Chapter 8. Binomial Theorem

Question-1

Expand each of the following expression $(1 - 2x)^5$

Solution:

By using Binomial Theorem, we have

$(1 - 2x)^5 = [1+(-2x)]^5$

$= {}^5C_0 + {}^5C_1(-2x) + {}^5C_2(-2x)^2 + {}^5C_3(-2x)^3 + {}^5C_4(-2x)^4 + {}^5C_5(-2x)^5$

$= 1 + 5(-2x) + 10(-2x)^2 + 10(-2x)^3 + 5(-2x)^4 + (-2x)^5$

$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

Question-2

Expand each of the following expression $\left(\frac{2}{x} - \frac{x}{2}\right)^5$

Solution:

By using Binomial Theorem, we have

$\left(\frac{2}{x} - \frac{x}{2}\right)^5 = \left[\frac{2}{x} + \left(-\frac{x}{2}\right)\right]^5$

$= {}^5C_0\left(\frac{2}{x}\right)^5\left(\frac{-x}{2}\right)^0 + {}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{-x}{2}\right)^1 + {}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{-x}{2}\right)^2 + {}^5C_3\left(\frac{2}{x}\right)^2\left(\frac{-x}{2}\right)^3 + {}^5C_4$
$\left(\frac{2}{x}\right)^1\left(\frac{-x}{2}\right)^4 + {}^5C_5\left(\frac{-x}{2}\right)^5$

$= \frac{32}{x^5} + 5\left(\frac{16}{x^4}\right)\left(\frac{-x}{2}\right) + 10\left(\frac{8}{x^3}\right)\left(\frac{x^2}{4}\right) + 10\left(\frac{4}{x^2}\right)\left(\frac{-x^3}{8}\right) + 5\left(\frac{2}{x}\right)\left(\frac{x^4}{16}\right) + \left(\frac{-x^3}{32}\right)$

$= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5x^2}{8} - \frac{x^5}{32}$

Question-3

Expand each of the following expression $(2x - 3)^6$

Solution:

By using Binomial Theorem, we have

$(2x - 3)^6 = [2x + (-3)]^6$

$= {}^6C_0 (2x)^6 (-3)^\circ + {}^6C_1(2x)^5 (-3)^1 + {}^6C_2 (2x)^4 (-3)^2 + {}^6C_3(2x)^3 (-3)^3 + {}^6C_4$
$(2x)^2 (-3)^4 + {}^6C_5(2x)^1 (-3)^5 + {}^6C_6(2x)^0 (-3)^6$

$= 1(2x)^6 + 6(2x)^5(-3) + 15(2x)^4 (-3)^2 + 20(2x)^3 (-3)^3 + 15(2x)^2(-3)^4 +$
$6(2x)(-3)^5 + (-3)^6$

$= 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$

Question-4

Expand each of the following expression $\left(\frac{x}{3}+\frac{1}{x}\right)^5$

Solution:

By using Binomial Theorem, we have;

$$\left(\frac{x}{3}+\frac{1}{x}\right)^5 = {}^5C_0\left(\frac{x}{3}\right)^5\left(\frac{1}{x}\right)^0 + {}^5C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + {}^5C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2 + {}^5C_3\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3 + {}^5C_4\left(\frac{x}{3}\right)^1\left(\frac{1}{x}\right)^4 + {}^5C_5\left(\frac{x}{3}\right)^0\left(\frac{1}{x}\right)^5$$

$$= \left(\frac{x}{3}\right)^5 + 5\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + 10\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2 + 10\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3 + 5\left(\frac{x}{3}\right)\left(\frac{1}{x}\right)^4 + \left(\frac{1}{x}\right)^5$$

$$= \frac{x}{243} + \frac{5x^3}{3} + \frac{10x}{3} + \frac{10}{3x} + \frac{5}{3x^3} + \frac{1}{x^5}$$

Question-5

Expand each of the following expression $\left(x+\frac{1}{x}\right)^6$

Solution:

By using Binomial Theorem, we have:

$$\left(x+\frac{1}{x}\right)^6 = {}^5C_0(x)^6\left(\frac{1}{x}\right)^0 + {}^6C_1(x)^5\left(\frac{1}{x}\right)^1 + {}^6C_2(x)^4\left(\frac{1}{x}\right)^2 + {}^6C_3(x)^3\left(\frac{1}{x}\right)^3 + {}^6C_4(x)^2\left(\frac{1}{x}\right)^4 + {}^6C_5(x)^1\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6$$

$$= x^6 + 6x^5\left(\frac{1}{x}\right) + 15x^4\frac{1}{x^2} + 20x^3.\frac{1}{x^3} + 15x^2.\frac{1}{x^4} + 6x.\frac{1}{x^5} + \frac{1}{x^6}$$

$$I = x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

Question-6

Using binomial theorem, evaluate each of the following $(96)^3$

Solution:

We express 96 as the sum or difference of two numbers whose powers are easier to calculate, and then use Binomial Theorem

Write 96 = 100 - 4

Therefore

$(96)^3 = (100 - 4)^3$

$= {}^3C_0\,(100)^3 - {}^3C_1(100)^2(4) + {}^3C_2(100)^1(4)^2 - {}^3C_3(4)^3$

$= 1000000 - 3\,(10000)\,(4) + 3\,(100)\,(16) - (64)$

$= 1000000 - 120000 + 4800 - 64$

$= 884736$

Question-7

Using binomial theorem, evaluate each of the following $(102)^5$

Solution:

$(102)^5 = (100 + 2)^5$

$= {}^5C_0(100)^5 + {}^5C_1(100)^4(2) + {}^5C_2(100)^3(2)^2 + {}^5C_3(100)^2(2)^3 + {}^5C_4(100)(2)^4 + {}^5C_5(2)^5$

$= 10000000000 + 5(100000000)(2) + 10(1000000)(4) + 10(10000)(8) + 5(100)(16) + 32$

$= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32$

$= 11040808032$

Question-8

Using binomial theorem, evaluate each of the following $(101)^4$

Solution:

$(101)^4 = (100 + 1)^4$

$= {}^4C_0(100)^4 + {}^4C_1(100)^3(1) + {}^4C_2(100)^2(1)^2 + {}^4C_3(100)^1(1)^3 + {}^4C_4(1)^4$

$= 100000000 + 4(1000000) + 6\ (10000) + 4\ (100) + 1$

$= 100000000 + 4000000 + 60000 + 400 + 1$

$= 104060401$

Question-9

Using binomial theorem, evaluate each of the following $(99)^5$

Solution:

$(99)^5 = (100 - 1)^5$

$= {}^5C_0(100)^5 - {}^5C_1(100)^4.1 + {}^5C_2(100)^3.(1)^2 - {}^5C_3(100)^2.(1)^2 + {}^5C_4(100)^1(1)^4 - {}^5C_5(1)^5$

$= (100)^5 - 5 \times (100)^4 + 10 \times (100)^3 - 10 \times (100)^2 + 5 \times 100 - 1$

$= 10000000000 - 500000000 + 10000000 - 100000 + 500 - 1$

$= 10010000500 - 500100001$

$= 9509900499$

Question-10

Using Binomial Theorem indicate which number is larger $(1.1)^{10000}$ or 1000.

