CBSE Class 11 Mathematics

Important Questions

Chapter 12

Introduction to Three Dimensional Geometry

1 Marks Questions

1. Name the octants in which the following lie. (5,2,3)

Ans. I

2. Name the octants in which the following lie. (-5,4,3)

Ans. II

3. Find the image of (-2,3,4) in the y z plane

Ans. (2, 3, 4)

4. Find the image of (5,2,-7) in the xy plane

Ans. (5, 2, 7)

5. A point lie on X –axis what are co ordinate of the point

Ans. $(a, 0, 0)$

6. Write the name of plane in which x axis and y - axis taken together.

Ans. XY Plane

7. The point $(4, -3, -6)$ lie in which octants

Ans. $VIII$

8. The point $(2, 0, 8)$ lie in which plane

Ans. XZ

9. A point is in the XZ plane. What is the value of y co-ordinates?

Ans. Zero

10. What is the coordinates of XY plane

Ans. $(x, y, 0)$

11. The point $(-4, 2, 5)$ lie in which octants.

Ans. II

12. The distance from origin to point (a, b, c) is:

Ans. $\sqrt{a^2 + b^2 + c^2}$

CBSE Class 12 Mathematics
Important Questions
Chapter 12
Introduction to Three Dimensional Geometry

4 Marks Questions

1.Given that P(3,2,-4), Q(5,4,-6) and R(9,8,-10) are collinear. Find the ratio in which Q divides PR

Ans. Suppose Q divides PR in the ratio λ :1. Then coordinator of Q are

$$\left(\frac{9\lambda+3}{\lambda+1}, \frac{8\lambda+2}{\lambda+1}, \frac{-10\lambda-4}{\lambda+1}\right)$$

But, coordinates of Q are (5,4,-6). Therefore

$$\frac{9\lambda+3}{\lambda+1}=5, \frac{8\lambda+2}{\lambda+1}=4, \frac{-10\lambda-4}{\lambda+1}=6$$

These three equations give

$$\lambda=\frac{1}{2}.$$

So Q divides PR in the ratio $\frac{1}{2}:1$ or 1:2

2. Determine the points in xy plane which is equidistant from these point A (2,0,3) B(0,3,2) and C(0,0,1)

Ans. We know that Z- coordinate of every point on xy-plane is zero. So, let $P(x, y, 0)$ be a point in xy-plane such that PA=PB=PC

Now, PA=PB

$\Rightarrow PA^2=PB^2$

$$\Rightarrow (x-2)^2+(y-0)^2+(0-3)^2=(x-0)^2+(y-3)^2+(0-2)^2$$

$$\Rightarrow 4x-6y=0 \text{ or } 2x-3y=0.......(i)$$

$PB=PC$

$$\Rightarrow PB^2=PC^2$$

$$\Rightarrow (x-0)^2+(y-3)^2+(0-2)^2=(x-0)^2+(y-0)^2+(0-1)^2$$

$$\Rightarrow -6y+12=0 \Rightarrow y=2.........(ii)$$

Putting $y=2$ in (i) we obtain $x=3$

Hence the required points (3,2,0).

3. Find the locus of the point which is equidistant from the point A(0,2,3) and B(2,-2,1)

Ans. Let $P(x,y,z)$ be any point which is equidistant from A(0,2,3) and B(2,-2,1). Then

PA=PB

$\Rightarrow PA^2=PB^2$

$$\Rightarrow \sqrt{(x-0)^2+(y-2)^2+(2-3)^2}=\sqrt{(x-2)^2+(y+2)^2+(z-1)^2}$$

$$\Rightarrow 4x-8y-42+4=0 \text{ or } x-2y-2+1=0$$

4. Show that the points A(0,1,2) B(2,-1,3) and C(1,-3,1) are vertices of an isosceles right angled triangle.

Ans. We have

$$AB=\sqrt{(2-0)^2+(-1-1)^2(+3-2)^2}=\sqrt{4+4+1}=3$$

$$BC=\sqrt{(1-2)^2+(-3+1)^2+(1-3)^2}=\sqrt{1+4+4}=3$$

And $CA=\sqrt{(1-0)^2+(-3-1)^2+(1-2)^2}=\sqrt{1+16+1}=3\sqrt{2}$

Clearly AB=BC and $AB^2+BC^2=AC^2$

Hence, triangle ABC is an isosceles right angled triangle.

5. Using section formula, prove that the three points A(-2,3,5), B(1,2,3), and C(7,0,-1) are collinear.

Ans. Suppose the given points are collinear and C divides AB in the ratio $\lambda:1$.

Then coordinates of C are

$$\left(\frac{\lambda-2}{\lambda+1},\frac{2\lambda+3}{\lambda+1},\frac{3\lambda+5}{\lambda+1}\right)$$

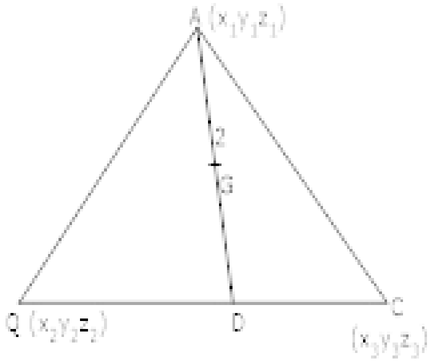
But, coordinates of C are (3,0,-1) from each of there equations, we get $\lambda=\frac{3}{2}$

Since each of there equation give the same value of V. therefore, the given points are collinear and C divides AB externally in the ratio 3:2.

