DCAM classes

Dynamic Classes for Academic Mastery

Physics

NCERT Exemplar Problems

Chapter 9

Ray Optics and Optical Instruments

Answers

9.1 (a)

9.2 (d)

9.3 (c)

9.4 (b)

9.5 (c)

9.6 (c)

9.7 (b)

9.8 (b)

9.9 (b)

9.10 (d)

9.11 (a)

9.12 (a), (b), (c)

9.13 (d)

9.14 (a), (d)

9.15 (a), (b)

9.16 (a), (b), (c)

9.17 As the refractive index for red is less than that for blue, parallel beams of light incident on a lens will be bent more towards the axis for blue light compared to red. Thus the focal length for blue light will be smaller than that for red.

9.18 The near vision of an average person is 25cm. To view an object with magnification 10,

$$m = \frac{D}{f} \Rightarrow f = \frac{D}{m} = \frac{25}{10} = 2.5 = 0.025\text{m}$$

$$P = \frac{1}{0.025} = 40 \text{ diopters.}$$

9.19 No. The reversibility of the lens makes equation.

9.20 Let the apparent depth be O_1 for the object seen from μ_2 then

$$O_1 = \frac{\mu_2}{\mu_1}\frac{h}{3}$$

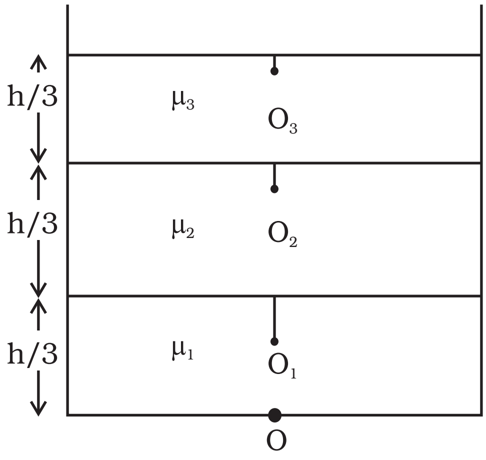
If seen from μ_3 the apparent depth is O_2.

$$O_2 = \frac{\mu_3}{\mu_2}\left(\frac{h}{3} + O_1\right) = \frac{\mu_3}{\mu_2}\left(\frac{h}{3} + \frac{\mu_2}{\mu_1}\frac{h}{3}\right) = \frac{h}{3}\left(\frac{\mu_3}{\mu_2} + \frac{\mu_3}{\mu_1}\right)$$

Seen from outside, the apparent height is

$$O_3 = \frac{1}{\mu_3}\left(\frac{h}{3}+O_2\right) = \frac{1}{\mu_3}\left[\frac{h}{3}+\frac{h}{3}\left(\frac{\mu_3}{\mu_2}+\frac{\mu_3}{\mu_1}\right)\right]$$

$$= \frac{h}{3}\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}+\frac{1}{\mu_3}\right)$$



9.21 At minimum deviation

$$\mu = \frac{\sin\left[\frac{(A+D_m)}{2}\right]}{\sin\left(\frac{A}{2}\right)}$$

$\therefore$ Given $D_m = A$

$$\therefore \ \mu = \frac{\sin A}{\sin \frac{A}{2}} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}} = 2\cos\frac{A}{2}$$

$$\therefore \ \cos\frac{A}{2} = \frac{\sqrt{3}}{2} \text{ or } \frac{A}{2} = 30° \ \therefore \ A = 60°$$

9.22 Let the two ends of the object be at distance $u_1 = u - L/2$ and $u_2 = u + L/2$, respectively, so that $|u_1 - u_2| = L$. Let the image of the two ends be formed at v_1 and v_2, so that the image length would be $L' = |v_1 - v_2|$. Since $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ or $v = \frac{fu}{u-f}$ the image of the two ends will be at $v_1 = \frac{f(u-L/2)}{u-f-L/2}$, $v_2 = \frac{f(u+L/2)}{u-f+L/2}$

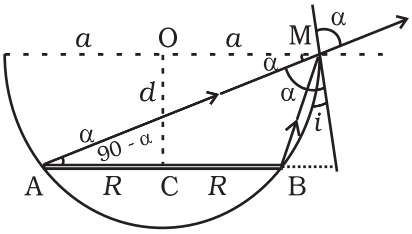
Hence

$$L' = |v_1 - v_2| = \frac{f^2 L}{(u-f)^2 \times L^2/4}$$

Since the object is short and kept away from focus, we have
$L^2/4 << (u-f)^2$
Hence finally

$$L' = \frac{f^2}{(u-f)^2} L.$$

9.23 Refering to the Fig., AM is the direction of incidence ray before liquid is filled. After liquid is filledm, BM is the direction of the incident ray. Refracted ray in both cases is same as that along AM.

