

Chapter 13. Mathematical Reasoning

Question-1

All natural numbers are integers.

Solution:

It is a statement. Its truth value is T.

Question-2

A square has five sides.

Solution:

It is a statement. Its truth value is F.

Question-3

The sky is blue.

Solution:

It is a statement. Its truth value is T.

Question-4

How are you?

Solution:

It is not a statement. It is a question.

Question-5

$7 + 2 < 10$.

Solution:

It is a statement. Its truth value is T.

Question-6

The set of rational numbers is finite.

Solution:

It is a statement. Its truth value is F.

Question-7

How beautiful you are?

Solution:

It is not a statement. It is an exclamatory.

Question-8

Wish you all success.

Solution:

It is not a statement. It is a command.

Question-9

Give me a cup of tea.

Solution:

It is not a statement. It is a question.

Question-10

2 is the only even prime

Solution:

It is a statement. Its truth value is T.

Question-11

All the sides of a rhombus are equal in length.

Solution:

True.

Question-12

Write down the truth value (T or F) of the following statement:

$1 + \sqrt{9}$ is an irrational number.

Solution:

True.

Question-13

Milk is white.

Solution:

True.

Question-14

The number 30 has four prime factors.

Solution:

False.

Question-15

Paris is in France.

Solution:

True.

Question-16

sin x is an even function.

Solution:

False.

Question-17

Every square matrix is non-singular.

Solution:

False.

Question-18

Jupiter is a planet.

Solution:

True.

Question-19

The product of a complex number and its conjugate is purely imaginary.

Solution:

False.

Question-20

Isosceles triangles are equilateral.

Solution:

False.

CBSE Class 11 Mathematics
Important Questions
Chapter 14
Mathematical Reasoning

4 Marks Questions

1. Give three examples of sentences which are not statements. Give reasons for the answers.

Ans. (i) The sentence “Rani is a beautiful girl” is not a statement. To some Rani may look beautiful and to other she may not look beautiful. We cannot say on logic whether or not this sentence is true.

(ii) The sentence ‘shut the door’ is not a statement. It is only an imperative sentence giving a direction to someone. There is no question of it being true or false.

(iii) The sentence ‘yesterday was Friday’ is not a statement. It is an ambiguous sentence which is true if spoken on Saturday and false if spoken on other days. Truth or falsehood of the sentence depends on the time at which it is spoken and not on mathematical reasoning.

2. Write the negation of the following statements

(i) Chennai is the capital of Tamil Nadu.

(ii) Every natural number is an integer.

Ans. (i) Chennai is not the capital of Tamil Nadu.

(ii) Every natural number is not an integer.

3. Find the component statements of the following compound statements and check whether they are true or false.

(i) The number 3 is prime or it is odd.

Ans. The component statements of the given statement are

p: "The number 3 is prime"

q: "number 3 is odd"

These two have been connected by using the connective "or"

The given statement is true as both the statements are true.

4. Check whether the following pair of statements are negations of each other Give reasons for your answer.

(i) $x + y = y + x$ is true for every real numbers x and y .

(ii) There exists real numbers x and y for which $x + y = y + x$.

Ans. The given statements are

p : " $x + y = y + x$ is true for every real number x and y "

q : " There exists real numbers x and y for which $x + y = y + x$."

These statements are not negations of each other as they can be true at the same time. Infact, negation of p is

$\sim p$: " There are real numbers x and y for which $x + y \neq y + x$."

Note that p is always true whatever x and y may be and $\sim p$ is always false.

5. Write the contra-positive and converse of the following statements.

(i) If x is a prime number, then x is odd.

(ii) if the two lines are parallel, then they do not intersect in the same plane.

Ans. If statement is $p \Rightarrow q$, then its contra-positive is $\sim q \Rightarrow \sim p$ and its converse is $q \Rightarrow p$.

(i) Contra-positive : “If x is not odd, then x is not a prime number.”

Converse : “If x is odd, then x is a prime number.”

(ii) Contra-positive : “If two lines intersect in the same plane, then they are not parallel.”

Converse: “If two lines do not intersect in the same plane, then they are parallel.”

6. Show that the statement

P : “If x is a real number such that $x^3 + 4x = 0$, then x is 0” is true by

(i) direct method, (ii) method of contradiction, (iii) method of contra-positive

Ans. Given statement is p: “If x is a real number such that $x^3 + 4x = 0$, then $x = 0$ ”

(i) Direct method: Let $x^3 + 4x = 0, x \in \mathbb{R}$

$$\Rightarrow x(x^2 + 4) = 0, x \in \mathbb{R} \Rightarrow x = 0 \left(\because \text{if } x \in \mathbb{R} \text{ then } x^2 + 4 \geq 4 \right)$$

Note that if the product of two numbers is zero then atleast one of them is surely zero.

