Unit 8 (Binomial Theorem)

Short Answer Type Questions:

Q1. Find the term independent of x, where x≠0, in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^{15}$

Sol. Given expansion is $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^{15}$

$$\therefore \quad T_{r+1} = {}^{15}C_r \left(\frac{3x^2}{2}\right)^{15-r} \left(-\frac{1}{3x}\right)^r$$

or $\quad T_{r+1} = {}^{15}C_r (-1)^r \, 3^{15-2r} 2^{r-15} \, x^{30-3r} \qquad$ (i)

For the term independent of x, $30 - 3r = 0 \Rightarrow r = 10$

$\therefore$ The term independent of x is

$$T_{10+1} = {}^{15}C_{10} \, 3^{-5} \, 2^{-5} \qquad \text{(Putting } r = 10 \text{ in (i))}$$

$$= {}^{15}C_{10} \left(\frac{1}{6}\right)^5$$

Q2. If the term free from x is the expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then find the value of k.

Sol: Given expansion is $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$

$$\therefore \quad T_{r+1} = {}^{10}C_r (\sqrt{x})^{10-r} \left(\frac{-k}{x^2}\right)^r = {}^{10}C_r (x)^{\frac{1}{2}(10-r)} (-k)^r x^{-2r}$$

$$= {}^{10}C_r (x)^{5-\frac{r}{2}-2r} (-k)^r = {}^{10}C_r x^{\frac{10-5r}{2}} (-k)^r$$

For the term free from x, $\frac{10-5r}{2} = 0 \quad \Rightarrow \quad r = 2$

So, the term free from x is $T_{2+1} = {}^{10}C_2 (-k)^2$.

$\Rightarrow \quad {}^{10}C_2 (-k)^2 = 405$

$\Rightarrow \quad \frac{10 \times 9 \times 8!}{2! \times 8!} (-k)^2 = 405$

$\Rightarrow \quad 45k^2 = 405 \quad \Rightarrow \quad k^2 = 9 \quad \therefore k = \pm 3$

Q3. Find the coefficient of x in the expansion of $(1 - 3x + 1x^2)(1 - x)^{16}$.

Sol: $(1 - 3x + 1x^2)(1 - x)^{16}$

$$= (1 - 3x + 7x^2)({}^{16}C_0 - {}^{16}C_1 x^1 + {}^{16}C_2 x^2 + \ldots + {}^{16}C_{16} x^{16})$$
$$= (1 - 3x + 7x^2)(1 - 16x + 120x^2 + \ldots)$$

$\therefore$ Coefficient of $x = -16 - 3 = -19$

Q4. Find the term independent of x in the expansion of $\left(3x - \frac{2}{x^2}\right)^{15}$

Sol: Given Expression $\left(3x - \frac{2}{x^2}\right)^{15}$

$$\therefore \; T_{r+1} = {}^{15}C_r (3x)^{15-r}\left(\frac{-2}{x^2}\right)^r = {}^{15}C_r\, 3^{15-r} x^{15-3r} (-2)^r$$

For the term independent of x, $15 - 3r = 0 \;\Rightarrow\; r = 5$

$\therefore$ The term independent of x is

$$T_{5+1} = {}^{15}C_5\, 3^{15-5} (-2)^5$$
$$= \frac{15\times14\times13\times12\times11\times10!}{5\times4\times3\times2\times1\times10!}\cdot 3^{10}\cdot 2^5$$
$$= -3003 \times 3^{10} \times 2^5$$

Q5. Find the middle term (terms) in the expansion of

(i) $\left(\frac{x}{a} - \frac{a}{x}\right)^{10}$ (ii) $\left(3x - \frac{x^3}{6}\right)^9$

Sol. (i) Given expression is $\left(\frac{x}{a} - \frac{a}{x}\right)^{10}$

Here index, $n = 10$ (even). So, there is one middle term which is $\left(\frac{10}{2}+1\right)$th term, i.e., 6th term.

