Chapter 2 (Units and Measurements)

Single Correct Answer Type

Q1. The number of significant figures in 0.06900 is

(a) 5 (b) 4 (c) 2 (d) 3

Sol: (b)

Key concept: Significant figures in the measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement. The reverse is also true.

The following rules are observed in counting the number of significant figures in a given measured quantity.

1. All non-zero digits are significant.
2. A zero becomes significant figure if it appears between two non¬zero digits.
3. Leading zeros or the zeros placed to the left of the number are never significant.
4. Trailing zeros or the zeros placed to the right of the number are significant.
5. In exponential notation, the numerical portion gives the number of significant figures.

Leading zeros or the zeros placed to the left of the number are never significant. In 0.06900 $\Rightarrow \underbrace{0.0}_{\text{not significant}}\ \underbrace{6900}_{\text{significant}}$

Hence, number of significant figures are four.

Q2. The sum of the numbers 436.32, 227.2 and 0.301 inappropriate significant figures is

(a) 663.821 (b) 664 (c) 663.8 (d) 663.82

Sol: (b) The result of an addition or subtraction in the number having different precisions should be reported to the same number of decimal places as present in the number having the least number of decimal places.

$$\begin{array}{rl} 436.32 & \\ +\ 227.2 & \leftarrow \text{(has only one decimal place)} \\ +\ \ 0.301 & \\ \hline 663.821 & \leftarrow \text{(answer should be reported to one decimal place)} \end{array}$$

The final result should, therefore, be rounded off to one decimal place, i.e. 664.

Q3. The mass and volume of a body are 4.237 g and 2.5 cm^3, respectively. The density of the material of the body in correct significant figures is

(a) 1. 6048 g cm^{-3}

(b) 1.69 g cm^{-3}

(c) 1.7 g cm^{3}

(d) 1.695 g cm^{-3}

Sol: (c) The answer to a multiplication or division is rounded off to the same number of significant figures as possessed by the least precise term used in the calculation. The final result should retain as many significant figures as are there in the original number with the least significant figures. In the given question, density should be reported to two significant figures

$$\text{Density} = \frac{4.237 \text{ g}}{2.5 \text{ cm}^3} = 1.6948$$

After rounding off the number, we get density =1.7

Q4. The numbers 2.745 and 2.735 on rounding off to 3 significant figures will give

(a) 2.75 and 2.74

(b) 2.74 and 2.73

(c) 2.75 and 2.73

(d) 2.74 and 2.74

Sol: (d)

Key concept: While rounding off measurements, we use the following rules by convention:

1. If the digit to be dropped is less than 5, then the preceding digit is left unchanged.
2. If the digit to be dropped is more than 5, then the preceding digit is raised by one.
3. If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one.
4. If digit to be dropped is 5 or 5 followed by zeros, then preceding digit is left unchanged, if it is even.
5. If digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd.

Units and Measurements

Let us round off 2.745 to 3 significant figures.
Here the digit to be dropped is 5, then preceding digit is left unchanged, if it is even.
Hence on rounding off 2.745, it would be 2.74.
Now consider 2.737, here also the digit to be dropped is 5, then the preceding digit is raised by one, if it is odd. Hence on rounding off 2.735 to 3 significant figures, it would be 2.74.

Q5. The length and breadth of a rectangular sheet are 16.2 cm and 10.1 cm, respectively. The area of the sheet in appropriate significant figures and error is

(a) 164 ±3 cm^2

(b) 163.62 ± 2.6 cm^2

(c) 163.6 ±2.6 cm^2

(d) 163.62 ±3 cm^2

Sol: (a)

Key concept: *Error in product of quantities:* Suppose $x = a \times b$

Let Δa = absolute error in measurement of a,

Δb = absolute error in measurement of b,

Δx = absolute error in calculation of x, i.e. product of a and b.

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

Percentage error in the value of x = (Percentage error in value of a) + (Percentage error in value of b)

According to the problem, length $l = (16.2 \pm 0.1)$ cm

Breadth $b = (10.1 \pm 0.1)$ cm

Area $A = l \times b = (16.2\text{ cm}) \times (10.1\text{ cm}) = 163.62\text{ cm}^2$

As per the rule area will have only three significant figures and error will have only one significant figure. Rounding off we get, area $A = 164\text{ cm}^2$

If ΔA is error in the area, then relative error is calculated as $\frac{\Delta A}{A}$.

$$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b} = \frac{0.1\text{ cm}}{16.2\text{ cm}} + \frac{0.1\text{ cm}}{10.1\text{ cm}}$$

$$= \frac{1.01 + 1.62}{16.2 \times 10.1} = \frac{2.63}{163.62}$$

$$\Rightarrow \quad \Delta A = A \times \frac{2.63}{163.62}\text{cm}^2 = 163.62 \times \frac{2.63}{163.62} = 2.63\text{ cm}^2$$

$\Delta A = 3\text{ cm}^2$ (By rounding off to one significant figure)

Area, $A = A \pm \Delta A = (164 \pm 3)\text{ cm}^2$

Q6. Which of the following pairs of physical quantities does not have same dimensional formula?

(a) Work and torque

(b) Angular momentum and Planck's constant

(c) Tension and surface tension

(d) Impulse and linear momentum

Sol. (c)

(a) Work $= F \times \Delta x = [MLT^{-2}][L] = [ML^2T^{-2}]$

Torque = force × distance= $[ML^2T^{-2}]$

(b) Angular momentum $= mvr = [M][LT^{-1}][L] = [ML^2T^{-1}]$

Planck's constant $= \frac{E}{V} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$

(c) Tension (force) $= [MLT^{-2}]$

Surface tension $= \frac{\text{force}}{\text{length}} = \frac{[MLT^{-2}]}{[L]} = [ML^0T^{-2}]$

(d) Impulse $= F \times \Delta t = [MLT^{-2}][T] = [MLT^{-1}]$

Momentum = mass × velocity $= [M][LT^{-1}] = [MLT^{-1}]$

So, among the above pairs only tension and surface tension does not have same dimensional formula. They both sound similar but they both have different meaning and different applications.

Q7. Measure of two quantities along with the precision of respective measuring instrument is $A = 2.5\ ms^{-1} \pm 0.5\ ms^{-1}$, $B = 0.10\ s \pm 0.01\ s$. The value of AB will be

(a) (0.25 ± 0.08) m

(b) (0.25 ± 0.5) m

(c) (0.25 ± 0.05) m

(d) (0.25 ± 0.135) m

Sol. (a) According to the problem,

$$A = 2.5\ \text{ms}^{-1} \pm 0.5\ \text{ms}^{-1},\ B = 0.10\ \text{s} \pm 0.01\ \text{s}$$

$$Z = AB = (2.5)(0.10) = 0.25\ \text{m}$$

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B},\ \Delta Z = Z\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$$

$$= 0.25\left(\frac{0.5}{2.5} + \frac{0.01}{0.10}\right) = 0.25(0.2 + 0.1) = 0.075$$

$\Delta Z = 0.075 = 0.08$ m (rounding off to two significant figures.)

