CBSE Class 11 physics

Important Questions

Chapter 4

Motion in A Plane

1 Marks Questions

1.What is “Trajectory of a projectile?

Ans: The path followed by a projectile is called trajectory of projectile e.g. parabola.

2.A projectile is fired at an angle of 30^0 with the horizontal with velocity 10m/s. At what angle with the vertical should it be fired to get maximum range?

Ans: Maximum range is obtained at an angle of 45^0.

3. What is the value of angular speed for 1 revolution?

Ans: For one complete revolution, $\theta = 2\pi$ in time period t = T , $W = 2\pi/T$

4. Give an example of a body moving with uniform speed but having a variable velocity and an acceleration which remains constant in magnitude but changes in direction

Ans:A body moving in a circular path.

5.What is the direction of centripetal force when particle is following a circular path?

Ans: The direction of the centripetal force is towards the centre of the circle.

6.Two vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other. What is the value of $\vec{A}\vec{B}$?

Ans: Since $\theta = 90^o$

$\vec{A}.\vec{B} = AB\cos\theta$

$\vec{A}.\vec{B} = 0$

7.What will be the effect on horizontal range of a projectile when its initial velocity is doubled, keeping the angle of projection same?

Ans: Four times the initial horizontal range.

8.What will be the effect on maximum height of a projectile when its angle of projection is changed from 30^{o} to 60^{o}, keeping the same initial velocity of projection?

Ans: Three times the initial vertical height.

9.What is the angular velocity of the hour hand of a clock?

Ans: $\pi/6$ radian per hour.

10.A body is moving on a curved path with a constant speed. What is the nature of its acceleration?

Ans: Acceleration must be perpendicular to the direction of motion and is called centripetal acceleration.

11. State, for each of the following physical quantities, if it is a scalar or a vector:

volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

Ans. Scalar: Volume, mass, speed, density, number of moles, angular frequency

Vector: Acceleration, velocity, displacement, angular velocity

A scalar quantity is specified by its magnitude only. It does not have any direction associated with it. Volume, mass, speed, density, number of moles, and angular frequency are some of the scalar physical quantities.

A vector quantity is specified by its magnitude as well as the direction associated with it. Acceleration, velocity, displacement, and angular velocity belong to this category.

12. Pick out the two scalar quantities in the following list:

force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.

Ans. Work and **current** are scalar quantities.

Work done is given by the dot product of force and displacement. Since the dot product of two quantities is always a scalar, work is a scalar physical quantity.

Current is described only by its magnitude. Its direction is not taken into account. Hence, it is a scalar quantity.

13. Pick out the only vector quantity in the following list:

Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

Ans. Impulse

Impulse is given by the product of force and time. Since force is a vector quantity, its product with time (a scalar quantity) gives a vector quantity.

CBSE Class 11 physics

Important Questions

Chapter 4

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2 Marks Questions

1.What is the angle between two forces of 2N and 3N having resultant as 4N?

Ans: Using $R=(A^2+B^2+2AB\cos\theta)^{½}$ we get

$$4=(2^2+3^2+2\times2\times3\cos\theta)^{½}$$

$$16=4+9+12\cos\theta$$

$$\cos\theta=\frac{3}{12}=\cos^{-1}0.25$$

$$\theta=75^{\circ}32'$$

2.What is the angle of projection at which horizontal range and maximum height are equal?

Ans: Equating,

$$\frac{\mu^2\sin 2\theta}{g}=\frac{u^2\sin^2\theta}{2g}$$

$$\because\frac{u^2 Sin2\theta}{g}=R(\text{Horizontal Range})$$

$$\sin 2\theta=½\sin^2\theta$$

$$\because\frac{u^2 Sin^2\theta}{2g}=hm(\text{Maximum Height})$$

$$2\sin\theta\cos\theta = \frac{1}{2}\sin^2\theta$$
$$ie \tan\theta = 4 \Rightarrow \theta = 75.96^\circ$$

3.Prove that for elevations which exceed or fall short of 45^{o} by equal amounts the ranges are equal?

Ans: We know $R = \dfrac{u^2 \sin 2\theta}{g}$

$$\theta_1 = 45+\alpha\ ;\ \theta_2 = 45-\alpha$$

$$R_1 = \frac{u^2 \sin 2\theta_1}{g} = \frac{u^2 \sin 2(45+\alpha)}{g}$$

$$R_1 = \frac{u^2 \sin(90+2\alpha)}{g}$$

$$R_1 = \frac{u^2 Cos 2\alpha}{g} \rightarrow (1)$$

$$R_2 = \frac{u^2 Sin 2\theta_2}{g} = \frac{u^2 Sin 2(45-\alpha)}{g}s$$

$$= \frac{u^2 Sin(90-2\alpha)}{g}$$

So,

$R_1 = R_2$

4.At what range will a radar set show a fighter plane flying at 3 km above its centre and at distance of 4 km from it?

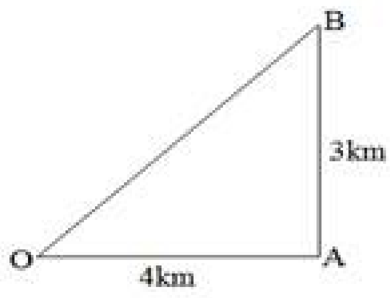
Ans: Here straight distance of the object form the radar = OB

$$OB = \sqrt{4^2 + 3^2}$$

$$OB = \sqrt{16 + 9}$$

$$OB = \sqrt{25} = 5$$

$$\Rightarrow Range = 5km$$



5.Two forces 5 and 10 kg wt are acting with an inclination of 120^{O} between them. What is the angle which the resultant makes with 10kg wt?

Ans: $F_1 = 5kgwt$

$$F_2 = 10kgwt$$

$$\theta = 120^\circ$$

$$\Rightarrow F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$$

$$and \tan\beta = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

$$\tan\beta = \frac{5 \sin 120^\circ}{10 + 5 \cos 120^\circ}$$

$$\tan\beta = \frac{5 \times \sqrt{3}/2}{10 - 5 \times 1/2}$$

$$\tan\beta = 1/\sqrt{3}$$

$$\Rightarrow \beta = \tan^{-1}\left(1/\sqrt{3} \right) = 30^\circ$$

6.A stone is thrown vertically upwards and then it returns to the thrower. Is it a projectile? Explain?

Ans:A stone cannot be considered as a projectile because a projectile must have two perpendicular components of velocities but in this case a stone has velocity in one direction while going up or coming downwards.

7.Which is greater the angular velocity of the hour hand of a watch or angular velocity of earth around its own axis?

Ans: In hour hand of a watch (T) =1 2h

$$W_H = \frac{2\pi}{12}$$

For rotation of earth T = 24h

$$We = \frac{2\pi}{24}$$

$$\Rightarrow W_H : We = 24/12 = 2$$

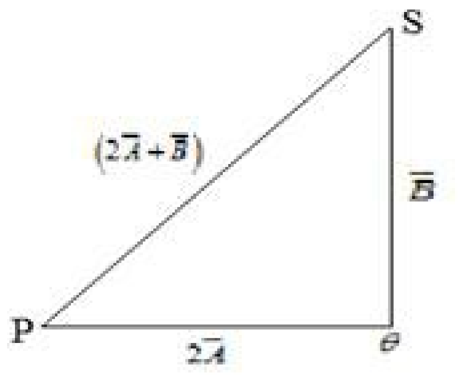
$$W_H = 2We$$

8.Why does the direction of motion of a projectile become horizontal at the highest point of its trajectory?

Ans: At the highest point vertical component of velocity becomes zero thus direction of motion of projectile becomes horizontal.

9.A vector $\vec{A}$ has magnitude 2 and another vector $\vec{B}$ have magnitude 3 and is perpendicular to each other. By vector diagram find the magnitude of $2\vec{A} + \vec{B}$ and

show its direction in the diagram.



Ans: Here $\left(\vec{PQ}\right) = 2\vec{A} = 4cm$

$\vec{QS} = \vec{B} = 3cm$

$\left|\vec{PS}\right| = \sqrt{PQ^2 + QS^2}$

$\left|\vec{PS}\right| = \sqrt{4^2 + 3^2}$

$\left|\vec{PS}\right| = 5cm$

10.Find a unit vector parallel to the resultant of the vectors
$\vec{A} = 2\hat{i} + 3\hat{J} + 4\hat{k}$ *and* $\vec{B} = 3\hat{i} - 5\hat{J} + \hat{k}$

Ans: We know $\hat{R} = \dfrac{\vec{R}}{\left|\vec{R}\right|}$

$\vec{R} = \vec{A} + \vec{B} = \left(2\hat{i} + 3\hat{J} + 4\hat{k}\right) + \left(3\hat{i} - 5\hat{J} + \hat{k}\right)$

$\vec{R} = 5\hat{i} - 2\hat{J} + 5\hat{k}$

$\left|\vec{R}\right| = \sqrt{(5)^2 + (-2)^2 + (5)^2}$

$$\left|\vec{R}\right| = \sqrt{25+4+25}$$

$$\left|\vec{R}\right| = \sqrt{54}$$

$$\Rightarrow \hat{R} = \frac{5\hat{i} - 2\hat{j} + 5\hat{k}}{\sqrt{54}}$$

11.A stone tied at the end of string is whirled in a circle. If the string breaks, the stone flies away tangentially. Why?

Ans: When a stone is moving around a circular path, its velocity acts tangent to the circle. When the string breaks, the centripetal force will not act. Due to inertia, the stone continues to move along the tangent to circular path, and flies off tangentially to the circular path.

12. What are the two angles of projection of a projectile projected with velocity 30m/s, so that the horizontal range is 45m. Take, g = 10m/s^2.

Ans:

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(30)^2 \sin 2\theta}{10} = 45$$

$$\Rightarrow \sin 2\theta = \frac{450}{(30)^2}$$

$$\sin 2\theta = 1/2$$

$$2\theta = 30^\circ \ or \ 150^\circ \Rightarrow \theta = 15^\circ \ or \ 75^\circ$$

13.The blades of an aeroplane propeller are rotating at the rate of 600 revolutions per minute. Calculate its angular velocity.

