Permutations and Combinations

1. Fundamental Principles of Counting

There are two fundamental principles of counting, which are as follow

(i) **Multiplication principle** Suppose an event A can be occur in m ways following which, another event B can be occur in n ways, then total number of performance of two events in the given order is $m \times n$ ways. This can be extended to any finite number of operations.

(ii) **Addition principle** If an event A can be occur in m ways and another event B, which is independent of A, can be occur in n ways, then A and B can be occurred in $(m + n)$ ways. This can be extended to any finite number of events.

2. *n* Factorial

The continued product of first n natural number is called factorial 'n'. It is denoted by $n!$ or $\underline{|n}$.

Note $n! = n(n-1)(n-2)\ldots 3 \times 2 \times 1$ and $0! = 1! = 1$

3. Permutations

A permutation is an arrangement in a definite order of a number of objects taken some or all at a time.

The number of arrangements of n different objects taken r at a time, where $0 < r \le n$, is denoted by ${}^nP_r$ and given by ${}^nP_r = \dfrac{n!}{(n-r)!}$.

Properties of permutations

(i) ${}^nP_n = n(n-1)(n-2)\ldots 3 \times 2 \times 1 = n!$

(ii) ${}^nP_0 = \dfrac{n!}{n!} = 1$

(iii) ${}^nP_1 = n$

(iv) ${}^nP_{n-1} = n!$

(v) ${}^nP_r = n \cdot {}^{n-1}P_{r-1} = n(n-1)\,{}^{n-2}P_{r-2}$

(vi) ${}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1} = {}^nP_r$ (vii) $\dfrac{{}^nP_r}{{}^nP_{r-1}} = n - r + 1$

4. Important Theorems Based on Permutations

(i) The number of permutations of n things taken r at a time, when repetition of object is allowed, is n^r.

(ii) The number of permutations of n objects, where p objects are of the sam kind or identical and other are distinct, is given by $\dfrac{n!}{p!}$.

(iii) The number of permutations of n objects, where p_1 are of one kind, p_2 are of second kind, ..., p_k are of kth kind such that $p_1 + p_2 + p_3 + \dots + p_k = n$, is $\dfrac{n!}{p_1!\,p_2!\,p_3!\dots p_k!}$.

5. Important Results Based on Permutation

(i) Number of permutations of n different objects taken r at a time,

(a) when a particular object is to be included in each arrangement, is $r \cdot {}^{n-1}P_{r-1}$.

(b) when a particular object is always excluded is ${}^{n-1}P_r$.

(ii) Number of permutations of n different objects taken all at a time, when m specified objects always come together, is $m!(n-m+1)!$.

(iii) Number of permutations of n different objects taken all at a time, when m specified objects never come together, is $n! - m!(n-m+1)!$.

6. Combinations

Each of the different selections, which is made by taking some or all of a number of objects irrespective of their arrangements is called combinations.

The number of selection of r objects from the given n distinct objects is denoted by ${}^nC_r$ and given by

$$ {}^nC_r = \frac{n!}{r!(n-r)!}. $$

Properties of combinations

(i) ${}^nC_0 = {}^nC_n = 1$

(ii) ${}^nC_1 = {}^nC_{n-1} = n$

(iii) ${}^nC_r = \dfrac{{}^nP_r}{r!}$

(iv) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(v) ${}^nC_r = {}^nC_{n-r}$

(vi) $r\ {}^nC_r = n\ {}^{n-1}C_{r-1}$

(vii) If ${}^nC_x = {}^nC_y$, then either $x = y$ or $x + y = n$