6. Show that coordinator of the centroid of triangle with vertices A($x_1 y_1 z_1$), B($x_2 y_2 z_2$), and C($x_3 y_3 z_3$) is $\left[\frac{x_1+y_1+z_1}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right]$

Ans. Let D be the mid point of AC. Then

Coordinates of D are $\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2},\frac{z_2+z_3}{2}\right)$.



Let G be the centroid of ΔABC. Then G, divides AD in the ratio 2:1. So coordinates of D are

$$\left(\frac{1.x_1+2\frac{(x_2+x_3)}{2}}{1+2},\frac{1.y_1+2\left(\frac{y_2+y_3}{2}\right)}{1+2},\frac{1.z_1+2\left(\frac{z_2+z_3}{2}\right)}{1+2}\right)$$

i.e. $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right)$

7. Prove by distance formula that the points $A(1,2,3)$, $B(-1,-1,-1)$ and $C(3,5,7)$ are collinear.

Ans. Distance

$$|AB|=\sqrt{(-1-1)^2+(-1-2)^2+(-1-3)^2}=\sqrt{4+9+16}=\sqrt{29}$$

Distance

$$|BC|=\sqrt{(3+1)^2+(5+1)^2+(7+1)^2}=\sqrt{16+36+64}=2\sqrt{29}$$

Distance

$$|AC|=\sqrt{(3-1)^2+(5-2)^2+(7-3)^2}=\sqrt{4+9+16}=\sqrt{29}$$

$$\therefore\ |BC| = |AB| + |Ac|$$

$\therefore$ The paints A.B.C. are collinear.

8. Find the co ordinate of the point which divides the join of $P(2,-1,4)$ and $Q(4,3,2)$ in the ratio $2:5$ (i) internally (ii) externally

Ans. Let paint $R(x,y,z)$ be the required paint.

(i) For internal division

$$x=\frac{2\times4+5\times2}{2+5}=\frac{8+10}{7}=\frac{18}{7}$$

$$y=\frac{2\times3+5\times-1}{2+5}=\frac{6-5}{7}=\frac{1}{7}$$

$$z=\frac{2\times2+5\times4}{2+5}=\frac{4+20}{7}=\frac{24}{7}$$

$\therefore$ Required paint $R\left(\frac{18}{7},\frac{1}{7},\frac{24}{7}\right)$

(ii) For external division.

$$x=\frac{2\times4-5\times2}{2-5}=\frac{8-10}{-3}=\frac{-2}{-3}=\frac{2}{3}$$

$$y=\frac{2\times3-5\times-1}{2-5}=\frac{6+5}{-3}=\frac{11}{-3}$$

$$z=\frac{2\times2-5\times4}{2-5}=\frac{4-20}{-3}=\frac{-16}{-3}=\frac{16}{3}$$

$\therefore$ Required point $R\left(\frac{2}{3},\frac{-11}{3},\frac{16}{3}\right)$

9. Find the co ordinate of a point equidistant from the four points

$0(0,0,0)$, $A(a,0,0)$, $B(0,b,0)$ **and** $C(0,0,c)$

Ans. Let $P(x,y,z)$ be the required point

According to condition

$OP = PA = PB = PC$

Now $OP = PA$

$\Rightarrow OP^2 = PA^2$

$\Rightarrow x^2 + y^2 + z^2 = (x-a)^2 + (y-0)^2 + (z-0)^2$

$\Rightarrow x^2 + y^2 + z^2 = x^2 - 2ax + a^2 + y^2 + z^2$

$2ax = a^2$

$\therefore x = \frac{a}{2}$

Similarly $OP = PB$

$\Rightarrow y = \frac{b}{2}$

$A(x,y,z,)$ $B(x_2,y_2,z_2)$ and $C(x_3,y_3,z_3)$ D,E and F are mid points of side $BC, CA,$ and AB respectively,

Then $\frac{x_1 + x_2}{2} = -1$

$x_1 + x_2 = -2......(1)$

$\frac{y_1 + y_2}{2} = 1$

$$y_1 + y_2 = 2......(2)$$

$$\frac{z_1 + z_2}{2} = -4$$

$$z_1 + z_2 = -8......(3)$$

$$\frac{x_2 + x_3}{2} = 1$$

$$x_2 + x_3 = 2......(4)$$

$$\frac{y_2 + y_3}{2} = 2$$

$$y2 + y3 = 4......(5)$$

$$\frac{z_2 + z_3}{2} = -3$$

$$z_2 + z_3 = -6......(6)$$

$$\frac{x_1 + x_3}{2} = 3$$

$$x_1 + x_3 = 6......(7)$$

$$\frac{y_1 + y_3}{2} = 0$$

$$y_1 + y_3 = 0......(8)$$

$$\frac{z_1 + z_3}{2} = 1$$

$$z_1 + z_3 = 2......(9)$$

Adding eq (1),(4) and (7) we get

$2(x_1 + x_2 + x_3) = -2 + 2 + 6$

Adding eq. (2),(5) and (8)

