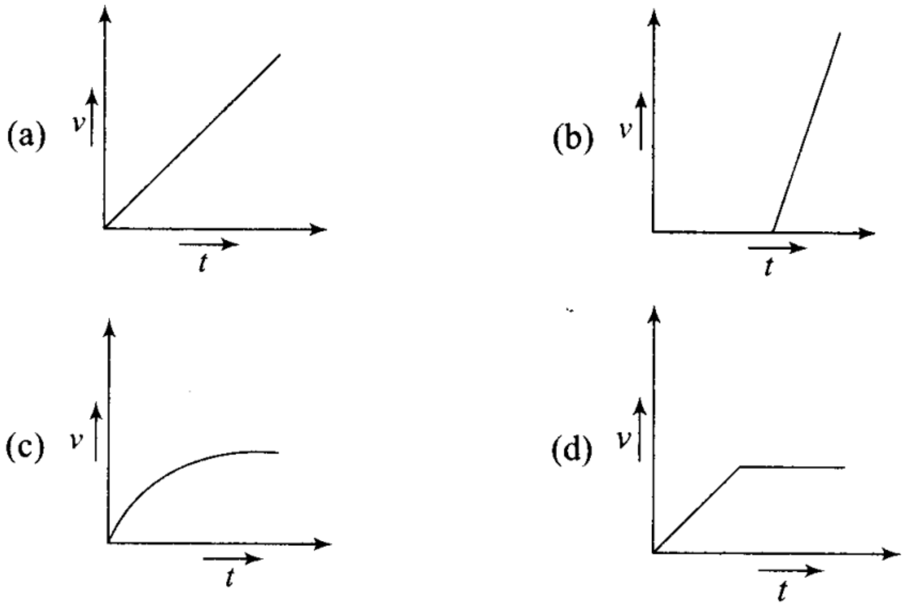
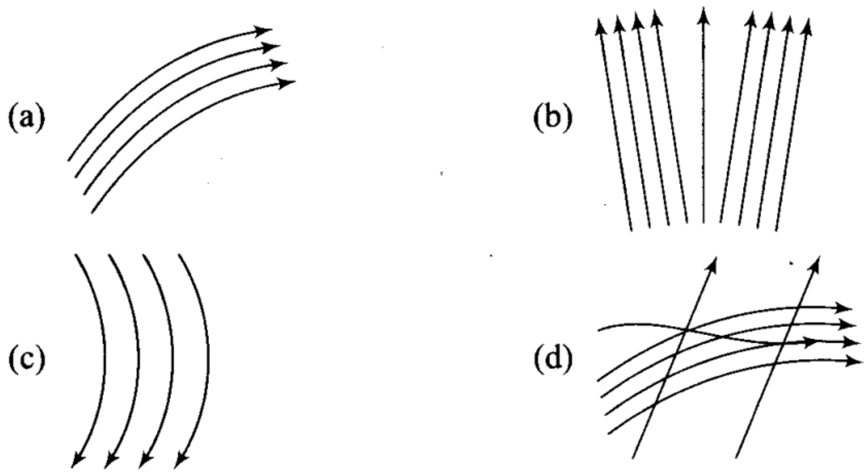
Chapter 10 (Mechanical Properties of

Q1. A tall cylinder is filled with viscous oil. A round pebble is dropped from the top with zero initial velocity. From the plot shown in figure, indicate the one that represents the velocity (v) of the pebble as a function of time (t).



Sol: (c) In fluids, when the pebble is dropped from the top of a tall cylinder filled with viscous oil, a variable force called viscous force will act which increases with increase in speed. And at equilibrium this velocity becomes constant, that constant velocity is called terminal velocity.
When the pebble is falling through the viscous oil, the viscous force is $F = 6\pi\eta rv$
where r is the radius of the pebble, v is instantaneous speed, η is coefficient of viscosity. As the force is variable, hence acceleration is also variable so v-t graph will not be a straight line. First velocity increases and then becomes constant known as terminal velocity.

Q2. Which of the following diagrams does not represent a streamline flow?



Sol. (d)

Streamline flow: Streamline flow of a liquid is that flow in which each element of the liquid passing through a point travels along the same path and with the same velocity as the preceding element passes through that point.

A streamline may be defined as the path, straight or curved, the tangent to which at any point gives the direction of the flow of liquid at that point.

The two streamlines cannot cross each other and the greater is the crowding of streamlines at a place, the greater is the velocity of liquid particles at that place. If we consider a cross¬sectional area, then a point on the area cannot have different velocities at the same time

Path ABC is streamline as shown in the figure and v1, v2 and v3 are the velocities of the liquid particles at A, B and C point respectively.

Q3. Along a streamline,

(a) the velocity of a fluid particle remains constant

(b) the velocity of all fluid particles crossing a given position is constant

(c) the velocity of all fluid particles at a given instant is constant

(d) ‘ the speed of a fluid particle remains constant,

Sol:(b) As discussed above for a streamline flow of a liquid velocity of each particle at a particular cross-section is constant, because Av = constant (law of continuity) between two cross-section of a tube of flow. So we can say that along a streamline, the velocity of every fluid particle while crossing a given position is the same.

