Chapter 11. Conic Sections

Question-1

Find the centre and radius of the following circles:
(i) $x^2 + y^2 = 1$
(ii) $x^2 + y^2 - 4x - 6y - 9 = 0$
(iii) $x^2 + y^2 - 8x - 6y - 24 = 0$
(iv) $3x^2 + 3y^2 + 4x - 4y - 4 = 0$
(v) $(x - 3)(x - 5) + (y - 7)(y - 1) = 0$

Solution:

(i) The general equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$.
$2g = 0, 2f = 0, c = -1$
$\therefore$ centre is $(-g, -f) = (0, 0)$
Radius = $\sqrt{g^2 + f^2 - c} = \sqrt{0^2 + 0^2 + 1} = 1$units

(ii) The general equation of circle is $x^2 + y^2 - 4x - 6y - 9 = 0$.
$2g = -4, 2f = -6, c = -9$
$g = -2, f = -3$
$\therefore$ centre is $(-g, -f) = (2, 3)$
Radius = $\sqrt{g^2 + f^2 - c} = \sqrt{(-2)^2 + (-3)^2 + 9} = \sqrt{22}$ units

(iii) The general equation of circle is $x^2 + y^2 - 8x - 6y - 24 = 0$.
$2g = -8, 2f = -6, c = -24$
$g = -4, f = -3$
$\therefore$ centre is $(-g, -f) = (4, 3)$
Radius = $\sqrt{g^2 + f^2 - c} = \sqrt{(-4)^2 + (-3)^2 + 24} = \sqrt{49} = 7 = 1$units

(iv) The general equation of circle is $3x^2 + 3y^2 + 4x - 4y - 4 = 0$.
$2g = 4/3, 2f = -4/3, c = -4/3$
$\therefore$ centre is $(-g, -f) = (-2/3, 2/3)$
Radius = $\sqrt{g^2 + f^2 - c} = \sqrt{\left(\frac{2}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 + \frac{4}{3}} = \sqrt{\frac{8}{9} + \frac{4}{3}} = \sqrt{\frac{20}{9}}$ unit

(v) $(x - 3)(x - 5) + (y - 7)(y - 1) = 0$
$x^2 - 8x + 15 + y^2 - 8y + 7 = 0$
The general equation of circle is $x^2 + y^2 - 8x - 8y + 22 = 0$.
$2g = -8, 2f = -8, c = 22$
$\therefore$ centre is $(-g, -f) = (4, 4)$
Radius = $\sqrt{g^2 + f^2 - c} = \sqrt{(-4)^2 + (-4)^2 - 22} = \sqrt{16 + 16 - 22} = \sqrt{10}$ units

Question-2

For what values of a and b does the equation $(a - 2)x^2 + by^2 + (b - 2)xy + 4x + 4y - 1 = 0$ represent a circle? Write down the resulting equation of the circle.

Solution:

The general condition for a second degree equation to represent a circle is

$x^2 + y^2 + 2gx + 2fy + c = 0.$

$2fy + c = 0.$

The equation $(a - 2)x^2 + by^2 + (b - 2)xy + 4x + 4y - 1 = 0$ when compared with the general equation we get,

Coefficient of x^2 = Coefficient of y^2

Thus, $a - 2 = b$ ....(i)

Coefficient of xy is zero (i.e.) $h = 0$

$b - 2 = 0$

$b = 2$ ....(ii)

Substitute (ii) in (i)

$a = b + 2 = 2 + 2 = 4$

∴ The required equation of circle is $2x^2 + by^2 + 4x + 4y - 1 = 0$.

Question-3

Find the equation of the circle passing through the point (2,3) and having its centre at (1, 2).

Solution:

If the centre is {h, k) and radius is r, then the equation of the circle is

$(x - h)^2 + (y - k)^2 = r^2$

Here $(h, k) = (1, 2)$

$(x - 1)^2 + (y - 2)^2 = r^2$ is the required equation of the circle.

The circle passes through (2, 3).

$\therefore (2 - 1)^2 + (3 - 2)^2 = r^2$ $\therefore (1)^2 + (1)^2 = r^2$ $\therefore r^2 = 2$

∴ The required equation of the circle is $(x - 1)^2 + (y - 2)^2 = 2$.

Question-4

$x + 2y = 7$, $2x + y = 8$ are two diameters of a circle with radius 5 units. Find the equation of the circle.

Solution:

$x + 2y = 7$ ....(i)

$2x + y = 8$ ....(ii)

(i) × 2 − (ii)

$2x + 4y = 14$

$3y = 6$

$y = 2$

$\therefore x = 7 - 2y = 7 - 2(2) = 7 - 4 = 3$

The two diameters of a circle intersect each other at a point which is the centre of the circle.

∴ (3, 2) is the centre of a circle.

∴ The equation of the circle $(x - 3)^2 + (y - 2)^2 = 5^2$.

Question-5

The area of a circle is 16π square units. If the centre of the circle is (7, - 3), find the equation of the circle.

Solution:

Area of a circle = 16π square units$\pi r^2 = 16\pi$ $r^2 = 16$

The equation of the circle is $(x - 7)^2 + (y + 3)^2 = 16$.

Question-6

Find the equation of the circle whose centre is (- 4, 5) and circumference is 8π units.

Solution:

Circumference of a circle = 8π units

$2\pi r = 16\pi$ $2r = 16$

r = 8 units

The equation of the circle is $(x + 4)^2 + (y - 5)^2 = 64$.

Question-7

Find the circumference and area of the circle $x^2 + y^2 - 6x - 8y + 15 = 0$.

Solution:

$x^2 + y^2 - 6x - 8y + 15 = 0$

$2g = -6, 2f = -8, c = 15$

$g = -3, f = -4$

$r^2 = g^2 + f^2 - c = 9 + 16 - 15 = 10$

Area of the circle = $\pi r^2 = 10\pi$ square units

Circumference of the circle = $2\pi r = \pi 2\sqrt{10}$ units

Question-8

Find the equation of the circle which passes through (2, 3) and whose centre is on x-axis and radius is 5 units.

Solution:

Centre is on x-axis.

∴ The equation of the circle is $(x - h)^2 + y^2 = r^2$.

$(2 - h)^2 + 3^2 = 5^2$

$4 - 4h + h^2 + 9 = 25$

$h^2 - 4h - 12 = 0$

$(h - 6)(h + 2) = 0$

h = 6, -2

$(x - 6)^2 + y^2 = 25$ or $(x + 2)^2 + y^2 = 25$

$x^2 - 12x + 36 + y^2 = 25$ or $x^2 + 4x + 4 + y^2 = 25$

$x^2 - 12x + y^2 + 11 = 0$ or $x^2 + 4x + y^2 - 21 = 0$ are the equations of the circle.

Question-9

Find the equation of the circle described on the line joining the points (1, 2) and (2, 4) as its diameter.

Solution:

The equation of the circle is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.

Here $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (2, 4)$

$\therefore (x - 1)(x - 2) + (y - 2)(y - 4) = 0$.

$x^2 + y^2 - 3x - 6y + 10 = 0$.

Question-10

Find the equation of the circle passing through the points (1, 0), (0, - 1) and (0,1).

Solution:

The general equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$.

The points (1, 0), (0, - 1) and (0,1) lie on the circle.

If (1, 0) lie on the circle x2 + y2 + 2gx + 2fy + c = 0, we get,

$\therefore 1 + 2g + c = 0$

$2g + c = 1$ ....(i)

If (0, -1) lie on the circle x2 + y2 + 2gx + 2fy + c = 0, we get,

$1 - 2f + c = 0$

$-2f + c = -1$ ....(ii)

If (0, -1) lie on the circle x2 + y2 + 2gx + 2fy + c = 0, we get,

$1 - 2f + c = 0$

$-2f + c = -1$ ....(ii)

If (0, 1) lie on the circle x2 + y2 + 2gx + 2fy + c = 0, we get,

$1 + 2f + c = 0$

$2f + c = -1$ ....(iii)

(ii) + (iii)

$2c = -2$

$c = -1$

Substituting c = -1 in (i)

$2g - 1 = 1$

$g = 1$

Substituting c = -1 in (ii)

$-2f - 1 = -1$

$f = 0$

The general equation of circle is $x^2 + y^2 + 2x - 1 = 0$.

Question-11

Find the equation of the circle passing through the points (1, 1), (2, -1) and (3, 2).

Solution:

The general equation of circle is x2 + y2 + 2gx + 2fy + c = 0.

The points (1, 1), (2, -1) and (3, 2) lie on the circle.

