Ray optics (solutions)

Explanations

1. The ability of lens to converge or diverge the rays of incident light on it. It is called the power of lens. Its SI unit is dioptre(D) or m^{-1}. **(1)**

2. During sunrise and sunset, the rays have to travel a larger part of the atmosphere because they are very close to the horizon. According to Rayleigh's law of scattering, scattering $\propto \frac{1}{\lambda^4}$, wavelength of red is large, hence it is least scattered.

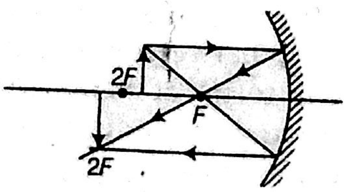
   Therefore, light rays other than red is mostly scattered away. Most of the red light, which is the least scattered, enters our eyes. Hence, the sun appears red at sunrise and sunset. **(1)**

3. The blue colour of the sky is due to the scattering of sunlight by the molecules of the atmosphere. According to Rayleigh's law of scattering, the intensity of scattered light, $I \propto \frac{1}{\lambda^4}$, blue colour having the short wavelength in the visible spectrum scatter the most. When we look at the sky, the scattered light enters our eyes and this light contains blue colour in a large proportion, so the sky appears blue. **(1)**

4. A concave lens is made up of certain material behaves as a diverging lens, when it is placed in a medium of refractive index less than the refractive index of the material of the lens and behaves as a converging lens, when it is placed in a medium of refractive index greater than the refractive index of the material of the lens.

   In the given case, concave lens is immersed in medium having refractive index greater than the refractive index of the material of the lens $(1 \cdot 65 > 1 \cdot 5)$. Therefore, it will behave as a converging lens. **(1)**

5. When an object is placed between f and $2f$ of a concave mirror, the image formed is real, inverted and magnified.



(1)

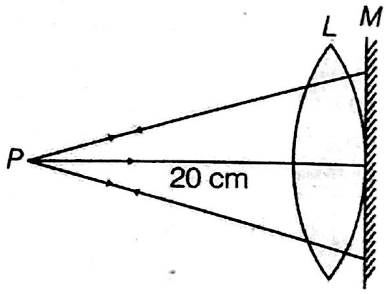
6. When a lens is placed in a liquid, where refractive index is more than that of the material of lens, then the nature of the lens changes. So, when a biconvex lens of refractive index 1.25 is immersed in water (refractive index 1.33), i.e. in the liquid of higher refractive index, its nature will change. So, biconvex lens will act as biconcave lens or diverging lens. **(1)**

7. A biconvex lens acts as a converging lens in air because the refractive index of air is less than that of the material of the lens. The refractive index of water is less than the refractive index of the material of the lens (1.5). So, its nature will not change.

   It behaves as a converging lens. **(1)**

8. The adjacent figure shows a convex lens L in contact with a plane mirror M. P is the point object kept in the front of this combination at a distance of 20 cm from it. **(1/2)**

   Since, the image is coinciding with the object itself, the rays from the object after refraction from the lens fall normally on the mirror M and form an image coinciding with the object itself. So, the image is formed at the focus of the lens. So, focal length of the lens is 20 cm.



(1/2)

9. The relation between the angle of incidence i, angle of prism A and the angle of minimum deviation, δ_m for a triangular prism is given by

$$i = \frac{A + \delta_m}{2}.$$

(1)

10. Focal length of the lens decrease when red light is replaced by blue light. **(1)**

11. > This question can be answered by considering the lens maker's formula.
    >
    > From the formula, we can identify which factor will change on changing the wavelength.

    The refractive index of the material of a lens increases with the decrease in wavelength of the incident light. So, focal length will decrease with decrease in wavelength according to the formula.

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Thus, when we replace red light with violet light then due to increase in wavelength the focal length of the lens will decrease. **(1)**

12. When refractive index of lens is equal to the refractive index of liquid. **(1)**

13. From Snell's law, $\mu = \sin i/\sin r = c/v$

$\Rightarrow$ $v \propto \sin r$ for given value of i

$\Rightarrow$ Smaller angle of refraction, smaller the velocity of light in medium.

Velocity of light is minimum in medium A as angle of refraction is minimum, i.e. 15°. **(1)**

14. Because refractive index for a given pair of media depends on the ratio of wavelengths or velocity of light in two medium and not on frequency. **(1)**

15. The refractive index of diamond is much higher than that of glass. Due to high refractive index, the critical angle for diamond air interface is low. The diamond is cut suitably so that the light entering the diamond from any face suffers multiple total internal reflections at the various surfaces. **(1)**

16. Frequency remains unchanged when light travels from one transparent medium to another transparent medium. **(1)**

17. *Following are the criteria for total internal reflection*

(i) Light must pass from a optically denser to a optically rarer medium.

(ii) Angle of incidence in denser medium is must be greater than critical angle for two media. **(1)**

18. When a lens is immersed in a liquid whose refractive index is more than that of the material of lens, then nature of lens changes, i.e. converging lens behaves like diverging lens and *vice-versa*.

Refractive index of the material of lens is less than the refractive index of water. **(1)**

19. Combined focal length of a lens combination

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$ (For two thin lenses in contact)

As, $f_2 = -f_1$

(focal lengths are equal, one is convex and other is concave)

$$\Rightarrow \frac{1}{f} = 0 \Rightarrow f = \infty.$$

i.e., combination of both lenses behave as a plane glass because focal length of plane glass is infinity. **(1)**

20. When a lens immersed in a liquid disappears then, $\mu_{\text{liquid}} = \mu_g = 1.45$ **(1)**

21. Critical angle is the angle of incidence for which angle of refraction becomes 90°. Here, in this case refractive index, $\mu = 1/\sin i_C$

$\because$ Refractive index, $\mu = \frac{c}{v} = \frac{1}{\sin i_C}$

$$\Rightarrow v = c \sin i_C = 3\times10^8 \times \sin 30^\circ$$

$$= 3\times10^8 \times 1/2 = 1.5\times10^8 \text{ m/s}$$ **(1)**

22. Refer to Sol. 3 on page 260. **(1)**

23. The condition for formation of rainbow is that the sun should be shining in one part of sky and it should be raining in the opposite part of sky. And the back of the observer should be toward the sun. **(1)**

24. Resultant power of the combination,

$$P = P_1 + P_2 = 6 - 2 = 4\text{ D}$$

$$\therefore \frac{1}{f} = 4 \Rightarrow f = \frac{1}{4}\text{ m} = 25\text{ cm}$$ **(1)**

25. Refer to Sol. 24 on page 261. (Ans. $f = 50$ cm). **(1)**

26. Refer to Sol. 24 on page 261.(Ans. $f = 40$ cm). **(1)**

27. Given, $\mu_1 = 1.4, \mu_2 = 1.5, P = -5$D

Using lens Maker's formula

$$P = \frac{1}{F} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$-5 = \left(\frac{1.5-1.4}{1.4}\right)\left(-\frac{1}{R} - \frac{1}{R}\right)$$ **(1)**

[For equi-concave lens, $R_1 = -R$ and $R_2 = R$]

$$-5 = \frac{0.1}{1.4}\left(-\frac{2}{R}\right)$$

$$\Rightarrow R = \frac{1}{14} \times \frac{2}{5} = \frac{1}{35} = 0.0286\text{ m} = 2.86\text{ cm}$$ **(1)**

28. Given, $A = 60^\circ$ (for equilateral prism)

$$\mu_1 = \frac{4\sqrt{2}}{5}, \mu_2 = 1.6,$$

The refractive index is given by

$$\frac{\mu_2}{\mu_1} = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$ **(1)**

where, D = angle of minimum deviation.

$$\frac{1.6\times5}{4\sqrt{2}} = \frac{\sin\left(\frac{60^\circ + D}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)}$$

$$\sqrt{2} \times \sin 30^\circ = \sin\left(\frac{60^\circ + D}{2}\right)$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \sin\left(\frac{60^\circ + D}{2}\right)$$

$$\Rightarrow \quad \sin 45^\circ = \sin\left(\frac{60^\circ + D}{2}\right)$$

$$\Rightarrow \quad 45^\circ = \frac{60^\circ + D}{2}$$

$$D = 90^\circ - 60^\circ = 30^\circ$$ **(1)**

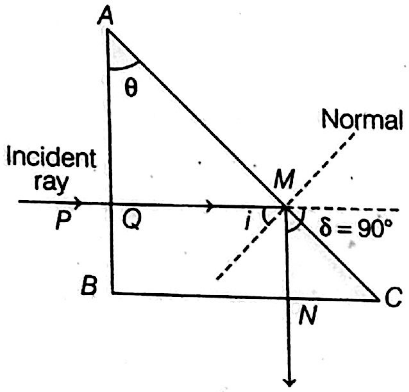
29. Condition for total internal reflection

(i) The ray of light passes from denser medium to rarer medium.

(ii) Angle of incidence should be greater than critical angle. **(1)**

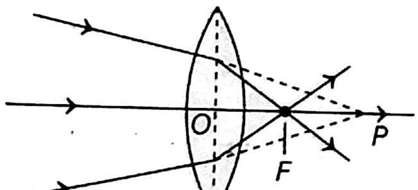
ABC is a right angled isosceles prism.

A ray of light *PQ* incident normally on surface *AB*, therefore refracted towards *AC* and *QM*.



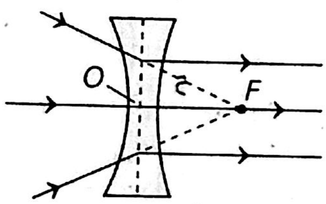
When angle of incident (i) is greater than critical angle for surface *AC*, then light ray incident on surface *AC* totally reflected along *MN*. In this case, angle of deviation is 90°, which is shown in figure. **(1)**

30. (i)



Convex lens **(1)**

(ii)

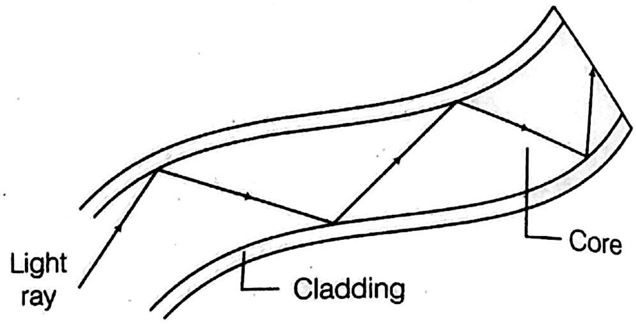


Concave lens **(1)**

31. Optical fibre works on the principle of total internal reflection.

When a light ray travelling from denser to a rarer medium is incident at an angle greater than the critical angle, then it is reflected back into the same medium. This phenomenon is called total internal reflection. **(1)**

Optical fibres are fabricated in such a way that light reflected at one side of the inner surface strikes the other at an angle larger than critical angle. Even, if fibre is bent, light can easily travel along the length. Optical fibre is used in communication system.



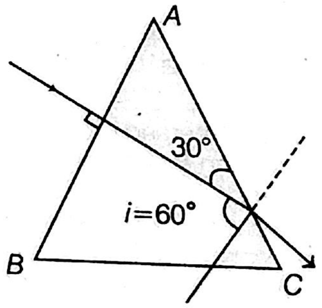
Optical Fibre **(1)**

32. (i) Refer to Sol. 3 **(1)**

(ii) Refer to Sol. 2 **(1)**

33. Given, refractive index of water, $\mu_w = 4/3$

Refractive index of glass prism, $\mu_g = \frac{3}{2}$



For total internal reflection occurrence the incident angle must be greater than critical angle. **(1)**

∴ Let us calculate critical angle C.

As we know that, $\sin C = \frac{1}{\mu}$

where, $\mu = \frac{\text{refractive index of glass } ({}_a\mu_g)}{\text{refractive index of water } ({}_a\mu_w)}$

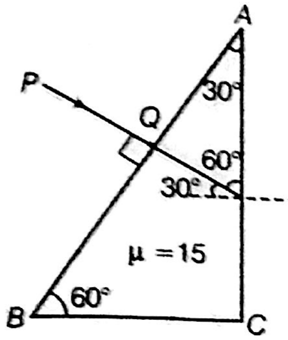
$$\therefore \quad \sin C = \frac{1}{\left(\frac{{}_a\mu_g}{{}_a\mu_w}\right)} = \frac{1}{\left(\frac{3/2}{4/3}\right)} = \frac{1}{9/8}$$

or $\sin C = \frac{8}{9} = 0.88 \Rightarrow C = 61.6^\circ$

[As $\sin 60^\circ = \sqrt{3}/2 = 0.86$]

As the critical angle, i.e. 61.6° is greater than the angle of incidence, i.e. 60°, hence TIR will not occurs. **(1)**

34. Given, refractive index of the material of the prism, $\mu = 1.5$



(1)

∴ Critical angle for the material,

$\sin C = \frac{1}{\mu} = \frac{1}{1.5} = 2/3$

$\Rightarrow C = \sin^{-1}(2/3) \approx 42°$.

