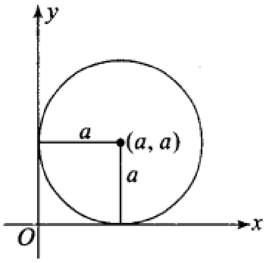
Unit 11 (Conic Sections)

Short Answer Type Questions

Q1. Find the equation of the circle which touches the both axes in first quadrant and whose radius is a.

Sol: Given that the circle of radius 'a' touches both axis. So, its centre is (a, a).



So, the equation of required circle is:

$$(x-a)^2+(y-a)^2=a^2$$

$$\Rightarrow \quad x^2-2ax+a^2+y^2-2ay+a^2=a^2$$

$$\Rightarrow \quad x^2+y^2-2ax-2ay+a^2=0$$

Q2. Show that the point (x, y) given by $x=\frac{2at}{1+t^2}$ and $y=\frac{1-t^2}{1+t^2}$ lies on a circle .

Sol. We have variable point as $x=\dfrac{2at}{1+t^2}$ and $y=\dfrac{a(1-t^2)}{1+t^2}$

On squaring and adding, we get

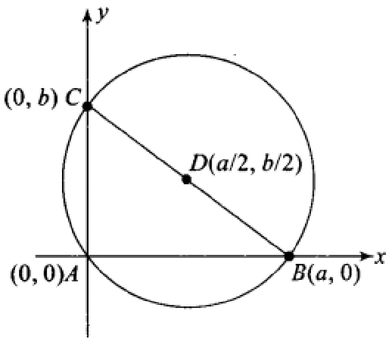
$$x^2+y^2=\frac{4a^2t^2}{(1+t^2)^2}+\frac{a^2(1-t^2)^2}{(1+t^2)^2}=\frac{a^2(4t^2+(1-t^2)^2)}{(1+t^2)^2}=\frac{a^2(1+t^2)^2}{(1+t^2)^2}$$

$\Rightarrow \quad x^2+y^2=a^2$, which is circle.

Q3. If a circle passes through the point (0, 0) (a, 0), (0, b) then find the coordinates of its centre.

Sol: We have circle through the point A(0, 0), B(a, 0) and C(0, b).

Clearly triangle is right angled at vertex A.



So, centre of the circle is the mid point of hypotenuse BC which is (a/2, b/2)

Q4. Find the equation of the circle which touches x-axis and whose centre is (1,2).

Sol: Given that, circle with centre (1,2) touches x-axis.

Radius of the circle is, r = 2

So, the equation of the required circle is:

$(x - l)^2 + (y - 2)^2 = 2^2$

$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 = 4$

$\Rightarrow x^2 + y^2 - 2x - 4y + 1 = 0$

Q5. If the lines 3x – 4y + 4 = 0 and 6x – 8y – 7 = 0 are tangents to a circle, then find the radius of the circle.

Sol: Given lines are 6x – 8y + 8 = 0 and 6x – 8y – 7 = 0.

These parallel lines are tangent to a circle.

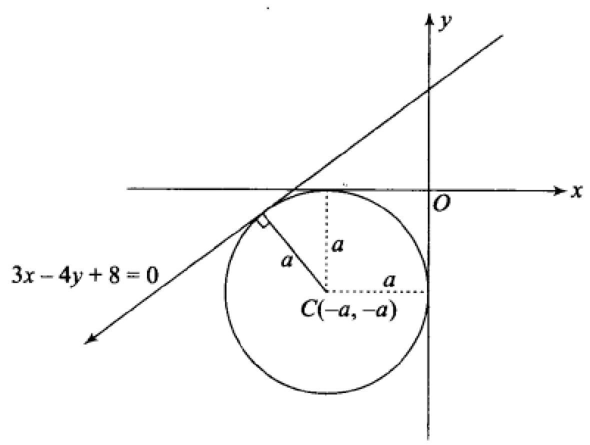
∴ Diameter of the circle = Distance between the lines

$$= \left|\frac{8-(-7)}{\sqrt{36+64}}\right| = \frac{15}{10} = \frac{3}{2}$$

∴ Radius of the circle = $\frac{3}{4}$

Q6. Find the equation of a circle which touches both the axes and the line 3x – 4y + 8 = 0 and lies in the third quadrant.

Sol.



Since circle touches both the axes, its centre is $C(-a, -a)$ and radius is a.

Also, circle touches the line $3x - 4y + 8 - 0$.

Distance from centre C to this line is radius of the circle.

∴ Radius of the circle, $a = \left|\frac{-3a + 4a + 8}{\sqrt{9+16}}\right| = \left|\frac{a+8}{5}\right|$

$\therefore \quad \frac{a+8}{5} = \pm a$

$\Rightarrow \quad a + 8 = 5a$ or $a + 8 = -5a$

$\Rightarrow \quad a = 2$ or $a = -4/3$

$\therefore \quad a = 2$

So, the equation of the required circle is:

$$(x+2)^2 + (y+2)^2 = 2^2$$

$\Rightarrow \quad x^2 + y^2 + 4x + 4y + 4 = 0$

Q7. If one end of a diameter of the circle $x^2 + y^2 -4x -6y + 11 = 0$ is (3,4), then find the coordinate of the other end of the diameter.

Sol: Given equation of the circle is:

$$x^2 + y^2 - 4x - 6y + 11 = 0$$

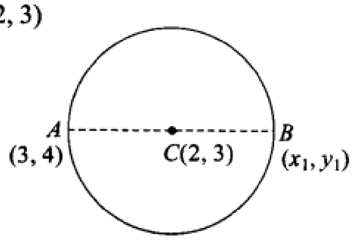
$\therefore \quad 2g = -4$ and $2f = -6$

So, the centre of the circle is $C(-g, -f) \equiv C(2, 3)$

$A(3, 4)$ is one end of the diameter.

Let the other end of the diameter be $B(x_1, y_1)$.

Here, mid point of AB is C.



$\therefore \quad 2 = \frac{3 + x_1}{2}$ and $3 = \frac{4 + y_1}{2}$

$\Rightarrow \quad x_1 = 1$ and $y_1 = 2$

So, the coordinates of other end of the diameter are (1, 2)

Q8. Find the equation of the circle having (1, -2) as its centre and passing through 3x +y= 14, 2x + 5y = 18.

Sol: Given lines are 3x + y = 14 and 2x + 5y= 18.

Solving these equations, we get point of intersection of the lines as $A(4, 2)$.

