Chapter 14 (Oscillations)

Multiple Choice Questions

Single Correct Answer Type

Q1. The displacement of a particle is represented by the equation

$$y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right).$$

. The motion of the particle is

(a) simple harmonic with period 2KUO

(b) simple hannonic with period nia)

(c) periodic but not simple harmonic

(d) non-periodic

Sol: (b)

Key concept:

- Acceleration is always directed towards the mean position and so is always opposite to displacement, *i.e.* $a \propto -y$ or $a = -\omega^2 y$
- The differential equation of S.H.M is $\frac{d^2y}{dt^2} + \omega^2 y = 0$ and the solution of this differential equation is $y = a\cos(\omega t + \varphi_0)$

Method 1: The displacement of the particle $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$
Velocity of the particle

$$v = \frac{dy}{dt} = \frac{d}{dt}\left[3\cos\left(\frac{\pi}{4} - 2\omega t\right)\right] = 6\omega\sin\left(\frac{\pi}{4} - 2\omega t\right)$$

Acceleration $a = \frac{dv}{dt} = \frac{d}{dt}\left[6\omega\sin\left(\frac{\pi}{4} - 2\omega t\right)\right]$

$$= -12\omega^2\cos\left(\frac{\pi}{4} - 2\omega t\right) = -4\omega^2\left[3\cos\left(\frac{\pi}{4} - 2\omega t\right)\right]$$

Hence $a = -4\omega^2 y = -(2\omega)^2 y$

It means acceleration, $a \propto -y$, the motion is SHM.

Hence angular frequency of S.H.M, $\omega' = 2\omega$

$$\omega' = 2\omega = \frac{2\pi}{T'} \quad \Rightarrow \quad T' = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

It means the motion is SHM with period $\frac{\pi}{\omega}$.

Method 2: Given the equation of displacement of the particle

$$y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$$

$$y = 3\cos\left[-\left(2\omega t - \frac{\pi}{4}\right)\right]$$

We know $\cos(-\theta) = \cos\theta$

Hence $y = 3\cos\left(2\omega t - \frac{\pi}{4}\right)$ ...(i)

Comparing with $y = a\cos(\omega t + \phi_0)$

Hence (i) represents simple harmonic motion with angular frequency 2ω.

Hence its time period, $T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$.

Q2. The displacement of a particle is represented by the equation y= $\sin^3 \omega t$The motion is

(a) non-periodic

(b) periodic but not simple harmonic

(c) simple harmonic with period $2\pi/\omega$

(d) simple harmonic with period π/ω

Sol: (b)

Key concept: There are certain motions that are repeated at equal intervals of time. Let the the interval of time in which motion is repeated. Then x(t) =x(t + T), where T is the minimum change in time. The function that repeats itself is known as a periodic function.

Given the equation of displacement of the particle, $y = \sin^3 \omega t$

We know $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

Hence $y = \frac{(3\sin\omega t - 4\sin 3\omega t)}{4}$

$$\Rightarrow \quad 4\frac{dy}{dt} = 3\omega\cos\omega t - 4\times[3\omega\cos 3\omega t]$$

$$\Rightarrow \quad 4\times\frac{d^2y}{dt^2} = -3\omega^2\sin\omega t + 12\omega\sin 3\omega t$$

$$\Rightarrow \quad \frac{d^2y}{dt^2} = -\frac{3\omega^2\sin\omega t + 12\omega^2\sin 3\omega t}{4}$$

$\Rightarrow \qquad \frac{d^2y}{dt^2}$ is not proportional to y.

Hence, motion is not SHM.

As the expression is involving sine function, hence it will be periodic.

Also $\sin^3 \omega t = (\sin \omega t)^3$

$$= [\sin(\omega t + 2\pi)]^3 = [\sin \omega(t + 2\pi/\omega)]^3$$

Hence, $y = \sin^3 \omega t$ represents a periodic motion with period $2\pi/\omega$.

Q3. The relation between acceleration and displacement of four particles are given below:

(a) $a_x = +2x$ (b) $a_x = +2x^2$ (c) $a_x = -2x^2$ (d) $a_x = -2x$

Which one of the particle is exempting simple harmonic motion?

Sol: (d)

Key Concept: In case of simple harmonic motion, the acceleration is always directed towards the mean position and so is always opposite to displacement, i.e. a α = -x or a = $-\omega^2 x$

In option (d) $a_x = -2x$ or a α -x, the acceleration of the particle is proportional to negative of displacement. Hence it represents S.H.M.

Q4. Motion of an oscillating liquid column in a U-tube is

(a) periodic but not simple harmonic

(b) non-periodic

(c) simple harmonic and time period is independent of the density of the liquid

(d) simple harmonic and time period is directly proportional to the density of the liquid

Sol: (c)

Key Concept: If the liquid in U-tube is filled to a height h and cross¬section of the tube is uniform and the liquid is incompressible and non- viscous. Initially the level of liquid in the two limbs will be at the same height equal to h. If the liquid is-pressed by y in one limb, it will rise by y along the length of the tube in the other limb, so the restoring force will be developed by hydrostatic pressure difference

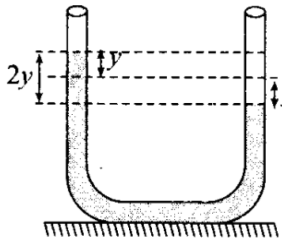
Restoring force F = Weight of liquid column of height $2y$

$\Rightarrow \qquad F = -(A \times 2y \times \rho) \times g = -2A\rho g y$

$\Rightarrow \qquad F \propto -y$

Motion is SHM with force constant

$$k = 2A\rho g$$



$\Rightarrow \qquad$ Timeperiod $T = 2\pi\sqrt{\frac{m}{k}}$

$$= 2\pi\sqrt{\frac{A \times 2h \times \rho}{2A\rho g}} = 2\pi\sqrt{\frac{h}{g}}$$

which is independent of the density of the liquid.

Q5. A particle is acted simultaneously by mutually perpendicular simple harmonic motion x = a cos ωt and y = a sin ωt. The trajectory of motion of the particle will be

(a) an ellipse (b) a parabola (c) a circle (d) a straight line

Sol: (c)

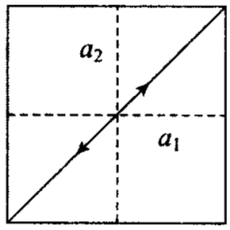
Key concept: If two S.H.M's act in perpendicular directions, then their resultant motion is in the form of a straight line or a circle or a parabola etc. depending on the frequency ratio of the two S.H.Ms and initial phase difference. These figures are called Lissajous figures.

