DCAM classes

Dynamic Classes for Academic Mastery

wave optics(solutions)

Explanations

1. Speed decreases due to decrease in wavelength of wave but energy carried by the light wave depends on the amplitude of the wave. **(1)**

2. **Wavefront** Refer to class notes
Law of reflection from Huygens' wave theory
Refer to

3. **Law of refraction from Huygens' wave theory** Refer to class notes

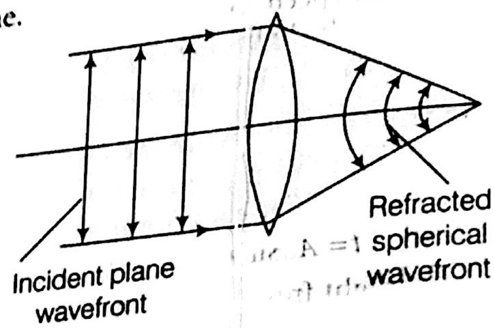
4. (i) Frequency is the characteristic of the sources while wavelength is the characteristic of the medium. When monochromatic light travels from one medium to another, its speed changes, so its wavelength changes but frequency remains same. Reflection and refracton arise through interaction of incident light with atomic constituents of matter which vibrate with the same frequency as that of the incident light. Hence frequency remains unchanged. **(1)**

(ii) Refer to Sol. 1 **(1)**

(iii) In the photon picture of light, intensity of a light is determined by the number of photons incident per unit area.
For a given frequency, intensity of light in the photon picture is determined by

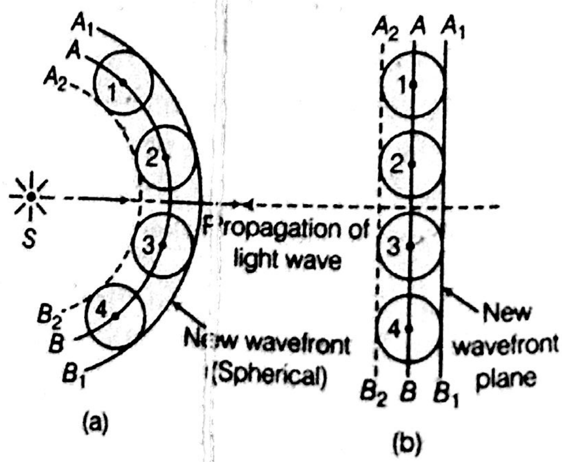
$$I = \frac{\text{Energy of photons}}{\text{area} \times \text{time}} = \frac{n \times h\nu}{A \times t}$$ **(1)**

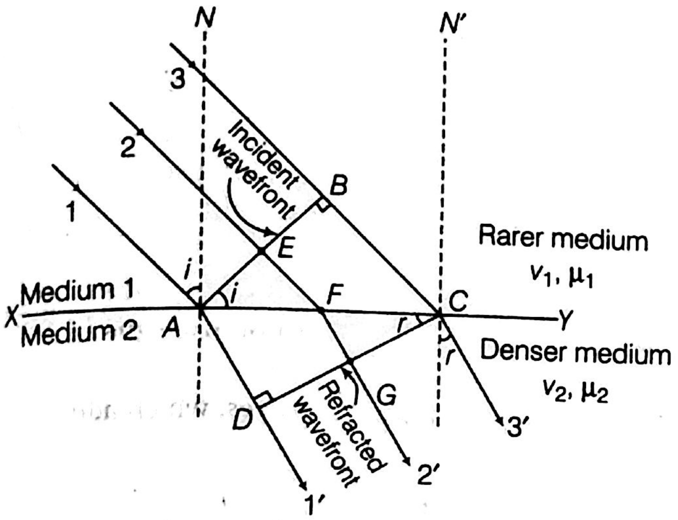
5. **Wavefront** Refer to class notes
Huygens' principle Each point on a wavefront is a source of secondary waves, which add up gives a shape to the wavefront at any time, and determines the shape of the wavefront at a later time.



When a plane wavefront (parallel rays) is incident on a thin convex lens, the emergent rays are focused on the focal point of the lens. Thus, the shape of emerging wavefront is spherical. **(2)**

6. According to Huygens' principle,
each point on the given wavefront (called primary wavefront) is the source of a secondary disturbance (called secondary wavelets) and the wavelets emanating from these point spread out in all the directions with the speed of the wave. **(1)**
A surface touching these secondary wavelets, tangentially in the forward direction at any instant gives the new wavefront at that instant. This is called secondary wavefront. **(1)**





If v_1, v_2 are the speeds of light into two media and t is the time taken by light to go from B to C or A to D or E to G through F, then $t = EF/v_1 + FG/v_2$

In ΔAFE , $\sin i = EF/AF$

In ΔFGC, $\sin r = FG/FC$

$\Rightarrow \quad t = AF \sin i/v_1 + FC \sin r/v_2$

$\Rightarrow \quad t = AC \sin r/v_2 + AF \ (\sin i/v_1 - \sin r/v_2)$

For rays of light from the different parts on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the refracted wavefront.

So, t should not depend on F. This is possible only, if $\quad \sin i/v_1 - \sin r/v_2 = 0$

or $\quad \sin i/\sin r = v_1/v_2 = \mu$

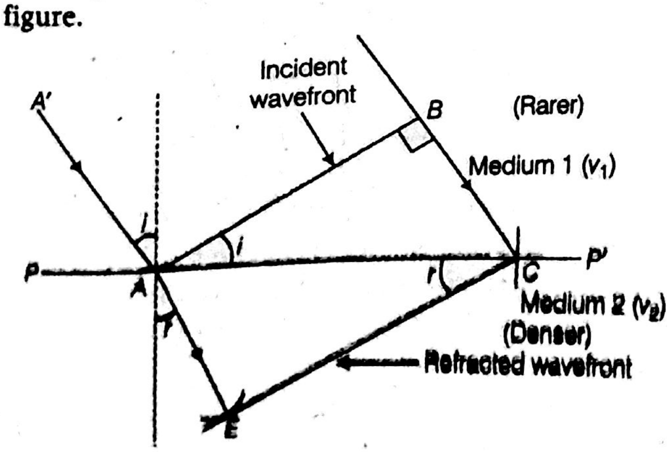
Now, if c represents the speed of light in vacuum, then $\mu_1 = c/v_1$ and $\mu_2 = c/v_2$ are known as the refractive index of medium 1 and medium 2 respectively. (1)

Then, $\quad \mu_1 \sin i = \mu_2 \sin r$

$\Rightarrow \quad \mu = \sin i/\sin r$

This is known as Snell's law of refraction. (1)

7. Snell's law of refraction : Let PP' represents the surface separating medium 1 and medium 2 as shown in figure.



Let v_1 and v_2 represent the speed of light in medium 1 and medium 2 respectively. We assume a plane wavefront AB propagating in the direction $A'A$ incident on the interface at an angle i. Let t be the time taken by the wavefront to travel the distance BC. (1)

$\therefore$

$BC = v_1 t$ [$\because$ distance = speed × time]

In order to determine the shape of the refracted wavefront, we draw a sphere, radius $v_2 t$ from the point A in the second medium (the speed of the wave in second medium is v_2). Let CE represents a tangent plane drawn from the point C.

Then $\quad AE = v_2 t$

$\therefore CE$ would represents the refracted wavefront.

In ΔABC and ΔAEC, we have

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC} \text{ and } \sin r = \frac{v_2 t}{AC}$$

where, i and r are the angles of incident and refraction respectively.

$$\frac{\sin i}{\sin r} = \frac{v_1 t}{AC} \cdot \frac{AC}{v_2 t}$$

$$\therefore \frac{\sin i}{\sin r} = \frac{v_1}{v_2} \quad (1)$$

If c represents the speed of light in vacuum, then

$$\mu_1 = \frac{c}{v_1} \text{ and } \mu_2 = \frac{c}{v_2}$$

$$\Rightarrow \quad v_1 = \frac{c}{\mu_1} \text{ and } v_2 = \frac{c}{\mu_2}$$

where, μ_1 and μ_2 are the refractive indices of medium 1 and medium 2.

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{c/\mu_1}{c/\mu_2}$$

$$\Rightarrow \quad \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

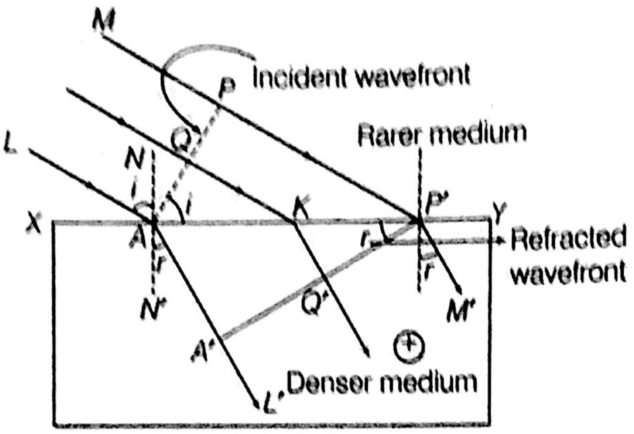
$$\Rightarrow \quad \mu_1 \sin i = \mu_2 \sin r$$

This is the Snell's law of refraction. (1)

8. Refer to Sol. 7

9. A wavefront is a surface through the points, having the same phase of disturbances. While a ray of light is a path along which light travels. It is always perpendicular to the wavefront and in outward direction from the source. (1)

Consider any point Q on the incident wavefront.



Suppose when disturbance from point P on incident wavefront reaches point P' on the refracted wavefront, the disturbance from point Q reaches the point Q' on the refracting surface XY. Since, $A'Q'P'$ represents the refracted wavefront, the time taken by light to travel from a point on incident wavefront to the corresponding point on refracted wavefront should always be the same. Now, time taken by light to go from Q to Q' will be

$$t = \frac{QK}{c} + \frac{kQ'}{v} \quad \text{...(i)}$$

(where, c and v are the velocities of light in two media)

In right angled ΔAQK, $\angle QAK = i$

$$\therefore \quad QK = AK \sin i \quad \text{...(ii)}$$

In right angled $\Delta P'Q'K$, $\angle Q'P'K = r$

$$KQ' = KP' \sin r \quad \text{...(iii)}$$

Substituting Eqs. (ii) and (iii) in Eq. (i), we get

$$t = \frac{AK \sin i}{c} + \frac{KP' \sin r}{v}$$

$$t = \frac{AK \sin i}{c} + \frac{(AP' - AK) \sin r}{v}$$

or
$$t = \frac{AP'}{v} \sin r + \left(\frac{\sin i}{c} - \frac{\sin r}{v}\right) AK \quad \text{...(iv)}$$

(1)

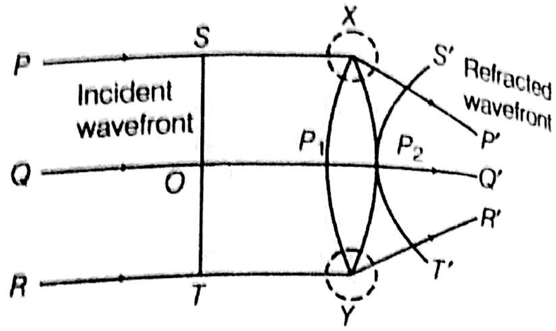
The rays from different points on the incident wavefront will take the same time to reach the corresponding points on the refracted wavefront, i.e., given by Eq. (iv) is independent of AK. It will happen so, if

$$\frac{\sin i}{c} - \frac{\sin r}{v} = 0 \Rightarrow \frac{\sin i}{\sin r} = \frac{c}{v}$$

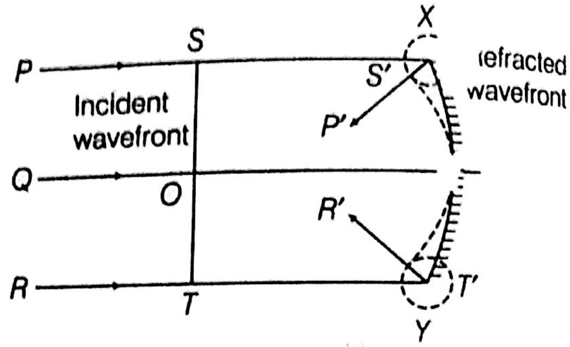
However, $\frac{c}{v} = n$

This is the Snell's law for refraction of light. **(1)**

10. (i) (a) **Behaviour of a converging lens**



(b) **Behaviour of a concave mirror**



(2)

(ii) Refer to Q. 4 (ii) **(1)**

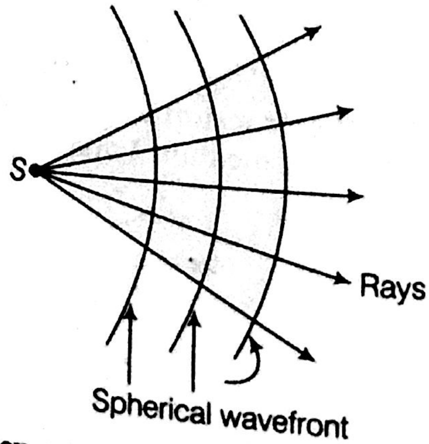
11. Laws of reflection Refer to text on page 297. **(3)**

12. Law of refraction Refer to text on pages 297 and 298. **(3)**

13. (i) When light is emitted from a source, then the particles present around it begins to vibrate. The locus of all such particles which are vibrating in the same phase is termed as wavefront. **(1)**

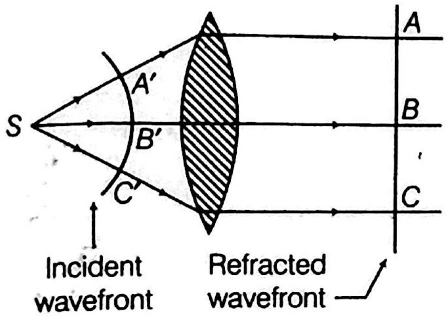
A line perpendicular to a wavefront is called a ray, it is the path along which light travels. **(1)**

(ii) (a) The wavefront will be spherical of increasing radius as shown in figure.



