Moving charges and magnetism(revision notes)

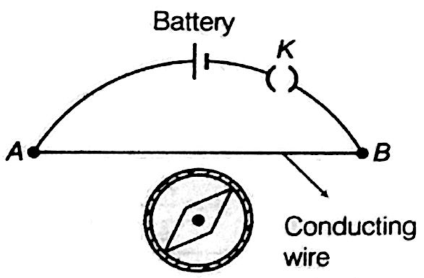
[TOPIC 1] Magnetic Field, Its Laws and their Applications

1.1 Magnetic Field

The space around a magnet or a current carrying conductor in which its magnetic influence can be experienced is called **magnetic field**. Its SI unit is **tesla** (T).

Oersted's Experiment

Oersted's experiment demonstrated that the current carrying conductor carrying moving charges produces magnetic field around it.



When key K is closed, then deflection occurs in the compass needle and *vice-versa*.

1.2 Laws of Magnetic Field

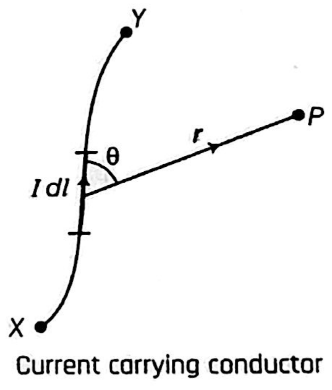
There are two laws of magnetic field

Biot-Savart's Law : Magnetic Field due to a Current Element

According to this law, the magnetic field due to small current carrying element dl at any nearby point P is given by

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin\theta}{r^2}$$

or $$d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{Id\mathbf{l} \times \mathbf{r}}{|\mathbf{r}|^2}$$



where, $\mu_0/4\pi = 10^{-7}$ T-m/A $= 10^{-7}$ H/m

Here μ_0 = permeability of free space or vacuum and r = distance of point P from current carrying element.

Its direction is given by right hand thumb rule.

Permittivity and Permeability

The relation between μ_0, ε_0 and c is

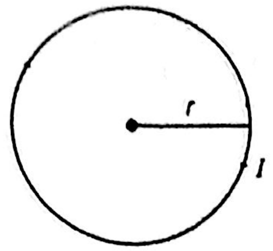
$$\frac{1}{\mu_0\varepsilon_0} = c^2$$

where, c is velocity of light, ε_0 is permittivity of free space and μ_0 is magnetic permeability.

Applications of Biot-Savart's Law

There are following applications of Biot-Savart's law

(i) Magnetic field at the centre of a circular current carrying conductor/coil.



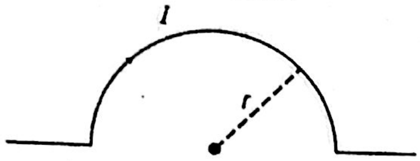
$$B = \frac{\mu_0 I}{2r}$$

where, r is the radius of a circular loop.

For N turns of coil, $B = \frac{\mu_0 N I}{2r}$

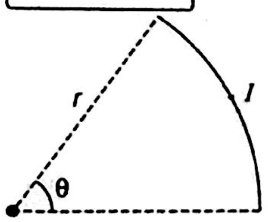
(ii) Magnetic field at the centre of semi-circular current carrying conductor

$$B = \frac{\mu_0 I}{4r}$$



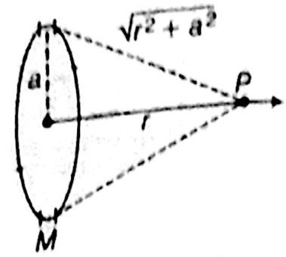
(iii) Magnetic field at the centre of an arc of circular current carrying conductor which subtends an angle θ at the centre

