Physics

NCERT Exemplar Problems

Chapter 2

Electrostatic Potential and Capacitance

2.1	(d)
2.2	(c)
2.3	(c)
2.4	(c)
2.5	(a)
2.6	(c)
2.7	(b), (c), (d)
2.8	(a), (b), (c)
2.9	(b), (c)
2.10	(b), (c)
2.11	(a), (d)
2.12	(a), (b)
2.13	(c) and (d)
2.14	More.

2.15 Higher potential.

2.16 Yes, if the sizes are different.

2.17 No.

2.18 As electric field is conservative, work done will be zero in both the cases.

2.19 Suppose this were not true. The potential just inside the surface would be different from that at the surface resulting in a potential gradient. This would mean that there are field lines pointing inwards or outwards from the surface. These lines cannot at the other end be again on the surface, since the surface is equipotential. Thus, this is possible only if the other end of the lines are at charges inside, contradicting the premise. Hence, the entire volume inside must be at the same potential.

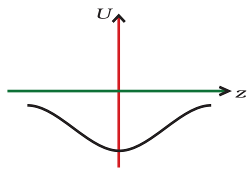
2.20 C will decrease

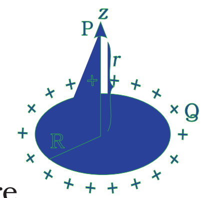
Energy stored = $\frac{1}{2}CV^2$ and hence will increase.

Electric field will increase.
Charge stored will remain the same.
V will increase.

2.21 Consider any path from the charged conductor to the uncharged conductor along the electric field. The potential will continually decrease along this path. A second path from the uncharged conductor to infinity will again continually lower the potential further. Hence this result.

2.22 $$U = \frac{-qQ}{4\pi\varepsilon_0 R\sqrt{1+z^2/R^2}}$$



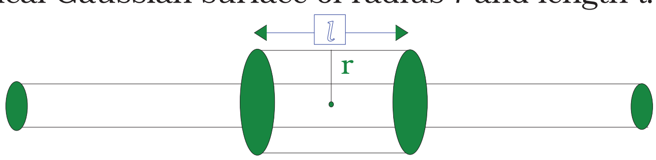


The variation of potential energy with z is shown in the figure.

The charge $-q$ displaced would perform oscillations. We cannot conclude anything just by looking at the graph.

2.23 $$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+z^2}}$$

2.24 To find the potential at distance r from the line consider the electric field. We note that from symmetry the field lines must be radially outward. Draw a cylindrical Gaussian surface of radius r and length l. Then



$$\oint \mathbf{E}.d\mathbf{S} = \frac{1}{\varepsilon_0}\lambda l$$

$$\text{Or } E_r 2\pi r l = \frac{1}{\varepsilon_0}\lambda l$$

$$\Rightarrow E_r = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Hence, if r_0 is the radius,

$$V(r) - V(r_0) = -\int_{r_0}^{r} \mathbf{E}.d\mathbf{l} = \frac{\lambda}{2\pi\varepsilon_0} ln\frac{r_0}{r}$$

For a given V,

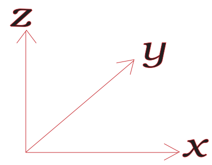
$$ln\ \frac{r}{r_0} = -\frac{2\pi\varepsilon_0}{\lambda}[V(r) - V(r_0)]$$

$$\Rightarrow r = r_0 e^{-2\pi\varepsilon_0 V r_0/\lambda}\ e^{+2\pi\varepsilon_0 V(r)/\lambda}$$

The equipotential surfaces are cylinders of radius

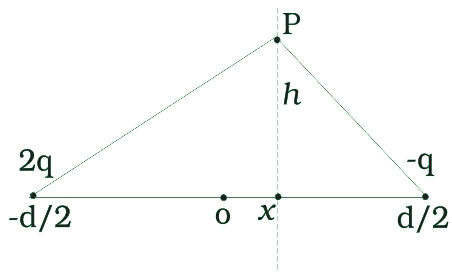
$$r = r_0 e^{-2\pi\varepsilon_0[V(r) - V(r_0)]/\lambda}$$

2.25 Let the plane be at a distance x from the origin. The potential at the point P is



$$\frac{1}{4\pi\varepsilon_0}\frac{q}{\left[(x+d/2)^2+h^2\right]^{1/2}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{\left[(x-d/2)^2+h^2\right]^{1/2}}$$

If this is to be zero.



$$\frac{1}{\left[(x+d/2)^2+h^2\right]^{1/2}} = \frac{1}{\left[(x-d/2)^2+h^2\right]^{1/2}}$$

Or, $(x-d/2)^2 + h^2 = (x+d/2)^2 + h^2$

$$\Rightarrow x^2 - dx + d^2/4 = x^2 + dx + d^2/4$$

Or, $2dx = 0$

$\Rightarrow\ x = 0$

The equation is that of a plane $x = 0$.