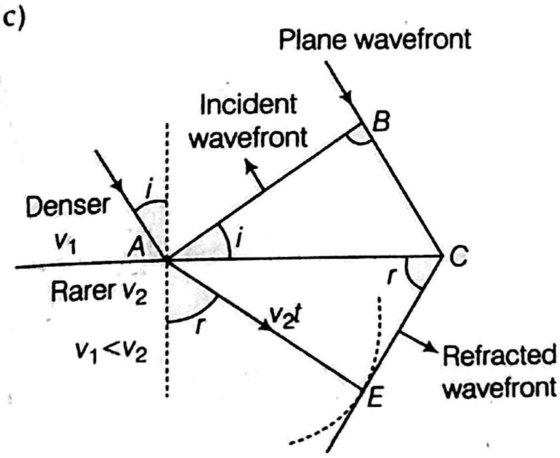
(1)

(b) When sources is at the focus, the rays coming out of the convex lens are parallel, so wavefront is plane as shown in figure.



(1)

(c)



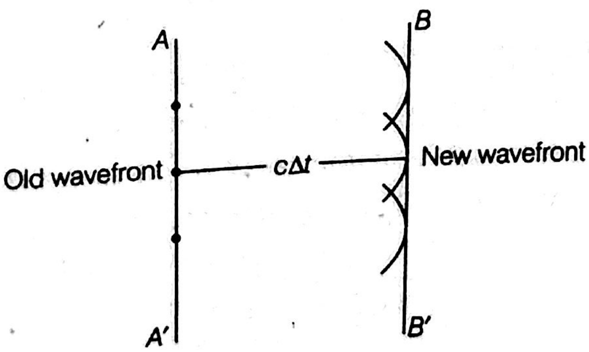
(1)

14. Huygens' Principle Each point on the primary wavefront acts as a source of secondary wavelets, sending out disturbance in all directions in a similar manner as the original source of light does. (1)

The new position of the wavefront at any instant (called secondary wavefront) is the envelope of the secondary wavelets at that instant. (1)

Law of Refraction Refer to class notes (3)

15. (i) Consider a plane wave moving through free space as shown in figure. At $t = 0$, the wavefront is indicated by the plane labelled AA'. According to Huygens' principle, each point on this wavefront is considered a point source. For clearity, only three point sources on AA' are as shown in figure below.

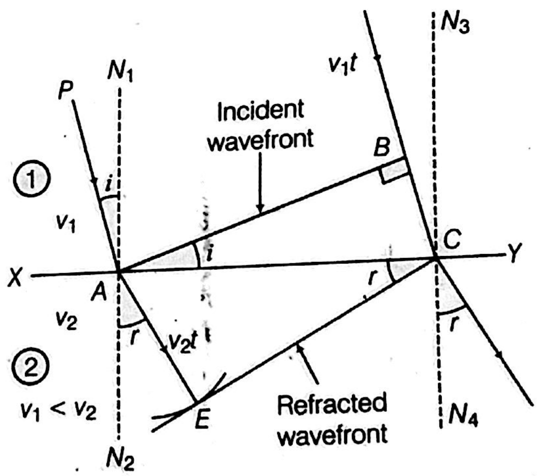


With these sources for the wavelets, we draw circular arcs, each of radius $c\Delta t$, where c is the speed of light in vacuum and Δt is sometime interval during which the wave propagates. The surface drawn tangent to these wavelets is the plane BB', which is the wavefront at a later time and is parallel to AA'. (1)

(ii) **Verification of Snell's Law** Refer to class notes. (2)

(iii) The reflection and refraction phenomenon occur due to interaction of corpuscles of incident light and the atoms of matter on receiving light energy, the atoms are forced to oscillate about their mean positions with the same frequency as incident light. According to Maxwell's classical theory, the frequency of light emitted by a charged oscillator is same as its frequency of oscillation. Thus, the frequency of reflected and refracted light is same as the incident frequency. (2)

16. (i) Let a plane wavefront AB is incident at the interface XY separating two media such that medium 1 is optically denser than medium 2. Let time t is taken by the wave to reach from B to C,



then $\quad BC = v_1 t \quad$ ...(i)

where, v_1 is the velocity of light in medium 1. In the duration of time t, the secondary wavelets emitted from point A gets spread over a hemisphere of radius, $AE = v_2 t$ ...(ii) in the medium 2 and $v_2 > v_1$.

The tangent plane CE from C over this hemisphere of radius $v_2 t$ will be the new refracted wavefront of AB.

It is evident that angle of refraction r is greater than angle of incidence i.

By geometry, $\angle N_2AE = \angle ECA = r$

(angle of refraction)

Also, $\quad \angle PAN_1 = \angle BAC = i$

(angle of incidence) (2)

(ii) Now, in ΔABC,

$\sin i = BC/AC = v_1 t/AC$ [from Eq. (i)]...(iii)

In ΔAEC, $\sin r = \frac{AE}{AC} = \frac{v_2 t}{AC}$ from Eq. (ii)] ...(iv)

Now, $$\frac{\sin i}{\sin r} = \frac{v_1 t/AC}{v_2 t/AC}$$

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \text{constant} = {}_1\mu_2$$

where, ${}_1\mu_2$ = refractive index of second medium w.r.t. first medium. **(2)**

Hence, Snell's law of refraction is verified.

(iii) No, energy carried by the wave does not depend on its speed instead, it depends on the frequency of wave. **(1)**

Explanations

1. (a) Given, $I_1 = I_0 \Rightarrow I_2 = 50\%$ of I_1 i.e., $I_2 = \frac{I_0}{2}$

Now, ratio of maximum and minimum intensity is given as, $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2$

$$= \left(\frac{\sqrt{I_0} + \sqrt{I_0/2}}{\sqrt{I_0} - \sqrt{I_0/2}}\right)^2 = \frac{\left(\sqrt{I_0} + \sqrt{I_0/2}\right)^2}{\left(\sqrt{I_0} - \sqrt{I_0/2}\right)^2}$$

$$= \frac{\left(1 + \frac{1}{\sqrt{2}}\right)^2}{\left(1 - \frac{1}{\sqrt{2}}\right)^2} = \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2 = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \left(\frac{3+2\sqrt{2}}{3-2\sqrt{2}}\right) \times \left(\frac{3+2\sqrt{2}}{3+2\sqrt{2}}\right)$$

$$= \frac{(3+2\sqrt{2})^2}{(3)^2 - (2\sqrt{2})^2} = 17 + 12\sqrt{2}$$

(1½)

(b) When a white light source is used, the interference patterns due to different component colours of white light overlap incoherently. The central bright fringe for different colours is at centre. So, central bright fringe is white. As $\lambda_{blue} < \lambda_{red}$, fringe closest on either side of central bright fringe is blue and the farthest is red. After few fringes, no clear pattern of fringes will be visible.

(1½)

2. (i) Angular width, $\theta = \frac{\lambda}{d}$ or $d = \frac{\lambda}{\theta}$

Here, $\lambda = 600 nm = 6 \times 10^{-7}$ m

$$\theta = \frac{0.1\pi}{180} \text{rad} = \frac{\pi}{1800} \text{rad}, d = ?$$

$$\therefore \quad d = \frac{6 \times 10^{-7} \times 1800}{\pi} = 3.44 \times 10^{-4} \text{ m}$$

(1½)

(ii) Frequency of a light depends on its source only. So, the frequencies of reflected and refracted light will be same as that of incident light.

Reflected light is in the same medium (air) so its wavelength remains same as 500 Å.

Wavelength of refracted light, $\lambda_r = \lambda/\mu_w$

μ_w = refractive index of water.

So, wavelength of refracted wave will be decreased. **(1½)**

3. (i) Two independent monochromatic sources cannot produce sustained interference pattern because the phase difference between the light waves from two independent sources keeps on changing continuously. **(1½)**

(ii) Refer to class notes **(1½)**

4. The intensity of light due to slit is directly proportional to width of slit.

$$\therefore \quad \frac{I_1}{I_2} = \frac{W_1}{W_2} = \frac{4}{1}$$

$$\Rightarrow \quad \frac{a_1^2}{a_2^2} = \frac{4}{1} \Rightarrow \frac{a_1}{a_2} = \frac{2}{1} \Rightarrow a_1 = 2a_2$$

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

$$= \frac{(2a_2 + a_2)^2}{(2a_2 - a_2)^2} = \frac{9a_2^2}{a_2^2} = 9:1$$

(3)

5. (i) Given, $y_1 = a\cos\omega t$

$y_2 = a\cos(\omega t + \phi)$

The resultant displacement is given by

$y = y_1 + y_2$

$= a\cos\omega t + a\cos(\omega t + \phi)$

$= a\cos\omega t + a\cos\omega t\cos\phi - a\sin\omega t\sin\phi$

$= a\cos\omega t\,(1 + \cos\phi) - a\sin\omega t\sin\phi$

Put, $R\cos\theta = a(1 + \cos\phi)$ ...(i)

$R\sin\theta = a\sin\phi$ ...(ii)

By squaring and adding Eqs. (i) and (ii), we get

$R^2 = a^2(1 + \cos^2\phi + 2\cos\phi) + a^2\sin^2\phi$

$= 2a^2(1 + \cos\phi) = 4a^2\cos^2\frac{\phi}{2}$

$$\therefore \quad I = R^2 = 4a^2\cos^2\frac{\phi}{2} = 4I_0\cos^2\frac{\phi}{2}$$ **(1½)**

(ii) For constructive interference, $\cos\frac{\phi}{2} = \pm 1$

$$\Rightarrow \quad \frac{\phi}{2} = n\pi$$

$$\Rightarrow \quad \phi = 2n\pi$$

For destructive interference,

$$\cos\frac{\phi}{2} = 0 \Rightarrow \frac{\phi}{2} = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \quad \phi = (2n+1)\pi$$ **(1½)**

6. For a single slit of width a the first minima of the interference pattern of a monochromatic light of wavelength λ occurs at an angle of (λ / a) because the light from centre of the slit differs by a half of wavelength.

Whereas a double slit experiment at the same angle of (λ / a) and slits separation a produces maxima because one wavelength difference in path length from these two slits is produced. **(1)**

7. Two monochromatic sources, which produce light waves, having a constant phase difference are known as coherent sources. **(1)**

8. (i) The essential condition, which must be satisfied for the sources to be coherent are:

(a) the two light waves should be of same wavelength.

(b) the two light waves should either be in phase or should have a constant phase difference. **(1½)**

(ii) Because coherent sources emit light waves of same frequency or wavelength and a stable phase difference. **(½)**

9. Fringe width, $\beta = \frac{D\lambda}{d}$

For given λ and d, $\beta \propto D$

Fringe width becomes double to that of original one. **(1)**

10. (i) The fringe width of interference pattern increases with the decrease in separation between $S_1 S_2$ as $\beta \propto \frac{1}{d}$ **(1/2)**

(ii) The fringe width decrease as wavelength gets reduced when interference set up is taken from air to water. **(1/2)**

11. Given, $\beta_1 = 7.2 \times 10^{-3}$ m,

$\beta_2 = 8.1 \times 10^{-3}$ m

and $\lambda_1 = 630 \times 10^{-9}$ m

$\because$ Fringe width, $\beta = \dfrac{D\lambda}{d}$

where, λ = wavelength, D = separation between slits and screen and d = separation between two slits.

$\Rightarrow \quad \beta_1/\beta_2 = \lambda_1/\lambda_2$ ($\because D$ and d are same) **(1/2)**

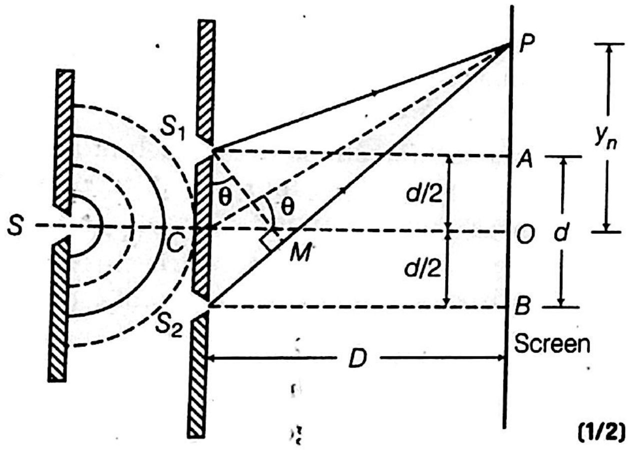
Wavelength of another source of laser light

$$\Rightarrow \quad \lambda_2 = \frac{\beta_2}{\beta_1} \times \lambda_1 = \frac{8.1 \times 10^{-3}}{7.2 \times 10^{-3}} \times 630 \times 10^{-9} \text{ m}$$

or $\quad \lambda_2 = 708.75 \times 10^{-9}$ m

$\therefore \quad \lambda_2 = 708.75$ nm **(1/2)**

12. Let two coherent sources of light, S_1 and S_2 (narrow slits) are derived from a source S. The two slits, S_1 and S_2 are equidistant from source, S. Now, suppose S_1 and S_2 are separated by distance d. The slits and screen are distance D apart.