Ans: $\nu = 600 \,\text{revolutions/min}$

$$v = \frac{600}{60} \text{ revolutions/sec.}$$

$$w = 2\pi v = 2 \times \pi \times \frac{600}{60}$$

$$w = 20\pi \ rad / s$$

14.What is a uniform circular motion? Explain the terms time period, frequency and angular velocity. Establish relation between them.

Ans: When an object moves in a circular path with constant speed then the motion is called uniform circular motion

Time period – The time taken by the object to complete one revolution

Frequency – The total number of revolutions in one second is called the frequency.

Angular velocity – It is defined as the time rate of change of angular displacement.

$$W = \frac{2\pi}{T} = 2\pi v \qquad \left(\because \frac{1}{T} = v \right)$$

15.A body of mass m is thrown with velocity $'\upsilon'$ at angle of 30^{o} to the horizontal and another body B of the same mass is thrown with velocity υ at an angle of 60^{o} to the horizontal. Find the ratio of the horizontal range and maximum height of A and B?

Ans:(1) When $\theta = 30^{\circ} R_A = \frac{\mu^2}{g} \sin 2\left(30^{\circ}\right)$

$$R_A = \frac{\mu^2}{g} \times \frac{\sqrt{3}}{2}$$

When $\theta = 60^{\circ} R_B = \frac{\mu^2}{g} \sin 2\left(60^{\circ}\right)$

$$R_B = \frac{\mu^2}{g} \times \frac{\sqrt{3}}{2}$$

$$R_A : R_B = 1:1$$

(2) When $\theta = 30^\circ \; H_A = \frac{\mu^2}{g} \sin^2 30^\circ$

$$H_A = \frac{\mu^2}{2g}\left(\frac{1}{4}\right)$$

When $\theta = 60^\circ \; H_B = \frac{\mu^2}{2g} \sin^2 60^\circ$

$$H_B = \frac{\mu^2}{g}\left(\tfrac{3}{4}\right)$$

$$H_A : H_B = 1:3$$

16. Read each statement below carefully and state with reasons, if it is true or false:

(a) The magnitude of a vector is always a scalar, (b) each component of a vector is always a scalar, (c) the total path length is always equal to the magnitude of the displacement vector of a particle. (d) the average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time, (e) Three vectors not lying in a plane can never add up to give a null vector.

Ans.

(a) True

(b) False

(c) False

(d) True

(e) True

Explanation:

(a) The magnitude of a vector is a number. Hence, it is a scalar.

(b) Each component of a vector is also a vector.

(c) Total path length is a scalar quantity, whereas displacement is a vector quantity. Hence, the total path length is always greater than the magnitude of displacement. It becomes equal to the magnitude of displacement only when a particle is moving in a straight line.

(d) It is because of the fact that the total path length is always greater than or equal to the magnitude of displacement of a particle.

(e) Three vectors, which do not lie in a plane, cannot be represented by the sides of a triangle taken in the same order.

17. State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful:

(a) adding any two scalars, (b) adding a scalar to a vector of the same dimensions, (c) multiplying any vector by any scalar, (d) multiplying any two scalars, (e) adding any two vectors, (f) adding a component of a vector to the same vector.

Ans. (a) Meaningful

(b) Not Meaningful

(c) Meaningful

(d) Meaningful

(e) Meaningful

(f) Meaningful

Explanation:

(a) The addition of two scalar quantities is meaningful only if they both represent the same physical quantity.

(b) The addition of a vector quantity with a scalar quantity is not meaningful.

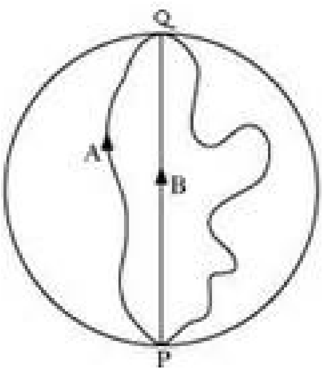
(c) A scalar can be multiplied with a vector. For example, force is multiplied with time to give impulse.

(d) A scalar, irrespective of the physical quantity it represents, can be multiplied with another scalar having the same or different dimensions.

(e) The addition of two vector quantities is meaningful only if they both represent the same physical quantity.

(f) A component of a vector can be added to the same vector as they both have the same dimensions.

18. Three girls skating on a circular ice ground of radius 200 m start from a point Pon the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Fig. 4.20. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of the path skated?



Ans.

Displacement is given by the minimum distance between the initial and final positions of a particle. In the given case, all the girls start from point P and reach point Q. The magnitudes of their displacements will be equal to the diameter of the ground.

Radius of the ground = 200 m

Diameter of the ground = 2×200 = 400 m

Hence, the magnitude of the displacement for each girl is 400 m. This is equal to the actual length of the path skated by girl B.

19. A man can swim with a speed of 4.0 km/h in still water. How long does he take to cross a river 1.0 km wide if the river flows steadily at 3.0 km/h and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?

Ans. Speed of the man, v_m = 4 km/h

Width of the river = 1 km

Time taken to cross the river $= \frac{\text{Width of the river}}{\text{Speed of the river}}$

$$= \frac{1}{4}\text{h} = \frac{1}{4} \times 60 = 15\text{min}$$

Speed of the river, v_r = 3 km/h

Distance covered with flow of the river = $v_r \times t$

$$= 3 \times \frac{1}{4} = \frac{3}{4}$$

$$= \frac{3}{4} \times 1000 = 750\text{m}$$

20. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?

Ans. Length of the string, l = 80 cm = 0.8 m

Number of revolutions = 14

Time taken = 25 s

Frequency, $\nu = \frac{\text{Number of revolutions}}{\text{Time taken}} = \frac{14}{25}\text{Hz}$

Angular frequency, $\omega = 2\pi\nu$

$$= 2 \times \frac{22}{7} \times \frac{14}{25} = \frac{88}{25}\text{rads}^{-1}$$

Centripetal acceleration, $a_c = \omega^2 r$

$$= \left(\frac{88}{25}\right)^2 \times 0.8$$

$$= 9.91 \text{m} / \text{s}^2$$

The direction of centripetal acceleration is always directed along the string, toward the centre, at all points.

21. An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity.

Ans.

Radius of the loop, *r*= 1 km = 1000 m

Speed of the aircraft, *v*= 900 km/h $= 900 \times \frac{5}{18} = 250 \text{m} / \text{s}$

Centripetal acceleration, $a_c = \frac{v^2}{r}$

$$= \frac{(250)^2}{1000} = 62.5 \text{m} / \text{s}^2$$

Acceleration due to gravity, g = $9.8 \, m / s^2$

$$\frac{a_c}{g} = \frac{62.5}{9.8} = 6.38$$

$$a_c = 6.38g$$

22. Read each statement below carefully and state, with reasons, if it is true or false:

(a) The net acceleration of a particle in circular motion is *always* along the radius of the circle towards the centre

(b) The velocity vector of a particle at a point is *always* along the tangent to the path of the particle at that point

(c) The acceleration vector of a particle in *uniform* circular motion averaged over one cycle is a null vector

Ans.

(a) False

The net acceleration of a particle in circular motion is not always directed along the radius of the circle toward the centre. It happens only in the case of uniform circular motion.

(b) True

At a point on a circular path, a particle appears to move tangentially to the circular path. Hence, the velocity vector of the particle is always along the tangent at a point.

(c) True

In uniform circular motion (UCM), the direction of the acceleration vector points toward the centre of the circle. However, it constantly changes with time. The average of these vectors over one cycle is a null vector.

23. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?

Ans.

Maximum horizontal distance, R = 100 m

The cricketer will only be able to throw the ball to the maximum horizontal distance when the angle of projection is 45°, i.e., θ= 45°.

The horizontal range for a projection velocity v, is given by the relation:

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$100 = \frac{u^2}{g} \sin 90^\circ$$

$$\frac{u^2}{g} = 100 \quad ....(i)$$

The ball will achieve the maximum height when it is thrown vertically upward. For such motion, the final velocity v is zero at the maximum height H.

Acceleration, a = $-g$ Using the third equation of motion:

$$v^2 - u^2 = -2gH$$

$$H = \frac{1}{2} \times \frac{u^2}{g} = \frac{1}{2} \times 100 = 50m$$

24. The position of a particle is given by $r = 3.0t\hat{i} - 2.0t^2\hat{j} + 4.0\hat{k}\,m$

Where t is in seconds and the coefficients have the proper units for r to be in metres.

(a) Find the v and a of the particle?

(b) What is the magnitude and direction of velocity of the particle at t = 2.0 s?

Ans.

(a) $\bar{v}(t) = \left(3.0\hat{i} - 4.0t\,\hat{j}\right)$

The position of the particle is given by:

$$\bar{r} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k}$$

Velocity $\bar{v}$, of the particle is given as:

$$\bar{v} = \frac{\overline{dr}}{dt} = \frac{d}{dt}\left(3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k}\right)$$

$$\therefore \bar{v} = 3.0\,\hat{i} - 4.0t\,\hat{j}$$

Acceleration $\bar{a}$, of the particle is given as:

$$\bar{a} = \frac{\overline{dv}}{dt} = \frac{d}{dt}\left(3.0\,\hat{i} - 4.0t\,\hat{j}\right)$$

$$\therefore \bar{a} = -4.0\,\hat{j}$$

(b) 8.54 m/s, 69.45°below the *x*-axis

We have velocity sector, $\bar{v} = 3.0\,\hat{i} - 4.0t\,\hat{j}$

At

$t = 2.0s:$

$$\bar{v} = 3.0\,\hat{i} - 8.0\,\hat{j}$$

The magnitude of velocity is given by:

$$|\bar{v}| = \sqrt{3^2 + (-8)^2} = \sqrt{73} = 8.54 m/s$$

$$\text{Direction}, \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

$$= \tan^{-1}\left(\frac{-8}{3}\right) = -\tan^{-1}(2.667)$$

$$= -69.45^{\circ}$$

The negative sign indicates that the direction of velocity is below the *x*-axis.