Solution:

Splitting 1.1 and using Binomial Theorem to write the first few terms we have

$(1.1)^{10000} = (1+0.1)^{10000}$

$= {}^{10000}C_0 + {}^{10000}C_1(0.1) + {}^{10000}C_2(0.1)^2$ + other positive terms.

$= 1 + 10000 \times (0.1)$ + other positive terms

$= 1 + 1000$ + other positive terms

>1000

Hence, $(1.1)^{10000} > 1000$.

Question-11

Find $(a + b)^4 - (a - b)^4$. Hence, evaluate $(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4$.

Solution:

$(a + b)^4 - (a - b)^4$

$= [{}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4] - [{}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3 ab^3 + {}^4C_4 b^4] = 2 \times {}^4C_1 a^3b + 2 \times {}^4C_3 ab^3$

$= 2[4a^3b + 4ab^3]$

$= 8ab [a^2 + b^2]$

Thus, $(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4$

$= 8 \sqrt{3} . \sqrt{2} [3 + 2] = 8\sqrt{6} (5) = 40\sqrt{6}$

Question-12

Find $(x + 1)^6 + (x - 1)^6$. Hence or otherwise evaluate $(\sqrt{2} + 1)^6 - (\sqrt{2} - 1)^6$

Solution:

$(x + 1)^6 + (x - 1)^6$

$= [{}^6C_0 x^6 + {}^6C_1 x^5 + {}^6C_2 x^4] + [{}^6C_3 x^3 + {}^6C_4 x^2 + {}^6C_5 x^1 - {}^6C_6] +$

$[{}^6C_0 x^6 - {}^6C_1 x^5 + {}^6C_2 x^4 - {}^6C_3 x^3 + {}^6C_4 x^2 + {}^6C_5x^1 - {}^6C_6]$

$= 2[{}^6C_0x^6 + {}^6C_2 x^4 + {}^6C_4 x^2 + {}^6C_6]$

$= 2[x^6 + 15x^4 + 15x^2 + 1]$

Thus, $(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$

$= 2 [(\sqrt{2})^6 + 15(\sqrt{2})^4 + 15 (\sqrt{2}) + 1]$

$= 2 [8 + 15 (4) + 15 (2) + 1]$

$= 2 [8 + 60 + 30 + 1] = 198.$

Question-13

Show that $9^{n+1} - 8n - 9$ is divisible by 64, whenever n is a positive integer.

Solution:

$n = 1 \Rightarrow 9^{n+1} - 8n - 9 = 9^2 - 8 - 9$

$= 81 - 17 = 64 = 1(64)$

$n = 2 \Rightarrow 9^{n+1} - 8n - 9 = 9^3 - 8(2) - 9$

$= 729 - 16 - 9 = 704 = 11\ (64)$

From $n = 3, 4, 5, \ldots 9^{n+1} - 8n - 9 = 9(1 + 8)n - 8n - 9$

$= 9\ [{}^nC_0 + {}^nC_1 \,.\, 8 + {}^nC_2.8^2 + \ldots {}^nC_n\, 8^n] - 8n - 9$

$= 9[1 + 8n + {}^nC_2.8^2 + \ldots {}^nC_n\, 8n] - 8n - 9$

$= 9 + 72n + 9.\ {}^nC_2.\ 8^2 + \ldots 9\ {}^nC_n\ 8^n - 8n - 9$

$= 8^2\ [n + 9\ ({}^nC_2 + {}^nC_3.8 + \ldots {}^nC_n\ 8^{n-2})]$

which is divisible by 64.

Question-14

Prove that $\sum_{r=0}^{n} 3^r\ {}^nC_r = 4^n$.

Solution:

L.H.S $= 3^0C(n,0) + 3^1\ C(n,1) + 3^2\ C(n,2) + \text{---}\ 3^r(n, r) + \text{---} + 3^n\ C(n, n)$

$= C(n,0) + C(n,1)\ 3^1 + C(n,2)\ 3^2 + C(n,3)\ 3^3 + \text{---} + C(n,n)3^n$

This is in the form of $(1+3)^n$

$= (1+3)^n = 4^n =$ R.H.S

Question-15

Prove that x^5 in $(x + 3)^8$

Solution:

Suppose x^5 occurs in the $(r + 1)$th term of the expansion $(x + 3)^8$

Now $T_{r+1} = {}^nC_r\, a^{n-r}\, b^r = {}^8C_r\, x^{8-r}\, 3^r$

Comparing the indices of x in x^5 and in T_{r+1}, we get $r = 3$

Thus, the coefficient of x^5 is

${}^8C_3(3)^3 = 1512$

Question-16

Prove that a b^7 in $(a-2b)^{12}$

Solution:

Let a^5b^7 occurs in the (r + 1)th term, in the expansion of $(a - 2b)^{12}$ given by ${}^{12}C_r.\ a^{12-r}(-2b)^r$. Then $12 - r = 5$. This gives r= 7.

Thus the coefficient of a^5b^7 is

$${}^{12}C_5\,(-2)^7 = \frac{12!}{5!7!} \times (-128) = (792)\,(-128) = -101376$$

Question-17

Prove that $(x^2 - y)^6$

Solution:

We have T_{r+1} in $(a + b)^n = {}^nC_r a^{n-r}.\ b^r, 0 \le r \le n$

T_{r+1} in $(x^2 - y)^6 = {}^6C_r(x^2)^{6-r}(-y)^r$

$= {}^6C_r\, x^{12-2r}\,(-y)^r$

Question-18

Prove that $(x^2 - yx)^{12}$, x ¹ 0

Solution:

T_{r+1} in $(x^2 - yx)^{12} = {}^{12}C_r\,(x^2)^{12-r}\,(-yx)^r$

$= {}^{12}C_r\, x^{24-2r}\,(-1)^r(y)^r(x)^r$

$= {}^{12}C_r\, x^{24-r}\, y^r(-1)^r$

Question-19

Find the 4th term in the expansion of $(x - 2y)^{12}$.

Solution:

4th term in $(x - 2y)^{12} = T_4 = T_{3+1}$

$= {}^{12}C_3(x)^{12-3}(-2y)^3$

$[T_{r+1}$ in $(a + b)^n = {}^nC_r\, a^{n-r}\, b^r]$

$= {}^{12}C_3(x)^9\,(-2)^3\,(y)^3$

$= \frac{12 \times 11 \times 10}{3 \times 2} x - 8 \times x^9 y^3$

$= -1760\, x^9\, y^3$

Question-20

Find the 13th term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$

Solution:

13th term in $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$ $= T_{13} = T_{12+1}$

$$= {}^{18}C_{12}(9x)^{18-12}\left(-\frac{1}{3\sqrt{x}}\right)^{12}$$

$$= {}^{18}C_{12}(9x)^{6}\left(-\frac{1}{3}\right)^{12}\left(-\frac{1}{\sqrt{x}}\right)^{12}$$

$$= {}^{18}C_{12}(9x)^{6}\left(\frac{-1}{3}\right)^{12}\left(x^{\frac{-1}{2}}\right)^{12}$$

$$= {}^{18}C_{12}(9x)^{6}\left(\frac{-1}{3}\right)^{12}(x)^{-6}$$

$$= {}^{18}C_{12}(3^2)^6(x)^6(-1)^{12}(3)^{-12}(x)^{-6}$$

$$= {}^{18}C_{12}(3^{12})(x)^6(3)^{-12}(x)^{-6}$$

$$= {}^{18}C_{12} = 18564$$

Question-21

Find the 13th term in the expansion of $\left(3 - \frac{x^3}{6}\right)^7$

Solution:

The index of $\left(3 - \frac{x^3}{6}\right)^7$, is 7, which is an odd natural number.