Now circle with centre $C(1, -2)$ passes through $A(4, 2)$.

$\therefore \quad \text{Radius} = AC = \sqrt{(4-1)^2 + (2+2)^2} = \sqrt{9+16} = 5$

So, equation of the required circle is:

$$(x-1)^2 + (y+2)^2 = 5^2$$

$\Rightarrow \quad x^2 - 2x + 1 + y^2 + 4y + 4 = 25 \Rightarrow x^2 + y^2 - 2x + 4y - 20 = 0$

Q9. If the line y= √3 x + k touches the circle $x^2 + y^2 = 16$, then find the value of

Sol: Given line is $y = \sqrt{3}x + k$ and the circle is $x^2 + y^2 = 16$.

Centre of the circle is (0, 0) and radius is 4.

Since the line $y = \sqrt{3}x + k$ touches the circle, perpendicular distance from (0, 0) to line is equal to the radius of the circle.

$$\therefore \quad \left|\frac{0-0+k}{\sqrt{3+1}}\right| = 4 \Rightarrow \pm\frac{k}{2} = 4 \Rightarrow k = \pm 8$$

Q10. Find the equation of a circle concentric with the circle $x^2 + y^2 - 6x + 12y + 15 = 0$ and has double of its area.

Sol: Given equation of the circle is:

$$x^2 + y^2 - 6x + 12y + 15 = 0$$

or $$(x-3)^2 + (y+6)^2 = (\sqrt{30})^2$$

Hence, centre is (3, –6) and radius is $\sqrt{30}$.

Since the required circle is concentric with above circle, centre of the required circle is (3, –6).

Let its radius be r.

Now it is given that,

Area of the required circle = 2 × Area of the given circle

$$\Rightarrow \quad \pi r^2 = 2 \times \pi(\sqrt{30})^2 \Rightarrow r^2 = 60 \Rightarrow r = \sqrt{60}$$

So, equation of the required circle is:

$$(x-3)^2 + (y+6)^2 = 60$$

$$\Rightarrow \quad x^2 + y^2 - 6x + 12y - 15 = 0$$

Q11. If the latus rectum of an ellipse is equal to half of minor axis, then find its eccentricity.

Sol. Consider the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

It is given that, length of latus rectum = half of minor axis

$$\Rightarrow \quad \frac{2b^2}{a} = b \Rightarrow a = 2b$$

Now, $b^2 = a^2(1-e^2)$

$$\Rightarrow \quad b^2 = 4b^2(1-e^2) \Rightarrow 1-e^2 = \frac{1}{4} \Rightarrow e^2 = \frac{3}{4} \therefore e = \frac{\sqrt{3}}{2}$$

Q12. Given the ellipse with equation $9X^2 + 25y^2 = 225$, find the eccentricity and foci.

Sol. Given equation of ellipse, $9x^2 + 25y^2 = 225$

or $$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

So, $a = 5, b = 3$

Now, $b^2 = a^2(1-e^2)$

$$\Rightarrow \quad 9 = 25(1-e^2) \Rightarrow \frac{9}{25} = 1 - e^2 \Rightarrow e^2 = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\therefore \quad e = \frac{4}{5}$$

Foci ≡ ($\pm ae$, 0) ≡ (±5 × (4/5), 0) ≡ (±4, 0)

Q13. If the eccentricity of an ellipse is 5/8 and the distance between its foci is 10, then find latus rectum of the ellipse.

Sol. Let equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b)$

Given that, eccentricity, $e=\frac{5}{8}$

Now the foci of this ellipse are $(\pm ae, 0)$.

Distance between foci = 10 (Given)

$\therefore \quad 2ae = 10$

$\Rightarrow \quad \frac{5}{8}a = 5 \quad \Rightarrow \quad a = 8$

We know that, $b^2 = a^2(1-e^2)$

$\Rightarrow \quad b^2 = 64\left(1-\frac{25}{64}\right) = 64-25 = 39$

$\therefore$ Length of latus rectum of ellipse $= \frac{2b^2}{a} = 2\times\frac{39}{8} = \frac{39}{4}$

Q14. Find the equation of ellipse whose eccentricity is 2/3, latus rectum is 5 and thecentre is (0, 0).

Sol. Let equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b)$

Given that, $e=\frac{2}{3}$ and latus rectum = 5

$\therefore \quad \frac{2b^2}{a} = 5 \Rightarrow b^2 = \frac{5a}{2}$

We know that, $b^2 = a^2(1-e^2)$

$\Rightarrow \quad \frac{5a}{2} = a^2\left(1-\frac{4}{9}\right) \Rightarrow \frac{5}{2} = \frac{5a}{9} \Rightarrow a = \frac{9}{2}$

$\therefore \quad b^2 = \frac{5\times 9}{2\times 2} = \frac{45}{4}$

So, the required equation of the ellipse is $\frac{4x^2}{81}+\frac{4y^2}{45}=1$.

Q15. Find the distance between the directrices of the ellipse $\frac{x^2}{36} + \frac{y^2}{20} = 1$

Sol. The equation of ellipse is $\frac{x^2}{36}+\frac{y^2}{20}=1$.

$\therefore \quad a = 6, b = 2\sqrt{5}$

We know that, $b^2 = a^2(1-e^2)$

$\Rightarrow \quad 20 = 36(1-e^2) \quad \Rightarrow \quad \frac{5}{9} = 1-e^2 \quad \Rightarrow \quad e^2 = \frac{4}{9}$

$\therefore \quad e = \frac{2}{3}$

Now, directrices are: $x = \pm\frac{a}{e}$

$\therefore$ Distance between direcrtrix $= \frac{2a}{e} = \frac{2\times 6}{2/3} = 18$

Q16. Find the coordinates of a point on the parabola $y^2 = 8x$ whose focal distance is 4.

Sol: Given parabola is $y^2 = 8x$

On comparing this parabola to the $y^2 = 4ax$, we get $a = 2$

Focal distance = distance of any point on parabola from the focus.

Here, focus is $S(2, 0)$.

Let any point on parabola be $P(x_1, y_1)$.