25: For any arbitrary motion in space, which of the following relations are true:

(a) $v_{average} = \left(\frac{1}{2}\right)(v(t_1) + v(t_2))$

(b) $v_{average} = \frac{[r(t_2) - r(t_1)]}{(t_2 - t_1)}$

(c) $v(t) = v(0) + at$

(d) $r(t) = r(0) + v(0)t + \frac{1}{2}at^2$

(e) $a_{average} = \frac{[v(t_2) - v(t_1)]}{(t_2 - t_1)}$

(The 'average' stands for average of the quantity over the time interval *t*1to *t*2)

Ans. (b)and **(e)**

(a) It is given that the motion of the particle is arbitrary. Therefore, the average velocity of the particle cannot be given by this equation.

(b) The arbitrary motion of the particle can be represented by this equation.

(c) The motion of the particle is arbitrary. The acceleration of the particle may also be non-uniform. Hence, this equation cannot represent the motion of the particle in space.

(d) The motion of the particle is arbitrary; acceleration of the particle may also be non-uniform. Hence, this equation cannot represent the motion of particle in space.

(e) The arbitrary motion of the particle can be represented by this equation.

26. Read each statement below carefully and state, with reasons and examples, if it is true or false:

A scalar quantity is one that
(a) is conserved in a process
(b) can never take negative values
c) must be dimensionless
(d) does not vary from one point to another in space
(e) has the same value for observers with different orientations of axes

Ans.(a) False
Despite being a scalar quantity, energy is not conserved in inelastic collisions.

(b) False
Despite being a scalar quantity, temperature can take negative values.

(c) False
Total path length is a scalar quantity. Yet it has the dimension of length.

(d) False
A scalar quantity such as gravitational potential can vary from one point to another in space.

(e) True
The value of a scalar does not vary for observers with different orientations of axes.

27. (a) A vector has magnitude and direction. Does it have a location in space? (b) Can it vary with time? (c) Will two equal vectors a and b at different locations in space necessarily have identical physical effects? Give examples in support of your answer.

Ans.(a) No; (b) Yes; (c) No

Generally speaking, a vector has no definite locations in space. This is because a vector remains invariant when displaced in such a way that its magnitude and direction remain the same. However, a position vector has a definite location in space.

A vector can vary with time. For example, the displacement vector of a particle moving with a certain velocity varies with time.

Two equal vectors located at different locations in space need not produce the same physical effect. For example, two equal forces acting on an object at different points can cause the body to rotate, but their combination cannot produce an equal turning effect.

28. (a) A vector has both magnitude and direction. Does it mean that anything that has magnitude and direction is necessarily a vector? (b) The rotation of a body can be specified by the direction of the axis of rotation, and the angle of rotation about the axis. Does that make any rotation a vector?

Ans.(a) No; (b) No
A physical quantity having both magnitude and direction need not be considered a vector. For example, despite having magnitude and direction, current is a scalar quantity. The essential requirement for a physical quantity to be considered a vector is that it should follow the law of vector addition.
Generally speaking, the rotation of a body about an axis is not a vector quantity as it does not follow the law of vector addition. However, a rotation by a certain small angle follows the law of vector addition and is therefore considered a vector.

29. Can you associate vectors with (a) the length of a wire bent into a loop, (b) a plane area, (c) a sphere? Explain.
Ans. (a) No; (b) Yes; (c) No
(a) One cannot associate a vector with the length of a wire bent into a loop.
(b) One can associate an area vector with a plane area. The direction of this vector is normal, inward or outward to the plane area.
(c) One cannot associate a vector with the volume of a sphere. However, an area vector can be associated with the area of a sphere.

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Important Questions

Chapter 4

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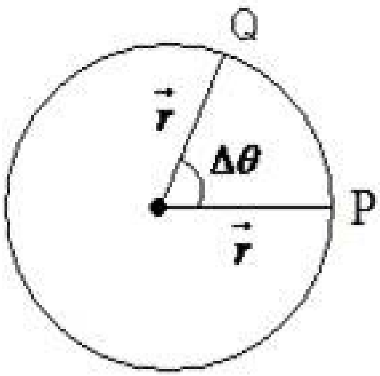
3 Marks Questions

1.Derive expressions for velocity and acceleration for uniform circular motion.

OR Derive expression for linear acceleration in uniform circular motion.



Ans: (1) IF $PQ = \Delta l$ then $\vartheta = \dfrac{\Delta l}{\Delta t}$ ------(1)

And angular velocity $W = \dfrac{\Delta\theta}{\Delta l}$

Using $\theta = l/r \Rightarrow \Delta\theta = \Delta l/r$ ------(2)

$\Delta l = V\Delta$ t and $\Delta\theta = W\Delta t$

Substituting in (1) $W\cancel{\Delta t} = V\cancel{\Delta t}/r \Rightarrow v = rw$

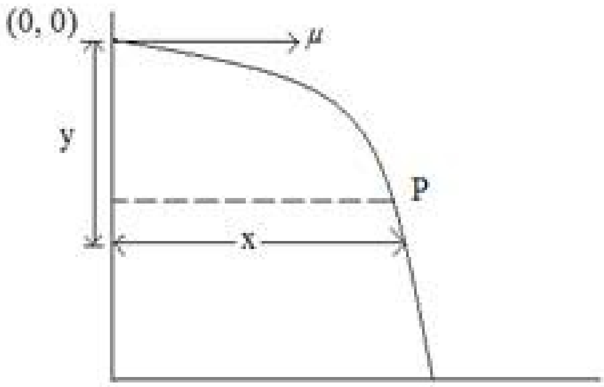
(2) Since $a = dv/dt = r\,dw/\mathrm{d}t = w\,dr/\mathrm{d}t = w\upsilon = \dfrac{\upsilon}{r}\times\upsilon = \upsilon^2/r$

$\Rightarrow a = \upsilon^2/r$

2.Derive an equation for the path of a projectile fired parallel to horizontal.

Ans: Let a projectile having initial uniform horizontal velocity u be under the influence of gravity, then at any instant t at position P the horizontal and vertical.

For horizontal motion



$$S = ut + \frac{1}{2}at^2$$

$$s = x,\ u = u,\ t = t$$

$$and\ a = 0$$

$$x = ut$$

$$t = x/u ----(1)$$

For vertical motion

$$S = ut + \frac{1}{2}at^2$$

$$s = -y,\ u = 0,\ t = t,\ a = -g$$

We get $-y = -\frac{1}{2}gt^2$

Or $y = \frac{1}{2} g t^2 - - - - - (2)$

Using equation (1) and (2)

$$y = \frac{1}{2} g \left(x/u \right)^2 = \frac{1}{2} g \frac{x^2}{u^2}$$

3.(a) Define time of flight and horizontal range?

(b) From a certain height above the ground a stone A is dropped gently. Simultaneously another stone B is fired horizontally. Which of the two stones will arrive on the ground earlier?

Ans: (a) **Time of flight** – The time taken by the projectile to complete its trajectory is called time of flight.

Horizontal Range – The maximum horizontal distance covered by the projectile form the foot of the tower to the point where projectile hits the ground is called horizontal range.

(b) Both the stones will reach the ground simultaneously because the initial vertical velocity in both cases is zero and both are falling with same acceleration equal to acceleration due to gravity.

4.At what point of projectile motion (i) potential energy maximum (ii) Kinetic energy maximum (iii) total mechanical energy is maximum

Ans:(1) P.E. Will be maximum at the highest point

(P.E.) highest point = mgH

$$(P.E)_H = mg \left(\frac{\mu^2 \sin^2 \theta}{2g} \right)$$

$$(P.E)_H = \frac{1}{2} m \mu^2 \sin^2 \theta$$

(2)K.E will be minimum at the highest point

$$(K.E.)_H = \frac{1}{2} m (v_H)^2$$

(Vertical component of velocity is zero)

$$(K.E.)_H = \frac{1}{2} m \vartheta^2 \cos^2 \theta$$

(3) Total mechanical energy

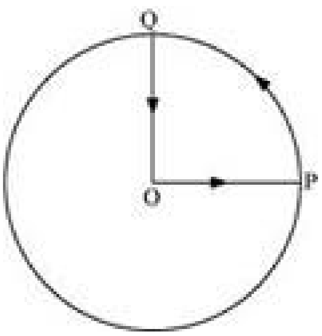
$$(K.E.)_H + (P.E.)_H$$

$$\frac{1}{2} m u^2 \cos^2 \theta + \frac{1}{2} m u^2 \sin^2 \theta$$

$$\frac{1}{2} m u^2 \left(\cos^2 \theta + \sin^2 \theta\right)$$

$$\frac{1}{2} m u^2$$

5. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge Pof the park, then cycles along the circumference, and returns to the centre along QO as shown in Fig. 4.21. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist?



Ans. (a) Displacement is given by the minimum distance between the initial and final positions of a body. In the given case, the cyclist comes to the starting point after cycling for 10 minutes. Hence, his net displacement is zero.

(b) Average velocity is given by the relation:

Average velocity $= \dfrac{\text{Net displacement}}{\text{Total time}}$

Since the net displacement of the cyclist is zero, his average velocity will also be zero.

(c) Average speed of the cyclist is given by the relation:

Average speed $= \dfrac{\text{Total path length}}{\text{Total time}}$

Total path length = OP + PQ + QO

$$= 1 + \frac{1}{4}(2\pi \times 1) + 1$$

$$= 2 + \frac{1}{2}\pi = 3.570\text{km}$$

Time taken = 10 min $= \dfrac{10}{60} = \dfrac{1}{6}\text{h}$

$\therefore$Average speed $= \dfrac{3.570}{\frac{1}{6}} = 21.42\text{km/h}$

6. A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity? Are the two equal?

Ans. (a) Total distance travelled = 23 km

Total time taken = 28 min $= \frac{28}{60}$h

$\therefore$Average speed of the taxi $= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$$= \frac{23}{\left(\frac{28}{60}\right)} = 49.29 \text{km} / \text{h}$$

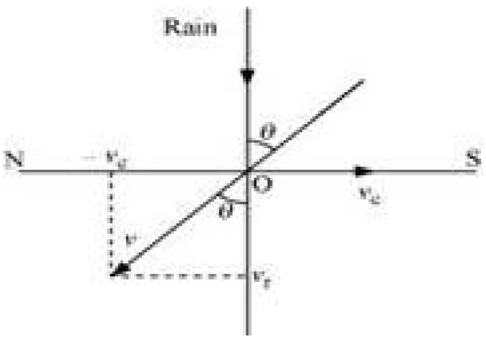
(b) Distance between the hotel and the station = 10 km = Displacement of the car

$\therefore$Average velocity $= \frac{10}{\frac{28}{60}} = 21.43 \text{km} / \text{h}$

Therefore, the two physical quantities (average speed and average velocity) are not equal.