Let the equations of two mutually perpendicular S.H.M's of same frequency be

$$x = a_1 \sin \omega t \quad \text{and} \quad y = a_2 \sin(\omega t + \phi)$$

Then the general equation of Lissajou's figure can be obtained as

$$\frac{x^2}{a_1^2} + \frac{y^2}{a_2^2} - \frac{2xy}{a_1 a_2}\cos\phi = \sin^2\phi$$

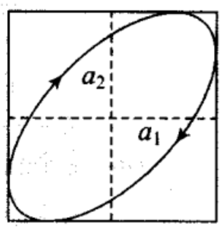
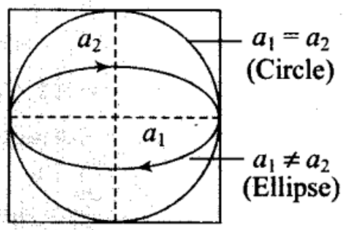
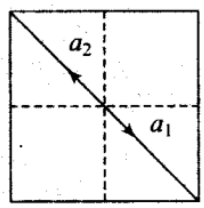


For $\phi = 0°$: $\frac{x^2}{a_1^2} + \frac{y^2}{a_2^2} - \frac{2xy}{a_1 a_2} = 0 \Rightarrow \left(\frac{x}{a_1} - \frac{y}{a_2}\right)^2 = 0$

$$\Rightarrow \quad \frac{x}{a_1} = \frac{y}{a_2} \Rightarrow y = \frac{a_2}{a_1}x$$

This is a straight line which passes through origin and its slope is $\frac{a_2}{a_1}$.

Lissajou's figures in other conditions $\left(\text{with } \frac{\omega_1}{\omega_2} = 1\right)$

Phase diff.(ϕ)	Equation	Figure
$\frac{\pi}{4}$	$\frac{x^2}{a_1^2} + \frac{y^2}{a_2^2} - \frac{\sqrt{2}xy}{a_1 a_2} = \frac{1}{2}$	Oblique ellipse 
$\frac{\pi}{2}$	$\frac{x^2}{a_1^2} + \frac{y^2}{a_2^2} = 1$	
π	$\frac{x}{a_1} + \frac{y}{a_2} = 0$ $\Rightarrow y = -\frac{a_2}{a_1}x$	Straight line 

We have to find resultant-displacement by adding x and y-components. According to variation of x and y, trajectory will be predicted.

Given, $x = a\cos\omega t = a\sin\left(\omega t + \frac{\pi}{2}\right)$ ...(i)

And $\quad y = a\sin\omega t$ ...(ii)

Here $a_1 = a_2 = a$ and $\phi = \frac{\pi}{2}$

The trajectory of motion of the particle will be a circle of radius a.

Q6. The displacement of a particle varies with time according to the relation

y = a sint + b cost

(a) The motion is oscillatory but not SHM

(b) The motion is SHM with amplitude a + b

(c) The motion is SHM with amplitude $a^2 + b^2$

(d) The motion is SHM with amplitude $\sqrt{a^2 + b^2}$

Sol. (d)

Key concept: The sum of two S.H.Ms of same frequencies is a S.H.M.

According to the question, the displacement

$$y = a \sin \omega t + b \cos \omega t$$

Given $\quad x = a \sin \omega t + b \cos \omega t \quad \ldots(i)$

Let $\quad a = A \cos \phi \quad \ldots(ii)$

and $\quad b = A \sin \phi \quad \ldots(iii)$

Squaring and adding (ii) and (iii), we get

$$a^2 + b^2 = A^2 \cos^2 \phi + A^2 \sin^2 \phi = A^2$$

$$= A^2 \Rightarrow A = \sqrt{a^2 + b^2}$$

$$y = A \sin \phi \cdot \sin \omega t + A \cos \phi \cdot \cos \omega t$$

$$= A \sin(\omega t + \phi)$$

$$\frac{dy}{dt} = A\omega \cos(\omega t + \phi)$$

$$\frac{d^2 y}{dt^2} = - A\omega^2 \sin(\omega t + \phi)$$

$$= - Ay\omega^2 = (-A\omega^2) y$$

$$\Rightarrow \quad \frac{d^2 y}{dt^2} \propto (-y)$$

Hence, it is an equation of SHM with amplitude

$$A = \sqrt{a^2 + b^2}$$

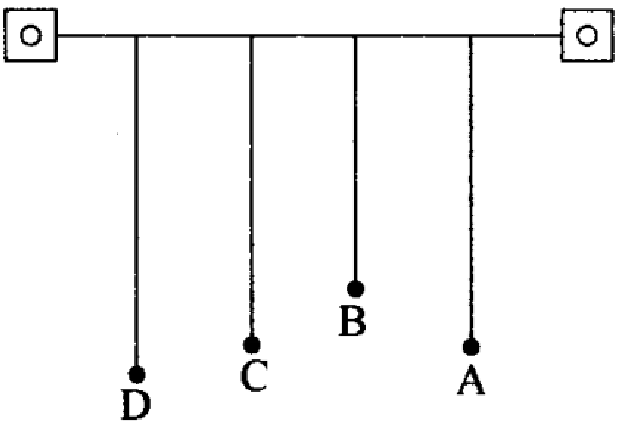
Q7. Four pendulums A, B, C and D are suspended from the same elastic support as shown in figure. A and C are of the same length, while B is smaller than A and D is larger than A. If A is given a transverse displacement,

(a) D will vibrate with maximum amplitude

(b) C will vibrate with maximum amplitude

(c) B will vibrate with maximum amplitude

(d) All the four will oscillate with equal amplitude



Sol: (b) Here A is given a transverse displacement. Through the elastic support the disturbance is transferred to all the pendulums.

A and C are having same length, hence they will be in resonance, because of their time period of oscillation. Since length of pendulums A and C is same and $T = 2\pi\sqrt{L/g}$, hence their time period is same and they will have

frequency of vibration. Due to it, a resonance will take place and the pendulum C will vibrate with maximum amplitude.

