[TOPIC 1] Electric Conduction, Ohm's Law and Resistance

1.1 Electric Current

The directed rate of flow of electric charge through any cross-section of a conductor is known as electric current.

If ΔQ charge flows in time Δt, then current at any time t is $I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$

Also $\boxed{I = \frac{q}{t} = \frac{ne}{t}}$ $[\because q = ne]$

where, n = number of charged particles constituting the electric current.

The direction of the current is in the flow of positive charge and opposite to the direction of flow of negative charge and it is represented by A.

$$1 \text{ A} = \frac{1 \text{ coulomb (C)}}{1 \text{ second (s)}} = 1 \text{C/s}$$

The current is a scalar quantity. The SI unit of current is ampere.

Current Density

The current density at a point in a conductor is the ratio of the current at that point to the area of cross-section of the conductor at that point.

If a current I is distributed uniformly over the cross-section A of a conductor, then the current density at that point is

$$\boxed{J = I/A.}$$

The current density is a vector quantity. Its SI unit is ampere/metre^2 (A/m^2).

Electric Current in Conductors

There are following case of electric current in conductors

(i) In Case of a Metallic Conductor

All metals (i.e. Cu, Fe, Ag, etc.) are good conductors of electricity. Free electrons are the cause of conductance of electricity in them.

(ii) In Case of a Solid Conductor

In case of solid conductor (i.e. Graphite, etc.) atoms are tightly bound to each other but there are large number of free electrons in them to conduct.

(iii) In Case of a Liquid Conductor

In case of a liquid conductor like electrolytic solutions, there are positive and negative charged ions which can move on applying electric field to conduct.

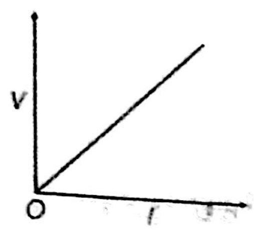
1.2 Ohm's Law

At constant temperature, the potential difference V across the ends of a given metallic wire (conductor) in an electric circuit is directly proportional to the current flowing through it.

$$V \propto I \quad \text{or} \quad V = IR$$

where, R = resistance of conductor.

The variation of current w.r.t. applied potential difference is shown with the help of following graph.



No effect of V and I on R because as V increase, I increase but R remains the same.

Note Those conductors which obey Ohm's law are called **ohmic conductors** while others are called **non-ohmic** conductors.

Resistance of a Conductor

It is defined as the ratio of potential difference applied across the ends of the conductor to the current flowing through it.

Mathematically, $R = V/I$

Its SI unit is ohm (Ω).

Resistance can also be written as, $R = \rho \frac{L}{A}$

where, L = length of the conductor, A = area of cross-section and ρ = constant, known as resistivity of the material. It depends upon the nature of the material and temperature of the conductor.

Effect of Temperature on Resistance

The resistance of a wire and its temperature are related as

$$R_2 = R_1[1 + \alpha(t_2 - t_1)]$$

where, R_1 and R_2 are the resistances at t_1°C and t_2°C, respectively and α is temperature coefficient of resistance averaged over the temperature range t_1°C to t_2°C is given by

$$\alpha = R_2 - R_1 / R_1 (t_2 - t_1)$$

Drift Velocity

It is defined as the average velocity with which the free electrons move towards the positive end of a conductor under the influence of an external electric field applied across the conductor.

Thus, $$\mathbf{v}_d = \frac{-e\mathbf{E}}{m}\tau$$

where, τ = average relaxation time of free electron, $\mathbf{E}$ = electric field, m = mass of the electron and e = electronic charge.

The drift velocity of electron is of the order of 10^{-4} ms^{-1}.

Mobility

The ratio of the drift velocity of the electrons to the applied electric field is known as mobility.

It is expressed as,

$$\mu = v_d / E = e\tau / m$$

Its SI unit is $m^2s^{-1}V^{-1}$.

1.3 Resistivity of Various Material

Resistivity of various material is defined as the resistance of unit cube of that material,

$$\rho = m / ne^2\tau$$

where, n = number of free electron per unit volume.

It depends upon the following factors

(i) $\rho = 1/n$ (ii) $\rho = 1/\tau$

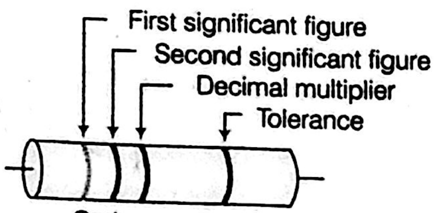
Colour Code of Carbon Resistance

The colour code of carbon resistor remains in the form of coaxial rings.