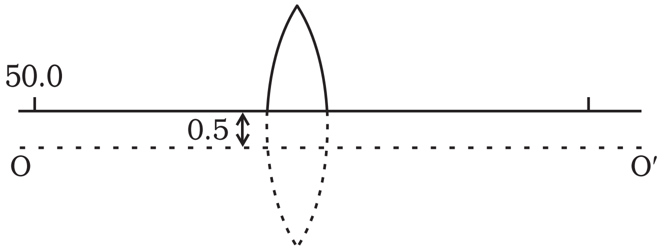


$$\frac{1}{\mu} = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin \alpha}$$

$$\sin i = \frac{a - R}{\sqrt{d^2 + (a - R)^2}} \quad \text{and} \quad \sin\alpha = \cos(90 - \alpha) = \frac{a + R}{\sqrt{d^2 + (a - R)^2}}$$

Substuting, we get $d = \dfrac{\mu(a^2 - R^2)}{\sqrt{(a + R)^2 - \mu(a - R)^2}}$

9.24



If there was no cut then the object would have been at a height of 0.5 cm from the principal axis 00′.
Consider the image for this case.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\therefore \frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{-50} + \frac{1}{25} = \frac{1}{50}$$

$\therefore v = 50$ cm.

Magnification is m = $\dfrac{v}{u} = -\dfrac{50}{50} = -1$.

Thus the image would have been formed at 50 cm from the pole and 0.5 cm below the principal axis.
Hence with respect to the X axis passing through the edge of the cut lens, the co-ordinates of the image are

(50 cm, –1 cm)

9.25 From the reversibility of u and v, as seen from the formula for lens,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

It is clear that there are two positions for which there shall be an image on the screen.
Let the first position be when the lens is at O.
Given $-u + v = D$
$\Rightarrow u = -(D - v)$
Placing it in the lens formula

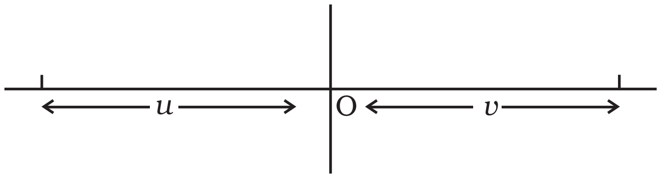
$$\frac{1}{D - v} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \frac{v+D-v}{(D-v)v} = \frac{1}{f}$$

$$\Rightarrow v^2 - Dv + \mathrm{D}f = 0$$

$$\Rightarrow v = \frac{D}{2} \pm \frac{\sqrt{\mathrm{D}^2 - 4Df}}{2}$$

$$u = -(D - v) = -\left(\frac{D}{2} \pm \frac{\sqrt{D^2 - 4Df}}{2}\right)$$



Thus, if the object distance is $\frac{D}{2} - \frac{\sqrt{D^2 - 4Df}}{2}$ then the image is at $\frac{D}{2} + \frac{\sqrt{D^2 - 4Df}}{2}$

If the object distance is $\frac{D}{2} + \frac{\sqrt{D^2 - 4Df}}{2}$, then the image is at $\frac{D}{2} - \frac{\sqrt{D^2 - 4Df}}{2}$.

The distance between the poles for these two object distances is

$$\frac{D}{2} + \frac{\sqrt{D^2 - 4Df}}{2} - \left(\frac{D}{2} - \frac{\sqrt{D^2 - 4\mathrm{D}f}}{2}\right) = \sqrt{D^2 - 4Df}$$

Let $d = \sqrt{D^2 - 4Df}$

If $u = \frac{D}{2} + \frac{d}{2}$ then the image is at $v = \frac{D}{2} - \frac{d}{2}$.

$\therefore$ The magnification $m_1 = \frac{\mathrm{D}-d}{\mathrm{D}+d}$

If $u = \frac{\mathrm{D}-d}{2}$ then $v = \frac{\mathrm{D}+d}{2}$

$\therefore$The magnification $m_2 = \frac{\mathrm{D}+d}{\mathrm{D}-d}$. Thus $\frac{m_2}{m_1} = \left(\frac{\mathrm{D}+d}{\mathrm{D}-d}\right)^2$.