Q8. Figure **shows the circular motion of a particle. The radius of the circle, the period, sense**

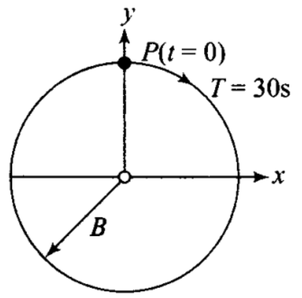
of revolution and the initial position are indicated on the figure. The simple harmonic motion of the x-projection of the radius vector of the rotating particle P is

(a) $x(t) = B\sin\left(\frac{2\pi t}{30}\right)$

(b) $x(t) = B\cos\left(\frac{\pi t}{15}\right)$

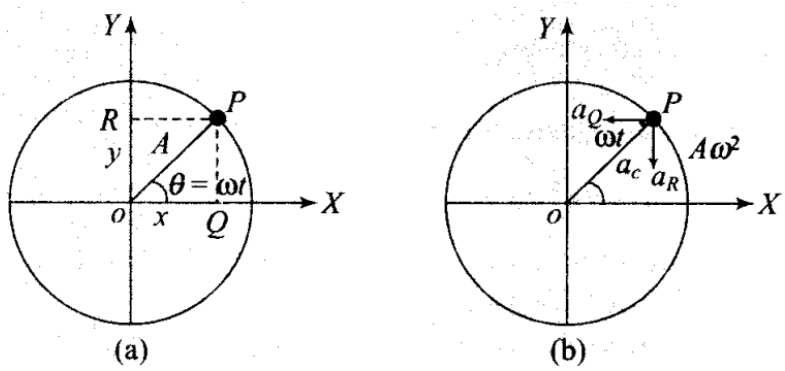
(c) $x(t) = B\sin\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$

(d) $x(t) = B\cos\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$



Sol: (a)

Key concept: Suppose a particle P is moving uniformly on a circle of radius A with angular speed. Q and R are the two feets of the perpendicular drawn from P on two diameters one along .Y-axis and the other along Y-axis.



Suppose the particle P is on the X-axis at t = 0. Radius OP makes an angle with the X-axis at time t, then

x = A cosωt and y = A sinωt

Here, x and v are the displacements of Q and R from the origin at time t, which are the displacement equations of SHM. It implies that although P is under uniform circular motion, Q and R are performing SHM about O with the same angular speed as that of P.

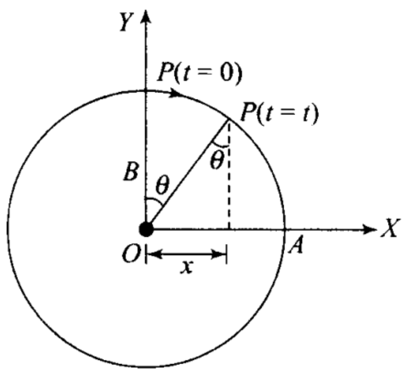
Let angular velocity of the particle executing circular motion is ω and when it is at P makes an angle θ as shown in the diagram.

As $\sin\theta = \frac{x}{OP} = \frac{x}{B}$,

Clearly, $\omega = \omega t$

$$x = B\sin\theta = B(\sin\omega t) = B\sin\left(\frac{\pi t}{15}\right)$$

$$x = B\sin\left(\frac{2\pi}{30}t\right)$$



Q9. The equation of motion of a particle is x = a cos(∝t)^2 . The motion

(a) periodic but not oscillatory

(b) periodic and oscillatory

(c) oscillatory but not periodic

(d) neither periodic nor oscillatory

Sol: (c) The equation of motion of a particle is

x = a cos(∝t)^2

is a cosine function and x varies between -a and +a, the motion is oscillatory. Now checking for periodic motion, putting t+T in place of t. T is supposed as period of the function ω(t).

$$x(t+T) = a\cos[\alpha(t+T)]^2$$
$$= a\cos[\alpha^2 t^2 + \alpha^2 T^2 + 2\alpha^2 tT] \neq x(t)$$

Hence, it is not periodic.

Q10. A particle executing SHM has a maximum speed of 30 cm/s and a maximum acceleration of 60 cm/s^2. The period of oscillation is

(a) sec
(b) /2 sec
(c) 2 sec
(d) /t

=

Sol. (a)

Key concept: Let equation of an SHM is represented by $y = a \sin \omega t$

$$v = \frac{dy}{dt} = a\omega \cos \omega t$$

Hence $(v)_{max} = a\omega$ ...(i)

Acceleration $(A) = \frac{dx^2}{dt^2} = -a\omega^2 \sin \omega t$

Hence $A_{max} = \omega^2 a$ ...(ii)

Maximum speed, $v_{max} = \omega A$ ...(i)

Maximum acceleration, $a_{max} = \omega^2 A$ ...(ii)

Divide eqn. (ii) by eqn. (i), we get

$$\frac{a_{max}}{v_{max}} = \frac{\omega^2 A}{\omega A} = \omega \qquad \therefore \frac{a_{max}}{v_{max}} = \frac{2\pi}{T}$$

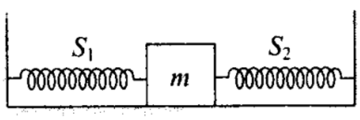
$$T = 2\pi\left(\frac{v_{max}}{a_{max}}\right)$$

Here, $v_{max} = 30 \text{ cms}^{-1}$, $a_{max} = 60 \text{ cms}^{-2}$

$$\therefore \quad T = 2\pi\left(\frac{30 \text{ cms}^{-1}}{60 \text{ cms}^{-2}}\right) = \pi \text{s}$$

Q11. When a mass in is connected individually to two springs S_1 and S_2, the oscillation frequencies are V_1 and V_2. If the same mass is attached to the two springs as shown in figure, the oscillation frequency would be

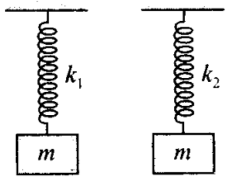
(a) $v_1 + v_2$
(b) $\sqrt{v_1^2 + v_2^2}$
(c) $\left(\frac{1}{v_1} + \frac{1}{v_2}\right)^{-1}$
(d) $\sqrt{v_1^2 - v_2^2}$



Sol. (b) When the mass is connected to the two springs individually,

$$v_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} \quad \ldots(i)$$

$$v_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}} \quad \ldots(ii)$$



Now the block is connected with two springs considered as parallel.