So, Middle terms are $T_{\frac{7+1}{2}}$ and $T_{\frac{7+3}{2}}$

$$T_{\frac{7+1}{2}} = T_4 = T_{3+1} = {}^7C_3(3)^{7-3}\left(-\frac{x^3}{6}\right)^3$$

$$= {}^7C_3(3)^4(x^3)^3(-1)(6^{-1})^3$$

$$= {}^7C_3(81)(x)^9\,(6)^{-3}(-1)^3$$

$$= \frac{-35 \times 81}{216}x^9 = \frac{-105}{8}x^9$$

$$T_{\frac{7+3}{2}} = T_5 = T_{4+1} = {}^7C_4(3)^{7-4}\left(-\frac{x^3}{6}\right)^4$$

$$= {}^7C_4(3)^3\,(-1)^4(x^3)(6)^{-4}$$

$$= {}^7C_4(27)\,(x)^{12}(6)^{-4}$$

$$= (35)(27)(6)^{-4}(x)^{12}$$

$$= \frac{35 \times 27}{1296}x^{12} = \frac{35}{48}x^{12}$$

Question-22

$\left(\frac{x}{3} + 9y\right)^{10}$

Solution:

The index $\left(\frac{x}{3} + 9y\right)^{10}$ is 10, which is an even natural number.

Hence, Middle term = $T_{\frac{10+2}{2}} = T_6 = T_{5+1}$

$= {}^{10}C_5\left(\frac{x}{3}\right)^{10-5}(9y)^5$

$= {}^{10}C_5\left(\frac{x}{3}\right)^5(9y)^5$

$= {}^{10}C_5(x)^5\left(\frac{1}{3}\right)^5(9)^5(y)^5$

$= {}^{10}C_5(3)^{-5}\left(3^2\right)^5(x)^5(y)^5$

$= {}^{10}C_5(3)^{-5}(3)^{10}x^5y^5$

$= {}^{10}C_5 3^5\, x^5\, y^5 = (252)(243)x^5\, y^5$

$= 61236\, x^5y^5$

Question-23

In the expansions of $(1 + a)^{m+n}$, using Binomial Theorem, prove that coefficients of a^m and a^n are equal.

Solution:

We have,

$(1 + a)^{m+n} = [{}^{m+n}C_0 + {}^{m+n}C_1a^1 + {}^{m+n}C_2a^2 + \ldots\ldots {}^{m+n}C_r$
$a^r + \ldots\ldots + {}^{m+n}C_{m+n}\, a^{m+n}$

Coefficient of $a^m = {}^{m+n}C_m = \frac{(m+n)!}{m!n!}$

Also the coefficient of a^n

$= {}^{m+n}C_n = \frac{(m+n)!}{n!m!}$

Clearly, ${}^{m+n}C_m = {}^{m+n}C_n$

Question-24

The coefficients of the $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ terms in the expansion of $(x+1)^n$ are in the ratio 1:3:5. Find both n and r.

Solution:

Coefficient of (r-1)th term = C(n, r-2)

Coefficient of rth term = C(n, r-1)

Coefficient of (r+1)th term = C(n, r)

Considering 1^{st} and ^{2nd}

$$\frac{C(n,r-2)}{C(n,r-1)} = \frac{n!}{(r-2)!(n-r+2)!} \times \frac{(r-1)!(n-r+1)!}{n!} = \frac{(r-1)!(n-r+1)!}{(r-2)!(n-r+2)(n-r+1)!} = \frac{(r-1)(r-2)!}{(r-2)!(n-r+2)} = \frac{(r-1)}{n-r+2} = \frac{1}{3}$$

3r - 3 = n - r + 2

n - 4r = -5 ----------(1)

Considering 2nd and 3rd

$$\frac{C(n,r-1)}{C(n,r)} = \frac{3}{5} = \frac{n!}{(r-1)!(n-r+1)!} \times \frac{r!(n-r)!}{n!} = \frac{(n-r)!r(r-1)!}{(n-r+1)(n-r)!(r-1)!} = \frac{r}{n-r+1} = \frac{3}{5}$$

5r = 3n - 3r +3

3n - 8r = -3 ---------(2)

2(n - 4r = -5)

2n - 8r = -10 ---------(3)

Subtract (3) from (2)

n = 7

Substitute n = 7 in (2)

We get r = 3

n = 7, r = 3

Question-25

Prove that the coefficient of x^n in the expansion of $(1+x)^{2n}$ is twice the coefficient of x^n in the expansion $(1 + x)^{2n-1}$.

Solution:

$(1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1 x^1 + {}^{2n}C_2 x^2 + \ldots + {}^{2n}C_n x^n$

$(1+x)^{2n-1} = {}^{2n-1}C_0 + {}^{2n-1}C_1 x^1 + {}^{2n-1}C_2 x^2 + \ldots + {}^{2n-1}C_n x^n$

Coefficient of x^n in $(1+x)^{2n-1}$ is $({}^{2n-1}C_n)$

$${}^{2n}C_n = \frac{(2n)!}{n!n!}$$

$$= \frac{(2n)(2n-1)(2n-2)!}{n(n-1)!n!}$$

$$= \frac{(2n)(2n-1)2(n-1)!}{n(n-1)!n!}$$

$$= 2\left[\frac{2(2n-1)}{n!}\right] \ldots\ldots.(i)$$

Now $${}^{2n-1}C_n = \frac{(2n-1)!}{n!(2n-1-n)!}$$

$$= \frac{(2n-1)!}{n!(n-1)!}$$

$$= \frac{(2n-1)(2n-2)!}{n!(n-1)!}$$

$$= \frac{(2n-1)2(n-1)!}{n!(n-1)!}$$

$$= \frac{2(2n-1)}{n!} \ldots\ldots.(ii)$$

From (i) and (ii) we have

${}^{2n}C_n = 2.\ {}^{2n-1}C_n$

CBSE Class 11 Mathematics

Important Questions

Chapter 8

Binomial Theorem

1 Marks Questions

1. What is The middle term in the expansion of $(1+x)^{2n+1}$

Ans. Since $(2n+1)$ is odd there is two middle term

$i.e\ {}^{2n+1}C_n x^{n+1}$ and ${}^{2n+1}C_{n+1} x^n$

2. When n is a positive integer, the no. of terms in the expansion of $(x+a)^n$ is

Ans. The no. of terms in the expansion of $(x+a)^n$ is one more than the index $n.\quad i.e\quad (n+1).$

3. Write the general term $(x^2-y)^6$

Ans. $T^{r+1} = {}^6C_r (x^2)^{6-r}.(-y)^r$

$= {}^6C_r (x)^{12-2r}.(-1)^r.(y)^r$

4. In the expansion of $\left(x+\frac{1}{x}\right)^6$, find the 3rd term from the end

Ans. 3rd term form end $=(6-3+2)^{th}$ term from beginning i.e $T_5 = {}^6C_4 (x)^{6-4} . \left(\frac{1}{x}\right)^4$

