Physics

NCERT Exemplar Problems

Chapter 12

Atoms

Answers

12.1 (c)

12.2 (c)

12.3 (a)

12.4 (a)

12.5 (a)

12.6 (a)

12.7 (a)

12.8 (a), (c)

12.9 (a), (b)

12.10 (a), (b)

12.11 (b), (d)

12.12 (b), (d)

12.13 (c), (d)

12.14 Einstein's mass-energy equivalence gives $E = mc^2$. Thus the mass of a H-atom is $m_p + m_e - \frac{B}{c^2}$ where B ≈ 13.6eV is the binding energy.

12.15 Because both the nuclei are very heavy as compared to electron mass.

12.16 Because electrons interact only electromagnetically.

12.17 Yes, since the Bohr formula involves only the product of the charges.

12.18 No, because accoding to Bohr model, $E_n = -\frac{13.6}{n^2}$,

and electons having different energies belong to different levels having different values of n. So, their angular momenta will be different, as $mvr = \frac{nh}{2\pi}$.

12.19 The 'm' that occurs in the Bohr formula $E_n = -\frac{me^4}{8\varepsilon_0 n^2 h^2}$ is the reduced mass. For H-atom $m \approx m_e$. For positronium $m \approx m_e/2$. Hence for a positonium $E_1 \approx -6.8$eV.

12.20 For a nucleus with charge 2e and electrons of charge $-e$, the levels are $E_n = -\frac{4me^4}{8\varepsilon_0{}^2 n^2 h^2}$. The ground state will have two electrons each of energy E, and the total ground state energy would by –(4×13.6)eV.

12.21 υ = velocity of electron

a_0= Bohr radius.

$\therefore$Number of revolutions per unit time $= \frac{2\pi a_0}{\upsilon}$

$\therefore$ Current $= \frac{2\pi a_0}{\upsilon} e.$

12.22 $\nu_{\text{mn}} = cRZ^2\left[\frac{1}{(n+p)^2} - \frac{1}{n^2}\right],$

where $m = n + p$, (p = 1, 2, 3, ...) and R is Rydberg constant.

For $p << n$.

$$\nu_{mn} = cRZ^2\left[\frac{1}{n^2}\left(1+\frac{p}{n}\right)^{-2} - \frac{1}{n^2}\right]$$

$$\nu_{mn} = cRZ^2\left[\frac{1}{n^2} - \frac{2p}{n^3} - \frac{1}{n^2}\right]$$

$$\nu_{mn} = cRZ^2\frac{2p}{n^3} ; \left(\frac{2cRZ^2}{n^3}\right)p$$

Thus, ν_{mn} are approximately in the order 1, 2, 3..........

12.23 H_γ in Balmer series corresponds to transition n = 5 to n = 2. So the electron in ground state n = 1 must first be put in state n = 5. Energy required = $E_1 - E_5$ = 13.6 – 0.54 = 13.06 eV.

If angular momentum is conserved, angular momentum of photon = change in angular momentum of electron

$= L_5 - L_2 = 5\text{h} - 2\text{h} = 3\text{h} = 3 \times 1.06 \times 10^{-34}$

= 3.18 × 10^{-34} kg m^2/s.

12.24 Reduced mass for $H = \mu_H = \frac{m_e}{1+\frac{m_e}{M}} ; m_e\left(1 - \frac{m_e}{M}\right)$

Reduced mass for $D = \mu_D$; $m_e\left(1-\frac{m_e}{2M}\right) = m_e\left(1-\frac{m_e}{2M}\right)\left(1+\frac{m_e}{2M}\right)$

$h\nu_{ij} = (E_i - E_j)\alpha\,\mu$. Thus, $\lambda_{ij}\,\alpha\,\frac{1}{\mu}$

If for Hydrogen/Deuterium the wavelength is λ_H / λ_D

$$\frac{\lambda_D}{\lambda_H} = \frac{\mu_H}{\mu_D} \; ; \; \left(1+\frac{m_e}{2M}\right)^{-1} \; ; \; \left(1-\frac{1}{2\times 1840}\right)$$

$$\lambda_D = \lambda_H \times (0.99973)$$

Thus lines are 1217.7 Å , 1027.7 Å, 974.04 Å, 951.143 Å.