The first band represents the first significant figure, second band represents second significant figure and third band represents multiplier (i.e. power of ten). The fourth band represents tolerance.

Black	Brown	Red	Orange	Yellow
B	B	R	O	Y
0	1	2	3	4

	Green	Blue		Violet	Grey	White
of	Great	Britain	had	Very	Good	Wife
	5	6		7	8	9



Carbon resistor

Tolerance Unit

Gold	5%
Silver	10%
No colour	20%

1.4 Conductance and Conductivity

Conductance

It is defined as the reciprocal of resistance of conductor.

$$G = 1/R$$

Its SI unit is mho (Ω^{-1}) or siemen (S). The dimensional formula of conductance is $[M^{-1}L^{-2}T^{-3}A^{2}]$.

Conductivity

It is defined as the reciprocal of resistivity of a conductor. It is expressed as, $\sigma = 1/\rho$.

Its SI unit is mho per metre $(\Omega\text{-m})^{-1}$.

Relation between J, σ and E

The relation between the current flowing through the conductor and drift velocity of electron is given by

$$I = nAev_d$$

From this, we can have,

$$J = \sigma E$$

It is **microscopic form of Ohm's law.**

Superconductivity

The resistivity of certain metals or alloys drops to zero when they are cooled below a certain temperature. This is called superconductivity. It was observed by Prof. Kamerlingh in 1911.

Some Important Units

(i) Resistance - Ohm (Ω)
(ii) Resistivity - Ohm-metre (Ω-m)
(iii) Conductance ($1/R$) - Mho or Ω^{-1} or Siemen (S)
(iv) Current density - A/m^2

NOTE If a conductor is stretched or compresses to n times of original length, then

$$l' = nl \Rightarrow R' = n^2R$$

where, R' = new resistance and R = original resistance.

1.5 Combinations of Resistors

There are two types of combinations of resistors

Series Combination

In this combination, different resistors are connected end to end.

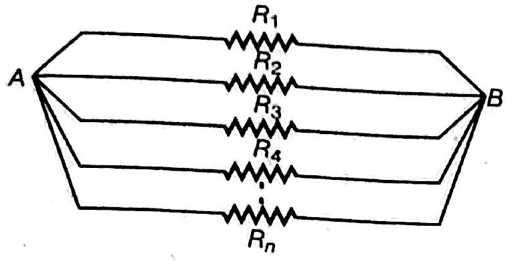
Equivalent resistance can be obtained by the formula,

$$R_{eq} = R_1 + R_2 + \ldots + R_n$$

NOTE The total resistance in the series combination is more than the greatest resistance in the circuit.

Parallel Combination

In this combination, first end of all the resistors are connected to one point and last end of all the resistors are connected to other point.



Equivalent resistance can be obtained by the formula,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots \frac{1}{R_n}$$

NOTE The total resistance in parallel combination is less than the least resistance of the circuit.

- If n identical resistors each of resistance r are connected in

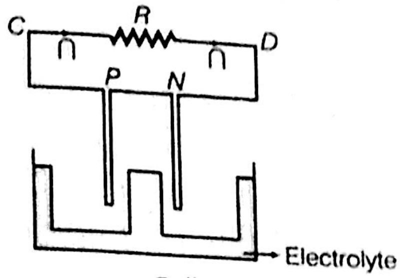
  (i) series combination, $R_{eq} = nr$

  (ii) parallel combination, $R_{eq} = \frac{r}{n}$

[TOPIC 2] Cell, Combination of Cells and Kirchhoff's Laws

2.1 Cell

A device which is used to maintain a steady current in an electric circuit is called cell or electrolytic cell. It has two electrodes positive (P) and negative (N) as shown in figure above.



Cell

Terms Related to Cell

There are following terms related to cell

EMF of a Cell

It is the maximum potential difference between two terminals of circuit, when circuit is open. EMF of the cell , $E = W/q$.

Internal Resistance (r)

Internal resistance of a cell is defined as the resistance offered by the electrolyte of the cell to the flow of current through it. It is denoted by r. Its SI unit is ohm (Ω).

Terminal Potential Difference (V)

The maximum potential difference between two terminals of circuit when the circuit is closed, is known as terminal voltage or terminal potential difference (V) of cell.

Relation between r, R, E and V

$$r = R(E/V - 1) \qquad \dots(i)$$

where, r = internal resistance,
R = external resistance,
E = emf of cell

and V = terminal voltage of cell.

Also, $V = E - Ir$ ...(ii)

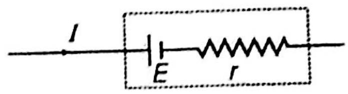
and $V = (E/R + r)R$

or $$V = \frac{E}{(1 + r/R)} \qquad \dots(iii)$$

The terminal voltage increases with the increase of external resistance R.