Here equivalent spring constant, $k_{eq} = k_1 + k_2$

Time period of oscillation of the spring block-system is

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

Hence frequency,

$$v = \frac{1}{T} = \frac{1}{2\pi} \times \sqrt{\frac{k_1 + k_2}{m}} \quad \ldots(iii)$$

$$v = \frac{1}{2\pi}\left[\frac{k_1}{m} + \frac{k_2}{m}\right]^{1/2}$$

From Eq. (i) $\frac{k_1}{m} = 4\pi^2 v_1^2$ and from Eq. (ii), $\frac{k_2}{m} = 4\pi^2 v_2^2$

$$\Rightarrow \qquad v = \frac{1}{2\pi}\left[\frac{4\pi^2 v_1^2}{1} + \frac{4\pi^2 v_2^2}{1}\right]^{1/2} = \frac{2\pi}{2\pi}[v_1^2 + v_2^2]^{1/2}$$

$$\Rightarrow \qquad v = \sqrt{v_1^2 + v_2^2}$$

More Than One Correct Answer Type

Q12. The rotation of earth about its axis is

(a) periodic motion

(b) simple harmonic motion

(c) periodic but not simple harmonic motion

(d) non-periodic motion

Sol: (a, c) Rotation of earth about its axis repeats its motion after a fixed interval of lime, so its motion is periodic.

The rotation of earth is obviously not a to and fro type of motion about a fixed point, hence its motion is not an oscillation. Also this motion does not follow S.H.M equation, a ∝ -x.

Hence, this motion is not a S.H.M.

Q13. Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower point is

(a) simple harmonic motion

(b) non-periodic motion

(c) periodic motion

(d) periodic but not SHM

Sol: (a, c) For small angular displacement, the situation is shown in the figure. Only one restoring force creates .

motion in ball inside bowl.

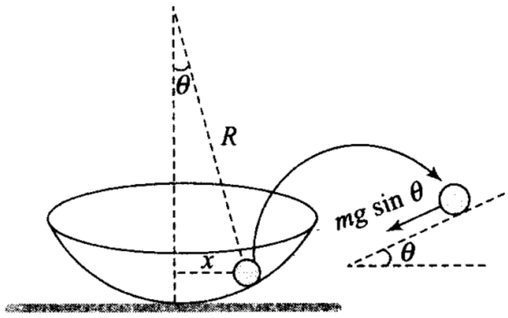
Only one restoring force creates motion in ball inside bowl.

$$F = -mg \sin\theta$$

As θ is small, $\sin\theta = \theta$

So, $ma = -mg\dfrac{x}{R}$

or, $a = -\left(\dfrac{g}{R}\right)x \Rightarrow a \propto -x$



So, motion of the ball is S.H.M and periodic.

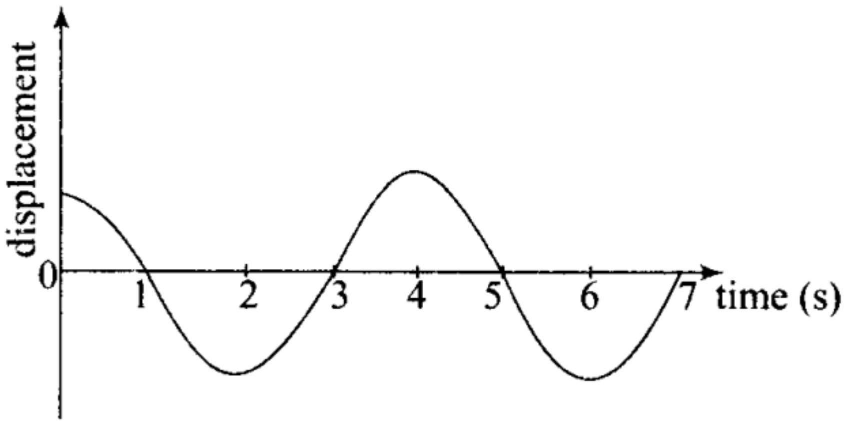
Q14. Displacement versus time curve for a particle executing SHM is shown in figure. Choose the correct statements.

(a) Phase of the oscillator is same at t = 0s and t = 2 s

(b) Phase of the oscillator is same at t = 2s and t = 6 s

(c) Phase of the oscillator is same at t = 1s and t = 7 s

(d) Phase of the oscillator is same at t = 1s and t = 5 s



Sol: (b,d)

Key concept:

Phase: The physical quantity which represents the state of motion of particle (e.g. its position and direction of motion at any instant).

The time varying quantity (t +) is called the phase of the motion, and the constant is called the phase constant (or phase angle). Phase determines the status of the particle at t = 0.

Suppose we choose t= 0, at an instant when the particle is passing through its mean position and is going towards the positive direction. The phase (t +)becomes zero.

=> = 0 and x= Asint and v = A*cos*t

If we choose t = 0, at an instant when the particle is at its position extreme position, then is π/2 at that instant.

Thus t + = π/2 at t = 0 ⇒ = π/2, or x = A sin (t + π/2) – A cost

of λ $\left(\text{say}\dfrac{\lambda}{2}, \dfrac{3\lambda}{2}, \ldots\right)$ or the time interval is an odd multiple of $(T/2)$.

Phase difference: If two particles performs S.H.M and their equation are

$$y_1 = a\sin(\omega t + \phi_1) \text{ and } y_2 = a\sin(\omega t + \phi_2)$$

then phase difference $\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1) = \phi_2 - \phi_1$

In option (a) at t =0 s and t = 2 s, the displacements are in opposite directions, hence phase of the oscillator is not same which makes option incorrect.

In option (b) it is clear from the curve that points corresponding to t = 2 s and t = 6 s are separated by a distance belonging to one time period. Hence, these points must be in same phase.
In option (c) t = 1 s and t – 7 s though the displacement is zero but the particle moves in opposite directions, downwards at t = 1 s and upwards at t — 7 s. Hence phase of the oscillator is not same which makes option incorrect.
In option (d) points belong to t = 1 s and t = 5 s are at separation of one time period, hence must be in phase.

