Physics

NCERT Exemplar Problems

Chapter 1

Electric Charges and Fields

1.1 (a) as q_1 is negative, so Q will attract q_1.

1.2 (a) as electric field is moves +ve to -ve and always normal to the surface.

1.3 (d) as the net charge enclosed by all figures is same and flux = q/ε

1.4 (b) as q2 and q4 are enclosed by the Gaussian Surface.

1.5 (c) as the Electric Field is not uniform.

1.6 (a) directed perpendicular to the plane and away from the plane.

1.7 (a) perpendicular to the diameter

1.8 (c), (d)

1.9 (b), (d)

1.10 (b), (d)

1.11 (c), (d)

1.12 (a), (c).

1.13 (a), (b), (c) and (d).

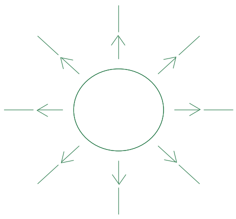
1.14 Zero.

1.15 (i) $\dfrac{-Q}{4\pi R_1^2}$ (ii) $\dfrac{Q}{4\pi R_2^2}$

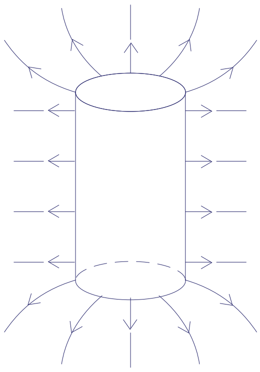
1.16 The electric fields bind the atoms to neutral entity. Fields are caused by excess charges. There can be no excess charge on the inter surface of an isolated conductor.

1.17 No, the field may be normal. However, the converse is true.

1.18



Top view



Side view

1.19 (i) $\dfrac{q}{8\varepsilon_0}$ (ii) $\dfrac{q}{4\varepsilon_0}$ (iii) $\dfrac{q}{2\varepsilon_0}$ (iv) $\dfrac{q}{2\varepsilon_0}$.

1.20 1 Molar mass M of Al has $N_A = 6.023 \times 10^{23}$ atoms.

$\therefore$ m = mass of Al paisa coin has $N = N_A \dfrac{m}{M}$ atoms

Now, $Z_{Al} = 13$, $M_{Al} = 26.9815$g

Hence $N = 6.02 \times 10^{23}$ atoms/mol $\times \dfrac{0.75}{26.9815\text{g/mol}}$

$= 1.6733 \times 10^{22}$ atoms

$\therefore q$ = +ve charge in paisa = $N\,Ze$

$= (1.67 \times 10^{22})(13)\,(1.60 \times 10^{-19}\text{C})$

$= 3.48 \times 10^{4}$ C.

q = 34.8 kC of ±ve charge.
This is an enormous amount of charge. Thus we see that ordinary neutral matter contains enormous amount of ± charges.

1.21 (i) $F_1 = \dfrac{|q|^2}{4\pi\varepsilon_0 r_1^2} = \left(8.99\times10^{9}\dfrac{\text{Nm}^2}{\text{C}^2}\right)\dfrac{(3.48\times10^{4}\text{C})}{10^{-4}\text{m}^2} = 1.1\times10^{23}\text{N}$

(ii) $\dfrac{F_2}{F_1} = \dfrac{r_1^2}{r_2^2} = \dfrac{(10^{-2}\text{m})^2}{(10^{2}\text{m})^2} = 10^{-8} \Rightarrow F_2 = F_1\times10^{-8} = 1.1\times10^{15}\text{N}$

(iii) $\dfrac{F_3}{F_1} = \dfrac{r_1^2}{r_3^2} = \dfrac{(10^{-2}\text{m})^2}{(10^{6}\text{m})^2} = 10^{-16}$

$F_3 = 10^{-16} F_1 = 1.1 \times 10^{7}$ N.

Conclusion: *When separated as point charges these charges exert an enormous force. It is not easy to disturb electrical neutrality.*

1.22 (i) Zero, from symmetry.

(ii) Removing a +ve Cs ion is equivalent to adding singly charged -ve Cs ion at that location.