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I\theta}{r}$$

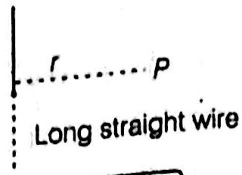


(iv) Magnetic field at any point on the axis of circular current loop

$$B = \frac{\mu_0 I a^2}{2(r^2 + a^2)^{3/2}}$$



(v) Magnetic field due to straight current carrying conductor at any point P at a distance r from the wire is given by



$$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r}$$

Ampere's Circuital Law

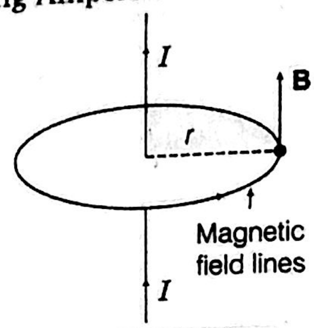
The line integral of the magnetic field $\mathbf{B}$ around any closed path in vacuum is equal to μ_0 times of the total current I threading through the closed circuit i.e. loop.

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

Applications of Ampere's Circuital Law

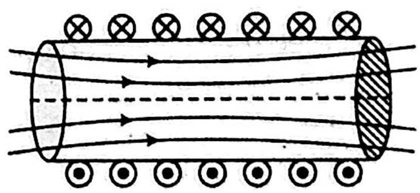
These are some applications of Ampere's circuital law

(i) Magnitude of magnetic field of a straight wire using Ampere's law



$$B = \frac{\mu_0 I}{2\pi r}$$

(ii) Magnetic field due to a straight solenoid



(a) At any point inside the solenoid,

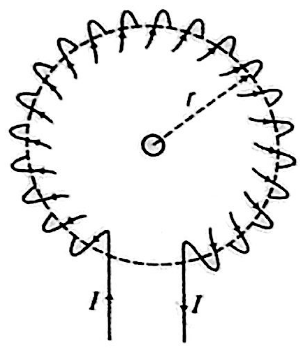
$$\boxed{B = \mu_0 n I}$$

where, n = number of turns per unit length.

(b) At points near the ends of air closed solenoid,

$$\boxed{B = \frac{1}{2}\mu_0 n I}$$

(iii) Magnetic field due to a toroidal solenoid



(a) Inside the toroidal solenoid,

$$\boxed{B = \mu_0 n I}$$

Here, $n = \dfrac{N}{2\pi r}$

and N = total number of turns

(b) In the open space, interior or exterior of toroidal solenoid, $B = 0$

[TOPIC 2] Lorentz Force and Cyclotron

2.1 Force on Moving Charge in a Uniform Magnetic Field

Force experienced by a single charged particle q moving with speed v in a uniform magnetic field at an angle θ with it is given as

$$\boxed{\mathbf{F} = q(\mathbf{v} \times \mathbf{B})} \quad \text{[in vector form]}$$

Magnitude of $F = qvB \sin\theta$ and direction of force is given by right hand palm rule or Fleming's left hand rule.

- Work done by this magnetic force on charge particle is zero as $\mathbf{F} \perp \mathbf{v}$, hence $\mathbf{F}$ is perpendicular to displacement.
- Magnetic force cannot increase the kinetic energy of charged particle.

SI Unit of Magnetic Field

Magnetic force, $F = qvB \sin\theta$

$\Rightarrow \quad B = F/qv \sin\theta$

If $\quad F = 1\,\text{N}, q = 1\,\text{C}, v = 1\,\text{ms}^{-1}$

$\sin\theta = 1 = \sin 90° \Rightarrow \theta = 90°$

$\therefore$ SI unit of $B = \dfrac{1\,\text{N}}{(1\text{C})\,(1\text{ms}^{-1})}$

$= 1\text{NA}^{-1}\text{m}^{-1} = 1\text{T}$

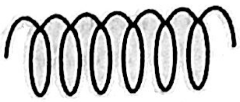
SI unit of magnetic field, $B = 1$ tesla (T).

Right Hand Palm Rule

If all four fingers of right hand together points in the direction of magnetic field and thumb points in the direction of motion of positive charge, then the palm of right hand faces in the direction of force.