(1/2)

Considering any arbitrary point P on the screen at a distance y_n from the centre O. The path difference between interfering waves is given by $S_2P - S_1P$

i.e. Path difference $= S_2P - S_1P = S_2M$

$$S_2P - S_1P = d \sin\theta$$

where, $\quad S_1M \perp S_2P$ **(1)**

[$\because \angle S_2S_1M = \angle OCP$ (by geometry)

$\Rightarrow \quad S_1P = PM \Rightarrow S_2P = S_2M$]

If θ is small, then $\sin\theta \approx \theta \approx \tan\theta$

$\therefore$ Path difference,

$$S_2P - S_1P = S_2M = d\sin\theta \approx d\tan\theta$$

$$\text{Path difference} = d\left(\frac{y_n}{D}\right) \quad \ldots\text{(i)}$$

[$\because$ In ΔPCO, $\tan\theta = \dfrac{OP}{CO} = \dfrac{y_n}{D}$]

For constructive interference

Path difference $= n\lambda$, where, $n = 0, 1, 2, \ldots$ [from Eq. (i)]

$$\Rightarrow \quad y_n = \frac{Dn\lambda}{d}$$

$$\Rightarrow \quad y_{n+1} = \frac{D(n+1)\lambda}{d}$$

$\because$ Fringe width of dark fringe $= y_{n+1} - y_n$

[$\because$ dark fringe exist between two bright fringes]

$$\beta = \frac{D\lambda}{d}(n+1) - \frac{Dn\lambda}{d}$$

$$= \frac{D\lambda}{d}(n + 1 - n) = \frac{D\lambda}{d}$$

Fringe width of dark fringe, $\beta = \dfrac{D\lambda}{d}$ ...(ii)

For destructive interference

Path difference $= (2n-1)\dfrac{\lambda}{2}$, where $n = 1, 2, 3, \ldots$

$$\Rightarrow \quad \frac{y'_n d}{D} = (2n-1)\frac{\lambda}{2} \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad y'_n = \frac{(2n-1)D\lambda}{2d}$$

where, y'_n is the separation of nth order dark fringe from central fringe.

$$\therefore \quad y'_{n+1} = (2n+1)\frac{D\lambda}{2d}$$ **(1)**

$\therefore$ Fringe width of bright fringe = Separation between $(n+1)$th and nth order dark fringe from centred fringe,

$$\Rightarrow \quad \beta = y'_{n+1} - y'_n$$

or $\quad \beta = \dfrac{(2n+1)D\lambda}{2d} - \dfrac{(2n-1)D\lambda}{2d}$

$$= \frac{D\lambda}{2d}[2n + 1 - 2n + 1] = \frac{D\lambda}{2d}$$

Fringe width of bright fringe, $\beta = \dfrac{D\lambda}{d}$ ...(iii)

From Eqs. (ii) and (iii), we can see that, fringe width of dark fringe

= fringe width bright fringe $\beta = \dfrac{D\lambda}{d}$ **(1/2)**

13.

> Intensity can be found out if, we know the phase difference. Phase difference can be calculated with the help of path difference. So first of all, path difference will be calculated.

Given, $OP = y_n$

The distance OP equals one-third of fringe width of the pattern

i.e. $\quad y_n = \dfrac{\beta}{3} = \dfrac{1}{3}\left(\dfrac{D\lambda}{d}\right) = \dfrac{D\lambda}{3d}$

$$\Rightarrow \quad \frac{dy_n}{D} = \frac{\lambda}{3}$$

Path difference, **(1)**

$$S_2P - S_1P = \frac{dy_n}{D} = \frac{\lambda}{3}$$

Now for phase difference corresponding to path difference.

Phase difference $= \frac{2\pi}{\lambda} \times$ Path difference $= \frac{2\pi}{\lambda} \times \frac{\lambda}{3}$

$\therefore$ Phase difference $= 2\pi/3$ **(1)**

If intensity at central fringe is I_0, then intensity at a point P, where phase difference is ϕ, is given by $I = I_0 \cos^2 \phi$

$$\Rightarrow \quad I = I_0 \left(\cos \frac{2\pi}{3}\right)^2$$

$$= I_0 \left(-\cos \frac{\pi}{3}\right)^2$$

$$= I_0 \left(-\frac{1}{2}\right)^2 = \frac{I_0}{4}$$

Hence, the intensity at point P would be $I_0/4$. **(1)**

14. Distance between the two sources

$$d = 0.15 \text{ mm} = 1.5 \times 10^{-4} \text{ m}$$

Wavelength, $\lambda = 450 \text{ nm} = 4.5 \times 10^{-7}$ m

Distance of screen from source, $D = 1$ m

(i) (a) The distance of nth order bright fringe from central fringe is given by

$$y_n = Dn\lambda/d$$

For second bright fringe, $y_2 = \frac{2D\lambda}{d}$

$$= \frac{2 \times 1 \times 4.5 \times 10^{-7}}{1.5 \times 10^{-4}}$$

$$y_2 = 6 \times 10^{-3} \text{ m}$$

The distance of the second bright fringe **(1)**

$$y_2 = 6 \text{ mm}$$

(b) The distance of nth order dark fringe from central fringe is given by $y'_n = (2n-1)\frac{D\lambda}{2d}$

For second dark fringe, $n = 2$

$$y'_n = (2 \times 2 - 1)\frac{D\lambda}{2d} = \frac{3D\lambda}{2d}$$

$$y'_n = \frac{3}{2} \times \frac{1 \times 4.5 \times 10^{-7}}{1.5 \times 10^{-4}}$$

The distance of the second dark fringe,

$$y'_n = 4.5 \text{ mm}$$ **(1)**

(ii) With increase of D, fringe width increases as

$$\beta = \frac{D\lambda}{d} \quad \text{or} \quad \beta \propto D$$ **(1)**

15.

> To find the point of coincidence of bright fringes, we can equate the distance of bright fringes from the central maxima, made by both the wavelengths of light.

Given, $D = 1$ m, $d = 4 \times 10^{-3}$m, $\lambda_1 = 560$ nm, and $\lambda_2 = 420$ nm

Let nth order bright fringe of λ_1 coincides with $(n+1)$th order bright fringe of λ_2.

$$\therefore \quad \frac{Dn\lambda_1}{d} = \frac{D(n+1)\lambda_2}{d} \qquad (\lambda_1 > \lambda_2)$$

$$\Rightarrow \quad n\lambda_1 = (n+1)\lambda_2$$ **(1)**

$$\Rightarrow \quad \frac{n+1}{n} = \frac{\lambda_1}{\lambda_2}$$

$$1 + \frac{1}{n} = \frac{560 \times 10^{-9}}{420 \times 10^{-9}}$$

$$\Rightarrow \quad 1 + \frac{1}{n} = \frac{4}{3} \Rightarrow \frac{1}{n} = \frac{1}{3} \Rightarrow n = 3$$ **(1)**

$\therefore$ Least distance from the central fringe where bright fringe of two wavelength coincides

= Distance of 3rd order bright fringe of λ_1

$$\Rightarrow \quad y_n = \frac{3D\lambda_1}{d} = \frac{3 \times 1 \times 560 \times 10^{-9}}{4 \times 10^{-3}}$$

$$y_n = 420 \times 10^{-6} \text{ m}$$

$$= 0.42 \times 10^{-3} \text{ m}$$

$$\therefore \quad y_n = 0.42 \text{ mm}$$ **(1)**

Thus, 3rd bright fringe of λ_1 and 4th bright fringe of λ_2 coincide at 0.42 mm from central fringe. **(1)**

16. (i) Refer to Sol. 5

(ii) For fringes to be seen, $s/S \leq \lambda/d$

Condition should be satisfied.

where, S = size of the source and

d = distance of the source from the plane of two slits. As, the source slit width increase, the fringe pattern get less and less sharp. **(1)**

When the source slit is so wide, then above condition is not satisfied and the interference pattern disappears. **(1)**

The interference pattern due to different colour components of white light overlap. The central bright fringes for different colours are at the same position. Therefore, the centre fringes are white. And on the either side of the central fringe i.e. central maxima, coloured bands will appear. **(1)**

The fringe closed on either side of central white fringe is red and the farthest will be blue. After a few fringes, no clear fringes pattern is seen. **(1)**

17. (i) Refer to Sol. 12 **(3)**

(ii) Given, $\frac{I_{min}}{I_{max}} = \frac{9}{25}$

But $\left[\frac{\sqrt{I_1} - \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}\right]^2 = \frac{9}{25}$

$\Rightarrow \quad \frac{\sqrt{I_1} - \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}} = \frac{3}{5}$

$\Rightarrow \quad 5\sqrt{I_1} - 5\sqrt{I_2} = 3\sqrt{I_1} + 3\sqrt{I_2}$

$2\sqrt{I_1} = 8\sqrt{I_2}$

$\Rightarrow \quad \sqrt{I_1/I_2} = 4$

Ratio of intensities $I_1/I_2 = 16/1$

Ratio of widths of the slits $d_1/d_2 = I_1/I_2 = 16/1$ **(2)**

18. (i) (a) Two independent monochromatic sources of light cannot produce a sustained interference pattern because their relative phases are changing randomly. When d is negligibly small fringe width β is proportional to $1/d$ may become too large. Even a single fringe may occupy the screen.

Hence, the pattern cannot be detected.

(b) Refer to Sol. 5 (i) **(2)**

(ii) Intensity, $I = 4I_0 \cos^2\frac{\phi}{2}$ ...(i)

where, I_0 is incident intensity and I is resultant intensity.

At a point where path difference is λ

Phase difference, $\phi = 2\pi/\lambda \times \lambda = 2\pi$

Substituting the value of ϕ in Eq.(i), we get

$I = 4I_0 \cos^2\frac{2\pi}{2} = 4I_0 \cos^2 \pi = 4I_0 = K$

At a point, where path difference is $\lambda/3$,

Phase difference, $\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{3} = \frac{2\pi}{3}$

$I_2 = 4I_0 \cos^2\frac{\phi}{2} = 4\left(\frac{K}{4}\right)\cos^2\frac{\pi}{3}$

$= 4\frac{K}{4} \times \frac{1}{4} = \frac{K}{4}$ **(3)**

19. (i) Refer to Ans. 12 **(3)**

(ii) Given, $\lambda_1 = 800$ nm, $\lambda_2 = 600$ nm

$D = 1.4$ m and $d = 0.28$ mm $= 2.8 \times 10^{-4}$ m

Let nth order bright fringe of $\lambda = 800$ nm coincide with $(n+1)$th order 600 nm wavelength.

$\therefore \quad \frac{Dn\lambda_1}{d} = \frac{D(n+1)\lambda_2}{d}$ **(1/2)**

$\Rightarrow \quad n\lambda_1 = (n+1)\lambda_2$ **(1/2)**

$n \times 800 \times 10^{-9} = (n+1) \times 600 \times 10^{-9}$

$\frac{n+1}{n} = \frac{4}{3} \Rightarrow \frac{1}{n} = \frac{4}{3} - 1 = \frac{1}{3}$

$n = 3$ **(1/2)**

$\therefore$ Least distance from central fringe, $y_n = \frac{Dn\lambda_1}{d}$

$y_n = \frac{1.4 \times 3 \times 800 \times 10^{-9}}{2.8 \times 10^{-4}} = 12 \times 10^{-3}$ m

$y_n = 12$ mm **(1/2)**

20. (i) (a) From the fringe width expression,

$\beta = \frac{\lambda D}{d}$

With the decrease in separation between two slits, the fringe width β increases. **(1)**

(b) For interference fringes to be seen, $\frac{s}{S} < \frac{\lambda}{d}$

Condition should be satisfied

where, s = size of the source,

S = distance of the source from the plane of two slits. As, the source slit width increases, fringe pattern gets less and less sharp. When the source slit is so wide, the above condition does not satisfied and the interference pattern disappears. **(1)**

(c) Refer to 16 (ii) 2nd paragraph on page 311.

(ii) Intensity at a point is given by

$I = 4I' \cos^2 \phi/2$

where, ϕ = phase difference,

I' = intensity produced by each one of the individual sources.