Thus, measured value of AB, *i.e.*, $= Z \pm \Delta Z = (0.25 \pm 0.08)$ m

Q8. You measure two quantities as A = 1.0 m ± 0.2 m, B = 2.0 m ± 0.2 m. We should report correct value for $\sqrt{AB}$ as

(a) 1.4 m± 0.4 m

(b) 1.41 m± 0.15 m

(c) 1.4 m + 0.3 m

(d) 1.4 m± 0.2 m

Sol. (d) According to the problem, $A = 1.0\ \text{m} \pm 0.2\ \text{m}$, $B = 2.0\ \text{m} \pm 0.2\ \text{m}$

Let, $Z = \sqrt{AB} = \sqrt{(1.0)(2.0)} = 1.414$ m

Rounding off to two significant digits $Z = 1.4$ m

As $\frac{\Delta Z}{Z} = \frac{1}{2}\frac{\Delta A}{A} + \frac{1}{2}\frac{\Delta B}{B}$

$$= \frac{1}{2}\left(\frac{0.2\ \text{m}}{1\ \text{m}}\right) + \frac{1}{2}\left(\frac{0.2\ \text{m}}{2\ \text{m}}\right) = 0.15$$

$\Rightarrow \Delta Z = Z(0.15) = 1.4\ \text{m}(0.15) = 0.212$

Rounding off to one significant digit, $\Delta Z = 0.2$ m

The correct value for $\sqrt{AB} = 1.4 \pm 0.2$ m.

Q9. Which of the following measurements is most precise?

(a) 5.00 mm

(b) 5.00 cm

(c) 5.00 m

(d) 5.00 km

Sol:(a)

Key concept: Precision is the degree to which several measurements provide answers very close to each other. It is an indicator of the scatter in the data. The lesser the scatter, higher the precision.

Let us first check the units. In all the options magnitude is same but units of measurement are different. As here 5.00 mm has the smallest unit. All given measurements are correct upto two decimal places. However, the absolute error in (a) is 0.01 mm which is least of all the four. So it is most precise.

Q10. The mean length of an object is 5 cm. Which of the following measurements is most accurate?

(a) 4.9 cm

(b) 4.805 cm

(c) 5.25 cm

(d) 5.4 cm

Sol: (a)

Key concept: Accuracy describes the nearness of a measurement to the standard or true value, i.e. a highly accurate measuring device will provide measurements very close to the standard, true or known values.

Example: In target shooting, a high score indicates the nearness to the bull's eye and is a measure of the shooter's accuracy.

According to the problem, length $l = 5$ cm

Let us first check the errors in each values by picking options one by one, we get

$\Delta l_1 = 5 - 4.9 = 0.1$ cm, $\Delta l_2 = 5 - 4.805 = 0.195$ cm,

$\Delta l_3 = 5.25 - 5 = 0.25$ cm and $\Delta l_4 = 5.4 - 5 = 0.4$ cm

Error Δl_1 is the least.

Hence 4.9 cm is most closer to true value. So, 4.9 is more accurate.

Q11. Young's modulus of steel is 1.9×10^{11} N/m^2. When expressed in CGS units of dyne/cm^2, it will be equal to (1 N = 10^5 dyne, 1 m^2 = 10^4 cm^2) .

(a) 1.9 xlO^{10}

(b) 1.9×10^{12}

(c) 1.9 xlO^{12}

(d) 1.9 xlO^{13}

Sol. **(c)** According to the problem,

Young's modulus, $Y = 1.9 \times 10^{11}$ N/m^2

1 N in SI system of units = 10^5 dyne in C.G.S system.

Hence, $Y = 1.9 \times 10^{11} \times 10^5$ dyne/m^2

In C.G.S. length is measured in unit 'cm', so we should also convert m into cm.

$$\therefore \quad Y = 1.9 \times 10^{11}\left(\frac{10^5 \text{dyne}}{10^4 \text{cm}^2}\right) \qquad [\because 1 \text{ m} = 100 \text{ cm}]$$

$$= 1.9 \times 10^{12} \text{ dyne/cm}^2$$

Q12. If momentum (p), area (A) and time (T) are taken to be fundamental quantities, then energy has the dimensional formula

(a) $[pA^{-1}T^{1}]$ (b) $[p^2AT]$ (c) $[pA^{-1/2}T]$ (d) $[pA^{1/2}T^{-1}]$

Sol. **(d)** According to the problem, fundamental quantities are momentum (p), area (A) and time (T) and we have to express energy in these fundamental quantities.

Let energy E,

$$E \propto p^a A^A T^c \quad \Rightarrow \quad E = kp^a A^A T^c$$

where, k is dimensionless constant of proportionality.

Dimensional formula of energy, $[E] = [ML^2T^{-2}]$ and $[p] = [MLT^{-1}]$

$$[A] = [L^2], [T] = [T] \text{ and } [E] = [K][p]^a[A]^b[T]^c$$

Putting all the dimensions, we get

$$ML^2T^{-2} = [MLT^{-1}]^a[L^2]^b[T]^c$$
$$= M^aL^{a+2b}T^{-a+c}$$

According to the principle of homogeneity of dimensions, we get

$$a = 1 \quad \text{...(i)}$$
$$a + 2b = 2 \quad \text{...(ii)}$$
$$-a + c = -2 \quad \text{...(iii)}$$

By solving these equations (i), (ii) and (iii), we get

$$a = 1, b = \frac{1}{2}, c = -1$$

Dimensional formula for E is $[p^1A^{1/2}t^{-1}]$.

More Than One Correct Answer Type

Q13. On the basis of dimensions, decide which of the following relations for the displacement of a particle undergoing simple harmonic motion is not correct?

(a) $y = a \sin 2\pi/T$ (b) $y = a \sin vt$

(c) $y = \frac{a}{T}\sin\left(\frac{t}{a}\right)$ (d) $y = a\sqrt{2}\left(\sin\frac{2\pi t}{T} - \cos\frac{2\pi t}{T}\right)$

Sol. **(b, c)** The argument of trigonometric functions (sin, cos etc.) should be dimensionless. y is displacement and according to the principle of homogeneity of dimensions LHS and RHS.

$$[Y] = [L], [a] = [L]$$

$$\left[\frac{2\pi t}{T}\right] = \frac{[T]}{[T]} = [T^0]$$

$$[vt] = [v][t] = [LT^{-1}][T] = [L]$$

$$\left[\frac{a}{T}\right] = \frac{[a]}{[T]} = \frac{[L]}{[T]} = [LT^{-1}]$$

$$\left[\frac{t}{a}\right] = [L^{-1}T]$$

$$[\text{LHS}] \neq [\text{RHS}]$$

Hence, (c) is not the correct option.