$\Rightarrow$ Focal distance of point $P = SP = \sqrt{(x_1 - 2)^2 + (y_1 - 0)^2}$

$= \sqrt{x_1^2 - 4x_1 + 4 + y_1^2}$

$= \sqrt{x_1^2 - 4x_1 + 4 + 8x_1}$ (as $y_1^2 = 8x_1$)

$= \sqrt{(x_1 + 2)^2} = |x_1 + 2|$

Given that, $|x_1 + 2| = 4$

$\Rightarrow$ $x_1 + 2 = \pm 4$

$\therefore$ $x_1 = 2, -6$

But $x \neq -6$

For $x = 2, y_1^2 = 8 \times 2 = 16$

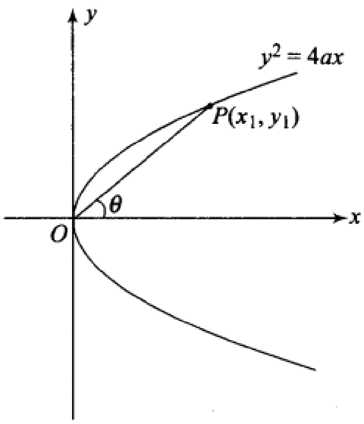
$\therefore$ $y_1 = \pm 4$

So, the points are (2, 4) and (2, –4).

Q17. Find the length of the line-segment joining the vertex of the parabola $y^2 = 4ax$ and a point on the parabola where the line-segment makes an angle 6 to the x-axis.

Sol: Given equation of the parabola is$y^2 = 4ax$.

Let the point on the parabola be $P(x_1,, y_1)$.



From the figure, slope of $OP = \tan\theta = \frac{y_1}{x_1}$ (i)

Also, $y_1^2 = 4ax_1$ (ii)

Now, $OP = \sqrt{x_1^2 + y_1^2} = \sqrt{x_1^2 + \tan^2\theta x_1^2} = \sqrt{x_1^2 \sec^2\theta} = x_1 \sec\theta$

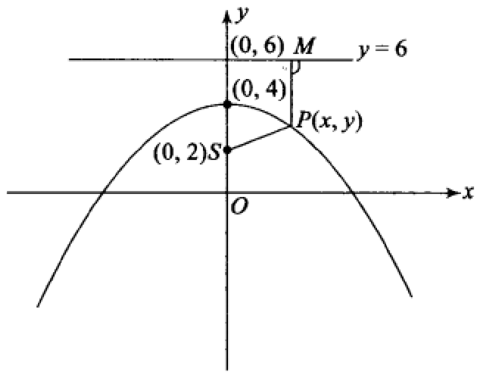
From (i) and (ii), we have

$$\tan^2\theta x_1^2 = 4ax_1 \quad \Rightarrow x_1 = \frac{4a}{\tan^2\theta}$$

$$\therefore \quad OP = \frac{4a\sec\theta}{\tan^2\theta} = \frac{4a\cos\theta}{\sin^2\theta}$$

Q18. If the points (0, 4) and (0, 2) are respectively the vertex and focus of a parabola, then find the equation of the parabola.

Sol.



Given that the vertex of the parabola is $A(0, 4)$ and its focus is $S(0, 2)$.
So, directrix of the parabola is $y = 6$.
Now by definition of the parabola for any point $P(x, y)$ on the parabola,

$$SP = PM$$

$$\Rightarrow \quad \sqrt{(x-0)^2+(y-2)^2} = \left|\frac{0+y-6}{\sqrt{0+1}}\right|$$

$$\Rightarrow \quad x^2 + y^2 - 4y + 4 = y^2 - 12y + 36 \quad \Rightarrow \quad x^2 + 8y = 32$$

Q19. If the line y = mx + 1 is tangent to the parabola $y^2 = 4x$ then find the value of m.

Sol: Given that, liney = mx + 1 is tangent to the parabola $y^2 = 4x$.

Solving line with parabola, we have

$$(mx+1)^2 = 4x$$

$$\Rightarrow \quad m^2x^2 + 2mx + 1 = 4x \quad \Rightarrow \quad m^2x^2 + x(2m-4) + 1 = 0$$

Since the line touches the parabola, above equation must have equal roots.

$$\therefore \quad \text{Discriminant, } D = 0$$

$$\Rightarrow \quad (2m-4)^2 - 4m^2 = 0 \quad \Rightarrow \quad 4m^2 - 16m + 16 - 4m^2 = 0$$

$$\Rightarrow \quad 16m = 16$$

$$\therefore \quad m = 1$$

Q20. If the distance between the foci of a hyperbola is 16 and its eccentricity is √2, then obtain the equation of the hyperbola.

Sol. Let the equation of the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Foci are $(\pm ae, 0)$.

Distance between foci $= 2ae = 16$ (given)

Also, $e = \sqrt{2}$ (Given)

$$\therefore \quad a = 4\sqrt{2}$$

We know that, $b^2 = a^2(e^2 - 1)$

$$\Rightarrow \quad b^2 = (4\sqrt{2})^2[(\sqrt{2})^2 - 1] = 16 \times 2(2-1) = 32$$

So, the equation of hyperbola is: $\frac{x^2}{32} - \frac{y^2}{32} = 1$ or $x^2 - y^2 = 32$

Q21. Find the eccentricity of the hyperbola $9y^2 - 4x^2 = 36$

Sol: We have the hyperbola:$9y^2 - 4x^2 = 36$

or $$\frac{x^2}{9} - \frac{y^2}{4} = -1$$

We know that $a^2 = b^2(e^2 - 1)$

$\therefore \quad 9 = 4(e^2 - 1)$

$$\Rightarrow \quad e = \sqrt{1 + \frac{9}{4}} = \frac{\sqrt{13}}{2}$$

Q22. Find the equation of the hyperbola with eccentricity 3/2 and foci at (±2, 0).

Sol. Let the equation of the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Given that eccentricity, $e = \frac{3}{2}$ and foci $(\pm ae, 0) \equiv (\pm 2, 0)$

$\therefore \quad ae = 2$

$$\Rightarrow \quad a \times \frac{3}{2} = 2 \Rightarrow a = \frac{4}{3}$$

We know that, $b^2 = a^2(e^2 - 1)$

$$\Rightarrow \quad b^2 = \frac{16}{9}\left(\frac{9}{4} - 1\right) = \frac{16}{9} \times \frac{5}{4} = \frac{20}{9}$$

So, the equation of hyperbola is:

$$\frac{x^2}{\frac{16}{9}} - \frac{y^2}{\frac{20}{9}} = 1 \quad \Rightarrow \quad \frac{x^2}{16} - \frac{y^2}{20} = \frac{1}{9}$$

Long Answer Type Questions

Q23. If the lines 2x – 3y = 5 and 3x-4y = 7 are the diameters of a circle of area 154 square units, then obtain the equation of the circle.