$$\therefore \; T_6 = T_{5+1} = {}^{10}C_5\left(\frac{x}{a}\right)^{10-5}\left(\frac{-a}{x}\right)^5$$
$$= -{}^{10}C_5\left(\frac{x}{a}\right)^5\left(\frac{a}{x}\right)^5$$
$$= -\frac{10\times9\times8\times7\times6\times5!}{5!\times5\times4\times3\times2\times1}\left(\frac{x}{a}\right)^5\left(\frac{x}{a}\right)^{-5} = -252$$

(ii) Given expression is $\left(3x - \frac{x^3}{6}\right)^9$

Here index, $n = 9$ (odd)

So, there are two middle terms, which are $\left(\frac{9+1}{2}\right)$th i.e., 5^{th} term and $\left(\frac{9+1}{2}+1\right)$th i.e., 6^{th} term.

$$\therefore \quad T_5 = T_{4+1} = {}^9C_4\,(3x)^{9-4}\left(-\frac{x^3}{6}\right)^4$$

$$= \frac{9\times8\times7\times6\times5!}{4\times3\times2\times1\times5!}3^5\,x^5\,x^{12}\,6^{-4}$$

$$= \frac{9\times8\times7\times6}{4\times3\times2\times1}\times\frac{3^5}{3^4\times2^4}x^{17} = \frac{189}{8}x^{17}$$

And $T_6 = T_{5+1} = {}^9C_5\,(3x)^{9-5}\left(-\frac{x^3}{6}\right)^5$

$$= -\frac{9\times8\times7\times6\times5!}{5!\times4\times3\times2\times1}\cdot3^4\cdot x^4\cdot x^{15}\cdot6^{-5} = -\frac{-21}{16}x^{19}$$

Q6. Find the coefficient of x^{15} in the expansion of $(x - x^2)^{10}$

Sol: Given expression is $(x - x^2)^{10}$

$\therefore \quad T_{r+1} = {}^{10}C_r\,x^{10-r}\,(-x^2)^r = (-1)^r\,{}^{10}C_r x^{10-r}x^{2r} = (-1)^r\,{}^{10}C_r\,x^{10+r}$

For the coefficient of x^{15}, we have

$10 + r = 15 \quad \Rightarrow \quad r = 5$

$\therefore \quad T_{5+1} = (-1)^5\,{}^{10}C_5\,x^{15}$

$\therefore \quad$ Coefficient of $x^{15} = -\frac{10\times9\times8\times7\times6\times5!}{5\times4\times3\times2\times1\times5!} = -252$

Q7. Find the coefficient of $\frac{1}{x^{17}}$ in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$

Sol. Given expression is $\left(x^4 - \frac{1}{x^3}\right)^{15}$

$$\therefore \quad T_{r+1} = {}^{15}C_r\,(x^4)^{15-r}\left(-\frac{1}{x^3}\right)^r = {}^{15}C_r\,x^{60-4r}\,(-1)^r\,x^{-3r} = {}^{15}C_r\,x^{60-7r}\,(-1)^r$$

For the coefficient x^{-17}, we have

$60 - 7r = -17 \quad \Rightarrow \quad 7r = 77 \quad \Rightarrow \quad r = 11$

$\therefore \quad T_{11+1} = {}^{15}C_{11}\,x^{60-77}(-1)^{11}$

$\therefore \quad$ Coefficient of $x^{-17} = \frac{-15\times14\times13\times12\times11!}{11!\times4\times3\times2\times1}$

$$= -15\times7\times13 = -1365$$

Q8. Find the sixth term of the expansion $(y^{1/2} + x^{1/3})^n$, if the binomial coefficient of the third term from the end is 45.

Sol. Given expression is $(y^{1/2} + x^{1/3})^n$.