=> LHS ≠ RHS.

So, option (b) is also not correct.

Q14. If P, Q, R are physical quantities, having different dimensions, which of the following combinations can never be a meaningful quantity?

(a) (P-Q)/R

(b) PQ-R

(c) PQ/R

(d) $(PR-Q^2)/R$

(e)(R + Q)/P

Sol: (a, e)

Key concept: Principle of Homogeneity of dimensions: It states that in a correct equation, the

dimensions of each term added or subtracted must be same. Every correct equation must have same dimensions on both sides of the equation.
According to the problem P, Q and R are having different dimensions, since, sum and difference of physical dimensions, are meaningless, i.e., (P – Q) and (R + Q) are not meaningful.
So in option (b) and (c), PQ may have the same dimensions as those of R and in option (d) PR and Q^2 may have same dimensions as those of R.
Hence, they cannot be added or subtracted, so we can say that (a) and (e) are not meaningful.

Q15. Photon is a quantum of radiation with energy E = hv, where v is frequency and h is Planck's constant. The dimensions of h are the same as that of
(a) Linear impulse
(b) Angular impulse
(c) Linear momentum
(d) Angular momentum

Sol. (b, d) We know that energy of radiation, $E = h\nu$. So, we have to compare h with dimensional formula of each option.

$$[h] = \frac{[E]}{[\nu]} = \frac{force \times displacement}{frequency}$$

$$= \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$

(a) Dimension of linear impulse

$$[I] = [Ft] = [MLT^{-2}][T] = [MLT^{-1}]$$

where, t is the time interval.

(b) Dimension of angular impulse

$$[J] = [I\omega] = [ML^2][T^{-1}] = [ML^2T^{-1}]$$

(c) Dimensión of linear momentum

$$[P] = [mv] = [M][LT^{-1}] = [MLT^{-1}]$$

(d) Dimension of angular momentum

$$[L] = [mvr] = [M][LT^{-1}][L] = [ML^2T^{-1}]$$

Hence, dimensión of angular impulse and angular momentum is same as Planck's constant (h).

Q16. If Planck's constant (h) and speed of light in vacuum (c) are taken as two fundamental quantities, which one of the following can, in addition, be taken to express length, mass and time in terms of the three chosen fundamental quantities?
(a) Mass of electron (m_e)
(b) Universal gravitational constant (G)
(c) Charge of electron (e)
(d) Mass of proton (m_p)

Sol. (a, b, d) We know that dimension of h

$$[h] = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}], [G] = [M^{-1}L^3T^{-2}], [c] = [LT^{-1}]$$

For mass, let $m = c^a h^b G^c$

where a, b and c are exponents of c, h and G respectively.

$\Rightarrow \quad [ML^0T^0] = [LT^{-1}]^a [ML^2T^{-1}]^b [M^{-1}L^3T^{-2}]^c$

Using the principle of homogeneity of dimensions,

$b - c = 1$ ...(i)

$a + 2b + 3c = 0$ ...(ii)

$-a - b - 2c = 0$ ...(iii)

Adding eqns. (i), (ii) and (iii),

$2b = 1 \Rightarrow b = 1/2$

From eqn. (i), $a = b - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$

From eqn. (iii), $a = -(b + 2c) = -\left(\frac{1}{2} - 1\right) = \frac{1}{2}$

Hence, $m = c^{1/2} h^{1/2} G^{-1/2} \Rightarrow m = \sqrt{\frac{ch}{G}}$

Similarly, for length, $L = \sqrt{\frac{hG}{c^3}}$ and for time, $T = \sqrt{\frac{hG}{c^5}}$

$\therefore$ Planck's length $= \sqrt{\frac{Gh}{c^3}}$; Planck's time $= \sqrt{\frac{Gh}{c^5}}$;

and Planck's mass $= \sqrt{\frac{ch}{G}}$

Mass can be expressed by m_e and m_p.

Hence, (a), (b) or (d) any can be used to express L, M and T in terms of three chosen fundamental quantities.

Q17. Which of the following ratios express pressure?

(a) Force/Area

(b) Energy/Volume

(c) Energy/Area

(d) Force/Volume

Sol: (a, b) Let us first express the relation of pressure with other physical quantities one by one with the help of dimensional analysis.

We know that pressure

(a) $\frac{\text{Force}}{\text{Area}} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

So, this ratio express pressure (In fact this ratio actually represents pressure).

(b) $\frac{\text{Energy}}{\text{Area}} = \frac{[ML^2T^{-2}]}{[L^2]} = [MT^{-2}]$

Dimensions of this ratio are not same as pressure, so this ratio does not express pressure.

(c) $\frac{\text{Energy}}{\text{Volume}} = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}]$

Dimensions of this ratio is the same as pressure, so this ratio also express pressure.

(d) $\frac{\text{Force}}{\text{Volume}} = \frac{[MLT^{-2}]}{[L^3]} = [ML^{-2}T^{-2}]$

Dimensions of this ratio are not same as pressure, so this ratio does not express pressure.

Q18. Which of the following are not a unit of time?

(a). Second

(b) Parsec

(c) Year

(d) Lightyear

Sol: (b, d) Parsec and light year are those practical units which are used to measure large distances. For example, the distance between sun and earth or other celestial bodies. So they are the units of length not time. Here, second and year represent time.
Important point: 1 light year (distance that light travels in 1 year with speed = 3 x 10^8 m/s.) = 9.46 x 10^{11} m And 1 par see = 3.08 x 10^{16} m

Very Short Answer Type Questions

Q19. Why do we have different units for the same physical quantity?

Sol: Magnitude of any given physical quantity may vary over a wide range, therefore, different units of same physical quantity are required.
For example:
1.Mass ranges from 10^{-30} kg (for an electron) to 10^{53} kg (for the known universe). We need different units to measure them like miligram, gram, kilogram etc.
2.The length of a pen can be easily measured in cm, the height of a tree can be measured in metres, the distance between two cities can be measured in kilometres and distance between two heavenly bodies can be measured in light year.

Q20. The radius of atom is of the order of 1 A and radius of nucleus is of the order of fermi. How many magnitudes higher is the volume of atom as compared to the volume of nucleus?