Sol: Given that lines 2x – 3y – 5 = 0 and 3x – 4y -1 = 0 are diameters of the circle. Solving these lines we get point of intersection as (1, -1), which is centre of the circle.

Also given that are of the circle is 154 sq. units.

Let the radius of the circle be r.

Then, $\pi r^2 = 154$

$$\Rightarrow \quad \frac{22}{7} \times r^2 = 154 \quad \Rightarrow \quad r^2 = \frac{154 \times 7}{22} = 49$$

$\therefore \quad r = 7$

So, the equation of circle is:

$$(x - 1)^2 + (y + 1)^2 = 49$$

$$\Rightarrow \quad x^2 - 2x + 1 + y^2 + 2y + 1 = 49 \quad \Rightarrow \quad x^2 + y^2 - 2x + 2y = 47$$

Q24. Find the equation of the circle which passes through the points (2, 3) and (4, 5) and the centre lies on the straight line y – 4x + 3 = 0.

Sol: Let the centre of the circle be C(h, k).

Given that the centre lies on the line y – 4x + 3 = 0.

k — 4h + 3 = 0 or k = 4h — 3

So, the centre is C(h, 4h – 3).

Now point $A(2, 3)$ and $B(4, 5)$ lies on the circle.

$\therefore \quad AC^2 = BC^2$

$\Rightarrow \quad (h-2)^2 + (4h-3-3)^2 = (h-4)^2 + (4h-3-5)^2$

$\Rightarrow \quad (h-2)^2 + (4h-6)^2 = (h-4)^2 + (4h-8)^2$

$\Rightarrow \quad -4h + 4 - 48h + 36 = -8h + 16 - 64h + 64 \quad \Rightarrow \quad 20h = 40$

$\Rightarrow \quad h = 2$

So, the centre is $C(2, 5)$.

and radius $= AC = \sqrt{(2-2)^2 + (3-5)^2} = 2$

Therefore, equation of the circle is:

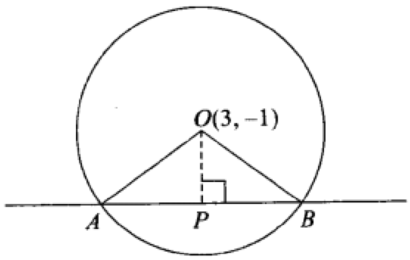
$$(x-2)^2 + (y-5)^2 = 4$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 10y + 25 = 0$$

Q25. Find the equation of a circle whose centre is (3, -1) and which cuts off a chord of length 6 units on the line 2x — 5y+ 18 = 0.

Sol: Given centre of the circle 0(3, -1)

Chord of the circle is AB.



Given that equation of AB is $2x - 5y + 18 = 0$.

Also, $AB = 6$

Perpendicular distance from O to AB is:

$$OP = \left|\frac{2(3) - 5(-1) + 18}{\sqrt{4+25}}\right| = \frac{29}{\sqrt{29}} = \sqrt{29}$$

In ΔOPB, we have

$$OB^2 = OP^2 + PB^2 \quad \Rightarrow \quad OB^2 = 29 + 9 = 38$$

So, the radius of circle is $\sqrt{38}$.

Thus, equation of the circle is:

$$(x-3)^2 + (y+1)^2 = 38$$

$$\Rightarrow \quad x^2 - 6x + 9 + y^2 + 2y + 1 = 38 \quad \Rightarrow \quad x^2 + y^2 - 6x + 2y = 28$$

Q26. Find the equation of a circle of radius 5 which is touching another circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at (5, 5).

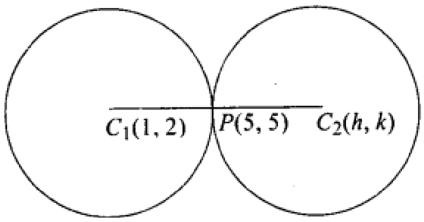
Sol. Given circle is $x^2 + y^2 - 2x - 4y - 20 = 0$

or $\quad (x-1)^2 + (y-2)^2 = 5^2$

Centre of the this circle is $C_1(1, 2)$.

Now, the required circle of radius '5' touches the above circle at $P(5, 5)$.

Let the centre of the required circle be $C_2(h, k)$.



Since the radius of the given circle and the required circle is same, point P is mid-point of C_1C_2.

$\therefore \quad 5 = \frac{1+h}{2} \Rightarrow h = 9$ and $5 = \frac{2+k}{2} \Rightarrow k = 8$

So, the equation of and required circle is:

$$(x-9)^2 + (y-8)^2 = 25$$

$$\Rightarrow \quad x^2 - 18x + 81 + y^2 - 16y + 64 = 25$$

$$\Rightarrow \quad x^2 + y^2 - 18x - 16y + 120 = 0$$

Q27. Find the equation of a circle passing through the point (7, 3) having radius 3 units and whose centre lies on the line y = x -1.

Sol: Given that circle passes through the point A(7, 3) and its radius is 3.

Also, centre of the circle lies on the line $y = x - 1$.

Therefore, centre of the circle is $C(h, h-1)$.

Now, radius of the circle is $AC = 3$ (given)

$$\therefore \quad (h-7)^2 + (h-1-3)^2 = 9$$

$$\Rightarrow \quad 2h^2 - 22h + 56 = 0 \quad \Rightarrow \quad h^2 - 11h + 28 = 0$$

$$\Rightarrow \quad (h-4)(h-7) = 0$$

$$\Rightarrow \quad h = 4, 7$$

Thus, centre of the circle is $C(4, 3)$ or $C(7, 6)$.

Hence, equation of the circle can be:

$$(x-4)^2 + (y-3)^2 = 9 \text{ and } (x-7)^2 + (y-6)^2 = 9$$

$$\Rightarrow \quad x^2 + y^2 - 8x - 6y + 16 = 0 \text{ and } x^2 + y^2 - 14x - 12y + 76 = 0$$

Q28. Find the equation of each of the following parabolas.

(i) Directrix, x = 0, focus at (6, 0)

(ii) Vertex at (0,4), focus at (0, 2)

(iii) Focus at (-1, -2), directrix x − 2y + 3 = 0

Sol: We know that the distance of any point on the parabola from its focus and its directrix is same.