Q4. An ideal fluid flows through a pipe of circular cross-section made of two sections with diameters 2.5 cm and 3.75 cm. The ratio of the velocities in the two pipes is

(a) 9:4 (b) 3:2 (c)√3: √2 (d)√2:√3

Sol: (a) The situation is shown in the diagram below in which an ideal fluid is flowing through

a pipe of circular cross sections.

d_1 = Diameter at 1st point is 3.75

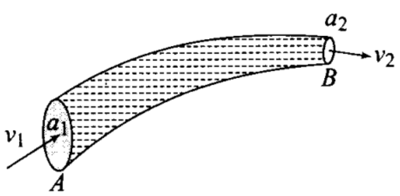
d_2 = Diameter at 2nd point is 2.5

According to equation of continuity

$$a_1 v_1 = a_2 v_2$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{a_2}{a_1} = \frac{\pi d_2^2/4}{\pi d_1^2/4} = \left(\frac{d_2}{d_1}\right)^2$$

$$= \left(\frac{3.75}{2.50}\right)^2 = \frac{9}{4}$$



Q5. The angle of contact at the interface of water-glass is 0°, ethylalcohol-glass is 0°, mercury-glass is 140° and methyliodide-glass is 30°. A glass capillary is put in a trough containing one of these four liquids. It is observed that the meniscus is convex. The liquid in the trough is

(a) water

(b) ethylalcohol

(c) mercury

(d) methyliodide

Sol. (c)

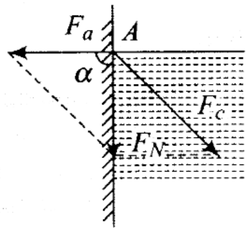
Key concept: *Shape of Liquid Meniscus:*

(i) Force of adhesion F_a (acts outwards at right angle to the wall of the tube).

(ii) Force of cohesion F_c (acts at an angle 45° to the vertical).
Resultant force F_N depends upon the value of F_a and F_c.
If resultant force F_N makes an angle α with F_a.

Then $\tan\alpha = \dfrac{F_c \sin 135°}{F_a + F_c \cos 135°} = \dfrac{F_c}{\sqrt{2}F_a - F_c}$

By knowing the direction of resultant force we can find out the shape of meniscus because the free surface of the liquid adjust itself at right angle to this resultant force.



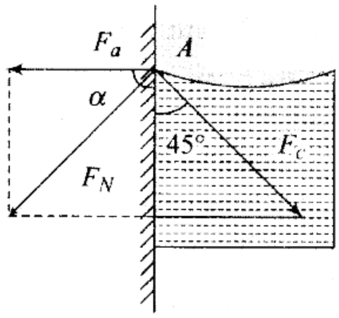
(1) If $F_c = \sqrt{2}F_a$,

$\tan\alpha = \infty \quad \therefore \alpha = 90°$

i.e., the resultant force acts vertically downwards. Hence the liquid meniscus must be horizontal.

(2) If $F_c < \sqrt{2}F_a$,

$\tan\alpha = \text{positive} \quad \therefore \alpha$ is acute angle



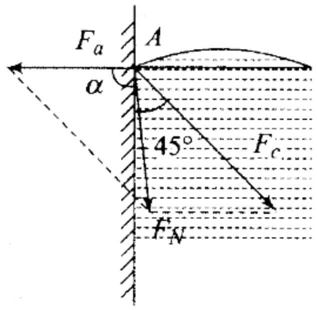
i.e., the resultant force directed outside the liquid. Hence the liquid meniscus must be concave upward.

(3) $F_c > \sqrt{2}F_a$

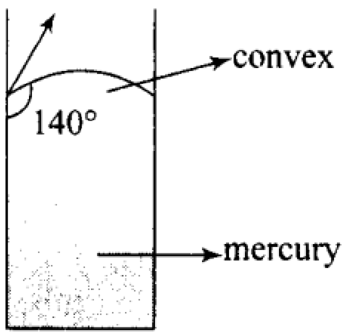
tan α = negative

$\therefore$ α is obtuse angle.

i.e., the resultant force directed inside the liquid. Hence the liquid meniscus must be convex upward.



According to the question, the observed meniscus of liquid in a capillary tube is of convex upward which is only possible when angle of contact is obtuse. It is so when one end of glass capillary tube is immersed in a trough of mercury. Hence, the combination will be of mercury-glass (140°) as shown in the figure.



More Than One Correct Answer Type

Q6. For a surface molecule,

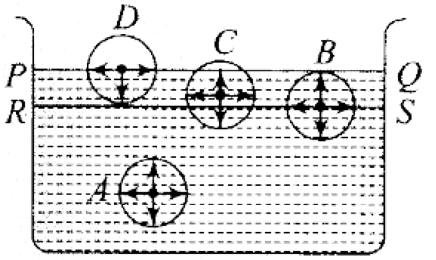
(a) the net force on it is zero

(b) there is a net downward force

(c) the potential energy is less than that of a molecule inside

(d) the potential energy is more than that of a molecule inside

Sol: (b, d)

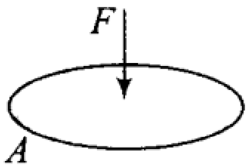


Key concept: To understand the concept of tension acting on the free
surface of a liquid, let us consider four liquid molecules like A, B, C and D. Their sphere of influence are shown in the figure.