Sol. According to the question,

Radius of atom $1\text{Å} = 10^{-10}$m   Radius of nucleus $\approx$ 1 fermi = 10^{-15} m

Volume of atom $V_{\text{Atom}} = \frac{4}{3}\pi R_A^3$ Volume of nucleus $V_{\text{nucleus}} = \frac{4}{3}\pi R_N^3$

$$\frac{V_{\text{Atom}}}{V_{\text{Nucleus}}} = \frac{\frac{4}{3}\pi R_A^3}{\frac{4}{3}\pi R_N^3} = \left(\frac{R_A}{R_N}\right)^3 = \left(\frac{10^{-10}}{10^{-15}}\right)^3 = 10^{15}$$

Mass of one mole of $_6C^{12}$ atom = 12 g

Number of atoms in one mole = Avogadro's number = 6.023×10^{23}

$\therefore$ Mass of one $_6C^{12}$ atom $= \frac{12}{6.023 \times 10^{23}}$ g

$$1 \text{ amu} = \frac{1}{12} \times \text{mass of one } _6C^{12} \text{ atom}$$

$$\therefore \quad 1 \text{ amu} = \left(\frac{1}{12} \times \frac{12}{6.023 \times 10^{23}}\right) \text{g} = 1.67 \times 10^{-24} \text{ g}$$

$$= 1.67 \times 10^{-27} \text{ kg} \qquad (\because 1 \text{ g} = 10^{-3} \text{ kg})$$

Q21. Name the device used for measuring the mass of atoms and molecules.

Sol: A mass spectrograph is a device which is used for measuring the mass of atoms and molecules.

Q22. Express unified atomic mass unit in kg.

Sol: The unified atomic mass unit is the standard unit that is used for indicating mass on an atomic or molecular scale (atomic mass). One unified atomic mass unit is approximately the mass of one nucleon (either a single proton or neutron) and is numerically equivalent to 1 g/mol. It is defined as one- twelfth of the mass of an unbound neutral atom of carbon-12 in its nuclear and electronic ground state.

23. A function $f(\theta)$ is defined as

$$f(\theta)=1-\theta+\frac{\theta^2}{2!}-\frac{\theta^3}{3!}+\frac{\theta^4}{4!}+.....$$

Why is it necessary for θ to be a dimensionless quantity?

Sol. We know, the function $f(\theta)$ is a sum of different powers of θ. And it is a dimensionless quantity. This is the limitation of dimensional analysis that we cannot add the powers of a dimensional quantity.
By principle of homogeneity as RHS is dimensionless, hence LHS should also be dimensionless.

Important point: To avoid confusion we can assume the similar expression to understand which is a function of x instead of θ.

$$f(x)=1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}+...$$

In fact in this case x has dimensions.

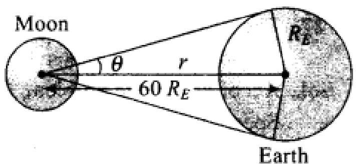
Q24. Why length, mass and time are chosen as base quantities in mechanics?

Sol: Normally each physical quantity requires a unit or standard for its specification, so it appears that there must be as many units as there are physical quantities. However, it is not so. It has been found that if in mechanics we choose arbitrarily units of any three physical quantities we can express the units of all other physical quantities in mechanics in terms of these. So, length, mass and time are chosen as base quantities in mechanics because
(i) Length, mass and time cannot be derived from one another, that is these quantities are independent.
(ii) All other quantities in mechanics can be expressed in terms of length, mass and time.

Short Answer Type Questions

25. (a) The earth-moon distance is about 60 earth radius. What will be the . diameter of the earth (approximately in degrees) as seen from the moon?
(b) Moon is seen to be of (1/2)° diameter from the earth. What must be the relative size compared to the earth?
(c) From parallax measurement, the sun is found to be at a distance of about 400 times the earth-moon distance. Estimate the ratio of sun-earth diameters.

Sol. (a) As the distance between moon and earth is greater than radius of earth, then radius of earth can be treated as an arc.



According to the problem ,
R_E = length of arc
Distance between moon and earth = $60R_E$
So, angle subtended at distance r due to an arc of length l is

$$\theta_E=\frac{l}{r}=\frac{2R_E}{60R_E}=\frac{1}{30}\text{rad}$$

$$=\frac{1}{30}\times\frac{180^\circ}{\pi}\text{degree}=\frac{6^\circ}{3.14}\text{degree}=1.9^\circ\approx 2^\circ$$

Hence, angle subtended by diameter of the earth $2\theta=2^\circ$.

(b) According to the problem, moon is seen as $\left(\frac{1}{2}\right)^\circ$ diameter from earth and earth is seen as 2° diameter from moon.

As θ is proportional to diameter,

Hence, $\frac{\text{Diameter of earth}}{\text{Diameter of moon}} = \frac{2}{\left(\frac{1}{2}\right)} = 4$

(c) From parallax measurement given that Sun is at a distance of about 400 times the earth-moon distance, hence, $\frac{r_{sun}}{r_{moon}} = 400$

(Suppose, here r stands for distance and D for diameter)
Sun and moon both appear to be of the same angular diameter as seen from the earth.

$$\therefore \quad \frac{D_{sun}}{r_{sun}} = \frac{D_{moon}}{r_{moon}} \quad \Rightarrow \quad \frac{D_{sun}}{D_{moon}} = 400$$

$$\text{But} \quad \frac{D_{earth}}{D_{moon}} = 4 \quad \Rightarrow \quad \frac{D_{sun}}{D_{earth}} = 100$$

Q26. Which of the following time measuring devices is most precise?

(a) A wallclock

(b) A stop watch

(c) A digital watch

(d) An atomic clock

Given reason for your answer.

Sol: Option (d) is correct because a clock can measure time correctly up to one second. A stop watch can measure time correctly up to a fraction of a second. A digital watch can measure time up to a fraction of second whereas an atomic clock is the most accurate timekeeper and is based on characteristic frequencies of radiation emitted by certain atoms having precision of about 1 second in 300,000 years. So an atomic clock can measure time most precisely as precision of this clock is about 1 s in 10^{13} s.

Q27. The distance of a galaxy is of the order of 10^{25} Calculate the order of magnitude of time taken by light to reach us from the galaxy.

Sol: According to the problem, distance of the galaxy = 10^{25}m.

Speed of light = 3×10^8 m/s

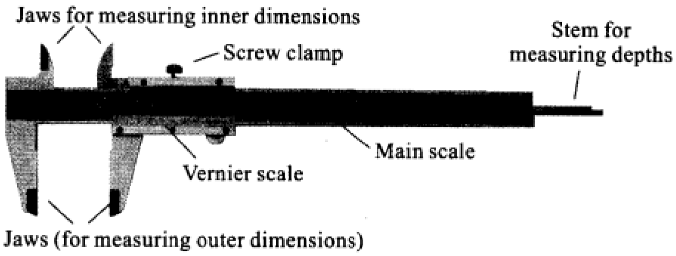
Hence, time taken by light to reach us from galaxy is

$$t = \frac{\text{Distance}}{\text{Speed}} = \frac{10^{25}\text{ m}}{3 \times 10^8 \text{ m/s}} \approx \frac{1}{3} \times 10^{17}$$

$$= \frac{10}{3} \times 10^{16} = 3.33 \times 10^{16}\text{ s}$$

Q28. The Vernier scale of a travelling microscope has 50 divisions which coincide with 49 main scale divisions. If each main scale division is 0.5 mm, calculate the minimum inaccuracy in the measurement of distance.