Q15. Which of the following statements is/are true for a simple hannonic oscillator?
(a) Force acting is directly proportional to displacement from the mean position and opposite to it
(b) Motion is periodic
(c) Acceleration of the oscillator is constant
(d) The velocity is periodic
Sol: (a, b, d)
Key concept: The simple harmonic motion is a type of periodic motion or oscillation motion where the restoring force is directly proportional to the displacement and acts in the direction opposite to that of displacement. When the system is displaced from its equilibrium position, a restoring force that obeys Hooke's law tends to restore the system to equilibrium. As a result, it accelerates and starts going back to the equilibrium position. An oscillation follows simple harmonic motion if it fulfils the following two rules:
1. Acceleration is always in the opposite direction to the displacement from the equilibrium position.
2. Acceleration is proportional to the displacement from the equilibrium position.

Let us write the equation for the SHM $x = a\sin(\omega t + \phi)$
Clearly, it is a periodic motion as it involves sine function.

Letusfindvelocityoftheparticle, $v = \frac{dx}{dt} = \frac{d}{dt}(a\sin(\omega t + \phi)) = a\omega\cos(\omega t + \phi)$

Velocity is also periodic because it is a cosine function.

Now let us find acceleration, $A = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -a\omega^2\sin(\omega t + \phi)$

The acceleration is a sine function, hence cannot be constant.

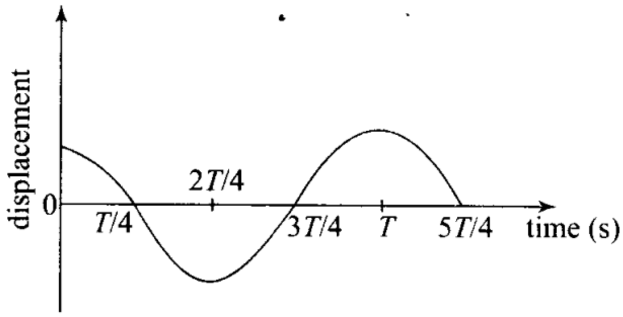
$\Rightarrow \qquad A = -(\omega^2 a)\sin(\omega t + \phi) = -\omega^2 x$

Force, F = mass × acceleration

$= mA = -m\omega^2 x$

Hence, force acting is directly proportional to displacement from the mean position and opposite to it.

Q16. The displacement-time graph of a particle executing SHM is shown in figure. Which of the following statement is/are true?



(a) The force is zero at $t = \frac{3T}{4}$

(b) The acceleration is maximum at $t = \frac{4T}{4}$

(c) The velocity is maximum at $t = \frac{T}{4}$

(d) The PE is equal to KE of oscillation at $t = \frac{T}{2}$

Sol. (a, b, c) Displacement of the particle at any time t,

$$y = A_0 \cos \omega t = A_0 \cos \frac{2\pi}{T} t$$

And acceleration $A = -\omega^2 y = \omega^2 A_0 \cos \frac{2\pi}{T} t$.

In option (a) Force $F = m\omega^2 y$.

At $$t = \frac{3T}{4},\ y = A_0 \cos\left(\frac{2\pi}{T} \times \frac{3T}{4}\right)$$

$$= A_0 \cos\left(\frac{3\pi}{2}\right) = 0$$

Hence $F = 0$,which makes option correct.

In option (b) acceleration $A = -\omega^2 y = \omega^2 A_0 \cos \frac{2\pi}{T} t$.

At $t = \frac{4T}{4},\ y = A_0 \cos\left(\frac{2\pi}{T} \times T\right) = A_0 \cos 2\pi = +A_0$

$\therefore a = \omega^2 A_0$, which is maximum value of a.

Hence option (b) is correct.

In option (c) The velocity $v = \omega\sqrt{A_0^2 - y^2}$

At $t = T/4$

$$y = A_0 \cos\left(\frac{2\pi}{T} \times \frac{T}{4}\right) = A_0 \cos\left(\frac{\pi}{2}\right) \Rightarrow y = 0$$

Hence, $v = \omega A_0$, which is maximum value of velocity.

Hence option (c) is correct.

In option (d) At $t = T/2$,

$$y = A_0 \cos\left(\frac{2\pi}{T} \times \frac{T}{2}\right) = -A_0$$

Displacement is maximum, i.e corresponds to extreme position, it means PE is maximum and KE is zero.

Q17. A body is performing SHM, then its

(a) average total energy per cycle is equal to its maximum kinetic energy

(b) average kinetic energy per cycle is equal to half of its maximum kinetic energy

(c) mean velocity over a complete cycle is equal to $2/\pi$ times of its maximumvelocity

(d) root mean square velocity is $1/\sqrt{2}$ times of its maximum velocity

Sol. (a, b, d) In case of S.H.M, average total energy per cycle

$$= \text{Maximum kinetic energy } (K_0)$$
$$= \text{Maximum potential energy } (U_0)$$

Average KE per cycle $= \dfrac{0+K_0}{2} = \dfrac{K_0}{2}$

Let us write the equation for the SHM $x = a \sin \omega t$.

Clearly, it is a periodic motion as it involves sine function.

Let us find velocity of the particle, $v = \dfrac{dx}{dt} = \dfrac{d}{dt}(a \sin \omega t) = a\omega \cos \omega t$

Mean velocity over a complete cycle,

$$v_{\text{mean}} = \frac{\int_0^{2\pi} \omega a \cos \theta d\theta}{2\pi} = \frac{\omega a[\sin \theta]_0^{2\pi}}{2\pi} = 0$$

So, $\qquad v_{\text{mean}} \neq \dfrac{2}{\pi} v_{\max}$

Root mean square speed,

$$v_{\text{rms}} = \sqrt{\frac{v_{\min}^2 + v_{\max}^2}{2}} = \sqrt{\frac{0 + v_{\max}^2}{2}}$$

$$v_{\text{rms}} = \frac{1}{\sqrt{2}} v_{\max}$$

Q18. A particle is in linear simple harmonic motion between two points A and B, 10 cm apart (figure). Take the direction from A to B as the positive direction and choose the correct statements. __ _

AO = OB = 5 cm

BC= 8 cm

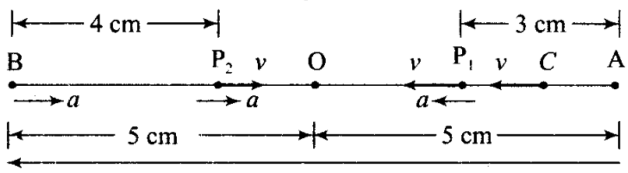
(a) The sign of velocity, acceleration and force on the particle when it is 3 cm away from A going towards B are positive