Thus, we find that p is a true statement.

(ii) Method of contradiction.

Let x be a nonzero real number

$$\Rightarrow x^2 > 0 \left(\because \text{Square of a non-zero real number is always positive} \right)$$

$$\Rightarrow x^2 + 4 > 4 \Rightarrow x^2 + 4 \neq 0$$

$$\Rightarrow x(x^2 + 4) \neq 0 \left(\because x \neq 0 \text{ and } x^2 + 4 \neq 0 \right)$$

$$\Rightarrow x^3 + 4x \neq 0, \text{ which is a contradiction.}$$

Hence, $x = 0$

(iii) Method of contra-positive:

Let q : " $x \in \mathbb{R}$ and $x^3 + 4x = 0$ "

r : " $x = 0$ "

\therefore Given statement p is $q \Rightarrow r$

Its contra-positive is $\sim r \Rightarrow \sim q$

i.e. "if x is a non-zero real number then $x^3 + 4x$ is also nonzero"

Now $x \neq 0, x \in \mathbb{R} \Rightarrow x^2 > 0 \Rightarrow x^2 + 4 > 4 \Rightarrow x^2 + 4 \neq 0$

$\Rightarrow x(x^2 + 4) \neq 0 \Rightarrow x^3 + 4x \neq 0$ i.e. $\sim r \Rightarrow \sim q$.

Thus the statement $\sim r \Rightarrow \sim q$ is always true

Hence, $q \Rightarrow r$ is always true

Note: Infact, 'Method of contradiction' is another form of 'contra-positive method' while proving an implication.

7. Given below are two statements

P: 25 is a multiple of 5.

q: 25 is a multiple of 8

Write the compound statements connecting these two statements with "and" and "OR".

In both cases check the validity of the compound statement.

Ans. Case I. Using the connective 'and', we obtain the compound statement " p and q ".

i.e., "25 is a multiple of 5 and 8".

It is false statement as q is always false. (\because 25 is not a multiple of 8)

Case II. Using the connective 'or', we obtain the compound statement " p or q ".

i.e., "25 is a multiple of 5 or 8".

It is a true statement as p always true. ($\because 25$ is a multiple of 5)

8. Write the following statement in five different ways, conveying the same meaning.

P : If a triangle is equiangular, then it is an obtuse angled triangle.

Ans. Given statement is

“If a triangle is equiangular, then it is an obtuse angled triangle”. Its five equivalents are as follows:

- (i) “A triangle is equiangular only if it is an obtuse angled triangle”.
- (ii) “If a triangle is not obtuse angled triangle then it is not an equiangular triangle.”
- (iii) “*equiangularity* is a sufficient condition for triangle to be obtuse angled.”
- (iv) “A triangle being obtuse angled, is necessary condition for it to be equiangular”.
- (v) A triangle is obtuse is obtuse angled if it is equiangular.

Mathematical Reasoning

- Which of the following sentences are statements? Justify
 - A triangle has three sides.
 - 0 is a complex number.
 - Sky is red.
 - Every set is an infinite set.
 - $15 + 8 > 23$.
 - $y + 9 = 7$.
 - Where is your bag?
 - Every square is a rectangle.
 - Sum of opposite angles of a cyclic quadrilateral is 180° .
 - $\sin^2 x + \cos^2 x = 0$
- Find the component statements of the following compound statements.
 - Number 7 is prime and odd.
 - Chennai is in India and is the capital of Tamil Nadu.
 - The number 100 is divisible by 3, 11 and 5.
 - Chandigarh is the capital of Haryana and U.P.
 - $\sqrt{7}$ is a rational number or an irrational number.
 - 0 is less than every positive integer and every negative integer.
 - Plants use sunlight, water and carbon dioxide for photosynthesis.
 - Two lines in a plane either intersect at one point or they are parallel.
 - A rectangle is a quadrilateral or a 5 - sided polygon.
- Write the component statements of the following compound statements and check whether the compound statement is true or false.
 - 57 is divisible by 2 or 3.
 - 24 is a multiple of 4 and 6.
 - All living things have two eyes and two legs.
 - 2 is an even number and a prime number.
- Write the negation of the following simple statements
 - The number 17 is prime.
 - $2 + 7 = 6$.