Sol. Diagram of Vernier caliper is shown below.



According to the problem, 50 divisions of Vernier scale which coincide with 49 main scale divisions.

$$50\ VSD = 49\ MSD$$

$$\Rightarrow \quad 1\ MSD = \frac{50}{49} VSD \quad \text{or} \quad 1\ VSD = \frac{49}{50} MSD$$

where MSD = Main scale division and VSD = Vernier scale division.

We know that

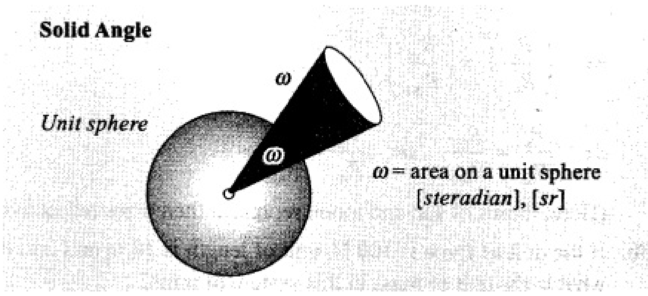
$$\begin{aligned}\text{Minimum inaccuracy} &= \text{Vernier constant}\\ &= 1\ MSD - 1\ VSD\\ &= 1\ MSD - \frac{49}{50} MSD = \frac{1}{50} MSD\end{aligned}$$

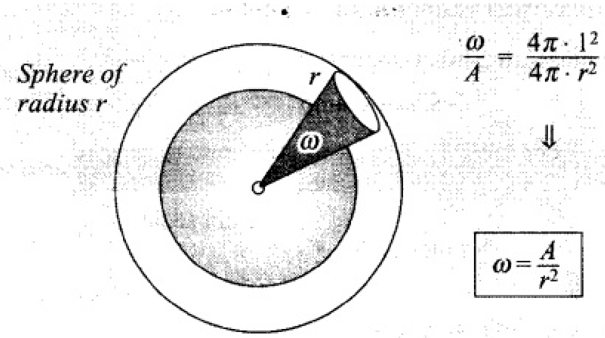
It is given in the problem that $1\ MSD = 0.5$ mm

Hence, minimum inaccuracy $= \frac{1}{50} \times 0.5 \text{ mm} = \frac{1}{100} = 0.01 \text{ mm}$

Q29. During a total solar eclipse the moon almost entirely covers the sphere of the sun. Write the relation between the distances and sizes of the sun and moon.

Sol: Key point: In geometry, a solid angle (symbol: Ω or w) is the twodimensional angle in three-dimensional space that an object subtends at a point. It is a measure of how large the object appears to an observer looking from that point. In the International System of Units (SI), a solid angle is expressed in a dimensionless unit called a steradian (symbol: sr).





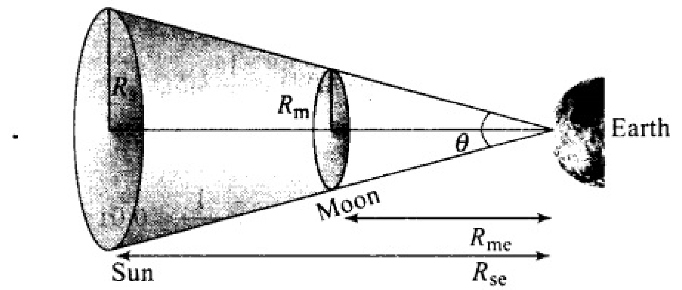
A small object nearby may subtend the same solid angle as a larger object farther away. For example, although the Moon is much smaller than the Sun, it is also much closer to Earth.

Diagram given below shows that moon almost entirely covers the sphere of the sun.

R_{me} = Distance of moon from earth

R_{se} = Distance of sun from earth

Let the solid angle made by sun and moon is $d\Omega$, we can write



$$d\Omega = \frac{A_{sun}}{R_{se}^2} = \frac{A_{moon}}{R_{me}^2}$$

Here, A_{sun} = Area of the sun

A_{moon} = Area of the moon

$$\Rightarrow \quad \theta = \frac{\pi R_s^2}{R_{se}^2} = \frac{\pi R_m^2}{R_{me}^2}$$

$$\Rightarrow \quad \left(\frac{R_s}{R_{se}}\right)^2 = \left(\frac{R_m}{R_{me}}\right)^2$$

$$\Rightarrow \quad \frac{R_s}{R_{se}} = \frac{R_m}{R_{me}} \quad \text{or} \quad \frac{R_s}{R_m} = \frac{R_{se}}{R_{me}}$$

(Here, radius of sun and moon represent their sizes respectively)

Q30. If the unit of force is 100 N, unit of length is 10 m and unit of time is 100 s, what is the unit of mass in this system of units?

Sol. First write dimension of each quantity and then relate them.

Force $[F] = [MLT^{-2}] = 100$ N ...(i)

Length $[L] = [L] = 10$ m ...(ii)

Time $[t] = [T] = 100$ s ...(iii)

Substituting values of L and T from Eqs. (ii) and (iii) in Eq. (i), we get

$$M \times 10 \times (100)^{-2} = 100$$

$$\Rightarrow \quad \frac{M \times 10}{100 \times 100} = 100$$

$$\Rightarrow \quad M = 100 \times 1000 \text{ kg} = 10^5 \text{ kg}$$

Q31. Give an example of

(a) a physical quantity which has a unit but no dimensions

(b) a physical quantity which has neither unit nor dimensions

(c) a constant which has a unit

(d) a constant which has no unit

Sol. (a) Solid angle $\Omega = \frac{A}{r^2}$ *steradian* and a Plane angle $\theta = \frac{L}{r}$ radian. Both are dimensionless but have units.

(b) Specific density $= \frac{\text{density of medium}}{\text{density of water at } 4°C}$

It is a ratio of two same quantities. So, it is a unitless and dimensionless constant.

(c) Gravitational constant $(G) = 6.67 \times 10^{-11}$ N-m²/kg²

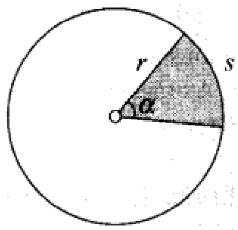
(d) Reynold's number is a constant which has no unit.

Q32. Calculate the length of the arc of a circle of radius 31.0 cm which subtends an angle of π/6 at the centre.

Sol.

Key concept:

Plane Angle $\alpha = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$



According to the problem, $\theta = \frac{\pi}{6} = \frac{s}{31}$ cm

Hence, length of arc $= s = 31 \times \frac{\pi}{6}$ cm $= \frac{31 \times 3.14}{6}$ cm $= 16.22$ cm

Rounding off to three significant figures it would be 16.2 cm.

Q33. Calculate the solid angle subtended by the periphery of an area of 1 cm^2 at a point situated symmetrically at a distance of 5 cm from the area.