Motion of a Charged Particle in a Uniform Magnetic Field

- The trajectory/path traversed by the charged particle in a uniform magnetic field is a
  - *(i)* straight line when angle between $\mathbf{v}$ and $\mathbf{B}$ is $0°$ or $180°$.
  - *(ii)* circle when angle between $\mathbf{v}$ and $\mathbf{B}$ is $90°$.
  - *(iii)* helix when angle between $\mathbf{v}$ and $\mathbf{B}$ is ar acute angle.



- When charged particle enter a magnetic field perpendicularly, then
  - *(i)* $mv^2/r = qvB$
  - *(ii)* radius, $r = mv/qB$
  - *(iii)* $T = \dfrac{2\pi m}{qB}$
  - *(iv)* $f = \dfrac{qB}{2\pi m}$
  - *(v)* $\text{KE} = \dfrac{q^2B^2r^2}{2m}$

- When angle between **v** and **B** is θ, then

(i) Radius of helical path,

$$r = \frac{mv \sin\theta}{qB} = \frac{mv_\perp}{qB}$$

where, $v_\perp = v\sin\theta$ = perpendicular component of velocity.

(ii) Time period,

$$T = \frac{2\pi}{v_\perp} = \frac{2\pi}{v\sin\theta} \cdot \frac{mv\sin\theta}{qB} = \frac{2\pi m}{qB}$$

(iii) Frequency, $f = \frac{qB}{2\pi m}$

(iv) Pitch $= \frac{2\pi m\, v\cos\theta}{qB} = \frac{2\pi m v_\parallel}{qB}$

where, $v_\parallel = v\cos\theta$ = parallel component of velocity.

2.2 Charge Moving in Combined Electric and Magnetic Field

The net force exerted on a charged particle moving in the presence of both electric and magnetic field is known as **Lorentz force.** It is given as

$$\mathbf{F}_{\text{Lorentz}} = \mathbf{F}_{\text{electric}} + \mathbf{F}_{\text{magnetic}} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$$

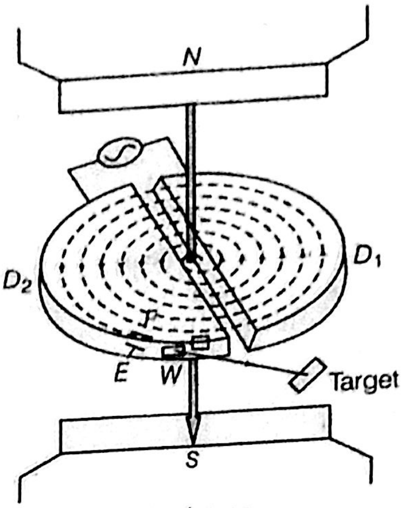
$$\Rightarrow \quad \mathbf{F}_{\text{Lorentz}} = q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$$

The condition for the charge to move in the fields undeflected is $qE = qvB$

or $\boxed{v = E/B}$

2.3 Cyclotron

It is a compact device used to accelerate the positively charged ions or particles (like protons, deuterons, α-particles) to very high speed.



Cyclotron

It is based on the principle of magnetic resonance. A charged particle can be accelerated to high speed by passing it again and again through small region of oscillating electrical field by making use of strong normal magnetic field.

- Cyclotron frequency is given as $f_{osc} = qB/2\pi m$
- KE_{max} of charged particle accelerated by cyclotron is $KE_{max} = q^2B^2R^2/2m$
  where, R = radius of circular track of charged particle.
- Cyclotron cannot accelerate electron and electrically neutral particles.

[TOPIC 3] Magnetic Force and Torque between Two Parallel Currents

3.1 Force on a Current Carrying Conductor in a Uniform Magnetic Field

The **magnetic force** experienced by a current-carrying conductor placed in a uniform magnetic field is given by

$$\boxed{\mathbf{F} = I\,(\mathbf{l} \times \mathbf{B})} \qquad \text{[vector form]}$$

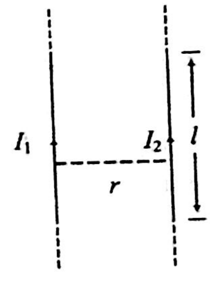
where, $\mathbf{l}$ = a vector whose magnitude is equal to length of the conductor and has identical direction in the flow of electric current I and $\mathbf{B}$ = magnetic field.