(i) Given that, directrix, x = 0 and focus = (6, 0)

So, for any point P(x, y) on the parabola

Distance of P from directrix = Distance of P from focus => $x^2 = (x - 6)^2 + y^2$

=> $y^2 - 12x + 36 = 0$

(ii) Given that, vertex = (0,4) and focus = (0, 2)

Now distance between the vertex and directrix is same as the distance between the vertex and focus.

Directrix is y − 6 = 0

For any point of P(x, y) on the parabola

Distance of P from directrix = Distance of P from focus

$$\Rightarrow \quad |y-6| = \sqrt{(x-0)^2 + (y-2)^2}$$

$$\Rightarrow \quad y^2 - 12y + 36 = x^2 + y^2 - 4y + 4$$

$$\Rightarrow \quad x^2 = 32 - 8y$$

(iii) Given that, focus at $(-1, -2)$ and directrix $x - 2y + 3 = 0$

So, the equation of parabola is

$$\sqrt{(x+1)^2 + (y+2)^2} = \left|\frac{x - 2y + 3}{\sqrt{1+4}}\right|$$

$$\Rightarrow \quad x^2 + 2x + 1 + y^2 + 4y + 4 = \frac{1}{5}[x^2 + 4y^2 + 9 + 6x - 4xy - 12y]$$

$$\Rightarrow \quad 4x^2 + 4xy + y^2 + 4x + 32y + 16 = 0$$

Q29. Find the equation of the set of all points the sum of whose distances from the points (3, 0) and (9, 0) is 12.

Sol: Let the coordinates of the variable point be (x, y).

Then according to the question,

$$\sqrt{(x-3)^2+y^2}+\sqrt{(x-9)^2+y^2}=12 \Rightarrow \sqrt{(x-3)^2+y^2}=12-\sqrt{(x-9)^2+y^2}$$

Squaring both sides, we get

$$x^2-6x+9+y^2=144+(x^2-18x+81+y^2)-24\sqrt{(x-9)^2+y^2}$$

$$\Rightarrow \quad x-18=-2\sqrt{(x-9)^2+y^2}$$

Again squaring both sides, we get

$$x^2-36x+324=4(x^2-18x+81+y^2)$$

$$\Rightarrow \quad 3x^2+4y^2-36x=0, \text{ which is an ellipse.}$$

Q30. Find the equation of the set of all points whose distance from (0,4) are 2/3 of their distance from the line y = 9.

Sol: Let the point be P(x, y).

According to the question

Distance of P from $(0, 4) = \frac{2}{3} \times$ (Distance from the line $y = 9$)

$$\Rightarrow \quad \sqrt{x^2+(y-4)^2}=\frac{2}{3}\left|\frac{y-9}{\sqrt{1}}\right|$$

$$\Rightarrow \quad x^2+y^2-8y+16=\frac{4}{9}(y^2-18y+81)$$

$$\Rightarrow \quad 9x^2+9y^2-72y+144=4y^2-72y+324$$

$$\Rightarrow \quad 9x^2+5y^2=180, \text{ which is an ellipse.}$$

Q31. Show that the set of all points such that the difference of their distances from (4, 0)and (-4, 0) is always equal to 2 represent a hyperbola.

Sol. Let the points be $P(x, y)$.

According to the question

Distance of P from $(4, 0)$ – Distance of P from $(-4, 0) = 2$

$$\Rightarrow \quad \sqrt{(x+4)^2+y^2}-\sqrt{(x-4)^2+y^2}=2$$

$$\Rightarrow \quad \sqrt{(x+4)^2+y^2}=2+\sqrt{(x-4)^2+y^2}$$

Squaring both sides, we get

$$x^2+8x+16+y^2=4+x^2-8x+16+y^2+4\sqrt{(x-4)^2+y^2}$$

$$\Rightarrow \quad (4x-1)=\sqrt{(x-4)^2+y^2}$$

Again squaring both sides we get

$$16x^2-8x+1=x^2+16-8x+y^2$$

$$\Rightarrow \quad 15x^2-y^2=15 \text{ which is a parabola.}$$

32. Find the equation of the hyperbola with

(a) Vertices $(\pm 5, 0)$, foci $(\pm 7, 0)$

(b) Vertices $(0, \pm 7)$, $e = \frac{7}{3}$

(c) Foci $(0, \pm\sqrt{10})$, passing through $(2, 3)$

Sol. (a) Given that, vertices = $(\pm 5, 0)$, foci = $(\pm 7, 0)$

$\therefore \quad a = 5$ and $ae = 7$

$\Rightarrow \quad e = \frac{7}{5}$

Now $b^2 = a^2(e^2 - 1) = 25\left(\frac{49}{25} - 1\right) = 49 - 25 = 24$

So, the equation of hyperbola is

$$\frac{x^2}{25} - \frac{y^2}{24} = 1$$

(b) Vertices = $(0, \pm 7)$, $e = \frac{4}{3}$

$\therefore \quad b = 7, e = \frac{4}{3}$

Now, $a^2 = b^2(e^2 - 1) = 49\left(\frac{16}{9} - 1\right) = \frac{343}{9}$

So, the equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$$\Rightarrow \quad \frac{x^2}{343/9} - \frac{y^2}{49} = -1 \quad \Rightarrow \quad 9x^2 - 7y^2 + 343 = 0$$

(c) Given that, foci = $(0, \pm\sqrt{10})$

$\therefore \quad be = \sqrt{10}$

Also $a^2 = b^2(e^2 - 1)$

$\Rightarrow \quad a^2 = b^2e^2 - b^2 = 10 - b^2$

$\therefore$ Equation of the hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \quad \text{or} \quad \frac{x^2}{10 - b^2} - \frac{y^2}{b^2} = -1$$

Since, hyperbola passes through the point (2, 3).

$$\therefore \quad \frac{4}{10 - b^2} - \frac{9}{b^2} = -1$$

$\Rightarrow \quad 4b^2 - 9(10 - b^2) = -b^2(10 - b^2)$

$\Rightarrow \quad b^4 - 23b^2 + 90 = 0 \quad \Rightarrow \quad (b^2 - 18)(b^2 - 5) = 0$

$\Rightarrow \quad b^2 = 5$ ($b^2 = 18$ not possible as $a^2 + b^2 = 10$)

$\therefore \quad a^2 = 10 - 5 = 5$

So, the equation of hyperbola is $\frac{x^2}{5} - \frac{y^2}{5} = -1$ or $y^2 - x^2 = 5$

True/False Type Questions

Q33. The line x + 3y = 0 is a diameter of the circle $x^2 + y^2 + 6x + 2y = 0$.