At central maxima, $\phi = 0$, the intensity at the central maxima, $I = I_0 = 4I'$

or $I' = I_0/4$ ...(i)

As, path difference $= \lambda/3$

Phase difference,

$\phi' = \frac{2\pi}{\lambda} \times$ path difference $= \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$

Now, intensity at this point

$I'' = 4I' \cos^2\frac{1}{2}\left(\frac{2\pi}{3}\right) = 4I' \cos^2\frac{\pi}{3}$

$= 4I' \times \frac{1}{4} = I'$

or $I'' = \frac{I_0}{4}$ [from Eq. (i)]

Hence proved. **(2)**

21. **Coherent source** Refer to Sol. 7 **(1)**

Condition for constructive and destructive interference Refer to Sol. 5 (ii) **(1)**

Fringe width Refer to Sol. 12

Other part of this question Refer to Sol. 10 (ii) **(3)**

22. (i) **For coherent sources** Refer to Sol. 7 **(1)**

For necessary condition of interference Refer to Sol. 5 (ii)

The two coherent sources are derived from the single source by division of wavefront. Two slits, namely S_1 and S_2 are kept at equidistant from source slit S, then S_1 and S_2 are lying on the same wavefront emitting secondary wavelets of constant phase difference. **(1)**

(ii) Given, the displacement of two coherent sources, Refer to Ans. 5 (i)

Condition for bright fringe or constructive interference

$\cos(\phi/2) = \pm 1$, then, $I = I_0 = 4a^2$

$$\Rightarrow \quad \phi/2 = n\pi$$

$$\phi = 2n\pi$$

Also path difference $= \dfrac{\lambda}{2\pi} \times$ Phase difference

or $$x = \frac{\lambda}{2\pi} \times 2n\pi$$

Path difference, $x = n\lambda$

Bright fringe obtained when path difference of interfering wave is $n\lambda$ and phase difference is $2n\pi$. **(1)**

Condition for dark fringe or destructive interference

$$I = 0 \Rightarrow \cos\frac{\phi}{2} = 0$$

or $$\cos\frac{\phi}{2} = 0 = \cos(2n+1)\frac{\pi}{2}$$

$$\Rightarrow \quad \frac{\phi}{2} = (2n+1)\frac{\pi}{2}, \text{ where, } n = 0, 1, 2\ldots$$

$$\Rightarrow \quad \phi = (2n+1)\pi, \text{ where, } n = 0, 1, 2, \ldots$$

Path difference,

$$x = \frac{\lambda}{2\pi} \times \phi = \frac{\lambda}{2\pi} \times (2n+1)\pi \Rightarrow x = (2n+1)\frac{\lambda}{2}$$

Path difference, $x = (2n+1)\dfrac{\lambda}{2}$

Dark fringes obtained when interfering wave have path difference is odd multiple of $\dfrac{\lambda}{2}$ and phase difference is odd multiple of π. **(2)**

Explanations

1. Refractive index $= \tan\theta = \frac{c}{v}$

$$\Rightarrow \quad v = \frac{c}{\tan\theta} = \frac{3\times10^8}{\tan 30^\circ}$$

$$= 3\sqrt{3}\times10^8\ \text{ms}^{-1}$$ **(1)**

2. According to Brewster's law, $\mu = \tan i_B$

Given, $i_B = 60^\circ$, then $\mu = \tan 60^\circ = \sqrt{3}$ or 1.732 **(1)**

3. Refer to Sol. 4 on page 322. **(1)**

4. Polarised light If the vibrations of a wave are present in one direction in a plane perpendicular to the direction of propagation, the waves is said to polarised. **(1/2)**

Unpolarised light A transverse wave in which vibrations are present in all direction in a plane perpendicular to direction of propagation is said to be unpolarised. **(1/2)**

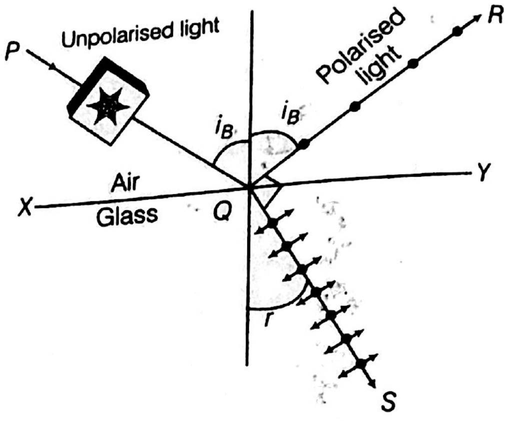
5. Heat waves can be polarised because heat waves are transverse waves whereas sound waves cannot be polarised because sound waves are longitudinal waves, oscillates only along the direction of its propagation. **(1)**

6. As, angular width, $\theta = \frac{\lambda}{d}$. Here, θ is independent of separation between slit and screen. Angular separation continues to be the same. **(1)**

7. When a wavefront strikes to the corner of an obstacle,light wave bends around the corner because every point on wavefront again behaves like a light source and emit secondary wavelets in all directions (Huygens' wave theory) including the region of geometrical shadow.This explains diffraction. **(1)**

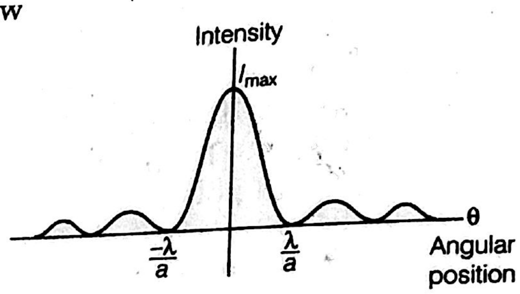
8. Angular width of central maxima is given by $2\theta = \frac{2\lambda}{d}$ Since $\lambda'_r > \lambda_b$, $d =$ slit width. Therefore, width of central maxima of red light is greater than the width of central maxima of blue light. **(1)**

9. When an unpolarised light is incident on the boundary between two transparent media, the reflected light is polarised with its electric field vector perpendicular to the plane of incidence when the reflected and refracted rays make a right angle with each other. **(1)**

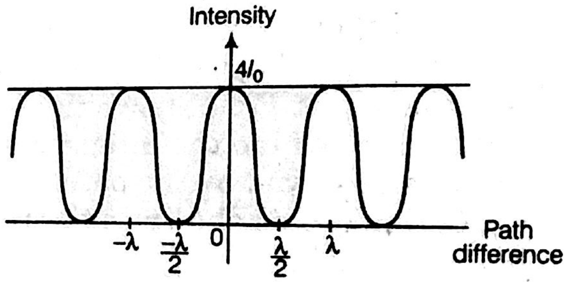


(1)

10. Intensity pattern for single slit diffraction is shown below



Intensity pattern for double slit interference pattern is shown.



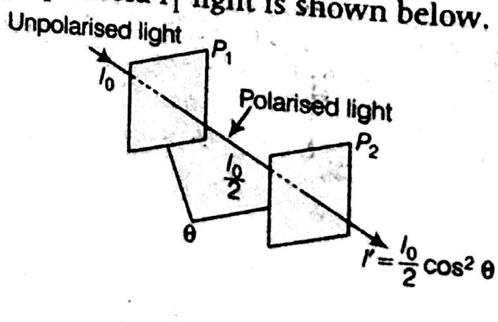
(1)

Difference between diffraction and interference patterns are

(i) In interference pattern, all maxima and all minima are of same width but in diffraction pattern, width of central maxima is maximum and for successive maxima, it goes on decreasing.

(ii) In interference pattern, each maxima have same intensity while in diffraction pattern, intensity of central maxima is largest and it decreases rapidly for successive maxima. **(1)**

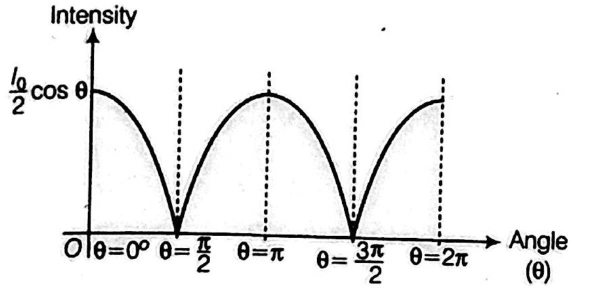
11. The figure when unpolarised light beam is passed through polaroid P_1 light is shown below.



By law of Malus,

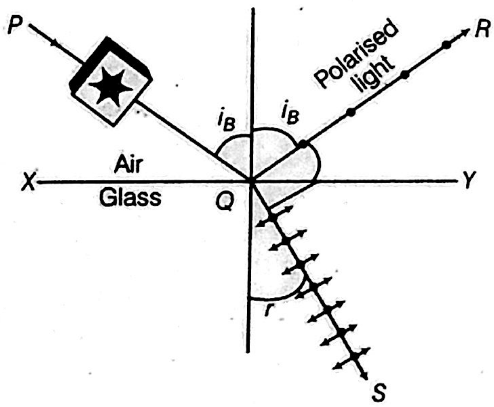
intensity received after $P_2 = I' = \frac{I_0}{2}\cos^2\theta$. (1)

Variation of intensity with angle θ is shown below.



(1)

12. According to Brewster's law, when a light ray falls on a surface of transparent medium in such a way that reflected ray is perpendicular to the refracted ray, the reflected ray is a totally polarised ray.
The angle of incidence in this case is called Brewster's angle i_B.



Polarisation by reflection (1)

The tangent of the polarising angle of incidence of a transparent medium is equal to its refractive index, i.e. $\mu = \tan(i_B)$

$\therefore$ Brewster's angle $i_B = \tan^{-1}\mu$

As we have seen, the value of i_B depends on the refractive index of the medium and the refractive index of a medium depends on the wavelength of incident light. Thus, the value of i_B will be different for different colours of light (i.e. wavelength of light). (1)

13. (i) Refer to Sol. 4 and 9 on page 322. (1)

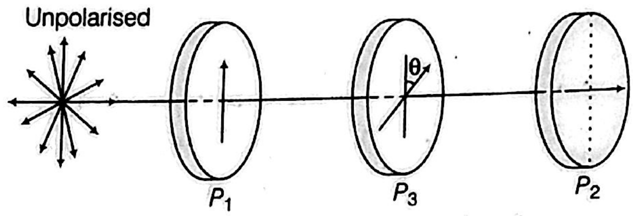
(ii) Cosine of the angle between direction of polarisation of light and axis of the polaroid.

The intensity of light transmitted can be given using Malus law as

$$I = I_0\cos^2\theta = I_0\cos^2 60° = \frac{I_0}{4}$$

Precentage of light is transmitted through the polaroid sheet is $= \dfrac{I_0 - \frac{I_0}{4}}{I_0} \times 100 = 75\%$ (1)

14. Let us consider two crossed polarisers, P_1 and P_2 with a polaroid sheet P_3 placed between them.



Let I_0 be the intensity of polarised light after passing through the first polariser P_1. If θ is the angle between the axes of P_1 and P_3, then the intensity of the polarised light after passing through P_3 will be $I = I_0\cos^2\theta$. As, P_1 and P_2 are crossed, the angle between the axes of P_1 and P_2 is 90°.

$\therefore$ Angle between the axes of P_2 and P_3 is $(90° - \theta)$. (1)

The intensity of light emerging from P_2 will be given by

$$I = [I_0\cos^2\theta]\cos^2(90° - \theta)$$

$$I = [I_0\cos^2\theta]\sin^2\theta \Rightarrow I = \frac{I_0}{4}(4\cos^2\theta\sin^2\theta)$$

$$\Rightarrow I = \frac{I_0}{4}(2\sin\theta\cos\theta)^2 \Rightarrow I = \frac{I_0}{4}\sin^2(2\theta)$$

The intensity of polarised light transmitted from P_2 will be maximum, when

$$\sin 2\theta = \text{maximum} = 1$$

$$\Rightarrow \sin 2\theta = \sin 90° \Rightarrow 2\theta = 90° \Rightarrow \theta = 45°$$

Also, the maximum transmitted intensity will be given by $I = \frac{I_0}{4}$. (1)

15. In diffraction, angular position $\theta = \dfrac{\Delta x}{a}$

For first minima, $\Delta x = \lambda$

$$\therefore \quad \theta = \frac{\lambda}{d}$$

In interference, $d = a$ (given) and angular position, $\theta = \dfrac{\Delta x}{d}$

$\therefore$ Angular position of first maxima

$$(\Delta x = \lambda),\ \theta = \frac{\lambda}{d}$$

(2)

16. The distance of the nth minimum from the centre of the screen is

$$x_n = \frac{nD\lambda}{d} \qquad \ldots(\text{i})$$

where, D = distance of slit from screen
λ = wavelength of the light
d = width of the slit for first minimum, $n = 1$

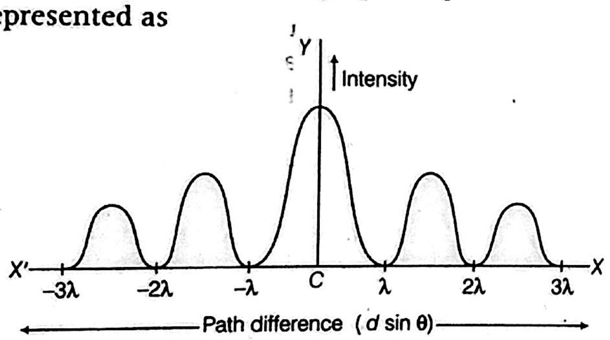
$$x_n = 2.5 \text{ mm}$$
$$= 2.5 \times 10^{-3} \text{m}, D = 1 \text{ m}$$
$$\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{m}$$

Putting this values in Eq. (i), we get

$$2.5 \times 10^{-3} = \frac{1(1)(500 \times 10^{-9})}{d}$$

$\Rightarrow$ $$d = 2 \times 10^{-4} \text{ m} = 0.2 \text{ mm}$$ **(2)**

17. In case of single slit The diffraction pattern obtained on the screen consists of a central bright band having alternate dark and weak bright band of decreasing intensity on both sides. The diffraction pattern can be graphically represented as



Points to compare the intensity distribution between interference and diffraction.

(a) In interference, all bright fringes have same intensity, but in diffraction all the bright fringes are not of same intensity.

(b) In interference, the widths of all the fringes are same but in diffraction fringes are of different widths.

The point C corresponds to the position of central maxima and the position $-3\lambda, -2\lambda, -\lambda, \lambda, 2\lambda, 3\lambda \ldots$ are secondary minima. The above conditions for diffraction maxima and minima are exactly reverse of mathematical conditions for interference maxima and minima. **(2)**

18. Given, $\lambda = 6000 \text{ Å} = 6 \times 10^{-7}$ m and $d = 1 \times 10^{-4}$ m
Separation between slit and screen, $D = 1.5$ m.