Charging of a Cell

During charging of the cell, $V = E + Ir$



So, $V < E$, when current is drawn from the cell, i.e. discharging. and $V > E$ when charging of cell takes place.

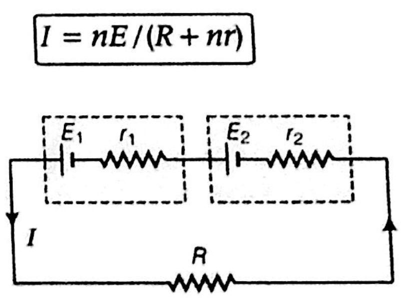
2.2 Combination of Cells

There are following types of combination of cells

Series Combination

In series combination, current is given by

$$\boxed{I = nE/(R + nr)}$$

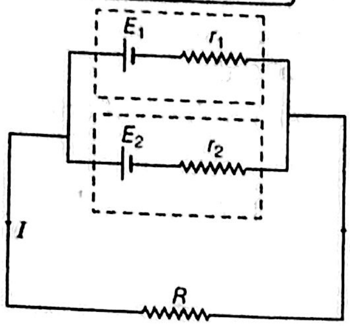


where, $r = r_1 + r_2$

Parallel Combination

In parallel combination, current is given by

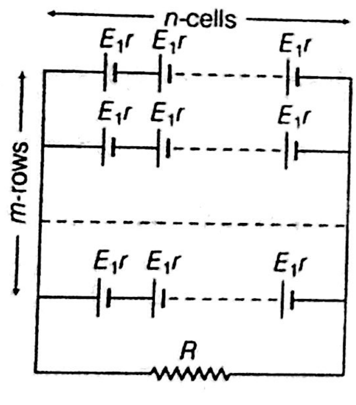
$$I = \frac{mE}{(r + mR)}$$



Mixed Grouping

It consists of m-rows in parallel combination such that each row contains n-cells of each of emf E and internal resistance r, then current in the circuit is given by

$$I = \frac{mnE}{mR + nr}$$



and maximum current is drawn from the battery, when external resistance matches with net internal resistance, i.e.

$$R = \frac{nr}{m} \text{ and } I_{max} = \frac{nE}{2(nr/m)} = \frac{mnE}{2nr} = \frac{mE}{2r}$$

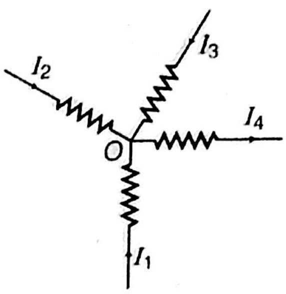
2.3 Kirchhoff's Laws

Kirchhoff has given two rules based on conservation of electric charge and of energy, these are known as Kirchhoff's laws.

There are two laws

Kirchhoff's First Law (Junction Rule)

The algebraic sum of electric currents at any junction of electric circuit is equal to zero, i.e. the sum of current entering into a junction is equal to the sum of current leaving the junction.



$$\Sigma I = 0, \; I_4 = I_1 + I_2 + I_3$$

Junction law supports law of conservation of charge because this is a point in a circuit which cannot act as a source or sink of charge(s).

NOTE The current flowing towards the junction of conductors is considered as positive and the current flowing away from the junction is taken as negative.

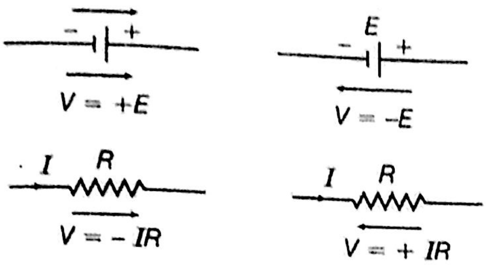
Kirchhoff's Second Law (Loop Rule)

In any closed loop of electrical circuit, the algebraic sum of emfs of cell and the product of currents and resistance is always equal to zero, i.e.

$$\Sigma \Delta V = 0 \text{ or } \Sigma E = \Sigma I R$$

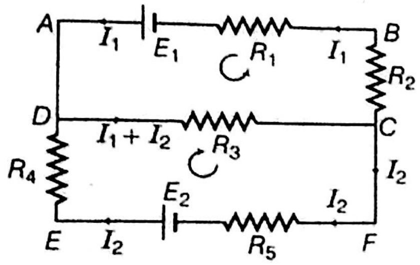
Kirchhoff's second law supports the law of conservation of energy, because the net change in the energy of a charge, after completing a closed path must be zero.