Sol: False

Given equation of the circle is $x^2 + y^2 + 6x + 2y = 0$

Centre = (-3, -1)

Clearly, it does not lie on the line x + 3y = 0 as -3 + 3(-l) = -6.

So, this line is not diameter of the circle.

Q34. The shortest distance from the point (2, -7) to the circle x +y^2 – 1 4jc – lOy- 151 = 0 is equal to 5.

Sol: False

Given circle is $x^2 + y^2 - 14x - 10y - 151 = 0$

$\therefore$ Centre $\equiv C(7, 5)$

And Radius $= \sqrt{49 + 25 + 151} = \sqrt{225} = 15$

Now distance between the point $P(2, -7)$ and centre

$$= \sqrt{(2-7)^2 + (-7-5)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

$\therefore$ Shortest distance of point P from the circle $= |13 - 15| = 2$

Q35. If the line lx + my = 1 is a tangent to the circle $x^2 + y^2 = a^2$, then the point (1, m) lies on a circle.

Sol. True

Given circle is $x^2 + y^2 = a^2$

$\therefore$ Radius $= a$ and centre $\equiv (0, 0)$

Now given that line $lx + my - 1 = 0$ is tangent to the circle

$\therefore$ Distance of (0, 0) from the line $lx + my - 1 = 0$ is equal to radius 'a'.

$$\Rightarrow \quad \frac{|0 + 0 - 1|}{\sqrt{l^2 + m}} = a \quad \Rightarrow \quad l^2 + m^2 = \frac{1}{a^2}$$

Thus, locus of (l, m) is $x^2 + y^2 = \frac{1}{a^2}$, which is circle.

Q36. The point (1,2) lies inside the circle $x^2 + y^2 - 2x + 6y + 1 = 0$.

Sol: False

Given circle is $x^2 + y^2 - 2x + 6y + 1 = 0$.

or $(x-1)^2 + (y+3)^2 = 3^2$

Centre is $C(1, -3)$ and radius is 3.

Distance of point $P(1, 2)$ from centre is 5.

Thus, $CP >$ radius

So, point P lies outside the circle.

Q37. The line lx+ my + n = 0 will touch the parabola^^{2} = 4 ax if ln = am^2.

Sol: True

Give line $lx + my + n = 0$ and parabola $y^2 = 4ax$

Solving line and parabola for their point of intersection, we get

$$\frac{l}{4a}y^2 + my + n = 0$$

Since line touches the parabola, above equation must have equal roots.

$\therefore$ Discriminant, $D = 0$

$$\therefore \quad m^2 - 4\left(\frac{l}{4a}\right)n = 0 \quad \Rightarrow \quad am^2 = nl$$

38. If P is a point on the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ whose foci are S and S', then $PS+PS'=8$.

Sol. False

We have equation of the ellipse is $\frac{x}{16}+\frac{y^2}{25}=1$

From the definition of the ellipse, we know that sum of the distances of any point P on the ellipse from the two foci is equal to the length of the major axis.

Here major axis $= 2b = 2 \times 5 = 10$

S and S' are foci, then $SP + S'P = 10$

39. The line $2x + 3y = 12$ touches the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=2$ at the point (3, 2).

Sol. True

Given line is $2x + 3y = 12$ and the ellipse is $4x^2 + 9y^2 = 72$.

Solving line and ellipse, we get

$$(12-3y)^2 + 9y^2 = 72$$

$\Rightarrow \quad (4-y)^2 + y^2 = 8 \quad \Rightarrow \quad 2y^2 - 8y + 8 = 0 \quad \Rightarrow \quad y^2 - 4y + 4 = 0$

$\Rightarrow \quad (y-2)^2 = 0 \quad \Rightarrow \quad y = 2$

$\Rightarrow \quad 2x = 12 - 3(2)$ (from the equation of line)

$\Rightarrow \quad x = 3$

So, point of contact is (3, 2).

40. The locus of the point of intersection of lines $\sqrt{3}x - y - 4\sqrt{3}k = 0$ and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$ for different value of k is a hyperbola whose eccentricity is 2.

Sol. True

Given equation of lines are:

$$\sqrt{3}x - y - 4\sqrt{3}k = 0 \quad \text{(i)}$$

and $$\sqrt{3}kx + ky - 4\sqrt{3} = 0 \quad \text{(ii)}$$

From Eq. (i), $k = \frac{\sqrt{3}x - y}{4\sqrt{3}}$

From Eq. (ii), $k = \frac{4\sqrt{3}}{\sqrt{3}x + y}$

Equating the values of k, we get

$$\frac{\sqrt{3}x - y}{4\sqrt{3}} = \frac{4\sqrt{3}}{\sqrt{3}x + y}$$

$\Rightarrow \quad 3x^2 - y^2 = 48$

$\Rightarrow \quad \frac{x^2}{16} - \frac{y^2}{48} = 1$, which is equation of hyperbola

$\therefore \quad a^2 = 16$ and $b^2 = 48$

$\Rightarrow \quad e^2 = 1 + \frac{48}{16} = 1 + 3 = 4$

$\Rightarrow \quad e = 2$

Fill in the Blanks Type Questions

Q41. The equation of the circle having centre at (3, -4) and touching the line 5x + 12y- 12 = 0 is ______.

Sol. The perpendicular distance from centre (3, –4) to the given line is,

$$r=\frac{|5(3)+12(-4)-12|}{\sqrt{25+144}}=\frac{45}{13}, \text{ which is radius of the circle}$$

So, the required equation of the circle is $(x-3)^2+(y+4)^2=\left(\frac{45}{13}\right)^2$.

Q42. The equation of the circle circumscribing the triangle whose sides are the lines y = x + 2, 3y = 4x, 2y = 3x is ______.

Given equation of line are:

$$y=x+2 \quad \text{(i)}$$
$$3y=4x \quad \text{(ii)}$$
$$2y=3x \quad \text{(iii)}$$

Solving these lines, we get points of intersection $A(6, 8)$, $B(4, 6)$ and $C(0, 0)$.