Magnitude of magnetic force, $F = IBl \sin\theta$

where, θ is the angle between current and magnetic field. The direction of force is given by Fleming's left hand rule.

Magnetic Force between Two Parallel Current Carrying Conductor

Magnetic force per unit length between two straight parallel current-carrying conductors is given by



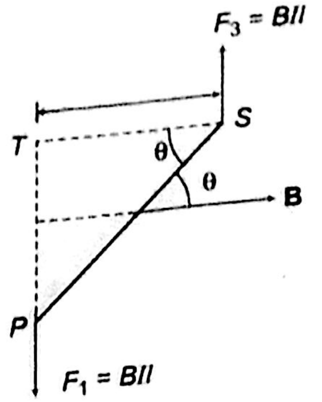
$$\boxed{\frac{F}{l} = \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{r}}$$

Force will be of attractive nature, if direction of flow of currents in the two conductors are in the same direction i.e. parallel. The force of repulsion will act when direction of flow of currents are in opposite directions. (antiparallel)

If both wires are of length l_1 and l_2 (if $l_1 > l_2$), then magnetic force between two straight current carrying wires/conductors is given by

$$F = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r} l_2$$

(l_2 small length)



3.2 Torque on Current Carrying Loop (Magnetic Dipole)

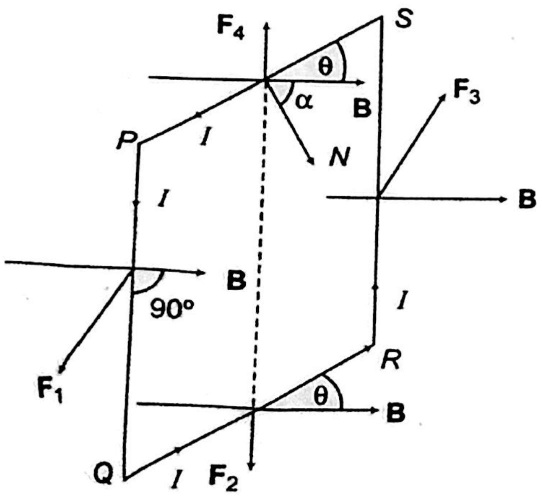
Torque is experienced by a current-carrying loop placed in a uniform magnetic field B as shown in the figure given below. It is given by $\tau = NIAB \sin\theta$

where, θ is the angle between the direction of magnetic field B and that of vector N drawn normal to the plane of the coil.

or $\quad \tau = \mathbf{M} \times \mathbf{B}$

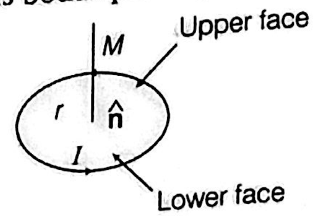
where, $\quad \mathbf{M} = NIA$

and $\mathbf{M}$ is known as magnetic dipole moment of coil. Its SI unit is A-m^2.



3.3 Circular Current Loop as a Magnetic Dipole

A current loop behaves as a magnetic dipole. If we look at the upper face, current is anti-clockwise, so it has North polarity. If we look at lower face, then current is clockwise, so it has South polarity.