From the ray diagram, it is clear that angle of incidence $i = 30° < C$.

Therefore the ray incident at the face AC will not suffer total internal reflection and merges out through this face, i.e. a AC. (1)

35. According to the mirror equation, we have

$$1/v + 1/u = 1/F$$

where, u = distance of the object from the mirror

v = distance of the image from the mirror hand

F = focal length of the mirror.

Applying new cartesian sign convention, we get

$$F = -\text{ve and } u = -\text{ve}$$

Given, $F < u < 2F$

When $u = -F$, we get $\frac{1}{v} = \frac{1}{(-F)} - \frac{1}{(-F)} = 0$

(1)

$\Rightarrow \quad v = \infty$

From the mirror formula, when $u = -2F$.

$\Rightarrow \quad \frac{1}{-2f} + \frac{1}{v} = \frac{1}{-f}$

$\Rightarrow \quad \frac{1}{v} = \frac{1}{2F} - \frac{1}{F} = \frac{-1}{2F}$ (1)

$\therefore \quad F < u < 2F, \infty < v < 2F$

36. The refractive index of a transparent medium is inversely proportional to the wavelength of incident light. The relationship between the two is given by,

$$\mu = \lambda_0/\lambda$$

where,

μ = Refractive index of medium

λ_0 = Wavelength of incident light in vacuum

λ = Wavelength of incident light in medium (1)

Given,

Velocity of light in air, $c = 3\times10^8$ m/s

Velocity of light in glass, $v_g = 2\times10^8$ m/s

The refractive index of glass is given by, $\mu_g = c/v_g$.

Where c is speed of light in vacuum.

The refractive index of air is given by, $\mu_a = \frac{c}{v_a}$

∴ The refractive index of glass w.r.t. air will be

$${}^a\mu_g = \frac{\mu_g}{\mu_a}$$

$$\Rightarrow \quad {}^a\mu_g = \frac{v_a}{v_g} = \frac{3\times10^8}{2\times10^8} = 1.5$$

We know $\quad {}^a\mu_g = 1/\sin C$

where, C is the critical angle for the interface

$\therefore \quad 1/\sin C = 1.5 \Rightarrow \sin c = 1/1.5$

$\Rightarrow \quad C = \sin^{-1}(0.66) \Rightarrow c = 41.3°$

∴ Critical angle, $C = 41.3°$ (1)

37. The focal length of original equiconvex lens is f.

Let the focal length of each part after cutting be F.

Here, $\quad \frac{1}{f} = \frac{1}{F} + \frac{1}{F} \Rightarrow \frac{1}{f} = \frac{2}{F}$

$\Rightarrow \quad f = \frac{F}{2} \Rightarrow F = 2f$

Power of each part will be given by

$$P = \frac{1}{F} \Rightarrow P = \frac{1}{2f}$$ (1)

From lens maker formula, we have

$$P = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

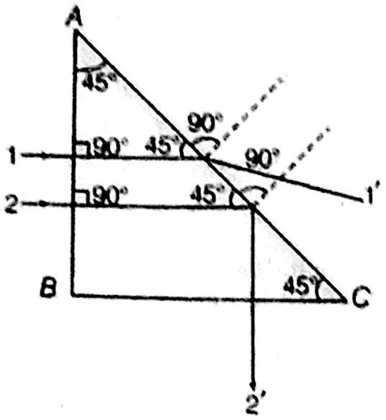
$$5 = (1.55 - 1)\left\{\frac{1}{R} - \left(\frac{1}{-R}\right)\right\} \quad \begin{bmatrix} R_1 = R \\ R_2 = -R \end{bmatrix}$$

or $\quad 5 = 0.55 \times \frac{2}{R}$

$$R = \frac{0.55\times2}{5}$$

$= 0.22$ m $= 22$ cm (1)

38. *The paths are shown as below:*



From the figure, it is clear that angle of incidence for ray 1 is 45°.

For ray 1, $\sin i = \sin 45° = \frac{1}{\sqrt{2}} = \frac{1}{1.414}$

For ray 1, the refractive index of the prism is

$$\mu = 1.35$$

$$\mu = \frac{1}{\sin c} \Rightarrow \sin c = \frac{1}{\mu} = \frac{1}{1.35}$$

Here, $\frac{1}{1.414} < \frac{1}{1.35}$ **(1)**

i.e. $\sin i < \sin c$ or $i < c$

So, ray 1 will be refracted by the prism.

For ray 2, angle of incidence, $i = 45°$

$$\sin i = \sin 45° = \frac{1}{\sqrt{2}} = \frac{1}{1.414}$$

For ray 2, the refractive index, $\mu = 1.45$

$$\mu = \frac{1}{\sin c} \Rightarrow \sin c = \frac{1}{\mu} = \frac{1}{1.45}$$

Here, $\frac{1}{1.414} > \frac{1}{1.45}$

i.e. $\sin i > \sin c$ or $i > c$

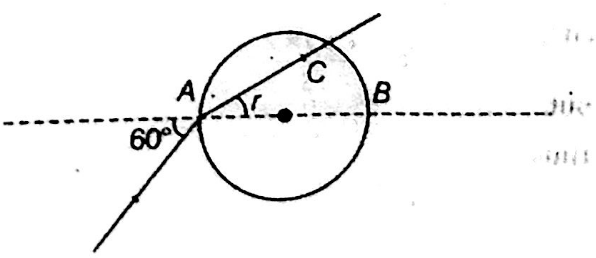
So, ray 2 will get total internally reflected. **(1)**

39. Given, $i = 60°, \mu = \sqrt{3}$

From Snell's law, we have

$$\frac{\sin i}{\sin r} = \mu \Rightarrow \frac{\sin 60°}{\sin r} = \sqrt{3}$$

$$\sin r = \frac{\sin 60°}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$$ **(1)**

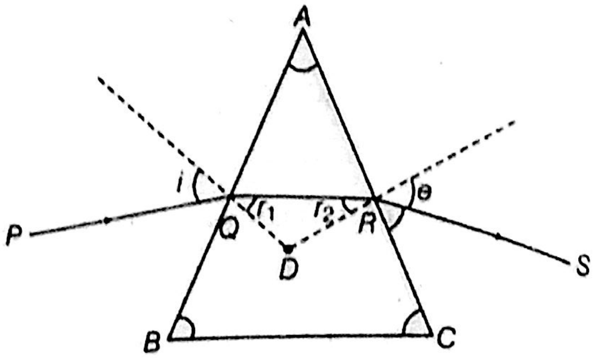


$$\sin r = 0.5$$

$$\Rightarrow r = \sin^{-1}(0.5)$$

$$\Rightarrow r = 30°$$ **(1)**

40. (i) When QR is parallel to the base BC, we have $i = e$ (prism is in the position of minimum deviation)



(1)

$$\Rightarrow r_1 = r_2 = r \quad \text{(let)} \quad ...(i)$$

We know that

$$r_1 + r_2 = A$$

From Eq. (i), we get

$$2r = A, r = A/2 \quad ...(ii)$$

$$\therefore r_1 = r_2 = A/2$$

(ii) Also, we have

$$A + D = i + e$$

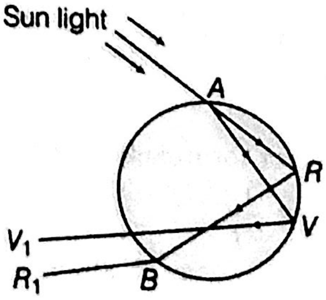
Substituting, $D = D_m$ and $e = i$

$$A + D_m = i + i$$

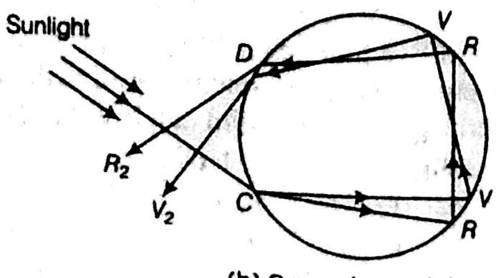
$$\therefore D_m = 2i - A$$ **(1)**

41. The conditions for observing a rainbow

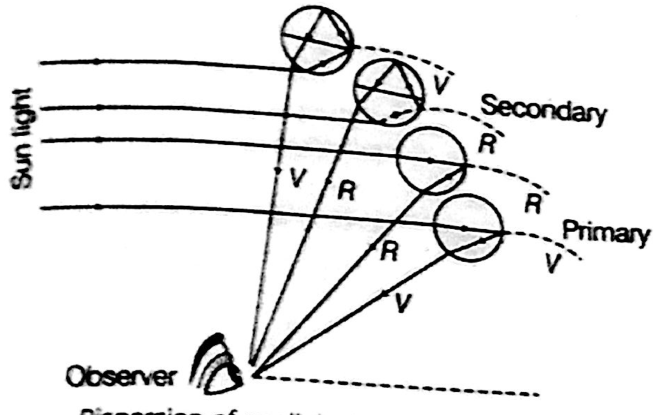
Refer to Sol. 23



(a) Primary rainbow



(b) Secondary rainbow **(1)**



Dispersion of sunlight by a single raindrop

Formation of rainbow

The rays of light reach the observer through a refraction followed by a total internal reflection, followed by a refraction.

Figure shows red light from drop 1 and violet light from drop 2, reaching the observer eye. **(1)**

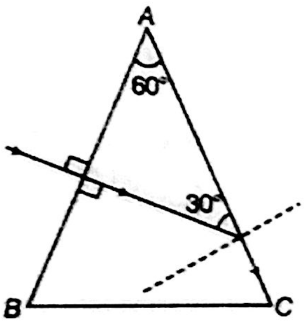
42. Now, given $A = 60°$, $i = 0°$

At the interface AB,

$$\frac{\sin i}{\sin r} = \frac{\mu_g}{\mu_a}$$

$$\Rightarrow \quad \sin r = \frac{\mu_a \sin i}{\mu_g}$$

$$\Rightarrow \quad r = 0$$



(1)

Angle of incidence at face AC of the prism is $60°$.

At the interface AC, $i = 60°$

$$\frac{\sin i}{\sin \phi} = \frac{\mu_a}{\mu_g} \Rightarrow \sin e = \frac{\mu_g \sin i}{\mu_a}$$

$$\sin e = \frac{2}{\sqrt{3}} \times \sin 60° = 1$$

$$\Rightarrow \quad e = 90°$$

Hence, refracted ray grazes the surface AC. Angle of emergence $= 90°$.

Angle of deviation $= 30°$. **(1)**

43. Focal length for convex lens $= f_1$

Focal length for concave lens $= -f_2$

The equivalent focal length of a combination of convex lens and concave lens is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{-f_2}$$

$$\Rightarrow \quad F = \frac{f_1 f_2}{f_2 - f_1}$$ **(1)**

44. (i) The frequency of reflected and refracted light remains same as the frequency of incident light because frequency only depends on the source of light. **(1)**

(ii) Since, the frequency remains same, hence there is no reduction in energy. **(1)**

45. (i) Refer to Sol. 17 **(1)**

(ii) ${}^a\mu_b = \frac{1}{\sin C}$, where a and b are the rarer and denser media, respectively. C is the critical angle for the given pair of optical media. **(1)**

46. Given, focal length of convex lens,

$f_1 = +25\text{cm} = +0.25$ m and focal length of concave lens, $f_2 = -20$ cm $= -0.20$ m

Equivalent focal length of convex and concave lens,

$$F = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{25} + \frac{1}{-20} = -\frac{1}{100}$$

$$\therefore \quad F = -100 \text{ cm} = -1 \text{ m}$$ **(1)**

Now, the power of lens, $P = \frac{1}{f}$

For convex lens, $P_1 = \frac{1}{f_1} = \frac{1}{0.25}$

For concave lens, $P_2 = \frac{1}{f_2} = \frac{1}{-0.20}$

Hence, power of the combination

$$P = P_1 + P_2 = \frac{1}{0.25} + \frac{1}{-0.20} = \frac{100}{25} + \frac{100}{-20}$$

$$= \frac{400 - 500}{100} = \frac{-100}{100} = -1 \text{ D}$$

Here, the focal length of the combination $= 100$ cm $= -1$ m

Since, the focal length is in negative, so the system will be diverging in nature. **(1)**

47.