Given that

Binomial coefficient of third term from the end = 45

$\Rightarrow \quad {}^nC_{n-2} = 45 \quad \Rightarrow \quad {}^nC_2 = 45 \quad \Rightarrow \quad \frac{n(n-1)(n-2)!}{2!(n-2)!} = 45$

$\Rightarrow \quad n(n-1) = 90 \quad \Rightarrow \quad n^2 - n - 90 = 0 \quad \Rightarrow \quad (n-10)(n+9) = 0$

$\Rightarrow \quad n = 10 \quad [\because n \neq -9]$

> Now, sixth term = ${}^{10}C_5 (y^{1/2})^{10-5} (x^{1/3})^5 = 252\, y^{5/2} \cdot x^{5/3}$

Q9. Find the value of r, if the coefficients of (2r + 4)th and (r – 2)th terms in the expansion of $(1 + x)^{18}$ are equal.

Sol. Given expression is $(1 + x)^{18}$.

Now, (2r + 4)th term, i.e., $T_{(2r+3)+1}$

$\therefore \quad T_{(2r+3)+1} = {}^{18}C_{2r+3}\,(x)^{2r+3}$

And (r – 2)th term, i.e., $T_{(r-3)+1}$

$\therefore \quad T_{(r-3)+1} = {}^{18}C_{r-3}\,x^{r-3}$

According to the question,

${}^{18}C_{2r+3} = {}^{18}C_{r-3}$

$\Rightarrow \quad 2r + 3 + r - 3 = 18 \qquad [\because {}^nC_x = {}^nC_y \Rightarrow x + y = n]$

$\Rightarrow \quad 3r = 18 \quad \therefore \quad r = 6$

Q10. If the coefficient of second, third and fourth terms in the expansion of $(1 + x)^{2n}$ are in A.P., then show that $2n^2 - 9n + 7 = 0$.

Sol. Given expression is $(1 + x)^{2n}$

Now, coefficient of 2nd, 3rd and 4th terms are ${}^{2n}C_1$, ${}^{2n}C_2$ and ${}^{2n}C_3$, respectively.

Given that, ${}^{2n}C_1$, ${}^{2n}C_2$ and ${}^{2n}C_3$ are in A.P.

Then, $\quad 2 \cdot {}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$

$$\Rightarrow \quad 2\left[\frac{2n(2n-1)(2n-2)!}{2 \times 1 \times (2n-2)!}\right] = 2n + \frac{2n(2n-1)(2n-2)(2n-3)!}{3!(2n-3)!}$$

$$\Rightarrow \quad n(2n-1) = n + \frac{n(2n-1)(n-1)}{3}$$

$\Rightarrow \quad 3(2n-1) = 3 + (2n^2 - 3n + 1)$

$\Rightarrow \quad 6n - 3 = 2n^2 - 3n + 4 \quad \Rightarrow \quad 2n^2 - 9n + 7 = 0$

Q11. Find the coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^{11}$.

Sol. Given expression is $(1 + x + x^2 + x^3)^{11}$

$= [(1 + x) + x^2(1 + x)]^{11} = [(1 + x)(1 + x^2)]^{11} = (1 + x)^{11} \cdot (1 + x^2)^{11}$

$= ({}^{11}C_0 + {}^{11}C_1x + {}^{11}C_2x^2 + {}^{11}C_3x^3 + {}^{11}C_4x^4 + \ldots)({}^{11}C_0 + {}^{11}C_1x^2 + {}^{11}C_2x^4 + \ldots)$

$\therefore \quad$ Coefficient of $x^4 = {}^{11}C_0 \times {}^{11}C_4 + {}^{11}C_1 \times {}^{11}C_2 + {}^{11}C_2 \times {}^{11}C_0$

$= 330 + 605 + 55 = 990$

Long Answer Type Questions

Q12. If p is a real number and the middle term in the expansion $\left(\frac{p}{2} + 2\quad\right)^8$ is 1120, then find the value of p.