Net force then is

$$F = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

where r = distance between the Cl ion and a Cs ion.

$$= \sqrt{(0.20)^2 + (0.20)^2 + (0.20)^2} \times 10^{-9} = \sqrt{3(0.20)^2} \times 10^{-9}$$

$$= 0.346 \times 10^{-9}\,\text{m}$$

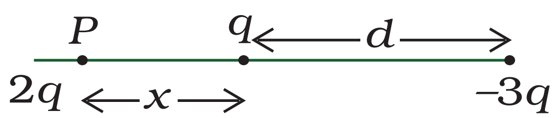
Hence, $F = \dfrac{(8.99\times10^{9})(1.6\times10^{-19})^2}{(0.346\times10^{-9})^2} = 192\times10^{-11}$

$$= 1.92 \times 10^{-9}\,\text{N}$$

Ans 1.92×10^{-9} N, directed from A to Cl^-

1.23 At P: on $2q$, Force due to q is to the left and that due to $-3q$ is to the right.

$$\therefore \frac{2q^2}{4\pi\varepsilon_0 x^2} = \frac{6q^2}{4\pi\varepsilon_0 (d+x)^2}$$



$\therefore (d + x)^2 = 3x^2$

$\therefore 2x^2 - 2dx - d^2 = 0$

$$x = \frac{d}{2} \pm \frac{\sqrt{3}d}{2}$$

(–ve sign would be between q and $-3q$ and hence is unaceptable.)

$$x = \frac{d}{2} + \frac{\sqrt{3}d}{2} = \frac{d}{2}(1+\sqrt{3}) \text{ to the left of } q.$$

1.24 (a) Charges A and C are positive since lines of force emanate from them.

(b) Charge C has the largest magnitude since maximum number of field lines are associated with it.

(c) (i) near A. There is no neutral point between a positive and a negative charge. A neutral point may exist between two like charges. From the figure we see that a neutral point exists between charges A and C. Also between two like charges the neutral point is closer to the charge with smaller magnitude. Thus, electric field is zero near charge A.

1.25 (a) (i) zero (ii) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$ along $\overrightarrow{OA}$ (iii) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{2q}{r^2}$ along $\overrightarrow{OA}$

(b) same as (a).

1.26 (a) Let the Universe have a radius R. Assume that the hydrogen atoms are uniformly distributed. The charge on each hydrogen atom is $e_H = -(1 + y)\, e + e = -\, ye = |ye|$

The mass of each hydrogen atom is ~ m_p (mass of proton). Expansion starts if the Coulumb repulsion on a hydrogen atom, at R, is larger than the gravitational attraction.

Let the Electric Field at R be $\mathbf{E}$. Then

$$4\pi R^2 E = \frac{4}{3\varepsilon_o}\pi R^3 N|ye| \quad \text{(Gauss's law)}$$

$$\mathbf{E}\,(R) = \frac{1}{3}\frac{N|ye|}{\varepsilon_o} R\,\hat{\mathbf{r}}$$

Let the gravitational field at R be G_R. Then

$$-\,4\pi R^2\, G_R = 4\,\pi G\; m_p \frac{4}{3}\pi R^3 N$$

$$G_R = -\frac{4}{3}\pi G m_\rho N R$$

$$\mathbf{G}_R(\mathbf{R}) = -\frac{4}{3}\pi G m_\rho N\; R\;\hat{\mathbf{r}}$$

Thus the Coulombic force on a hydrogen atom at R is

$$ye\boldsymbol{E}(R) = \frac{1}{3}\frac{Ny^2e^2}{\varepsilon_o}R\,\hat{\mathbf{r}}$$

The gravitional force on this atom is

$$m_p\mathbf{G}_R\,(R) = -\frac{4\pi}{3}GNm_p^2R\,\hat{\mathbf{r}}$$

The net force on the atom is

$$\mathbf{F} = \left(\frac{1}{3}\frac{Ny^2e^2}{\varepsilon_o}R - \frac{4\pi}{3}GNm_p^2R\right)\hat{\mathbf{r}}$$

The critical value is when

$$\frac{1}{3}\frac{Ny_C^2e^2}{\varepsilon_o}R = \frac{4\pi}{3}GNm_p^2R$$

$$\Rightarrow y_c^2 = 4\pi\varepsilon_o G\frac{m_p^2}{e^2}$$

$$\approx \frac{7\times10^{-11}\times1.8^2\times10^6\times81\times10^{-62}}{9\times10^9\times1.6^2\times10^{-38}}$$

$$\approx 63\times10^{-38}$$

$\therefore y_C \approx 8\times10^{-19} \approx 10^{-18}$

(b) Because of the net force, the hydrogen atom experiences an acceleration such that

$$m_p \frac{d^2R}{dt^2} = \left(\frac{1}{3}\frac{Ny^2e^2}{e_o} R - \frac{4p}{3} GNm_p^2 R \right)$$