$\therefore$ 1 + 1 + 2g + 2f + c = 0

2g + 2f + c = -2 ....(i)

4 + 1 + 4g - 2f + c = 0

4g - 2f + c = -5 ....(ii)

9 + 4 + 6g + 4f + c = 0

6g + 4f + c = -13 ....(iii)

(i) - (ii)

-2g + 4f = 3 ....(iv)

(i) − (iii)

-4g - 2f = 11 ....(v)

(iv) − [2(v)]

-10g = 25

$g = \frac{25}{-10} = \frac{-5}{2}$

Substitute $g = \frac{-5}{2}$ in (iv)

-2g + 4f = 3

$-2\left(\frac{-5}{2}\right) + 4f = 3$

5 + 4f = 3

4f = 3 − 5

= -2

$f = \frac{-2}{4} = \frac{-1}{2}$

Substitute $g = \frac{-5}{2}$ and $f = \frac{-1}{2}$ in (i)

2g + 2f + c = -2

$2\ \frac{-5}{2} + 2\ \frac{-1}{2} + c = -2$

-5 +(- 1) + c = -2

c = -2 + 6 = 4

By substituting $g = \frac{-5}{2}$, $f = \frac{-1}{2}$ and c = 4 in x2 + y2 + 2gx + 2fy + c = 0 we get,

$\therefore$ The general equation of circle is x2 + y2 - 5x - y + 4 = 0.

Question-12

Find the equation of the circle that passes through the points (4, 1) and (6, 5) and has its centre on the line 4x + y = 16.

Solution:

The general equation of the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

(4, 1) lies on the circle.

$16 + 1 + 8g + 2f + c = 0$

$8g + 2f + c = -17$ ....(i)

(6, 5) lies on the circle.

$36 + 25 + 12g + 10f + c = 0$

$12g + 10f + c = -61$ ....(ii)

(ii) – (i)

$4g + 8f = -44$

$g + 2f = -11$ ....(iii)

Let (-g, -f) be the centre of the circle lying on 4x + y = 16.

$\therefore -4g - f = 16$ ....(iv)

2(iv) + (iii)

$-7g = 21$

$g = -3$

Substitute g = -3 in (iv)

$-4(-3) - f = 16$

$f = -4$

Substitute in (i)

$8(-3) + 2(-4) + c = -17$

$-24 - 8 + c = -17$

$c = -17 + 32$

$c = 15$ $\therefore$ The general equation of the circle is $x^2 + y^2 - 6x - 8y + 15 = 0$.

Question-13

Find the equation of the circle whose centre is on the line x = 2y and which passes through the points (- 1, 2) and (3, -2).

Solution:

The general equation of the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$. (-g, -f) is centre of the circle.

Centre (-g, -f) lies on the line x = 2y.

$\therefore -g = -2f$

$g = 2f$ ....(i)

(-1, 2) lies on the circle.

$\therefore 1 + 4 - 2g + 4f + c = 0$

$1 + 4 - 2(2f) + 4f + c = 0$ (from i)

$\therefore c = -5$ ....(ii)

(3, -2) lies on the circle.

$\therefore 9 + 4 + 6g - 4f + c = 0$

$\therefore 9 + 4 + 6(2f) - 4f - 5 = 0$ (from i and ii)

$8f + 8 = 0$

$f = -1$ ....(iii)

$g = -2$ ....(iv)$\therefore$ The required equation of the circle is $x^2 + y^2 - 4x - 2y - 5 = 0$.

Question-14

Find the cartesian equation of the circle whose parametric equations are $x = \frac{1}{4}\cos\theta$, $y = \frac{1}{4}\sin\theta$ and $0 \le \theta \le 2\pi$.

Solution:

$\cos\theta = 4x$

$\sin\theta = 4y$

$\cos^2\theta + \sin^2\theta = 1$

$\therefore 16x^2 + 16y^2 = 1$ is the required cartesian equation of the circle.

Question-15

Find the parametric equation of the circle $4x^2 + 4y^2 = 9$.

Solution:

$r^2 = 9/4$

\ r = 3/2

$\therefore$ The parametric equations of the given circle $4x^2 + 4y^2 = 9$ are $x = \frac{3}{2}\cos\theta$ and $y = \frac{3}{2}\sin\theta$, $0 \le \theta \le 2\pi$.

Question-16

Find the coordinates of foci, equations of the directrices, and the length of the latusrectum of the parabola: $y^2 = 12x$.

Solution:

The given problem $y^2 = 12x$ is of the form $y^2 = 4ax$, where $4a = 12$. (i.e.) $a = 3$. The coordinates of the focus $(a, 0)$ is $(3, 0)$. The equation of the directrix is $x = -a$ (i.e.) $(-3, 0)$ Length of the latusrectum $= 4a = 12$.

Question-17

Find the foci, vertices of the parabola: $y = -4x^2+3x$.

Solution:

$y = -4x^2 + 3x$

$4x^2 - 3x = -y$

$x^2 - \frac{3}{4}x = \frac{-y}{4}$

$x^2 - 2x\left(\frac{3}{8}\right) + \left(\frac{3}{8}\right)^2 = \frac{-y}{4} + \left(\frac{3}{8}\right)^2$

$\left(x - \frac{3}{8}\right)^2 = -1\left[\frac{y}{4} - \left(\frac{3}{8}\right)^2\right]$

$= \frac{-1}{4}\left[y - \frac{9}{16}\right]$

$\left(x - \frac{3}{8}\right)^2 = 4 \times \left(\frac{-1}{16}\right)\left(y - \frac{9}{16}\right)$ .... (i)

Shifting the origin to the point $\left(\frac{3}{8}, \frac{9}{16}\right)$ without rotating the axes and denoting the new coordinates w.r.t. these axes by X and Y we have,

Put $X = \left(x - \frac{3}{8}\right)^2$

$Y = \left(y - \frac{9}{16}\right)$

In original coordinates :vertex is X =0

$\left(x - \frac{3}{8}\right) = 0 \Rightarrow x = \frac{3}{8}$

Y = 0

$\left(y - \frac{9}{16}\right) = 0 = 0 \Rightarrow y = \frac{9}{16}$

$\therefore$ Vertex is $\left(\frac{3}{8}, \frac{9}{16}\right)$

Focus: The coordinates of the focus with respect to the new axes are (X = 0, Y = a)

In original coordinates,

Focus $\left(x - \frac{3}{8}\right) = 0 \Rightarrow x = \frac{3}{8}$

$\left(y - \frac{9}{16}\right) = \frac{-1}{16}$

$y = \left(\frac{9}{16} - \frac{1}{16}\right) = \frac{8}{16} = \frac{1}{2}$

$\therefore$ Focus is $\left(\frac{3}{8}, \frac{1}{2}\right)$

Question-18

Find the equation of the parabola whose directrix is $x = 0$ and focus at (6, 0).

Solution:

Let P(x, y) be a point on the parabola. Join SP.

$SP = \sqrt{(x-6)^2+(y-0)^2} = \sqrt{x^2-12x+36+y^2}$

PM = x

SP = PM

$x^2 = (x - 6)^2 + y^2$

$x^2 = x^2 - 12x + 36 + y^2$

$y^2 - 12x + 36 = 0$

$y^2 = 12x - 36$

Question-19

Does the point (7, -11) lie inside or outside the circle $x^2 + y^2 - 10x = 0$?

Solution:

By substituting the point (7, -11) in the equation $x^2 + y^2 - 10x$, we get

$7^2 + (-11)^2 - 10(7) = 49 + 121 - 70$

$= 170 - 70 = 100 > 0$

Thus the point (7, -11) lies outside the circle.

Question-20

Determine whether the points (-2, 1), (0, 0) and (4, -3) lie outside, on or inside the circle $x^2 + y^2 - 5x + 2y - 5 = 0$.

Solution:

By substituting (-2, 1) in the equation x2 + y2 – 5x + 2y – 5 we get,

(-2)2 + (1)2 – 5(-2) + 2(1) – 5 = 4 + 1 + 10 + 2 - 5

= 12 > 0.

Thus the point (0, 0) lie on the circle .

By substituting (0, 0) in the equation x2 + y2 – 5x + 2y – 5 we get,

(0)2 + (0)2 – 5(0) + 2(0) – 5 = 0 + 0 + 0 + 2 - 5

= -5 < 0.

Thus the point (0, 0) lie inside the circle.

