

Revision notes for Linear programming

Linear Programming

It is a mathematical technique for maximizing and minimizing a linear function of several variables subject to the constraints on the variables that form linear inequality or equation.

Linear Programming Problem

A linear programming problem is one that in which we have to find optimal value (maximum or minimum) of a linear function of several variables subject to the certain conditions that the variables are non-negative and satisfying by a set of linear inequalities.

NOTE The variables occurring in the problem.

Mathematical form of LPP

The mathematical form of linear programming problem is as follows

Objective function, $Z = C_1x + C_2y$

Subject to constraints are $a_1x + b_1y \leq d_1$,
 $a_2x + b_2y \leq d_2$ etc. and $x \geq 0, y \geq 0$

Terms Related to Linear Programming

1. **Constraints** The linear inequalities or inequations or restrictions on the variables of the linear programming problem are called

constraints. The conditions $x \geq 0, y \geq 0$ are called non-negative restrictions.

2. **Objective Function** A linear function $Z = px + qy$ (p and q are constants) which has to be maximised or minimised, is called an objective function.
3. **Optimal Value** The maximum or minimum value of an objective function is known as its optimal value.
4. **Feasible and Infeasible Region** The common region determined by all the constraints including non-negative constraints $x, y \geq 0$ of a linear programming problem is called the feasible region for the problem. The region other than the feasible region is called an infeasible region. The feasible region is always a convex polygon.
5. **Feasible Solutions** Points within and on the boundary of the feasible region represent feasible solutions of the constraints. Any point outside the feasible region is called an infeasible solution.
6. **Optimal Feasible Solution** Any point in the feasible region that gives the optimal value of the objective function is called the optimal feasible solution.
7. **Bounded and Unbounded Region** A feasible region of a system of linear inequalities is said to be bounded, if it can be enclosed within circle. Otherwise, it is called unbounded.

Fundamental Theorems for Solving Linear Programming Problem

Theorem 1 Let R be the feasible region (convex polygon) for a linear programming problem and let $Z = ax + by$ be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, then this optimal value must occur at a corner point (vertex) of the feasible region.

NOTE A corner point of a feasible region is a point in the region which is the intersection of two boundary lines.

Theorem 2 Let R be the feasible region for a linear programming problem and let $Z = ax + by$ be the objective function. If R is bounded, then Z has both maximum and minimum value on R and each of these occurs at a corner point (vertex) of R .

NOTE Maximum or minimum value of the objective function may or may not exist, if the feasible region is unbounded.

Working Rule to Solve a LPP

The following steps are used to solve a LPP:

- (i) First, write the given LPP in mathematical form by using mathematical formulation (if it is not given in mathematical form).
- (ii) Consider all constraints as linear equations.
- (iii) Draw the graph of each linear equation obtained in step II.
- (iv) Find the feasible region of the linear programming problem and determine its corner points either by inspection or by solving the two equations of the lines intersecting at that point and check that feasible region is bounded or unbounded.
- (v) Evaluate the objective function $Z = ax + by$ at each corner point. Let M and m be, respectively denote the largest and the smallest values of Z at these points.
- (vi) If the feasible region is bounded, then M and m , respectively are the maximum and minimum values of the objective function.
If the feasible region is unbounded, then
 - (a) M is the maximum value of objective function Z , if the open half plane determined by $ax + by > M$ has no point in

common with the feasible region. Otherwise, Z has no maximum value.

- (b) m is the minimum value of Z , if the open half plane determined by $ax + by < m$ has no point in common with the feasible region. Otherwise, Z has no minimum value.

- NOTE**
- (i) Sometimes it may be possible that feasible region lies only on a line segment. Then, the required maximum (or minimum) value of Z is on the line segment.
 - (ii) If two corner points of the feasible region gives optimal solutions of the same type, i.e. both produce the same maximum or minimum value, then any point on the line segment joining these two points is also gives an optimal solution of the same type.