$= {}^6C_4 x^2 .x^{-4}$

$= 15^{x-2}$

$= \frac{15}{x^2}$

5. Expand $(1+x)^n$

Ans. $(1+x)^n = 1 + {}^nC_1 (x)^1 + {}^nC_2 (x)^2 + {}^nC_3 (x)^3 + ...x^n$

6. The middle term in the expansion of $(1+x)^{2n}$ **is**

Ans. ${}^{2n}C_n .x^n$

7. Find the no. of terms in the expansions of $\left(1-2x+x^2\right)^7$

Ans. $\left(1-2x+x^2\right)^7$

$= \left(x^2 - 2x + 1\right)^7$

$= \left[(x-1)^2\right]^7$

$= (x-1)^{14}$

No. of term is 15

8. Find the coeff of x^5 **in** $(x+3)^9$

Ans. $T_{r+1} = {}^{9}C_{r}(x)^{9-r}.(3)^{r}$

Put $9 - r = 5$

$r = 4$

$T_{5} = {}^{9}C_{4}(x)^{5}.(3)^{4}$

Coeff of x^5 is ${}^{9}C_{4}(3)^{4}$

9. Find the term independent of x $\left(x+\frac{1}{x}\right)^{10}$

Ans. $T_{r+1} = {}^{10}C_{r}(x)^{10-r}.\left(\frac{1}{x}\right)^{r}$

$= {}^{10}C_{r}(x)^{10-r}.(x)^{-r}$

$= {}^{10}C_{r}(x)^{10-2r}$

Put $10 - 2r = 0$

$r = 5$

Independent term is ${}^{10}C_{5}$

10. Expand $(a+b)^{n}$

Ans. $(a+b)^{n} = {}^{n}C_{0}a^{n} + {}^{n}C_{1}a^{n-1}b + {}^{n}C_{2}a^{n-2}b^{2} + ... + {}^{n}C_{n}b^{n}$

CBSE Class 12 Mathematics

Important Questions

Chapter 8

Binomial Theorem

4 Marks Questions

1. Which is larger $(1.01)^{10,00000}$ **or** $10,000$

Ans. $(1.01)^{10,00000} = (1+0.01)^{10,00000}$

$= {}^{10,00000}C_0 + {}^{10,00000}C_1(0.01) + \text{other positive term}$

$= 1 + 10,00000 \times 0.01 + \text{other positive term}$

$= 1 + 10,000$

$= 10,001$

Hence $(1.01)^{10,00000} > 10,000$

2. Prove that $\sum_{r=0}^{n} 3^r \;\; {}^nC_r = 4^n$

Ans.

$\sum_{r=0}^{n} 3^r \;\; {}^nC_r = \sum_{r=0}^{n} {}^nC_r . 3^r$

$= {}^nC_0 + {}^nC_1 . 3 + {}^nC_2 . 3^2 + ... + {}^nC_n 3^n$

$\left[\because (1+a)^n = 1 + {}^nC_1 . a + {}^nC_2 a^2 + {}^nC_3 a^3 + ... + a^n\right]$

$= (1+3)^n$

$= (4)^n$

$H.P$

3. Using binomial theorem, prove that $6^n - 5n$ always leaves remainder 1 when divided by 25.

Ans. Let $6^n = (1+5)^n$

$= 1 + {}^nC_1 5^1 + {}^nC_2 5^2 + {}^nC_3 5^3 + ... + 5^n$

$= 1 + 5n + 5^2\left({}^nC_2 + {}^nC_3 .5 + ... + 5^{n-2}\right)$

$6^n - 5n = 1 + 25\left({}^nc_2 + {}^nc_3 .5 + ... + 5^{n-2}\right)$

$= 1 + 25k\left[\text{where } k = {}^nc_2 + {}^nc_3 .5 + ... 5^{n-2}\right]$

$= 25k + 1$

$H.P$

4.Find the 13th term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$, $x \neq 0$

Ans. The general term in the expansion of

$\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$ is

$T_{r+1} = {}^{18}C_r (9x)^{18-r}\left(-\frac{1}{3\sqrt{x}}\right)^r$

For 13th term, $r + 1 = 13$

$r = 12$

$$= {}^{18}C_{12}(9x)^6\left(-\frac{1}{3\sqrt{x}}\right)^{12}$$

$$= {}^{18}C_{12}(3)^{12}.x^6\left(-\frac{1}{3}\right)^{12}.(x)^{-6}$$

$$= {}^{18}C_{12}(3)^{12}.(-1)^{12}.(3)^{-12}$$

$$= {}^{18}C_{12}$$

$$= 18564$$

5. Find the term independent of x in the expansion of $\left(\sqrt[3]{x}+\frac{1}{2\sqrt[3]{x}}\right)^{18}, x > 0$

Ans.

$$T_{r+1} = {}^{18}C_r\left(\sqrt[3]{x}\right)^{18-r}\left(\frac{1}{2\sqrt[3]{x}}\right)^r$$

$$= {}^{18}C_r(x)^{\frac{18-r}{3}}.\left(\frac{1}{2}\right)^r.x^{\frac{-r}{3}}$$

$$= {}^{18}C_r(x)^{\frac{18-r-r}{3}}.\left(\frac{1}{2}\right)^r$$

For independent term $\frac{18-2r}{3} = 0$

$r = 9$

The req. term is ${}^{18}C_9\left(\frac{1}{2}\right)^9$

6. Find the coefficient of x^5 in the expansion of the product $(1+2x)^5(1-x)^7$

Ans.

$$(1+2x)^6(1-x)^7=\left(1+{}^6C_1(2x)+{}^6C_2.(2x)^2+{}^6C_3(2x)^3+{}^6C_4(2x)^4+{}^6C_5(2x)^5+{}^6C_6(2x)^6\right)$$

$$\left(1-{}^7C_1x+{}^7C_2(x)^2-{}^7C_3(x)^3+{}^7C_4(x)^4-{}^7C_5(x)^5+{}^7C_6(x)^6-{}^7C_7(x)^7\right)$$

$$=\left(1+12x+60x^2+160x^3+240x^4+192x^5+64x^6\right).\left(1-7x+21x^2-35x^3+35x^4-21x^5+7x^6-x^7\right)$$

Coeff of x^5 is

$$=1\times(-21)+12\times35+60(-35)+160\times21+240\times(-7)+192\times1$$
$$=171$$

7. Compute $(98)^5$

Ans. $(98)^5=(100-2)^5$

$$={}^5C_0(100)^5-{}^5C_1(100)^4.2+{}^5C_2(100)^3.2^2$$

$$-{}^5C_3(100)^2.(2)^3+{}^5C_4(100)(2)^4-{}^5C_5(2)^5$$

$$=10000000000-5\times100000000\times2+10\times1000000\times4$$
$$-10\times10000\times8+5\times100\times16-32$$

$$=100\ 4\ 000\ 8000-1000\ 8000\ 32=9039207968$$

8. Expand $\left(x+\frac{1}{x}\right)^6$

Ans.