(1) Molecule A is well within the liquid, so it is attracted equally in all directions. Hence the net

force on this molecule is zero and it moves freely inside the liquid.
(2) Molecule B is little below the free surface of the liquid and it is atso attracted equally in all directions. Hence the resultant force acts on it is also zero
(3) Molecule C is just below the upper surface of the liquid film and the part of its sphere of influence is outside the free liquid surface. So the number of molecules in the upper half (attracting the molecules upward) is less than the number of molecule in the lower half (attracting the molecule downward). Thus the molecule C experiences a net downward force.
(4) Molecule D is just on the free surface of the liquid. The upper half of the sphere of influence has no liquid molecule. Hence the molecule D experiences a maximum downward force.
Thus all molecules lying on surface film experiences a net downward force. Therefore, free surface of the liquid behaves like a stretched membrane.
From the key concept, it is clear from point (4) that molecules on the surface experiences a net downward force. As shown in figure above, molecule D experiences a net downward force. Because on the above side of this molecule there is no liquid molecule. So, the potential energy is more than that of a molecule inside. Hence option (b) and (d) are correct.

Q7. Pressure is a scalar quantity, because
(a) it is the ratio of force to area **and both force and area are vectors.**
(b) it is the ratio of the magnitude of the force to area.
(c) it is the ratio of the component of the force normal to the area.
(d) ‘it does not depend on the size of the area chosen.
Sol: (b, c) Pressure is defined as the ratio of magnitude of component of the force normal to the area and the area under consideration.

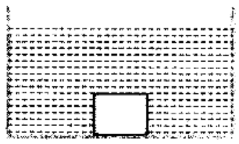


i.e P = F/A
Pressure is a scalar quantity. Pressure acts normal to a surface and it is always compressive in nature, therefore, only its magnitude is required for its complete description.

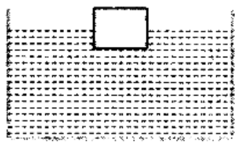
Q8. A wooden block with a coin placed on its top, floats in water as shown in figure. The distance / and h are shown in the figure. After sometime, the coin falls into the water. Then,
(a) l decreases
(b) h decreases
(c) l increases
(d) h increases
Sol: (a, b)
Key concept: When a body of density ρ and volume V is immersed in a liquid of density σ, the forces acting on the body are:
1. Weight of body W = mg = Vρg, acting vertically downwards through centre of gravity of the body

2. Upthrust force = $V\sigma g$ acting vertically upwards through the centre of gravity of the displaced liquid i.e., centre of buoyancy.

Case 1: If density of body is greater than that of liquid $\rho > \sigma$.



Case 2: If density of body is lesser than that of liquid $\rho < \sigma$.



When coin is in water, volume of water displaced by coin is equal to the volume of coin V_1(say).
When the coin falls into the water, weight of the (block + coin) system decreases, which was balanced by the upthrust force earlier. As weight of the system decreases, block moves up. Hence Idecreases.
When coin is at the top of wooden block, it displaces a volume of water V_2, which is more than V_1 Because,
Weight of coin = Weight of volume of water displaced by coin (when coin is at .the top of wooden block)

$$\Rightarrow \quad \rho_c V_1 g = \rho_l V_2 g$$

$$\Rightarrow \quad \frac{V_2}{V_1} = \frac{\rho_c}{\rho_l}$$

Since, $\rho_c > \rho_l$
So, $V_2 > V_1$

Hence, upthrust force will also decrease. As volume of water displaced by the block decreases, hence h decreases.

Q9. With increase in temperature, the viscosity of
(a) gases decreases
(b) liquids increases
(c) gases increases
(d) liquids decreases

Sol: (c, d) The viscosity of gases increases with increase of temperature, because on increasing temperature the rate of diffusion increases.
The viscosity of liquid decreases with increase of temperature, because the cohesive force between the liquid molecules decreases with increase of temperature.
Relation between coefficient of viscosity and temperature (Andrade formula)

$$\eta = \frac{Ae^{C\rho/T}}{\rho^{-1/3}}$$

where T = Absolute temperature of liquid, p = density of liquid, A and C are constants.
Important point: With increase in temperature, the coefficient of viscosity of liquids decreases but that of gases increases. The reason is that as temperature rises, the atoms of the liquid become more mobile, whereas in case of a gas, the collision frequency of atoms increases as

their motion becomes more random.

Q10. Streamline flow is more likely for liquids with
(a) high density (b) high viscosity
(c) low density (d) low viscosity
Sol: (b, c) Streamline flow is more likely for liquids having low density. We know that greater the coefficient of viscosity of a liquid more will be the velocity gradient, hence each line of flow can be easily differentiated. Streamline flow is related with critical velocity. The critical velocity is that velocity of liquid flow up to which its flow is streamlined and above which its flow becomes turbulent.
As the critical velocity is related to viscosity (η) and density (ρ) of the liquid as:

$(V_c) \propto \eta/\rho$

Hence if the density will be low and viscosity will be high, the value of critical velocity will be more. So, option (b) and (c) are correct.

Very Short Answer Type Questions
Q11. Is viscosity a vector?
Sol: Viscosity is not a vector quantity. It is a scalar quantity because viscosity is a property of liquid as it does not have any direction.

Q12. Is surface tension a vector?
Sol: Surface tension of a liquid is measured by the force acting per unit length on either side of an imaginary line drawn on the free surface of liquid, the direction of this force being perpendicular to the line and tangential to the free surface of liquid. So if F is the force acting on one side of imaginary line of length L, then T= (F/L)
It depends only on the nature of liquid and is independent of the area of surface or length of line considered.
It is a scalar quantity as it has a unique direction which is not to be specified

13. Iceberg floats in water with part of it submerged. What is the fraction of the volume of iceberg submerged, if the density of ice is $\rho_l = 0.917 \text{ g cm}^{-3}$?