2.26 Let the final voltage be U: If C is the capacitance of the capacitor without the dielectric, then the charge on the capacitor is

$Q_1 = CU$

The capacitor with the dielectric has a capacitance εC. Hence the charge on the capacitor is

$$Q_2 = \varepsilon U \quad = \quad \alpha C U^2$$

The initial charge on the capacitor that was charged is

$Q_0 = CU_0$

From the conservation of charges,

$Q_0 = Q_1 + Q_2$

Or, $CU_0 = CU + \alpha CU^2$

$$\Rightarrow \alpha U^2 + U - u_0 = 0$$

$$\therefore U = \frac{-1 \pm \sqrt{1+4\alpha U_0}}{2\alpha}$$

$$= \frac{-1 \pm \sqrt{1+624}}{4}$$

$$= \frac{-1 \pm \sqrt{625}}{4} \text{ volts}$$

As U is positive

$$U = \frac{\sqrt{625}-1}{4} = \frac{24}{4} = 6\text{V}$$

2.27 When the disc is in touch with the bottom plate, the entire plate is a equipotential. A change q' is transferred to the disc.

The electric field on the disc is

$$= \frac{V}{d}$$

$$\therefore q' = -\varepsilon_0 \frac{V}{d} \pi r^2$$

The force acting on the disc is

$$-\frac{V}{d} \times q' = \varepsilon_0 \frac{V^2}{d^2} \pi r^2$$

If the disc is to be lifted, then

$$\varepsilon_0 \frac{V^2}{d^2} \pi r^2 = mg$$

$$\Rightarrow V = \sqrt{\frac{mgd^2}{\pi \varepsilon_0 r^2}}$$

2.28 $$\text{U} = \frac{1}{4\pi\varepsilon_0} \left\{ \frac{q_d q_d}{r} - \frac{q_u q_d}{r} - \frac{q_u q_d}{r} \right\}$$

$$= \frac{9\times10^9}{10^{-15}} \left(1.6\times10^{-19}\right)^2 \left\{(1/3)^2 - (2/3)(1/3) - (2/3)(1/3)\right\}$$

$$= 2.304 \times 10^{-13} \left\{\frac{1}{9} - \frac{4}{9}\right\} = -7.68\times10^{-14}\text{J}$$

$$= 4.8 \times 10^5 \text{ eV} = 0.48 \text{ MeV} = 5.11 \times 10^{-4} \, (m_n c^2)$$

2.29 Before contact

$$Q_1 = \sigma.4\pi R^2$$

$$Q_2 = \sigma.4\pi(2R^2) = 4\left(\sigma.4\pi R^2\right) = 4Q_1$$

After contact :

$$Q_1' + Q_2' = Q_1 + Q_2 = 5Q_1,$$

$$= 5(\sigma.4\pi R^2)$$

They will be at equal potentials:

$$\frac{Q_1'}{R} = \frac{Q_2'}{2R}$$

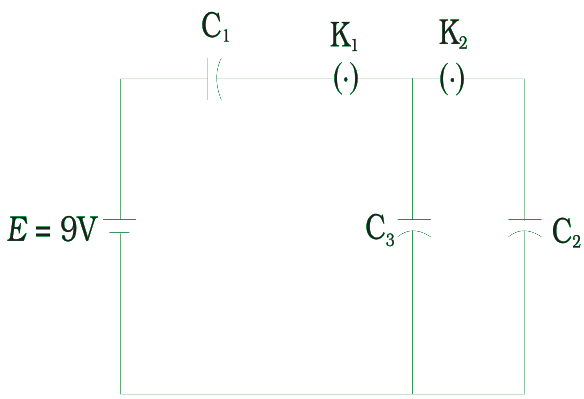
$$\therefore Q_2' = 2Q'.$$

$$\therefore 3Q_1' = 5\left(\sigma.4\pi R^2\right)$$

$$\therefore Q_1' = \frac{5}{3}\left(\sigma.4\pi R^2\right) \text{ and } Q_2' = \frac{10}{3}\left(\sigma.4\pi R^2\right)$$

$$\therefore \sigma_1 = 5/3\,\sigma \quad \text{and} \quad \therefore \sigma_2 = \frac{5}{6}\sigma.$$

2.30 Initially : $V \propto \dfrac{1}{C}$ and $V_1 + V_2 = E$



$\Rightarrow V_1 = 3\,V$ and $V_2 = 6V$

$\therefore Q_1 = C_1 V_1 = 6C \times 3 = 18\,\mu C$

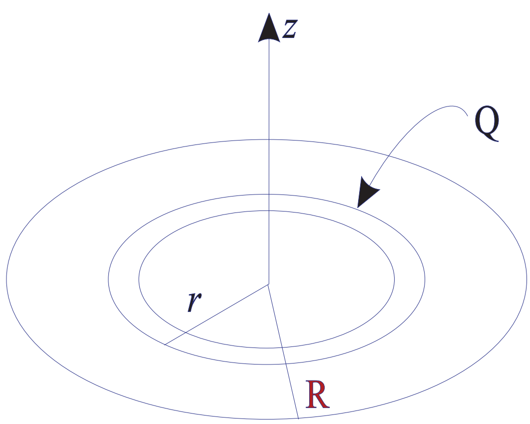
$Q_2 = 9\,\mu C$ and $Q_3 = 0$

Later : $Q_2 = Q_2' + Q_3$

with $C_2 V + C_3 V = Q_2$ $\quad \Rightarrow V = \dfrac{Q_2}{C_2 + C_3} = (3/2)\,V$