12.25 Taking into account the nuclear motion, the stationary state energies shall be, $E_n = -\frac{\mu Z^2 e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n^2}\right)$. Let μ_H be the reduced mass of Hydrogen and μ_D that of Deutrium. Then the frequency of the 1st Lyman line in Hydrogen is $h\nu_H = \frac{\mu_H e^4}{8\varepsilon_0^2 h^2}\left(1-\frac{1}{4}\right) = \frac{3}{4}\frac{\mu_H e^4}{8\varepsilon_0^2 h^2}$. Thus the wavelength of the transition is $\lambda_H = \frac{3}{4}\frac{\mu_H e^4}{8\varepsilon_0^2 h^3 c}$. The wavelength of the transition for the same line in Deutrium is $\lambda_D = \frac{3}{4}\frac{\mu_D e^4}{8\varepsilon_0^2 h^3 c}$.

$$\therefore \Delta\lambda = \lambda_D - \lambda_H$$

Hence the percentage difference is

$$100\times\frac{\Delta\lambda}{\lambda_H} = \frac{\lambda_D - \lambda_H}{\lambda_H}\times 100 = \frac{\mu_D - \mu_H}{\mu_H}\times 100$$

$$= \frac{\dfrac{m_e M_D}{(m_e + M_D)} - \dfrac{m_e M_H}{(m_e + M_H)}}{m_e M_H / (m_e + M_H)}\times 100$$

$$= \left[\left(\frac{m_e + M_H}{m_e + M_D}\right)\frac{M_D}{M_H} - 1\right]\times 100$$

Since $m_e << M_H < M_D$

$$\frac{\Delta\lambda}{\lambda_H}\times 100 = \left[\frac{M_H}{M_D}\times\frac{M_D}{M_H}\left(\frac{1+m_e/M_H}{1+m_e/M_D}\right)-1\right]\times 100$$

$$= \left[(1+m_e/M_H)(1+m_e/M_D)^{-1}-1\right]\times 100$$

$$; \left[(1+\frac{m_e}{M_H}-\frac{m_e}{M_D}-1\right]\times 100$$

$$\approx m_e\left[\frac{1}{M_H}-\frac{1}{M_D}\right]\times 100$$

$$= 9.1\times 10^{-31}\left[\frac{1}{1.6725\times 10^{-27}}-\frac{1}{3.3374\times 10^{-27}}\right]\times 100$$

$$= 9.1\times 10^{-4}[0.5979-0.2996]\times 100$$

$$= 2.714 \times 10^{-2}\,\%$$

12.26 For a point nucleus in H-atom:

$$\text{Ground state: } mvr = \text{h}, \frac{mv^2}{r_B} = -\frac{e^2}{{r_B}^2}.\frac{1}{4\pi\varepsilon_0}$$

$$\therefore m\frac{\text{h}^2}{m^2{r_B}^2}.\frac{1}{r_B} = +\left(\frac{e^2}{4\pi\varepsilon_0}\right)\frac{1}{{r_B}^2}$$

$$\therefore \frac{\hbar^2}{m}.\frac{4\pi\varepsilon_0}{e^2} = r_B = 0.51\,\text{Å}$$

Potential energy

$$-\left(\frac{e^2}{4\pi r_0}\right).\frac{1}{r_B} = -27.2eV; K.E = \frac{mv^2}{2} = \frac{1}{2}m.\frac{\hbar^2}{m^2 r_B^2} = \frac{\hbar}{2mr_B^2} = +13.6\text{eV}$$

For an spherical nucleus of radius R,

If $R < r_B$, same result.

If $R >> r_B$: the electron moves inside the sphere with radius r'_B (r'_B = new Bohr radius).

$$\text{Charge inside } r_B'^{\,4} = e\left(\frac{r_B'^3}{R^3}\right)$$

$$\therefore r'_B = \frac{h^2}{m}\left(\frac{4\pi\varepsilon_0}{e^2}\right)\frac{R^3}{r'^3_B}$$

$$r'^4_B = (0.51\,\mathring{A}).R^3. \qquad R = 10\,\mathring{A}$$

$$= 510(\mathring{A})^4$$

$$\therefore r'_B \approx (510)^{1/4}\,\mathring{A} < R.$$

$$K.E = \frac{1}{2}mv^2 = \frac{m}{2}.\frac{h}{m^2 r'^2_B} = \frac{h}{2m}.\frac{1}{r'^2_B}$$