$2(y_1 + y_2 + y_3) = 6$

$y_1 + y_2 + y_3 = 3......(11)$

And $OP = PC$

$\Rightarrow z = \frac{c}{2}$

Hence co-ordinate of $P\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

10. Find the ratio in which the join the $A(2,1,5)$ and $B(3,4,3)$ is divided by the plane $2x + 2y - 2z = 1$ Also find the co-ordinate of the point of division

Ans. Suppose plane $2x + 2y - 2z = 1$ divides $A(2,1,5)$ and $B(3,4,5)$ in the ratio $\lambda : 1$ at pain C

Then co-ordinate of paint C

$\left(\frac{3\lambda + 2}{\lambda + 1}, \frac{4\lambda + 1}{\lambda + 1}, \frac{3\lambda + 5}{\lambda + 1}\right)$

$\because$ Point C lies on the plane $2x + 2y - 2z = 1$

$\therefore$ Points C must satisfy the equation of plane

$2\left(\frac{3\lambda + 2}{\lambda + 1}\right) + 2\left(\frac{4\lambda + 1}{\lambda + 1}\right) - 2\left(\frac{3\lambda + 5}{\lambda + 1}\right) = 1$

$\Rightarrow 8\lambda - 4 = \lambda + 1$

$\Rightarrow \lambda = \frac{5}{7}$

$\therefore$ Required ratio 5:7

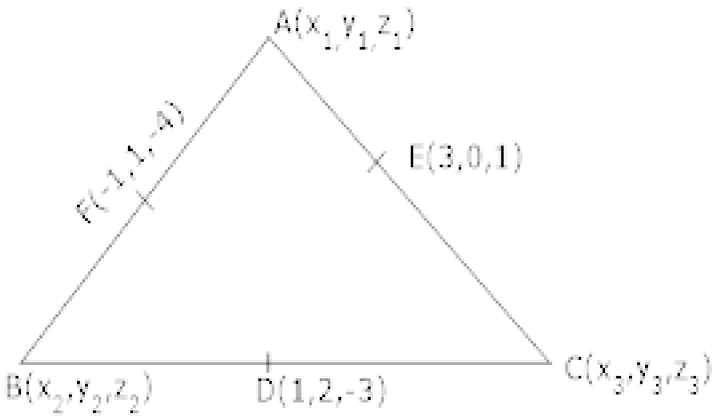
11. Find the centroid of a triangle, mid points of whose sides are $(1,2,-3)$, $(3,0,1)$ and $(-1,1,-4)$

Ans. Suppose co-ordinate of vertices of ΔABC are

Adding eq. (3), (6) and (9)

$$2(z_1 + z_2 + z_3) = -8 - 6 + 2$$

$$z_1 + z_2 + z_3 = -6 \ldots\ldots(12)$$



Co-ordinate of centroid

$$x = \frac{x_1 + x_2 + x_3}{3} = \frac{3}{3} = 1$$

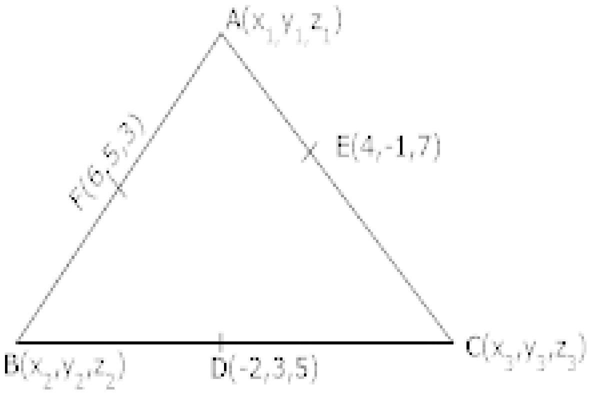
$$y = \frac{y_1 + y_2 + y_3}{3} = \frac{3}{3} = 1$$

$$z = \frac{z_1 + z_2 + z_3}{3} = \frac{-6}{3} = -2$$

$(1, 1, -2)$

12. The mid points of the sides of a ΔABC are given by $(-2,3,5)$, $(4,-1,7)$ and $(6,5,3)$ find the co ordinate of A, B and C

Ans. Suppose co-ordinate of point A,B,C, are (x_1,y_1,z_1), (x_2,y_2,z_2) and (x_3,y_3,z_3) respectively let D,E and F are mid points of side BC,CA and AB respectively



$$\therefore \frac{x_1+x_2}{2}=6$$

$$x_1+x_2=12......(1)$$

$$\frac{y_1+y_2}{2}=5$$

$$y_1+y_2=10......(2)$$

$$\frac{z_1+z_2}{2}=3$$

$$z_1+z_2=6......(3)$$

$$\frac{x_2+x_3}{2}=-2$$

$$x_2+x_3=-4......(4)$$

$$\frac{y_2+y_3}{2}=3$$

$$y_2+y_3=6......(5)$$

$$\frac{z_1 + z_2}{2} = 5$$

$$z_1 + z_2 = 10......(6)$$

$$\frac{x_1 + x_3}{2} = 4$$

$$x_1 + x_3 = 8......(7)$$

$$\frac{y_1 + y_3}{2} = -1$$

$$y_1 + y_3 = -2......(8)$$

$$\frac{z_1 + z_3}{z} = 7$$

$$z_1 + z_3 = 14......(9)$$

Adding eq. (1), (4) and (7)