By substituting (4, -3) in the equation x2 + y2 – 5x + 2y – 5 we get,

(4)2 + (-3)2 – 5(4) + 2(-3) – 5 = 16 + 9 + 20 - 6 - 5

= 34 > 0.

Thus the point (4, -3) lie on the circle.

Question-21

For the following ellipses find the lengths of major and minor axes, coordinates of foci and vertices and eccentricity 3x2 + 2y2 = 6.

Solution:

$3x^2 + 2y^2 = 6.$

Dividing by 6 we get,

$$\frac{3x^2}{6} + \frac{2y^2}{6} = 1$$
$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{3} = 1$$

This equation is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a^2 = 2$ and $b^2 = 3$.(i.e.,) a = $\sqrt{2}$ and b = $\sqrt{3}$

Here a < b, so the major and minor axes of the given ellipse are along y and x – axes respectively.

Length of the major axis = 2b = $2\sqrt{3}$, Length of the minor axis = 2a = $2\sqrt{2}$.

The coordinates of the vertices are (0, b) and (0, -b).

(i.e.,) (0, $\sqrt{3}$) and (0, -$\sqrt{3}$)

The eccentricity e of the ellipse is given by e = $\sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{2}{3}} = \frac{1}{\sqrt{3}}$.

The coordinates of the foci are (0, be) and (0, -be) (i.e.,) (0, 1) and (0, -1).

Question-22

The foci of an ellipse are $(\pm 8, 0)$ and its eccentricity is $\frac{1}{2}$, find its equation.

Solution:

Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The coordinates of foci are $(\pm ae, 0)$.

ae = 8

$$a \times \frac{1}{2} = 8$$
$$\Rightarrow a = 16$$

Thus

$$b^2 = a^2(1 - e^2)$$
$$= 256\left(1 - \frac{1}{4}\right)$$
$$= 192$$

Hence, the equation of the ellipse is $\frac{x^2}{256} + \frac{y^2}{192} = 1$

Question-23

Find the equation of the ellipse whose foci are (4, 3), (-4, 3) and whose semi −minor axis is 3.

Solution:

Let S and S' be two foci of the required ellipse. Then the coordinates of S and S' are (4, 3) and (-4, 3) respectively.

SS' = 8

Let 2a and 2b be the lengths of the axes of the ellipse and e be the eccentricity.

Then SS' = 2ae, but 2ae = 8

Thus, ae = 4.

Now, $b^2 = a^2(1 - e^2)$

$9 = a^2 - 4^2$

$9 + 16 = a^2$

a = 5

Let P(x, y) be any point on the ellipse.

Then,

SP + S'P = 2 a

$$\sqrt{(x-4)^2 + (y-3)^2} + \sqrt{(x+4)^2 + (y-3)^2} = 10$$

$$\left[(x-4)^2 + (y-3)^2\right] - \left[(x+4)^2 + (y-3)^2\right] = 100 - 20\sqrt{(x+4)^2 + (y-3)^2}$$

$$-16x = 100 - 20\sqrt{(x+4)^2 + (y-3)^2}$$

$$16x + 100 = 20\sqrt{(x+4)^2 + (y-3)^2}$$

Dividing by 4 we get,

$$4x + 25 = 5\sqrt{(x+4)^2 + (y-3)^2}$$

Squaring we get,

$$(4x + 25)^2 = 25\left[(x+4)^2 + (y-3)^2\right]$$

$$16x^2 + 200x + 625 = 25x^2 + 200x + 400 + 25y^2 - 150y + 225$$

$9x^2 + 25y^2 - 150y = 0.$

Question-24

Solution:

(i) Let P be the point on x-axis where it touches the circle.
Centre C is (5, 6) and P is (5, 0).

r = CP = $\sqrt{(5-5)^2+(6-0)^2}$ = 6.

The equation of the circle is $(x - 5)^2 + (y - 6)^2 = 6^2$.

$x^2 + y^2 - 10x - 12y + 25 + 36 = 36$

$x^2 + y^2 - 10x - 12y + 25 = 0$

(ii) Let P be the point on y-axis where it touches the circle.
Centre C is (5, 6) and P is (0, 6).

r = CP = $\sqrt{(5-0)^2+(6-6)^2}$ = 5.

The equation of the circle is $(x - 5)^2 + (y - 6)^2 = 5^2$.

$x^2 + y^2 - 10x - 12y + 36 = 0$

CBSE Class 11 Mathematics

Important Questions

Chapter 11

Conic Sections

1 Marks Questions

1. Find the equation of a circle with centre (P,Q) & touching the y axis

$(A)\, x^2 + y^2 + 2Qy + Q^2 = 0$

$(B)\, x^2 + y^2 - 2px + 2Qy + Q^2 = 0$

$(C)\, x^2 + y^2 - 2px + 2Qy + Q^2 = 0$

(D) none of these

Ans. $x^2 + y^2 - 2px + 2Qy + Q^2 = 0$

2.Find the equations of the directrix & the axis of the parabola $\Rightarrow 3x^2 = 8y$

$(A)\, 3y - 4 = 0, x = 0$

$(B)\, 3x - 4 = 0, y = 0$

$(C)\, 3y - 4x = 0$

(D) none of these

Ans. $3y - 4 = 0, x = 0$

3.Find the coordinates of the foci of the ellipse $\Rightarrow x^2 + 4y^2 = 100$

$(A)\, F\left(\pm 5\sqrt{3}, 0\right)$

$(B)\, F\left(\pm 3\sqrt{5}, 0\right)$

$(C)\, F\left(\pm 4\sqrt{5}, 0\right)$

(D) none of these

Ans. $F\left(\pm 5\sqrt{3}, 0\right)$

4.Find the eccentricity of the hyperbola: $3x^2 - 2y^2 = 6$

$(A)\, e = \sqrt{\frac{5}{2}}$ $(B)\, e = \frac{\sqrt{5}}{2}$ $(C)\, e = \frac{\sqrt{2}}{5}$ (D) **none of these**

Ans. $e = \sqrt{\frac{5}{2}}$

5.Find the equation of a circle with centre (b, a) **& touching** $x-$ **axis?**

$(A)\, x^2 + y^2 - 2bx + 2ay + b^2 = 0$

$(B)\, x^2 + y^2 + 2bx - 2ay + b^2 = 0$

$(C)\, x^2 + y^2 - 2bx - 2ay + b^2 = 0$

(D) none of these

Ans. $x^2 + y^2 - 2bx - 2ay + b^2 = 0$

6.Find the lengths of axes of $3x^2 - 2y^2 = 6$?

$(A)\, 2\sqrt{2}$ & $2\sqrt{5}$ units

$(B)\, 2\sqrt{2}$ & $2\sqrt{3}$ units

$(C) 2\sqrt{5}$ & $2\sqrt{2}$ units

(D) none of these

Ans. $2\sqrt{2}$ Units & $2\sqrt{3}$ units

7.Find the length of the latus rectum of $3x^2 + 2y^2 = 18$?

(A) 2 units (B) 3 units (C) 4 units (D) none of these

Ans.4 units

8.Find the length of the latus rectum of the parabola $3y^2 = 8x$

$(A) \frac{4}{3}$ units $(B) \frac{8}{3}$ units $(C) \frac{2}{3}$ units (D) none of these

Ans. $\frac{8}{3}$ units

9.The equation $x^2 + y^2 - 12x + 8y - 72 = 0$ **represent a circle find its centre**

$(A)(-6,-4)$ $(B)(6,-4)$ $(C)(6,4)$ $(D)(-6,4)$

Ans. $(6,-4)$

10.Find the equation of the parabola with focus $F(4,0)$ **& directrix** $x = -4$

$(A) y^2 = 32x$ $(B) y^2 = -16x$ $(C) y^2 = 8x$ $(D) y^2 = 16x$

Ans. $y^2 = 16x$

11.Find the coordinates of the foci of $\frac{x^2}{8} + \frac{y^2}{4} = 1$

$(A) F_1(2,0) \& F_2(-2,0)$

$(B) F_1(-2,0) \& F_2(2,0)$

$(C) F_1(-2,0) \& F_2(-2,0)$

(D) none of these

Ans. $F_1(-2,0) \& F_2(2,0)$

12.Find the coordinates of the vertices of $x^2 - y^2 = 1$

$(A) A(-1,0), B(-1,0)$

$(B) A(-1,0), B(1,0)$

$(C) A(1,0), B(-1,0)$

(D) none of these

Ans. $A(-1,0), B(1,0)$

13.Find the coordinates of the vertices of $x^2 - y^2 = 1$

$(A)\ A(-1,0)\ \ \&\ B(5,0)$

$(B) A(-5,0)\ \ \&\ B(-1,0)$

$(C) A(-1,0)\ \ \&\ B(-5,0)$

(D) none of these

Ans. $A(-1,0) \& B(5,0)$

14.Find the eccentricity of ellipse $4x^2+9y^2=1$

$(A)\ e=\frac{\sqrt{5}}{3}\quad (B)\ e=\frac{-\sqrt{5}}{3}\quad (C)\ e=\frac{\sqrt{3}}{5}\ (D)\ e=\frac{3}{\sqrt{5}}$

