.

However, β and α are very small.

$\therefore$ $\beta \approx \tan\beta$

or $\alpha \approx \tan\alpha$

$\Rightarrow$ $M = \frac{\tan\beta}{\tan\alpha}$ **(1)**

I is the image formed by the objective, f_o and f_e are the focal length of objective and eyepiece, respectively.

$$\tan\alpha = \frac{I}{f_o}$$

or $\tan\beta = \frac{I}{-f_e}$

$\therefore$ $M = \frac{-I/f_e}{I/f_o}$ or $M = -f_o/f_e$ **(2)**

(ii) Given, $f_o + f_e = 105$, $f_o = 20\,f_e$

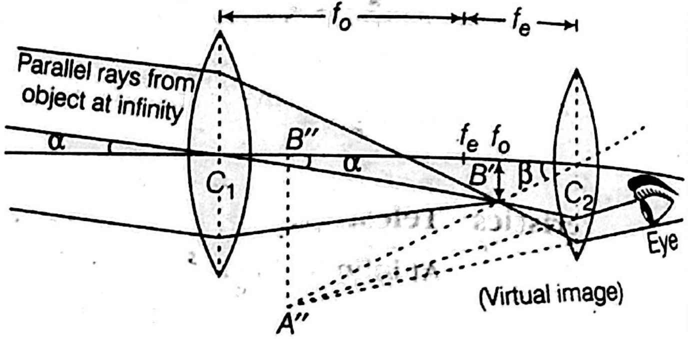
$$f_e = \frac{105}{21} = 5$$

$\Rightarrow$ $f_o = 20 \times 5 = 100\text{ cm}$

$$M = \frac{f_o}{f_e} = \frac{100}{5} = 20$$ **(2)**

29. (i) In astronomical telescope for normal adjustment final image is formed at infinity and it is virtual.

The labelled ray diagram to obtain one of the real image formed by the astronomical telescope is as follows.



Magnifying power is defined as the ratio of the angle subtended at the eye by the focal image as seen through the telescope to the angle subtended at the eye by the object seen directly, when both the image and the object lies at infinity. **(3)**

(ii) (a) We know objective lens of a telescope should have larger focal length and eyepiece lens should have smaller focal length. And focal length is inverse of power, so lens of power ($P = 1/f$) 10 D can be used as eyepiece and lens of power 0.5 D can be used as objective lens.

(b) The objective lens of a telescope should have larger aperture, in order to form bright image of an distant objects, so that it can gather sufficient light rays from the distant objects. **(2)**

30. Refer to Sol. 19 **(2)**

For telescope

Focal length of objective lens, $f_o = 150\text{ cm}$

Focal length of eye lens, $f_e = 5\text{ cm}$

When final image forms at $D = 25\text{ cm}$

$\therefore$ Magnification, $M = -f_o/f_e\,(1 + f_e/D)$

$$= -\frac{150}{5}\left(1 + \frac{5}{25}\right) = -\frac{150}{5} \times \frac{6}{5}$$

$$M = -36$$

Let height of final image is h cm (1½)

$\therefore \quad \tan\beta = h/25$

β = visual angle formed by final image at eye

α = visual angle subtended by object at objective

$$\tan\alpha = \frac{100\text{ m}}{3000\text{ m}} = \frac{1}{30}$$

But, $$M = \frac{\tan\beta}{\tan\alpha}$$

$$-36 = \frac{(h/25)}{(1/30)}$$

$$\Rightarrow \quad -36 = \frac{h}{25} \times 30 = \frac{6h}{5}$$

$$h = -\frac{36 \times 5}{6} = -30\text{ cm}$$ (1½)

Negative sign indicates inverted image.

31. Differences between telescope and microscope are given as below:

Characteristics	Telescope	Microscope
Position of object	At infinity	Near objective at a distance lying between f_o and $2f_o$
Position of image	Focal plane of objective	Beyond $2f_o$ when f_o is the focal length of objective.

(1 / 2 × 2 = 1)

For microscope

$f_o = 1.25$ cm, $f_e = 5$ cm

When final image forms at infinity, then magnification produced by eye lens is given by

$$M = -\frac{L}{f_o}\cdot\frac{D}{f_e} \Rightarrow -30 = -\frac{L}{1.25} \times \frac{25}{5}$$

$$L = \frac{30 \times 1.25}{5} \Rightarrow L = 7.50\text{ cm}$$ (1)

For objective lens

$v_o = L = 7.5$ cm

$f_o = 1.25$ cm, $u_o = ?$

Applying lens formula

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o} \Rightarrow \frac{1}{1.25} = \frac{1}{7.5} - \frac{1}{u_o}$$

$$\frac{1}{u_o} = \frac{1}{7.5} - \frac{1}{1.25} = \frac{1.25 - 7.5}{7.5 \times 1.25} = -\frac{6.25}{7.5 \times 1.25}$$

$$\Rightarrow \quad u_o = -\frac{7.5 \times 1.25}{6.25}$$

$$= -1.5\text{ cm}$$

The object must be at a distance of 1.5 cm from objective lens. (3)

32. Refer to Sol. 7 (1)

The objective lens forms real, inverted magnified image $A'B'$ of object AB in such a way that AB' fall some where between pole and focus of eye lens.

So, $A'B'$ acts as an object for eye lens and its virtual magnified image $A''B''$ formed by the lens. (1)

The magnifying power of a compound microscope is defined as the ratio of the visual angle subtended by final image at eye (β) and the visual angle subtended by object at naked eye when both are at the least distance of distinct vision from the eye.

$$m \Rightarrow \frac{\text{Visual angle with instrument } (\beta)}{\text{Visual angle when object is placed at least distance of distinct vision } (\alpha)}$$

$$\Rightarrow \quad m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha}$$

$$= \frac{B'A'/u_e}{BA/D} = \left(\frac{B'A'}{BA}\right) \times \frac{D}{u_e}$$

$$= m_o m_e u_e$$

$m = m_o m_e$, where m_o and m_e are magnification produced by objective and eye lens, respectively.

Now, $$m_o = \frac{B'A'}{BA} = \frac{v_o}{-u_o}$$

$$m_e = \frac{D}{u_e} = 1 + \frac{D}{f_e} \quad \text{[By lens formula]}$$

$$\therefore \quad m = -\left(\frac{v_o}{u_o}\right)\left(1 + \frac{D}{f_e}\right)$$ (1)

This is the required expression.

Also, $u_o = +1.5$ cm

$\Rightarrow \quad f_o = 1.25$ cm, $f_e = 5$ cm

$v_e = -D = -25$ cm

For objective lens,

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o} \Rightarrow \frac{1}{1.25} = \frac{1}{v_o} + \frac{1}{1.5}$$

$$\Rightarrow \quad \frac{1}{v_o} = \frac{1}{1.25} - \frac{1}{1.5} = \frac{1.5 - 1.25}{1.5 \times 1.25}$$

$$= \frac{0.25}{1.5 \times 1.25} = \frac{1}{7.50}$$

$$v_o = 7.5\text{ cm}$$ (1)

$\therefore$ Magnifying power,

$$m = -\left(\frac{v_o}{u_o}\right)\left(1 + \frac{D}{f_e}\right)$$

$$= -\left(\frac{7.5}{1.5}\right)\left(1 + \frac{25}{5}\right) = -5 \times 6$$

$$m = -30.$$ (1)