Or, $\frac{d^2R}{dt^2} = a^2 R$ where $\alpha^2 = \frac{1}{m_p}\left(\frac{1}{3}\frac{Ny^2e^2}{e_o} - \frac{4p}{3} GNm_p^2 \right)$

This has a solution $R = Ae^{at} + Be^{-at}$

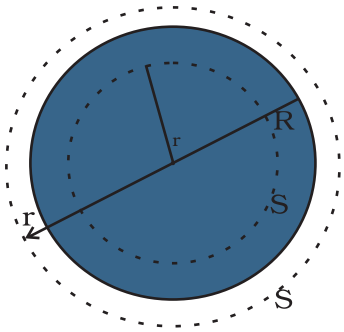
As we are seeking an expansion, $B = 0$.

$$\therefore R = Ae^{\alpha t}$$

$$\Rightarrow \dot{R} = \alpha A e^{\alpha t} = \alpha R$$

Thus, the velocity is proportional to the distance from the centre.

1.27 (a) The symmetry of the problem suggests that the electric field is radial. For points $r < R$, consider a spherical Gaussian surfaces. Then on the surface



$$\oiint \mathbf{E_r.dS} = \frac{1}{\varepsilon_o}\int_V \rho dv$$

$$4\pi r^2 E_r = \frac{1}{\varepsilon_o} 4\pi k \int_o^r r'^3 dr'$$

$$= \frac{1}{\varepsilon_o}\frac{4\pi k}{4} r^4$$

$$\therefore E_r = \frac{1}{4\varepsilon_o} k r^2$$

$$\mathbf{E}(r) = \frac{1}{4\varepsilon_o} k r^2 \hat{\mathbf{r}}$$

For points $r > R$, consider a spherical Gaussian surfaces' of radius r,

$$\oiint \mathbf{E}_r.d\mathbf{S} = \frac{1}{\varepsilon_o}\int_V \rho dv$$

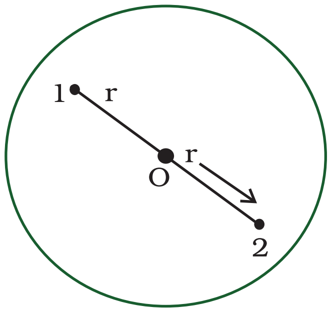
$$4\pi r^2 E_r = \frac{4\pi k}{\varepsilon_o}\int_o^R r^3 dr$$

$$= \frac{4\pi k}{\varepsilon_o}\frac{R^4}{4}$$

$$\therefore E_r = \frac{k}{4\varepsilon_o}\frac{R^4}{r^2}$$

$$\mathbf{E}(r) = (k / 4\varepsilon_o)\,(R^4 / r^2)\hat{\mathbf{r}}$$

(b) The two protons must be on the opposite sides of the centre along a diameter. Suppose the protons are at a distance r from the centre.



Now, $4\pi\int_o^R kr'^3 dr = 2e$

$$\therefore \frac{4\pi k}{4}R^4 = 2e$$

$$\therefore k = \frac{2e}{\pi R^4}$$

Consider the forces on proton 1. The attractive force due to the charge distribution is

$$-e\mathbf{E}_r = -\frac{e}{4\varepsilon_o}kr^2\hat{\mathbf{r}} = -\frac{2e^2}{4\pi\varepsilon_o}\frac{r^2}{R^4}\hat{\mathbf{r}}$$

The repulsive force is $\dfrac{e^2}{4\pi\varepsilon_o}\dfrac{1}{(2r)^2}\hat{\mathbf{r}}$

Net force is $\left(\dfrac{e^2}{4\pi\varepsilon_o 4r^2} - \dfrac{2e^2}{4\pi\varepsilon_o}\dfrac{r^2}{R^4}\right)\hat{\mathbf{r}}$

This is zero such that

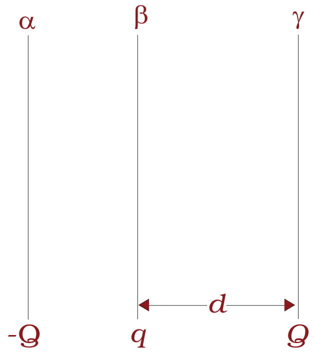
$$\frac{e^2}{16\pi\varepsilon_o r^2} = \frac{2e^2}{4\pi\varepsilon_o}\frac{r^2}{R^4}$$

$$\text{Or, } r^4 = \frac{4R^4}{32} = \frac{R^4}{8}$$

$$\Rightarrow r = \frac{R}{(8)^{1/4}}$$

Thus, the protons must be at a distance $r = \dfrac{R}{\sqrt[4]{8}}$ from the centre.