- (iii) Violets are blue.
 - (iv) $\sqrt{5}$ is a rational number.
 - (v) 2 is not a prime number.
 - (vi) Every real number is an irrational number.
 - (vii) Cow has four legs.
 - (viii) A leap year has 366 days.
 - (ix) All similar triangles are congruent.
 - (x) Area of a circle is same as the perimeter of the circle.
5. Translate the following statements into symbolic form
- (i) Rahul passed in Hindi and English.
 - (ii) x and y are even integers.
 - (iii) 2, 3 and 6 are factors of 12.
 - (iv) Either x or $x + 1$ is an odd integer.
 - (v) A number is either divisible by 2 or 3.
 - (vi) Either $x = 2$ or $x = 3$ is a root of $3x^2 - x - 10 = 0$
 - (vii) Students can take Hindi or English as an optional paper.
6. Write down the negation of following compound statements
- (i) All rational numbers are real and complex.
 - (ii) All real numbers are rationals or irrationals.
 - (iii) $x = 2$ and $x = 3$ are roots of the Quadratic equation $x^2 - 5x + 6 = 0$.
 - (iv) A triangle has either 3-sides or 4-sides.
 - (v) 35 is a prime number or a composite number.
 - (vi) All prime integers are either even or odd.
 - (vii) $|x|$ is equal to either x or $-x$.
 - (viii) 6 is divisible by 2 and 3.
7. Rewrite each of the following statements in the form of conditional statements
- (i) The square of an odd number is odd.
 - (ii) You will get a sweet dish after the dinner.
 - (iii) You will fail, if you will not study.

- (iv) The unit digit of an integer is 0 or 5 if it is divisible by 5.
- (v) The square of a prime number is not prime.
- (vi) $2b = a + c$, if a , b and c are in A.P.
8. Form the biconditional statement $p \leftrightarrow q$, where
- (i) p : The unit digit of an integer is zero.
 q : It is divisible by 5.
- (ii) p : A natural number n is odd.
 q : Natural number n is not divisible by 2.
- (iii) p : A triangle is an equilateral triangle.
 q : All three sides of a triangle are equal.
9. Write down the contrapositive of the following statements:
- (i) If $x = y$ and $y = 3$, then $x = 3$.
- (ii) If n is a natural number, then n is an integer.
- (iii) If all three sides of a triangle are equal, then the triangle is equilateral.
- (iv) If x and y are negative integers, then xy is positive.
- (v) If natural number n is divisible by 6, then n is divisible by 2 and 3.
- (vi) If it snows, then the weather will be cold.
- (vii) If x is a real number such that $0 < x < 1$, then $x^2 < 1$.
10. Write down the converse of following statements :
- (i) If a rectangle 'R' is a square, then R is a rhombus.
- (ii) If today is Monday, then tomorrow is Tuesday.
- (iii) If you go to Agra, then you must visit Taj Mahal.
- (iv) If the sum of squares of two sides of a triangle is equal to the square of third side of a triangle, then the triangle is right angled.
- (v) If all three angles of a triangle are equal, then the triangle is equilateral.
- (vi) If $x : y = 3 : 2$, then $2x = 3y$.
- (vii) If S is a cyclic quadrilateral, then the opposite angles of S are supplementary.
- (viii) If x is zero, then x is neither positive nor negative.
- (ix) If two triangles are similar, then the ratio of their corresponding sides are equal.

11. Identify the Quantifiers in the following statements.
- (i) There exists a triangle which is not equilateral.
 - (ii) For all real numbers x and y , $xy = yx$.
 - (iii) There exists a real number which is not a rational number.
 - (iv) For every natural number x , $x + 1$ is also a natural number.
 - (v) For all real numbers x with $x > 3$, x^2 is greater than 9.
 - (vi) There exists a triangle which is not an isosceles triangle.
 - (vii) For all negative integers x , x^3 is also a negative integers.
 - (viii) There exists a statement in above statements which is not true.
 - (ix) There exists a even prime number other than 2.
 - (x) There exists a real number x such that $x^2 + 1 = 0$.
12. Prove by direct method that for any integer ' n ', $n^3 - n$ is always even.
[Hint: Two cases (i) n is even, (ii) n is odd.]
13. Check the validity of the following statement.
- (i) p : 125 is divisible by 5 and 7.
 - (ii) q : 131 is a multiple of 3 or 11.
14. Prove the following statement by contradiction method.
 p : The sum of an irrational number and a rational number is irrational.
15. Prove by direct method that for any real numbers x, y if $x = y$, then $x^2 = y^2$.
16. Using contrapositive method prove that if n^2 is an even integer, then n is also an even integers.