Let the equation of circle circumscribing the given triangle be

$$x^2+y^2+2gx+2fy+c=0$$

Since the points $A(6, 9)$, $B(4, 6)$ and $C(0, 0)$ lie on this circle, we have

$$36+64+12g+16f+c=0$$
$$\Rightarrow \quad 12g+16f+c=-100 \quad \text{(iv)}$$

Also, $16+36+8g+12f+c=0$

$$\Rightarrow \quad 8g+12f+c=-52 \quad \text{(v)}$$

And $c=0$ (vi)

Putting $c=0$ in Eqs. (iv) and (v), we get

$$3g+4f=-25$$

and $2g+3f=-13$

On solving these, we get $g=-23$ and $f=11$.

So, the equation of circle is:

$$x^2+y^2-46x+22y+0=0$$
$$\Rightarrow \quad x^2+y^2-46x+22y=0$$

Q43. An ellipse is described by using an endless string which is passed over two pins. If the axes are 6 cm and 4 cm, the length of the string and distance between the pins are _____ .

Sol. Let equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

According to the question, $a=3$ and $b=2$

Now, $b^2=a^2(1-e^2)$

$$\therefore \quad e^2=1-\frac{b^2}{a^2}=1-\frac{4}{9}=\frac{5}{9}$$

$$\therefore \quad e=\frac{\sqrt{5}}{3}$$

From the definition of the ellipse, for any point P on the ellipse, we have

$$SP+S'P=2a, \quad \text{where } S \text{ and } S' \text{ are foci.}$$

$\therefore$ Length of the endless string $=SP+S'P+SS'$

$$=2a+2ae=2(3)+2(3)\times\frac{\sqrt{5}}{3}=6+2\sqrt{5}$$

Q44. The equation of the ellipse having foci (0,1), (0, -1) and minor axis of length 1 is ___ .

Sol. Given that, foci of the ellipse are $(0, \pm be) \equiv (0, \pm 1)$

$\therefore \quad be = 1$

Length of minor axis, $2a = 1 \Rightarrow a = \frac{1}{2}$

Now $a^2 = b^2(1 - e^2)$

$\Rightarrow \quad \frac{1}{4} = b^2 - b^2e^2 = b^2 - 1 \Rightarrow b^2 = \frac{5}{4}$

So, the equation of ellipse is $\frac{x^2}{1/4} + \frac{y^2}{5/4} = 1$ or $4x^2 + \frac{4y^2}{5} = 1$

Q45. The equation of the parabola having focus at (-1, -2) and the directrix $x - 2y + 3 = 0$ is______ .

Sol: Given that, focus at S(-l, -2) and directrix is $x - 2y + 3 = 0$

Let any point on the parabola be $P(x, y)$.

$\therefore$ Length of perpendicular from S on the directrix = SP

$$\Rightarrow \quad \frac{(x - 2y + 3)^2}{5} = (x + 1)^2 + (y + 2)^2$$

$$\Rightarrow \quad 5[x^2 + 2x + 1 + y^2 + 4y + 4] = x^2 + 4y^2 + 9 - 4xy - 12y + 6x$$

$$\Rightarrow \quad 4x^2 + y^2 + 4x + 32y + 16 = 0$$

Q46. The equation of the hyperbola with vertices at (0, ±6) and eccentricity 5/3 ________ and its foci are _____ .

Sol. Let the equation of the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$.

Vertices are $(0, \pm b) \equiv (0, \pm 6)$

$\therefore \quad b = 6$

Also, $e = \frac{5}{3}$

Now, $a^2 = b^2(e^2 - 1)$

$$\Rightarrow \quad a^2 = 36\left(\frac{25}{9} - 1\right) = 64$$

So, the equation of hyperbola is:

$$\frac{x^2}{64} - \frac{y^2}{36} = -1$$

So, foci = $(0, \pm be) \equiv \left(0, \pm\frac{5}{3} \times 6\right) = (0, \pm 10)$

Objective Type Questions

Q47. The area of the circle centred at (1,2) and passing through (4, 6) is

(a) 5π (b) 10π (c) 25π (d) none of these

Sol. **(c)** Centre of the circle is $C(1, 2)$.

Also, circle passes through the point $P(4, 6)$.

$$\text{Radius} = CP = \sqrt{(4 - 1)^2 + (6 - 2)^2} = \sqrt{9 + 16} = 5$$

$\therefore \quad$ Area of the circle $= \pi r^2 = 25\pi$ sq. units.

Q48. Equation of a circle which passes through (3, 6) and touches the axes is

(a) $x^2 + y^2 + 6x + 6y + 3 = 0$ (b) $x^2 + y^2 - 6x - 6y - 9 = 0$
(c) $x^2 + y^2 - 6x - 6y + 9 = 0$ (d) none of these

Sol. **(c)** Given that the circle touches both axes.

Therefore, equation of the circle is: $(x - a)^2 + (y - a)^2 = a^2$

Circle passes through the point (3, 6).

$\therefore \quad (3 - a)^2 + (6 - a)^2 = a^2$

$\Rightarrow \quad a^2 - 18a + 45 = 0 \quad \Rightarrow \quad (a - 3)(a - 15) = 0$

$\therefore \quad a = 3, a = 15$

For $a = 3$, the equation of circle is:

$(x - 3)^2 + (y - 3)^2 = 9$

$\Rightarrow \quad x^2 + y^2 - 6x - 6y + 9 = 0$

Q49. Equation of the circle with centre on the j-axis and passing through the origin and the point (2, 3) is

(a) $x^2 + y^2 + 13y = 0$ (b) $3x^2 + 3y^2 + 13x + 3 = 0$
(c) $6x^2 + 6y^2 - 13x = 0$ (d) $x^2 + y^2 + 13x + 3 = 0$

Sol. **(None)** Centre of the circle lies on the y-axis.

So, let the centre be $C(0, k)$.

Circle passes through $O(0, 0)$ and $A(2, 3)$.