$$2(x_1 + x_2 + x_3) = 12 - 4 + 8$$

$$x_1 + x_2 + x_3 = \frac{16}{2} = 8......(10)$$

Similarly $y_1 + y_2 + y_3 = 7......(11)$

$$z_1 + z_2 + z_3 = 15......(12)$$

Subtracting eq. (1), (4) and (7) from (10)

$$x_3 = -4, \quad x_1 = 12, \quad x_2 = 0$$

Now subtracting eq. (2), (5) and (8) from (11)

$$y_3 = -3, \quad y_1 = 1, \quad y_2 = 9$$

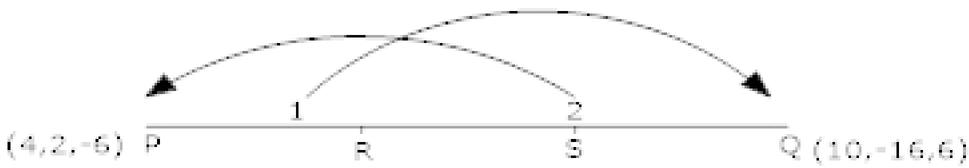
Similarly $z_3 = 9, \quad z_1 = 5, \quad z_2 = 1$

$\therefore$ co-ordinate of point $A, B,$ and C are

$A(12, 0, -4)$, $B(1, 9, -3)$, and $C(5, 1, 9)$

13. Find the co-ordinates of the points which trisects the line segment PQ formed by joining the point $P(4, 2, -6)$ and $Q(10, -16, 6)$

Ans. Let R and S be the points of trisection of the segment PO. Then



$\therefore PR = RS = SQ$

$\Rightarrow 2PR = RQ$

$\Rightarrow \frac{PQ}{RQ} = \frac{1}{2}$

$\therefore$ R divides PQ in the ratio 1:2

$\therefore$ Co-ordinates of point

$$R\left[\frac{1(10)+2\times 4}{1+2}, \frac{1(-16)+2\times 2}{1+2}, \frac{1\times 6+2(-6)}{1+2}\right]$$

$= R(6, -4, -2)$

Similarly $PS = 2SQ$

$\Rightarrow \frac{PS}{SQ} = \frac{2}{1}$

$\therefore$ S divider PQ in the ratio 2:1

$\therefore$ co-ordinates of point S

$$\left[\frac{2(10)+1(4)}{1+2}, \frac{2(-16)+1(2)}{1+2}, \frac{2(6)+1(-6)}{1+2}\right]$$

$\therefore S(8,-10,2)$

14. Show that the point $P(1,2,3)$, $Q(-1,-2,-1)$, $R(2,3,2)$ and $S(4,7,6)$ taken in order form the vertices of a parallelogram. Do these form a rectangle?

Ans. Mid point of PR is $\left(\frac{1+2}{2}, \frac{2+3}{2}, \frac{3+2}{2}\right)$

i.e. $\left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}\right)$

also mid point of QS is $\left(\frac{-1+4}{2}, \frac{-2+7}{2}, \frac{-1+6}{2}\right)$

i.e. $\left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}\right)$

Then PR and QS have same mid points.

$\therefore$ PR and QS bisect each other. It is a Parallelogram.

Now $PR=\sqrt{(2-1)^2+(3-2)^2+(2-3)^2}=\sqrt{3}$ and

$QS=\sqrt{(4+1)^2+(7+2)^2+(6+1)^2}=\sqrt{155}$

$\therefore PR \neq QS$ diagonals an not equal

$\therefore PQRS$ are not rectangle.

15. A point R with x co-ordinates 4 lies on the line segment joining the points $P(2,-3,4)$ and $Q(8,0,10)$ find the co-ordinates of the point R

Ans. Let the point. R divides the line segment joining the point P and Q in the ratio $\lambda:1$, Then co-ordinates of Point R

$$\left[\frac{8\lambda+2}{\lambda+1}, \frac{-3}{\lambda+1}, \frac{10\lambda+4}{\lambda+1}\right]$$

The x co-ordinates of point R is 4

$$\Rightarrow \frac{8\lambda+2}{\lambda+1} = 4 \quad , \lambda = \frac{1}{2}$$

$\therefore$ co-ordinates of point R

$$\left[4, \frac{-3}{\frac{1}{2}+1}, \frac{10\times\frac{1}{2}+4}{\frac{1}{2}+1}\right] \quad \text{i.e.}(4, -2, 6)$$

16. If the points $P(1, 0, -6)$, $Q(-3, P, q)$ and $R(-5, 9, 6)$ are collinear, find the values of P and q

Ans. Given points

$P(1, 0, -6)$, $Q(-3, P, q)$ and $R(-5, 9, 6)$ are collinear

Let point Q divider PR in the ratio K:1

$\therefore$ co-ordinates of point $P\left(\frac{1-5K}{K+1}, \frac{0+9K}{K+1}, \frac{-6+6K}{K+1}\right)$

$Q(-3, P, q)$

$$\frac{1-5K}{K+1}=-3$$

$$1-5K=-3K-3$$

$$-2K=-4$$

$$K=\frac{-4}{-2}$$

$$K=2$$

$\therefore$ the value of P and q are 6 and 2.