9.26 Let d be the diameter of the disc. The spot shall be invisible if the incident rays from the dot at O to the surface at $\frac{d}{2}$ are at the critical angle.

Let i be the angle of incidence.

Then $\sin i = \dfrac{1}{\mu}$

Now, $\dfrac{d/2}{h} = \tan i$

$$\Rightarrow \frac{d}{2} = h \tan i = h\left[\sqrt{\mu^2 - 1}\right]^{-1}$$

$$\therefore d = \frac{2h}{\sqrt{\mu^2 - 1}} .$$

9.27 (i) Let the power at the far point be P_f for the normal relaxed eye.

Then $P_f = \dfrac{1}{f} = \dfrac{1}{0.1} + \dfrac{1}{0.02} = 60$ D

With the corrective lens the object distance at the far point is ∞. The power required is

$$P_f' = \frac{1}{f'} = \frac{1}{\infty} + \frac{1}{0.02} = 50\,\text{D}$$

The effective power of the relaxed eye with glasses is the sum of the eye and that of the glasses P_g.

$\therefore\ P_f' = P_f + P_g$

$\therefore\ P_g = -10$ D.

(ii) His power of accomadation is 4 diopters for the normal eye. Let the power of the normal eye for near vision be P_n.

Then $4 = P_n - P_f$ or $P_n = 64$ D.

Let his near point be x_n, then

$$\frac{1}{x_n} + \frac{1}{0.02} = 64 \ \text{ or } \frac{1}{x_n} + 50 = 64$$

$$\frac{1}{x_n} = 14,$$

$$\therefore\ x_n = \frac{1}{14};\ \ 0.07\text{m}$$

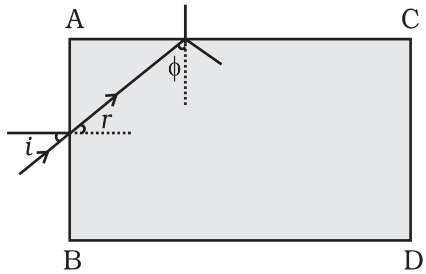
(iii) With glasses $P_n' = P_f' + 4 = 54$

$$54 = \frac{1}{x_n'} + \frac{1}{0.02} = \frac{1}{x_n'} + 50$$

$$\frac{1}{x_n'} = 4,$$

$$\therefore\ x_n' = \frac{1}{4} = 0.25\text{m} .$$

9.28 Any ray entering at an angle i shall be guided along AC if the angle the ray makes with the face AC (ϕ) is greater than the critical angle.



$\Rightarrow \sin \geq \dfrac{1}{\mu}$

$\Rightarrow \cos r \geq \dfrac{1}{\mu}$

Or, $1 - \cos^2 r \leq 1 - \dfrac{1}{\mu^2}$

i.e. $\sin^2 r \leq 1 - \dfrac{1}{\mu^2}$

Since $\sin i = \mu \sin r$

$\dfrac{1}{\mu^2}\sin^2 i \leq 1 - \dfrac{1}{\mu^2}$

Or, $\sin^2 i \leq \mu^2 - 1$

The smallest angle ϕ shall be when $i = \dfrac{\pi}{2}$. If that is greater than the critical angle then all other angles of incidence shall be more than the critical angle.

Thus $1 \leq \mu^2 - 1$

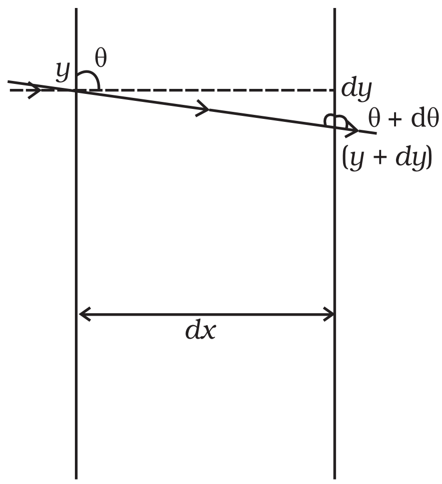
Or, $\mu^2 \geq 2$

$\Rightarrow \mu \geq \sqrt{2}$

9.29 Consider a portion of a ray between x and $x + dx$ inside the liquid. Let the angle of incidence at x be θ and let it enter the thin column at height y. Because of the bending it shall emerge at $x + dx$ with an angle $\theta + d\theta$ and at a height $y + dy$. From Snell's law