Ans. $e=\frac{\sqrt{5}}{3}$

15.Find the length of the latus rectum of $9x^2+y^2=36$

$(A)\frac{1}{3}$ units $(B)\frac{1}{5}$ units $(C)1\frac{1}{3}$ units $(D)\frac{1}{6}$ units

Ans. $1\frac{1}{3}\ units$

16.Find the length of minor axis of $x^2+4y^2=100$

(A)10 units (B)12 units (C)14 units (D)8 units

Ans. $10\ units$

17.Find the centre of the circles $x^2+(y-1)^2=2$

$(A)\ (1,0)\quad (B)\ (0,1)\quad (C)\ (1,2)\quad (D)$ **None of these**

Ans. $(0,1)$

18.Find the radius of circles $x^2+(y-1)^2=2$

$(A)\sqrt{2}\quad (B)\ 2\quad (C)\ 2\sqrt{2}\quad (D)$ **None of these**

Ans. $\sqrt{2}$

19.Find the length of latcus rectum of $x^2 = -22y$

$(A)11 \quad (B)-22 \quad (C)22 \quad (D)$ **None of these**

Ans.22

20.Find the length of latcus rectum of $25x^2 + 4y^2 = 100$

$(A)\frac{3}{5}\text{units} \quad (B)\frac{1}{5}\text{units} \quad (C)\frac{8}{5}\text{units} \quad (D)$ **None of these**

Ans. $\frac{8}{5}$ Units

CBSE Class 12 Mathematics
Important Questions
Chapter 11
Conic Sections

4 Marks Questions

1.Show that the equation $x^2+y^2-6x+4y-36=0$ represent a circle, also find its centre & radius?

Ans. This is of the form $x^2+y^2+2gx+2Fy+c=0,$

where $2g=-6, 2f=4 \& c=-36$

$\therefore q=-3, f=2 \& c=-36$

So, centre of the circle $=(-g,-f)=(3,-2)$

&

Radius of the circle $=\sqrt{q^2+f^2-c}=\sqrt{9+4+36}$

$=7$ units

2.Find the equation of an ellipse whose foci are $(\pm 8, 0)$ & the eccentricity is $\frac{1}{4}$?

Ans. Let the required equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$ where $a^2>b^2$

let the foci be $(\pm c, 0), c=8$

&

$$e=\frac{c}{a}\Leftrightarrow a=\frac{c}{e}=\frac{8}{\frac{1}{4}}=32$$

Now $c^2=a^2-b^2 \Leftrightarrow b^2=a^2-c^2=1024-64=960$

$\therefore a^2=1024 \quad \& \quad b^2=960$

Hence equation is $\frac{x^2}{1024}+\frac{y^2}{960}=1$

3.Find the equation of an ellipse whose vertices are $(0,\pm 10)$ & $e=\frac{4}{5}$

Ans. Let equation be $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

& its vertices are $(0,\pm a) \quad \& \quad a=10$

Let $c^2=a^2-b^2$

Then $e=\frac{c}{a} \quad \Rightarrow \quad c=ae=10\times\frac{4}{5}=8$

Now $c^2=a^2-b^2 \Leftrightarrow b^2=\left(a^2-c^2\right)=100-64=36$

$\therefore a^2=(10)^2=100 \quad \& \quad b^2=36$

Hence the equation is $\frac{x^2}{36}+\frac{y^2}{100}=1$

4.Find the equation of hyperbola whose length of latus rectum is 36 & foci are $(0,\pm 12)$

Ans. Clearly C = 12

Length of cat us rectum $= 36 \Leftrightarrow \frac{2b^2}{a} = 36$

$\Rightarrow b^2 = 18a$

Now $c^2 = a^2 + b^2 \Leftrightarrow a^2 = c^2 - b^2 = 144 - 18a$

$a^2 + 18a - 144 = 0$

$(a+24)(a-6) = 0 \Leftrightarrow a = 6 \quad [\because a \text{ is non negative}]$

This $a^2 = 6^2 = 36 \;\; \& \;\; b^2 = 108$

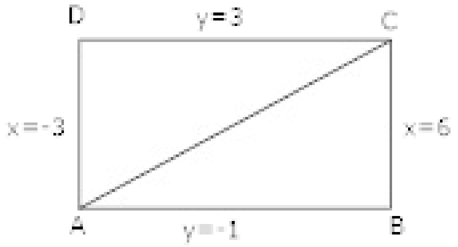
Hence, $\frac{x^2}{36} + \frac{y^2}{108} = 1$

5.Find the equation of a circle drawn on the diagonal of the rectangle as its diameter, whose sides are $x = 6,\ x = -3,\ y = 3 \;\; \& \;\; y = -1$

Ans. Let ABCD be the given rectangle &

$AD = x = -3,\ BC = x = 6,\ AB = y = -1 \;\&\; CD = y = -3$

Then $A(-3,-1) \;\; \& \;\; c(6,3)$



So the equation of the circle with AC as diameter is given as

$(x+3)(x-6) \; + \; (y+1)(y-3) = 0$

$\Rightarrow x^2 + y^2 - 3x - 2y - 21 = 0$

6.Find the coordinates of the focus & vertex, the equations of the diretrix & the axis &

length of latus rectum of the parabola $x = -8y$

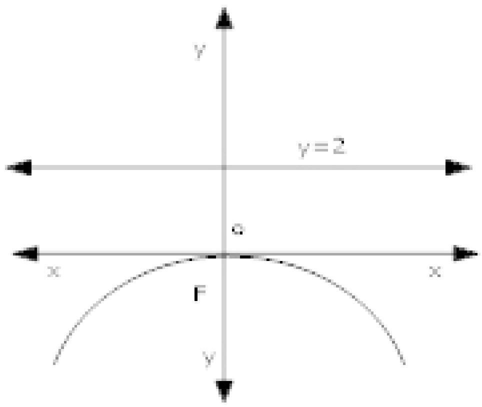
Ans. $x^2 = -8y$

& $x^2 = -4ay$

So, $4a = 8 \Leftrightarrow a = 2$

So it is case of downward parabola

o, foci is $F(0, -a)$ *ie* $F(0, -2)$



Its vertex is $0(0, 0)$

So, $y = a = 2$

Its axis is y – axis, whose equation is $x = 0$ length of lotus centum

$= 4a = 4 \times 2 = 8$ units.

7.Show that the equation $6x^2 + 6y^2 + 24x - 36y - 18 = 0$ **represents a circle. Also find its centre & radius.**

Ans. $6x^2 + 6y^2 + 24x - 36y + 18 = 0$

So $x^2 + y^2 + 4x - 6y + 3 = 0$

Where, $2g = 4, 2f = -6 \& C = 3$

$\therefore g=2, f=-3 \& C=3$

Hence, centre of circle $=(-g,-f)=(-2,3)$

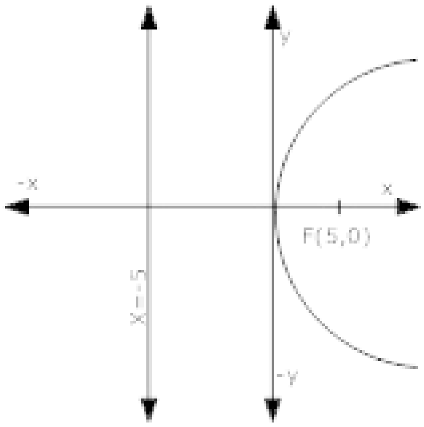
&

Radius of circle $=\sqrt{4+9+9}=\sqrt{20}$

$=2\sqrt{5}$ units

8.Find the equation of the parabola with focus at $F(5,0)$ & directrix is $x=-5$

Ans. Focus $F(5,0)$ lies to the right hand side of the origin



So, it is right hand parabola.