The separation between two dark lines on either side of the central maxima = fringe width of central maxima $= \frac{2D\lambda}{d}$

$$= \frac{2 \times 1.5 \times 6 \times 10^{-7}}{1 \times 10^{-4}} = 18 \times 10^{-3} \text{ m} = 18 \text{ mm}$$ **(1)**

$$\text{Angular Spread} = \lambda / d = \frac{6000 \times 10^{-10}}{1 \times 10^{-4}}$$
$$= 6 \times 10^{-3} \text{m} = 6 \text{ mm}$$ **(1)**

19. Resolving power $= D/1.22\lambda$
where, A is the aperture of the objective lens of the telescope.

$$\therefore \quad \frac{(\text{RP})_1}{(\text{RP})_2} = \frac{A_1}{A_2}$$ **(1)**

The telescope with objective of aperture A_1 should be prefered due to following reasons

(i) It gives a better resolution.
(ii) It has a highlight gathering power. **(1)**

20. (a) Refer to Sol. 11 (i) on pages 322 and 323. **(1½)**
Refer to Sol. 11. (ii) on pages 322 and 323.
(b) Refer to Sol. 11 on pages 322 and 323. **(1½)**

21. (i) According to law of Malus, when a beam of completely plane polarised light is incident on an analyser, the resultant intensity of light I transmitted from the analyser varies directly as the square of the cosine of the angle θ between the plane of transmission of analyser and polariser.

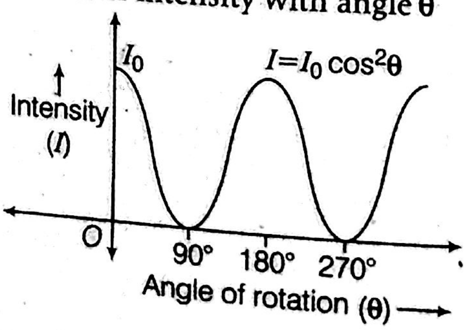
i.e. $$I \propto \cos^2\theta$$

This rule is also called **cosine square rule.**

$$\therefore \quad I = I_0 \cos^2\theta$$ **(1)**

where, I_0 is the intensity of the polarised light after passing through P_1.

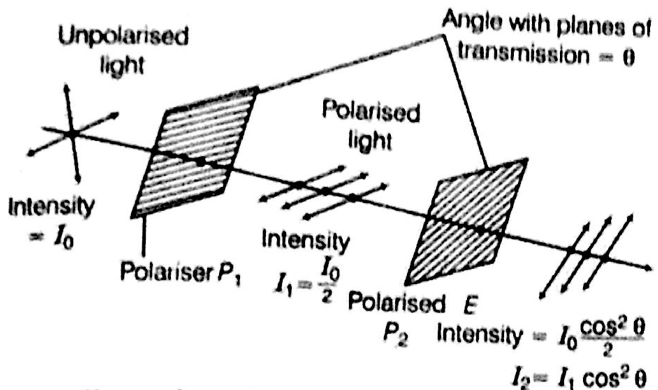
(ii) Variation of intensity with angle θ



(1)

(iii) As we know, $\mu = \tan i_p$
where, i_p = polarising angle,
μ = refractive index
$\mu = \tan 60° = \sqrt{3}$ **(1)**

22. (i)



According to law of Malus, when a beam of completely plane polarised light is incident on an analyser, the resultant intensity of light (I) transmitted from the analyser varies directly as the square of the cosine of angle θ between the plane of transmission of analyser and polariser.

i.e. $\quad I \propto \cos^2\theta \Rightarrow I = I_0 \cos^2\theta$

When polariser and analyser are parallel, $\theta = 0°$ or $180°$.

So that, $\quad \cos\theta = \pm 1 \Rightarrow I = I_0$

When polariser and analyser are perpendicular to each other, then $\theta = 90°$

$\Rightarrow \quad \cos\theta = \cos 90° = 0 \Rightarrow I = 0$ **(1)**

In unpolarised light, vibrations are probable in all the direction in a plane perpendicular to the direction of propagation.

Therefore, θ can have any value from 0 and 2π.

$$\therefore \quad [\cos^2\theta]_{av} = \frac{1}{2\pi}\int_0^{2\pi} \cos^2\theta \, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{(1+\cos 2\theta)d\theta}{2}$$

$$= \frac{1}{2\pi \times 2}\left[0 + \frac{\sin 2\theta}{2}\right]_0^{2\pi} = \frac{1}{2}$$

Using law of Malus, $I = I_0 \cos^2\theta$

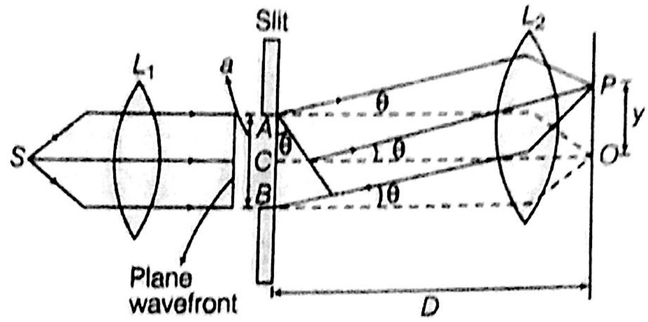
$$\Rightarrow \quad I = I_0 \times \frac{1}{2} = \frac{1}{2} I_0$$

As per the question, I_0 is the intensity of incident unpolarised light and I_1 and I_2 are the intensities of polaroids P_1 and P_2 respectively, then we can say that when unpolarised light of intensity I_0 get polarised on passing through a polaroid P_1 its intensity become half, i.e. $I_1 = \frac{I_0}{2}$ **(1)**

(ii) When this polarised light of intensity I_1 passes through polaroid P_2, then its intensity will be given by $\quad I_2 = I_1 \cos^2\theta$

This is a required relation between intensities I_1 and I_2. **(1)**

23. (i)

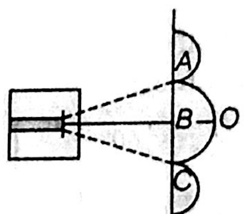


Consider a parallel beam of light from a lens falling on a slit AB. As, diffraction occurs, the pattern is focused on the screen with the help of lens L_2. We will obtain a diffraction pattern that is central maximum at the centre O, flanked by a number of dark and bright fringes called secondary maxima and minima.

Each point on the plane wavefront AB sends out the secondary wavelets in all directions.

The waves from points equidistant from the centre C, lying on the upper and lower half, reach point O with zero path difference and hence, reinforce each other producing maximum intensity at O. **(1)**

(ii) Let λ and d be the wavelength and slit width of diffracting system, respectively. Let O be the position of central maximum.



(1)

Condition for the first minimum is given by

$$d\sin\theta = m\lambda \quad \text{...(i)}$$

Let θ be the angle of diffraction.

As, diffraction angle is small

$\therefore \quad \sin\theta \approx \theta$

For first diffraction minimum,

$$\theta = \theta_1 \quad \text{(let)}$$

For the first minimum, take $m = 1$

$$d\theta_1 = \lambda \Rightarrow \theta_1 = \lambda/d$$

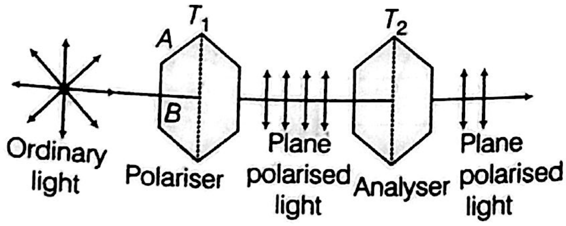
Now, angular width, $AB = \theta_1$

Angular width, $\quad BC = \theta_1$

Angular width, $\quad AC = 2\theta_1$ **(1)**

(iii) On increasing the value of n, the part of slit contributing to the maximum decreases. Hence, the maximum becomes weaker.

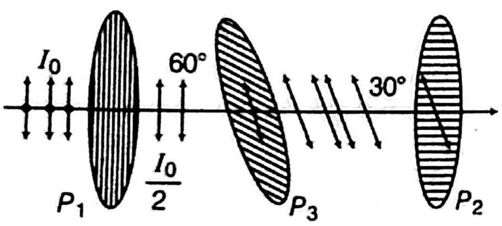
24. (i) The following set up can be arranged to show the transverse nature of light waves



We observe that intensity and character of light transmitted by T_1 and T_2 remain unaffected only when T_1 and T_2 are set with their axes parallel. If both T_1 and T_2 are rotated with same angular velocity in the same direction, no change in intensity of transmitted light is observed.

The phenomenon can be explained only when we assume that light waves are transverse. **(1)**

(ii)



Intensity through $P_1 = \dfrac{I_0}{2}$

($\because$ After polarisation intensity of light becomes half) **(1)**

Intensity through

$$P_3 = \frac{I_0}{2}\cos^2 60° = \frac{I_0}{2}\cdot\left(\frac{1}{2}\right)^2 = \frac{I_0}{8}$$

Intensity through

$$P_2 = \frac{I_0}{8}\cos^2 30° = \frac{I_0}{8}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3I_0}{32}$$ **(1)**

25. (i) In diffraction pattern, intensity will be minimum at an angle $\theta = n\lambda/d$

where, d = width of slit and $n = 1, 2, 3, \ldots$

$\therefore$ There will be a first minimum at an angle

$$\theta = \lambda/d$$

On either side of central maximum.

$\therefore$ Width of central maxima $= 2\lambda/d$

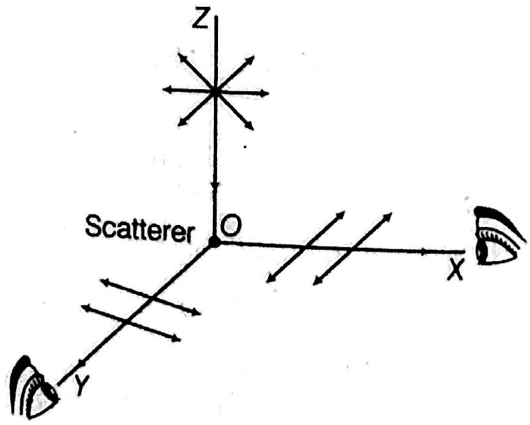
whereas the width of other minimum/maximum $\approx \lambda/d$ **(1½)**

(ii) The intensity of maxima decreases as the order (n) or diffraction maxima increases. This is because, on dividing the slit into odd number of parts, the contributions of the corresponding (outermost) pairs cancel each other, leaving behind the contribution of only the innermost segment. e.g. for first maxima, slit is divided into three parts; out of these three parts of the slit, the contribution from first two parts cancel each other, only $\frac{1}{3}$rd position of slit contributes to the maxima of intensity. Similarly, for second maxima, slit is divided into five parts, contribution of first four parts will be zero. The remaining $\frac{1}{5}$th portion will only contribute for maxima and so on. **(1½)**

26. (i) Polarisation also occur when light is scattered while travelling through a medium. When light strikes the atoms of a material, it will often set the electrons of those atoms into vibration.

The vibrating electrons then produce their own electromagnetic wave that is radiated outward in all directions. This newly generated wave strikes neighbour atoms, forcing their electrons into vibrations at the same original frequency.

These vibrating electrons produce another electromagnetic wave that is once more radiated outward in all directions. This absorption and re-emission of light waves cause the light to be scattered about the medium. **(1)**



This process of scattering contributes to the blueness of our sky. This scattered light is partially polarised.

(ii) $I_0/2, I_0/4, I_0/8$

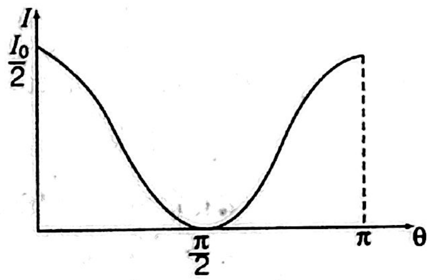
Refer to Sol. 24(ii) on page 326.

27. (i) In an unpolarised light, light waves vibrate in more than one plane, whereas in linearly polarised light, vibration of light waves occurs in a single plane.

(ii) A polaroid consists of long chain molecule aligned in a particular direction. The electric vectors (associated with the propagating light wave) along the direction of aligned molecules get absorbed. Thus, if an unpolarised light wave is incident on such a polaroid, then the light wave will get linearly polarised with the electric vector oscillating along a direction perpendicular to the aligned molecule. **(1)**

(iii) Unpolarised light scattering from air molecules sets their electrons into vibration perpendicular to the direction of the original ray. The scattered light therefore has a polarisation perpendicular to the original direction. Refer to Sol. 26(i) on page 326. **(1)**

28. (i) The required graph would have the form as shown in figure below:



(1)

Using $I_2 = I_1 \cos^2\theta$

(ii) Given, I_1 = light transmitted by P_1

I_2 = light transmitted by P_2

I_3 = light transmitted by P_3

According to Malus law

$$I_3 = I_1 \cos^2\beta \quad \ldots\text{(i)}$$

$$I_2 = I_3 \cos^2(\theta - \beta) \quad \ldots\text{(ii)}$$

According to question, $I_2 = I_3$

Substituting the value of I_2 and I_3 from Eq. (i) and Eq. (ii), we get

$$I_3 \cos^2(\theta - \beta) = I_1 \cos^2\beta$$

Substituting the value of I_3 from Eq. (i),

$$I_1 \cos^2\beta \cos^2(\theta - \beta) = I_1 \cos^2\beta$$

$$\cos^2(\theta - \beta) = 1$$

$$(\theta - \beta) = \cos^{-1}(1)$$

$$(\theta - \beta) = 0$$

$$\theta = \beta \quad \ldots\text{(iii)}$$

According to question $I_1 = I_2$

Substituting the value of I_2 from Eq. (ii),

$$I_1 = I_3 \cos^2(\theta - \beta)$$

Substituting the value of I_3 from Eq. (i),

$$I_1 = I_1 \cos^2\beta \cos^2(\theta - \beta)$$

or $\quad \cos^2\beta = 1 \quad$ [From Eq. (iii), $\theta = \beta$]

$$\beta = 0° \text{ or } \pi$$ **(2)**

29. **For linearly polarised light** Refer to Sol. 27(i) on page 326.