7. Rain is falling vertically with a speed of 30 m s^{-1}. A woman rides a bicycle with a speed of 10 m s^{-1} in the north to south direction. What is the direction in which she should hold her umbrella?

Ans. The described situation is shown in the given figure.



Here,

v_c = Velocity of the cyclist

v_r = Velocity of falling rain

In order to protect herself from the rain, the woman must hold her umbrella in the direction of the relative velocity (*v*) of the rain with respect to the woman.

$$v = v_t + (-v_c)$$

$$= 30 + (-10) = 20 m/s$$

$$\tan\theta = \frac{v_c}{v_r} = \frac{10}{30}$$

$$\theta = \tan^{-1}(0.333) \approx 18^\circ$$

Hence, the woman must hold the umbrella toward the south, at an angle of nearly 18° with the vertical.

8. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of $40\ ms^{-1}$ can go without hitting the ceiling of the hall?

Ans. Speed of the ball, *u* = 40 m/s

Maximum height, *h* = 25 m

In projectile motion, the maximum height reached by a body projected at an angle *θ*, is given by the relation:

$$h = \frac{u^2 \sin^2\theta}{2g}$$

$$25 = \frac{(40)^2}{2 \times 9.8}$$

$\sin^2\theta$ = 0.30625

sin *θ* = 0.5534

∴*θ* = $\sin^{-1}$ (0.5534) = 33.60°

Horizontal range, $R = \frac{u^2 \sin 2\theta}{g}$

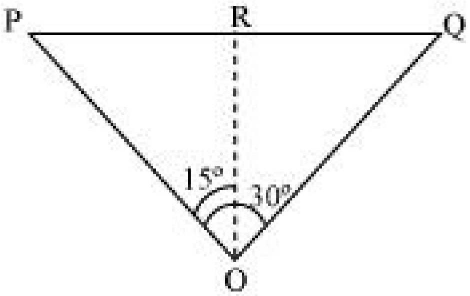
$= \frac{(40)^2 \times \sin 2 \times 33.60}{9.8}$

$= \frac{1600 \times \sin 67.2}{9.8}$

$= \frac{1600 \times 0.922}{9.8} = 150.53\text{m}$

9. An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is 30°, what is the speed of the aircraft?

Ans. The positions of the observer and the aircraft are shown in the given figure.



Height of the aircraft from ground, OR = 3400 m

Angle subtended between the positions, ∠POQ = 30

Time = 10 s

In ΔPRO:

$\tan 15^\circ = \frac{PR}{OR}$

$PR = OR \tan 15^\circ$

$= 3400 \times \tan 15^\circ$

ΔPRO is similar to ΔRQO.

∴PR = RQ

PQ = PR + RQ

= 2PR = 2 × 3400 tan 15°

= 6800 × 0.268 = 1822.4 m

∴Speed of the aircraft $= \frac{1822.4}{10} = 182.24 m/s$

10. A bullet fired at an angle of 30° with the horizontal hits the ground 3.0 km away. By adjusting its angle of projection, can one hope to hit a target 5.0 km away? Assume the *muzzle* speed to the fixed, and neglect air resistance.

Ans. No

Range, R = 3 km

Angle of projection, θ = 30°

Acceleration due to gravity, g = $9.8\ m/s^2$

Horizontal range for the projection velocity u_0, is given by the relation:

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$3 = \frac{u_0^2}{g} \sin 60^\circ$$

$$\sqrt{3} \times \sqrt{3} = \frac{u_0^2}{g} \times \frac{\sqrt{3}}{2} \qquad \left[\because 3 = \sqrt{3} \times \sqrt{3}\right]$$

$$\frac{u_0^2}{g} = 2\sqrt{3} \qquad \ldots(1)$$

The maximum range (R_{max}) is achieved by the bullet when it is fired at an angle of 45°with the horizontal, that is,

$$R_{max} = \frac{u_0^2}{g} \qquad \ldots(2)$$

On comparing equations (*1*) and (*2*), we get:

$$R_{max} = 2\sqrt{3} = 2 \times 1.732 = 3.46 km$$

Hence, the bullet will not hit a target 5 km away.

CBSE Class 11 physics

Important Questions

Chapter 4

Motion in A Plane

4 Marks Questions

1. Given a + b + c + d = 0, which of the following statements are correct:

(a) a, b, c, and d must each be a null vector,

(b) The magnitude of (a + c) equals the magnitude of (b+ d),

(c) The magnitude of *a* can never be greater than the sum of the magnitudes of b, c, and d,

(d) b + c must lie in the plane of a and d if a and d are not collinear, and in the line

Ans. (a) Incorrect
In order to make **a** + **b** + **c** + **d** = **0**, it is not necessary to have all the four given vectors to be null vectors. There are many other combinations which can give the sum zero.

(b) Correct
a + **b** + **c** + **d** = **0**

a + **c** = **-(b** + **d)**

Taking modulus on both the sides, we get:

$$|a+c| = |-(b+d)| = |b + d|$$

Hence, the magnitude of (**a** + **c**) is the same as the magnitude of (**b** + **d**).

(c) Correct
a + **b** + **c** + **d** = **0**
a = **(b** + **c** + **d)**
Taking modulus both sides, we get:

| **a** | = | **b** + **c** + **d** |

$|a| \leq |a| + |b| + |c|$ ... (*i*)

Equation (*i*) shows that the magnitude of **a** is equal to or less than the sum of the magnitudes of **b**, **c**, and **d**.

Hence, the magnitude of vector *a* can never be greater than the sum of the magnitudes of **b**, **c**, and **d**.

(d) Correct

For **a** + **b** + **c** + **d** = 0

a + **(b** + **c)** + **d** = 0

The resultant sum of the three vectors **a**, (**b** + **c**), and **d** can be zero only if (**b** + **c**) lie in a plane containing **a** and **d**, assuming that these three vectors are represented by the three sides of a triangle.

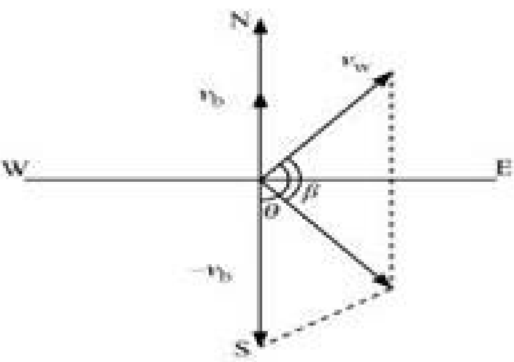
If **a** and **d** are collinear, then it implies that the vector (**b** + **c**) is in the line of **a** and **d**. This implication holds only then the vector sum of all the vectors will be zero.

2. In a harbour, wind is blowing at the speed of 72 km/h and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at a speed of 51 km/h to the north, what is the direction of the flag on the mast of the boat?

Ans. Velocity of the boat, v_b = 51 km/h

Velocity of the wind, v_w = 72 km/h

The flag is fluttering in the north-east direction. It shows that the wind is blowing toward the north-east direction. When the ship begins sailing toward the north, the flag will move along the direction of the relative velocity (v_{wb}) of the wind with respect to the boat.



The angle between v_w *and* $(-v_b)$ = 90° + 45°

$$\tan\beta = \frac{51\sin(90+45)}{72+51\cos(90+45)}$$

$$= \frac{51\sin 45}{72+51(-\cos 45)} = \frac{51\times\frac{1}{\sqrt{2}}}{72-51\times\frac{1}{\sqrt{2}}}$$

$$= \frac{51}{72\sqrt{2}-51} = \frac{51}{72\times 1.414-51} = \frac{51}{50.800}$$

$$\therefore \beta = \tan^{-1}(1.0038) = 45.11^0$$

Angle with respect to the east direction = $45.11° - 45°$ = 0.11°

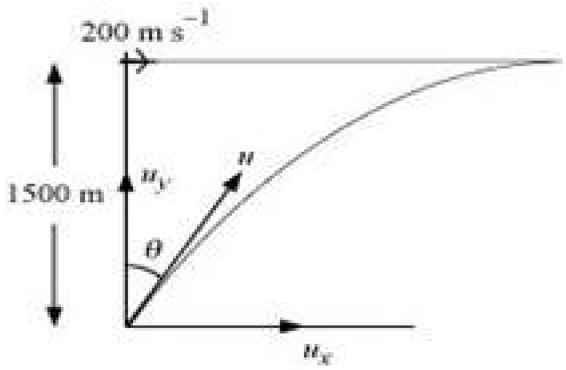
Hence, the flag will flutter almost due east.

3. A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 km/h passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600 m s^{-1} to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit? (Take *g* = 10 m s^{-2}).

Ans. Height of the fighter plane = 1.5 km = 1500 m

Speed of the fighter plane, *v* = 720 km/h = 200 m/s

Let *θ* be the angle with the vertical so that the shell hits the plane. The situation is shown in the given figure.



Muzzle velocity of the gun, *u* = 600 m/s

Time taken by the shell to hit the plane = *t*

Horizontal distance travelled by the shell = $u_x t$

Distance travelled by the plane = *vt*

The shell hits the plane. Hence, these two distances must be equal.