Let the required equation be

$y^2=4ax \quad \& \quad a=5$

So, $y^2=20x$

9.Find the equation of the hyperbola with centre at the origin, length of the transverse axis 18 & one focus at (0,4)

Ans. Let its equation be $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

Clearly, C = 4.

length of the transverse axis $= 8 \Leftrightarrow 2a = 18$

$a = 9$

Also, $C^2 = \left(a^2 + b^2\right)$

$b^2 = c^2 - a^2 = 16 - 81 = -65$

So, $a^2 = 81 \;\; \& \;\; b^2 = -65$

So, equation is $\frac{y^2}{81} + \frac{x^2}{65} = 1$

10.Find the equation of an ellipse whose vertices are $(0, \pm 13)$ & the foci are $(0, \pm 5)$

Ans. Let the equation be $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

& a = 13

Let its foci be $(0, \pm c)$, then $c = 5$

$\therefore b^2 = a^2 - c^2 = 169 - 25 = 144$

So, $a^2 = 169 \;\; \& \;\; b^2 = 144$

So, equation be $\frac{x^2}{144} + \frac{y^2}{169} = 1$

11.Find the equation of the ellipse whose foci are $(0, \pm 3)$ & length of whose major axis is 10

Ans. Let the required equation be $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Let $c^2 = a^2 - b^2$

Its foci are $(0, \pm c)$ & $c = 3$

Also, a = length of the semi- major axis = $\frac{1}{2} \times 10 = 5$

Now, $c^2 = a^2 - b^2 \Rightarrow b^2 = a^2 - c^2 = 25 - 3 = 16.$

Then, $a^2 = 25$ & $b^2 = 16$

Hence the required equation is $\frac{x^2}{16} + \frac{y^2}{25} = 1.$

12.Find the equation of the hyperbola with centre at the origin, length of the transverse axis 8 & one focus at (0,6)

Ans. Let its equation by $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Clearly, C = 6

& length of the transverse axis $= 8 \Rightarrow 2a = 8 \Rightarrow a = 4$

Also, $c^2 = a^2 + b^2 \Leftrightarrow b^2 = c^2 - a^2 \Rightarrow 36 - 16 = 20$

So, $a^2 = 16$ & $b^2 = 20$

Hence, the required equation is $\frac{y^2}{16} - \frac{x^2}{20} = 1$

13.Find the equation of the hyperbola whose foci are at $(0, \pm B)$ & the length of whose conjugate axis is $2\sqrt{11}$

Ans. Let it equation be $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Let it foci be $\left(0,\pm C\right)$

$\therefore C=8$

Length of conjugate axis $=2\sqrt{11}$

$\Rightarrow 2b=2\sqrt{11}\Rightarrow b=\sqrt{11}\Rightarrow b^2=11$

Also, $C^2=\left(a^2+b^2\right)=\left(c^2-b^2\right)=64-11=53$

$a^2=53$

Hence, required equation is $\frac{y^2}{53}-\frac{x^2}{11}=1$

14.Find the equation of the hyperbola whose vertices are $\left(0,\pm 3\right)$ & foci are $\left(0,\pm 8\right)$

Ans. The vertices are $\left(0\pm a\right)$

But it is given that the vertices are $\left(0\pm 3\right)$

$\therefore a=3$

Let its foci be $\left(0,\pm c\right)$

But it is given that the foci are $\left(0,\pm 8\right)$

$\therefore c=8$

Now $b^2=\left(c^2-a^2\right)=8^2-3^2=64-9=55$

Then $a^2=3^2=9\,\&\,b^2=55$

Hence the required equation is $\frac{y^2}{9}-\frac{x^2}{55}=1$

15.Find the equation of the ellipse for which $e=\frac{4}{5}$ & whose vertices are $(0,\pm 10)$.

Ans. Its vertices are $(0,\pm a)$ & therefore a =10

Let $c^2=(a^2-b^2)$

Then, $e=\frac{c}{a}\Rightarrow c=ae=\left[10\times\frac{4}{5}\right]=8$

Now, $c^2=(a^2-b^2)\Rightarrow b^2=(a^2-c^2)=(100-64)=36$

$\therefore a^2=(10)^2=100 \,\&\, b^2=36$

Hence the required equation is $\frac{x^2}{36}+\frac{y^2}{100}=1$

16.Find the equation of the ellipse, the ends of whose major axis are $(\pm 7,0)$ & the ends of whose minor axis are $(0,\pm 2)$

Ans. Its vertices are $(\pm a,0)$ & therefore, a = 5 ends of the minor axis are $c(0,-5)\,\&\,D(0,5)$

$\therefore CD=25$ i.e length of minor axis = 25 units

$\therefore 2b=25\Rightarrow \frac{25}{2}=12.5$

Now, $a=5 \;\&\; b=12.5 \;\Rightarrow\; a^2=25 \;\&\; b^2=156.25$

Hence, the required equation $\frac{x2}{25}+\frac{y^2}{156.25}=1$

16.Find the equation of the parabola with vertex at the origin & y+5 = 0 as its directrix. Also, find its focus

Ans. Let the vertex of the parabola be $o(0, 0)$

Now $y+5=0 \Rightarrow y=-5$

Then the directrix is a line parallel

To the x axis at a distance of 5 unite below the x axis so the focus is $F(0, 5)$

Hence the equation of the parabola is

$x^2 = 4ay$ Where a = 5i.e, $x^2 = 20y$

17.Find the equation of a circle, the end points of one of whose diameters are $A(2, -3)$ & $B(-3, 5)$.

Ans. Let the end points of one of whose diameters are (x_1, y_1) & (x_2, y_2) is given by

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

Hence $x_1 = 2,\ y_1 = -3\ \ \&\ \ x_2 = -3,\ y_2 = 5$

$\therefore$ The required equation of the circle is

$(x-2)(x+3)+(y+3)(y-5)=0$

$\Rightarrow x^2 + y^2 + x - 2y - 21 = 0$

18.Find the equation of ellipse whose vertices are $(0, \pm 13)$ & the foci are $(0, \pm 5)$

Ans. Let the required equation be $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ 5.

Its vertices are $(0 \pm a)$ & therefore a = 13

Let its foci be $(0 \pm C)$ then C = 5

$\therefore b^2 = a^2 - c^2 = 169 - 25 = 144$

This $b^2 = 144 \,\&\, a^2 = 169$

Hence, the required equation is $\frac{x^2}{144} + \frac{y^2}{169} = 1$

19.Find the equation of the hyperbola whose foci are $(\pm 5, 0)$ & the transverse axis is of length 8.

Ans. Let the required equation be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Length of its Trans verse axis =2a

$\therefore 2a = 8 \Leftrightarrow a = 4 \Leftrightarrow a^2 = 16$

Let its foci be $(\pm C, 0)$

Then C = 5

$\therefore b^2 = (c^2 - a^2) = 5^2 - 4^2 = 9$

This $a^2 = 16 \,\&\quad b^2 = 9$

Hence, the required equation is $\frac{x^2}{16} - \frac{y^2}{9} = 1$

20.Find the equation of a circle, the end points of one of whose diameters are $A(-3, 2) \,\&\, B(5, -3)$.

Ans. Let the equation be $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Hence $x_1 = -3, y_1 = 2 \,\&\, x_2 = 5, y_2 = -3$

So $(x + 3)(x - 5) + (y - 2)(y + 3) = 0$

$$x^2 - 2x - 15 + y^2 + y - 6 = 0$$
$$x^2 + y^2 - 2x + y - 21 = 0$$

21.If eccentricity is $\frac{1}{5}$ & foci are $(\pm 7, 0)$ find the equation of an ellipse.