$$\left(x+\frac{1}{x}\right)^6 = {}^6C_0(x)^6 + {}^6C_1(x^5)\left(\frac{1}{x}\right) + {}^6C_2(x^4)\left(\frac{1}{x}\right)^2 +$$

$${}^6C_3(x^3)\left(\frac{1}{x}\right)^3 + {}^6C_4(x^2)\left(\frac{1}{x}\right)^4 + {}^6C_5(x)^1\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6$$

$$= x^6 + 6x^5\left(\frac{1}{x}\right) + 15x^4\left(\frac{1}{x^2}\right) + 20x^3.\left(\frac{1}{x^3}\right) + 15x^2.\frac{1}{x^4} + 6x.\frac{1}{x^5} + \frac{1}{x^6}$$

$$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

9. Find the fourth term from the end in the expansion of $\left(\frac{x}{x^2} - \frac{x^3}{3}\right)^9$.

Ans. Fourth term from the end would be equal to $(9-4+2)^{th}$ term from the beginning

$$T_7 = T_{6+1} = {}^9C_6\left(\frac{3}{x^2}\right)^{9-6}.\left(\frac{-x^3}{3}\right)^6$$

$$= {}^9C_6(3)^3.(x)^{-6}.(x)^{18}.(3)^{-6}$$

$$= \frac{9!}{6!3!}.(3)^{-3}.(x)^{12}$$

$$= \frac{28}{9}x^{12}$$

10. Find the middle term of $\left(2x - \frac{x^2}{4}\right)^9$.

Ans. $n = 9$ so there are two middle term i.e $\left(\frac{9+1}{2}\right)^{th}$ term and $\left(\frac{9+1}{2}+1\right)^{th}$ term

$$T_5 = T_{4+1} = {}^9C_4 (2x)^{9-4} \cdot \left(\frac{-x^2}{4}\right)^4$$

$$= \frac{63}{4} x^{13}$$

$$T_6 = T_{5+1} = {}^9C_5 (2x)^{9-5} \left(\frac{-x^2}{4}\right)^5$$

$$= -{}^9C_4 (2)^4 \cdot x^4 \ \frac{(x)^{10}}{(4)^5}$$

$$= \frac{-63}{32}\ x^{14}$$

11. Find the coefficient of $a^5 b^7$ in $(a-2b)^{12}$.

Ans.

$$T_{r+1} = {}^{12}C_r (a)^{12-r} \cdot (-2b)^r$$

Put $12 - r = 5$

$r = 7$

$$T_8 = {}^{12}C_7 (a)^5 \cdot (-2b)^7$$

$= {}^{12}C_7 (a)^5 \cdot (-2)^7 \cdot b^7$ s

coeff. Of a^5 b^7 is ${}^{12}C_7\ (-2)^7$

12. Find a positive value of m for which the coefficient of x^2 in the expansion $(1+x)^m$ is 6.

Ans. $T_{r+1} = {}^mC_r (1)^{m-r} . (x)^r$

$= {}^mC_r (x)^r$

Put $r = 2$

ATQ ${}^mC_2 = 6$

$$\frac{m!}{2!(m-2)!} = \frac{6}{1}$$

$$\frac{m(m-1)\cancel{(m-2)!}}{2\times1\times\cancel{(m-2)!}} = \frac{6}{1}$$

$m^2 - m = 12$

$m^2 - m - 12 = 0$

$m(m-4) = 3(m-4) = 0$

$(m-4)(m-3) = 0$

$m = 4$

$m = -3(\text{neglect})$

13. Show that the coefficient of the middle term in the expansion of $(1+x)^{2n}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1+x)^{2n-1}$.

Ans. As $2n$ is even so the expansion $(1+x)^{2n}$ has only one middle term which is

$\left(\frac{2n}{2}+1\right)^{th}$ term i.e $(n+1)^{th}$ term

$T_{r+1} = {}^{2n}C_r \ (1)^{2n-r} .(x)^r$

Coeff. of x^n is ${}^{2n}C_r$

And $(2n-1)$ is odd so two middle term

$\left(\frac{2n-1+1}{2}\right)^{th}$ and $\left(\frac{2n-1+1}{2}+1\right)^{th}$

i.e n^{th} and $(n+1)^{th}$ term

The coefficients of these terms are ${}^{2n-1}C_{n-1}$ and ${}^{2n-1}C_n$

Now ATQ

$${}^{2n-1}C_{n-1} + {}^{2n-1}C_n = {}^{2n}C_n \quad \left[\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r\right]$$

$H.P$

14. Find a if the coeff. of x^2 and x^3 in the expansion of $(3+ax)^9$ are equal

Ans. $T_{r+1} = {}^9C_r (3)^{9-r} .(ax)^r$

ATQ

$${}^9C_2 (3)^7 .a^2 = {}^9C_3 (3)^6 . a^3$$

$${}^9C_2 (3)^1 = {}^9C_3 .a$$

$$\frac{9!}{2!7!}\times 3=\frac{9!}{3!6!}a$$

$$\frac{3!6!\times 3}{2!7!}=a$$

$$\frac{3\times 2\times 1\times 6!\times 3}{2\times 1\times 7\times 6!}=a$$

$$\frac{9}{7}=a$$

15. Find $(a+b)^4-(a-b)^4$. **Hence evaluate** $\left(\sqrt{3}+\sqrt{2}\right)^4+\left(\sqrt{3}-\sqrt{2}\right)^4$.

Ans. $(a+b)^4-(a-b)^4=\left({}^4C_0a^4+{}^4C_1a^3b+{}^4C_2a^2b^2+{}^4C_3ab^3+{}^4C_4b^4\right)$

$$-\left(a^4-{}^4C_1a^3b+{}^4C_2a^2b^2-{}^4C_3ab^2+{}^4C_4b^4\right)$$

$$=2\left({}^4C_1a^3b+{}^4C_3ab^3\right)$$

$$=2\left(4a^3b+4ab^3\right)$$

$$=8ab\left(a^2+b^2\right)$$

Put $a=\sqrt{3}$, $b=\sqrt{2}$

$$\left(\sqrt{3}+\sqrt{2}\right)^4-\left(\sqrt{3}-\sqrt{2}\right)^4=8\sqrt{3}.\sqrt{2}\,(3+2)$$

$$=40\sqrt{6}$$

16. Show that $9^{n+1}-8n-9$ **is divisible by 64, whenever n is positive integer.**

Ans. $(9)^{n+1}=(1+8)^{n+1}$

$$=1+{}^{n+1}C_1 8^1+{}^{n+1}C_2 .8^2+{}^{n+1}C_3 8^3+........+{}^{n+1}C_{n+1} 8^{n+1}$$

$$=1+(n+1).8+8^2\left[{}^{n+1}C_2+{}^{n+1}C_3 .8+.....+8^{n-1}\right]$$

$$9^{n+1}-8n-9=64\left[{}^{n+1}C_2+{}^{n+1}C_3 .+.......+8^{n-1}\right]$$

$$9^{n+1}-8n-9=64k,\ \text{where } k=\left[{}^{n+1}C_2+{}^{n+1}C_3 .8+....+8^{n+1}\right]$$