Sol. Solid angle, $\Omega = \frac{\text{Area}}{(\text{Radial distance})^2}$

$$= \frac{1\text{ cm}^2}{(5\text{ cm})^2} = \frac{1}{25} = 4 \times 10^{-2} \text{ steradian}$$

($\because$ Area = 1 cm^2, distance = 5 cm)

Important point: Please keep in mind that solid angle is for 3-D figure like sphere, cone etc and plane angle is for plane objects or 2-D figures like circle, arc etc.

Q34. The displacement of a progressive wave is represented by y =A sin(wt− kx), where x is distance and / is time. Write the dimensional formula of (i) w and (ii) k

Sol: We have to apply principle of homogeneity to solve this problem. Principle of homogeneity states that in a correct equation, the dimensions of each term added or subtracted must be same, i.e., dimensions of LHS and RHS should be equal.

According to the problem

$$y = A \sin(\omega t - kx)$$

Here $y = [L]$ hence

$$A \sin(\omega t - kx) = [L]$$

Here $A = [L]$, which peak value of y

So, $\omega t - kx$ should be dimensionless,

(i) $[\omega t]$ = constant

$\Rightarrow \quad [\omega] = [T^{-1}]$

(ii) $[kx]$ = constant

$\Rightarrow \quad [k] = [L^{-1}]$

Q35. Time for 20 oscillations of a pendulum is measured as t_1 =39.6 s; t_2 = 39.9 s and t_3 = 39.5 s. What is the precision in the measurements? What is the accuracy of the measurement?

Sol: According to the problem, time for 20 oscillations of a pendulum,

t_1 = 39.6 s, t_2 = 39.9 s and t_3 = 39.5 s

It is quite obvious from these observations that the least count of the watch is 0.1 s. As measurements have only one decimal place. Precision in the measurement = Least count of the measuring instrument= 0.1 s

Precision in 20 oscillations = 0.1

Therefore, Precision in 1 oscillation $= \frac{0.1}{20} = 0.005$

(ii) Mean value of time for 20 oscillations is given by

$$t = \frac{t_1 + t_2 + t_3}{3}$$

$$= \frac{39.6 + 39.9 + 39.5}{3} = 39.66 \text{ s}$$

Mean time period of the second pendulum $= \frac{39.66}{20} \approx 1.98$ s

Rounding off the time period of second pendulum = 2 s

Measured time period of the second pendulum = 2 – 0.005 = 1.995 s

Accuracy of measurement is the máximum observed error, and is given by

$= 1.995 - 1.980 = 0.015$ s

Long Answer Type Questions

Q36. A new system of units is proposed in which unit of mass is α kg, unit of length β m and unit of time γ s). How much will 5J measure in this new system?

Sol: For solving this problem, dimensions of physical quantity will remain same whatever be the system of units of its measurement.

Let the physical quantity be $Q = n_1u_1 = n_2u_2$
Let M_1, L_1, T_1, and M_2, L_2, T_2 are units of mass, length and time in given tw systems.

$$\text{So, } n_2 = n_1\left[\frac{M_1}{M_2}\right]^a \times \left[\frac{L_1}{L_2}\right]^b \times \left[\frac{T_1}{T_2}\right]^c$$

We know that dimension of energy $[U] = [ML^2T^{-2}]$
According to the problem, $M_1 = 1$ kg, $L_1 = 1$ m, $T_1 = 1$ s
$M_2 = \alpha$ kg, $L_2 = \beta$ m, $T_2 = \gamma$ s
Substituting the values, we get

$$n_2 = 5\left[\frac{M_1}{M_2}\right] \times \left[\frac{L_1}{L_2}\right]^2 \times \left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 5\left[\frac{1}{\alpha}\text{kg}\right] \times \left[\frac{1}{\beta}\text{m}\right]^2 \times \left[\frac{1}{\gamma}\text{s}\right]^{-2}$$

$$= 5 \times \frac{1}{\alpha} \times \frac{1}{\beta^2} \times \frac{1}{\gamma^{-2}} = \frac{5\gamma^2}{\alpha\beta^2}\text{ J}$$

This is the required value of energy in the new system of units.

37. The volume of a liquid flowing out per second of a pipe of length l and radius r is written by a student as $V = \frac{\pi}{8}\frac{pr^4}{\eta l}$ where p is the pressure difference between the two ends of the pipe and η is coefficient of viscosity of the liquid having dimensional formula $[ML^{-1}T^{-1}]$. Check whether the equation is dimensionally correct.

Sol. If dimensions of LHS of an equation is equal to dimensions of RHS, then equation is said to be dimensionally correct.
According to the problem, the volume of a liquid flowing out per second of a pipe is given by $V = \frac{\pi}{8}\frac{pr^4}{\eta l}$
(where, V = rate of volume of liquid per unit time)

Dimension of given physical quantities,

$$[V] = \frac{\text{Dimension of volume}}{\text{Dimension of time}} = \frac{[L^3]}{[T]} = [L^3T^{-1}], [p] = [ML^{-1}T^{-2}],$$

$$[\eta] = [ML^{-1}T^{-1}], [l] = [L], [r] = [L]$$

$$\text{LHS} = [V] = \frac{[L^3]}{[T]} = [L^3T^{-1}]$$

$$\text{RHS} = \frac{[ML^{-1}T^{-2}] \times [L^4]}{[ML^{-1}T^{-1}] \times [L]} = [L^3T^{-1}]$$

Dimensionally, L.H.S. = R.H.S.
Therefore, equation is correct dimensionally.

38. A physical quantity X is related to four measurable quantities a, b, c and d as follows $X = a^2 b^3 c^{5/2} d^{-2}$. The percentage error in the measurement of a, b, c and d are 1%, 2%, 3% and 4%, respectively. What is the percentage error in quantity X? If the value of X calculated on the basis of the above relation is 2.763, to what value should you round off the result.

Sol. Percentage error in quantity X is given by, $\frac{\Delta x}{x} \times 100$

According to the problem, physical quantity is $X = a^2 b^3 c^{5/2} d^{-2}$

percentage error in $a = \left(\frac{\Delta a}{a} \times 100\right) = 1\%$

percentage error in $b = \left(\frac{\Delta b}{b} \times 100\right) = 2\%$

percentage error in $c = \left(\frac{\Delta c}{c} \times 100\right) = 3\%$

percentage error in $d = \left(\frac{\Delta d}{d} \times 100\right) = 4\%$

Maximum percentage error in X is

$$\frac{\Delta X}{X} \times 100 = \pm\left[2\left(\frac{\Delta a}{a} \times 100\right) + 3\left(\frac{\Delta b}{b} \times 100\right) + \frac{5}{2}\left(\frac{\Delta c}{c} \times 100\right) + 2\left(\frac{\Delta d}{d} \times 100\right)\right]$$

$$= \pm\left[2(1) + 3(2) + \frac{5}{2}(3) + 2(4)\right]\%$$

$$= \pm\left[2 + 6 + \frac{15}{2} + 8\right] = \pm 23.5\%$$

$\therefore$ Percentage error in quantity $X = \pm 23.5\%$

Mean absolute error in $X = \pm 0.235 = \pm 0.24$ (rounding-off upto two significant digits)

On the basis of these values, the value of X should have two significant digits only.