$\mu(y) \sin \theta = \mu(y+dy) \sin (\theta+d\theta)$

or $\mu(y) \sin\theta \; ; \left(''(y) + \dfrac{d\mu}{dy} dy \right) (\sin\theta \cos d\theta + \cos\theta \sin d\theta)$



$; \;\; \mu(y) \sin\theta + \mu(y) \cos\theta d\theta + \dfrac{d\mu}{dy} dy \sin\theta$

or $\mu(y) \cos\theta d\theta \; ; \; \dfrac{-d\mu}{dy} dy \sin\theta$

$d\theta \; ; \; \dfrac{-1}{\mu} \dfrac{d\mu}{dy} dy \tan\theta$

But $\tan\theta = \dfrac{dx}{dy}$ (from the fig.)

$$\therefore\ d\theta = \frac{-1}{\mu}\frac{d\mu}{dy}dx$$

$$\therefore\ \theta = \frac{-1}{\mu}\frac{d\mu}{dy}\int_o^d dx = \frac{-1}{\mu}\frac{d\mu}{dy}d$$

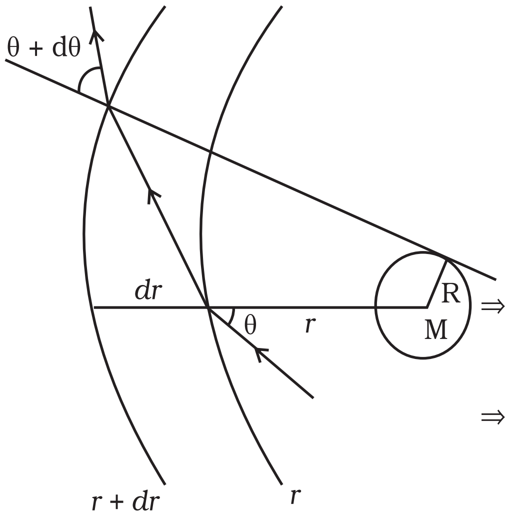
9.30 Consider two planes at r and $r + dr$. Let the light be incident at an angle θ at the plane at r and leave $r + dr$ at an angle θ $+d\theta$

Then from Snell's law

$n(r)\sin\theta = n(r + dr)\sin(\theta + d\theta)$

$$\Rightarrow n(r)\sin\theta \simeq \left(n(r) + \frac{dn}{dr}dr\right)(\sin\theta\cos d\theta + \cos\theta\ \sin d\theta)$$

$$\simeq \left(n(r) + \frac{dn}{dr}dr\right)(\sin\theta + \cos\theta\ d\theta)$$



Neglecting products of differentials

$$n(r)\sin\theta \simeq n(r)\sin\theta + \frac{dn}{dr}dr\sin\theta + n(r)\cos\theta d\theta$$

$$\Rightarrow -\frac{dn}{dr}\tan\theta = n(r)\frac{d\theta}{dr}$$

$$\Rightarrow \frac{2GM}{r^2c^2}\tan\theta = \left(1 + \frac{2GM}{rc^2}\right)\frac{d\theta}{dr} \approx \frac{d\theta}{dr}$$

$$\therefore \int_0^{\theta_o} d\theta = \frac{2GM}{c^2}\int_{-\infty}^{\infty}\frac{\tan\theta dr}{r^2}$$

Now $r^2 = x^2 + R^2$ and $\tan\theta = \dfrac{R}{x}$

$2rdr = 2xdx$

$$\int_0^{\theta_o} d\theta = \frac{2GM}{c^2}\int_{-\infty}^{\infty}\frac{R}{x}\frac{xdx}{(x^2+R^2)^{\frac{3}{2}}}$$

Put $x = R\tan\phi$

$dx = R\,\mathrm{Sec}^2\phi\, d\phi$

$$\therefore\quad \theta_0 = \frac{2GMR}{c^2}\int_{-\pi/2}^{\pi/2}\frac{R\sec^2\phi d\phi}{R^3\sec^3\phi}$$

$$= \frac{2GM}{Rc^2}\int_{-\pi/2}^{\pi/2}\cos\phi\, d\phi = \frac{4GM}{Rc^2}$$

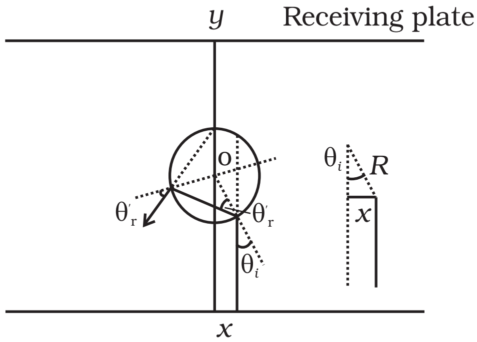
9.31 As the material is of refractive index –1, θ_r is negative and θ_r' positive.