17. Three consecutive vertices of a parallelogram ABCD are $A(3,-1,2)$, $B(1,2,-4)$ and $C(-1,1,2)$ find forth vertex D

Ans. Given vertices of 11gm ABCD

$A(3,-1,2)$, $B(1,2,-4)$, $C(-1,1,2)$

Suppose co-or dine of forth vertex $D(x,y,z)$

Mid point of $AC\left(\frac{3-1}{2},\frac{-1+1}{2},\frac{2+2}{2}\right)$

$=(1,0,2)$

Mid point of $BD\left(\frac{x+1}{2},\frac{y+2}{2},\frac{-4+z}{2}\right)$

Mid point of AC = mid point of BD

$$\frac{x+1}{2}=1\Rightarrow x=1$$

$$\frac{y+2}{2}=0\Rightarrow y=-2$$

$$\frac{-4+z}{2}=2 \Rightarrow z=8$$

Co-ordinates of point $D(1,-2,8)$

18. If A and B be the points $(3,4,5)$ and $(-1,3,7)$ respectively. Find the eq. of the set points P such that $PA^2+PB^2=K^2$ where K is a constant

Ans. Let co-ordinates of point P be

(x,y,z)

$PA^2=(x-3)^2+(y-4)^2+(z-5)^2$

$=x^2-6x+9+y^2-8y+16+z^2-10z+25$

$=x^2+y^2+z^2-6x-8y-10z+50$

$PB^2=(x+1)^2+(y-3)^2+(z-7)^2$

$=x^2+2x+1+y^2-6y+9+z^2-14+49$

$=x^2+y^2+z^2+2x-6y-14z+59$

$PA^2+PB^2=K^2$

$2(x^2+y^2+z^2)-4x-14y-24z+109=K^2$

$$x^2+y^2+z^2-2x-7y-12z=\frac{K^2-109}{2}$$

CBSE Class 12 Mathematics

Important Questions

Chapter 12

Introduction to Three Dimensional Geometry

6 Marks Questions

1. Prove that the lines joining the vertices of a tetrahedron to the centroids of the opposite faces are concurrent.

Ans. Let ABCD be tetrahedron such that the coordinates of its vertices are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ and $D(x_4, y_4, z_4)$

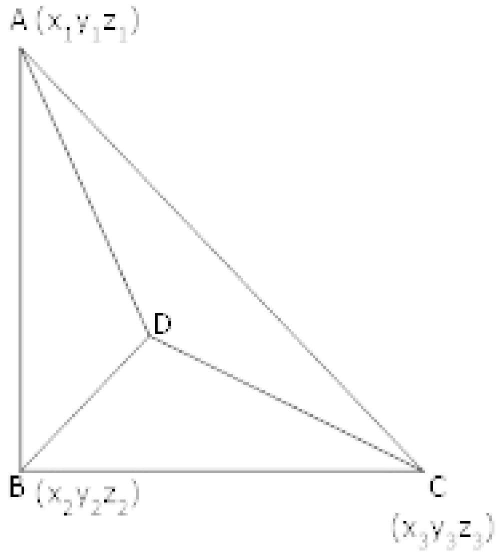
The coordinates of the centroids of faces ABC, DAB, DBC and DCA respectively

$$G_1\left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right]$$

$$G_2\left[\frac{x_1+x_2+x_4}{3}, \frac{y_1+y_2+y_4}{3}, \frac{z_1+z_2+z_4}{3}\right]$$

$$G_3\left[\frac{x_2+x_3+x_4}{3}, \frac{y_2+y_3+y_4}{3}, \frac{z_2+z_3+z_4}{3}\right]$$

$$G_4\left[\frac{x_4+x_3+x_1}{3}, \frac{y_4+y_3+y_1}{3}, \frac{z_4+z_3+z_1}{3}\right]$$



Now, coordinates of point G dividing DG1 in the ratio 3:1 are

$$\left[\frac{1.x_4+3\left(\frac{x_1+x_2+x_3}{3}\right)}{1+3}, \frac{1.y_4+3\left(\frac{y_1+y_2+y_3}{3}\right)}{1+3}, \frac{1.z_4+3\left(\frac{z_1+z_2+z_3}{3}\right)}{1+3}\right]$$

$$=\left[\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right]$$

Similarly the point dividing CG2, AG3 and BG4 in the ratio 3:1 has the same coordinates.

Hence the point $G\left[\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right]$ is common to DG1, CG2, AG3 and BG4.

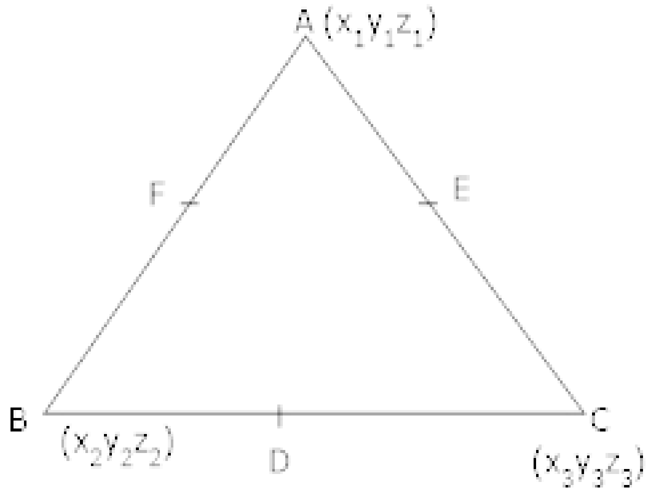
Hence they are concurrent.