$\therefore$ $X = 2.8$

39. In the expression $P = El^2 m^{-5} G^{-2}$; E, m, l and G denote energy, mass, angular momentum and gravitational constant, respectively. Show that P is a dimensionless quantity.

Sol. According to the problem, expression is $P = El^2 m^{-5} G^{-2}$

where E is energy $[E] = [ML^2T^{-2}]$, m is mass $[m] = [M]$,

L is angular momentum $[L] = [ML^2 T^{-1}]$, G is gravitational constant $[G] = [M^{-1}L^3T^{-2}]$

Substituting dimensions of each physical quantity in the given expression,

$$[P] = [ML^2T^{-2}] \times [ML^2T^{-1}]^2 \times [M]^{-5} \times [M^{-1}L^3T^{-2}]^{-2}$$

$$= [M^{1+2-5+2} L^{2+4-6} T^{-2-2+4}]$$

$$= [M^0L^0T^0]$$

This shows that P is a dimensionless quantity.

Q40. If velocity of light c, Planck's constant h and gravitational constant G are taken as fundamental quantities, then express mass, length and time in terms of dimensions of these quantities.

Sol: We have to apply principle of homogeneity to solve this problem. Principle of homogeneity states that in a correct equation, the dimensions of each term added or subtracted must be same, i.e., dimensions of LHS and RHS should be equal,

We know that, dimensions of

$[h] = [ML^2T^{-1}]$, $[c] = [LT^{-1}]$, $[G] = [M^{-1}L^3T^{-2}]$

(i) Let $m \propto c^x h^y G^z$

$$\Rightarrow \quad m = kc^a h^b G^c \qquad \ldots(i)$$

where, k is a dimensionless constant of proportionality.

Substituting dimensions of each term in Eq. (i), we get

$$[ML^0T^0] = [LT^{-1}]^a \times [ML^2T^{-1}]^b[M^{-1}L^3T^{-2}]^c$$

Comparing powers of same terms on both sides, we get

$$b - c = 1 \qquad \ldots(ii)$$

$$a + 2b + 3c = 0 \qquad \ldots(iii)$$

$$-a - b - 2c = 0 \qquad \ldots(iv)$$

Adding Eqs. (ii), (iii) and (iv), we get

$$2b = 1 \quad \Rightarrow \quad b = \frac{1}{2}$$

Substituting value of b in Eq. (ii), we get

$$c = -\frac{1}{2}$$

From Eq. (iv)

$a = -b - 2c$

Substituting values of b and c, we get

$$a = -\frac{1}{2} - 2\left(-\frac{1}{2}\right) = \frac{1}{2}$$

Putting values of a, b and c in Eq. (i), we get

$$m = kc^{1/2}h^{1/2}G^{-1/2} = k\sqrt{\frac{ch}{G}}$$

(ii) Let $L \propto c^a h^b G^c$

$$\Rightarrow \quad L = kc^a h^b G^c \qquad \ldots(v)$$

where k is a dimensionless constant.

Substituting dimensions of each term in Eq. (v), we get

$$[M^0LT^0] = [LT^{-1}]^a \times [ML^2T^{-1}]^b \times [M^{-1}L^3T^{-2}]^c$$

$$= [M^{b-c}L^{a+2b+3c}T^{-a-b-2c}]$$

On comparing powers of same terms, we get

$$b - c = 0 \qquad \ldots(vi)$$

$$a + 2b + 3c = 1 \qquad \ldots(vii)$$

$$-a - b - 2c = 0 \qquad \ldots(viii)$$

Adding Eqs. (vi), (vii) and (viii), we get

$$2b = 1 \quad \Rightarrow \quad b = \frac{1}{2}$$

Substituting value of b in Eq. (vi), we get

$$c = \frac{1}{2}$$

From Eq. (viii), $a = -b - 2c$

Substituting values of b and c, we get

$$a = -\frac{1}{2} - 2\left(\frac{1}{2}\right) = -\frac{3}{2}$$

Putting values of a, b and c in Eq. (v), we get

$$L = kc^{-3/2}h^{1/2}G^{1/2} = k\sqrt{\frac{hG}{c^3}}$$

(iii) Let $T \propto c^a h^b G^c$

$\Rightarrow \quad T = c^a h^b G^c$ ...(ix)

where, k is a dimensionless constant.

Substituting dimensions of each term in Eq. (ix), we get

$$[M^0L^0T^1] = [LT^{-1}]^a \times [ML^2T^{-1}]^b \times [M^{-1}L^3T^{-2}]^c$$
$$= [M^{b-c} L^{a+2b+3c} T^{-a-b-2c}]$$

On comparing powers of same terms, we get

$b - c = 0$ ...(x)

$a + 2b + 3c = 1$ ...(xi)

$-a - b - 2c = 1$ ...(xii)

Adding Eqs. (x), (xi) and (xii), we get

$$2b = 1 \quad \Rightarrow \quad b = \frac{1}{2}$$

Substituting the value of b in Eq. (x), we get

$$c = b = \frac{1}{2}$$

From Eq. (xii),

$$a = -b - 2c - 1$$

Substituting values of b and c, we get

$$a = -\frac{1}{2} - 2\left(\frac{1}{2}\right) - 1 = -\frac{5}{2}$$

Putting values of a, b and c in Eq. (ix), we get

$$T = kc^{-5/2}h^{1/2}G^{1/2} = k\sqrt{\frac{hG}{c^5}}$$

Q41. An artificial satellite is revolving around a planet of mass M and radius R, in a circular orbit of radius r. From Kepler's third law about the period of a satellite around a common central body, square of the period of revolution T is proportional to the cube of the radius of the orbit r. Show using dimensional

analysis, that $T = \frac{k}{R}\sqrt{\frac{r^3}{g}}$ where k is a dimensionless constant and g is acceleration due to gravity.

Sol. According to Kepler's third law, $T^2 \propto a^3$ i.e., square of time period (T^2) of a satellite revolving around a planet, is proportional to the cube of the radius of the orbit (a^3).

We have to apply Kepler's third law,

$$T^2 \propto r^3 \Rightarrow T \propto r^{3/2}$$

Also, T depends on R and g.

Let $T \propto r^{3/2} g^a R^b$

$\Rightarrow \quad T = kr^{3/2}R^a g^b$ ...(i)

where, k is a dimensionless constant of proportionality.