Sol. Given expansion is $\left(\frac{p}{2}+2\right)^8$

Since index is $n = 8$, there is only one middle term, i.e., $\left(\frac{8}{2}+1\right)$th $= 5^{th}$ term

$$T_5 = T_{4+1} = {}^8C_4\left(\frac{p}{2}\right)^{8-4} \cdot 2^4$$

$\Rightarrow \quad 1120 = {}^8C_4 p^4 \qquad \Rightarrow \quad 1120 = \frac{8\times7\times6\times5\times4!}{4!\times4\times3\times2\times1}p^4$

$\Rightarrow \quad 1120 = 7\times2\times5\times p^4 \qquad \Rightarrow \quad p^4 = \frac{1120}{70}$

$\Rightarrow \quad p^4 = 16 \qquad \Rightarrow \quad p^2 = 4 \quad \Rightarrow \quad p = \pm2$

13. Show that the middle term in the expansion of $\left(x-\frac{1}{x}\right)^{2n}$ is $\frac{1\times3\times5\times\ldots\times(2n-1)}{n!}\times(-2)^n$.

Sol. Given, expression is $\left(x-\frac{1}{x}\right)^{2n}$.

Since the index is $2n$, which is even. So, there is only one middle term, i.e., $\left(\frac{2n}{2}+1\right)$th term $= (n + 1)$th term

$$T_{n+1} = {}^{2n}C_n(x)^{2n-n}\left(-\frac{1}{x}\right)^n = {}^{2n}C_n(-1)^n = (-1)^n\frac{(2n!)}{n!\cdot n!}$$

$$= (-1)^n\frac{1\cdot2\cdot3\cdot4\cdot5\ldots(2n-1)\cdot(2n)}{n!\cdot n!} = (-1)^n\frac{[1\cdot3\cdot5\ldots(2n-1)]\cdot[2\cdot4\cdot6\ldots(2n)]}{(1\cdot2\cdot3\ldots n)\cdot n!}$$

$$= (-1)^n\frac{[1\cdot3\cdot5\ldots(2n-1)]\cdot2^n[1\cdot2\cdot3\ldots n]}{(1\cdot2\cdot3\ldots n)\cdot n!} = (-2)^n\frac{[1\cdot3\cdot5\ldots(2n-1)]\cdot2^n}{n!}$$

14. Find n in the binomial $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$, if the ratio of 7^{th} term from the beginning to the 7^{th} term from the end is $\frac{1}{6}$.

Sol. Given expression is $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$

Now, 7^{th} term from beginning, $T_7 = T_{6+1} = {}^nC_6(\sqrt[3]{2})^{n-6}\left(\frac{1}{\sqrt[3]{3}}\right)^6$ (i)

And 7^{th} term from end is same as 7^{th} term from the beginning of $\left(\frac{1}{\sqrt[3]{3}}+\sqrt[3]{2}\right)^n$

i.e., $\quad T_7 = {}^nC_6\left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}(\sqrt[3]{2})^6$ (ii)

Given that, $\dfrac{{}^nC_6(\sqrt[3]{2})^{n-6}\left(\frac{1}{\sqrt[3]{3}}\right)^6}{{}^nC_6\left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}(\sqrt[3]{2})^6} = \frac{1}{6}$

$\Rightarrow \quad \dfrac{(\sqrt[3]{2})^{n-12}}{\left(\frac{1}{\sqrt[3]{3}}\right)^{n-12}} = \frac{1}{6} \quad \Rightarrow \quad (\sqrt[3]{2}\sqrt[3]{3})^{n-12} = 6^{-1} \quad \Rightarrow \quad 6^{\frac{n-12}{3}} = 6^{-1}$

$\Rightarrow \quad \frac{n-12}{3} = -1 \quad \Rightarrow \quad n = 9$

Q15. In the expansion of $(x + a)^n$, if the sum of odd term is denoted by 0 and the sum of even term by Then, prove that