> While tracing the path of the ray, we should remember that prism bends the incident rays towards its base.

Refractive index of glass, $\mu_g = \sqrt{3}$

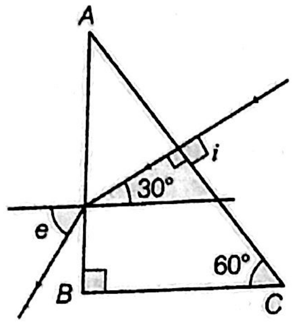
Since, $i = 0$

At the interface AC, we have (according to Snell's law) $\frac{\sin i}{\sin r} = \frac{\mu_g}{\mu_a}$

But, $\sin i = \sin 0° = 0$

Thus, $\sin r = \frac{\mu_a \sin i}{\mu_g} = 0$

(1)



Hence, $r = 0$

This ray pass unrefracted at AC interface and reaches AB interface. Here, we can see angle of incidence becomes 30°.

Thus, applying Snell's law

$$\frac{\sin 30°}{\sin e} = \frac{\mu_a}{\mu_g} = \frac{1}{\sqrt{3}}$$

$$\sin e = \sqrt{3} \times \sin 30° = \sqrt{3}/2$$

Thus, $e = 60°$

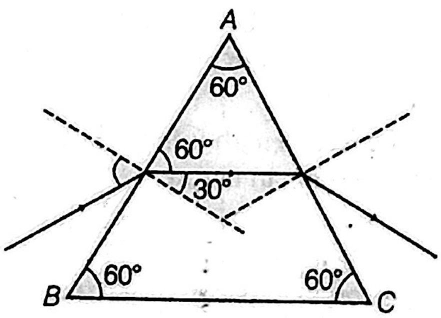
Hence, angle of emergence is 60°. **(1)**

48. To draw the ray diagram for the refraction from the prism. Following things should be kept in the mind.

(i) Draw normal to the point of incidence.

(ii) Consider each boundary of the prism as separate interface and draw the ray diagram for the refraction taking place.

The reflection of light through prism is shown as below:



By geometry Angle of refraction, $r = 30°$ **(1/2)**

Given, refractive index, $\mu = \sqrt{3}$

Using Snell's law, $\mu = \sin i / \sin r$ **(1/2)**

$$\Rightarrow \sin i = \mu \sin r$$

$$= (\sqrt{3}) \sin (30°) = \sqrt{3}/2$$ **(1/2)**

Angle of incidence, $i = 60° = \pi/3$

$$\therefore i = \pi/3$$ **(1/2)**

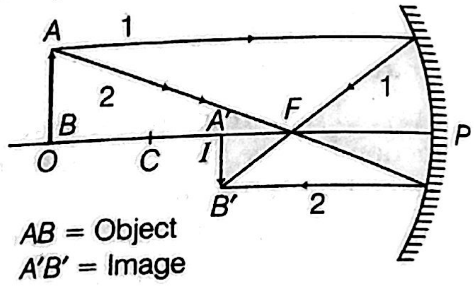
49. To draw the ray diagram for image formation, the following are the rules to form the image from spherical mirror.

(i) The ray parallel to principal axis passes through the focus after reflection.

(ii) The ray passing through the focus becomes parallel to principal axis after reflection.

(iii) The ray passing through the centre of curvature returns on the same path after reflection.

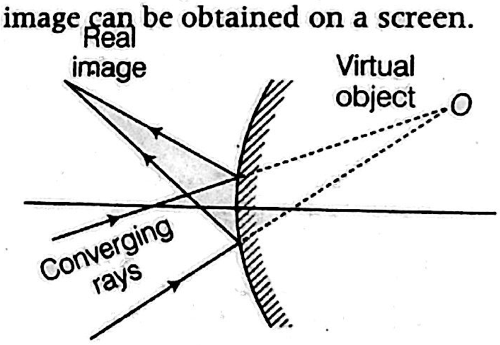
(i) The ray diagram showing the image formation of the object **(1)**



(ii) The position of the image remains same whereas intensity of image reduces.

(1/2+1/2 = 1)

50. (i) If a plane or a convex mirror is placed in the path of rays converging to a point the rays get reflected to a point in front of the mirror. Real image can be obtained on a screen. **(1)**



(ii) Because convex mirror forms virtual, erect and smaller image of object irrespective of relative position of object from mirror and therefore, its field of view is very wide. **(1)**

51. (i)

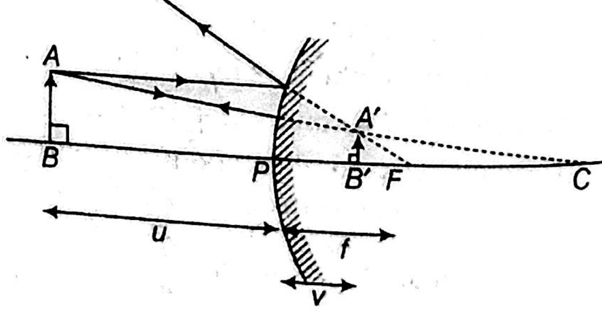


Figure shows the formation of image $A'B'$ of a finite object AB by a convex mirror, virtual, erect and diminished.

(ii) Now, $\Delta ABP \sim \Delta A'B'P$ **(1)**

$$\therefore \quad \frac{A'B'}{AB} = \frac{PB'}{PB}$$

Applying the new cartesian sign convention,

$$A'B' = h_2,\ AB = h_1$$
$$PB' = v,\ PB = -u$$
$$\therefore \quad \frac{h_2}{h_1} = \frac{v}{-u}$$

Linear magnification

$$m = \frac{h_2}{h_1} = -\frac{v}{u}$$ **(1)**

52. We know $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$f \propto \frac{1}{(\mu - 1)} \text{ and } \mu_V > \mu_R$$ **(1)**

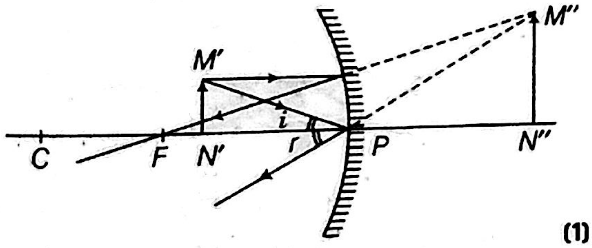
The increase in refractive index would result in decrease of focal length of lens. Hence, we can say that replacing red light with violet light, decreases the focal length of the lens used. **(1)**

53. Net power, $P = P_1 + P_2 = -4 + 2 = -2\text{D}$ **(1)**

Focal length, $f = \frac{1}{P} = \frac{1}{-2}\text{m}$

$$= -0.5\text{ m} = -50\text{ cm}$$ **(1)**

54. Ray diagram of image formation by a concave mirror.



(1)

$\Delta M'N'P$ and $M''N''P$ are similar triangles

$$\therefore \quad \frac{M''N''}{M'N'} = \frac{N''P}{N'P}$$

By sign convention, $PN' = -u$, $PN'' = +v$

$$M'N' = h_1$$

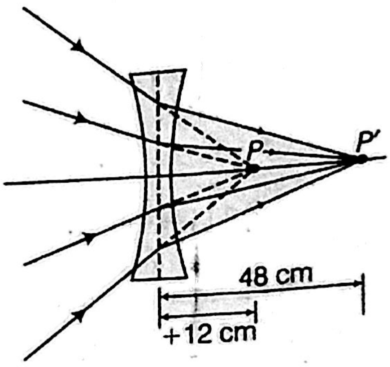
and

$$M''N'' = h_2$$

$$\therefore \quad \frac{h_2}{h_1} = \frac{+v}{-u}$$

$\therefore$ Linear magnification, $m = \frac{h_2}{h_1}$ $\quad m = -\frac{v}{u}$ **(1)**

55. Ray diagram



(1)

Given, $u = +12$ cm, $f = -16$ cm, $v = ?$

Using lens equation, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow -\frac{1}{16} = \frac{1}{v} - \frac{1}{12}$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{12} - \frac{1}{16} = \frac{4-3}{48} = \frac{1}{48}$$

$$v = +48 \text{ cm}$$

The image of virtual object at P forms at P' at a distance 48 cm from the lens. **(1)**

56. **For ray diagram for the formation of rainbow** **(1)**

Refer to Sol. 41

The primary rainbow is formed by those rays which suffer one total internal reflection and two refraction and comes out of the rain drop at angle of minimum deviation.

The violet and red light colours emerge as cone of rays at 41° and 43° respectively and can be viewed by observer. **(1)**

57. Given, $R_1 = +10$ cm, $R_2 = -15$cm, $f = +12$cm, $\mu = ?$

Lens maker's formula, **(1)**

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{12} = (\mu - 1)\left(\frac{1}{10} + \frac{1}{15}\right) = (\mu - 1)\left(\frac{5}{30}\right)$$

$$\Rightarrow \quad \mu - 1 = \frac{1}{2}$$

$$\therefore \quad \mu = \frac{3}{2}$$ **(1)**

58. Given, $f = \frac{2}{3}R$, $R_1 = +R$, $R_2 = -R$

$\because$ Using Lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{\frac{2R}{3}} = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R}\right) \quad \textbf{(1)}$$

$$\frac{3}{2R} = (\mu - 1)\left(\frac{2}{R}\right) \Rightarrow \mu - 1 = \frac{3}{4}$$

$$\mu = 1 + \frac{3}{4} \Rightarrow \mu = \frac{7}{4} = 1.75 \quad \textbf{(1)}$$

59. (i) Refer to Sol. 2 **(1)**

(ii) Refractive index μ of prism is maximum for violet and minimum for red colour. **(1)**

60. For a plano-convex lens, $R_1 = \infty$

$$R_2 = -R \quad f = 0.3 \text{ m} = 30 \text{ cm}$$

$$\mu = 1.5 \Rightarrow R = ?$$

Lens Maker's formula,

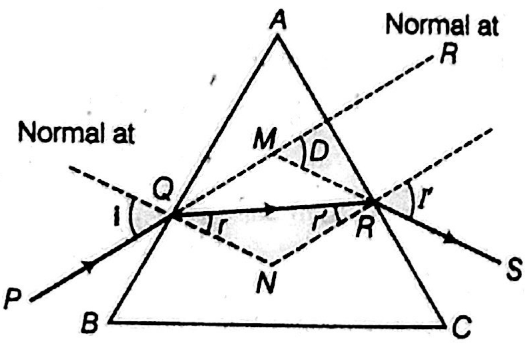
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \textbf{(1)}$$

$$\Rightarrow \quad \frac{1}{30} = (\mu - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$$

$$\frac{1}{30} = \frac{(1.5 - 1)}{R} \Rightarrow R = 15 \text{ cm} \quad \textbf{(1)}$$

61. (i) From the table angle of minimum deviation $= 40°$. The corresponding value of $i = 52°$. When prism is adjusted at an angle of minimum deviation, then the angle of incidence is equal to the angle of emergence because $i + e = A + \delta$. **(1)**

(ii) *The ray diagram in the condition of minimum deviation is shown as below:*



(1)

62. Given, $\alpha = 60°$ (for isosceles triangle)

$$r_1 = 90° - \beta \text{ and } r_2 = \beta - 30°$$

For minimum deviation, $r_1 = r_2$

$$\Rightarrow \quad 90° - \beta = \beta - 30°$$

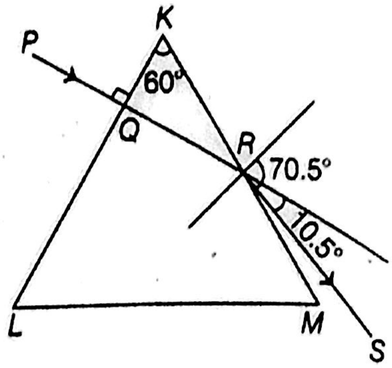
$$\Rightarrow \quad 2\beta = 120° \text{ or } \beta = 60° = \alpha \quad \textbf{(1½)}$$