Sol. According to the problem, density of ice (ρ_{ice}) = 0.917 g/cm^3, Density of water (ρ_w) = 1 g/cm^3
Let V_i = Volume of iceberg,
V_w = Volume of water displaced by iceberg,
Weight of iceberg, $W = \rho_i V_i g$,
Upthrust, $F_B = \rho_w V_w g$

At equilibrium, Weight of the iceberg = Weight of the water displaced by the submerged part by ice

$$\Rightarrow \quad \rho_w V_w g = \rho_i V_i g$$

$$\Rightarrow \quad \frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = \frac{0.917}{1} = 0.917$$

Q14. A vessel filled with water is kept on a weighing pan and the scale adjusted to zero. A block of mass Mand density p is suspended by a massless spring of spring constants k. This block is submerged inside into the water in the vessel. What is the reading of the scale?
Sol: As shown in the diagram, A block of mass M and density p is suspended by a massless spring of spring constants k. This block is submerged into the water in the vessel.
The scale is adjusted to zero, therefore, when the block suspended to a spring is immersed in water, then the reading of the scale will be equal to the upthrust on the block due to water.

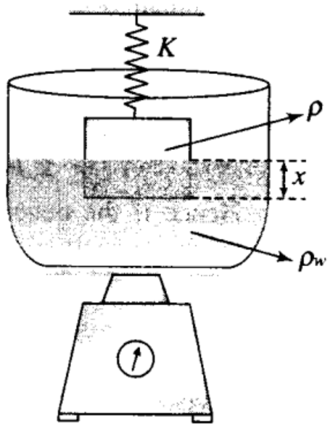
Upthrust experienced by the block = Weight of water displaced = $V\rho_w g$ (where V is volume of the block and ρ_w is density of water).
Let x be the compression on the spring. As the block is in equilibrium,

$$Mg - (kx + V\rho_w g) = 0$$

The reading in the pan is the force applied by the water on the pan, i.e.

$$m_{vessel} + m_{water} + V\rho_w g$$

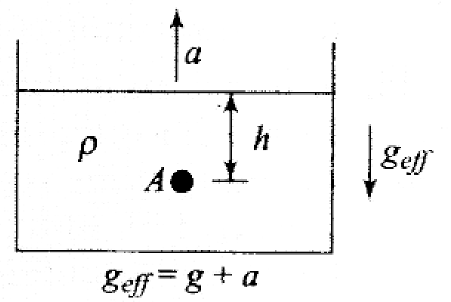
Since the scale has been adjusted to zero without the block, the new reading is $V\rho_w g$.



Important point: If the scale is not adjusted to zero as said earlier, reading on the scale will be different. Then weight of vessel and weight of water is also added in the reading.

Q15. A cubical block of density ρ is floating on the surface of water. Out of its height L, fraction x is submerged in water. The vessel is in an elevator accelerating upward with acceleration a. What is the fraction immersed?

Sol: Key concept: When a fluid is subjected to constant vertical acceleration, its free surface remains horizontal as the net effective gravity is acting vertically. But the magnitude of pressure at a point in the fluid increases or decreases depending upon the direction of acceleration.

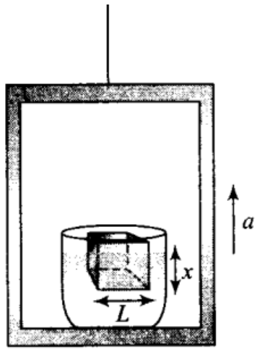


Free surface remains horizontal. Pressure at every point increases. At the point A, P= $\rho(g + a)h$

As the elevator is accelerating upward, then we can solve this problem with respect to elevator by applying pseudo force.
As of block is accelerating upward, so pseudo force on the container as well as on the block must be downward.

As of block is accelerating upward, so pseudo force on the container as well as on the block must be downward.

The situation is shown in the diagram above.



According to the problem,

Given, density of block = ρ, Height of block = L,

Density of water = ρ_w, Volume of the block (V) = L^3,

Mass of the block (m) = $V\rho = L^3\rho$,

Weight of the block = $mg = L^3\rho g$

Let x be the height of cube submerged in water.

1st case:

Volume of part of cube submerged in water = xL^2

$\therefore$ Weight of water displaced by block = $xL^2 \times \rho_w g$

As block is floating, so

$\therefore$ weight of block = weight of water displaced by block

$\Rightarrow \quad L^3\rho g = lL^2\rho_w g$

$\Rightarrow \quad \frac{l}{L} = \frac{\rho}{\rho_w} = x$

2nd case:

When vessel is placed in an elevator moving upward with acceleration a, then effective acceleration = $(g + a)$ ($\because$ Pseudo force is downward)

Then, weight of the block = $m(g + a)$

$= L^3\rho\,(g + a)$

$\therefore$ Effective weight of block = $m(g + a)$

Let x_1 = new fraction of block submerged in water. Since block is still floating in water, then

$$m(g + a) = (x_1 L^3)\rho_w(g + a)$$

$$x_1 = \frac{m}{L^3\rho_w} = \frac{L^3\rho}{L^3\rho_w} = \frac{\rho}{\rho_w} = x$$

Hence, the fraction of the block submerged is independent of any type of acceleration.