(i) $O^2 - E^2 = (x^2 - a^2)^n$

(ii) $4OE = (x + a)^{2n} - (x - a)^{2n}$

Sol. (i) We have $(x + a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a^1 + {}^nC_2x^{n-2}a^2 + {}^nC_3x^{n-3}a^3 + \dots + {}^nC_n a^n$

Sum of odd terms, $O = {}^nC_0x^n + {}^nC_2x^{n-2}a^2 + \dots$

And sum of even terms, $E = {}^nC_1x^{n-1}a + {}^nC_3x^{n-3}a^3 + \dots$

Since $(x + a)^n = O + E$ (i)

$(x - a)^n = O - E$ (ii)

$\therefore$ $(O + E)(O - E) = (x + a)^n(x - a)^n$

$\Rightarrow$ $O^2 - E^2 = (x^2 - a^2)^n$

(ii) $4OE = (O + E)^2 - (O - E)^2 = [(x + a)^n]^2 - [(x - a)^n]^2 = (x + a)^{2n} - (x - a)^{2n}$

16. If x^p occurs in the expansion of $\left(x^2 + \frac{1}{x}\right)^{2n}$, then prove that its coefficient is

$$\frac{(2n)!}{\left(\frac{4n-p}{3}\right)!\left(\frac{2n+p}{3}\right)!}.$$

Sol. Given expression is $\left(x^2 + \frac{1}{x}\right)^{2n}$.

$$\therefore \qquad T_{r+1} = {}^{2n}C_r(x^2)^{2n-r}\left(\frac{1}{x}\right)^r = {}^{2n}C_r x^{4n-3r}$$

If x^p occurs in the expansion,

Let $4n - 3r = p$

$$\Rightarrow \qquad 3r = 4n - p \;\Rightarrow\; r = \frac{4n-p}{3}$$

$$\therefore \qquad \text{Coefficient of } x^p = {}^{2n}C_r = \frac{(2n)!}{r!\,(2n-r)!} = \frac{(2n)!}{\left(\frac{4n-p}{3}\right)!\left(2n - \frac{4n-p}{3}\right)!}$$

$$= \frac{(2n)!}{\left(\frac{4n-p}{3}\right)!\left(\frac{2n+p}{3}\right)!}$$

Q17. Find the term independent ofx in the expansion of $(1 + x + 2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

Sol. Given expansion is $(1 + x + 2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

Now, consider $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

$$T_{r+1} = {}^9C_r\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r = {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r}$$

Hence, the general term in the expansion of $(1 + x + 2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is:

$$ {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r} + {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{19-3r}$$
$$+ 2\cdot{}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{21-3r}$$

For term independent of x, putting $18 - 3r = 0$, $19 - 3r = 0$ and $21 - 3r = 0$, we get

$r = 6, r = 7$

Hence, second term is not independent of x. Therefore, term independent of x is:

$$ {}^9C_6\left(\frac{3}{2}\right)^{9-6}\left(-\frac{1}{3}\right)^6 + 2\cdot{}^9C_7\left(\frac{3}{2}\right)^{9-7}\left(-\frac{1}{3}\right)^7$$
$$= \frac{9\times8\times7\times6!}{6!\times3\times2}\cdot\frac{1}{2^3\cdot3^3} - 2\cdot\frac{9\times8\times7!}{7!\times2\times1}\cdot\frac{3^2}{2^2}\cdot\frac{1}{3^7}$$
$$= \frac{84}{8}\cdot\frac{1}{3^3} - \frac{36}{4}\cdot\frac{2}{3^5} = \frac{21-4}{54} = \frac{17}{54}$$

Objective Type Questions

Q18. The total number of terms in the expansion of $(x + a)^{100} + (x - a)^{100}$ after simplification is

(a) 50
(b) 202
(c) 51
(d) none of these

Sol. **(c)** We have, $(x + a)^{100} + (x - a)^{100}$

$$= ({}^{100}C_0x^{100} + {}^{100}C_1x^{99}a + {}^{100}C_2x^{98}a^2 + \ldots)$$
$$+ ({}^{100}C_0x^{100} - {}^{100}C_1x^{99}a + {}^{100}C_2x^{98}a^2 + \ldots)$$
$$= 2({}^{100}C_0x^{100} + {}^{100}C_2x^{98}a^2 + \ldots + {}^{100}C_{100}a^{100})$$

So, there are 51 terms.