$u_x t = vt$

$u \sin\theta = v$

$$\sin\theta = \frac{v}{u}$$

$$= \frac{200}{600} = \frac{1}{3} = 0.33$$

$\theta = \sin^{-1}(0.33)$

$= 19.5^\circ$

In order to avoid being hit by the shell, the pilot must fly the plane at an altitude (*H*) higher than the maximum height achieved by the shell.

$$\therefore H = \frac{u^2 \sin^2(90-\theta)}{2g}$$

$$= \frac{(600)^2 \cos^2\theta}{2g}$$

$$= \frac{360000 \times \cos^2 19.5}{2 \times 10}$$

$= 18000 \times (0.943)^2$

$= 16006.42\text{m}$

$\approx 16\text{km}$

CBSE Class 11 physics

Important Questions

Chapter 4

Motion in A Plane

5 Marks Questions

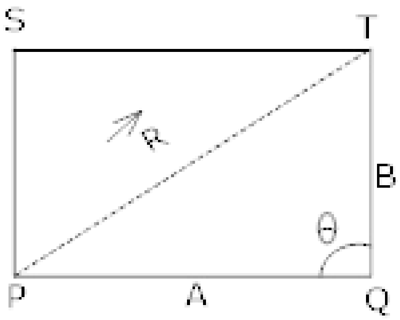
1.(a) What is the angle between $\vec{A}$ and $\vec{B}$ if $\vec{A}$ and $\vec{B}$ denote the adjacent sides of a parallelogram drawn form a point and the area of the parallelogram is $\frac{1}{2}AB$?

(b) State and prove triangular law of vector addition?

Ans:(a) Area of a parallelogram = $\left|\vec{A}\times\vec{B}\right|$

Area of parallelogram = A B Sin θ (∵ Applying cross product)

Given, area of parallelogram = $\frac{1}{2}AB$



So, $\frac{1}{2} = AB = AB\ Sin\theta$

$\frac{1}{2} = Sin\theta$

$\theta = Sin^{-1}\left(\frac{1}{2}\right)$

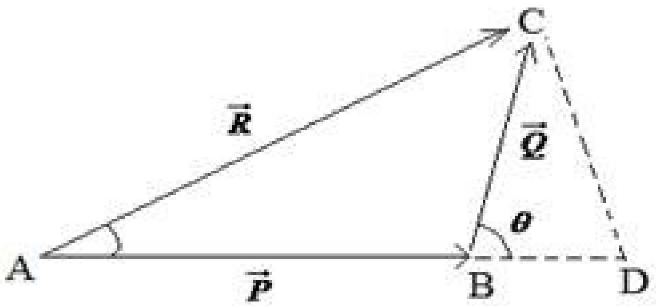
$\theta = 30^0$

(b) Triangular law of vector addition states that if two vectors can be represented both in magnitude and direction by the sides of a triangle taken in order then their resultant is given by the third side of the triangle taken in opposite order.

Proof à in Δ ADC

$$(AC)^2 = (AD)^2 + (DC)^2$$

$$(AC)^2 = (AB + BD)^2 + (DC)^2$$



$$(AC)^2 = (AB)^2 + (BD)^2 + 2(AB)(BD) + (DC)^2$$

$$(AC)^2 = (P^2) + (Q\cos\theta)2 + 2(P)(Q\cos\theta) + (Q\sin\theta)^2$$

$$(AC)^2 = P^2 + Q^2(\sin^2\theta + \cos^2\theta)\left(\because \frac{BD}{BC} = \cos\theta\right)$$

$$+ 2PQ\cos\theta\left(\because \frac{CD}{BC} = \sin\theta\right)$$

$$(R)^2 = P^2 + Q + 2PQ\cos\theta\left(\because \sin^2\theta + \cos^2 Q\right)$$

$$R = \sqrt{P^2 + Q + 2P\theta\cos\theta}$$

2. Establish the following vector inequalities geometrically or otherwise:

(a) $|a + b| \le |a| + |b|$

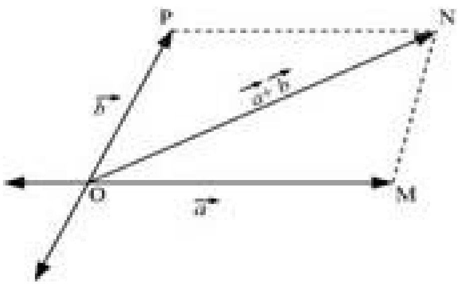
(b) $\left|a+b\right| \geq \left|\left|a\right|-\left|b\right|\right|$

(c) $\left|a-b\right| \leq \left|a\right|+\left|b\right|$

(d) $\left|a-b\right| \geq \left|\left|a\right|-\left|b\right|\right|$

When does the equality sign above apply?

Ans.(a) Let two vectors $\bar{a}$ and $\bar{b}$ be represented by the adjacent sides of a parallelogram OMNP, as shown in the given figure.



Here, we can write:

$$\left|\overline{OM}\right| = \left|\bar{a}\right| \text{ .....(i)}$$

$$\left|\overline{MN}\right| = \left|\overline{OP}\right| = \left|\bar{b}\right| \text{ .....(ii)}$$

$$\left|\overline{ON}\right| = \left|\bar{\alpha}+\bar{b}\right| \text{ ......(iii)}$$

In a triangle, each side is smaller than the sum of the other two sides.

Therefore, in ΔOMN, we have:

ON < (OM + MN)

$$\left|\bar{a}+\bar{b}\right| \leq \left|\bar{a}\right|+\left|\bar{b}\right| \text{ .....(iv)}$$

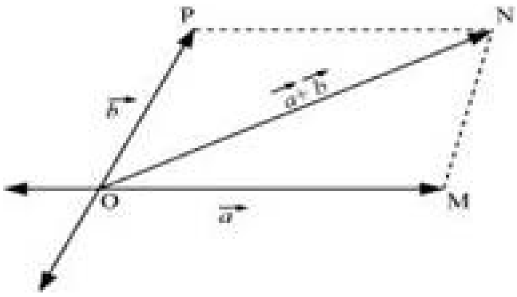
If the two vectors $\bar{a}$ and $\bar{b}$ act along a straight line in the same direction, then we can write:

$$\left|\bar{a}+\bar{b}\right| = \left|\bar{a}\right|+\left|\bar{b}\right| \text{.....(v)}$$

Combining equations (*iv*) and (*v*), we get:

$$\left|\bar{a}+\bar{b}\right| \le \left|\bar{a}\right| + \left|\bar{b}\right|$$

(b) Let two vectors $\bar{a}$ and $\bar{b}$ be represented by the adjacent sides of a parallelogram OMNP, as shown in the given figure.



Here, we have:

$$\left|\overline{OM}\right| = \left|\bar{\alpha}\right| \text{....(i)}$$

$$\left|\overline{MN}\right| = \left|\overline{OP}\right| = \left|\bar{b}\right| \text{.....(ii)}$$

$$\left|\overline{ON}\right| = \left|\bar{a}+\bar{b}\right| \text{......(iii)}$$

In a triangle, each side is smaller than the sum of the other two sides.

Therefore, in ΔOMN, we have:

$$ON + MN > OM$$

$$ON + OM > MN$$

$$\left|\overline{ON}\right| > \left|\overline{OM} - \overline{OP}\right| \quad (\because OP = MN)$$

$$\left|\bar{a}+\bar{b}\right| > \left|\left|\bar{a}\right| - \left|\bar{b}\right|\right| \text{ ... } (iv)$$

If the two vectors $\bar{a}$ nd $\bar{b}$ act along a straight line in the same direction, then we can write:

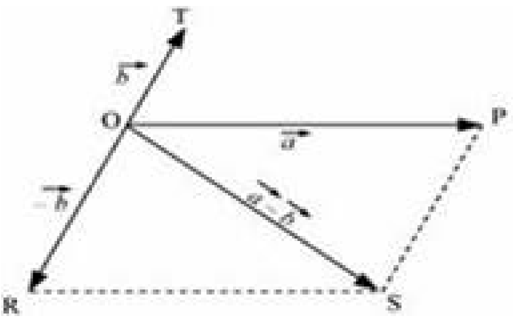
$$\left|\bar{a}+\bar{b}\right| = \left|\left|\bar{a}\right| - \left|\bar{b}\right|\right| \ldots (v)$$

Combining equations (*iv*) and (*v*), we get:

$$\left|\bar{a}+\bar{b}\right| \geq \left|\left|\bar{a}\right| - \left|\bar{b}\right|\right|$$

$$\left|\bar{a}+\bar{b}\right| > \left|\left|\bar{a}\right| - \left|\bar{b}\right|\right|$$

(c) Let two vectors $\bar{a}$ and $\bar{b}$ be represented by the adjacent sides of a parallelogram PORS, as shown in the given figure.



Here we have:

$$\left|\overline{OR}\right| = \left|\overline{PS}\right| = \left|\bar{b}\right| \ldots\ldots(i)$$

$$\left|\overline{OP}\right| = \left|\bar{a}\right| \ldots\ldots(ii)$$

In a triangle, each side is smaller than the sum of the other two sides. Therefore, in $\triangle OP$ we have:

OS<OP+PS

$$\left|\bar{a}-\bar{b}\right| < \left|\bar{a}\right| + \left|-\bar{b}\right| \quad \left|\bar{a}-\bar{b}\right| < \left|\bar{a}\right| + \left|\bar{b}\right| \ldots\ldots(iii)$$

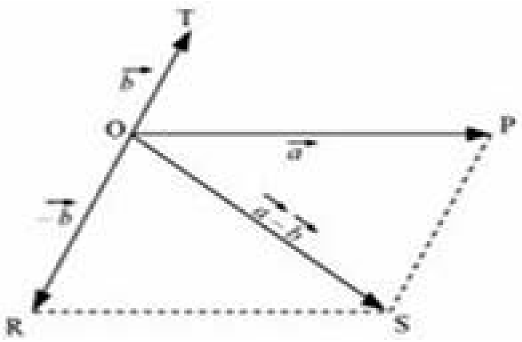
If the two vectors act in a straight line but in opposite directions, then we can write:

$$\left|\bar{a}-\bar{b}\right| < \left|\bar{a}\right| + \left|\bar{b}\right| \ldots\ldots(iv)$$

Combining equations (iii) and (iv) we get:

$$\left|\bar{a}-\bar{b}\right| \le \left|\bar{a}\right|+\left|\bar{b}\right|$$

(d) Let two vectors $\bar{a}$ and $\bar{b}$ be represented by the adjacent sides of a parallelogram PORS, as shown in the given figure.