Ans. Let the required equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Let its foci be $(\pm C, 0)$, Then C =7

Also,

$$e = \frac{c}{a} \Leftrightarrow a = \frac{c}{e} = \frac{7}{\frac{1}{5}} = 35$$

Now $c^2 = \left(a^2 - b^2\right)$

$$b^2 = a^2 - c^2 = (35)^2 - 49 = 1225 - 49 = 1176$$
$$\therefore a^2 = 1225 \,\&\, b^2 = 1176$$

Hence the required equation is $\frac{x^2}{1225} + \frac{y^2}{1176} = 1$

22.Find the equation of the hyperbola where foci are $(\pm 5, 0)$ & the transverse axis is of length

Ans. Let the required equation be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Length of its transverse axis $= 2a$

$\therefore 2a = 8 \Leftrightarrow a = \frac{8}{2} = 4$

$a^2 = 16$

Let its foci be $(\pm C, 0)$

Then C = 5

$\therefore b^2 = c^2 - a^2 = 25 - 16 = 9$

Hence the required equation is $\frac{x^2}{16} - \frac{y^2}{9} = 1$

23.Find the length of axes & coordinates of the vertices of the hyperbola $\frac{x^2}{49} - \frac{y^2}{64} = 1$

Ans. The equation of the given hyperbola is $\frac{x^2}{49} - \frac{y^2}{64} = 1$

Comparing the given equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get

$a^2 = 49 \,\&\, b^2 = 64$

$\therefore C^2 = (a^2 + b^2) = 49 + 64 = 113$

Length of transverse axis = $2a = 2 \times 7 = 14$ units

Length of conjugate axis = $2b = 2 \times 8 = 16$ units

The coordinators of the vertices are $A(-a, 0) \,\&\, B\,(a - 0)\, ie\, A(-7, 0) \,\&\, B\,(7, 0)$

24.Find the lengths of axes & length of lat us rectum of the hyperbola, $\frac{y^2}{9} - \frac{x^2}{16} = 1$

Ans. The given equation is $\frac{y^2}{9} - \frac{x^2}{16} = 1$ means hyperbola

Comparing the given equation with $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we get

$a^2 = 9 \quad \& \quad b^2 = 16$

Length of transverse axis $= 2a = 2 \times 3 = 6$ units

Length of conjugate axis $= 2b = 2 \times 4 = 8$ units

The coordinates of the vertices are $A(0, -a) \quad \& \quad B(0, a)$ i.e $A(0, -3) \quad \& \quad B(0, 3)$

25.Find the eccentricity of the hyperbola of $\frac{y^2}{9} - \frac{x^2}{16} = 1$

Ans. As in above question

$a = 3 \quad \& \quad b = 4$

&

$c^2 = a^2 + b^2 = 9 + 16 = 25$

So, c = 5

Then $e = \frac{c}{a} = \frac{5}{3}$

26.Find the equation of the hyperbola with centre at the origin, length of the trans verse axis 6 & one focus at $(0, 4)$

Ans. Let its equation be $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Clearly c = 4

Length of transverse axis $= 6 \Leftrightarrow 2a = 6 \Leftrightarrow a = 3$.

Also, $c^2 = a^2 + b^2 \Leftrightarrow b^2 = c^2 - a^2 = 4^2 - 3^2 = 16 - 9 = 7$

Then $a^2 = 3^2 = 9 \quad \& \quad b^2 = 7$

Hence, the required equation is $\frac{y^2}{9} - \frac{x^2}{7} = 1$

27.Find the equation of the ellipse, the ends of whose major axis are $(\pm 3, 0)$ & at the ends of whose minor axis are $(0, \pm 4)$

Ans. Let the required equation be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Its vertices are $(\pm a, 0) \quad \& \quad a = 3$

Ends of minor axis are $C(0, -4) \quad \& \quad D(0, 4)$

$\therefore CD = 8$ i.e length of the minor axis = 8 units

Now, $2b = 8 \Leftrightarrow b = 4$

$\therefore a = 3 \quad \& \quad b = 4$

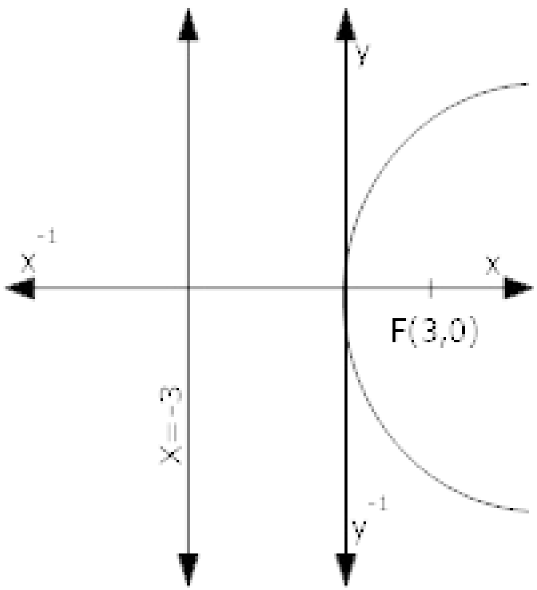
Hence the required equation is $\frac{x^2}{9} + \frac{y^2}{16} = 1$

28.Find the equation of the parabola with focus at $F(4, 0)$ & directrix $x = -3$

Ans. Focus $F(4, 0)$ lies on the axis hand side of the origin so, it is a right handed parabola. Let the required equation be $y^2 = 4ax$.

Than, a = 4

Hence, the required equation is $y^2 = 16x$



29.If $y = 2x$ is a chord of the circle $x^2 + y^2 - 10x = 0$, find the equation of the circle with this chord as a diameter

Ans. $y = 2x \quad \& \quad x^2 + y^2 - 10x = 0$

Putting $y = 2x$ in $x^2 + y^2 - 10x = 0$ we get

$5x^2 - 10x = 0 \Leftrightarrow 5x(x-2) = 0 \Leftrightarrow x = 0$ or $x = 2$

Now, $x = 0 \Rightarrow y = 0 \quad \& \quad x = 2 \Rightarrow y = 4$

$\therefore$ the points of intersection of the given chord & the given circle are

$A(0,0) \quad \& \quad B(2,4)$

$\therefore$ the required equation of the circle with AB as diameter is

$(x-0)(x-2)+(y-0)(y-4) = 0$

$\Rightarrow x^2 + y^2 - 2x - 4y = 0$

CBSE Class 12 Mathematics
Important Questions
Chapter 11
Conic Sections

6 Marks Questions

1. Find the length of major & minor axis- coordinate's of vertices & the foci, the eccentricity & length of latus rectum of the ellipse $16x^2 + y^2 = 16$

Ans. $16x^2 + y^2 = 16$

Dividing by 16,

$$x^2 + \frac{y^2}{16} = 1$$

So $b^2 = 1 \ \& \ a^2 = 16 \ \& \ b = 1 \ \& \ a = 4$

&

$$c = \sqrt{a^2 - b^2} = \sqrt{16 - 1}$$

$$= \sqrt{15}$$

Thus $a = 4, \ b = 1 \ \& \ c = \sqrt{15}$

(i) Length of major axis $= 2a = 2 \times 4 = 8$ units

Length of minor axis $= 2b = 2 \times 1 = 2$ units

(ii) Coordinates of the vertices are $A(-a, 0) \ \& \ B(a, 0) \ ie \ A(-4, 0) \ \& \ B(4, 0)$

(iii) Coordinates of foci are $F_1(-c, 0) \ \& \ F_2(c, 0) \ ie \ F_1(-\sqrt{15}, 0) \ \& \ F_2(\sqrt{15, 0})$

(iv) Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{15}}{4}$

(v) Length of latus rectum $= \frac{2b^2}{a} = \frac{2}{4} = \frac{1}{2}$ units

2. Find the lengths of the axis , the coordinates of the vertices & the foci the eccentricity & length of the lat us rectum of the hyperbola $25x^2 - 9y^2 = 225$

Ans. $25x^2 - 9y^2 = 225 \Rightarrow \frac{x^2}{9} - \frac{y^2}{25} = 1$

So, $a^2 = 9 \quad \& \quad b^2 = 25$

& $c = \sqrt{a^2 + b^2} = \sqrt{9+25} = \sqrt{34}$

(i) Length of transverse axis $= 2a = 2 \times 3 = 6$ units

Length of conjugate axis $= 2b = 2 \times 5 = 10$ units

(ii) The coordinates of vertices are $A(-a,0) \quad \& \quad B(a,0) \quad ie \quad A(-3,0) \quad \& \quad B(3,0)$

(iii) The coordinates of foci are

$F_1.(-c,0) \quad \& \quad F_2(c,0) \quad ie \quad F_1\left(-\sqrt{34},0\right) \quad \& \quad F_2\left(\sqrt{34},0\right)$

(iv) Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{34}}{3}$

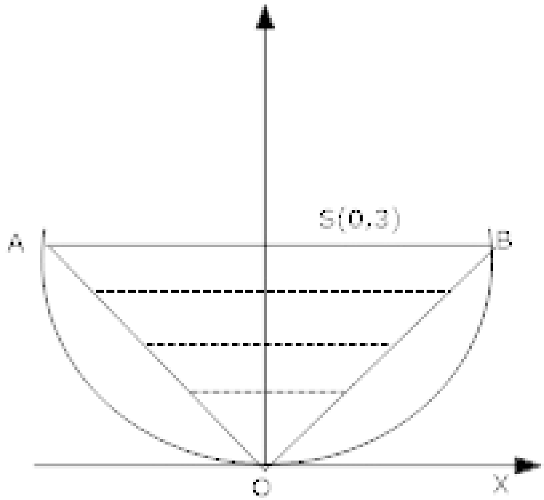
(v) Length of the lat us rectum $= \frac{2b^2}{a} = \frac{50}{3}$ units

3. Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ **to the ends of its latus rectum.**

Ans. The vertex of the parabola $x^2 = 12y \quad ie \quad o(0,0)$.