For total internal reflection, $\frac{1}{\sin i_C} \leq \mu$

$$\frac{1}{\sin 30°} \leq \mu \qquad [\because r_2 = i_C = 30°]$$

$$\Rightarrow \quad \mu_2 \geq 2 \quad \textbf{(1½)}$$

63. Given, $A = 60°$, $\mu = \frac{2}{\sqrt{3}}$, $i = 0°$



(1½)

$$\text{As, } \mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\frac{2}{\sqrt{3}} = \frac{\sin\left(\frac{60° + \delta_m}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$$

$$\Rightarrow \quad \frac{2}{\sqrt{3}} \times \frac{1}{2} = \sin\left(\frac{60° + \delta_m}{2}\right)$$

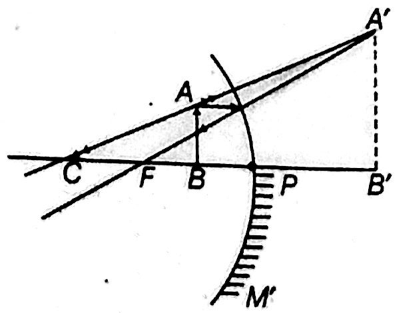
$$\Rightarrow \quad \frac{60° + \delta_m}{2} = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) = 35.3°$$

Angle of deviation, $\delta_m = 10.5°$

Also, $\quad A + \delta_m = i + e$

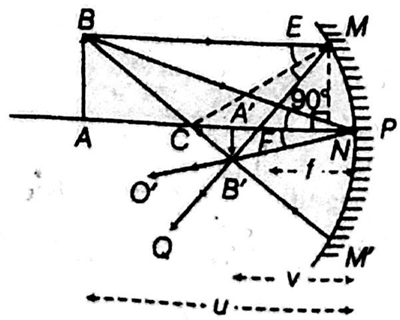
Angle of deviation, $e = 60° + 10.5° - 0° = 70.5°$ **(1½)**

64. (i)



When object is placed between the pole and focus, then a virtual, erect and magnified image is formed as shown in above figure. **(1)**

(ii) The ray incident at any angle at the pole is reflected following the laws of reflection.



In the above figure, the ray diagram is considering three rays for image formation by a concave mirror. In the figure, triangles $A'B'F$ and NEF are similar.

Then, $\frac{A'B'}{NE} = \frac{A'F}{NF}$

As, the aperture of the concave mirror is small and the points N and P lie very close to each other, then

$$NF \approx PF \text{ and } NE = AB.$$

$$\Rightarrow \quad \frac{A'B'}{AB} = \frac{A'F}{PF}$$

Since, all the distances are measured from the pole of the concave mirror, we have,

$$A'F = PA' - PF$$

$$\therefore \quad \frac{A'B'}{AB} = \frac{PA' - PF}{PF} \quad \ldots\text{(i)}$$

Also, triangles ABP and $A'B'P$ are similar, then

$$\frac{A'B'}{AB} = \frac{PA'}{PA} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{PA' - PF}{PF} = \frac{PA'}{PA} \quad \ldots\text{(iii)}$$

Applying the new Cartesian sign conventions, we have,

$$PA = -u$$

[∵ distance of object is measured against incident ray]

$$PA' = -v$$

[∵ distance of image is measured against incident ray]

$$PF = -f$$

[∵ focal length of concave mirror is measured against incident ray]

Substituting these values in Eq. (iii), we have

$$\frac{-v-(-f)}{-f} = \frac{-v}{-u} \Rightarrow \frac{v-f}{f} = \frac{v}{u} \Rightarrow \frac{v}{f} - 1 = \frac{v}{u}$$

Dividing both sides by v, we get

$$\therefore \quad \boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v}}$$

Magnification, from figure

$$m = \frac{A'B'}{AB} = \frac{PA'}{PA}$$

$$\frac{-h'}{h} = \frac{-v}{-u}$$

$$\Rightarrow \quad m = \frac{-v}{u} \quad \textbf{(2)}$$

65. First measurement gives the focal length ($f_{eq} = x$) combination of the convex lens and the plano-convex liquid lens. Second measurement gives the focal length ($f_1 = y$) of the convex lens. Focal length (f_2) of plano-convex lens is given by

$$\frac{1}{f_2} = \frac{1}{f_{eq}} - \frac{1}{f_1} = \frac{1}{x} - \frac{1}{y}$$

$$\Rightarrow \quad f_2 = \frac{xy}{y-x} \quad \ldots\text{(i)}$$

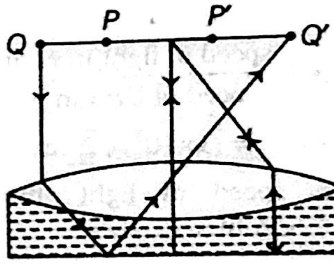
For equiconvex glass lens using Lens Maker's formula, we get

$$\frac{1}{f_1} = (n_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{y} = (1.5-1)\left(\frac{2}{R}\right)$$

(As $R_1 = R$ and $R_2 = -R$) **(1)**

$$\Rightarrow \quad \frac{1}{y} = \frac{1}{2} \times \frac{2}{R} \Rightarrow R = y$$



Now, we apply Lens Maker's formula for plano-convex lens.

Here $R_1 = R$ and $R_2 = \infty$ and let n_l = refractive index of liquid

$$\frac{1}{f_2} = (n_l - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$$\Rightarrow \quad \frac{1}{f_2} = (n_l - 1)\left(\frac{1}{R}\right) \quad \textbf{(1)}$$

$$\Rightarrow \quad n_l = 1 + \frac{R}{f_2} = 1 + \frac{y}{\left(\frac{xy}{y-x}\right)}$$

$$= 1 + \frac{y-x}{x} = \frac{y}{x} \quad \textbf{(1)}$$

66. (i) In refraction, frequency remains same so,

$$f_{\text{refracted beam}} = f_{\text{incident beam}}$$

Also, $\mu_{21} = \frac{v_1}{v_2} = \frac{f\lambda_1}{f\lambda_2} = \frac{\lambda_1}{\lambda_2}$ [∵ $v = f\lambda$]

$$\Rightarrow \quad v_2 = \frac{v_1}{\mu_{21}} = \frac{3 \times 10^8}{1.33} = 225 \times 10^8 \text{ ms}^{-1}$$

$$\therefore \quad \lambda_2 = \frac{\lambda_1}{\mu_{21}} = \frac{589}{1.33} = 442.85 \approx 443 \text{nm}$$

So, wavelength of reflected beam ≈ 443nm and its speed = 2.25×10^8 ms^{-1}. **(1½)**

(ii) For a biconvex lens, using lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $f = 20$ cm, $\mu = 1.55 \Rightarrow R_1 = +R$ and $R_2 = -R$

$\therefore$ We have, $\frac{1}{f} = (\mu - 1)\frac{2}{R}$

$\Rightarrow R = 2(\mu - 1)f = 2 \times (1.55 - 1) \times 20 = 22$cm

$\therefore$ Radius of 22 cm is required. **(1½)**

67. (i) Given, angle of minimum deviation, $\delta_m = 30°$

$\therefore$ Angle of prism, $A = 60°$

By prism formula, reflected index

$$\mu = \frac{\sin\frac{\delta_m + A}{2}}{\sin A/2} = \frac{\sin\frac{30° + 60°}{2}}{\sin 30°} = \frac{\sin 45°}{\sin 30°}$$

$$= \frac{1}{\sqrt{2}} \times 2 = \sqrt{2}$$

Also, $\mu = \frac{\text{speed of light in vacuum } (c)}{\text{speed of light in prism } (v)}$

$\Rightarrow v = c/\mu = (3 \times 10^8/\sqrt{2})$ m/s

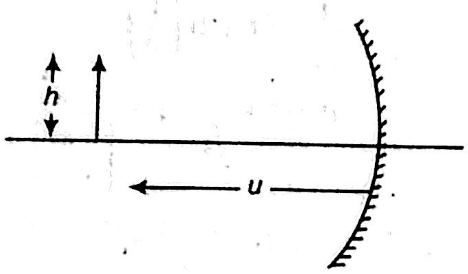
Hence, speed of light through prism is $(3 \times 10^8/\sqrt{2})$ m/s. **(2)**

(ii) As the emergent ray grazes along the face AC, $e = 90°$. At the interface AC, using Snell's law, $\sin i/\sin e = \sqrt{2}$

$\Rightarrow \quad \sin i = \sqrt{2} \sin e = \sqrt{2} \times \sin 90°$

$i = \sin^{-1}(\sqrt{2})$ **(1)**

68. (i) According to question,



Given, magnification $(m) = -2$, $R = -20$ cm

$f = -10$ cm

i.e. $\quad \frac{h_2}{h_1} = -2 = \frac{-v}{u}$

$\Rightarrow \quad u = \frac{v}{2}$ or $v = 2u$

Now, using mirror formula **(1)**

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{2u} + \frac{1}{u} = \frac{1}{-10}$$

$$\frac{1+2}{2u} = -\frac{1}{10} \qquad \left(\because f = \frac{R}{2}\right)$$

$$\frac{3}{2u} = -\frac{1}{10} \Rightarrow u = \frac{-10 \times 3}{2} = -15\text{cm}$$

$$v = 2 \times u = 2 \times -15 = -30 \text{ cm}$$ **(1)**

Hence, the object distance and image distance are -15 cm and -30 cm respectively in front of the mirror.

(ii) According to mirror formula, i.e. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\Rightarrow \quad v = \frac{uf}{u - f}$$

And we know, the value of u and f for a convex mirror are always negative and positive respectively. So, the value of v will always be positive it means convex mirror always forms a virtual image. **(1)**

69. As per the figure,

The virtual image formed by lens L_1 is at P.

Therefore, using lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

As per the parameters given in the question

$u = -15$cm, $f_{L_1} = 20$ cm

So, the image distance will be,

$$\frac{1}{v} - \frac{1}{(-15)} = \frac{1}{20}$$

$$v = -60\text{cm}$$ **(1)**

Now, this image is acting as an object for the lens L_2. We can again use the lens formula and other parameters given in the question and question figure to find the focal length of lens L_2. **(1)**

$$\frac{1}{v_{L_2}} - \frac{1}{u_{L_2}} = \frac{1}{f_{L_2}}$$

Here, $u_{L_2} = v + (-20) = -60 - 20 = -80$cm

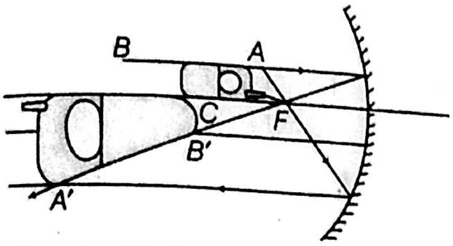
$v_{L_2} = 80$ cm

$$\frac{1}{80} - \frac{1}{(-80)} = \frac{1}{f_{L_2}}$$

$$f_{L_2} = 40\text{cm}$$

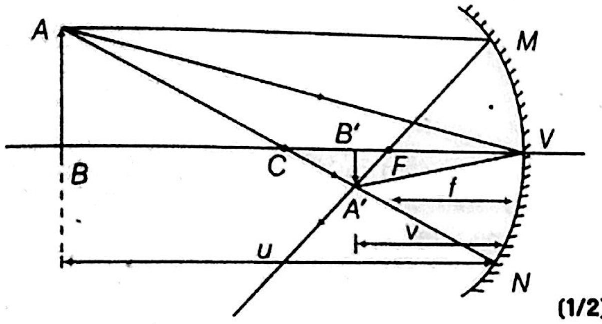
So, the focal length of the lens $L_2 = 40$cm. **(1)**

70. (i) The ray diagram for the formation of the image of the phone is shown as below.
The image of the part which is on the plane perpendicular to the principal axis will be on the same plane. It will be of the same size,
i.e. $B'C = BC$ **(1/2)**



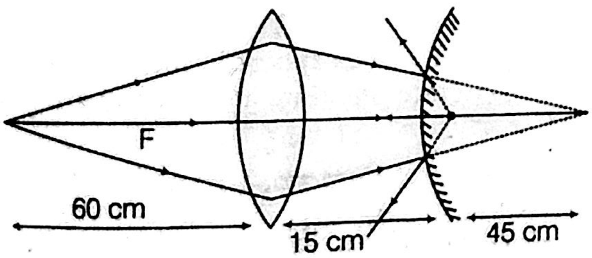
(1)

(ii) We may think that the image will now show only half of the object, but considering the laws of reflection to be true for all points of the remaining part of the mirror, the image will be that of the whole object. However, as the area of the reflecting surface has been reduced, the intensity of the image will be low, i.e. half. **(1)**