(b) The sign of velocity of the particle at C going towards B is negative

(c) The sign of velocity, acceleration and force on the particle when it is 4 cm away from B going towards A are negative

(d) The sign of acceleration and force on the particle when it is at point B is negative

Sol. (a, c, d)

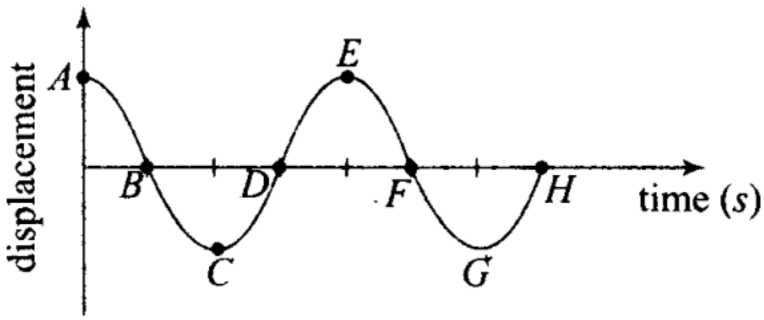


⊕ Positive direction from A to B

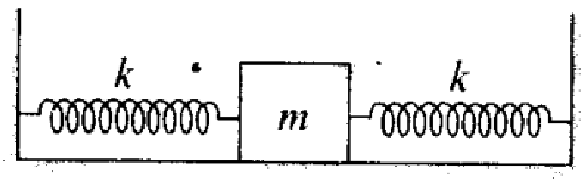
In option (a): When the particle is 3 cm away from A going towards B. So, velocity is towards AB, i.e. positive. In SHM, acceleration is always towards mean position (O) it means both

force and acceleration act towards O, have positive sign.
Hence option (a) is correct.
In option (b): When the particle is at C, velocity is towards B hence positive. Hence option (b) is not correct.
In option (c): When the particle is 4 cm away from B going towards A velocity is negative and acceleration is towards mean position (O), hence negative. Hence option (c) is correct.
In option (d): Acceleration is always towards mean position (O). When the particle is at B, acceleration and force are towards BA that is negative. Hence option (d) is correct.

Very Short Answer Type Questions
Q19. Displacement versus time curve for a particle executing SHM is shown in figure**. Identify the points marked at which (i) velocity of the oscillator is zero, (ii) speed of the oscillator is maximum.**
Sol: Key concept: In displacement-time graph of SHM, zero displacement values correspond to mean position; where velocity of the oscillator is maximum. Whereas the crest and troughs represent amplitude positions, where displacement is maximum and velocity of the oscillator is zero.
(i) The points A, C, E, G lie at extreme positions (maximum displacement, y = A). Hence the velocity of the oscillator is zero.
(ii) The points B, D, F, H lie at mean position (zero displacement, y = 0). We know the speed is maximum at mean position.



Q20. Two identical springs of spring constant k are attached to a block of mass m and to fixed supports as shown in figure. When the mass is displaced from equilibrium position by a distance x towards right**, find the restoring force.**



When mass is displaced from equilibrium position by a distance x towards right, the right spring gets compressed by x developing a restoring force kx towards left on the block. The left spring is stretched by an amount x developing a restoring force kx left on the block.

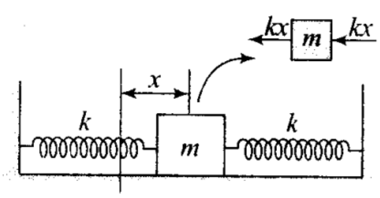
developing a restoring force kx towards left on the block.

$F_1 = -kx$ (for left spring) and

$F_2 = -kx$ (for right spring)

Restoring force, $F = F_1 + F_2 = -2kx$

$\therefore \quad F = 2kx$ towards left.



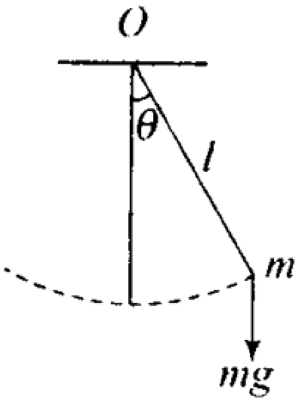
Q21. What are the two basic characteristics of a simple harmonic motion?
Sol: The two basic characteristics of a simple harmonic motion
(i) Acceleration is directly proportional to displacement.

(ii) The direction of acceleration is always towards the mean position, that is opposite to displacement.

Q22. When will the motion of a simple pendulum be simplehannonic**?**

Sol: Simple pendulum perform angular S.H.M. Consider the bob of simple pendulum is displaced through an angle θ shown. Q
The restoring torque about the fixed point O is

τ = mgl sinθ
If θ is small angle in radians, then sin θ = 0
=> mglθ
In vector form τ ∝ θ
Hence, motion of a simple pendulum is SHM for small angle of oscillations.



Q23. What is the ratio of maximum acceleration to the maximum velocity of a simple harmonic oscillator?

Sol. Let us write the displacement equation for the SHM $x = a\sin(\omega t + \phi)$.
We have to find velocity by differentiating the equation representing displacement and acceleration by differentiating the equation relating velocity and time.

The velocity of the particle, $v = \frac{dx}{dt} = \frac{d}{dt}(a\sin(\omega t + \phi)) = a\omega\cos(\omega t + \phi)$

Maximum velocity $|v|_{max} = a\omega$

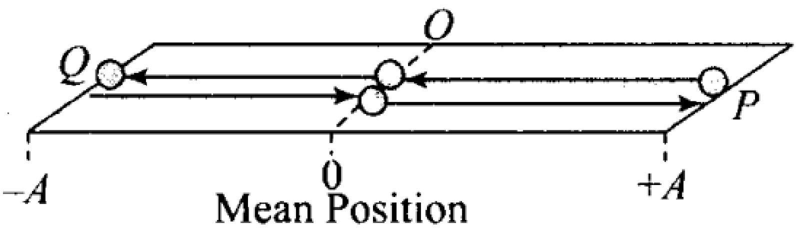
Now let us find acceleration, $A = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -a\omega^2\sin(\omega t + \phi)$

Maximum acceleration $|A|_{max} = \omega^2 a$

$$\frac{|v|_{max}}{|A|_{max}} = \frac{\omega a}{\omega^2 a} = \frac{1}{\omega} \Rightarrow \frac{a_{max}}{v_{max}} = \omega$$

Q24. What is the ratio between the distance travelled by the oscillator in one time period and amplitude?