0	0	1
6	3	1
-6	3	1

Comparing $x^2 = 12y$ with $x^2 = 4ay$, we get $a = 3$ the coordinates of its focus S are $(0,3)$.



Clearly, the ends of its latus rectum are : $A(-2a,a)$ & $B(2a,a)$

Ie $A(-6,3)$ & $B(6,3)$

$\therefore$ area of $\Delta OBA = \frac{1}{2}$

$= \frac{1}{2}\left[1\times(18+18)\right]$

$= 18$ units.

4. A man running in a race course notes that the sum of the distances of the two flag posts from him is always 12 m & the distance between the flag posts is 10 m. find the equation of the path traced by the man.

Ans. We know that on ellipse is the locus of a point that moves in such a way that the sum of its distances from two fixed points (caked foci) is constant.

So, the path is ellipse.

Let the equation of the ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$$

where $b^2=a^2\left(1-c^2\right)$

Clearly, $2a=12 \quad \& \quad 2ae=10$

$$\Rightarrow a=b \quad \& \quad e=\frac{5}{6}$$

$$\Rightarrow b^2=a^2\left(1-e^2\right)=36\left(1-\frac{25}{36}\right)$$

$$\Rightarrow b^2=11$$

Hence, the required equation is $\frac{x^2}{36}+\frac{y^2}{11}=1$

5. An equilateral triangle is inscribed in the parabola $y^2=4ax$ so that one angular point of the triangle is at the vertex of the parabola. Find the length of each side of the triangle.

Ans. Let ΔPQR be an equilateral triangle inscribed in the parabola $y^2=4ax$

Let QP = QP = QR = PR = C

Let ABC at the $x-$ axis at M.

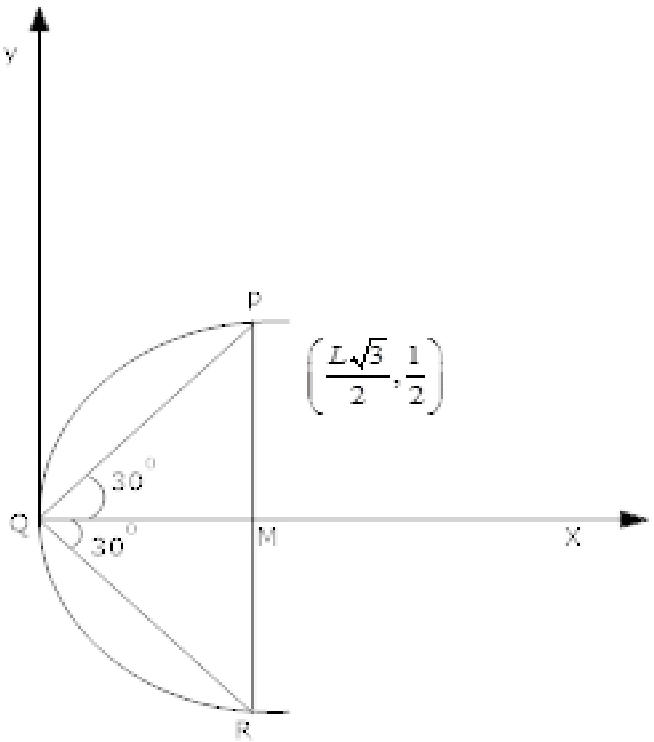
Then , $\angle\ \angle PQM=\angle RQWM=30^\circ$

$$\therefore \frac{QM}{QP}=\cos 30^\circ \Rightarrow QM=\angle \cos 30^\circ$$

$$\Rightarrow \frac{L\sqrt{3}}{2}$$

$$\Rightarrow \frac{PM}{QP} = \sin 30^\circ \Rightarrow PM = \angle \sin 30^\circ$$

$$\Rightarrow \frac{L}{2}$$



$\therefore$ the coordinates of are $\left[\frac{L\sqrt{3}}{2}, \frac{L}{2}\right]$

Since P lies on the parabola $y^2 = 4ax,$ we have

$$l^2 = 4a \times \frac{L\sqrt{3}}{2} \Rightarrow l = 8a\sqrt{3}$$

Hence length of each side of the triangle is $8a\sqrt{3}$ units.

6. Find the equation of the hyperbola whose foci are at $\left(0, \pm\sqrt{10}\right)$ & which passes through the points $(2, 3)$

Ans. Let it equation be $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1......(i)$

Let its foci be $(0,\pm C)$

But the foci are $(0,\pm\sqrt{10})$

$\therefore C=\sqrt{10}\Leftrightarrow C^2=10\Leftrightarrow(a^2+b^2)=10......(ii)$

Since (i) passes through (2,3), we have $\frac{9}{a^2}-\frac{4}{b^2}=1$

Now

$\frac{9}{a^2}-\frac{4}{b^2}=1\Leftrightarrow\frac{9}{a^2}-\frac{4}{(10-a^2)}=1......(iii)$

$\Rightarrow 9(10-a^2)-4a^2=a^2(10-a^2)$

$\Rightarrow a^2-23a^2+90=0$

$\Rightarrow(a^2-18)(a^2-5)=0\Leftrightarrow a^2=5$

[$\because a^2=18\Rightarrow b^2=-8$, which is not possible]

Then $a^2=5$ & $b^2=5$

Hence, the required equation is $\frac{y^2}{5}-\frac{x^2}{5}=1$,

i.e. $y^2-x^2=5$

7. Find the equation of the curve formed by the set of all these points the sum of whose distance from the points $A(4,0,0)$ & $B(-4,0,0)$ is 10 units.

Ans. Let $P(x,y,z)$ be an arbitrary point on the given curve

Then $PA + PB = 10$

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} + \sqrt{(x+4)^2 + y^2 + z^2} = 10$$

$$= \sqrt{(x+4)^2 + y^2 + z^2} = 10 - \sqrt{(x-4)^2 + y^2 + z^2} \ldots\ldots(i)$$

Squaring both sides

$$\Rightarrow (x+4)^2 + y^2 + z^2 = 100 - (x-4)^2 + y^2 + z^2 - 20\sqrt{(x-4)^2 + y^2 + z^2}$$

$$\Rightarrow 16x = 100 - 20\sqrt{(x-4)^2 + y^2 + z^2}$$

$$\Rightarrow 5\sqrt{(x-4)^2 + y^2 + z^2} = 25 - 4x$$

$$\Rightarrow 25\left[(x-4)^2 + y^2 + z^2\right] = 625 + 16x^2 - 200x$$

$$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$$

Hence, the required equation of the curve is

$$9x^2 + 25x^2 + 25z^2 - 225 = 0$$

8. Find the equation of the hyperbola whose foci are at $\left(0, \pm\sqrt{10}\right)$ & which passes through the point $(2, 3)$.

Ans. Let its equation be $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \ldots\ldots(i)$

Let its foci be $(0, \pm c)$

But, the foci are $\left(0, \pm\sqrt{10}\right)$

$$\therefore C = \sqrt{10} \Leftrightarrow C^2 = 10$$

& $a^2 + b^2 = 10 \ldots\ldots (ii)$

Since (i) passes through $(2, 3)$, we have

$$\frac{9}{a^2} - \frac{4}{b^2} = 1$$

Now

$$\frac{9}{a^2} + \frac{4}{b^2} = 1 \Leftrightarrow \frac{9}{a^2} - \frac{4}{(10 - a^2)} = 1$$

$$\Rightarrow a^4 - 23a^2 + 90 = 0$$

$$\Rightarrow (a^2 - 18)(a^2 - 5) = 0$$

$$\Rightarrow a^2 = 5$$

Then $a^2 = 5 = b^2$

Hence, the required equation is $\frac{y^2}{5} - \frac{x^2}{5} = 1$

i.e. $y^2 - x^2 = 5$

9. Find the equation of the ellipse with centre at the origin, major axis on the y – axis & passing through the points $(3, 2)$ & $(1, 6)$

Ans. Let the required equation be $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \ldots\ldots (i)$

Since $(3, 2)$ lies on (i) we have $\frac{9}{b^2} + \frac{4}{a^2} = 1 \ldots\ldots (ii)$

Also, since $(1, 6)$ lies on (i), we have $\frac{1}{b^2} + \frac{36}{a^2} = 1 \ldots\ldots (iii)$

Putting $\frac{1}{b^2} = u \quad \& \quad \frac{1}{a^2} = v$ these equations become:

$9u + 4v = 1......(iv)$ & $u + 36v = 1......(v)$

On multiplying (v) by 9 & subtracting (iv) from it we get

$$320v = 8 \Leftrightarrow v = \frac{8}{320} = \frac{1}{40} \Leftrightarrow \frac{1}{a^2} = \frac{1}{40} \Leftrightarrow a^2 = 40$$

Putting $v = \frac{1}{40}$ in (v) we get

$$u + \left[36 \times \frac{1}{40}\right] = 1 \Leftrightarrow u = \left[1 - \frac{9}{10}\right] = \frac{1}{10} \Leftrightarrow \frac{1}{b^2} = \frac{1}{10} \Leftrightarrow b^2 = 10$$

Then, $b^2 = 10 \quad \& \quad a^2 = 40$

Hence the required equation is $\frac{x^2}{10} + \frac{y^2}{40} = 1$

10. Prove that the standard equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Where a & b are the lengths of the semi major axis & the semi- major axis respectively & a > b.