(1/2)

71. *The ray diagram showing the image formation is shown as below:*



(1)

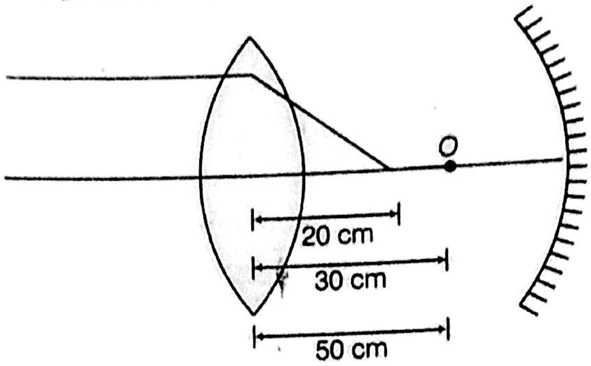
O is at $2f$ of lens so it will form image at $2f$, i.e. 60 cm from lens so position of object for mirror is at (60 − 15) cm = 45 cm behind the mirror. **(1)**

For mirror

$$f = +10\text{ cm}$$
$$u = +45\text{ cm}$$
$$v = ?$$
$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{45} = \frac{1}{10}$$
$$\therefore \quad v = +\frac{90}{7}\text{ cm} \quad \text{(behind the mirror)}$$

(1)

72. Image formed by the lens will be f at focus.



(1)

For mirror, $u = -30$, $f = -10$

According to lens formula,
$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$
$$\Rightarrow \frac{1}{-10} = \frac{1}{v} - \frac{1}{30}$$
$$\Rightarrow \frac{1}{v} = \frac{1}{30} - \frac{1}{10}$$
$$\Rightarrow \frac{1}{v} = \frac{1-3}{30}$$
$$\Rightarrow v = -15\text{cm}$$

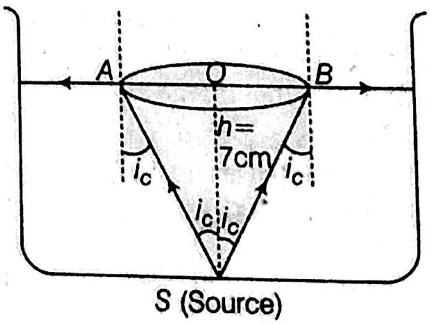
(2)

73. The angle of incidence in denser medium for which the angle of refraction in rarer medium is 90° is called the critical angle (i_c) for the pair of media. **(1)**

The light rays emerge through a circle of radius r.

Because radius $r = h\tan i_c$
$$= h \cdot \frac{\sin i_c}{\cos i_c} = h\frac{1/\mu}{\sqrt{1 - 1/\mu^2}}$$

Hence, area of water surface
$$= \frac{\pi h^2}{\mu^2 - 1} = \frac{22}{7} \times \frac{(7)^2}{(1.33)^2 - 1}$$
$$= 200.28\text{cm}^2$$



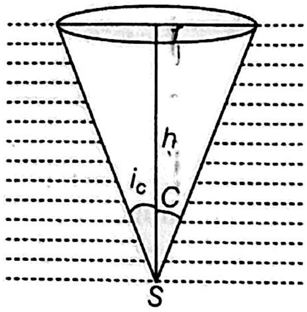
(2)

74. The light rays starting from bulb can pass through the surface if angle of incidence at surface is less than or equal to critical angle (C) for water air interface. If h is the depth of bulb from the

surface, the light will emerge only through a circle of radius r **(1)**

given by $r = \frac{h}{\sqrt{\mu^2 - 1}}$

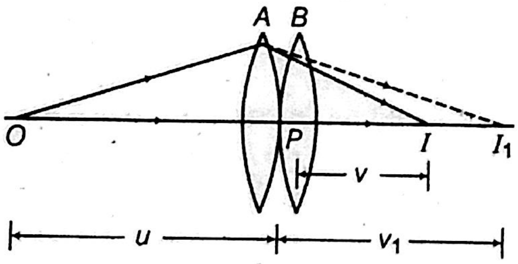
As $r = h\tan i_c = h\cdot\frac{\sin i_c}{\cos i_c} = h\cdot\frac{1/\mu}{\sqrt{1-\frac{1}{\mu^2}}}$ **(1)**



Area of water surface $= \frac{\pi h^2}{\mu^2 - 1}$ $\left[\text{as } \mu = \frac{4}{3} = 1.33\right]$

or $A = \frac{22}{7}\times\frac{(0.80)^2}{(1.33)^2 - 1} = 2.6\,m^2$ **(1)**

75. The power of a lens is equal to the reciprocal of its focal length when it is measured in metre. Power of a lens, $P = 1/f$(metre). Its SI unit is dioptre (D).



(1/2)

Consider two lenses A and B of focal lengths, f_1 and f_2 placed in contact with each other. An object is placed at a point O beyond the focus of the first lens A.

The first lens produces an image (real image) at I_1, which serves as a virtual object for the second lens B producing the final image at I. **(1/2)**

Since, the lenses are thin, we assume the optical centres P of the lenses to be coincident. For the image formed by the first lens A, we obtain

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \quad ...(i)$$

For the image formed by the second lens B, we obtain,

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \quad ...(ii)$$

Adding Eqs. (i) and (ii), we obtain

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \quad ...(iii)$$

(1)

If the two lenses system is regarded as equivalent to a single lens of focal length f, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad ...(iv)$$

From Eqs. (iii) and (iv), we obtain

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}$$

(1)

76. For lens L_1, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ **(1/2)**

Given, $u = -15$ cm, $v = ?$, $f = +10$ cm

$$\frac{1}{10} = \frac{1}{v} + \frac{1}{15}$$

(1/2)

Distance of image from lens L_1,

$\Rightarrow$ $v = 30$ cm

For lens L_3, $\frac{1}{f''} = \frac{1}{v''} - \frac{1}{u''}$

Distance of image from lens L_3,

$v'' = 10$ cm

$$\frac{1}{10} = \frac{1}{10} + \frac{1}{u''} \Rightarrow u'' = \infty$$

(1)

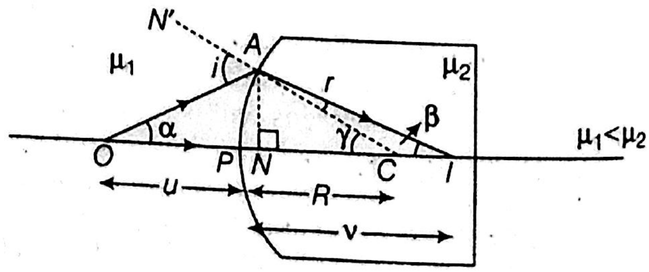
The refracted rays from lens L_1 become parallel to principal axis. It is possible only, when image formed by L_1 lies at first focus of L_2 i.e. at a distance of 10 cm from L_2.

$\therefore$ Separation between L_1 and L_2 is

$= 30 + 10 = 40$ cm

The distance between L_2 and L_3 may take any value. **(1)**

77. Refraction at convex spherical surface. When object is in rarer medium and image formed is real. **(1)**



In ΔOAC, $i = \alpha + \gamma$ and

In ΔAIC, $\gamma = r + \beta$ or $r = \gamma - \beta$

$\therefore$ By Snell's law, ${}^1\mu_2 = \frac{\sin i}{\sin r} \approx \frac{i}{r} = \frac{\alpha + \gamma}{\gamma - \beta}$

or $\frac{\mu_2}{\mu_1} = \frac{\alpha+\gamma}{\gamma-\beta}$ or $n_2\gamma - n_2\beta = n_1\alpha + n_1\gamma$

or $(n_2 - n_1)\gamma = n_1\alpha + n_2\beta$ ...(i) (1)

As α, β and γ are small and P and N lie close to each other.

So, $\alpha \approx \tan\alpha = \frac{AN}{NO} \approx \frac{AN}{PO}$

$\beta \approx \tan\beta = \frac{AN}{NI} \approx \frac{AN}{PI}$

$\gamma \approx \tan\gamma = \frac{AN}{NC} \approx \frac{AN}{PC}$

On using them in Eq. (i), we get

$$(\mu_2 - \mu_1)\frac{AN}{PC} = \mu_1 \frac{AN}{PO} + \mu_2 \frac{AN}{PI}$$

or $$\frac{\mu_2 - \mu_1}{PC} = \frac{\mu_1}{PO} + \frac{\mu_2}{PI} \quad \text{...(ii)}$$

where, $PC = +R$, radius of curvature

$PO = -u$, object distance

$PI = +v$, image distance

So, $\frac{\mu_2 - \mu_1}{R} = \frac{\mu_1}{-u} + \frac{\mu_2}{v}$

or $\frac{n_2 - n_1}{R} = \frac{n_2}{v} - \frac{n_1}{u}$

This gives formula for refraction at spherical surface, when object is in rarer medium. (1)

78. (i) From lens maker's formula,

$$\frac{1}{f} = ({}_m\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{{}_a\mu_g}{{}_a\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(i)}$$

As, ${}_a\mu_g < {}_a\mu_m$ (1.65) for the first medium with refractive index 1.65. (1)

(ii) And ${}_a\mu_g > {}_a\mu_m$(1.33) for the second medium with refractive index 1.33.

Hence, the value of focal length f will be negative in the first medium.

(a) So, the convex lens will behave as the diverging lens for first medium and will behave as the converging lens for the second medium as the sign of the focal length will not change in second case. (1)

Given, ${}_a\mu_g = 1.5$, ${}_a\mu_w = \frac{4}{3}$

$${}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{1.5}{4/3} = \frac{4.5}{4}$$

As, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

(b) $\therefore \frac{f_2}{f_1} = \left(\frac{{}_a\mu_g - 1}{{}_w\mu_g - 1}\right)$

$$= \frac{(1.5 - 1)}{\left(\frac{4.5}{4} - 1\right)} = \frac{0.5}{\frac{0.5}{4}} = 4$$

$f_2/f_1 = 4 \Rightarrow f_2 = 4f_1$

Change in focal length $= 4f_1 - f_1 = 3f_1$

Change in focal length is equal to thrice of its original focal length. (1)

(1)

79. (i) Refer to Sol. 35

(ii) For convex mirror, $f > 0$

Also, $u < 0$

But, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{v} - \frac{1}{|u|}$ (taking u with sign)

$\frac{1}{v} = \frac{1}{f} + \frac{1}{|u|}$

For f and $|u|$ to be positive, $1/v > 0 \Rightarrow v > 0$

$\Rightarrow$ Virtual image formed corresponding to the object. (1)

(iii) $\because$ $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

For concave mirror, $f < 0, u < 0$

$\Rightarrow -\frac{1}{|f|} = \frac{1}{v} - \frac{1}{|u|} \Rightarrow \frac{1}{v} = \frac{1}{|u|} - \frac{1}{|f|}$

$|f| > |u| > 0$

When object is between pole and focus.

$\because$ $|u| < |f| \Rightarrow \frac{1}{|u|} > \frac{1}{|f|}$

$\Rightarrow$ $\frac{1}{v} > 0 \Rightarrow v > 0$

Image is formed on RHS of mirror, i.e. virtual image.

Also, $\frac{1}{f} = \frac{1}{|v|} - \frac{1}{|u|}$

For concave mirror f is negative.

$\Rightarrow$ $\frac{1}{|v|} < \frac{1}{|u|} \Rightarrow \frac{|v|}{|u|} > 1 \Rightarrow m > 1$

Enlarged, virtual image formed on the other side of mirror. (1)

80. According to the diagram,

Given, for lens of focal length 10 cm.

$f = +10$ cm, $u = -30$ cm

Using lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{10} = \frac{1}{v} - \frac{1}{(-30)}$

$\Rightarrow$ $v = 15$ cm (1)

The image formed by first lens acts as an virtual object for plano-concave lens.