$$= \left(\frac{h^2}{2mr_B{}^2}\right).\left(\frac{r_B^2}{r'^2_B}\right) = (13.6\text{eV})\frac{(0.51)^2}{(510)^{1/2}} = \frac{3.54}{22.6} = 0.16\text{eV}$$

$$P.E = +\left(\frac{e^2}{4\pi\varepsilon_0}\right).\left(\frac{r'^2_B - 3R^2}{2R^3}\right)$$

$$= +\left(\frac{e^2}{4\pi\varepsilon_0}.\frac{1}{r_B}\right).\left(\frac{r_B(r'^2_B - 3R^2}{R^3}\right)$$

$$= +(27.2\text{eV})\left[\frac{0.51(\sqrt{510} - 300)}{1000}\right]$$

$$= +(27.2\text{eV}).\frac{-141}{1000} = -3.83\text{eV}.$$

12.27 As the nucleus is massive, recoil momentum of the atom may be neglected and the entire energy of the transition may be considered transferred to the Auger electron. As there is a single valence electron in Cr, the energy states may be thought of as given by the Bohr model.

The energy of the nth state $E_n = -Z^2 R\frac{1}{n^2}$ where R is the Rydberg constant and Z = 24.

The energy released in a transition from 2 to 1 is $\Delta E = Z^2 R\left(1 - \frac{1}{4}\right) = \frac{3}{4}Z^2 R$. The energy required to eject a n = 4 electron is $E_4 = Z^2 R\frac{1}{16}$.

Thus the kinetic energy of the Auger electron is

$$K.E = Z^2 R\left(\frac{3}{4} - \frac{1}{16}\right) = \frac{1}{16} Z^2 R$$

$$= \frac{11}{16} \times 24 \times 24 \times 13.6\,\text{eV}$$

= 5385.6 eV

12.28 $m_p c^2$ = 10^{-6} × electron mass × c^2

$$\approx 10^{-6} \times 0.5\,\text{MeV}$$

$$\approx 10^{-6} \times 0.5 \times 1.6 \times 10^{-13}$$

$$\approx 0.8 \times 10^{-19}\,\text{J}$$

$$\frac{h}{m_p c} = \frac{hc}{m_p c^2} = \frac{10^{-34} \times 3 \times 10^8}{0.8 \times 10^{-19}} \approx 4 \times 10^{-7}\,\text{m} >> \text{Bohr radius.}$$

$$|\mathbf{F}| = \frac{e^2}{4\pi\varepsilon_0}\left[\frac{1}{r^2} + \frac{\lambda}{r}\right]\exp(-\lambda r)$$

$$\text{where } \lambda^{-1} = \frac{\hbar}{m_p c} \approx 4 \times 10^{-7}\,\text{m} >> r_B$$

$$\therefore \lambda << \frac{1}{r_B} \; i.e\; \lambda r_B << 1$$

$$U(r) = -\frac{e^2}{4\pi\varepsilon_0} \cdot \frac{\exp(-\lambda r)}{r}$$

$$mvr = h \therefore v = \frac{h}{mr}$$

$$\text{Also}: \frac{mv^2}{r} =\approx \left(\frac{e^2}{4\pi\varepsilon_0}\right)\left[\frac{1}{r^2} + \frac{\lambda}{r}\right]$$

$$\therefore \frac{h^2}{mr^3} = \left(\frac{e^2}{4\pi\varepsilon_0}\right)\left[\frac{1}{r^2} + \frac{\lambda}{r}\right]$$

$$\therefore \frac{h^2}{m} = \left(\frac{e^2}{4\pi\varepsilon_0}\right)[r + \lambda r^2]$$

If $\lambda = 0; r = r_B = \frac{h}{m}.\frac{4\pi\varepsilon_0}{e^2}$

$$\frac{h^2}{m} = \frac{e^2}{4\pi\varepsilon_0}.r_B$$

Since $\lambda^{-1} >> r_B$, put $r = r_B + \delta$

$\therefore r_B = r_B + \delta + \lambda(r_B^2 + \delta^2 + 2\delta r_B)$; negect δ^2

or $0 = \lambda r_B^2 + \delta(1 + 2\lambda r_B)$

$$\delta = \frac{-\lambda r_B{}^2}{1+2\lambda r_B} \approx \lambda r_B{}^2(1-2\lambda r_B) = -\lambda r_B{}^2 \text{ since } \lambda r_B << 1$$

$$\therefore V(r) = -\frac{e^2}{4\pi\varepsilon_0}.\frac{\exp(-\lambda\delta - \lambda r_B)}{r_B + \delta}$$

$$\therefore V(r) = -\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r_B}\left[\left(1 - \frac{\delta}{r_B}\right).(1 - \lambda r_B)\right]$$