$Q_2' = (9/2)\,\mu C$ and $Q_3' = (9/2)\,\mu C$

2.31 $\sigma = \dfrac{Q}{\pi R^2}$



$$dU = \frac{1}{4\pi\varepsilon_0}\,\frac{\sigma.2\pi r\,dr}{\sqrt{r^2 + z^2}}$$

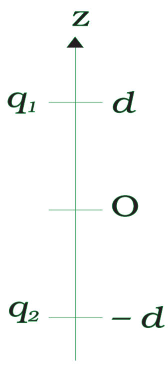
$$\therefore U = \frac{\pi\sigma}{4\pi\varepsilon_0}\int_0^R \frac{2r dr}{\sqrt{r^2 + z^2}}$$

$$= \frac{2\pi\sigma}{4\pi\varepsilon_0}\left[\sqrt{r^2+z^2}\right]_0^R = \frac{2\pi\sigma}{4\pi\varepsilon_0}\left[\sqrt{R^2+z^2}-z\right]$$

$$= \frac{2Q}{4\pi\varepsilon_0 R^2}\left[\sqrt{R^2+z^2}-z\right]$$

2.32 $$\frac{q_1}{\sqrt{x^2+y^2+(z-d)^2}} + \frac{q_2}{\sqrt{x^2+y^2+(z+d)^2}} = 0$$

$$\therefore \frac{q_1}{\sqrt{x^2+y^2+(z-d)^2}} = \frac{-q_2}{\sqrt{x^2+y^2+(z+d)^2}}$$



Thus, to have total potential zero, q_1 and q_2 must have opposite signs. Squaring and simplifying, we get.

$$x^2+y^2+z^2+\left[\frac{(q_1/q_2)^2+1}{(q_1/q_2)^2-1}\right](2zd)+d^2=0$$

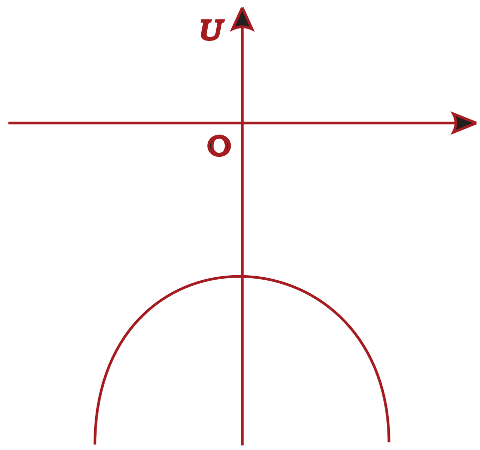
This is the equation of a sphere with centre at $\left(0,0,-2d\left[\frac{q_1^{\,2}+q_1^{\,2}}{q_1^{\,2}-q_1^{\,2}}\right]\right)$.

Note : if $q_1 = -q_2 \Rightarrow$ Then z = 0, which is a plane through mid-point.

2.33 $$U=\frac{1}{4\pi\varepsilon_0}\left\{\frac{-q^2}{(d-x)}+\frac{-q^2}{(d-x)}\right\}$$

$$U=\frac{-q^2}{4\pi\varepsilon_0}\,\frac{2d}{(d^2-x^2)}$$

$$\frac{dU}{dx}=\frac{-q^2.2d}{4\pi\in_0}.\,\frac{2x}{(d^2-x^2)^2}$$



$$U_0=\frac{2q^2}{4\pi\varepsilon_0 d} \qquad \frac{dU}{dx}=0 \text{ at } x=0$$

$x = 0$ is an equilibrium point.

$$\frac{d^2U}{dx^2}=\left(\frac{-2dq^2}{4\pi\in_0}\right)\left[\frac{2}{(d^2-x^2)^2}-\frac{8x^2}{(d^2-x^2)^3}\right]$$

$$= \left(\frac{-2dq^2}{4\pi \in_0} \right) \frac{1}{(d^2 - x^2)^3} \left[2\left(d^2 - x^2\right)^2 - 8x^2 \right]$$

At $x = 0$

$$\frac{d^2U}{dx^2} = \left(\frac{-2dq^2}{4\pi \in_0} \right) \left(\frac{1}{d^6} \right) (2d^2)$$, which is < 0.

Hence, unstable equilibrium.