Q19. If the integers $r > 1$, $n > 2$ and coefficients of $(3r)$th and $(r + 2)$nd terms in the binomial expansion of $(1 + x)^{2n}$ are equal, then

(a) $n = 2r$
(b) $n = 3r$
(c) $n = 2r + 1$
(d) none of these

Sol. **(a)** The given expression is $(1+x)^{2n}$

$\therefore \quad T_{3r} = T_{(3r-1)+1} = {}^{2n}C_{3r-1}\, x^{3r-1}$

and $\quad T_{r+2} = T_{(r+1)+1} = {}^{2n}C_{r+1} x^{r+1}$

Given, ${}^{2n}C_{3r-1} = {}^{2n}C_{r+1}$

$\Rightarrow \quad 3r - 1 + r + 1 = 2n \qquad [\because {}^{n}C_x = {}^{n}C_y \Rightarrow x + y = n]$

$\therefore \quad n = 2r$

Q20. The two successive terms in the expansion of $(1 + x)^{24}$ whose coefficients are in the ratio 1 : 4 are

(a) 3rd and 4th

(b) 4th and 5th

(c) 5th and 6th

(d) 6th and 7th

Sol. **(c)** Let the two successive terms in the expansion of $(1+x)^{24}$ be $(r+1)$th and $(r+2)$th terms.

Now, $T_{r+1} = {}^{24}C_r x^r$ and $T_{r+2} = {}^{24}C_{r+1} x^{r+1}$

Given that, $\dfrac{{}^{24}C_r}{{}^{24}C_{r+1}} = \dfrac{1}{4}$

$$\Rightarrow \frac{\dfrac{(24)!}{r!(24-r)!}}{\dfrac{(24)!}{(r+1)!(24-r-1)!}} = \frac{1}{4} \Rightarrow \frac{(r+1)r!(23-r)!}{r!(24-r)(23-r)!} = \frac{1}{4} \Rightarrow \frac{r+1}{24-r} = \frac{1}{4}$$

$\Rightarrow \quad 4r + 4 = 24 - r \quad \Rightarrow \quad r = 4$

$\therefore \quad T_{4+1} = T_5$ and $T_{4+2} = T_6$

Hence, 5th and 6th terms.

Q21. The coefficients of x^n in the expansion of $(1 + x)^{2n}$ and $(1 + x)^{2n\sim1}$ are in the ratio

(a) 1 : 2

(b) 1 : 3

(c) 3 : 1

(d) 2:1

Sol. **(d)** Coefficient of x^n in the expansion of $(1+x)^{2n} = {}^{2n}C_n$

Coefficient of x^n in the expansion of $(1+x)^{2n-1} = {}^{2n-1}C_n$

$$\therefore \quad \frac{{}^{2n}C_n}{{}^{2n-1}C_n} = \frac{\dfrac{(2n)!}{n!n!}}{\dfrac{(2n-1)!}{n!(n-1)!}} = \frac{(2n)!n!\,(n-1)!}{n!\,n!\,(2n-1)!}$$

$$= \frac{2n(2n-1)!n!(n-1)!}{n!n(n-1)!(2n-1)!} = \frac{2n}{n} = \frac{2}{1} = 2:1$$

Q22. If the coefficients of 2nd, 3rd and the 4th terms in the expansion of $(1 + x)^n$ are in A.P., then the value of n is

(a) 2

(b) 7

(c) 11

(d) 14

Sol. **(b)** $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + \ldots + {}^nC_n x^n$

So, coefficients of 2nd, 3rd and 4th terms are ${}^nC_1$, ${}^nC_2$ and ${}^nC_3$, respectively.