(i) The incident sunlight is unpolarised. The dot and double arrows show the polarisation in the perpendicular and in the plane of the figure. Under the influence of the electric field of the incident wave, the electrons in the molecules of the atmosphere acquire components of motion in both these directions. **(1)**

An observer looking at 90° to the direction of the sun, the charges accelerating parallel to the double arrows do not radiate energy towards this observer since their acceleration has no transverse component.

The radiation scattered by the molecule is therefore represented by dots. It is linearly polarised perpendicular to the plane of the figure. Refer diagram of Sol. 26 on page 326.

(ii) If the unpolarised light is incident on a polaroid the intensity is reduced by half. Even if the polaroid is rotated by an angle θ, the average value of $\cos^2\theta$ for polaroid will be

$$[\cos^2\theta]_{av} = \frac{1}{2}.$$

Thus, from Malus law

$$I = I_0 \cos^2\theta \quad \text{or} \quad \langle I \rangle = \langle I_0 \cos^2\theta \rangle$$

$$= I_0 \langle \cos^2\theta \rangle = I_0/2$$ **(2)**

Thus, the intensity of the transmitted light remains unchanged when the polaroid is rotated.

30. (i) If the width of each slit is comparable to the wavelength of light used, the interference pattern thus obtained in the double slit experiment is modified by diffraction from each of the two slits. **(1)**

(ii) Given that, wavelength of the light beam,

$$\lambda_1 = 590 \text{ nm} = 5.9 \times 10^{-7} \text{m}$$

Wavelength of another light beam,

$$\lambda_2 = 596 \text{ nm} = 5.96 \times 10^{-7} \text{m}$$

Distance of the slit from the screen,

$$D = 1.5 \text{ m}$$

Aperture of the slit $= d = 2 \times 10^{-4}$ m

For the first secondary maxima,

$$\sin\theta = \frac{3\lambda_1}{2d} = \frac{x_1}{D} \text{ or } x_1 = \frac{3\lambda_1 D}{2d}$$

and $$x_2 = \frac{3\lambda_2 D}{2d}$$

∴ Spacing between the positions of first secondary maxima of two sodium lines.

$$x_2 - x_1 = \frac{3D}{2d}(\lambda_2 - \lambda_1)$$

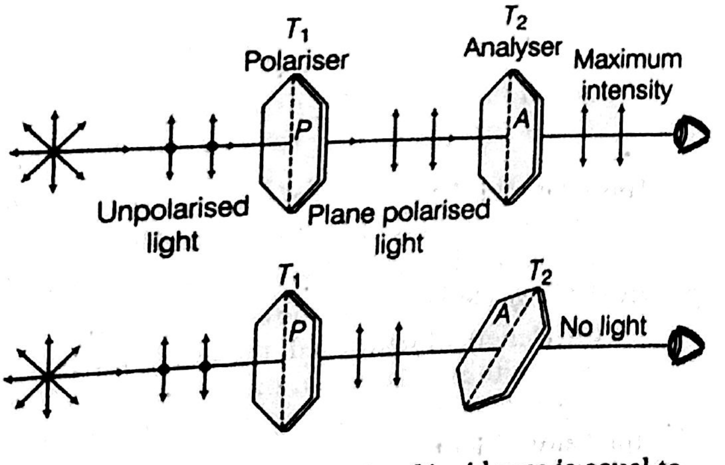
Substituting the value of all elements

$$= \frac{3\times 1.5}{2\times 2\times 10^{-4}}(5.96 - 5.9)\times 10^{-7}$$

$$= 6.75\times 10^{-5}\text{m}$$ **(2)**

31. (i) When an unpolarised light incident on tourmaline crystal T_1 (polariser), then intensity of transmitted light cut to its half. Let, another crystal, T_2 be placed in the path of transmitted light by T_1 and one full rotation is given to it, gradually increases intensity of transmitted light is observed.

The intensity of transmitted light is maximum when the axes of the two crystals, T_1 and T_2 are parallel and minimum when they are perpendicular to each other. Since, the intensity of polarised light on passing through a tourmaline crystal changes with the relative orientation of its crystallographic axis with that of polariser this implies, light wave must be of transverse in nature. **(1½)**



(ii) It happen when angle of incidence is equal to polarising angle falling on a transparent surface, then reflected light is completely polarised. In this situation, refractive index of refracting surface is given by $\mu = \tan i_p$

Also, the reflected and refracted light wave are mutually perpendicular to each other. **(1½)**

32. Here $v = 2.25\times 10^8$ m/s

Speed of light in vacuum, $c = 3\times 10^8$ m/s

$$\mu = \frac{c}{v} = \frac{3\times 10^8}{2.25\times 10^8} = 1.33$$ **(1)**

(a) If C is the critical angle, then $\mu = \frac{1}{\sin C}$

$$1.33 = \frac{1}{\sin C}$$

$$\sin C = \frac{1}{1.33} = 0.75$$

$$\Rightarrow \quad C = \sin^{-1}(0.75) = 48.5^\circ$$ **(1)**

(b) If i_B is the polarising angle then, $\tan i_B = \mu$ or $\tan i_B = 1.33$

$$\therefore \quad i_B = \tan^{-1}(1.33) \Rightarrow 53.06^\circ$$ **(1)**

33. (i) By Malus' law, intensity of emergent light from P_2, $I = I_0\cos^2\theta$, where θ is the angle between P_1 and P_2

when $\theta = 90^\circ$ $\Rightarrow I = I_0 \times 0$ $(\because \cos\theta = 0)$

Intensity of emergent light,

$$I = 0$$ **(1)**

(ii) The intensity of emergent light from P_3

$$I = \frac{I_0}{2}\cos^2\beta$$

The intensity of emergent light from P_2

$$\frac{I_0}{8} = I\cos^2(\theta - \beta)$$

$$\frac{I_0}{8} = \frac{I_0}{2}\cos^2\theta\cos^2(\theta - \beta)$$

$$\sin 2\beta = 1$$

$$\Rightarrow \quad 2\beta = 90^\circ$$

$$\Rightarrow \quad \beta = 45^\circ$$ **(2)**

34. (i) Refer to Sol 22 (i) on page 325. **(1)**

(ii) The intensity of transmitted light from

$$P_1 = \frac{I_0}{2} \quad (\because \text{Average of } \cos^2\theta = \frac{1}{2})$$

The intensity of transmitted light from P_2,

$$= \frac{I_0}{2}(\cos^2 30^\circ) = \frac{I_0}{2}\times\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3I_0}{8}$$ **(1)**

∴ The intensity of transmitted light from P_3, (third polaroid)

$$= \frac{3I_0}{8}[\cos(90^\circ - 30^\circ)]^2 = \frac{3I_0}{8}(\cos 60^\circ)^2$$

$$= \frac{3I_0}{8}\times\left(\frac{1}{2}\right)^2 = \frac{3I_0}{8}\times\frac{1}{4} = \frac{3I_0}{32}$$ **(1)**

35. (i) Refer to Sol. 24 (ii) on page 318. **(1)**

(ii) Because light wave is transverse and sound waves are longitudinal. The longitudinal waves can't be polarised, therefore, sound wave can't be polarised. **(1)**

(iii) Because it reduces the glare to half of that of incident unpolarised light from the sun on goggles. **(1)**

36. To calculate the intensity of transmitted radiation, we can use the formula,

$$I = I_0 \cos^2 \theta$$

where, θ is the angle between the plane of the polariser and that of an analyser.

Refer to Ans. 11. **(1/2)**

If I_0 be the intensity of unpolarised light on first polaroid P_1 become $\frac{I_0}{2}$. **(1/2)**

$\therefore$ The intensity of transmitted light from polaroid.

(a) P_3, IIIrd polaroid whose pass axis makes an angle θ with P_1 is given by $= \frac{I_0}{2}(\cos^2\theta)$ **(1/2)**

(b) From P_2 such that the angle between P_3 and P_2 is $(90° - \theta)$ is given by

$$= \left(\frac{I_0}{2}\cos^2\theta\right)[\cos(90° - \theta)]^2$$

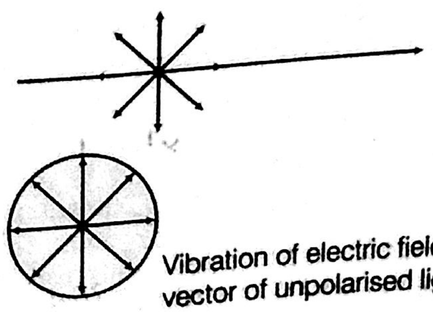
$$= \frac{I_0}{2}\cos^2\theta \cdot \sin^2\theta \quad \textbf{(1/2)}$$

Transmitted intensity from P_2 is

(i) minimum when P_1 and P_2 have their pass axes perpendicular to each other.

(ii) maximum when angle between pass axes of P_1 and P_2 is 45°. **(1)**

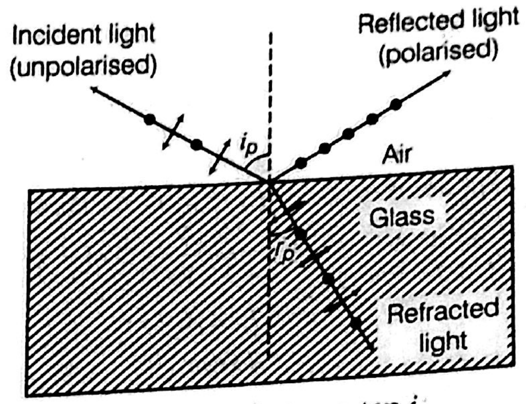
37. Unpolarised light When the vibrations of electric field vector are symmetrically distributed in a direction perpendicular to the direction of propagation of light wave, then the light wave is termed as unpolarised light. **(1)**



Vibration of electric field vector of unpolarised light

Polarisation by reflection from a transparent medium

When an unpolarised light beam is incident on a refracting transparent medium at a particular angle of incidence, known as polarising angle i_p, the reflected light is plane polarised. Thus, plane polarised light is produced by reflection. **(1)**



Brewster's law states that, $\mu = \tan i_p$

where, μ = refractive index of transparent and denser medium and i_p = polarising angle. **(1)**

38. The figure of the polarisation by reflection

Refer to Sol. 37 on page 329.

The reflected, completely plane polarised light makes an angle of 90° with the direction of corresponding refracted light. **(1)**

Brewster's law According to Brewster, the polarising angle i_p, the reflected plane polarised light and refracted rays are perpendicular to each other, then $i_p + r = 90°$

where, r = angle of refraction

$r = 90° - i_p$ **(1)**

$\because$ Snell's law, $\mu = \frac{\sin i_p}{\sin r} = \frac{\sin i_p}{\sin(90° - i_p)}$

$$\mu = \frac{\sin i_p}{\cos i_p}$$

$\Rightarrow \quad \mu = \tan i_p$

This relation is known as Brewster's law. **(1)**

39. Refer to Sol. 26 (i) on page 326.

Intensity of light through $P_1 = \frac{I_0}{2}$

Intensity of light through $P_3 = \frac{I_0}{2}\cos^2 45° = \frac{I_0}{4}$

Intensity of light through $P_2 = \frac{I_0}{8}\cos^2(90° - 45°)$

$$= \frac{I_0}{8}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I_0}{16} \quad \textbf{(2)}$$

40. (a) Two pin holes with two sodium lamps cannot produce coherent sources of light, so the phenomenon of interference cannot be observed.

(b) Refer to Sol. 12 (i) on page 310 (Topic 2).

(c) Given, $\lambda_1 = 590\,\text{nm}$, $\lambda_2 = 596\,\text{nm}$,

$d = 2 \times 10^{-6}\,\text{m}$, $D = 1.5\,\text{m}$

Distance of secondary maxima from centre,

$$x = \frac{3}{2}\frac{D\lambda}{d}.$$

Spacing between the first two maxima of sodium light

$$\Rightarrow x_2 - x_1 = \frac{3D}{2d}(\lambda_2 - \lambda_1)$$

$$= \frac{3 \times 1.5}{2 \times 2 \times 10^{-6}}(596 - 590) \times 10^{-9}$$

$$= 6.75 \times 10^{-3} \text{ m}$$

$$= 6.75 \text{mm}$$

41. Difference between Interference and Diffraction **(2)**

(i) The interference pattern has a number of equally spaced bright and dark bands. Where as the diffraction pattern has a central bright maximum, which is twice as wide as the other maxima. The intensity falls as we go to successive maxima away from the centre on either side.

(ii) We calculate the interference pattern by superposing two waves originating from the two narrow slites. The diffraction patter is a superposition of a continuous family of waves originating from each point on a single slit.