$\cong (-27.2\text{eV})$ remains unchanged.

$$K.E = -\frac{1}{2}mv^2 = \frac{1}{2}m.\frac{h^2}{mr^2} = \frac{h^2}{2(r_B+\delta)^2} = \frac{h^2}{2r_B{}^2}\left(1 - \frac{2\delta}{r_B}\right)$$

$= (13.6\text{eV})[1 + 2\lambda r_B]$

Total energy $= -\frac{e^2}{4\pi\varepsilon_0 r_B} + \frac{h^2}{2r_B{}^2}[1 + 2\lambda r_B]$

$= -27.2 + 13.6[1 + 2\lambda r_B]\text{eV}$

Change in energy $= 13.6 \times 2\lambda r_B\text{eV} = 27.2\lambda r_B\text{eV}$

12.29 Let $\varepsilon = 2 + \delta$

$$F = \frac{q_1q_2}{4\pi\varepsilon_0}.\frac{R_0^\delta}{r^{2+\delta}} = \wedge\frac{R_0^\delta}{r^{2+\delta}}, \text{ where } \frac{q_1q_2}{4\pi_0\varepsilon} = \wedge, \wedge = (1.6\times10^{-19})^2\times9\times10^9$$

$= 23.04\times10^{-29}$

$$= \frac{mv^2}{r}$$

$$v^2 = \frac{\wedge R_0^{\delta}}{m r^{1+\delta}}$$

(i) $mvr = n\text{h}$, $r = \frac{n\text{h}}{mv} = \frac{n\text{h}}{m}\left[\frac{m}{\wedge R_0^{\delta}}\right]^{1/2} r^{1/2+\delta/2}$

Solving this for r, we get $r_n = \left[\frac{n^2\hbar^2}{m \wedge R_0^{\delta}}\right]^{\frac{1}{1-\delta}}$

For $n = 1$ and substituting the values of constant, we get

$$r_1 = \left[\frac{\hbar^2}{m \wedge R_0^{\delta}}\right]^{\frac{1}{1-\delta}}$$

$r_1 = \left[\frac{1.05^2 \times 10^{-68}}{9.1 \times 10^{-31} \times 2.3 \times 10^{-28} \times 10^{+19}}\right]^{\frac{1}{2.9}} = 8 \times 10^{-11} = 0.08$ nm (< 0.1 nm)

(ii) $v_n = \frac{n\hbar}{mr_n} = n\hbar\left(\frac{m \wedge R_0^{\delta}}{n^2\hbar^2}\right)^{\frac{1}{1-\delta}}$. For $n = 1$, $v_1 = \frac{\hbar}{mr_1} = 1.44 \times 10^6$ m/s

(iii) K.E. $= \frac{1}{2}mv_1^2 = 9.43 \times 10^{-19}$ J=5.9eV

P.E. till $R_0 = -\frac{\wedge}{R_0}$

P.E. from R_0 to r $= + \wedge R_0^{\delta}\int_{R_0}^{r}\frac{dr}{r^{2+\delta}} = +\frac{\wedge R_0^{\delta}}{-1-\delta}\left[\frac{1}{r^{1+\delta}}\right]_{R_0}^{r}$

$$= -\frac{\wedge R_0^{\delta}}{1+\delta}\left[\frac{1}{r^{1+\delta}} - \frac{1}{R_0^{\,1+\delta}}\right]$$

$$= -\frac{\wedge}{1+\delta}\left[\frac{R_0^{\delta}}{r^{1+\delta}} - \frac{1}{R_0}\right]$$

$$P.E. = -\frac{\wedge}{1+\delta}\left[\frac{R_0^{\delta}}{r^{1+\delta}} - \frac{1}{R_0} + \frac{1+\delta}{R_0}\right]$$

$$P.E. = -\frac{\wedge}{-0.9}\left[\frac{R_0^{-1.9}}{r^{-0.9}} - \frac{1.9}{R_0}\right]$$

$$= \frac{2.3}{0.9} \times 10^{-18}[(0.8)^{0.9} - 1.9] \text{ J} = -17.3 \text{ eV}$$

Total energy is (–17.3 + 5.9) = –11.4 eV.