The following relations can be written for the given parallelogram.

$$OS + PS > OP \quad .....(i)$$

$$OS > OP - PS \quad .....(ii)$$

$$\left|\bar{a}-\bar{b}\right| > \left|\bar{a}\right|-\left|\bar{b}\right| \quad ....(iii)$$

The quantity on the LHS is always positive and that on the RHS can be positive or negative. To make both quantities positive, we take modulus on both sides as:

$$\left\|\bar{a}-\bar{b}\right\| > \left\|\bar{a}\right|-\left|\bar{b}\right\|$$

$$\left|\bar{a}-\bar{b}\right| > \left\|\bar{a}\right|-\left|\bar{b}\right\| \quad ......(iv)$$

If the two vectors act in a straight line but in the opposite directions, then we can write:

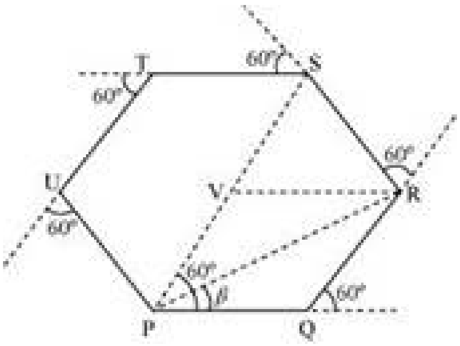
$$\left|\bar{a}-\bar{b}\right| = \left\|\bar{a}\right|-\left|\bar{b}\right\| \quad .....(v)$$

Combining equations (iv) and (v), we get:

$$\left|\bar{a}-\bar{b}\right| \ge \left\|\bar{a}\right|-\left|\bar{b}\right\|$$

3. On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

Ans.



The path followed by the motorist is a regular hexagon with side 500 m, as shown in the given figure

Let the motorist start from point P.

The motorist takes the third turn at S.

∴Magnitude of displacement = PS = PV + VS = 500 + 500 = 1000 m

Total path length = PQ + QR + RS = 500 + 500 +500 = 1500 m

The motorist takes the sixth turn at point P, which is the starting point.

∴Magnitude of displacement = 0

Total path length = PQ + QR + RS + ST + TU + UP

= 500 + 500 + 500 + 500 + 500 + 500 = 3000 m

The motorist takes the eight turn at point R

∴Magnitude of displacement = PR

$$= \sqrt{PQ^2 + QR^2 + 2(PQ).(QR)\cos 60^\circ}$$

$$= \sqrt{500^2 + 500^2 + (2 \times 500 \times 500 \times \cos 60^\circ}$$

$$= \sqrt{250000 + 25000 + \left(500000 \times \frac{1}{2}\right)}$$

$$= 866.03\text{m}$$

$$\beta = \tan^{-1}\left(\frac{500 \sin 60^\circ}{500 + 500 \cos 60^\circ}\right) = 30^\circ$$

Therefore, the magnitude of displacement is 866.03 m at an angle of 30° with PR.

Total path length = Circumference of the hexagon + PQ + QR

= 6 × 500 + 500 + 500 = 4000 m

The magnitude of displacement and the total path length corresponding to the required turns is shown in the given table

Turn	**Magnitude of displacement (m)**	**Total path length (m)**
Third	1000	1500
Sixth	0	3000
Eighth	866.03; 30°	4000

4. A particle starts from the origin at *t* = 0 s with a velocity of $10.0\hat{j}\,m/s$ and moves in the *x*-*y* plane with a constant acceleration of $(8.0\hat{i} + 2.0\hat{j})m\,s^{-2}$.

(a) At what time is the *x*-coordinate of the particle 16 m? What is the *y*-coordinate of the particle at that time?

(b) What is the speed of the particle at the time?

Ans.

Velocity of the particle, $\bar{v} = 10.0\hat{j}\,m/s$

Acceleration of the particle $\bar{a} = (8.0\hat{i} + 2.0\hat{j})$

Also,

But, $\bar{a} = \frac{\overline{dv}}{dt} = 8.0\hat{i} + 2.0\hat{j}$

$d\bar{v} = (8.0\hat{i} + 2.0\hat{j})dt$

Integrating both sides:

$\bar{v}(t) = 8.0t\,\hat{i} + 2.0t\,\hat{j} + \bar{u}$

Where,

$\bar{u}$ = Velocity vector of the particle at $t = 0$

$\bar{v}$ = Velocity vector of the particle at time t

But, $\bar{v} = \frac{dr}{dt}$

$d\bar{r} = \bar{v}dt = (8.0t\,\hat{i} + 2.0t\,\hat{j} + \bar{u})dt$

Integrating the equations with the conditions: at $t = 0$; $r = 0$ and at $t = t$; $r = r$

$\bar{r} = \bar{u}t + \frac{1}{2}8.0t^2\,\hat{i} + \frac{1}{2} \times 2.0t^2\,\hat{j}$

$= \bar{u}t + 4.0t^2\,\hat{i} + t^2\,\hat{j}$

$= (10.0\,\hat{j})t + 4.0t^2\,\hat{i} + t^2\,\hat{j}$

$x\,\hat{i} + y\,\hat{j} = 4.0t^2\,\hat{i} + (10t + t^2)\,\hat{j}$

Since the motion of the particle is confined to the *x*-*y* plane, on equating the coefficients of

$\hat{i}$ *and* $\hat{j}$, we get:

$x = 4t^2$

$t = \left(\frac{x}{4}\right)^{\frac{1}{2}}$

And $y = 10t + t^2$

(a) When x = 16 m:

$t = \left(\frac{16}{4}\right)^{\frac{1}{2}} = 2s$

$\therefore y = 10 \times 2 + (2)^2 = 24$ m

(b) Velocity of the particle is given by:

$\bar{v}(t) = 8.0t\,\hat{i} + 2.0t\,\hat{j} + \bar{u}$

at $t = 2s$

$\bar{v}(t) = 8.0 \times 2\,\hat{i} + 2.0 \times 2\,\hat{j} + 10\,\hat{j}$

$= 16\,\hat{i} + 14\,\hat{j}$

$\therefore$ Speed of the particle:

$|\bar{v}| = \sqrt{(16)^2 + (14)^2}$

$= \sqrt{256 + 196} = \sqrt{452}$

$= 21.26 m/s$

5. $\hat{i}$ and $\hat{j}$ are unit vectors along *x*- and *y*-axis respectively. What is the magnitude and direction of the vectors $\hat{i} + \hat{j}$, and $\hat{i} - \hat{j}$? What are the components of a vector $A = 2\hat{i} + 3\hat{j}$ along the directions of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$? [You may use graphical method]

Ans.

Consider a vector $\bar{P}$, given as:

$$\bar{P} = \hat{i} + \hat{j}$$

$$P_x \hat{i} + P_y \hat{j} = \hat{i} + \hat{j}$$

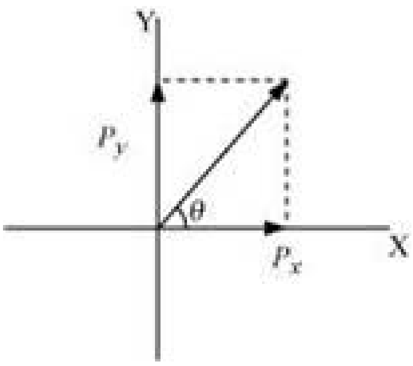
On comparing the components on both sides, we get:

$$P_x = P_y = 1$$

$$\left|\bar{P}\right| = \sqrt{P_x^2 + P_y^2} = \sqrt{1^2 + 1^2} = \sqrt{2} \quad \text{....(i)}$$

Hence, the magnitude of the vector $\hat{i} + \hat{j}$ is $\sqrt{2}$.

Let θ be the angle made by the vector $\bar{P}$, with the *x*-axis, as shown in the following figure.



$$\therefore \tan\theta = \left(\frac{P_y}{P_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{1}{1}\right) = 45^\circ \quad ....(ii)$$

Hence, the vector $\hat{i} + \hat{j}$ makes an angle of 45° with the x-axis.

$$\text{Let } \overline{Q} = \hat{i} - \hat{j}$$

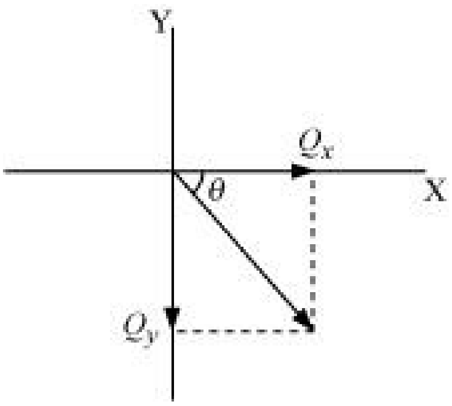
$$Q_x \hat{i} - Q_y \hat{j} = \hat{i} - \hat{j}$$

$$Q_x = Q_y = 1$$

$$\left|\overline{Q}\right| = \sqrt{Q_x^2 + Q_y^2} = \sqrt{2} \quad ....(iii)$$

Hence, the magnitude of the vector $\hat{i} - \hat{j}$ is $\sqrt{2}$.

Let θ be the angle made by the vector $\overline{Q}$, with the *x*- axis, as shown in the following figure.



$$\therefore \tan\theta = \left(\frac{Q_y}{Q_x}\right)$$

$$\theta = -\tan^{-1}\left(-\frac{1}{1}\right) = 45^\circ \quad ....(iv)$$

Hence, the vector $\hat{i} - \hat{j}$ makes an angle of 45°

It is given that:

$\overline{A} = 2\hat{i} + 3\hat{j}$

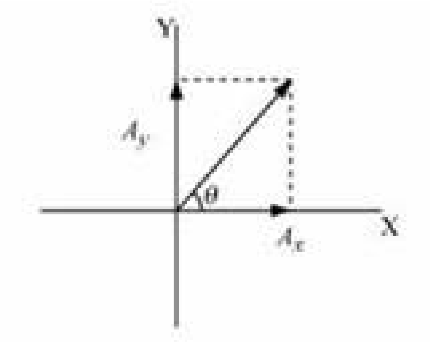
$A_x\hat{i} + A_y\hat{j} = 2\hat{i} + 3\hat{j}$

On comparing the coefficients of $\hat{i}$ and $\hat{j}$, we have:

$A_x = 2$ and $A_y = 3$

$|\overline{A}| = \sqrt{2^2 + 3^2} = \sqrt{13}$

Let $\overline{A}_x$ make an angle θ with the x-axis , as shown in the following figure.