2. The mid points of the sides of a triangle are (1,5,-1), (0,4,-2) and (2,3,4). Find its vertices.

Ans. Suppose vertices of ΔABC are $A(x_1y_1z_1)$, $B(x_2y_2z_2)$ and $C(x_3y_3z_3)$ respectively

Given coordinates of mid point of side BC, CA, and AB respectively are D(1,5,-1), E(0,4,-2) and

F(2,3,4)



$\therefore \frac{x_2+x_3}{2}=1 \quad \frac{y_2+y_3}{2}=5 \quad \frac{z_2+z_3}{2}=-1$

$x_2+x_3=2......(i)$

$\frac{x_1+x_3}{2}=0$

$y_2+y_3=10......(ii)$

$\frac{y_1+y_3}{2}=4$

$z_2+z_3=2......(iii)$

$\frac{z_1+z_3}{2}=-2$

$x_1+x_3=0.......(iv)$

$\frac{x_1+x_2}{2}=2$

$y_1+y_2=8.......(v)$

$\frac{y_1+y_2}{2}=3$

$z_1 + z_3 = -4 ....(vi)$

$\frac{z_1 + z_2}{2} = 4$

$x_1 + x_2 = 4 ....(vii)$

$y_1 + y_2 = 6 ....(viii)$

$z_1 + z_2 = 8 ....(ix)$

Adding eq. $(i), (iv), \& (vii)$

$2(x_1 + x_2 + x_3) = 6$

$x_1 + x_2 + x_3 = 3 ....(x)$

Subtracting eq. $(i), (iv), \& (vii)$ from (x) we get

$x_1 = 1, \ x_2 = 3, \ x_3 = -1$

Similarly, adding eq. $(ii), (v)$ and $(viii)$

$y_1 + y_2 + y_3 = 12 ......(xi)$

Subtracting eq. $(ii), (v)$ and $(viii)$ from (xi)

$y_1 = 2, \ y_2 = 4, \ y_3 = 6$

Similarly $z_1 + z_2 + z_3 = 3$

$z_1 = 1, \ z_2 = 7, \ z_3 = -5$

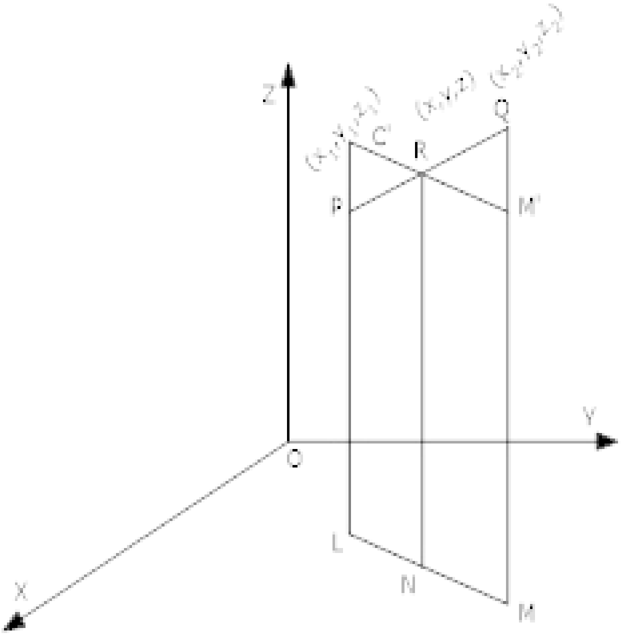
$\therefore$ Coordinates of vertices of Δ ABC are A(1,3,-1), B(2,4,6) and C(1,7,-5)

3. Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points in space find co ordinate of point R

which divides P and Q in the ratio $m_1 : m_2$ by geometrically

Ans. Let co-ordinate of Point R be (x, y, z) which divider line segment joining the point $P\ Q$ in the ratio $m_1 : m_2$

Clearly $\Delta PRL' \sim \Delta QRM'$ $[By\ AA \text{ sinilsrity}]$



$$\therefore \frac{PL'}{MQ'} = \frac{PR}{RQ}$$

$$\Rightarrow \frac{LL' - LP}{MQ - MM'} = \frac{m_1}{m_2}$$

$$\Rightarrow \frac{NR - LP}{MQ - NR} = \frac{m_1}{m_2} \qquad \begin{bmatrix} \because LL' = NR \\ \text{and } MM' = NR \end{bmatrix}$$

$$\Rightarrow \frac{z - z_1}{z_2 - z} = \frac{m_1}{m_2}$$

$$\Rightarrow z = \frac{m_1 . z_2 + m_2 z_1}{m_1 + m_2}$$

Similarly $x=\frac{m_1x_2+m_2x_1}{m_1+m_2}$ and

$$y=\frac{m_1y_2+m_2y_1}{m_1+m_2}$$

4. Show that the plane $ax+by+cz+d=0$ divides the line joining the points (x_1,y_1,z_1) and (x_2,y_2,z_2) in the ratio $\frac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d}$ s