For plano-concave lens,

$$u = +10 \text{ cm}, f = -10 \text{ cm}, v = ?$$

Using lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow -\frac{1}{10} = \frac{1}{v} - \frac{1}{10}$

$$1/v = 0 \Rightarrow v = \infty.$$ **(1)**

The refracted ray becomes parallel to principal axis for convex lens of focal length 30 cm.

$$u = -\infty, v = ?, f = 30 \text{ cm}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{30} = \frac{1}{v} - \frac{1}{(-\infty)}$$

$$\Rightarrow \quad v = 30 \text{ cm}$$

So, final image is formed at a distance of 30 cm from second convex lens on the other side of it. **(1)**

81. Given, $f_1 = +20$ cm,

$${}_a\mu_g = 1.6, \ {}_a\mu_w = 1.3$$

$$\Rightarrow \quad {}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{1.6}{1.3}$$ **(1)**

Using lens maker's formula (in water) for converging lens,

$$\frac{1}{f_2} = ({}_w\mu_g - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad \text{...(i)}$$

In air, $\frac{1}{f_1} = ({}_a\mu_g - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad \text{...(ii)}$ **(1)**

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{f_2}{f_1} = \frac{({}_a\mu_g - 1)}{({}_w\mu_g - 1)} = \frac{(1.6 - 1)}{\left(\frac{1.6}{1.3} - 1\right)} = \frac{0.6 \times 1.3}{0.3}$$

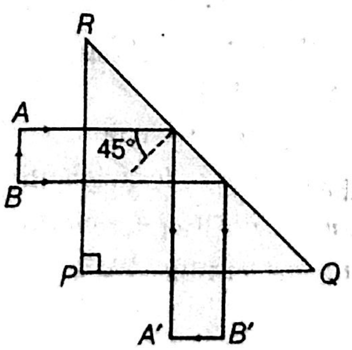
$$\frac{f_2}{f_1} = 2.6$$

New focal length, $f_2 = 2.6 \times f_1 = 2.6 \times 20$

$$f_2 = 52 \text{ cm}$$ **(1)**

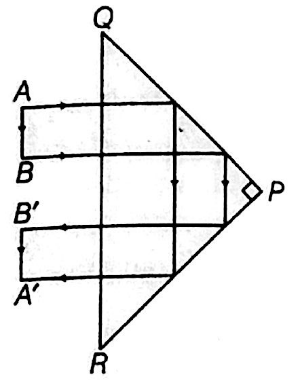
82. Refer to Sol. 17 **(1)**

(i) **Deviation of light rays through 90°**



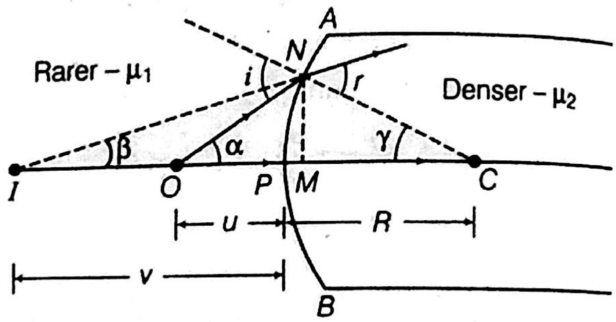
(1)

(ii) **Deviation of light rays through 180°**



(1)

83. Let an object O is placed at a distance u from convex spherical refracting surface whose virtual image formed at I at a distance v from surface. Let R is the radius of curvature of surface.



(1)

In ΔONC, $\quad i = \alpha + \gamma \quad$ ...(i)

In ΔINC, $\quad r = \beta + \gamma \quad$ ...(ii)

Also, for small angles α, β and γ

$$\alpha \simeq \tan \alpha = \frac{NM}{OM} \simeq \frac{NM}{PO} = \frac{h}{-u}$$

[Minimum close to P]

where, $h = NM$

$$\beta \simeq \tan \beta = \frac{NM}{IM} \simeq \frac{NM}{PI} = \frac{h}{-v} \quad \text{...(iii)}$$

Also, $\gamma \simeq \tan \gamma = \frac{NM}{MC} \simeq \frac{NM}{PC} = \frac{h}{+R}$ **(1)**

But by Snell's law, $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$

where μ_2, μ_1 are the refractive indices of denser medium and rarer medium, respectively.

$\because$ Angles i and r are small

$\therefore \quad \sin i \approx i, \sin r \approx r$

$$\Rightarrow \quad \frac{i}{r} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \quad \mu_1 i = \mu_2 r$$

$$\therefore \quad \mu_1(\alpha + \gamma) = \mu_2(\beta + \gamma)$$

[From Eqs. (i) and (ii)]

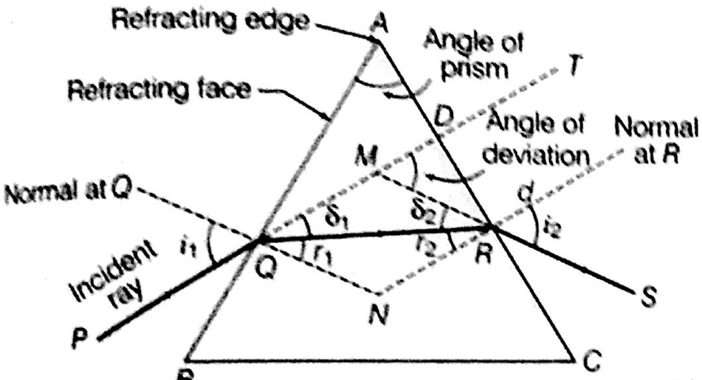
$$\Rightarrow \mu_1\alpha - \mu_2\beta = \gamma(\mu_2 - \mu_1)$$

$$\mu_1\left(\frac{h}{-u}\right) - \mu_2\left(\frac{h}{-v}\right) = \left(\frac{h}{+R}\right)(\mu_2 - \mu_1) \text{ [From Eq. (iii)]}$$

$$\Rightarrow \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

This is the required expression known as Lens makers formula. **(1)**

84. When a ray after passing through a prism suffers minimum deviation, the ray will travel parallel to the base of the prism inside the prism.



(1)

Let PQ and RS are incident and emergent rays. Let incident ray get deviated by δ by prism, i.e.

$$\angle TMS = \delta$$

Suppose δ_1 and δ_2 are deviation produced at refractors taking place at AB and AC, respectively.

$\therefore \quad \delta = \delta_1 + \delta_2 \Rightarrow \delta = (i_1 - r_1) + (i_2 - r_2)$

$\delta = (i_1 + i_2) - (r_1 + r_2)$ ...(i) **(1/2)**

Also, in quadrilateral $AQNR$,

$A + \angle QNR = 180°$

[$\because$ QN and RN are normal on two surfaces]

Also, In ΔQNR, $\angle QNR + r_1 + r_2 = 180°$

$\Rightarrow \quad A = r_1 + r_2$ ...(ii)

From Eqs. (i) and (ii), we get

$\delta = (i_1 + i_2) - A$ ...(iii) **(1/2)**

Angle of deviation produced by prism varies with angle of incidence. When prism is adjusted at angle of minimum deviation, then [say]

$i_1 = i_2 = i$

At $\delta = \delta_m$ [say]

$\Rightarrow r_1 = r_2 = r$

From Eqs. (i) and (ii), we have

$\delta_m = 2i - 2r$ and $2r = A$

$\Rightarrow \quad i = A + \delta_m/2 \Rightarrow r = A/2$

$\therefore$ Refractive index of material of prism is

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

This is the required expression. **(1)**

85. For convex lens: For erect image $u = -\text{ve}$, $v = +\text{ve}$

Magnification, $m = \frac{I}{O} = \frac{-v}{u}$

where, O = length of object

I = length of image

Given, $f = +20$ cm, $I = 4 \times$ length of object

$$\Rightarrow \frac{I}{O} = 4 \Rightarrow \frac{v}{-u} = 4 \Rightarrow v = -4u$$ **(1)**

Using lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$ **(1)**

$$\frac{1}{f} = \frac{1}{(-4u)} - \frac{1}{(-u)} \Rightarrow \frac{1}{f} = -\frac{1}{4u} + \frac{1}{u}$$

$$\Rightarrow \frac{1}{20} = \frac{4-1}{4u} = \frac{3}{4u} \Rightarrow u = \frac{20 \times 3}{4} = 15 \text{ cm}$$

$$u = 15 \text{ cm}, v = 4u = 15 \times 4 = 60 \text{ cm}$$

Distance of the object, $u = 15$ cm

Distance of the image, $v = 60$ cm

The image is on the same side of the object. **(1)**

86. Since, real and inverted image of an object can be taken on the screen.

Given, $v = +10$ cm

and magnification, $u = -\text{ve}$ (for real image)

$m = -19$, $f = ?$

$$\therefore \quad m = \frac{I}{O} = \frac{v}{u} \Rightarrow -19 = \frac{v}{u} \Rightarrow v = -19u$$

$$\Rightarrow \quad u = -\frac{v}{19}$$ **(1)**

Using lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{-\left(\frac{v}{19}\right)}$

$$\frac{1}{f} = \frac{1}{v} + \frac{19}{v} \Rightarrow \frac{1}{f} = \frac{20}{v}$$

$\because \quad v = 10$ cm

$\therefore \quad f = \frac{1}{2}$ cm $\Rightarrow f = 0.5$ cm **(2)**

87. As the image of the object is formed by the lens on the screen, therefore the image is real.

Let the object is placed at a distance x from the lens. As the distance between the object and the screen is 90 cm. Therefore, the distance of the image from the lens is $(90-x)$. **(1)**

According to new cartesian sign conventions,

$$u=-x,\ v=+(90-x)$$

Magnification $m=v/u$

$\therefore \quad -2=(90-x)/-x \Rightarrow x=30$ cm

$\therefore \quad u=-30$ cm, $v=60$ cm

Let f be focal length of the lens.

According to thin lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f};\ \frac{1}{60}-\frac{1}{-(30)}=\frac{1}{f}$$

$$\frac{1}{60}+\frac{1}{30}=\frac{1}{f} \Rightarrow f=+20 \text{ cm}$$

A convex lens of focal length 20 cm is required. **(2)**

88. (i) Focal length of spherical mirror does not get affected with the increase of wavelength. **(1)**

(ii) Using lens maker's formula,

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\therefore \quad \frac{f_w}{f_a}=\frac{({}_a\mu_g-1)}{({}_w\mu_g-1)}$$ **(1)**

$$\frac{f_w}{20}=\frac{1.5-1}{(1.5/1.33-1)}=\frac{0.5\times 1.33\times 20}{0.17}$$

$$f_w=78.2 \text{ cm}$$ **(1)**

89. (i) No change as f of mirror depends only on its radius of curvature. **(1)**

(ii) Refer to Sol. 88 (ii)
($f_w=39.11$ cm) **(2)**

90. Given, length of object $O=+3$ cm

$$u=-60 \text{ cm},\ f=+30 \text{ cm}$$

$\because \quad 1/f=1/v+1/u$ [mirror formula] **(1)**

or $\quad 1/30=1/v+1/(-60)$

or $\quad \dfrac{1}{v}=\dfrac{1}{30}+\dfrac{1}{60}=\dfrac{2+1}{60}$

$\Rightarrow \quad v=20$ cm **(1)**

$\therefore \quad \dfrac{I}{O}=-\dfrac{v}{u} \Rightarrow \dfrac{I}{(+3)}=-\dfrac{(+20)}{(-60)}$

$\Rightarrow \quad I=1$ cm **(1)**

So, the virtual, erect and diminished image will be formed on the other side of the mirror. **(3)**

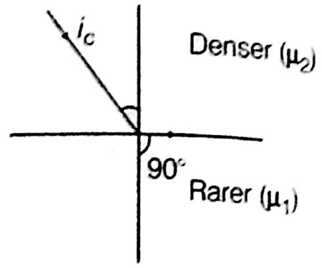
91. Refer to Sol. 90

92. (i) Refer to Sol. 17

From Snell's law, $\mu_2 \sin i_c=\mu_1 \sin 90°$

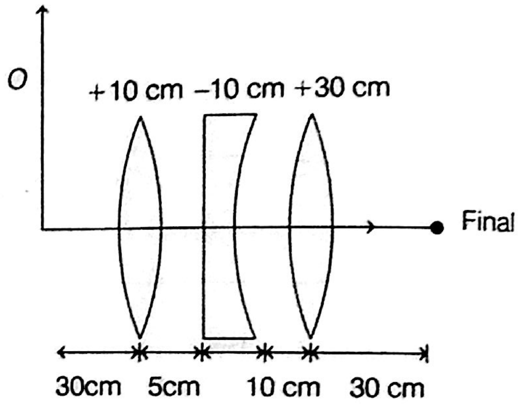
$$\frac{\mu_2}{\mu_1}=\frac{1}{\sin i_c}$$

$$_1\mu_2=\frac{1}{\sin i_c}$$



(2½)