Short Answer Type Questions

Q16. The sap in trees, which consists mainly of water in summer, rises in a system of capillaries of radius r = 2.5×10^{-5} m. The surface-tension of sap is T = 7.28×10^{-2} Nm^{-1} and the angle of contact is 0°. Does surface tension alone account for the supply of water to the top of all trees?

Sol: According to the problem, radius (r) = 2.5×10^{-5} m Surface tension (S) = 7.28×10^{-2} N/m Angle of contact (θ) = 0°

Density (ρ) = 10^3 kg m^{-3}

If h is the maximum height to which sap can rise in trees through capillarity action, is given by

$$h = \frac{2S\cos\theta}{r\rho g}$$

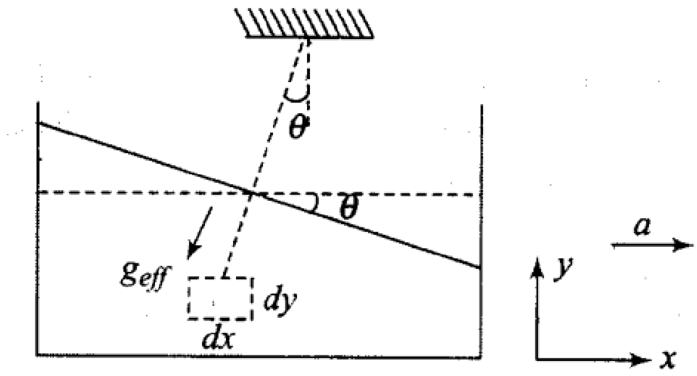
where S = surface tension, ρ = density and r = radius

$$= \frac{2 \times 7.28 \times 10^{-2} \times \cos 0°}{2.5 \times 10^{-5} \times 1 \times 10^{-3} \times 9.8} = 0.6 \text{ m}$$

This is the maximum height to which the sap can rise due to surface tension. Many trees have heights much greater than 0.6 m, so only this action is not sufficient for supply of water to the top of such long tree.

Q17. The free surface of oil in a tanker, at rest, is horizontal. If the tanker starts accelerating the free surface will be titled **by an angle θ. If the acceleration is a ms $^{-2}$, what will be the slope of the free surface?**

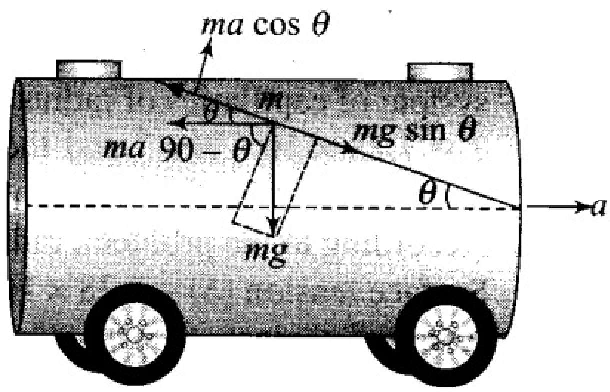
Sol: Key concept: The behaviour of a liquid contained in a horizontally accelerated vessel can be understood by understanding the behaviour of a pendulum suspended from the ceiling of a horizontally accelerated trolley.



Every fluid element attains an equilibrium position under the action of gravity and pseudo force. The free surface of the liquid orients itself perpendicular to the direction of net effective gravity.

tan θ = a/g

Suppose tanker accelerates along x-axis with acceleration a, free surface of the tanker will not be horizontal because pseudo force acts as shown in the diagram.



Consider an elementary particle of the oil of mass m.

The acting forces on the particle with respect to the tanker are shown in the figure alongside.

Now, balancing forces (as the particle is in equilibrium) along the inclined direction of surface.

ma = pseudo force

mg = weight of small part of oil.

Along free surface,

Net force = 0

=> ma cos θ = mg sin θ

=> a = g tan θ

=> θ = $\tan^{-1}(a/g)$

Q18.Two mercury droplets of radii 0.1 cm and 0.2 cm collapse into one single drop. What

amount of energy is released? The surface tension of mercury T=5 x 10^{-3} Nm^{-1}

Sol: When two drops form a bigger drop, volume remains conserved.

According to the problem, there is two mercury droplets of different radii collapse into one single drop.

Radius of smaller drop = r, = 0.1 cm = 10^{-3} m,

Radius of bigger drop = r_2 = 0.2 cm = 2 x 10^{-3} m

Surface tension (7) = 435.5 x 10^{-3} N/m

Let V_1 and V_2 be the volumes of these two mercury droplets and volume of big drop formed by collapsing is V.