Young's double slit experiment Refer to text on page 305 (Intensity of the Fringes)

(b) Given, $\lambda = 620 \text{ nm} = 620 \times 10^{-9}$ m

Aperture of slit, $b = 3\text{mm} = 3 \times 10^{-3}$ m

Distance between source and screen,

$$D = 1.5 \text{ m}$$

The distane of first order minima from central maxima, $x_1 = \frac{n\lambda D}{b} = \frac{\lambda D}{b} (n = 1)$

$$= \frac{620 \times 10^{-9} \times 1.5}{3 \times 10^{-3}}$$

$$= 310 \times 10^{-6} \text{ m}$$

The distance at third order maxima from central maxima,

$$x_3' = \frac{(2n+1)\lambda D}{2b} = \frac{7\lambda D}{2b}$$

$$= \frac{7 \times 620 \times 10^{-9} \times 1.5}{2 \times 3 \times 10^{-3}}$$

$$= 1085 \times 10^{-6} \text{ m}$$

Thus, distance between x_1 and x_3 is

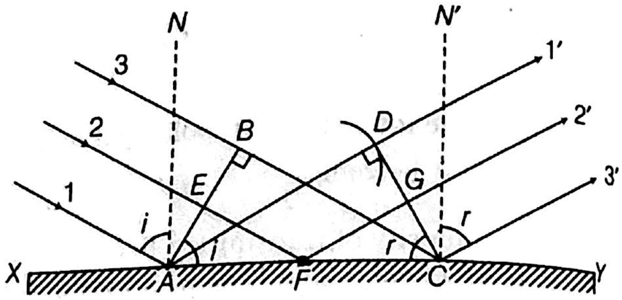
$$x = x_3 - x_1 = (1085 - 310) \times 10^{-6}$$

$$= 775 \times 10^{-6} \text{ m}$$

42. (a) **Wavefront** It is the locus of points (wavelets) having the same phase (a surface of constant phase) of oscillations. **(1)**

Laws of reflection at a plane surface (On Huygens' principle)

Let 1, 2, 3 be the incident rays and 1′, 2′, 3′ be the corresponding reflected rays.



Laws of reflection by Huygens' principle

If c is the speed of the light, t is the time taken by light to go from B to C or A to D or E to G through F, then

$$t = \frac{EF}{c} + \frac{FG}{c} \quad \text{...(i)}$$

In ΔAEF, $\sin i = \frac{EF}{AF}$

In ΔFGC, $\sin r = \frac{FG}{FC}$

or $t = \frac{AF \sin i}{c} + \frac{FC \sin r}{c}$

or $t = \frac{AC \sin r + AF(\sin i - \sin r)}{c}$

$(\because FC = AC - AF)$

For rays of light from different parts on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the reflected wavefront.

So, t should not depend upon AF. This is possible only if $\sin i - \sin r = 0$

i.e.

$$\sin i = \sin r \text{ or}$$

$$\angle i = \angle r$$

which is the **first law of reflection.** ...(ii)

Further, the incident wavefront AB, the reflecting surface XY and the reflected wavefront CD are all perpendicular to the plane of the paper.

Therefore, incident ray, normal to the mirror XY and reflected ray all lie in the plane of the paper. This is **second law of reflection.** **(2)**

(b) We know that, width of central maximum is given as $2y = 2D\lambda/a$

where, a = width of slit. When $a = 2a$

$\therefore$ Width of central maximum $= \frac{2D\lambda}{2a} = \frac{\lambda D}{a}$

Thus, the width of central maximum became half. But in case of diffraction, intensity of central maxima does not changes with slit width. Thus, the intensity remains same in both cases. **(1)**

(c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the obstacle because the waves diffracted from the edge of circular obstacle interfere constructively at the centre of the shadow resulting in the formation of a bright spot. **(1)**

43. Refer to Sol. 3 on page 322. **(3)**

Polarisation also occurs when light is scattered while travelling through a medium. When light strikes the atoms of a material, it will often set the electrons of those atoms into vibration. The vibrating electrons then produce its own electromagnetic wave that is radiated outward in all directions. This newly generated wave strikes neighbour atoms, forcing their electrons into vibrations at the same original frequency. **(1)**

These vibrating electrons produce another electromagnetic wave that is once again radiated outward in all directions. This absorption and re-emission of light wave causes the light to be scattered about the medium. The scattered light is partially polarised.

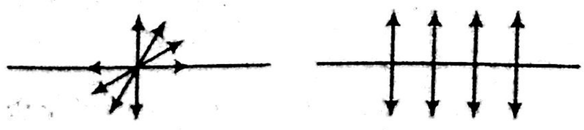
From Brewster's law, $\mu = \tan i_p$

where, i_p = Brewster's angle

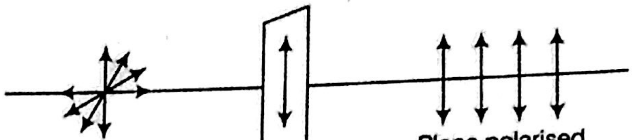
Given, $\mu = 1.5 \Rightarrow 1.5 = \tan i_p$

$$i_p = \tan^{-1}\left(\frac{3}{2}\right)$$ **(1)**

44. (i) A polarised light has plane of vibration only in one plane whereas, in unpolarised light, plane of vibration is spreaded in all directions as shown below. Here, vibration relates to electric and magnetic fields.



Polaroid is a special crystalline solid which contains a special axis called optic axis. When we make an unpolarised light to fall on a polaroid, only vibration parallel to optic axis is allowed to pass through and all other vibration are cut. Thus, the output is a plane polarised light. **(2½)**



(ii) Intensity coming out of a single polaroid is half of the incident intensity so, intensity of transmitted light from polaroid P_1, $I = \frac{I_0}{2}$

By using law of Malus, intensity of emergent light from polaroid P_2 is $= I' = I\cos^2\theta$

where, θ = angle between P_1 and P_2.

Intensity of light after transmission from polaroid

$$I' = I\cos^2\theta$$

$$= \frac{I_0}{2} \times (\cos 60^\circ)^2$$

$$= \frac{I_0}{2} \times \left(\frac{1}{2}\right)^2 = \frac{I_0}{8}$$ **(2½)**

45. (i) The features to distinguish is given as

(a) In Young's experiment, width of all the fringes are equal but in diffraction fringes, width of central fringe is twice the other fringes.

(b) The intensity of all the fringes are equal in interference fringe but intensity of fringes go on decreasing in diffraction as we go away from the central fringe. **(2½)**

(ii) Given, wavelength $(\lambda) = 500\text{ nm} = 500\times10^{-9}$ m

Width of single slit $(d) = 0.2\text{ mm} = 0.2\times10^{-3}$ m

Angular width of central fringe $= 2\times\frac{\lambda}{d}$

$$= \frac{2\times500\times10^{-9}}{0.2\times10^{-3}} = \frac{10^{-6}}{2\times10^{-4}}$$

$$= \frac{1}{200} = 5\times10^{-3} \text{ radian}$$

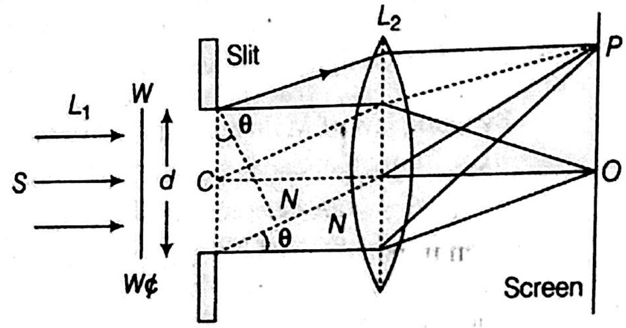
Let distance between the slit and screen be 1m. (which is not given in the problem but this data is necessary to solve the problem).

Linear width of central fringe of single slit
$$= 5\times10^{-3}\times10^{3}\text{ mm} = 5\text{mm}$$

Number of double slit fringe accommodated in central fringe $= \frac{50}{5} = 10$ fringes. **(2½)**

46. Phenomenon is diffraction All the secondary wavelets going straight across the slit are focussed at the central point of the screen. The wavelets from any two corresponding points of the two halves of the slit reach the point O in the same phase, they add constructively to produce a central bright fringe.

(i) The condition for first dark fringe is $d\sin\theta_1 = \lambda$. Similarly, the condition for second dark fringe will be $d\sin\theta_2 = 2\lambda$



Hence, the condition for nth dark fringe can be written as $d\sin\theta_n = n\lambda$, where $n = 1, 2, 3, \ldots$

The directions of various minima are given by
$$\theta_n \approx \sin\theta_n = n\frac{\lambda}{d} \quad [\text{As } \lambda << d, \text{ so } \sin\theta_n = \theta_n]$$

Suppose the point P is so located that $\theta = \frac{3\lambda}{2}$

When $\theta = \theta_1'$, then $a\sin\theta_1' = \frac{3}{2}\lambda$

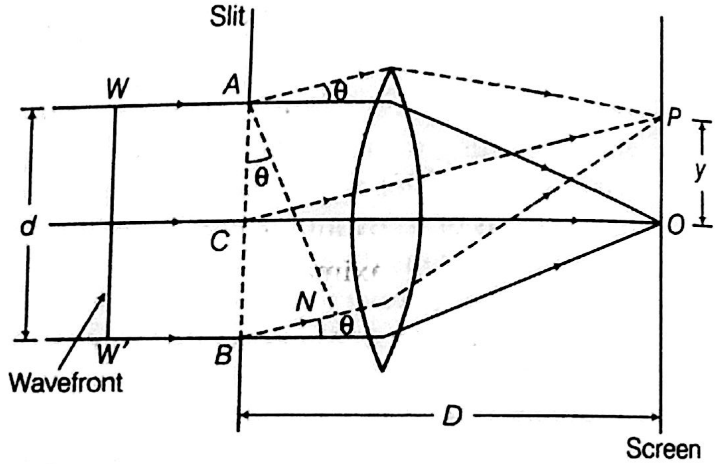
We can divide the slit into three equal parts. The path difference between two corresponding points of the first two parts will be $\frac{\lambda}{2}$. The wavelets from these points will interfere destructively.

However, the wavelets from the third part of the slit will contribute to some intensity forming a secondary maximum. The intensity of this maximum is much less than that of the central maximum. Thus, the condition for the first secondary maximum is $a\sin\theta_1' = \frac{3}{2}\lambda$ **(2½)**

(ii) The reason is that the intensity of the central maximum is due to the constructive interference of wavelets from all parts of the slit, the first secondary maximum is due to the contribution of wavelets from one-third part of the slit (wavelets from remaining two parts interfere destructively), the second secondary maximum is due to the contribution of wavelets from the one fifth part only (the remaining four parts interfere destructively) and so on. Hence, the intensity of secondary maximum decreases with the increases with the order n of the maximum. **(2)**

(iii) Width of central band halves. **(1/2)**

47. (i) **Diffraction of light at a single slit** A parallel beam of light with a plane wavefront WW' is made to fall on a single slit AB. As width of the slit $AB = d$ is of the order of wavelength of light, therefore, diffraction occurs on passing through the slit.



The wavelets from the single wavefront reach the centre O on the screen in same phase and hence, interfere constructively to give central maximum (bright fringe). **(1)**

The diffraction pattern obtained on the screen consists of a central bright band having alternate dark and weak bright band of decreasing intensity on both sides.

Consider a point P on the screen at which wavelets travelling in a direction making an angle θ with CO are brought to focus by the lens. The wavelets from points A and B will have a path difference equal to BN.

From the right angled ΔANB, we have
$$BN = AB\sin\theta$$
or
$$BN = d\sin\theta.$$

To establish the condition for secondary minima, the slit is divided into 2,4,6... equal parts such that corresponding wavelets from parts such that corresponding wavelets from successive regions interfere with path difference of $\lambda/2$ **(2)**

or for **nth secondary minimum**, the slit can be divided into $2n$ equal parts.

Hence, for nth secondary minimum, path difference $= d\sin\theta_n = n\lambda$

or $$\sin\theta_n = \frac{n\lambda}{d} \quad (n = 1, 2, 3, \ldots)$$

To establish the condition for secondary maxima, the slit is divided into 3, 5, 7... equal parts such that corresponding wavelets from alternate regions interfere with path difference of $\lambda / 2$

or for **nth secondary maximum**, the slit can be divided into $(2n+1)$ equal parts.

Hence, for nth secondary maximum

$$d\sin\theta_n = (2n+1)\frac{\lambda}{2} \quad (n = 1, 2, 3, \ldots)$$

or $$\sin\theta_n = (2n+1)\frac{\lambda}{2d}$$ **(1)**

(ii) For $\lambda_1 = 590$ mm

Location of I maxima

$$y_1 = (2n+1)\frac{D\lambda_1}{2d}$$

If $$n = 1 \Rightarrow y_1 = \frac{3D\lambda_1}{2d}$$

For $\lambda_2 = 596$ nm

Location of III maxima

$$y_2 = (2n+1)\frac{D\lambda_2}{2d}, \text{ if } n = 1$$

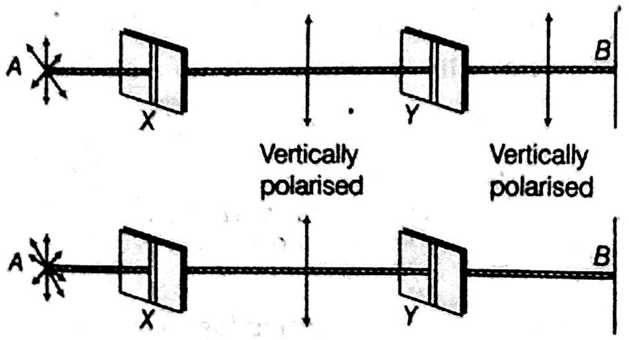
$$\Rightarrow \quad y_2 = \frac{3D\lambda_2}{2d}$$

$$\therefore \text{Path difference} = y_2 - y_1 = \frac{3D}{2d}(\lambda_2 - \lambda_1)$$

$$= \frac{3 \times 1.5}{2 \times 2 \times 10^{-6}}(596 - 590) \times 10^{-9}$$

$$= 6.75 \times 10^{-3} \text{ m}$$ **(1)**

48. (i) A polarised wave is a wave which vibrates in one plane only.