Ans. Suppose the plane $ax+by+cz+d=0$ divides the line joining the points (x_1,y_1,z_1) and (x_2,y_2,z_2) in the ratio $\lambda:1$

$$\therefore\ x=\frac{\lambda x_2+x_1}{\lambda+1},\quad y=\frac{\lambda y_2+y_1}{\lambda+1},\quad z=\frac{\lambda z_2+z_1}{\lambda+1}$$

$\because$ Plane $ax+by+cz+d=0$ Passing through (x,y,z)

$$\therefore\ Q\frac{(\lambda x_2+x_1)}{\lambda+1}+b\frac{(\lambda y_2+y_1)}{\lambda+1}+c\frac{(\lambda z_2+z_1)}{\lambda+1}+d=0$$

$$a(\lambda x_2+x_1)+b(\lambda y_2+y_1)+c(\lambda z_2+z_1)+d(\lambda+1)=0$$

$$\lambda(ax_2+by_2+cz_2+d)+(ax_1+by_1+cz_1+d)=0$$

$$\lambda=-\frac{(ax_1+by_1+cz_1+d)}{(ax_2+by_2+cz_2+d)}$$

Hence Proved.

5. Prove that the points $0(0,0,0)$, $A(2,0,0)$, $B(1,\sqrt{3},0)$, and $C\left(1,\frac{1}{\sqrt{3}},\frac{2\sqrt{2}}{\sqrt{3}}\right)$ are the vertices of a regular tetrahedron.

Ans. To prove O, A, B, C are vertices of regular tetrahedron.

We have to show that

$|OA| = |OB| = |OC| = |AB| = |BC| = |CA|$

$|OA| = \sqrt{(0-2)^2 + 0^2 + 0^2} = 2$ unit

$|OB| = \sqrt{(0-1)^2 + \left(0-\sqrt{3}\right)^2 + 0^2} = \sqrt{1+3} = \sqrt{4} = 2$ unit

$|OC| = \sqrt{(0-1)^2 + \left(0-\frac{1}{\sqrt{3}}\right) + \left(0-\frac{2\sqrt{2}}{3}\right)^2}$

$= \sqrt{1+\frac{1}{3}+\frac{8}{3}}$

$= \sqrt{\frac{12}{3}} = \sqrt{4} = 2$ unit

$|AB| = \sqrt{(2-1)^2 + \left(0-\sqrt{3}\right)^2 + (10-0)^2} = \sqrt{1+3+0}$

$= \sqrt{4} = 2$ unit

$|BC| = \sqrt{(1-1)^2 + \left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)^2 + \left(0-\frac{2\sqrt{2}}{\sqrt{3}}\right)^2}$

$= \sqrt{0+\left(\frac{2}{\sqrt{3}}\right)^2 + \frac{8}{3}}$

$= \sqrt{\frac{12}{3}} = 2$ unit

|CA|= $\sqrt{(1-2)^2+\left(\frac{1}{\sqrt{3}}-0\right)^2+\left(\frac{2\sqrt{2}}{\sqrt{3}}-0\right)^2}$

$=\sqrt{1+\frac{1}{3}+\frac{8}{3}}$

$=\sqrt{\frac{12}{3}}=2$ unit

∴ |AB| = |BC| = |CA| = |OA| = |OB| = |OC| = 2 unit

∴ O, A, B, C are vertices of a regular tetrahedron.

6. If A and B are the points $(-2,2,3)$ and $(-1,4,-3)$ respectively, then find the locus of P such that 3|PA| = 2|PB|

Ans. Given points $A(-2,2,3)$ and $B(-1,4,-3)$

Supper co-ordinates of point $P(x,y,z)$

|PA|= $\sqrt{(x+2)^2+(y-2)^2+(2-3)^2}$

|PA|= $\sqrt{x^2+y^2+z^2+4x-4y-6z+17}$

|PB|= $\sqrt{(x+1)^2+(y-4)^2+(z+3)^2}$

|PB|= $\sqrt{x2+y2+z2+2x-8y+6z+26}$

∵ 3|PA| = 2|PB|

9 PA2=4 PB2

$9\left(x^2+y^2+z^2+4x-4y-6z+17\right)=4\left(x^2+y^2+z^2+2x-8y+6z+26\right)$

$5x^2+5y^2+5z^2+28x-4y-30z+49=0$

Introduction to 3-D Geometry

1. Locate the following points:

   (i) (1, – 1, 3), (ii) (– 1, 2, 4)
   (iii) (– 2, – 4, –7) (iv) (– 4, 2, – 5).

2. Name the octant in which each of the following points lies.

   (i) (1, 2, 3), (ii) (4, – 2, 3), (iii) (4, –2, –5) (iv) (4, 2, –5)
   (v) (– 4, 2, 5) (vi) (–3, –1, 6) (vii) (2, – 4, – 7) (viii) (– 4, 2, – 5).