1.28 (a) The electric field at γ due to plate α is $-\dfrac{Q}{S2\varepsilon_o}\hat{\mathbf{x}}$



The electric field at γ due to plate β is $\dfrac{q}{S2\varepsilon_o}\hat{\mathbf{x}}$

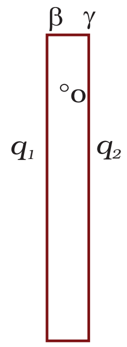
Hence, the net electric field is

$$\mathbf{E}_1 = \frac{(Q-q)}{2\varepsilon_o S}(-\hat{\mathbf{x}})$$

(b) During the collision plates β & γ are together and hence must be at one potential. Suppose the charge on β is q_1 and on γ is q_2. Consider a point O. The electric field here must be zero.

Electric field at 0 due to $\alpha = -\dfrac{Q}{2\varepsilon_o S}\hat{\mathbf{x}}$

Electric field at 0 due to $\beta = -\dfrac{q_1}{2\varepsilon_o S}\hat{\mathbf{x}}$



Electric Field at 0 due to $\gamma = -\dfrac{q_2}{2\varepsilon_o S}\hat{\mathbf{x}}$

$$\therefore \frac{-(Q+q_2)}{2\varepsilon_o S} + \frac{q_1}{2\varepsilon_o S} = 0$$

$$\Rightarrow q_1 - q_2 = Q$$

Further, $q_1 + q_2 = Q + q$

$$\Rightarrow q_1 = Q + q/2$$

and $q_2 = q/2$

Thus the charge on β and γ are $Q + q/2$ and $q/2$, respectively.

(c) Let the velocity be v at the distance d after the collision. If m is the mass of the plate γ, then the gain in K.E. over the round trip must be equal to the work done by the electric field.

After the collision, the electric field at γ is

$$\mathbf{E}_2 = -\frac{Q}{2\varepsilon_o S}\hat{\mathbf{x}} + \frac{(Q+q/2)}{2\varepsilon_o S}\hat{\mathbf{x}} = \frac{q/2}{2\varepsilon_o S}\hat{\mathbf{x}}$$

The work done when the plate γ is released till the collision is $F_1 d$ where F_1 is the force on plate γ.

The work done after the collision till it reaches d is $F_2 d$ where F_2 is the force on plate γ.

$$F_1 = E_1 Q = \frac{(Q-q)Q}{2\varepsilon_o S}$$

and $F_2 = E_2 q/2 = \dfrac{(q/2)^2}{2\varepsilon_o S}$

$\therefore$ Total work done is

$$\frac{1}{2\varepsilon_o S}\left[(Q-q)Q + (q/2)^2\right]d = \frac{1}{2\varepsilon_o S}(Q-q/2)^2 d$$

$$\Rightarrow (1/2)mv^2 = \frac{d}{2\varepsilon_o S}(Q-q/2)^2$$

$$\therefore v = (Q-q/2)\left(\frac{d}{m\varepsilon_o S}\right)^{1/2}$$

1.29 (i) $F = \dfrac{Q_q}{r^2} = 1\,\text{dyne} = \dfrac{[1\,\text{esu of charge}]^2}{[1\,\text{cm}]^2}$

Or,

1 esu of charge = 1 $(\text{dyne})^{1/2}$ (cm)

Hence, [1 esu of charge] = $[F]^{1/2} L = [MLT^{-2}]^{1/2} L = M^{1/2} L^{3/2} T^{-1}$

[1 esu of charge] = $M^{1/2} L^{3/2} T^{-1}$

Thus charge in cgs unit is expressed as fractional powers (1/2) of M and (3/2) of L.

(ii) Consider the coloumb force on two charges, each of magnitude 1 esu of charge separated by a distance of 1 cm:
The force is then 1 dyne = 10^{-5} N.
This situation is equivalent to two charges of magnitude x C separated by 10^{-2}m.
This gives:

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{x^2}{10^{-4}}$$

which should be 1 dyne = 10^{-5} N. Thus

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{x^2}{10^{-4}} = 10^{-5} \Rightarrow \frac{1}{4\pi\varepsilon_0} = \frac{10^{-9}}{x^2} \frac{\text{Nm}^2}{\text{C}^2}$$