Volume of big drop = Volume of small droplets

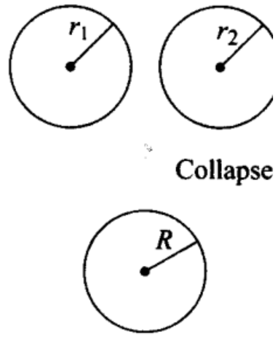
$$V = V_1 + V_2$$

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3$$

or $$R^3 = r_1^3 + r_2^3$$

$$= (0.1)^3 + (0.2)^3 = 0.001 + 0.008 = 0.009$$

or $$R = 0.21 \text{ cm} = 2.1 \times 10^{-3} \text{ m}$$



∴ **Decrease in surface area,**

$$\Delta A = 4\pi R^2 - (4\pi r_1^2 + 4\pi r_2^2)$$

$$= 4\pi[R^2 - (r_1^2 + r_2^2)]$$

Energy released,

$$E = T \times \Delta A$$

$$= T \times 4\pi[R^2 - (r_1^2 + r_2^2)]$$

$$= 435.5 \times 10^{-3} \times 4 \times 3.14[(2.1 \times 10^{-3})^2 - (1 \times 10^{-6} + 4 \times 10^{-6})]$$

$$= 435.5 \times 4 \times 3.14[4.41 - 5] \times 10^{-6} \times 10^{-3}$$

$$= -32.23 \times 10^{-7}$$

(Negative sign shows absorption)

Therefore, 3.22 x K^{-6} J energy will be absorbed. So, the surface area of the water decreases means surface area of bigger drop is less than the sum of surface area of two smaller drops.

Q19. If a drop of liquid breaks into smaller droplets, it results in lowering of temperature **of the droplets. Let a drop of radius R, breaks into N small droplets each of radius r. Estimate the drop in temperature.**

Sol: The volume remains conserved, when a big drop, breaks into N small droplets.

Volume of liquid drop of radius R = (Volume of liquid droplet of radius r)xN

$$\frac{4}{3}\pi R^3 = N \times \frac{4}{3}\pi r^3$$

or $$R^3 = Nr^3$$

or $$N = \frac{R^3}{r^3} \quad \ldots\text{(i)}$$

Energy released,

$$\Delta U = T \times \Delta A = T[4\pi R^2 - N(4\pi r^2)]$$

$$= 4\pi T(R^2 - Nr^2)$$

Due to releasing of this energy, the temperature is lowered.

If c is specific heat of liquid and its temperature is lowered by ΔT, then Energy released, ΔU –

me ΔT

$$\Delta T = \frac{\Delta U}{mc} = \frac{4\pi T(R^2 - Nr^2)}{\left(\frac{4\pi}{3}\right)R^3 \rho c} \qquad [\because \rho = \text{density of liquid}]$$

$$\Rightarrow \quad \Delta T = \frac{3T}{\rho c}\left(\frac{1}{R} - N\frac{r^2}{R^3}\right)$$

$$\Rightarrow \quad \Delta T = \frac{3T}{\rho c}\left(\frac{1}{R} - \frac{1}{r}\right) \qquad [\text{Using eqn. (i)}]$$

$$\because \quad R > r \Rightarrow \frac{1}{R} < \frac{1}{r} \Rightarrow \left(\frac{1}{R} - \frac{1}{r}\right) < 0$$

$\therefore$ ΔT will be negative. Hence, temperature of droplet falls.

Q20. The surface tension and vapour pressure of water at 20°C is 7.28 x 10^{-2} Nm^{-1} and 2.33 x 10^3 Pa, respectively. What is the radius of the smallest spherical water droplet which can form without evaporating at 20°C?

Sol: According to the problem, surface tension of water, T = 7.28 x 10^{-2} N m^{-1} Vapour pressure P = 2.33 x 10^3 Pa Let r = radius of drop, which formed without evaporating.

The excess pressure (2T/r) should be greater than the vapour pressure. Then, the drop will evaporate.

Vapour pressure = Excess pressure in drop

$$\frac{2T}{r} = P \Rightarrow r = \frac{2T}{P}$$

$$\Rightarrow \quad r = \frac{2 \times 7.28 \times 10^{-2}}{2.33 \times 10^3} = 6.25 \times 10^{-5} \text{ m}$$

Long Answer Type Questions

21. (a) Pressure decreases as one ascends the atmosphere. If the density of air is ρ, what is the change in pressure dp over a differential height dh?

(b) Considering the pressure p to be proportional to the density, find the pressure p at a height h if the pressure on the surface of the earth is p_0.

(c) If $p_0 = 1.03 \times 10^5$ Nm^{-2}, $\rho_0 = 1.29$ kg m^{-3} and $g = 9.8$ ms^{-2}, at what height will be pressure drop to (1/10) the value at the surface of the earth?

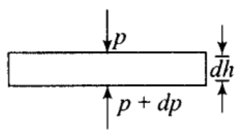
(d) This model of the atmosphere works for relatively small distances. Identify the underlying assumption that limits the model.

Sol. (a) Pressure will decrease as we are going up the atmosphere, because the thickness of the gases above us decreases.

Let us consider a horizontal layer of air with cross-section A and height dh.

Let, the Pressure at the top of the layer is P and Pressure at the bottom of the layer is $p + dp$.

Where, dp is the change in pressure of top most layer and bottom layer.



The layer is in equilibrium. So, the net upward force must be balanced by the weight,

$\therefore$ net upward force = net downward force

$$\Rightarrow \quad (p + dp)A - pA = -\rho g\, Adh$$

($\therefore$ Weight = Density × Volume × g)

$$\Rightarrow \quad dp = -\rho g dh$$

Negative sign shows that pressure decreases with height.