17. Find the general term in the expansion of $(x^2-yx)^{12}$

Ans. $T_{r+1}={}^{12}C_r\left(x^2\right)^{12-r}.(-yx)^r$

$$={}^{12}C_r(x)^{24-r}.(-1)^r.y^r.x^r$$

$$={}^{12}C_r(-1)^r.y^r.(x)^{24-2r}$$

18. In the expansion of $(1+a)^{m+n}$, **prove that coefficients of** a^m **and** a^n **are equal.**

Ans. $T_{r+1}={}^{m+n}C_r(1)^{m+n-r}.(a)^r$

$$T_{r+1}={}^{m+n}C_r(a)^r\ ....(i)$$

Put $r=m$ and $r=n$ respectively

$$T_{m+1}={}^{m+n}C_m a^m$$

Coeff of a^m is ${}^{m+n}C_m\Rightarrow\dfrac{(m+n)!}{m!n!}$

Coeff of a^n is ${}^{m+n}C_m \Rightarrow \frac{(m+n)!}{n!m!}$ H.P

19. Expand $\left(1-x+x^2\right)^4$

Ans. $\left(1-x+x^2\right)^4 = \left[(1-x)+x^2\right]^4$

$= {}^4C_0(1-x)^4 + {}^4C_1(1-x)^3.\left(x^2\right) + {}^4C_2(1-x)^2.\left(x^2\right)^2 + {}^4C_3(1-x)^1\left(x^2\right)^3 + {}^4C_4\left(x^2\right)^4$

$= (1-x)^4 + 4(1-x)^3.x^2 + 6(1-x)^2.x^4 + 4(1-x).x^6 + 1.x^8$

$= \left(1-4x+6x^2-4x^3+x^4\right) + 4\left(1-3x+3x^2-x^3\right)x^2 + 6\left(1-2x+x^2\right)\left(x^4\right) + 4(1-x).x^6 + x^8$

$= 1-4x+10x^2-16x^3+19x^4-16x^5+10x^6-4x^7+x^8$

20. Find the sixth term of the expansion $\left(y^{\frac{1}{2}}+x^{\frac{1}{3}}\right)^n$, if the binomial coefficient of the third term from the end is 45.

Ans. The binomial coeff of the third term from end = binomial coeff of the third term from beginning = ${}^nC_2$

$${}^nC_2 = 45$$

$$\frac{n(n-1)}{1.2} = 45$$

$$n^2 - n - 90 = 0$$

$$n = 10$$

$$T_{r+1} = {}^{10}C_r\left(y^{\frac{1}{2}}\right)^{10-r}.\left(x^{\frac{1}{3}}\right)^r$$

$$r = 5$$

$T_6 = {}^{10}C_5 \left(y^{\frac{1}{2}}\right)^5 . \left(x^{\frac{1}{3}}\right)^5$

$= 252 y^{\frac{5}{2}} . x^{\frac{5}{3}}$

21. Find a if the 17th and 18th terms of the expansion $(2+a)^{50}$ are equal.

Ans. $T_{r+1} = {}^{50}C_r (2)^{50-r} . (a)^r$

ATQ put $r = 16$ and 17

$\Rightarrow {}^{50}C_{16} (2)^{34} . a^{16} = {}^{50}C_{17} (2)^{33} . a^{17}$

$a = \dfrac{{}^{50}C_{16} \times 2}{{}^{50}C_{17}}$

$a = 1$

22. Find the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^6$

Ans. $T_{r+1} = {}^{6}C_r \left(\frac{3}{2}x^2\right)^{6-r} . \left(\frac{-1}{3x}\right)^r$

$= {}^{6}C_r \left(\frac{3}{2}\right)^{6-r} . (x)^{12-2r} . \left(\frac{-1}{3}\right)^r x^{-r}$

$= {}^{6}C_r \left(\frac{3}{2}\right)^{6-r} . \left(\frac{-1}{3}\right)^r . (x)^{12-3r}$

Put $12 - 3r = 0$

$r = 4$

$= {}^{6}C_{4}\left(\frac{3}{2}\right)^{2} \cdot \left(\frac{-1}{3}\right)^{4}$.

$= \frac{5}{12}$

23. If the coeff of $(r-5)^{th}$ and $(2r-1)^{th}$ terms in the expansion of $(1+x)^{34}$ are equal find r

Ans. $T_{r+1} = {}^{34}C_{r}(1)^{34-r} \cdot (x)^{r}$

$T_{r+1} = {}^{34}C_{r}(x)^{r}$ ....(i)

Coeff are

${}^{34}C_{r-6}$ and ${}^{34}C_{2r-2}$

ATQ ${}^{34}C_{r-6} = {}^{34}C_{2r-2}$

$r - 6 = 2r - 2$

$r = -4$ (neglect)

$r - 6 = 34 - (2r - 2)$ $\left[\begin{matrix} \because {}^{n}C_{r} = {}^{n}C_{p} \\ r = p \ or \ n = r + p \end{matrix}\right]$

$r = 14$

24. Show that the coeff of the middle term in the expansion of $(1+x)^{2n}$ is equal to the sum of the coeff of two middle terms in the expansion of $(1+x)^{2n-1}$

Ans. As $2n$ is even so the expansion $(1+x)^{2n}$ has only one middle term which is

$\left(\frac{2n}{2}+1\right)^{th}$ $i.e\,(n+1)^{th}$ term

Coeff of x^n is ${}^{2n}C_n$

Similarly $(2n-1)$ being odd the other expansion has two middle term i.e

$\left(\frac{2n-1+1}{2}\right)^{th}$ and $\left(\frac{2n-1+1}{2}+1\right)^{th}$ term

i.e n^{th} and $(n+1)^{th}$

The coeff are ${}^{2n-1}C_{n-1}$ and ${}^{2n-1}C_n$

$${}^{2n-1}C_{n-1}+{}^{2n-1}C_n={}^{2n}C_n\left[\because {}^{n}C_{r-1}+{}^{n}C_r={}^{n+1}C_r\right]$$

25. Find the value of r, if the coeff of $(2r+4)^{th}$ and $(r-2)^{th}$ terms in the expansion of $(1+x)^{18}$ are equal.

Ans. $T_{r+1}={}^{18}C_r(1)^{18-r}.(x)^r$

$T_{r+1}={}^{18}C_r x^r$

Put $r=r-3$

And $2r+3$

ATQ ${}^{18}C_{2r+3}={}^{18}C_{r-3}$

$18=2r+3+r-3$

$r=6$

26. Find the 13th term in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}, x \neq 0$

Ans. $T_{r+1} = {}^{18}C_{r}(9x)^{18-r} \cdot \left(\frac{-1}{3\sqrt{x}}\right)^{r}$

Put $r = 12$

$T_{13} = {}^{18}C_{12}(9x)^{18-12} \cdot \left(\frac{-1}{3}\right)^{12} \cdot (x)^{\frac{-12}{2}}$

$= {}^{18}C_{12}(3)^{12} \cdot x^{6} \cdot (3)^{-12} \cdot (-1)^{12} \cdot x^{-6}$

$= {}^{18}C_{12}$

CBSE Class 12 Mathematics

Important Questions

Chapter 8

Binomial Theorem

6 Marks Questions

1.Find n , if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$

Ans. Fifth term from the beginning in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is

$$T_{4+1} = {}^{n}C_{4}\left(\sqrt[4]{2}\right)^{n-4}.\left(\frac{1}{\sqrt[4]{3}}\right)^{4}$$

$$T_5 = {}^{n}C_{4}(2)^{\frac{n-4}{4}}.(3)^{-1}\ldots\ldots(i)$$

How fifth term from the end would be equal to $(n-5+2)$ in term from the beginning

$$T_{(n-4)+1} = {}^{n}C_{n-4}\left(\sqrt[4]{2}\right)^{n-(n-4)}.\left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}$$

$$= {}^{n}C_{n-4}(2)^{1}(3)^{\frac{n-4}{4}}\ldots\ldots..(ii)$$

ATQ $\dfrac{{}^{n}C_{4}.(2)^{\frac{n-4}{4}}(3)^{-1}}{{}^{n}C_{n-4}(2)^{1}(3)^{\frac{n-4}{-4}}} = \dfrac{\sqrt{6}}{1}$

$$\frac{(2)^{\frac{n-8}{4}}}{(3)^{\frac{-(n-8)}{4}}} = (6)^{\frac{1}{2}}$$

$$(6)^{\frac{n-8}{4}} = (6)^{\frac{1}{2}}$$

$$\frac{n-8}{4} = \frac{1}{2}$$

$$\Rightarrow 2n - 16 = 4$$

$$n = 10$$

2.The coefficients of three consecutive terms in the expansion of $(1+a)^n$ are in the ratio 1:7:42. Find n