(ii) $\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f}$



Here, for first lens,

$u=-30$ cm, $f=+10$ cm

$$\frac{1}{v_1}=\frac{1}{f}+\frac{1}{u}=\frac{1}{10}-\frac{1}{30}$$

$$\frac{1}{v_1}=\frac{2}{30} \Rightarrow v_1=15 \text{cm}$$

For second lens, $u=+10$ cm, $f=-10$ cm

$$\frac{1}{v_2}=\frac{1}{f}+\frac{1}{u}=\frac{1}{10}-\frac{1}{10}=\infty$$

Thus, for last lens the object is at infinity, hence the image formed at the focus of the lens, which is at a distance of 30 cm. **(2½)**

93. (i) Given focal length of convex lens, $f_1=30$ cm

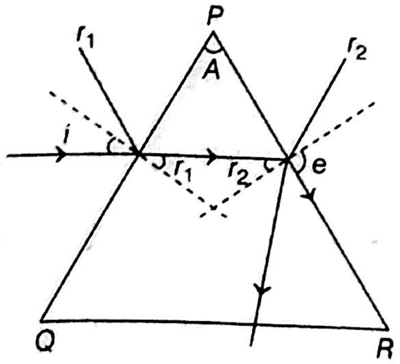
Focal lenth of concave lens, $f_2=-20$ cm 96 f be the combined focal length, then

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{30}-\frac{1}{20}$$

$$=\frac{2-3}{60}=-\frac{1}{60} \Rightarrow f=-60 \text{ cm}$$

since, F is negative, therefore, combined system is diverging. **(2½)**

(ii) Let the ray travel along the face PR for an angle of incidence i_C (critical angle)



$\therefore \quad e = 90°$

From Snell's law, $\dfrac{\sin i_c}{\sin r_1} = \mu = \dfrac{\sin e}{\sin r_2}$ ...(i)

$\Rightarrow \dfrac{\sin 90°}{\sin r_2} = \mu$

$\Rightarrow \quad \sin r_2 = \dfrac{1}{\mu} \Rightarrow r_2 = \sin^{-1}\left(\dfrac{1}{\mu}\right)$

Also, $r_1 + r_2 = A$

$\Rightarrow \quad r_1 = A - \sin^{-1}\left(\dfrac{1}{\mu}\right)$

From Eq. (i),

$\Rightarrow \dfrac{\sin i_c}{\sin\left[A - \sin^{-1}\left(\dfrac{1}{\mu}\right)\right]} = \mu$

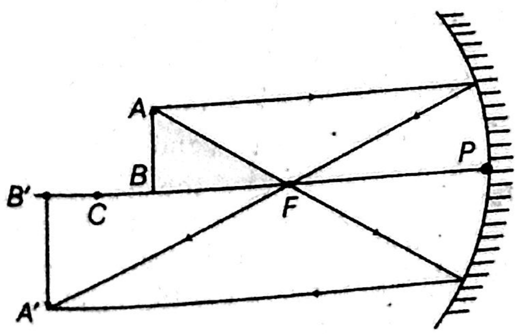
$\Rightarrow \quad \sin i_c = \mu \sin\left[A - \sin^{-1}\left(\dfrac{1}{\mu}\right)\right]$

$$i_c = \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$$

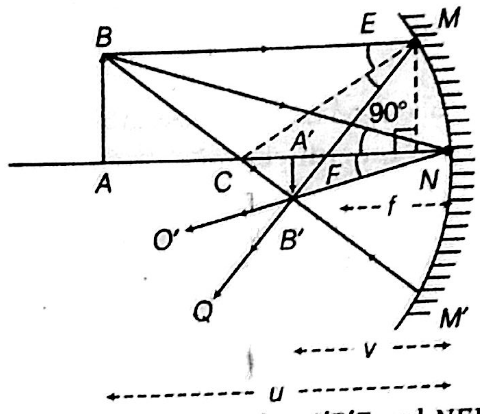
For total internal reflection, the angle of incidence is

$$i \geq i_c = \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$$ (2½)

94. (i) Concave mirror form real, inverted and magnified image of an object when it is placed between C and F. The ray diagram is given as



(ii) In the given figure, the ray diagram considering three rays for image formation by a concave mirror.



In the figure, triangles $A'B'F$ and NEF are similar.

Then, $\dfrac{A'B'}{NE} = \dfrac{A'F}{NF}$

As the aperture of the concave mirror is small, the points N and P lie very close to each other.

$$NF \approx PF \text{ and } NE = AB$$

$$\frac{A'B'}{AB} = \frac{A'F}{PF}$$

Since, all the distances are measured from the pole of the concave mirror, we have

$$A'F = PA' - PF$$

$\therefore \quad \dfrac{A'B'}{AB} = \dfrac{PA' - PF}{PF}$ ...(i)

Also, triangles ABP and $A'B'P$ are similar, then

$\dfrac{A'B'}{AB} = \dfrac{PA'}{PA}$ ...(ii)

From Eqs. (i) and (ii), we get

$\dfrac{PA' - PF}{PF} = \dfrac{PA'}{PA}$ ...(iii)

Applying new Cartesian sign convention, we have

$$PA = -u$$

($\because$ distance of object is measured against incident ray)

$$PA' = -v$$

($\because$ distance of image is measured against incident ray)

$$PF = -f$$

($\because$ focal length of concave mirror is measured against incident ray)

Substituting these values in Eq. (iii), we get

$$\frac{-v - (-f)}{-f} = \frac{-v}{-u}$$

$$\Rightarrow \quad \frac{v-f}{f}=\frac{v}{u} \Rightarrow \frac{v}{f}-1=\frac{v}{u}$$

Dividing both sides by v, we get

$$\therefore \quad \frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

The above relation is called **mirror formula.**

Linear magnification **(3)**

The ratio of the height of the image (h') formed by a spherical mirror to the height of the object (h) is called the linear magnification produced by the spherical mirror.

It is denoted by m.

$$m=\frac{h'}{h}$$

(iii) **Advantages of reflecting telescope over refracting telescope**

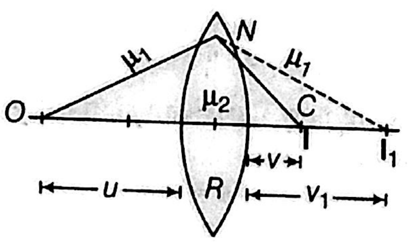
(a) In reflecting telescope, image formed is free from chromatic aberration defect. So, it is sharper than image formed by a retracting type telescope.

(b) A mirror is easier to produce with a large diameter, so that it can intercept rays crossing a large area and direct them to the eye-piece. **(2)**

95. (i) Refer to Sol. 77 **(1)**

Lens Maker's formula

If a convex lens is made up of two convex spherical refracting surfaces. The final images formed after two refractions. Let μ_2 be the refractive index of the material of the lens and n_1 be the refractive index of the rarer medium around the lens.



Let R_1 be the radius of curvature of second surface of the lens, I_1 would have been a real image of O formed after refraction, then from Eq. (iv),

$$\frac{\mu_2}{v_1}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R_1} \quad \dots \text{(v)}$$

Let R_2 be the radius of curvature of the second surface of the lens. Refraction is now taking place from denser to rarer medium

$$\frac{\mu_1}{v}-\frac{\mu_2}{v_1}=\frac{\mu_2-\mu_1}{R_2} \quad \dots \text{(vi)}$$

Adding Eqs. (v) and (vi), we get

$$\frac{\mu_1}{v}-\frac{\mu_1}{u}=(\mu_2-\mu_1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \dots \text{(vii)}$$

Put $\frac{\mu_2}{\mu_1}=\mu=$ refractive index of material of the lens with respect to surrounding medium

$$\frac{1}{v}-\frac{1}{u}=\left(\frac{\mu_2}{\mu_1}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right] \quad \dots \text{(viii)}$$

When object on the left of lens is at infinity, then image is formed at the principal focus of the lens.

$\therefore$ When $u=\infty$, $v=f=$ focal length of the lens.

$$\therefore \quad \frac{1}{f}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

This is the lens maker's formula. **(2)**

(ii) According to question,

$\mu_1=1$ [Given]

$\mu_2=1.5 \Rightarrow R=20\text{ cm}$

$\Rightarrow \quad u=-100\text{ cm}$

So, from surface formula

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R} \Rightarrow \frac{1.5}{v}+\frac{1}{100}=\frac{1.5-1}{20}$$

$$\Rightarrow \frac{1.5}{v}=\frac{0.5}{20}-\frac{1}{100} \Rightarrow \frac{1.5}{v}=\frac{5}{200}-\frac{1}{100}$$

$$\Rightarrow \frac{1.5}{v}=\frac{5-2}{200}=\frac{3}{200} \Rightarrow \frac{1.5}{v}=\frac{3}{200}$$

$$\Rightarrow \quad v=\frac{200\times1.5}{3}=\frac{300}{3}=100\text{ cm}$$ **(2)**

96. (i) Let a spherical surface separate a rarer medium of refractive index μ_1 from the second medium of refractive index μ_2. Let C be the centre of curvature and $R=MC$ be the radius of the surface.

Consider a point object O lying on the principal axis of the surface. Let a ray starting from O incident normally on the surface along OM and pass straight. Let another ray of light incident on NM along ON and refract along NI.

From M, draw MN perpendicular to OI.

The above figure shows the geometry of the formation of image I of an object O and the principal axis of a spherical surface with centre of curvature C and radius of curvature R. **(2)**

Here, *we have to make following assumptions,*

(a) the aperture of the surface is small as compared to the other distance involved.

(b) NM will be taken as nearly equal to the length of the perpendicular from the point N on the principal axis. **(½ × 2)**

$$\tan\angle NOM = \frac{MN}{OM}, \quad \tan\angle NCM = \frac{MN}{MC}$$

$$\tan\angle NIM = \frac{MN}{MI}$$

For ΔNOC, is the exterior angle.

$$\therefore \quad \angle i = \angle NOM + \angle NCM$$

For small angles,

$$i = \frac{MN}{OM} + \frac{MN}{NC} \quad \text{...(i)}$$

Similarly, $r = \angle NCM - \angle NIM$

$$\Rightarrow \quad r = \frac{MN}{NC} - \frac{MN}{NI} \quad \text{...(ii)}$$

By Snell's law, we get

$$\mu_1 \sin i = \mu_2 \sin r$$

For small angles, $n_1 i = n_2 r$

Put the values of i and r from Eqs. (i) and (ii), we get

$$n_1\left(\frac{MN}{OM} + \frac{MN}{MC}\right) = n_2\left(\frac{MN}{MC} - \frac{MN}{MI}\right)$$

$$\Rightarrow \quad \frac{\mu_1}{OM} + \frac{\mu_2}{MI} = \frac{\mu_2 - \mu_1}{MC} \quad \text{...(iii)}$$

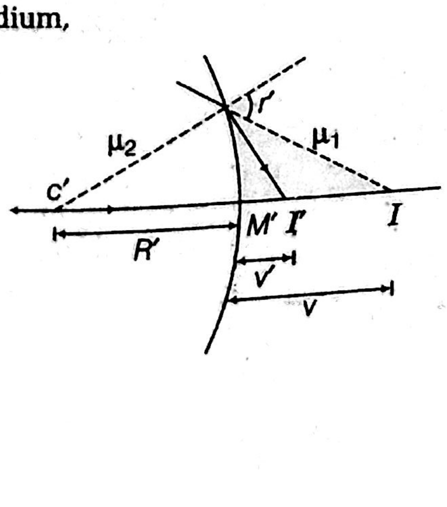
Applying new cartesian sign conventions, we get

$OM = -u$, $MI = +v$ and $MC = +R$

Substituting this in Eq. (iii), we get

$$\frac{\mu_2}{v} - \frac{\mu_2}{u} = \frac{\mu_2 - \mu_1}{R} \quad \text{...(iv)}$$

(ii) Now, the image I' acts as a virtual object for the second surface that will form a real at I. As, refraction takes place from denser to rarer medium,



$$\therefore \quad \frac{-\mu_2}{v} + \frac{\mu_1}{v'} = \frac{\mu_2 - \mu_1}{-R'} \quad \text{...(v)}$$

On adding Eqs. (iv) and (v), we get

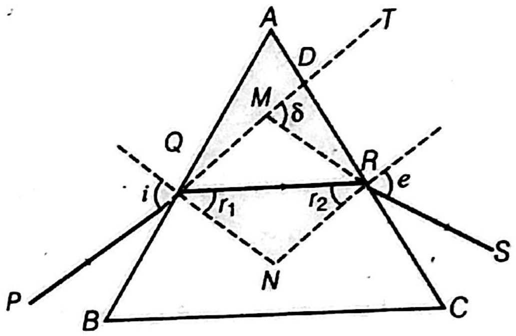
$$\frac{1}{f} = (\mu_{21} - 1)\left(\frac{1}{R} - \frac{1}{R'}\right)$$

$$\left\{\because \mu_{21} = \frac{\mu_2}{\mu_1}, \frac{1}{f} = \frac{1}{v} - \frac{1}{u}\right\} \quad \textbf{(2)}$$

97. (i) Let PQ and RS are incident and emergent rays. Let incident ray get deviated by δ by the prism.

i.e. $\angle TMS = \delta$

Let δ_1 and δ_2 are deviation produced at refractions taking place at AB and AC, respectively.