NOTE Sign convention for Kirchhoff's second law.



An Application based on Kirchhoff's Law

Let us consider a circuit diagram



Current distribution is shown in given circuit using **Kirchhoff's first law**.

Now, apply loop rule for the mesh *DCBAD*,

$$E_1 - (I_1 + I_2)R_3 - I_1(R_1 + R_2) = 0$$

$$\Rightarrow -I_1(R_1 + R_2 + R_3) - I_2R_3 + E_1 = 0 \quad \text{...(i)}$$

Similarly, applying loop rule for the mesh *CDEFC*,

$$E_2 - I_2R_4 - (I_1 + I_2)R_3 - I_2R_5 = 0$$

$$\Rightarrow I_1(R_3) + I_2(R_3 + R_4 + R_5) = E_2$$

$$\Rightarrow I_1(R_3) + I_2(R_3 + R_4 + R_5) - E_2 = 0 \quad \text{...(ii)}$$

By Eqs. (i) and (ii), we can calculate I_1 and I_2.

[TOPIC 3] Electrical Devices and Heating Effects of Current

3.1 Potentiometer

It is an electrical device which can

(i) measure the potential difference with greater accuracy.

(ii) measure the emf of a cell.

(iii) compare the emfs of two cells.

(iv) be used to determine the internal resistance of a primary cell.

Working Principle

The potentiometer works on the principle that potential difference across any two points of uniform current carrying conductor is directly proportional to the length between the two points, i.e. $V \propto l$

Potential Gradient

It is the potential drop per unit length of wire of potentiometer, i.e.

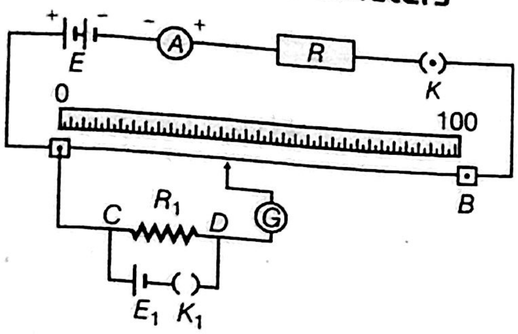
$$\boxed{K = V/l}$$

where V and l are potential difference applied by driving cell and length of wire of potentiometer, respectively.

Application of Potentiometer

There are following applications of potentiometer

(i) Measurement of Potential Difference Using Potentiometers



If r is the resistance of potentiometer wire of length L, then current through potentiometer wire is $I = \dfrac{E}{R+r}$.

Potential drop across potentiometer wire $= Ir = \left(\dfrac{E}{R+r}\right)r$

Potential gradient of potentiometer wire,

$$K = (E/R+r)\, r/L$$

$$\Rightarrow \boxed{V = Kl = (E/R+r)r/L\,(l)}$$

The potentiometer is a better device to measure potential difference than a voltmeter as null point method is used and hence it can measure even the emf of cell but voltmeter cannot. It measures potential difference with greater accuracy.

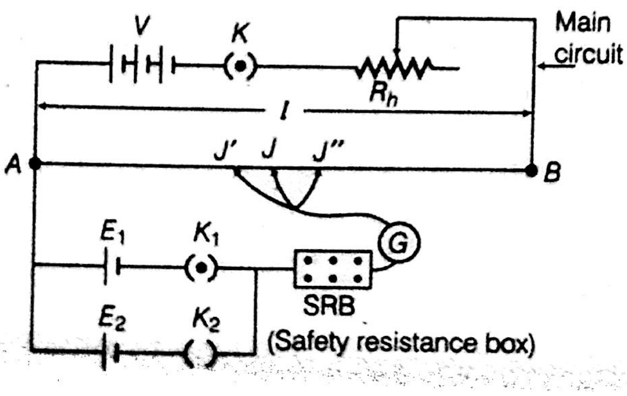
(ii) Comparing EMF of Two Cells

The emfs of two primary cells can be compared using potentiometer as $E_1/E_2 = l_1/l_2$

where, l_1 and l_2 are the balancing lengths corresponding to cells of emfs E_1 and E_2, respectively.