Ans. Let three consecutive terms in the expansion of $(1+a)^n$ are $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ term

$$T_{r+1} = {}^nC_r (1)^{n-r}.(a)^r$$

$$T_{r+1} = {}^nC_r\ (a)^r$$

Coefficients are

${}^nC_{r-2}$, ${}^nC_{r-1}$ and ${}^nC_r$ respectively

ATQ $\frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{1}{7}$

$$\Rightarrow n - 8r + 9 = 0 \ldots\ldots(i)$$

$$\frac{{}^{n}C_{r-1}}{{}^{n}C_{r}} = \frac{7}{42}$$

$$\Rightarrow n - 7r + 1 = 0 ......(ii)$$

On solving eq. (i) and (ii) we get $n = 55$

3. The second, third and fourth terms in the binomial expansion $(x+a)^n$ are 240, 720 and 1080 respectively. Find x, a and n.

Ans. $T_2 = 240$

$${}^{n}C_{1}\, x^{n-1}.a = 240 ....(i)$$

$${}^{n}C_{2}\, x^{n-2}.a^2 = 720 ....(ii)$$

$${}^{n}C_{3}\, x^{n-3}.a^3 = 1080 ....(iii)$$

Divide (ii) by (i) and (iii) by (ii)

We get

$\frac{a}{x} = \frac{6}{n-1}$ and $\frac{a}{x} = \frac{9}{2(n-2)}$

$$\Rightarrow n = 5$$

On solving we get

$$x = 2$$
$$a = 3$$

4.If a and b are distinct integers, prove that a-b is a factor of $a^n - b^n$, whenever n is positive.

Ans. Let $a^n = (a-b+b)^n$

$a^n = (b+a-b)^n$

$= {}^nC_0 b^n + {}^nC_1 b^{n-1}(a-b) + {}^nC_2 b^{n-2}.(a-b)^2 + {}^nC_3 b^{n-3}.(a-b)^3 + ... + {}^nC_n (a-b)^n$

$a^n = b^n + (a-b)\left[{}^nC_0 b^n + {}^nC_1 b^{n-1}(a-b) + {}^nC_2 b^{n-2}.(a-b)^2 + {}^nC_3 b^{n-3}.(a-b)^3 + ... + {}^nC_n (a-b)^n\right]$

$a^n - b^n = (a-b)k$

Where

${}^nC_1 b^{n-1} + {}^nC_2 b^{n-2}(a-b) + ... + (a-b)^{n-1} = k$

$H.P$

5. The sum of the coeff. 0f the first three terms in the expansion of $\left(x - \frac{3}{x^2}\right)^m$ m **being natural no. is 559. Find the term of expansion containing** x^3

Ans. The coeff. Of the first three terms of $\left(x - \frac{3}{x^2}\right)^m$ are ${}^mC_0, (-3)\,{}^mC_1$ and $9\;\;{}^mC_2$.

Therefore, by the given condition

${}^mC_0 - 3\,{}^mC_1 + 9\,{}^mC_2 = 559$

$1 - 3m + \frac{9m(m-1)}{2} = 559$

On solving we get $m = 12$

$T_{r+1} = {}^{12}C_r (x)^{12-r}\left(\frac{-3}{x^2}\right)^r$

$= {}^{12}C_r (x)^{12-r} . (-3)^r . (x)^{-2r}$

$= {}^{12}C_r (x)^{12-3r} . (-3)^r$

$12-3r=3 \Rightarrow r=3,\ req.$ term is $-5940\, x^3$

6.Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1.3.5.....(2n-1)}{n!} 2^n . x^n$.

Ans. As $2n$ is even, the middle term of the expansion $(1+x)^{2n}$ is $(n+1)^{th}$ term

$T_{n+1} = {}^{2n}C_n (1)^{2n-n} . x^n$

$= {}^{2n}C_n x^n$

$= \frac{(2n)!}{n!n!} . x^n$

$= \frac{(2n)(2n-1)(2n-2)......4.3.2.1}{n!n!} x^n$

$= \frac{1.2.3.4......(2n-2)(2n-1)(2n)}{n!n!} x^n$

$= \frac{[1.3.5.....(2n-1)][2.4.6.....(2n)]}{n!n!} x^n$

$= \frac{[1.3.5....(2n-1).2^n.(1.2.3.....n)]}{n!n!} x^n$

$= \frac{[1.3.5....(2n-1)].2^n.n!}{n!n!} . x^n$

$$= \frac{[1.3.5...(2n-1)].2^n.x^n}{n!}$$

7. In the expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$ **, the ratio of 7th term from the beginning to the 7th term the end is 1:6 find** n

Ans. $T_7 = {}^nC_6\left(\sqrt[3]{2}\right)^{n-6}.\left(\frac{1}{\sqrt[3]{3}}\right)^6$

$$= {}^nC_6(2)^{\frac{n-6}{3}}.(3)^{-2} ......(i)$$

7th term from end $=(n-7+2)$ term from beginning

$$T_{n-6+1} = {}^nC_{n-6}\left(\sqrt[3]{2}\right)^{n-n+6}.\left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}$$

$$= {}^nC_{n-6}(2)^2.(3)^{\frac{n-6}{-3}} ......(ii)$$

ATQ

$$\frac{{}^nC_6(2)^{\frac{n-6}{2}}.(3)^{-2}}{{}^nC_{n-6}(2)^2(3)^{\frac{6-n}{3}}} = \frac{1}{6}$$

$$\frac{(2)^{\frac{n-12}{3}}}{(3)^{\frac{12-n}{3}}} = \frac{1}{6}$$

$$(6)^{\frac{n-12}{3}} = (6)^{-1}$$

$$\frac{n-12}{3} = \frac{-1}{1}$$

$n-12=-3$

$n=9$

8.If the coeff. Of 5th 6th and 7th terms in the expansion of $(1+x)^n$ are in A.P, then find the value of n .