(b) Let ρ_0 be the density of air on the surface of the earth.
According to the problem, pressure is proportional to density

$$p \propto \rho$$

$$\therefore \quad \frac{\text{Pressure at some height } (p)}{\text{Pressure at the surface of Earth } (P_0)} = \frac{\rho}{\rho_0}$$

$$\Rightarrow \quad \frac{p}{p_0} = \frac{\rho}{\rho_0} \Rightarrow \rho = \frac{\rho_0}{p_0} p$$

$$\therefore \quad dp = -\frac{\rho_0 g}{p_0} p dh \qquad [\because dp = -\rho g dh]$$

$$\Rightarrow \quad \frac{dp}{p} = -\frac{\rho_0 g}{p_0} dh$$

$$\Rightarrow \quad \int_{p_0}^{p} \frac{dp}{p} = -\frac{\rho_0 g}{p_0} \int_0^h dh \qquad \left[\begin{array}{l}\because \text{at } h = 0, r = p_0 \\ \text{and at } h = h, p = p\end{array}\right]$$

$$\Rightarrow \quad [\ln p]_{p_0}^{p} = -\frac{\rho_0 g}{p_0} [h]_0^h$$

$$\Rightarrow \quad \ln \frac{p}{p_0} = -\frac{\rho_0 g}{p_0} h \qquad \ldots \text{(i)}$$

$$\therefore \quad p = p_o e^{\left(-\frac{\rho_o g h}{p_o}\right)}$$

(c) As $p = p_0 e^{-\frac{\rho_0 g h}{p_0}}$

And it is given that, $p_0 = 1.013 \times 10^5 \text{ Nm}^{-2}$

$$\rho_0 = 1.29 \text{ kg m}^{-3}, g = 9.8 \text{ ms}^{-2}, p = \frac{p_0}{10}, h = ?$$

$$\rho_0 = 1.29 \text{ kg m}^{-3}, g = 9.8 \text{ ms}^{-2}, p = \frac{p_0}{10}, h = ?$$

By substituting the values in the above relation (i.e., eq. (i)), we get

$$\Rightarrow \quad \ln \frac{p}{p_0} = -\frac{\rho_0 g}{p_0} h$$

$$\Rightarrow \quad \ln\left(\frac{\frac{1}{10} p_0}{p_0}\right) = -\frac{\rho_0 g}{p_0} h$$

$$\Rightarrow \quad \ln \frac{1}{10} = -\frac{1.29 \times 9.8}{1.013 \times 10^5} \times h$$

$$\Rightarrow \quad -2.30 = -\frac{12.642}{1.013 \times 10^5} h$$

$$\Rightarrow \quad h = \frac{2.3 \times 1.013 \times 10^5}{12.642} = 0.1843 \times 10^5$$

$$\therefore \quad h = 18.43 \text{ km.}$$

(d) It is $p \propto \rho$, but this relation is valid only when the temperature is uniform in the air, i.e., in isothermal situation. And which is valid only near the surface of the earth, not at greater heights.

Q22. Surface tension is exhibited by liquids due to force of attraction between molecules of the liquid. The surface tension decreases with increase in temperature and vanishes at boiling point. Given that the latent heat of vaporisation for water L_v = 540 kcal kg^{-1}, the mechanical equivalent of heat J = 4.2 Jeal^{-1}, density of water $\rho_w = 10^3$ kg^{-1}, Avogadro's number

N_a = 6.0 xlO^{26} kmole'^{1} and the molecular weight of water M_A = 10 kg for 1 kmole.

(a) Estimate the energy required for one molecule of water to evaporate

(b) Show that the inter-molecular distance for water is $d = \left[\frac{M_A}{N_A} \times \frac{1}{\rho_w}\right]^{1/3}$ and find its value.

(c) 1 g of water in the vapour state at 1 atm occupies 1601 cm^3. Estimate the inter-molecular distance at boiling point, in the vapour state.

(d) During vaporisation a molecule overcomes a force F, assumed constant, to go from an inter-molecular distance d to Estimate the value of F.

(e) Calculate Fid, which is a measure of the surface tension.

Sol. (a) According to the problem, latent heat of vaporisation for water

$$(L_v) = 540 \text{ kcal} = 540 \times 10^3 \times 4.2 \text{ J} = 2268 \times 10^3 \text{ J}$$

Therefore, energy required to evaporate 1 kmol (18 kg) of water

$$= (2268 \times 10^3 \text{ J})(18)$$

$$= 40824 \times 10^3 \text{ J} = 4.0824 \times 10^7 \text{ J}$$

Since, there are N_A molecules in M_A kg of water, the energy required for 1 molecule to evaporate is

$$U = \frac{M_A L_v}{N_A} \text{ J}$$

[where $N_A = 6 \times 10^{26}$ = Avogadro number]

$$U = \frac{4.0824 \times 10^7}{6 \times 10^{26}} \text{ J} = 0.68 \times 10^{-19} \text{ J}$$

$$= 6.8 \times 10^{-20} \text{ J}$$

(b) Let the water molecules to be at points and are placed at a distance d from each other,

Volume of N_A molecule of water $= \frac{M_A}{\rho_w}$

Thus, the volume around one molecule is

$$= \frac{\text{volume of 1 kmol}}{\text{number of molecules/kmol}} = \frac{M_A}{N_A \rho_w}$$