Now $|\theta_i| = |\theta_r| = |\theta_r'|$

The total deviation of the outcoming ray from the incoming ray is $4\theta_i$. Rays shall not reach the receiving plate if

$$\frac{\pi}{2} \le 4\theta_i \le \frac{3\pi}{2} \quad \text{(angles measured clockwise from the } y \text{ axis)}$$

$$\frac{\pi}{8} \le \theta_i \le \frac{3\pi}{8}$$



Now $\sin\theta_i = \frac{x}{R}$

$$\frac{\pi}{8} \le \sin^{-1}\frac{x}{R} \le \frac{3\pi}{8}$$

Or, $\frac{\pi}{8} \le \frac{x}{R} \le \frac{3\pi}{8}$

Thus for $\frac{R\pi}{8} \le x \le \frac{R3\pi}{8}$ light emitted from the source shall not reach the receiving plate.

9.32 (i) The time required to travel from S to P_1 is

$$t_1 = \frac{SP_1}{c} = \frac{\sqrt{u^2+b^2}}{c} ; \quad \frac{u}{c}\left(1+\frac{1}{2}\frac{b^2}{u^2}\right) \text{ assuming } b << u_0$$

The time required to travel from P_1 to O is

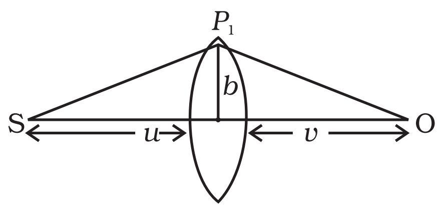
$$t_2 = \frac{P_1O}{c} = \frac{\sqrt{v^2+b^2}}{c} ; \quad \frac{v}{c}\left(1+\frac{1}{2}\frac{b^2}{v^2}\right)$$

The time required to travel through the lens is

$$t_l = \frac{(n-1)w(b)}{c} \quad \text{where } n \text{ is the refractive index.}$$

Thus the total time is

$$t = \frac{1}{c}\left[u+v+\frac{1}{2}b^2\left(\frac{1}{u}+\frac{1}{v}\right)+(n-1)w(b)\right]. \text{ Put } \frac{1}{D} = \frac{1}{u}+\frac{1}{v}$$



Then $t = \frac{1}{c}\left(u+v+\frac{1}{2}\frac{b^2}{D}+(n-1)\left(w_0+\frac{b^2}{\alpha}\right)\right)$

Fermet's principle gives

$$\frac{dt}{db} = ∩ = \frac{b}{CD} \quad \frac{2(n-1)b}{c\alpha}$$

$$\alpha = 2(n-1)D$$

Thus a convergent lens is formed if $\alpha = 2(n-1)D$. This is independant of b and hence all paraxial rays from S will converge at O (i.e. for rays $b << n$ and $b << v$).

Since $\frac{1}{D} = \frac{1}{u} + \frac{1}{v}$, the focal length is D.

(ii) In this case

$$t = \frac{1}{c}\left(u + v + \frac{1}{2}\frac{b^2}{\mathrm{D}} + (n-1)k_1 ln\left(\frac{k_2}{b}\right)\right)$$

$$\frac{dt}{db} = 0 = \frac{b}{D} - (n-1)\frac{k_1}{b}$$

$\Rightarrow b^2 = (n-1)\ k_1 D$

$\therefore$ b = $\sqrt{(n-1)k_1 D}$

Thus all rays passing at a height b shall contribute to the image. The ray paths make an angle

$$\beta ;\ \frac{b}{v} = \frac{\sqrt{(n-1)k_1 D}}{v^2} = \sqrt{\frac{(n-1)k_1 uv}{v^2(u+v)}} = \sqrt{\frac{(n-1)k_1 u}{(u+v)v}}\ .$$