Ans. $T_{r+1}={}^nC_r(1)^{n-r}.(x)^r$

$T_{r+1}={}^nC_r x^r$ ....(i)

Coeff of 5th , 6th, 7th terms in the expansion of $(1+x)^n$ are ${}^nC_4$, ${}^nC_5$, and ${}^nC_6$

ATQ $2.{}^nC_5={}^nC_4+{}^nC_6$

$$2.\frac{n!}{5!(n-5)!}=\frac{n!}{4!(n-4)!}+\frac{n!}{6!(n-6)!}$$

$n=7,14$

9. If P be the sum of odd terms and Q that of even terms in the expansion of $(x+a)^n$ prove that

$(i)\quad P^2-Q^2=\left(x^2-a^2\right)^n$

$(ii)\quad 4PQ=(x+a)^{2n}-(x-a)^{2n}$

$(iii)\quad 2\left(P^2+Q^2\right)=\left[(x+a)^{2n}+(x-a)^{2n}\right]$

Ans. $(x+a)^n={}^nC_0x^n+{}^nC_1x^{n-1}a+{}^nC_2x^{n-2}a^2+.....+{}^nC_na^n$

$=t_1+t_2+t_3+....+t_n+t_{n+1}$

$=(t_1+t_3+t_5+....)+(t_2+t_4+t_6+....)$

$=P+Q.......(i)$

$(x-a)^n=(t_1-t_2+t_3-t_4+.......)$

$=(t_1+t_3+t_5)-(t_2+t_4+t_6.....)$

$=P-Q.......(ii)$

$(i)\times(ii)$

$P^2-Q^2=(x^2-a^2)^n$

Sq. (i) and (ii) and subt.

$\left[(x+a)^{2n}-(x-a)^{2n}\right]=4PQ$

Sq. and adding we get

$\left[(x+a)^{2n}+(x-a)^{2n}\right]=2(P^2+Q^2)$

10.If three successive coeff. In the expansion of $(1+x)^n$ are 220,495 and 792 then find n

Ans. Let coeff are ${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$

ATQ ${}^nC_{r-1}=220......(i)$

${}^nC_r=495......(ii)$

${}^nC_{r+1}=792........(iii)$

Dividing (ii) by (i)

$$\frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{495}{220}$$

$$\frac{n-r+1}{r} = \frac{9}{4}$$

$$4n - 13r + 4 = 0 \ldots\ldots (iv)$$

Dividing (iii) by (ii)

$$\frac{{}^{n}C_{r+1}}{{}^{n}C_{r}} = \frac{792}{495}$$

$$\frac{n-r}{r+1} = \frac{8}{5}$$

$$5n - 13r - 8 = 0 \ldots\ldots (v)$$

On solving (iv) and (v) we get $n = 12$

Binomial Theorem

1. Find the term independent of x, $x \neq 0$, in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^{15}$.

2. If the term free from x in the expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, find the value of k.

3. Find the coefficient of x in the expansion of $(1 - 3x + 7x^2)(1 - x)^{16}$.

4. Find the term independent of x in the expansion of, $\left(3x - \frac{2}{x^2}\right)^{15}$.

5. Find the middle term (terms) in the expansion of

(i) $\left(\frac{x}{a} - \frac{a}{x}\right)^{10}$ (ii) $\left(3x - \frac{x^3}{6}\right)^{9}$

6. Find the coefficient of x^{15} in the expansion of $(x - x^2)^{10}$.

7. Find the coefficient of $\frac{1}{x^{17}}$ in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$.

8. Find the sixth term of the expansion $\left(y^{\frac{1}{2}} + x^{\frac{1}{3}}\right)^n$, if the binomial coefficient of the third term from the end is 45.

[Hint: Binomial coefficient of third term from the end = Binomial coefficient of third term from beginning = ${}^nC_2$.]

9. Find the value of r, if the coefficients of $(2r + 4)^{th}$ and $(r - 2)^{th}$ terms in the expansion of $(1 + x)^{18}$ are equal.

10. If the coefficient of second, third and fourth terms in the expansion of $(1 + x)^{2n}$ are in A.P. Show that $2n^2 - 9n + 7 = 0$.

11. Find the coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^{11}$.

12. If p is a real number and if the middle term in the expansion of $\left(\frac{p}{2}+2\right)^8$ is 1120, find p.

13. Show that the middle term in the expansion of $\left(x-\frac{1}{x}\right)^{2n}$ is

$$\frac{1\times 3\times 5\times \ldots (2n-1)}{\lfloor n}\times(-2)^n.$$

14. Find n in the binomial $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$ if the ratio of 7^{th} term from the beginning to the 7^{th} term from the end is $\frac{1}{6}$.

15. In the expansion of $(x+a)^n$ if the sum of odd terms is denoted by O and the sum of even term by E.
Then prove that
(i) $O^2 - E^2 = (x^2 - a^2)^n$ (ii) $4OE = (x+a)^{2n} - (x-a)^{2n}$

16. If x^p occurs in the expansion of $\left(x^2+\frac{1}{x}\right)^{2n}$, prove that its coefficient is

$$\frac{\lfloor 2n}{\left\lfloor \frac{4n-p}{3} \right. \left\lfloor \frac{2n+p}{3} \right.}.$$

17. Find the term independent of x in the expansion of $(1+x+2x^3)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$.

Objective Type Questions

Choose the correct answer from the given options in each of the Exercises 18 to 24 (M.C.Q.).

18. The total number of terms in the expansion of $(x+a)^{100}+(x-a)^{100}$ after simplification is

(A) 50 (B) 202 (C) 51 (D) none of these

19. Given the integers $r>1, n>2$, and coefficients of $(3r)^{th}$ and $(r+2)^{nd}$ terms in the binomial expansion of $(1+x)^{2n}$ are equal, then

(A) $n=2r$ (B) $n=3r$ (C) $n=2r+1$ (D) none of these

20. The two successive terms in the expansion of $(1+x)^{24}$ whose coefficients are in the ratio 1:4 are

(A) 3^{rd} and 4^{th} (B) 4^{th} and 5^{th} (C) 5^{th} and 6^{th} (D) 6^{th} and 7^{th}

[Hint: $\frac{{}^{24}C_r}{{}^{24}C_{r+1}}=\frac{1}{4}\Rightarrow\frac{r+1}{24-r}=\frac{1}{4}\Rightarrow 4r+4=24-4\Rightarrow \boxed{r=4}$]

21. The coefficient of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ are in the ratio.

(A) 1 : 2 (B) 1 : 3 (C) 3 : 1 (D) 2 : 1

[Hint : ${}^{2n}C_n : {}^{2n-1}C_n$

22. If the coefficients of 2^{nd}, 3^{rd} and the 4^{th} terms in the expansion of $(1+x)^n$ are in A.P., then value of n is

(A) 2 (B) 7 (c) 11 (D) 14

[Hint: $2\,{}^nC_2={}^nC_1+{}^nC_3\Rightarrow n^2-9n+14=0\Rightarrow n=2$ or 7

23. If A and B are coefficient of x^n in the expansions of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then $\frac{A}{B}$ equals

(A) 1 (B) 2 (C) $\frac{1}{2}$ (D) $\frac{1}{n}$

[Hint: $\frac{A}{B}=\frac{{}^{2n}C_n}{{}^{2n-1}C_n}=2$]

24. If the middle term of $\left(\frac{1}{x}+x\sin x\right)^{10}$ is equal to $7\frac{7}{8}$, then value of x is

(A) $2n\pi + \frac{\pi}{6}$ (B) $n\pi + \frac{\pi}{6}$ (C) $n\pi + (-1)^n \frac{\pi}{6}$ (D) $n\pi + (-1)^n \frac{\pi}{3}$

[**Hint**: $T_6 = {}^{10}C_5 \frac{1}{x^5} \cdot x^5 \sin^5 x = \frac{63}{8} \Rightarrow \sin^5 x = \frac{1}{2^5} \Rightarrow \sin = \frac{1}{2}$

$\Rightarrow x = n\pi + (-1)^n \frac{\pi}{6}$]