Ans. Let the equation of the given curve be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \& \; let$

$P(x, y)$ be an arbitrary point on this curve

Then,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow y^2 = b^2\left[1 - \frac{x^2}{a^2}\right]$$

$$\Rightarrow y^2 = \frac{b^2\left[a^2 - x^2\right]}{a^2} \ldots\ldots(i)$$

Also, let $\left(a^2 - b^2\right) = c^2 \ldots\ldots(ii)$

Let $F_1(-c, 0)$ & $F_2(c, 0)$ be two fixed points on the x- axis, than

$$PF_1 = \sqrt{(x+c)^2 + y^2}$$

$$= \sqrt{(x+c)^2 + \frac{b^2\left(a^2 - x^2\right)}{a^2}} \text{ using } (i)$$

$$= \sqrt{(x+c)^2 + \frac{\left(a^2 - c^2\right)\left(a^2 - x2\right)}{a^2}} \text{ using } (ii)$$

$$= \sqrt{a^2 + 2cx + \frac{c^2x^2}{a^2}}$$

$$= \sqrt{\left[a + \frac{cx}{a}\right]^2} = \left[a + \frac{cx}{a}\right]$$

Similarly, $PF_2 = \left[a - \frac{cx}{a}\right]$

$$\therefore PF_1 + PF_2 = \left[a + \frac{cx}{a} + a - \frac{cx}{a}\right]$$

$$\Rightarrow PF_1 + PF_2 = 2a$$

This shows that the given curve is an ellipse

Hence the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Conic Sections

1. Find the equation of the circle which touches the both axes in first quadrant and whose radius is a.

2. Show that the point (x, y) given by $x = \frac{2at}{1+t^2}$ and $y = \frac{a(1-t^2)}{1+t^2}$ lies on a circle for all real values of t such that $-1 \leq t \leq 1$ where a is any given real numbers.

3. If a circle passes through the point $(0, 0)$ $(a, 0)$, $(0, b)$ then find the coordinates of its centre.

4. Find the equation of the circle which touches x-axis and whose centre is $(1, 2)$.

5. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then find the radius of the circle.

   [Hint: Distance between given parallel lines gives the diameter of the circle.]

6. Find the equation of a circle which touches both the axes and the line $3x - 4y + 8 = 0$ and lies in the third quadrant.

   [Hint: Let a be the radius of the circle, then $(-a, -a)$ will be centre and perpendicular distance from the centre to the given line gives the radius of the circle.]

7. If one end of a diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is $(3, 4)$, then find the coordinate of the other end of the diameter.

8. Find the equation of the circle having $(1, -2)$ as its centre and passing through $3x + y = 14$, $2x + 5y = 18$

9. If the line $y = \sqrt{3}\,x + k$ touches the circle $x^2 + y^2 = 16$, then find the value of k.

   [Hint: Equate perpendicular distance from the centre of the circle to its radius].

10. Find the equation of a circle concentric with the circle $x^2 + y^2 - 6x + 12y + 15 = 0$ and has double of its area.

    [Hint: concentric circles have the same centre.]

11. If the latus rectum of an ellipse is equal to half of minor axis, then find its eccentricity.

12. Given the ellipse with equation $9x^2 + 25y^2 = 225$, find the eccentricity and foci.

13. If the eccentricity of an ellipse is $\frac{5}{8}$ and the distance between its foci is 10, then find latus rectum of the ellipse.

14. Find the equation of ellipse whose eccentricity is $\frac{2}{3}$, latus rectum is 5 and the

centre is (0, 0).

15. Find the distance between the directrices of the ellipse $\frac{x^2}{36}+\frac{y^2}{20}=1$.

16. Find the coordinates of a point on the parabola $y^2=8x$ whose focal distance is 4.

17. Find the length of the line-segment joining the vertex of the parabola $y^2=4ax$ and a point on the parabola where the line-segment makes an angle θ to the x-axis.

18. If the points (0, 4) and (0, 2) are respectively the vertex and focus of a parabola, then find the equation of the parabola.

19. If the line $y = mx + 1$ is tangent to the parabola $y^2 = 4x$ then find the value of m.
[Hint: Solving the equation of line and parabola, we obtain a quadratic equation and then apply the tangency condition giving the value of m].

20. If the distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$, then obtain the equation of the hyperbola.

21. Find the eccentricity of the hyperbola $9y^2 - 4x^2 = 36$.

22. Find the equation of the hyperbola with eccentricity $\frac{3}{2}$ and foci at $(\pm 2, 0)$.

23. If the lines $2x - 3y = 5$ and $3x - 4y = 7$ are the diameters of a circle of area 154 square units, then obtain the equation of the circle.

24. Find the equation of the circle which passes through the points (2, 3) and (4, 5) and the centre lies on the straight line $y - 4x + 3 = 0$.

25. Find the equation of a circle whose centre is (3, –1) and which cuts off a chord of length 6 units on the line $2x - 5y + 18 = 0$.

    [Hint: To determine the radius of the circle, find the perpendicular distance from the centre to the given line.]

26. Find the equation of a circle of radius 5 which is touching another circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at (5, 5).

27. Find the equation of a circle passing through the point (7, 3) having radius 3 units and whose centre lies on the line $y = x - 1$.

28. Find the equation of each of the following parabolas

    (a) Directrix $x = 0$, focus at (6, 0) (b) Vertex at (0, 4), focus at (0, 2)

    (c) Focus at (–1, –2), directrix $x - 2y + 3 = 0$

29. Find the equation of the set of all points the sum of whose distances from the points (3, 0) and (9, 0) is 12.

30. Find the equation of the set of all points whose distance from (0, 4) are $\frac{2}{3}$ of their distance from the line $y = 9$.

31. Show that the set of all points such that the difference of their distances from (4, 0) and (– 4, 0) is always equal to 2 represent a hyperbola.

32. Find the equation of the hyperbola with

    (a) Vertices (± 5, 0), foci (± 7, 0) (b) Vertices (0, ± 7), $e = \frac{4}{3}$

    (c) Foci $(0, \pm\sqrt{10})$, passing through (2, 3)

True or False Type Questions

33. The line $x + 3y = 0$ is a diameter of the circle $x^2 + y^2 + 6x + 2y = 0$.

34. The shortest distance from the point $(2, -7)$ to the circle $x^2 + y^2 - 14x - 10y - 151 = 0$ is equal to 5.

    [Hint: The shortest distance is equal to the difference of the radius and the distance between the centre and the given point.]

35. If the line $lx + my = 1$ is a tangent to the circle $x^2 + y^2 = a^2$, then the point (l, m) lies on a circle.

    [Hint: Use that distance from the centre of the circle to the given line is equal to radius of the circle.]

36. The point $(1, 2)$ lies inside the circle $x^2 + y^2 - 2x + 6y + 1 = 0$.

37. The line $lx + my + n = 0$ will touch the parabola $y^2 = 4ax$ if $ln = am^2$.

38. If P is a point on the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ whose foci are S and S′, then PS + PS′ = 8.

39. The line $2x + 3y = 12$ touches the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 2$ at the point $(3, 2)$.

40. The locus of the point of intersection of lines $\sqrt{3}x - y - 4\sqrt{3}k = 0$ and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$ for different value of k is a hyperbola whose eccentricity is 2.

    [Hint: Eliminate k between the given equations]