Writing the dimensions of various quantities on both the sides, we get

$$[M^0L^0T] = [L]^{3/2}[LT^{-2}]^a[L]^b$$
$$= [M^0L^{a+b+3/2}T^{-2a}]$$

On comparing the dimensions of both sides, we get

$a + b + \frac{3}{2} = 0$ ...(ii)

$-2a = 1 \quad \Rightarrow \quad a = -1/2$ ...(iii)

From Eq. (ii), we get

$$b - \frac{1}{2} + \frac{3}{2} = 0 \Rightarrow b = -1$$

Substituting the values of a and b in Eq. (i), we get

$$T = kr^{3/2}R^{-1}g^{-1/2} \Rightarrow T = \frac{k}{R}\sqrt{\frac{r^3}{g}}$$

Q42. In an experiment to estimate 'the size of a molecule of oleic acid, 1 mL of oleic acid is dissolved in 19 mL of alcohol. Then 1 mL of this solution is diluted to 20 mL by adding alcohol. Now, 1 drop of this diluted solution is placed on **water in a shallow trough. The solution spreads over the surface of water forming one molecule thick layer. Now, lycopodium powder is sprinkled evenly over the film and its diameter is measured. Knowing the volume of the drop and area of the film we can calculate the thickness of the film which will give us the size of** oleic **acid molecule.**

Read the passage carefully and answer the following questions.

- **Why do we dissolve oleic acid in alcohol?**
- **What is the role of lycopodium powder?**
- **What would be the volume of oleic acid in each mL of** solution **prepared?**
- **How will you calculate the volume of n drops of this solution of oleic**
- **What will be the volume of oleic acid in one drop of this solution?**

Sol: (a) Since Oleic acid does not dissolve in water, hence it is dissolved in alcohol.

(b)Lycopodium powder spreads on the entire surface of water when it is sprinkled evenly. When a drop of prepared solution of oleic acid and alcohol is dropped on water, oleic acid does not dissolve in water. Instead it spreads on the water surface pushing the lycopodium powder away to clear a circular area where the drop falls. We can thus be able to measure the area over which oleic acid spreads.

(c)Since 20 mL (1 mL oleic acid + 19 mL alcohol) contains 1 mL of oleic acid, oleic acid in each mL of the solution =1/20 mL. Further, as this 1 mL is diluted to 20 mL by adding alcohol. In each mL of solution prepared, volume of oleic acid = 1/20 mL x 1/20 = 1/400 mL

(d) Volume of n drops of this solution of oleic acid can be calculated by means of a burette (used to make solution in the form of countable drops) and measuring cylinder and measuring the number of drops.

(e) As 1 mL of solution contains n number of drops, then the volume of oleic acid in one drop will be = 1/(400)n mL

Q43. (a) How many astronomical units (AU) make 1 parsec?
(b) Consider the sun like a star at a distance of 2 parsecs. When it is seen through a telescope with 100 magnification, what should be the angular size of the star? Sun appears to be $(1/2)^\circ$ from the earth. Due to atmospheric fluctuations, eye **cannot resolve objects smaller than 1 arc minute.**
(c) Mars has approximately half of the earth's diameter. When it is closest to the earth it is at about 1/2 AU from the earth. Calculate at what size it will appear when seen through the same telescope.

Ans: (a) According to the definition, 1 parsec is equal to the distance at which 1 AU long arc subtends an angle of 1 s.

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But $1'' = \frac{1}{3600} \times \frac{\pi}{180}$ rad

$\therefore \quad 1 \text{ parsec} = \frac{3600 \times 180}{\pi} AU$

$= 206265 AU \approx 2 \times 10^5 AU$

(b) Sun's angular diameter from the earth is $\left(\frac{1}{2}\right)^\circ$ at 1 AU.

Angular diameter of the sun like star at a distance of 2 parsecs

$$= \frac{(1/2)^\circ}{2 \times 2 \times 10^5} = \left(\frac{1}{8} \times 10^{-5}\right)^\circ$$

$$= \left(\frac{1}{8} \times 10^{-5}\right)^\circ \times 60' = 7.5 \times 10^{-5} \text{ arcmin}$$

When the sun like star is seen through a telescope with magnification 100, the angular diameter of the star

$= 100 \times 7.5 \times 10^{-5} = 7.5 \times 10^{-3}$ arcmin

But eye cannot resolve smaller than 1 arcmin due to atmospheric fluctuations. So angular size of sun like star appears as 1 arcmin.

(c) Given that $\frac{D_{mars}}{D_{earth}} = \frac{1}{2}$ ...(i)

where D represents diameter.

We know that, $\frac{D_{earth}}{D_{sun}} = \frac{1}{100}$

$\therefore \quad \frac{D_{mars}}{D_{sun}} = \frac{1}{2} \times \frac{1}{100}$ [from Eq. (i)]

At 1AU Sun's diameter $= \left(\frac{1}{2}\right)^\circ$

$\therefore$ Diameter of Mars $= \frac{1}{2} \times \frac{1}{200} = \left(\frac{1}{400}\right)^\circ$

At $\frac{1}{2}$AU, Mars' diameter

$$= \frac{1}{400} \times 2^\circ = \left(\frac{1}{200}\right)^\circ$$

With 100 magnification, Mars' diameter

$$= \frac{1}{200} \times 100^\circ = \left(\frac{1}{2}\right)^\circ = 30'$$

This is larger than resolution limit due to atmospheric fluctuations. Hence, it looks magnified.

Q44. Einstein's mass-energy relation emerging out of his famous theory of relativity relates mass (m) to energy (E) as E= mc^2, where c is speed of light in vacuum. At the nuclear level, the magnitudes of energy are very small. The energy at nuclear level is usually measured in MeV, where 1 MeV = 1.6 x 10^{-13} J; the masses are measured in unified atomic mass unit (u) where, 1 u = 67 x 10^{-27} kg.

(a) Show that the energy equivalent of 1 u is 931.5 MeV.

(b) A student writes the relation as 1 u = 931.5 MeV. The teacher points out that the relation is dimensionally incorrect. Write the correct relation.

Sol: (a) We can apply Einstein's mass-energy relation in this problem, E = mc^2, to calculate the energy equivalent of the given mass.

Here

$1 \text{ amu} = 1\, u = 1.67 \times 10^{-27}$ kg

Applying $E = mc^2$

Energy $E = (1.67 \times 10^{-27})(3 \times 10^8)^2 \text{ J} = 1.67 \times 9 \times 10^{-11} \text{ J}$

$$E = \frac{1.67 \times 9 \times 10^{-11}}{1.6 \times 10^{-13}} \text{ MeV}$$

$$= 939.4 \text{ MeV} \approx 931.5 \text{ MeV}$$

(b) As $E = mc^2 \Rightarrow m = \frac{E}{c^2}$

According to this, $1u = \frac{931.5 \text{ MeV}}{c^2}$

Hence the dimensionally correct relation $1 \text{ amu} \times c^2 = 1u \times c^2$

$= 931.5$ MeV