$\therefore \quad OC^2 = AC^2$

$\Rightarrow \quad k^2 = (2 - 0)^2 + (3 - k)^2 \quad \Rightarrow k = 13/6$

$\therefore$ Centre $\equiv$ (0, 13/6) and radius = 13/6

So, equation of the required circle is:

$(x - 0)^2 + (y - 13/6)^2 = (13/6)^2$

$\Rightarrow \quad 3x^2 + 3y^2 - 13y = 0$

Q50. The equation of a circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length 3a is

(a) $x^2 + y^2 = 9a^2$ (b) $x^2 + y^2 = 16a^2$
(c) $x^2 + y^2 = 4a^2$ (d) $x^2 + y^2 = a^2$

Sol. **(c)** Given that, length of the median = $3a$

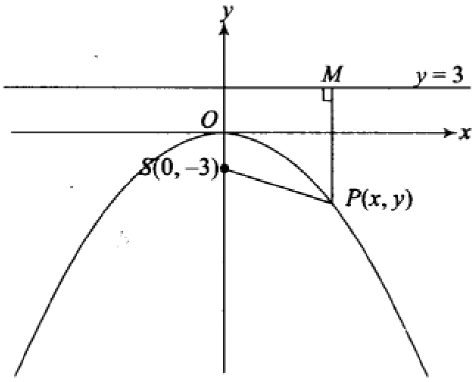
Now, Radius of circle = $\frac{2}{3} \times$ Length of median = $\frac{2}{3} \times 3a = 2a$

So, the equation of the circle is $x^2 + y^2 = 4a^2$.

Q51. If the focus of a parabola is (0, -3) and its directrix is y = 3, then its equation is

(a) $x^2 = -12y$ (b) $x^2 = 12y$ (c) $y^2 = -12x$ (d) $y^2 = 12x$

Sol. (a)



Given that, focus of parabola is at $S(0, -3)$ and equation of directrix is $y = 3$.

For any point $P(x, y)$ on the parabola, we have

$$SP = PM$$

$$\Rightarrow \quad \sqrt{(x-0)^2 + (y+3)^2} = |y-3| \quad \Rightarrow \quad x^2 + y^2 + 6y + 9 = y^2 - 6y + 9$$

$$\Rightarrow \quad x^2 = -12y$$

Q52. If the parabola $y^2 = 4ax$ passes through the point (3, 2), then the length of its latus rectum is

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{1}{3}$ (d) 4

Sol. **(b)** Parabola $y^2 = 4ax$, passes through the point (3, 2).

$$\therefore \quad 4 = 4a(3)$$

$$\therefore \quad \text{Length of latus rectum} = 4a = \frac{4}{3}$$

Q53. If the vertex of the parabola is the point (-3, 0) and the directrix is the line x + 5 = 0, then its equation is

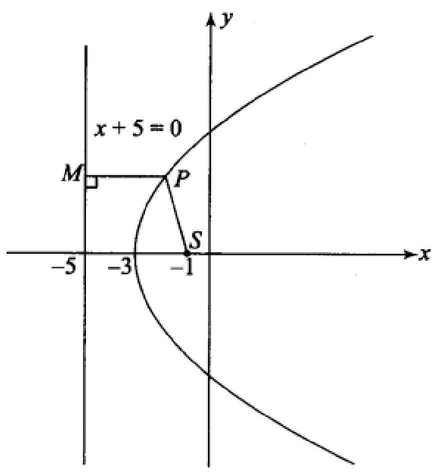
(a) $y^2 = 8(x + 3)$

(b) $x^2 = 8(y + 3)$

(c) $y^2 = -8(x + 3)$

(d) $y^2 = 8(x + 5)$

Sol. **(a)** Given that vertex $\equiv (-3, 0)$ and directrix, $x + 5 = 0$



So, focus $\equiv S(-1, 0)$

For any point of parabola $P(x, y)$, we have

$$SP = PM$$

$$\Rightarrow \quad \sqrt{(x+1)^2 + y^2} = |x+5| \quad \Rightarrow \quad x^2 + 2x + 1 + y^2 = x^2 + 10x + 25$$

$$\Rightarrow \quad y^2 = 8x + 24 \quad \Rightarrow \quad y^2 = 8(x+3)$$

Q54. The equation of the ellipse whose focus is (1, -1), the directrix the line x-y-3 = 0 and eccentricity 1/2 is

(a) $7x^2 + 2xy + 7y^2 - 10x + 10y + 7 = 0$
(b) $7x^2 + 2xy + 7y^2 + 7 = 0$
(c) $7x^2 + 2xy + 7y^2 + 10x - 10y - 7 = 0$
(d) none of these

Sol. **(a)** Given that, focus of the ellipse is $S(1, -1)$ and the equation of directrix is $x - y - 3 = 0$

Also, $e = \frac{1}{2}$

From definition of ellipse, for any point $P(x, y)$ on the ellipse, we have $SP = ePM$, where M is foot of the perpendicular from point P to the directrix.

$\therefore \quad \sqrt{(x-1)^2 + (y+1)^2} = \frac{1}{2}\frac{|x-y-3|}{\sqrt{2}}$

$\Rightarrow \quad 8x^2 - 16x + 16 + 8y^2 + 16y = x^2 + y^2 + 9 - 2xy + 6y - 6x$

$\Rightarrow \quad 7x^2 + 7y^2 + 2xy - 10x + 10y + 7 = 0$

Q55. The length of the latus rectum of the ellipse $3x^2 + y^2 = 12$ is

(a) 4

(b) 3

(c) 8

(d) 4/√3

Sol. **(d)** Given ellipse is:

$3x^2 + y^2 = 12$

$\Rightarrow \quad \frac{x^2}{4} + \frac{y^2}{12} = 1$

$\therefore \quad a^2 = 4 \Rightarrow a = 2$

and $\quad b^2 = 12 \Rightarrow b = 2\sqrt{3}$

Since $b > a$, length of latus rectum $= \frac{2a^2}{b} = \frac{2 \times 4}{2\sqrt{3}} = \frac{4}{\sqrt{3}}$

56. If e is the eccentricity of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (where, $a < b$), then

(a) $b^2 = a^2(1-e^2)$ (b) $a^2 = b^2(1-e^2)$

(c) $a^2 = b^2(e^2-1)$ (d) $b^2 = a^2(e^2-1)$

Sol. **(b)** Given that, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a < b$

We know that, $a^2 = b^2(1-e^2)$

57. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half of the distance between the foci is

(a) $\frac{4}{3}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) none of these

Sol. **(c)** Let the equation of the hyperbola be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

Length of latus rectum = 8

$\therefore \quad \frac{2b^2}{a}=8 \Rightarrow b^2 = 4a$ (i)

Conjugate axis = half of the distance between the foci

$\therefore \quad 2b = ae$ (ii)

Now, $\quad b^2 = a^2(e^2-1)$ (iii)

From Eqs. (i) and (iii), we get

$$\frac{a^2e^2}{4} = a^2(e^2-1)$$

$$\Rightarrow \quad e^2 = 4e^2 - 4 \Rightarrow e^2 = \frac{4}{3} \Rightarrow e = \frac{2}{\sqrt{3}}$$