(1)

$$\therefore \quad \delta = \delta_1 + \delta_2 = (i - r_1) + (e - r_2)$$
$$= (i + e) - (r_1 + r_2) \quad \text{...(i)}$$

But in ΔFNR,

$$\angle QNR + \angle RQN + \angle QRN = 180°$$

or $\angle QNR = 180° - (r_1 + r_2)$ ...(ii)

In $\square QARNF$, $\angle AQN$ and $\angle ARN$ are right angles.

So, $\angle QNR = 180° - A$ ...(iii)

where, A is angle of prism.

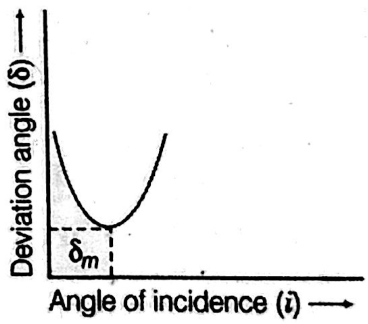
From Eqs. (ii) and (iii), we have

$$A = r_1 + r_2 \quad \text{...(iv)}$$

From Eqs. (i) and (iv), we have

$$\delta = (i + e) - A \quad \text{...(v)}$$

i - δ graph is shown in the figure



(1)

The conditions for the angle of minimum deviation are given as below:

(a) Angle of incidence (i) and angle of emergence (e) are equal.

i.e. $\angle i = \angle e$

(b) In equilateral prism, the refracted ray is parallel to base of prism.

(c) The incident and emergent rays are bent on same angle from refracting surfaces of the prism.

i.e. $\angle r_1 = \angle r_2$

For minimum deviation position,

putting $r = r_1 = r_2$ and $i = e$ in Eq. (iv)

$$2r = A \Rightarrow r = \frac{A}{2} \quad \text{...(vi)}$$

From Eq. (i), $\delta_m = 2i - A$

$$i = \frac{A + \delta_m}{2} \quad \text{...(vii)}$$

∴ Refractive index of material of prism is

$$\mu = \frac{\sin i}{\sin r}$$

From Eqs. (vi) and (vii), we get

$$\Rightarrow \quad \mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin A/2}$$ **(1)**

(ii) As per the question,

Angle of prism (A) = Angle of minimum deviation (δ_m)

i.e. $\angle A = \angle \delta_m$ ...(ii)

Substituting the value of $\angle \delta_m$ from Eq. (ii) to Eq. (i), we get

$$\Rightarrow \mu = \frac{\sin(A + A/2)}{\sin(A/2)} \Rightarrow \mu = \frac{\sin A}{\sin(A/2)}$$

$$\Rightarrow \mu = \frac{2\sin(A/2)\cdot\cos(A/2)}{\sin(A/2)} \Rightarrow \mu = 2\cos(A/2)$$

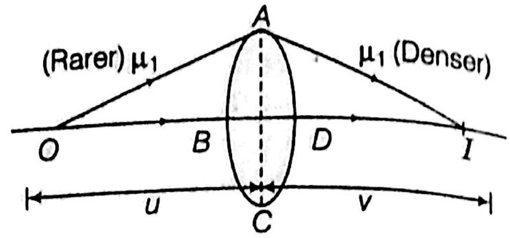
This is a required relation between refractive index of the glass prism and angle of prism.

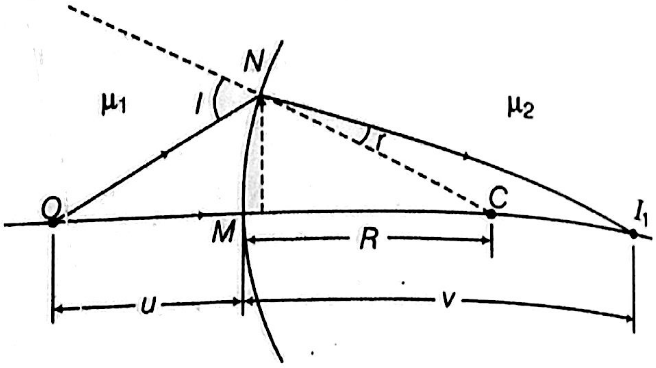
Since, $\angle A = 60°$ (given)

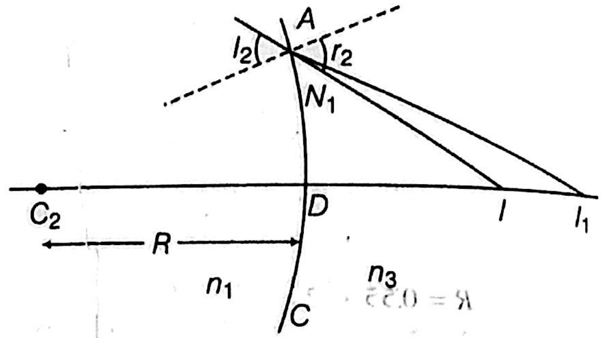
$$\Rightarrow \mu = 2\cos\left(\frac{60°}{2}\right) \Rightarrow \mu = 2\cos 30°$$

$$\Rightarrow \mu = 2 \times \frac{\sqrt{3}}{2} \Rightarrow \mu = \sqrt{3} \Rightarrow \mu = 1.732$$ **(2)**

98. (i) The incident rays coming from the object O kept in the rarer medium of refractive index μ_1, incident on the refracting. Surface NM produces the real image at I.







(1)

From the diagram,

$$\angle i = \angle NOM + \angle NCM = \frac{NM}{OM} + \frac{NM}{MC}$$

$$\angle r = \angle NCM - \angle NIM = \frac{NM}{OM} - \frac{NM}{NI}$$ **(1)**

From Snell's law,

$$\therefore \quad \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} \sim \frac{i}{r} \quad \text{(for small angle, } \sin\theta \approx \theta)$$

$$\therefore \quad \mu_2 r = \mu_1 i$$

$$\Rightarrow \mu_2\left(\frac{NM}{MC} - \frac{NM}{NI}\right) = \mu_1\left(\frac{NM}{OM} + \frac{NM}{MC}\right)$$

$$\Rightarrow \quad \mu_2\left(\frac{1}{R} - \frac{1}{v}\right) = \mu_2\left(\frac{1}{-v} + \frac{1}{R}\right)$$

$$\Rightarrow \quad \frac{\mu_2 - \mu_1}{R} = \frac{\mu_2}{v} - \frac{\mu_1}{v}$$ **(1)**

The first refracting surface ABC forms the image I_1 of the object O. The image I_1 acts as a virtual object for the second refracting surface ADC which forms the real image I as shown in the diagram.

For refraction at ABC,

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \quad \text{...(i)}$$

For refraction of ADC,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_1 - \mu_2}{R_{12}} \quad ...(ii)$$

Adding Eq. (i) and Eq. (ii), we get

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

We know that, if $u = \infty$, $v = f$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \; \frac{1}{f} = (\mu_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

(ii) Given, $\mu = 1.55$

$f = 20$ cm.

We know that, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{1}{20} = (1.55 - 1)\left[\frac{1}{R} - \left(\frac{1}{-R}\right)\right]$$

$$\Rightarrow \quad \frac{1}{20} = 0.55 \times \frac{2}{R}$$

$$R = 0.55 \times 2 \times 20$$

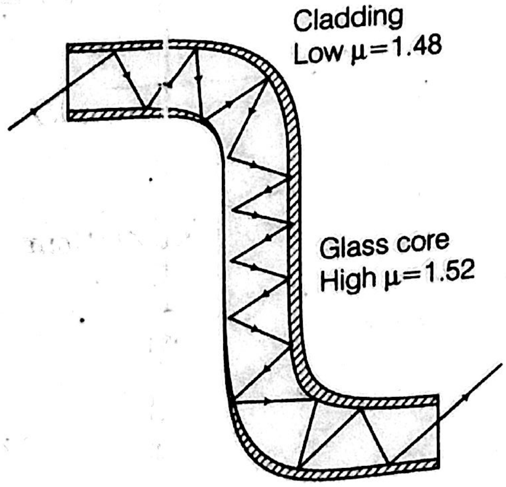
$$\Rightarrow \quad R = 22 \text{ cm}$$ **(2)**

99. Refer to Sol. 77 and 98 (i) **(5)**

100. (i) Refer to Sol. 97 (i) on pages 279 and 280. **(2)**

(ii) When light is incident on one end of the optical fibre at an angle of incidence greater than the critical angle for the glass cladding pair of media.

The light suffers repeated total internal reflection and light travels through the optical fibre without any loss of energy from one place to other inside the optical fibre. **(2)**



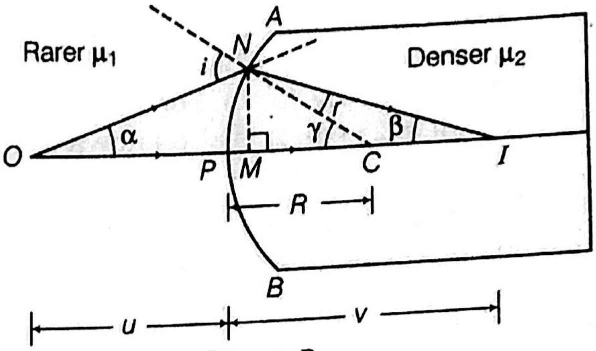
(1)

101. (i) Refer to Sol. 77 **(3)**

(ii) Refer to Sol. 54 **(2)**

102. Let, $NM = h$

The convex spherical refracting surface forms the image of object O at I. The radius of curvature is R **(1)**



(1)

$PC = +R$

$PI = +v \Rightarrow PO = -u$

In ΔNCO, $i = \gamma + \alpha$ ...(i)

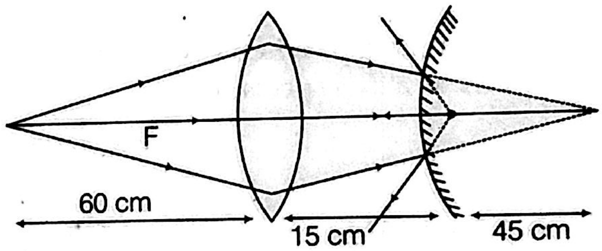
In ΔNCI, $\gamma = r + \beta$

$\Rightarrow \quad r = \gamma - \beta$ ...(ii)

For small angles α, β and γ, we have

$$\alpha \simeq \tan \alpha = \frac{MN}{MO} = \frac{MN}{PO} = \frac{+h}{-u}$$ **(1)**

$$\beta \simeq \tan \beta = \frac{MN}{MI} = \frac{MN}{PI} = \frac{h}{-v} \quad ...(iii)$$



$$\gamma \simeq \tan \gamma = \frac{MN}{MC} = \frac{MN}{PC} = \frac{h}{+R}$$

Assuming M is very close to P.

By Snell's law, $\frac{\mu_2}{\mu_1} = \mu = \frac{\sin i}{\sin r}$ **(1)**

For small i and r, $\frac{\mu_2}{\mu_1} = \frac{i}{r}$ or $r\mu_2 = i\mu_1$

$\mu_2(\gamma - \beta) = (\alpha + \gamma)\mu_1$ [From Eqs. (i) and (ii)]

$(\mu_2 - \mu_1)\gamma = \mu_1\alpha + \mu_2\beta$

$$(\mu_2 - \mu_1)\left(\frac{h}{R}\right) = \mu_1\left(\frac{h}{-u}\right) + \mu_2\left(\frac{h}{v}\right)$$

[From Eq. (iii)]

$$\Rightarrow \quad \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$ **(1)**