Magnetic dipole moment of the loop, $M = IA$

$$\Rightarrow \quad |\mathbf{M}| = I\pi r^2$$

The magnitude of magnetic field on axis of a circular loop of radius r carrying steady current I is given by

$$B = \frac{\mu_0 I r^2}{2(x^2 + r^2)^{3/2}} \quad \text{for } x >> r$$

$$B = \frac{\mu_0 I r^2}{2x^3} \Rightarrow \boxed{\mathbf{B} = \frac{\mu_0 \mathbf{M}}{2\pi x^3}}$$

If loop has N turns, $\mathbf{M} = NIA$

Magnetic Dipole Moment of a Revolving Electron

An electron being a charged particle, constitutes a current while moving in its circular orbit around the nucleus. The magnetic dipole moment

$$\boxed{M = \frac{e}{4\pi m_e} nh}$$

where, m_e = mass of electron,

h = Planck's constant and $n = 1, 2, 3, ...$

For $n = 1$, M will be minimum

$$\therefore M_{min} = \frac{eh}{4\pi m_e}$$

This value is known as **Bohr's magneton.**

3.4 Moving Coil Galvanometer

It is a device used to detect the current in electrical circuit. It is based on the principle that a current carrying loop placed in a uniform magnetic field experiences torque, the magnitude of which depends on the strength of current.

In equilibrium position of the coil of MCG,

Restoring torque = Deflecting torque

$$k\theta = NIBA$$

[Torque τ = Force × perpendicular distance

$= NIbB \times a \sin 90^\circ = NIBA$]

$$\Rightarrow \boxed{I = G\theta}$$

Current sensitivity, $I_s = \frac{\theta}{I} = \frac{NBA}{k}$

where, $G = \frac{NBA}{k}$

Its SI unit is rad/A or div/A.

Voltage sensitivity, $V_s = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{I_s}{R} = \frac{NBA}{kR}$

Its SI unit is rad/V or div/V.

Clearly, $\boxed{V_s = \frac{I_s}{R}}$

The current sensitivity and voltage sensitivity of galvanometer depends on number of turns of coil magnetic field B, area A of the coil and torsion constant k of the spring or suspension wire.

Conversion of Galvanometer into an Ammeter

A galvanometer can be converted into an ammeter by connecting a very low resistance (shunt S) in parallel with galvanometer whose value is given by

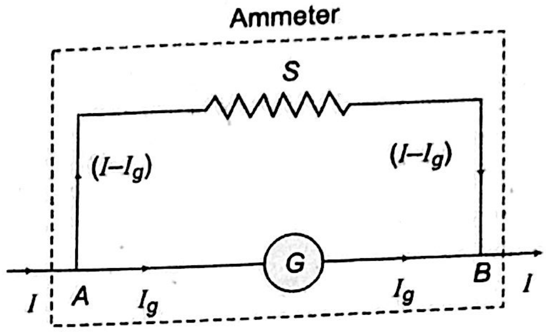
$$\boxed{S = \frac{I_g G}{I - I_g}}$$

where, G = resistance of galvanometer

I_g = current through galvanometer

I = total current in circuit and

S = resistance of the shunt (low resistance).



Conversion of a Galvanometer to Voltmeter

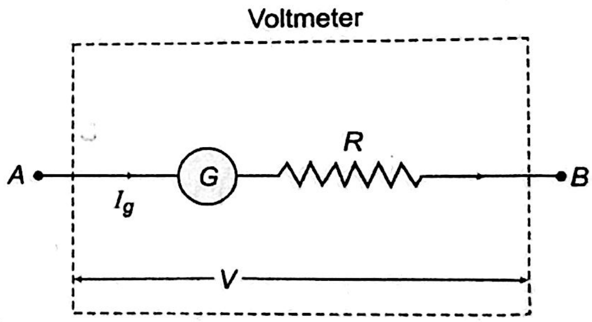
A galvanometer can be converted into voltmeter by connecting a very high resistance R in series with galvanometer which is given by

$$\boxed{R = \frac{V}{I_g} - G}$$

where, I_g = current through the galvanometer

G = resistance of galvanometer and

V = potentials difference across the terminal A and B.



NOTE The resistance of an ideal ammeter is zero and an ideal voltmeter is infinite. Ammeter is always connected in series with electrical circuit, whereas voltmeter is connected in parallel with the circuit.