$\therefore \tan\theta = \left(\frac{A_y}{A_x}\right)$

$\theta = \tan^{-1}\left(\frac{3}{2}\right)$

$= \tan^{-1}(1.5) = 56.31^\circ$

Angle between the vectors

$(2\hat{i} + 3\hat{j})$ and $(\hat{i} + \hat{j})$, $\theta = 56.31 - 45 = 11.31^\circ$

Component of vector $\overline{A}$, along the direction of $\overline{P}$, making an angle θ

$$= (A\cos\theta')\hat{P} = (A\cos 11.31)\frac{(i+j)}{\sqrt{2}}$$

$$= \sqrt{13}\times\frac{0.9806}{\sqrt{2}}(\hat{i}+\hat{j})$$

$$= 2.5(\hat{i}+\hat{j})$$

$$= \frac{25}{10}\times\sqrt{2}$$

$$= \frac{5}{\sqrt{2}} \text{ .....(v)}$$

Let θ be the angle between the vectors $(2\hat{i}+3\hat{j})$ and $\left(\hat{i}-\hat{j}\right)$.

$$\theta = 45 + 56.31 = 101.31^{\circ}$$

Component of vector $\overline{A}$, along the direction of $\overline{Q}$, making an angle θ

$$= (A\cos\theta)\overline{Q} = (A\cos\theta)\frac{\hat{i}-\hat{j}}{\sqrt{2}}$$

$$= \sqrt{13}\cos(901.31^{\circ})\frac{\left(\hat{i}-\hat{j}\right)}{\sqrt{2}}$$

$$= -\sqrt{\frac{13}{2}}\sin 11.30^{\circ}(\hat{i}-\hat{j})$$

$$= -2.550\times 0.1961(\hat{i}-\hat{j})$$

$$= -0.5(\hat{i}-\hat{j})$$

$$= -\frac{5}{10} \times \sqrt{2}$$

$$= -\frac{1}{\sqrt{2}} \text{ ......(vi)}$$

6. A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.50 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

Ans. 0.86 m/s2; 54.46° with the direction of velocity

Speed of the cyclist, $v = 27 km/h = 7.5 m/s$

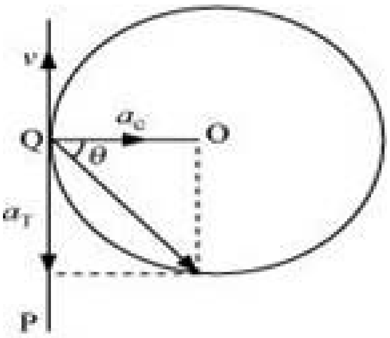
Radius of the circular turn, *r* = 80 m

Centripetal acceleration is given as:

$$a_c = \frac{v^2}{r}$$

$$= \frac{(7.5)^2}{80} = 0.7 m/s^2$$

The situation is shown in the given figure:



Suppose the cyclist begins cycling from point P and moves toward point Q. At point Q, he applies the breaks and decelerates the speed of the bicycle by $0.5\ m/s^2$.

This acceleration is along the tangent at Q and opposite to the direction of motion of the cyclist.

Since the angle between a_c and a_T is 90°, the resultant acceleration *a* is given by:

$$a = \sqrt{a_c^2 + a_T^2}$$
$$= \sqrt{(0.7)^2 + (0.5)^2}$$
$$= \sqrt{0.74} = 0.86 m/s^2$$
$$\tan\theta = \frac{a_c}{a_T}$$

Where θ is the angle of the resultant with the direction of velocity

$$\tan\theta = \frac{0.7}{0.5} = 1.4$$
$$\theta = \tan^{-1}(1.4)$$
$$= 54.46^{\circ}$$

7. (a) Show that for a projectile the angle between the velocity and the *x*-axis as a function of time is given by

$$\theta(t) = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{0x}}\right)$$

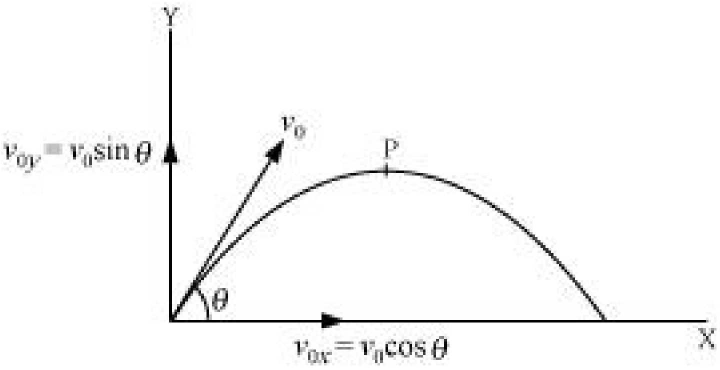
(b) Show that the projection angle θ_0 for a projectile launched from the origin is given by

$$\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

Where the symbols have their usual meaning.

Ans. (a) Let v_{ox} and v_{oy} respectively be the initial components of the velocity of the projectile along horizontal (*x*) and vertical (*y*) directions.

Let v_x and v_y respectively be the horizontal and vertical components of velocity at a point P.



Time taken by the projectile to reach point P = *t*

Applying the first equation of motion along the vertical and horizontal directions, we get:

$$v_y = v_{oy} = gt$$

$$\text{And} v_x = v_{0x}$$

$$\therefore \tan\theta = \frac{v_y}{v_x} = \frac{v_{oy} - gt}{v_{ox}}$$

$$\theta = \tan^{-1}\left(\frac{v_{oy} - gt}{v_{ox}}\right)$$

(b) Maximum vertical height, $h_m = \frac{u_0^2 \sin^2\theta}{2g}$ ...(i)

Horizontal range, $R = \frac{u_0^2 \sin^2 2\theta}{g}$ ....(ii)

Solving equations (i) and (ii), we get:

$$-\frac{h_m}{R} = \frac{\sin^2\theta}{2\sin^2\theta}$$

$$= \frac{\sin\theta \times \sin\theta}{2 \times 2\sin\theta\cos\theta}$$

$$= \frac{1}{4}\frac{\sin\theta}{\cos\theta} = \frac{1}{4}\tan\theta$$

$$\tan\theta = \left(\frac{4h_m}{R}\right)$$

$$\theta = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

Motion In a Plane

Very Short Answer Type Questions

1. A cyclist starts from centre O of a circular park of radius 1km and moves along the path OPRQO as shown Fig. 4.3. If he maintains constant speed of $10ms^{-1}$, what is his acceleration at point R in magnitude and direction?

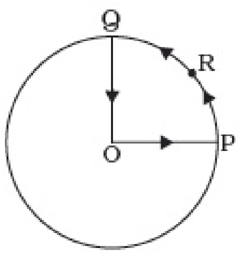


Fig. 4.3

2. A particle is projected in air at some angle to the horizontal, moves along parabola as shown in Fig. 4.4, where x and y indicate horizontal and vertical directions, respectively. Show in the diagram, direction of velocity and acceleration at points A, B and C.

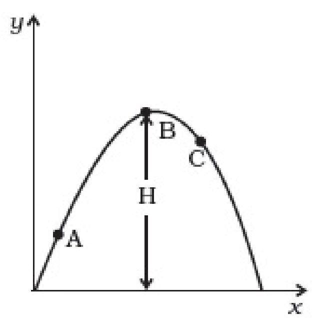


Fig. 4.4

3. A ball is thrown from a roof top at an angle of 45° above the horizontal. It hits the ground a few seconds later. At what point during its motion, does the ball have
   - (a) greatest speed.
   - (b) smallest speed.
   - (c) greatest acceleration? Explain

4. A football is kicked into the air vertically upwards. What is its
   - (a) acceleration, and
   - (b) velocity at the highest point?

5. A, B and C are three non-collinear, non co-planar vectors. What can you say about direction of A × (B × C)?

Short Answer Type Questions

1. A boy travelling in an open car moving on a levelled road with constant speed tosses a ball vertically up in the air and catches it back. Sketch the motion of the ball as observed by a boy standing on the footpath. Give explanation to support your diagram.
2. A boy throws a ball in air at 60° to the horizontal along a road with a speed of 10 m/s (36 km/h). Another boy sitting in a passing by car observes the ball. Sketch the motion of the ball as observed by the boy in the car, if car has a speed of (18km/h). Give explanation to support your diagram.
3. In dealing with motion of projectile in air, we ignore effect of air resistance on motion. This gives trajectory as a parabola as you have studied. What would the trajectory look like if air resistance is included? Sketch such a trajectory and explain why you have drawn it that way.
4. A fighter plane is flying horizontally at an altitude of 1.5 km with speed 720 km/h. At what angle of sight (w.r.t. horizontal) when the target is seen, should the pilot drop the bomb in order to attack the target?
5. (a) Earth can be thought of as a sphere of radius 6400 km. Any object (or a person) is performing circular motion around the axis of earth due to earth's rotation (period 1 day). What is acceleration of object on the surface of the earth (at equator) towards its centre? what is it at latitude θ ? How does these accelerations compare with $g = 9.8\ m/s^2$?
   (b) Earth also moves in circular orbit around sun once every year with on orbital radius of $1.5 * 10^{11}$m . What is the acceleration of

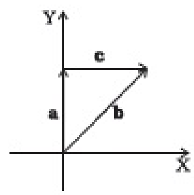
earth (or any object on the surface of the earth) towards the centre of the sun? How does this acceleration compare with g = 9.8 m/s^2?

$\left(Hint: acceleration \frac{V^2}{R} = \frac{4\pi^2 R}{T^2} \right)$

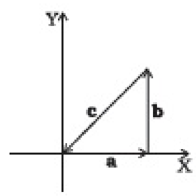
6. Given below in column I are the relations between vectors a, b and c and in column II are the orientations of a, b and c in the XY plane. Match the relation in column I to correct orientations in column II.