Given that, ${}^nC_1$, ${}^nC_2$ and ${}^nC_3$ are in A.P.

$\therefore \quad 2\,{}^nC_2 = {}^nC_1 + {}^nC_3$

$\Rightarrow \quad 2\left[\dfrac{n!}{(n-2)!2!}\right] = n + \dfrac{n!}{3!(n-3)!}$

$\Rightarrow \quad 2\left[\dfrac{n(n-1)}{2!}\right] = n + \dfrac{n(n-1)(n-2)}{3!} \Rightarrow (n-1) = 1 + \dfrac{(n-1)(n-2)}{6}$

$\Rightarrow \quad 6n-6 = 6+n^2-3n+2 \Rightarrow n^2-9n+14=0 \Rightarrow (n-7)(n-2)=0$

$\therefore \quad n = 2$ or $n = 7$

Since $n = 2$ is not possible, so $n = 7$.

Q23. If A and B are coefficients of x^n in the expansions of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then A/B equals to

(a) 1 (b) 2 (c) $\dfrac{1}{2}$ (d) $\dfrac{1}{n}$

Sol. **(b)** The coefficient of x^n in the expansion of $(1+x)^{2n}$ is ${}^{2n}C_n$.

$\therefore \quad A = {}^{2n}C_n$

The coefficient of x^n in the expansion of $(1+x)^{2n-1}$ is ${}^{2n-1}C_n$.

$\therefore \quad B = {}^{2n-1}C_n$

$\therefore \quad \dfrac{A}{B} = \dfrac{{}^{2n}C_n}{{}^{2n-1}C_n} = \dfrac{2}{1} = 2$

24. If the middle term of $\left(\dfrac{1}{x} + x\sin x\right)^{10}$ is equal to $7\dfrac{7}{8}$, then the value of x is

(a) $2n\pi + \dfrac{\pi}{6}$ (b) $n\pi + \dfrac{\pi}{6}$ (c) $n\pi + (-1)^n \dfrac{\pi}{6}$ (d) $n\pi + (-1)^n \dfrac{\pi}{3}$

Sol. **(c)** Given expression is $\left(\dfrac{1}{x} + x\sin x\right)^{10}$.

Since, $n = 10$ (even), so there is only one middle term which is, 6th term.

$\therefore \quad T_6 = T_{5+1} = {}^{10}C_5 \left(\dfrac{1}{x}\right)^{10-5} (x\sin x)^5$

$\Rightarrow \quad \dfrac{63}{8} = {}^{10}C_5 \sin^5 x$ (given)

$\Rightarrow \quad \dfrac{63}{8} = 252 \times \sin^5 x \Rightarrow \sin^5 x = \dfrac{1}{32} \Rightarrow \sin x = \dfrac{1}{2} \Rightarrow \sin x = \sin\dfrac{\pi}{6}$

$\Rightarrow \quad x = n\pi + (-1)^n \dfrac{\pi}{6}$

25. The largest coefficient in the expansion of $(1 + x)^{30}$ is _______.

Sol. Largest coefficient in the expansion of $(1 + x)^{30} = {}^{30}C_{30/2} = {}^{30}C_{15}$

26. The number of terms in the expansion of $(x + y + z)^n$ _______.

Sol. $(x + y + z)^n = [x + (y + z)]^n$

$$= {}^nC_0x^n + {}^nC_1x^{n-1}(y + z) + {}^nC_2x^{n-2}(y + z)^2 + \ldots + {}^nC_n(y + z)^n$$

$\therefore$ Number of terms in expansion $= 1 + 2 + 3 + \ldots + n + (n + 1)$

$$= \frac{(n+1)(n+2)}{2}$$

27. In the expansion of $\left(x^2 - \frac{1}{x^2}\right)^{16}$, the value of constant term is ______.

Sol. $T_{r+1} = {}^{16}C_r(x^2)^{16-r}\left(-\frac{1}{x^2}\right)^r = {}^{16}C_r x^{32-4r}(-1)^r$

For constant term, $32 - 4r = 0 \Rightarrow r = 8$

$\therefore \quad T_{8+1} = {}^{16}C_8$

28. If the seventh term from the beginning and the end in the expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$ are equal, then n equals to _______.