With $x = \frac{1}{[3] \times 10^9}$, this yields

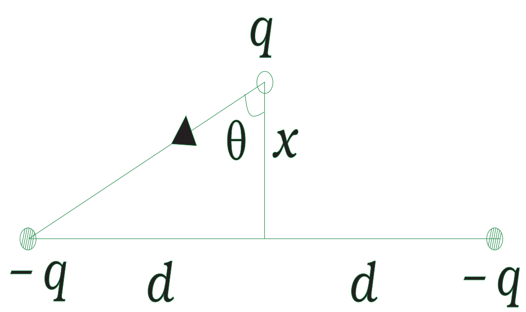
$$\frac{1}{4\pi\varepsilon_0} = 10^{-9} \times [3]^2 \times 10^{18} = [3]^2 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$$

With [3] $\rightarrow$ 2.99792458, we get

$$\frac{1}{4\pi\varepsilon_0} = 8.98755.... \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \text{ exactly}$$

1.30 Net force F on q towards the centre O

$$F = 2\frac{q^2}{4\pi\varepsilon_0 r^2}\cos\theta = -\frac{2q^2}{4\pi\varepsilon_0 r^2} \cdot \frac{x}{r}$$



$$F = \frac{-2q^2}{4\pi\varepsilon_0} \frac{x}{(d^2 + x^2)^{3/2}}$$

$$\approx \frac{-2q^2}{4\pi\varepsilon_0 d^3} x = -kx \text{ for } x << d.$$

Thus, the force on the third charge q is proportional to the displacement and is towards the centre of the two other charges. Therefore, the motion of the third charge is harmonic with frequency

$$\omega = \sqrt{\frac{2q^2}{4\pi\varepsilon_0 d^3 m}} = \sqrt{\frac{k}{m}}$$

and hence $T = \frac{2\pi}{\omega}\left[\frac{8\pi^3\varepsilon_0 m d^3}{q^2}\right]^{1/2}$.

1.31 (a) Slight push on q along the axis of the ring gives rise to the situation shown in Fig (b). A and B are two points on the ring at the end of a diameter.

Force on q due to line elements $\frac{-Q}{2\pi R}$ at A and B is

$$F_{A+B} = 2.\frac{-Q}{2\pi R}.q.\frac{1}{4\pi\varepsilon_0}.\frac{1}{r^2}.\cos\theta$$

$$= \frac{-Qq}{\pi R.4\pi\varepsilon_0}.\frac{1}{(z^2+R^2)}.\frac{z}{(z^2+R^2)^{1/2}}$$

Total force due to ring on q = $(F_{A+B})(\pi R)$

$$= \frac{-Qq}{4\pi\varepsilon_0}\frac{z}{(z^2+R^2)^{3/2}}$$

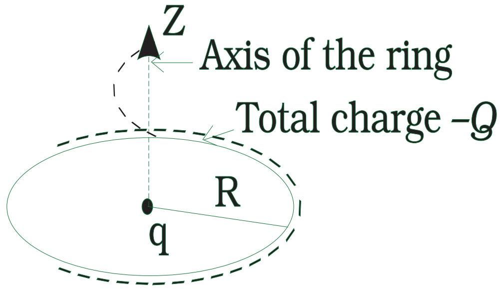
$$\approx \frac{-Qq}{4\pi\varepsilon_0}\frac{z}{R^3} \text{ for } z << R$$

Thus, the force is propotional to negative of displacement. Motion under such forces is harmonic.

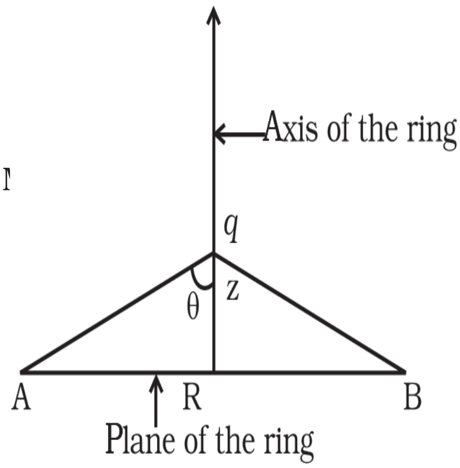
(b) From (a)

$$m\frac{d^2z}{dt^2} = -\frac{Qqz}{4\pi\varepsilon_0 R^3} \text{ or } \frac{d^2z}{dt^2} = -\frac{Qq}{4\pi\varepsilon_0 mR^3}z$$

That is, $\omega^2 = \frac{Qq}{4\pi\varepsilon_0 mR^3}$. Hence $T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mR}{Qq}}$



(a)



(b)