And also volume around one molecule $= d^3$

Thus, by equating these, we get

$$d^3 = \frac{M_A}{N_A \rho_w}$$

$$\therefore \quad d = \left(\frac{M_A}{N_A \rho_w}\right)^{1/3} = \left(\frac{18}{6 \times 10^{26} \times 10^3}\right)^{1/3}$$

$$= (30 \times 10^{-30})^{1/3} m \approx 3.1 \times 10^{-10} \text{ m}$$

(c) Volume occupied by 1 kmol (18 kg) of water molecules

$$= \frac{1601 \times 10^{-6} \text{ m}^3}{\text{g}} (18 \times 10^3 \text{ g})$$

$$= 28818 \times 10^{-3} \text{ m}^3$$

Since 6×10^{26} molecules occupies $18 \times 1601 \times 10^{-3}$ m^3

$\therefore$ Volume occupied by 1 molecule

$$= \frac{28818 \times 10^{-3}\ \text{m}^3}{6 \times 10^{26}} = 48030 \times 10^{-30}\ \text{m}^3$$

If d' is the intermolecular distance, then

$$(d')^3 = 48030 \times 10^{-30}\ \text{m}^3$$

So, $d' = 36.3 \times 10^{-10}$ m $= 36.3 \times 10^{-10}$ m

(d) Work done to change the distance from d to d' is $U = F(d' - d)$,
This work done is equal to energy required to evaporate 1 molecule.

$\therefore$ $F(d' - d) = 6.8 \times 10^{-20}$

or
$$F = \frac{6.8 \times 10^{-20}}{d' - d}$$

$$= \frac{6.8 \times 10^{-20}}{(36.3 \times 10^{-10} - 3.1 \times 10^{-10})}$$

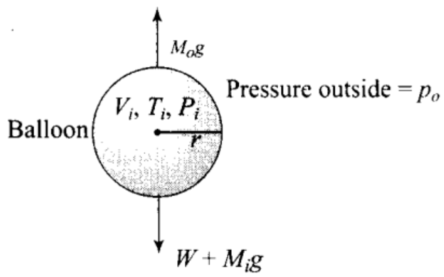
$$= 2.05 \times 10^{-11}\ \text{N}$$

(e) Surface tension $= \frac{F}{d} = \frac{2.05 \times 10^{-11}}{3.1 \times 10^{-10}} = 6.6 \times 10^{-2}$ N/m

Q23. A hot air balloon is a sphere of radius 8 m. The air inside is at a temperature of 60°C. How large a mass can the balloon lift when the outside temperature is20°C? Assume air in an ideal gas, R = 8.314 J mole^{-1} K ‘, 1 atm = 1.013 x 10^{S} Pa, the membrane tension is 5 Nm $^{-1}$.

Sol: Pressure inside the curved surface Will be greater than of outside pressure

Let the pressure inside the balloon be p_i and the outside pressure be p_o, then excess pressure is $p_i - p_o = \frac{2T}{r}$



where, T = Surface tension

r = radius of balloon

Let us consider, for the sake of simplicity of calculations, that air behaves like an ideal gas.

For n_i moles of air inside the balloon,

$$P_i V = n_i R T_i \text{ (equation of ideal gas)}$$

or
$$n_i = \frac{P_i V}{R T_i}$$

mass of air inside the balloon,

$$M_i = n_i M_A = \left(\frac{P_i V}{R T_i}\right) M_A$$

$$= \frac{M_A V}{R}\left(\frac{P_i}{T_i}\right)$$

where, M_i is the mass of air inside and M_A is the molar mass of air and $n_o = \frac{p_o V}{RT_o} = \frac{M_o}{M_A}$

where, M_o is the mass of air outside that has been displaced. If W is the load it can raise, then

$$W + M_i g = M_o g$$

$$\Rightarrow \quad W = M_o g - M_i g$$

$$= \frac{M_A V}{R}\left(\frac{P_o}{T_o} - \frac{P_i}{T_i}\right)g$$

Further, pressure inside the balloon due to membrane tension $(T) = \frac{2T}{r}$

$$= \frac{2 \times 5\ \text{N m}^{-1}}{8\ \text{m}} = 1.25\ \text{N/m}^2$$

It is given that $T_0 = 20 + 273 = 293$ K, $T_i = 60 + 273 = 333$ K

$$R = 8\ \text{m},\ V = \frac{4\pi}{3} r^3 = 2.144 \times 10^3\ \text{m}^3$$

$$P_0 = P_i = 1.013 \times 10^5\ \text{N m}^{-2}$$

And in atmosphere 21% O_2 and 79% N_2 is present

$$M_A = 21\% \text{ of } O_2 + 79\% \text{ of } N_2$$

$$= 0.21 \times 32\ \text{g} + 0.79 \times 28\ \text{g}$$

$$= 28.84\ \text{g/mole}$$

$$= 28.84 \times 10^{-3}\ \text{kg/mol}$$

Hence, we can find the weight raised by the balloon

$$W = \frac{M_A V}{R}\left(\frac{p_o}{T_o} - \frac{p_i}{T_i}\right)g$$

By substituting values, we get

$$W = \left[\frac{(28.84 \times 10^{-3}\ \text{kg/mol})(2.144 \times 10^3\ \text{m}^3)}{8.31\ \text{J/mol K}}\right]$$

$$\times \left[\frac{1.013 \times 10^5\ \text{N/m}^2}{293\ \text{K}} - \frac{1.013 \times 10^5\ \text{N/m}^2}{333\ \text{K}}\right](9.8\ \text{m/s}^2)$$

$$= 7.44 \times (1.013 \times 10^5)(4 \times 10^{-4})(9.8)\text{N} = 3014.69\ \text{N}$$

$\therefore$ Mass lifted by the balloon

$$= \frac{W}{g} = \frac{3014.69}{10} \approx 301.46\ \text{kg}.$$

$$= 301\ \text{kg}$$