Sol. Given expression is $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$.

According to the question,

$$\frac{{}^nC_6(\sqrt[3]{2})^{n-6}\left(\frac{1}{\sqrt[3]{3}}\right)^6}{{}^nC_6\left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}(\sqrt[3]{2})^6} = 1 \qquad \text{(Refer the solution of Q. 14)}$$

$\Rightarrow \quad (\sqrt[3]{2}\sqrt[3]{3})^{n-12} = 1 \Rightarrow 6^{\frac{n-12}{3}} = 6^0 \Rightarrow \frac{n-12}{3} = 0$

$\Rightarrow \quad n = 12$

29. The coefficient of $a^{-6}b^4$ in the expansion of $\left(\frac{1}{a} - \frac{2b}{3}\right)^{10}$ is ______.

Sol. Given expression is $\left(\frac{1}{a} - \frac{2b}{3}\right)^{10}$.

$\therefore \quad T_{r+1} = {}^{10}C_r\left(\frac{1}{a}\right)^{10-r}\left(-\frac{2b}{3}\right)^r$

For coefficient of $a^{-6}b^4$, $10 - r = 6 \Rightarrow r = 4$

$\therefore \quad$ Coefficient of $a^{-6}b^4 = {}^{10}C_4(-2/3)^4 = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6!}{6!\cdot 4\cdot 3\cdot 2\cdot 1}\cdot\frac{2^4}{3^4} = \frac{1120}{27}$

30. Middle term in the expansion of $(a^3 + ba)^{28}$ is _______.

Sol. Given expression is $(a^3 + ba)^{28}$.

Since index is $n = 28$ (even)

So, there is only one middle term which is $\left(\frac{28}{2} + 1\right)$th term or 15^{th} term.

$\therefore$ Middle term, $T_{15} = T_{14+1} = {}^{28}C_{14}(a^3)^{28-14}(ba)^{14}$

$$= {}^{28}C_{14}a^{42}b^{14}a^{14} = {}^{28}C_{14}a^{56}b^{14}$$

31. The ratio of the coefficients of x^p and x^q in the expansion of $(1 + x)^{p+q}$ is _______.

Sol. Given expression is $(1 + x)^{p+q}$.

$\therefore$ Coefficient of $x^p = {}^{p+q}C_p$

And coefficient of $x^q = {}^{p+q}C_q$

$$\therefore \quad \frac{{}^{p+q}C_p}{{}^{p+q}C_q} = \frac{{}^{p+q}C_p}{{}^{p+q}C_p} = 1:1$$

32. The position of the term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$ is _______.

Sol. Given expression is $\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$.

$$\therefore \quad T_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \left(\frac{3}{2x^2}\right)^r$$

For constant term, $10 - 5r = 0 \Rightarrow r = 2$

Hence, third term is independent of x.

33. If 25^{15} is divided by 13, then the remainder is ______.

Sol. $25^{15} = (26 - 1)^{15} = {}^{15}C_0 26^{15} - {}^{15}C_1 26^{14} + \ldots + {}^{15}C_{14} 26 - {}^{15}C_{15}$

$= ({}^{15}C_0 26^{15} - {}^{15}C_1 26^{14} + \ldots + {}^{15}C_{14} 26 - 13) + 12$

Clearly, when 25^{15} is divided by 13, then remainder will be 12.