3. Let A, B, C be the feet of perpendiculars from a point P on the x, y, z-axis respectively. Find the coordinates of A, B and C in each of the following where the point P is :

   (i) A = (3, 4, 2) (ii) (–5, 3, 7) (iii) (4, – 3, – 5)

4. Let A, B, C be the feet of perpendiculars from a point P on the xy, yz and zx-planes respectively. Find the coordinates of A, B, C in each of the following where the point P is

   (i) (3, 4, 5) (ii) (–5, 3, 7) (iii) (4, – 3, – 5).

5. How far apart are the points (2, 0, 0) and (–3, 0, 0)?

6. Find the distance from the origin to (6, 6, 7).

7. Show that if $x^2 + y^2 = 1$, then the point $(x, y, \sqrt{1-x^2-y^2})$ is at a distance 1 unit from the origin.

8. Show that the point A (1, – 1, 3), B (2, – 4, 5) and (5, – 13, 11) are collinear.

9. Three consecutive vertices of a parallelogram ABCD are A (6, – 2, 4), B (2, 4, – 8), C (–2, 2, 4). Find the coordinates of the fourth vertex.

   [**Hint**: Diagonals of a parallelogram have the same mid-point.]

10. Show that the triangle ABC with vertices A (0, 4, 1), B (2, 3, – 1) and C (4, 5, 0) is right angled.

11. Find the third vertex of triangle whose centroid is origin and two vertices are (2, 4, 6) and (0, –2, –5).

12. Find the centroid of a triangle, the mid-point of whose sides are D (1, 2, – 3), E (3, 0, 1) and F (– 1, 1, – 4).

13. The mid-points of the sides of a triangle are (5, 7, 11), (0, 8, 5) and (2, 3, – 1). Find its vertices.

14. Three vertices of a Parallelogram ABCD are A (1, 2, 3), B (– 1, – 2, – 1) and C (2, 3, 2). Find the fourth vertex D.

15. Find the coordinate of the points which trisect the line segment joining the points

A (2, 1, – 3) and B (5, – 8, 3).

16. If the origin is the centriod of a triangle ABC having vertices A $(a, 1, 3)$, B $(-2, b, -5)$ and C $(4, 7, c)$, find the values of a, b, c.

17. Let A (2, 2, – 3), B (5, 6, 9) and C (2, 7, 9) be the vertices of a triangle. The internal bisector of the angle A meets BC at the point D. Find the coordinates of D.

18. Show that the three points A (2, 3, 4), B (–1, 2, – 3) and C (– 4, 1, – 10) are collinear and find the ratio in which C divides AB.

19. The mid-point of the sides of a triangle are (1, 5, – 1), (0, 4, – 2) and (2, 3, 4). Find its vertices. Also find the centriod of the triangle.

20. Prove that the points (0, – 1, – 7), (2, 1, – 9) and (6, 5, – 13) are collinear. Find the ratio in which the first point divides the join of the other two.

21. What are the coordinates of the vertices of a cube whose edge is 2 units, one of whose vertices coincides with the origin and the three edges passing through the origin, coincides with the positive direction of the axes through the origin?

Objective Type Questions

Choose the correct answer from the given four options inidcated against each of the Exercises from 22 (M.C.Q.).

22. The distance of point P(3, 4, 5) from the yz-plane is

(A) 3 units (B) 4 units (C) 5 units (D) 550

23. What is the length of foot of perpendicular drawn from the point P (3, 4, 5) on y-axis

(A) $\sqrt{41}$ (B) $\sqrt{34}$ (C) 5 (D) none of these

24. Distance of the point (3, 4, 5) from the origin (0, 0, 0) is

(A) $\sqrt{50}$ (B) 3 (C) 4 (D) 5

25. If the distance between the points $(a, 0, 1)$ and $(0, 1, 2)$ is $\sqrt{27}$, then the value of a is

(A) 5 (B) ± 5 (C) -5 (D) none of these

26. x-axis is the intersection of two planes

(A) xy and xz (B) yz and zx (C) xy and yz (D) none of these

27. Equation of y-axis is considered as

(A) $x = 0, y = 0$ (B) $y = 0, z = 0$ (C) $z = 0, x = 0$ (D) none of these

28. The point $(-2, -3, -4)$ lies in the

(A) First octant (B) Seventh octant

(C) Second octant (D) Eighth octant

29. A plane is parallel to yz-plane so it is perpendicular to :

(A) x-axis (B) y-axis (C) z-axis (D) none of these

30. The locus of a point for which $y = 0, z = 0$ is

(A) equation of x-axis (B) equation of y-axis

(C) equation at z-axis (D) none of these

31. The locus of a point for which $x = 0$ is

(A) xy-plane (B) yz-plane (C) zx-plane (D) none of these

32. If a parallelopiped is formed by planes drawn through the points (5, 8, 10) and (3, 6, 8) parallel to the coordinate planes, then the length of diagonal of the parallelopiped is

(A) $2\sqrt{3}$ (B) $3\sqrt{2}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

33. L is the foot of the perpendicular drawn from a point P (3, 4, 5) on the xy-plane. The coordinates of point L are

(A) (3, 0, 0) (B) (0, 4, 5) (C) (3, 0, 5) (D) none of these

34. L is the foot of the perpendicular drawn from a point (3, 4, 5) on x-axis. The coordinates of L are

(A) (3, 0, 0) (B) (0, 4, 0) (C) (0, 0, 5) (D) none of these