Sol: In the diagram shown a particle is executing SHM between P and Q. The particle starts from mean position 'O' moves to amplitude position 'P', then particle turn back and moves from 'P' to iQ\ Finally the particle turns back again and return to mean position 'O'. In this way the particle completes one oscillation in one time period.



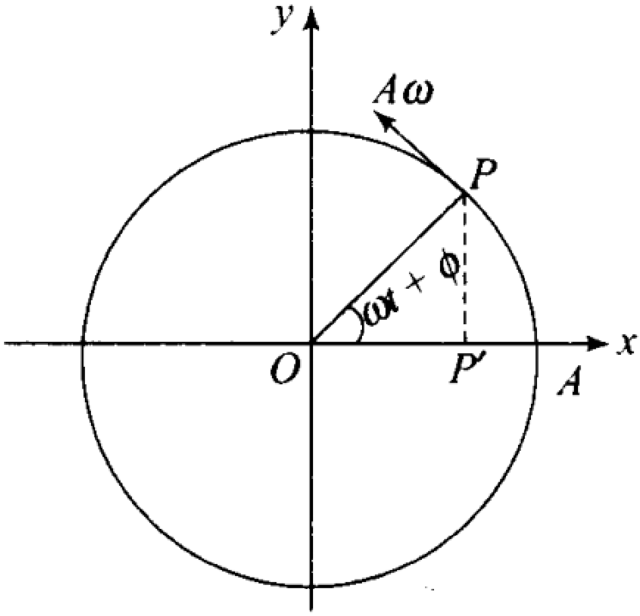
Total distance travelled while it goes from O → P → O → Q → O

= OP + PO + OQ+QO = A+A+A+A=4A

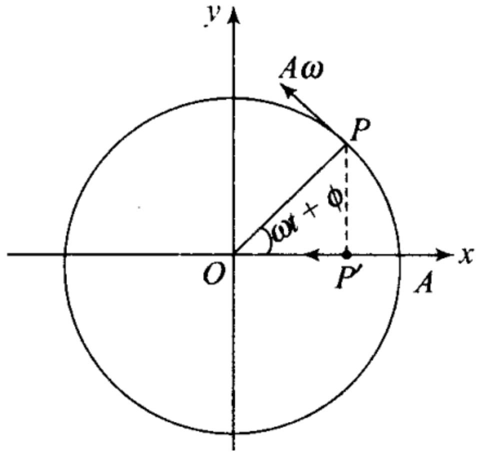
Amplitude = OP = A

Hence, ratio of distance and amplitude = 4A/A = 4

Q25. In figure, what will be the sign of the velocity of the point P', which is the projection of the velocity of the reference particle P. P is moving in a circle of radius R in anti-clockwise direction.



Sol. As the particle on reference circle moves in anti-clockwise direction. The projection will move from P' to O towards left, i.e. from right to left, hence sign is negative.



Q26. Show that for a particle executing SHM, velocity and displacement have a phase difference of π/2 .

Sol. Let the displacement equation of SHM

$$x = a \cos \omega t$$

velocity $v = \frac{dx}{dt} = a\omega(-\sin \omega t) = -a\omega \sin \omega t$

$$\Rightarrow \quad v = a\omega \cos\left(\frac{\pi}{2} + \omega t\right)$$

Now, phase of displacement $\phi_1 = \omega t$

Phase of velocity $\phi_2 = \frac{\pi}{2} + \omega t$

∴ Difference in phase of velocity to that of phase of displacement

$$\Delta\phi = \phi_2 - \phi_1 = \left(\frac{\pi}{2} + \omega t\right) - (\omega t) = \frac{\pi}{2}$$

Q27. Draw a graph to show the variation of PE, KE and total energy of a simple harmonic oscillator with displacement.

Sol. The potential energy (PE) of a simple harmonic oscillator is

$$\text{PE} = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2 \quad \text{...(i)}$$

When, PE is plotted against displacement x, we will obtain a parabola.
When $x = 0$, PE = 0.
When $x = \pm A$, PE = maximum

$$= \frac{1}{2}m\omega^2A^2$$

The kinetic energy (KE) of a simple harmonic oscillator $\text{KE} = \frac{1}{2}mv^2$

But velocity of oscillator $v = \omega\sqrt{A^2 - x^2}$

$$\Rightarrow \quad \text{KE} = \frac{1}{2}m[\omega\sqrt{A^2 - x^2}]^2$$

or

$$\text{KE} = \frac{1}{2}m\omega^2(A^2 - x^2) \quad \text{...(ii)}$$

This is also parabola, if we plot KE against displacement x.

i.e., KE = 0 at $x = \pm A$

and $\text{KE} = \frac{1}{2}m\omega^2A^2 at\ x = 0$

Now, total energy of the simple harmonic oscillator = PE + KE
[using Eqs. (i) and (ii)]

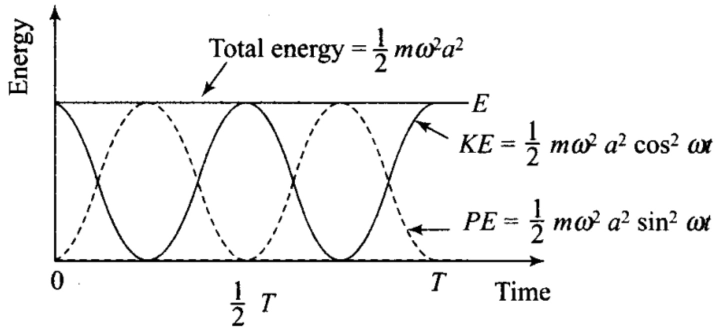
$$= \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$= \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2A^2 - \frac{1}{2}m\omega^2x^2$$

$$\text{TE} = \frac{1}{2}m\omega^2A^2 = \text{constant}$$

which is a constant and independent of x.

Plotting under the above guidelines KE, PE and TE versus displacement x-graph as follows:



Important point: From the graph we note that potential energy or kinetic energy completes two vibrations in a time during which S.H.M. completes one vibration. Thus the frequency of potential energy or kinetic energy is double than that of S.H.M.