(1)

The diagram shows an unpolarised wave on the left (it vibrates in all directions) and two polarising filters. Light from an **incandescent** source (something which emits light when heated) is **unpolarised**, i.e. the electric and magnetic fields are oscillating in many different planes.

If light from such a source is passed through a substance called a **polaroid** the emerging rays are now **polarised**, i.e. oscillate in one plane only.

If this light is then passed through a second polaroid, it only gets through if the second polaroid is parallel to that of the first.

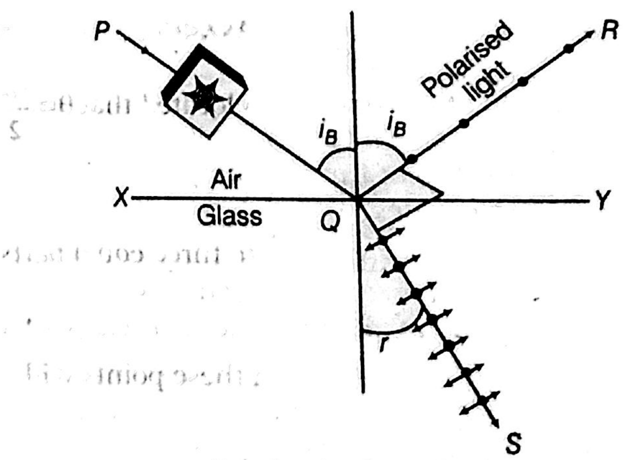
If the second polaroid is then rotated through 90°, no light gets through. **(1)**

(ii) When unpolarised light is reflected from a surface, the reflected light may be completely polarised, partially polarised or unpolarised. This would depend on the angle of incidence.

If the angle of incidence is 0° or 90°, then reflected beam remains unpolarised.

The angle of incidence at which the reflected light is completely plane polarised is called **polarising angle** or **Brewster's angle.**

When unpolarised light is incident on the boundary between two transparent media, the reflected light is polarised with its electric vector perpendicular to the plane of incidence when the reflected and refracted rays make a right angle with each other. **(1)**



Polarisation by reflection **(1)**

When unpolarised light is incident on the boundary between two transparent media, the reflected light is polarised with its electric vector perpendicular to the plane of incidence when the reflected and refracted rays make a right angle with each other.

Thus, when reflected wave is perpendicular to the refracted wave, the reflected wave is a totally polarised wave. The angle of incidence in this case is called **Brewster's angle i_B**.

Since, $i_B + r = \frac{\pi}{2}$, from Snell's law

$$\mu = \frac{\sin i_B}{\sin r} = \frac{\sin i_B}{\sin\left(\frac{\pi}{2} - i_B\right)}$$

$$= \frac{\sin i_B}{\cos i_B} = \tan i_B \quad \therefore \quad \mu = \tan i_B$$

Polarisation of light by reflection upto $i_B + r = 90°$. **(1)**

49. (i) **Linearly polarised light** Refer to Sol. 27(i) on page 326.

Unpolarised light Refer to Sol. 27(i) on page 326.. **(3)**

(ii) Refer to Sol. 31 (i) on page 328. **(1)**

(iii) The intensity of a clear blue portion of the sky show a rise and fall due to when unpolarised light is passed through polaroid it's intensity varies directly as the square of cosine of the angle θ between the plane of transmission of analyser and polarismer (Malus Law). **(1)**

50. (i) (a) Interference is the result of interaction of light coming from two different wavefronts originating from two coherent sources, whereas diffraction pattern is the result of interaction of light coming from different parts of the same wavefront. **(1)**

(b) In interference,the fringes may or may not be the same width, while in diffraction, the fringes are always of varying widths. **(1)**

(c) In interference,the bright fringes are of the same intensity, while in diffraction, the bright fringes are of varying intensities. **(1)**

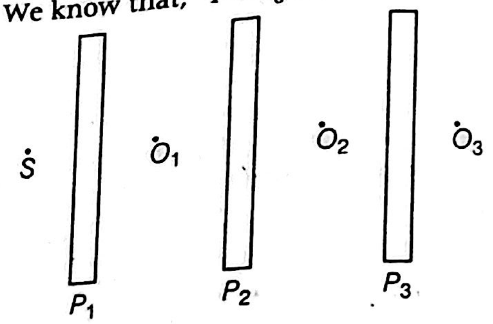
(ii) Distance of first minimum, $y = \frac{\lambda}{d}$

Slit width, $d = \frac{\lambda}{y}$

$$\therefore \quad a = \frac{500 \times 10^{-9}}{2.5 \times 10^{-4}} = 0.002\,\text{m}.$$ **(2)**

51. (i) When an unpolarised light beam is incident on a polaroid then only those vibration of electric vector which are parallel to crystallographic axis of polaroid are transmitted through polaroid and other vibrations are being absorbed by it. This selective absorption of electric field vector which are not parallel to axis is termed as dichroism and hence plane polarised light is produced by polaroid. **(2)**

(ii) We know that, $I = I_0 \cos^2\theta$



Intensity at O_1,

$$I = I_0 \cos^2\theta$$

Intensity at O_2,

$$I_1 = I \cos^2\theta_1$$

$$= I_0 \cos^2\theta \cos^2 60° \qquad [\because \theta_1 = 60°]$$

$$I_1 = \frac{I_0 \cos^2\theta}{4}$$

Intensity at O_3,

$$I_2 = I_1 \cos^2\theta_2$$

$$= \frac{I_0 \cos^2\theta}{4} \times \cos^2 90° \qquad [\because \theta_2 = 90°]$$

$$= 0$$ **(3)**

52 (i) When an unpolarised light beam is incident on a polaroid,then only those electric vectors which are parallel to pass axis of polaroid, passes through the polaroid and all vibration which are not along the pass axis of polaroid gets absorbed by it due to property of dichroism. Thus, the transmitted light is plane polarised. **(1)**

Refer to Sol. 36 on page 329.

(ii) Let, θ be the angle between the pass axis A and C. **(1½)**

Intensity of light passing through $A = \frac{I_0}{2}$

Intensity of light passing through C

$$= \frac{I_0}{2} \cos^2\theta$$

Intensity of light passing through B

$$= \left(\frac{I_0}{2} \cos^2\theta\right) \cos^2(90 - \theta)$$

$$= \frac{I_0}{2} (\sin\theta \cos\theta)^2$$

But, $\frac{I_0}{2} (\sin\theta \cos\theta)^2 = \frac{I_0}{8}$ **(1½)**

(according to problem)

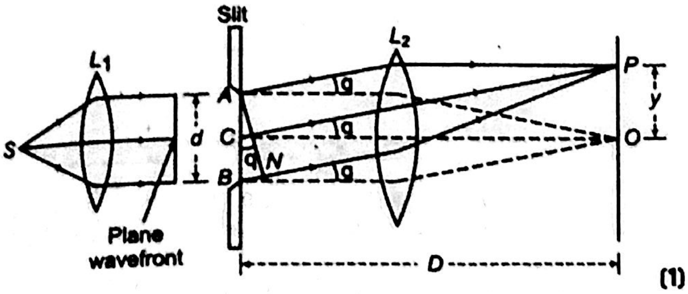
$(\sin\theta \cos\theta)^2 = \frac{1}{4}$ **(1/2)**

$$\therefore \quad \left(\frac{\sin 2\theta}{2}\right)^2 = \frac{1}{4} \quad \text{or} \quad \frac{\sin^2 2\theta}{4} = \frac{1}{4}$$

or $\sin 2\theta = 1$

$\Rightarrow \quad 2\theta = 90° \Rightarrow \theta = 45°$ **(1/2)**

53. (i) Consider a point P on the screen at which wavelets travelling in a direction,



(1)

making an angle θ with CO, are brought to focus by the lens. The wavelets from points A and B will have a path difference equal to BN.

From the right angled ΔANB, we have

$$BN = AB \sin\theta$$

$$BN = d \sin\theta \quad \ldots(i)$$

Suppose, $BN = \lambda$ and $\theta = \theta_1$

Then, the above equation gives

[slit width $d = a$]

$$\lambda = d \sin\theta_1$$

$$\Rightarrow \quad \sin\theta_1 = \frac{\lambda}{d} \quad \ldots(ii)$$

Such a point on the screen will be the position of first secondary minimum.

If $BN = 2\lambda$ and $\theta = \theta_2$, then

$$2\lambda = d \sin\theta_2$$

$$\sin\theta_2 = \frac{2\lambda}{d} \quad \ldots(iii)$$

Such a point on the screen will be the position of second secondary minimum.

In general, for nth minimum at point P,

$$\sin\theta_n = \frac{n\lambda}{d} \quad \ldots(iv)$$

If y_n is the distance of the nth minimum from the centre of the screen, then from right angled ΔCOP, we have

$$\tan\theta_n = \frac{OP}{CO} = \frac{y_n}{D} \quad \ldots(v)$$

In case of θ_n is small, $\sin\theta_n \approx \tan\theta_n$

There Eqs. (iv) and (v) are given as

$$\frac{y_n}{D} = \frac{n\lambda}{d}$$

$$\Rightarrow \quad y_n = \frac{n\lambda D}{d}$$ **(2)**

If $BN = \frac{3\lambda}{2}$ and $\theta = \theta_1'$, then from Eq. (i), we have

$$\sin\theta_1' = \frac{3\lambda}{2d}$$

Such a point on the screen will be the position of the first secondary maximum.

Corresponding to path difference, if, $BN = \frac{5\lambda}{2}$ and $\theta = \theta_2'$, the second secondary maximum is produced.

In general, for the nth maximum at point P,

$$\sin\theta_n' = \frac{(2n+1)\lambda}{2d} \quad \ldots(vi)$$

where, $n = 1, 2, 3, \ldots$ an integer.

If y_n is the distance of nth maximum from the centre of the screen, then the angular position of the nth maximum is given by,

$$\tan\theta_n' = \frac{y_n}{D} \quad \ldots(vii)$$

In case of θ_n' is small, $\sin\theta_n' \approx \tan\theta_n'$

$$\Rightarrow \quad y_n' = \frac{(2n+1)\lambda D}{2d}$$

For $\quad n = 1, \ \theta' = \frac{3\lambda}{2d}$

[From Eq. (vi), small angle approximation,

$$\sin\theta' = \theta' = \frac{(2n+1)\lambda}{2d}$$ **(1)**

This angle is midway between the two dark fringes. Divide the slit into three equal parts. If we take the first two-third part of the slit, then path difference between the two ends would be

$$\frac{2}{3} d \times \theta' = \lambda$$

The first two-third is divided into two halves which have path difference $\lambda/2$. The contribution due to these two halves is 180° out of phase and gets cancel.

Only the remaining one-third part of the slit contributes to the intensity at a point between the two minima which will be much weaker than the intensity of central maxima. Thus, with increase in the intensity, the maxima gets weaker.

(ii) As, the number of point sources increases, their contribution towards intensity also increases. Intensity varies as square of the slit width.

Thus, when the width of the slit is made double the original width, intensity will get four times of its original value.

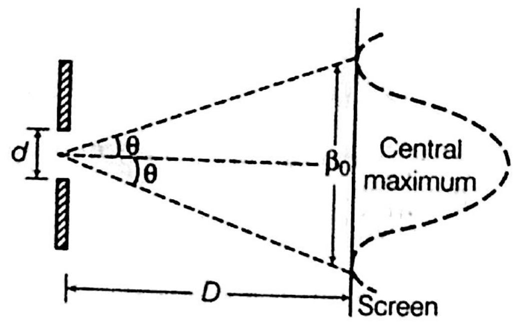
Width of central maximum is given by

$$\beta = 2D\lambda/d$$

where, D = distance between screen and slit

λ = wavelength of the light

d = size of slit.



So, with the increase in size of slit, the width of central maxima decreases. Hence, double the size of the slit would results as half the width of the central maxima. **(1)**

54. (i) **For Huygens' theory** Refer to text on page 296. **(1)**

Diffraction pattern Refer to Ans. 46 **(1)**

(ii) The angular width of central maximum is the angular separation between the directions of the first minima on the two sides of the central maxima.

The angular width of first minima on either side of central maximum is given by

$$\theta = \lambda / d \qquad \textbf{(1)}$$

$\therefore$ The angular width of central maxima

$$= 2\lambda / d \qquad \ldots(i)$$

$\therefore$ Angular separation of nth secondary minimum, $\theta_n = n\lambda / d$

Angular separation of $(n + 1)$th secondary minima, $\theta_{n+1} = (n+1)\dfrac{\lambda}{d}$

Therefore, the angular width of secondary maxima of nth order is equal to the difference of angular separation of $(n + 1)$th and nth order secondary minima.

$\therefore$ Angular width of secondary maxima

$$= \theta_{n+1} - \theta_n$$

$$= (n+1)\frac{\lambda}{d} - n\frac{\lambda}{d} = \frac{\lambda}{d} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

Angular width of first diffraction fringe is half of that of the central fringe. **(1)**

(iii) If a monochromatic source of light is replaced by white light, then coloured fringe pattern is obtained on the screen. **(1)**

The central maximum will be white but other bands will be coloured. As bandwidth $\propto \lambda$, therefore red bandwidth will be wider than the violet bandwidth.

55. (i) Refer to Sol. 53 (i) on page 335. **(3)**

(ii) Size of central diffraction band is inversely proportional to the slit width i.e. size of central diffraction band $= \dfrac{2\lambda}{d}$ **(2)**