Column I **Column II**

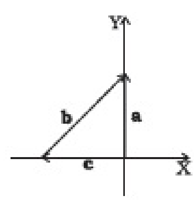
(a) $\mathbf{a} + \mathbf{b} = \mathbf{c}$ (i)



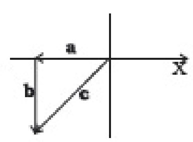
(b) $\mathbf{a} - \mathbf{c} = \mathbf{b}$ (ii)



(c) $\mathbf{b} - \mathbf{a} = \mathbf{c}$ (iii)



(d) $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$ (iv)



7. If |A| = 2 and |B| = 4 , then match the relations in column I with the angle θ between A and B in column II.

Column I	Column II
(a) $\mathbf{A}.\mathbf{B} = 0$	(i) $\theta = 0$
(b) $\mathbf{A}.\mathbf{B} = +8$	(ii) $\theta = 90°$
(c) $\mathbf{A}.\mathbf{B} = 4$	(iii) $\theta = 180°$
(d) $\mathbf{A}.\mathbf{B} = -8$	(iv) $\theta = 60°$

8. If |A| = 2 and |B| = 4, then match the relations in column I with the angle θ between A and B in column II.

Column I	Column II
(a) $\lvert \mathbf{A} \times \mathbf{B} \rvert = 0$	(i) $\theta = 30°$
(b) $\lvert \mathbf{A} \times \mathbf{B} \rvert = 8$	(ii) $\theta = 45°$
(c) $\lvert \mathbf{A} \times \mathbf{B} \rvert = 4$	(iii) $\theta = 90°$
(d) $\lvert \mathbf{A} \times \mathbf{B} \rvert = 4\sqrt{2}$	(iv) $\theta = 0°$

Long Answer Type Questions

1. A hill is 500 m high. Supplies are to be sent across the hill using a canon that can hurl packets at a speed of 125 m/s over the hill. The canon is located at a distance of 800m from the foot of hill and can be moved on the ground at a speed of 2 m/s; so that its distance from the hill can be adjusted. What is the shortest time in which a packet can reach on the ground across the hill ? Take g =10 m/s^2.
2. A gun can fire shells with maximum speed v_o and the maximum horizontal range that can be achieved is $R = v_o^2 / g$.

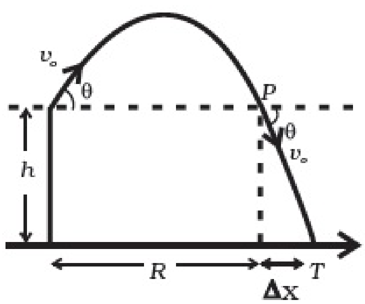


Fig 4.5

3. If a target farther away by distance ΔDx (beyond R) has to be hit with the same gun (Fig 4.5), show that it could be achieved by raising the gun to a height at least

$$h = \Delta x\left[1 + \frac{\Delta x}{R}\right]$$

Hint : This problem can be approached in two different ways:

- (i) Refer to the diagram: target T is at horizontal distance $x = R + \Delta x$ and below point of projection $y = -h$.
- (ii) From point P in the diagram: Projection at speed v_o at an angle θ below horizontal with height h and horizontal range Δx.

4. A particle is projected in air at an angle β to a surface which itself is inclined at an angle a to the horizontal (Fig. 4.6).
   - (a) Find an expression of range on the plane surface (distance on the plane from the point of projection at which particle will hit the surface).
   - (b) Time of flight.
   - (c) β at which range will be maximum.

     Hint : This problem can be solved in two different ways:
   - (i) Point P at which particle hits the plane can be seen as intersection of its trajectory (parabola) and straight line. Remember particle is projected at an angle (α + β) w.r.t. horizontal.
   - (ii) We can take x-direction along the plane and y-direction perpendicular to the plane. In that case resolve g (acceleration due to gravity) in two different components, g_x along the plane and g_y perpendicular to the plane. Now the problem can be solved as two independent motions in x and y directions respectively with time as a common parameter.

5. A particle falling vertically from a height hits a plane surface inclined to horizontal at an angle θ with speed v_o and rebounds elastically (Fig 4.7). Find the distance along the plane

where if will hit second time.

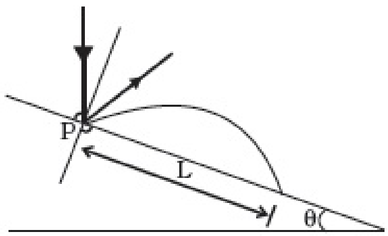


Fig 4.7

Hint:

- (i) After rebound, particle still has speed v_o to start.
- (ii) Work out angle particle speed has with horizontal after it rebounds.
- (iii) Rest is similar to if particle is projected up the incline.

6. A girl riding a bicycle with a speed of 5 m/s towards north direction, observes rain falling vertically down. If she increases her speed to 10 m/s, rain appears to meet her at 45° to the vertical. What is the speed of the rain? In what direction does rain fall as observed by a ground based observer?

   (*Hint:* Assume north to be $\hat{\mathbf{i}}$ direction and vertically downward to be $-\hat{\mathbf{j}}$. Let the rain velocity $\mathbf{v}_r$ be $a\hat{\mathbf{i}}+b\hat{\mathbf{j}}$. The velocity of rain as observed by the girl is always $\mathbf{v}_r - \mathbf{v}_{girl}$. Draw the vector diagram/s for the information given and find *a* and *b*. You may draw all vectors in the reference frame of ground based observer.)

7. A river is flowing due east with a speed 3m/s. A swimmer can swim in still water at a speed of 4 m/s (Fig. 4.8).

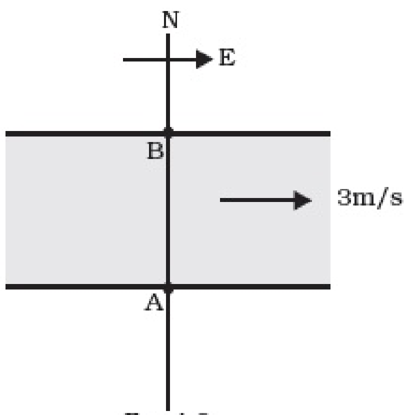


Fig. 4.8

- (a) If swimmer starts swimming due north, what will be his resultant velocity (magnitude and direction)?
- (b) If he wants to start from point A on south bank and reach opposite point B on north bank,
- (a) which direction should he swim?
- (b) what will be his resultant speed?
- (c) From two different cases as mentioned in (a) and (b) above, in which case will he reach opposite bank in shorter time?

8. A cricket fielder can throw the cricket ball with a speed v_o. If he throws the ball while running with speed u at an angle θ to the horizontal, find
   - (a) the effective angle to the horizontal at which the ball is projected in air as seen by a spectator.
   - (b) what will be time of flight?
   - (c) what is the distance (horizontal range) from the point of projection at which the ball will land?
   - (d) find θ at which he should throw the ball that would maximise the horizontal range as found in (iii).
   - (e) how does θ for maximum range change if $u > v_o$, $u = v_o$, $u < v_o$?

- (f) how does θ in (v) compare with that for u = 0 (i.e.45°)?
-

Motion in two dimensions, in a plane can be studied by expressing position, velocity and acceleration as vectors in Cartesian co-ordinates $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$ where $\hat{\mathbf{i}}$ *and* $\hat{\mathbf{j}}$ are unit vector along x and y directions, respectively and A_x and A_y are corresponding components of **A** (Fig. 4.9). Motion can also be studied by expressing vectors in circular polar co-ordinates as $\mathbf{A} = A_r\hat{\mathbf{r}} + A_\theta\hat{\boldsymbol{\theta}}$

where $\hat{\mathbf{r}} = \frac{\mathbf{r}}{r} = \cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}$ and $\hat{\boldsymbol{\theta}} = -\sin\theta\,\hat{\mathbf{i}} + \cos\theta\,\hat{\mathbf{j}}$ are unit vectors along direction in which 'r' and 'θ' are increasing.

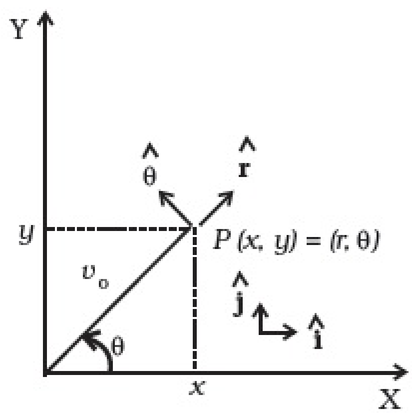


Fig. 4.9

(a) Express $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ in terms of $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$

(b) Show that both $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ are unit vectors and are perpendicular to each other.

(c) Show that $\frac{d}{dt}(\hat{\mathbf{r}}) = \omega\hat{\theta}$ where

$$\omega = \frac{d\theta}{dt} \text{ and } \frac{d}{dt}(\boldsymbol{\theta}) = -\omega\,\hat{\mathbf{r}}$$

(d) For a particle moving along a spiral given by $\mathbf{r} = a\theta\,\hat{\mathbf{r}}$, where a = 1 (unit), find dimensions of 'a'.

(e) Find velocity and acceleration in polar vector represention for particle moving along spiral described in (d) above.

9. A man wants to reach from A to the opposite corner of the square C (Fig. 4.10). The sides of the square are 100 m. A central square of 50m * 50m is filled with sand. Outside this square, he can walk at a speed 1 m/s. In the central square, he can walk only at a speed of v m/s (v < 1). What is smallest value of v for which he can reach faster via a straight path through the sand than any path in the square outside the sand?

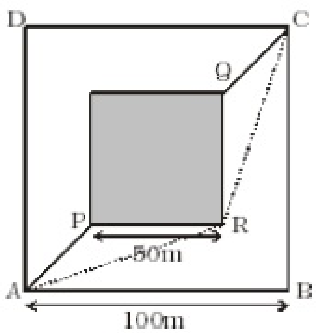


Fig. 4.10