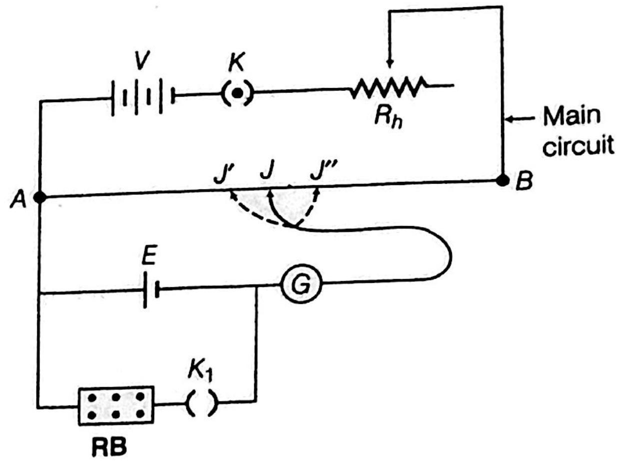
Circuit diagram

For comparing the emfs of two primary cells.



(iii) To Measure Internal Resistance of a Cell

The internal resistance can be determined using potentiometer.



If l_1, l_2 are the balancing lengths when key K_1 is opened and closed respectively and resistance R is applied in Resistance Box (RB), then internal resistance of primary cell of emfs is given by

$$\boxed{r = R\left(\frac{l_1}{l_2} - 1\right)}$$

The potentiometer works only when

(i) the terminal voltage applied by driving cell is greater than the emf of primary cell.

(ii) the positive terminals of driving cell and primary cell are connected at the zero end of potentiometer wire.

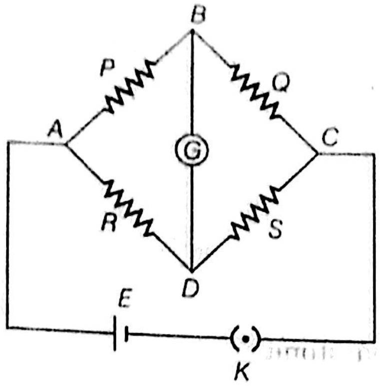
Sensitivity of a Potentiometer

It refers to the capability of measuring very small potential difference and exhibit change in balancing length even on very small change in potential difference.

The sensitivity of potentiometer can be increased by increasing the length of wire of potentiometer and hence decreasing the value of potential gradient.

3.2 Wheatstone Bridge

It is an arrangement of four resistances connected to form the arms of quadrilateral $ABCD$. A battery with key and galvanometer are connected along its two diagonals, respectively.



$$P/Q = R/S$$

where, P, Q = ratio arms,
R = known resistance
and S = unknown resistance.

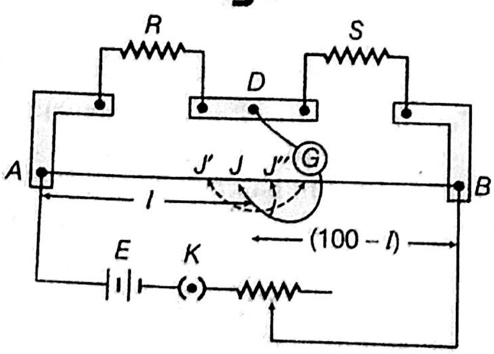
The bridge is said to be balanced, when

(i) $V_B = V_D$

(ii) There is no flow of current through galvanometer.

The Wheatstone bridge is said to be sensitive, if it gives ample deflection in the galvanometer even on slight change of resistance. For sensitivity of galvanometer, the magnitude of four resistances P, Q, R, S should be of same order.

3.3 Meter Bridge



It is an electrical device used to determine the resistance and hence specific resistance of material of given wire/conductor.

It is based on the principle of balanced Wheatstone bridge.

For a uniform wire, resistance of wire ∝ length of conductor

At balanced situation of bridge,

$$\frac{P}{Q} = \frac{R}{S} \text{ or } \frac{l}{100 - l} = \frac{R}{S}$$

$$\Rightarrow \quad S = \left(\frac{100 - l}{l}\right) \times R$$

where, l is the balancing length.

NOTE Meter bridge is also known as **slide wire bridge**.

3.4 Joule's Law of Heating

The amount of heat produced in a current carrying conductor, H is given by

$$H \propto I^2 Rt$$

or $$H = I^2 Rt$$

$$H = VIt \quad (\because V = IR)$$

or $$H = \frac{V^2}{R} t$$

where, R = resistance of conductor
I = amount of current flowing
and t = time.

Electric Power

The rate of consumption of electrical energy or production of heat energy is known as electric power. SI unit of electric power is watt (W).

Electric power, $P = I^2 R = V^2/R \quad [\because V = IR]$

$$P = VI$$

- Unit in the commercial unit of electric energy, 1 unit = 1kWh = 3.6×10^6 J
- If V_s = specified voltage and W is wattage of bulb or appliance, then resistance of bulb of appliance, $R = \frac{V_s^2}{W}$.

If V_a is applied voltage, then actual power consumed, $P_a = \frac{V_a^2}{R} = \frac{V_a^2}{V_s^2} W$