Q28. The length of a second's pendulum on the surface of earth is 1 m. What will be the length of a second's pendulum on the moon?

Sol.

Key concept: A second's pendulum means a simple pendulum having time period $T = 2$ s.

The time period of a simple pendulum, $T = 2\pi\sqrt{\dfrac{l}{g}}$

where, l = length of the pendulum and

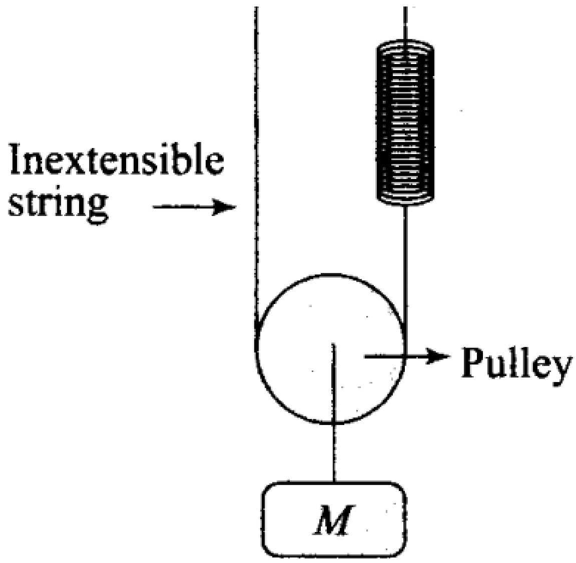
g = acceleration due to gravity on surface of the earth.

Thus, $\dfrac{l_{\text{Moon}}}{g_{\text{Moon}}} = \dfrac{l_{\text{Earth}}}{g_{\text{Earth}}}$

or $l_{\text{Moon}} = (l_{\text{Earth}})\dfrac{g_{\text{Moon}}}{g_{\text{Earth}}} = (1\text{ m})\left(\dfrac{1}{6}\right) = \dfrac{1}{6}\text{m}$

Short Answer Type Questions

Q29. Find the time period of mass M when displaced from its equilibrium position and then released for the system as shown in figure.

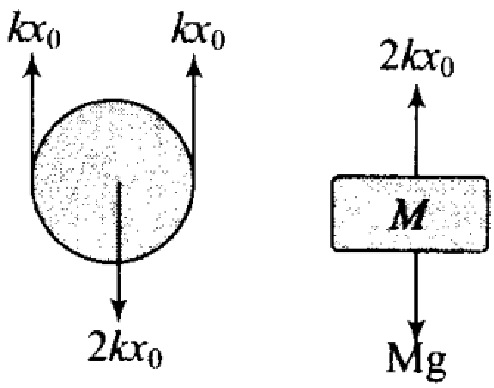


Sol: Key concept: For observing oscillation, we have to displace the block slightly beyond equilibrium position and find the acceleration due to the restoring force.

Let in the equilibrium position, the spring has extended by an amount x_0.

Tension in the spring = kx_0

For equilibrium of the mass M, Mg = $2kx_0$

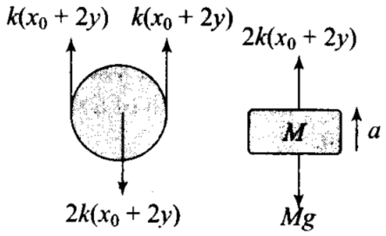


Let the mass be pulled through a distance y and then released. But, string is inextensible, hence the spring alone will contribute the total extension y + y = 2y, to lower the mass down by y from initial equilibrium mean position x_0. So, net extension in the spring (x_0 + 2y). From F.B.D of the block,

$$2k(x_0 + 2y) - Mg = Ma$$

$$2kx_0 + 4ky - Mg = Ma \Rightarrow Ma = 4ky$$

$$\vec{a} = -\left(\frac{4k}{M}\right)\vec{y}$$



K and M being constant.

$\therefore\ a \propto -x$. Hence, motion is SHM.

Comparing the above acceleration expression with standard SHM equation $a = -\omega^2 x$, we get

$$\omega^2 = \frac{4k}{M} \Rightarrow \omega = \sqrt{\frac{4k}{M}}$$

Q30.Show that the motion of a particle represented by y = sin ax – cos cot is simple harmonic with a period of 2π/ω

Sol: The given equation is in the form of combination of two harmonic functions. We can write this equation in the form of a single harmonic (sine or cosine) function.

We have displacement function: $y = \sin \omega t - \cos \omega t$

We have displacement function: $y = \sin \omega t - \cos \omega t$

$$y = \sqrt{2}\left(\frac{1}{\sqrt{2}} \cdot \sin \omega t - \frac{1}{\sqrt{2}} \cdot \cos \omega t\right)$$

$$= \sqrt{2}\left[\cos\left(\frac{\pi}{4}\right) \cdot \sin \omega t - \sin\left(\frac{\pi}{4}\right) \cdot \cos \omega t\right]$$

$$= \sqrt{2}\left[\sin\left(\omega t - \frac{\pi}{4}\right) = \sqrt{2}\left[\sin\left(\omega t - \frac{\pi}{4}\right)\right]\right]$$

$$\Rightarrow \qquad y = \sqrt{2} \sin\left(\omega t - \frac{\pi}{4}\right)$$

Comparing with standard equation of S.H.M,

$y = a \sin(\omega t + \phi)$ we get angular frequency of S.H.M, $\omega = \dfrac{2\pi}{T} \Rightarrow T = \dfrac{2\pi}{\omega}$

Hence the function represents SHM with a period T = 2π/ω

Q31. Find the displacement of a simple harmonic oscillator at which its PE is half of the maximum energy of the oscillator.

Sol: Let us assume that the required displacement where PE is half of the maximum energy of the oscillator be x.

The potential energy of the oscillator at this position,

$$PE = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2 \quad ...(i)$$

Maximum energy of the oscillator = Maximum potential energy = Total energy

$$TE = \frac{1}{2}m\omega^2 A^2 \quad ...(ii)$$

where, A = amplitude of motion

We are given, $PE = \frac{1}{2}TE$

$$\Rightarrow \quad \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}\left[\frac{1}{2}m\omega^2 A^2\right]$$

$$\Rightarrow \quad x^2 = \frac{A^2}{2} \text{ or } x = \sqrt{\frac{A^2}{2}} = \pm\frac{A}{\sqrt{2}}$$