Unit 7 (Permutations And Combinations)

Short Answer Type Questions

Q1. Eight chairs are numbered 1 to 8. Two women and 3 men wish to occupy one chair each. First the women choose the chairs from amongst the chairs 1 to 4 and then men select from the remaining chairs. Find the total number of possible arrangements.

Sol. First the women choose the chairs from amongst the chairs numbered 1 to 4.

Two women can be arranged in 4 chairs in $^4P_2$ ways.

In remaining 6 chairs, 3 men can be arranged in $^6P_3$ ways.

$\therefore$ Total number of possible arrangements $= {}^4P_2 \times {}^6P_3 = \frac{4!}{2!} \times \frac{6!}{3!}$

$= 4 \times 3 \times 6 \times 5 \times 4 = 1440$

Q2. If the letters of the word RACHIT are arranged in all possible ways as listed in dictionary, then what is the rank of the word RACHIT?

Sol: The alphabetical order of the letters of the word RACHIT is: A, C, H, I, R, T. Number of words beginning with A = 5!

Number of words beginning with C = 5!

Number of words beginning with H = 5!

Number of words beginning with 1 = 5!

Clearly, the first word beginning with R is RACHIT.

.•. Rank of the word RACHIT in dictionary = 4×5! + 1= 4 x120+1= 481

Q3. A candidate is required to answer 7 questions out of 12 questions, which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. Find the number of different ways of doing questions.

Sol: Since the candidate cannot attempt more than 5 questions from either group, he is able to attempt minimum two questions from either group.

The possible number of questions attempted from each group will be as given in the following table:

Group I	5	4	3	2
Group II	2	3	4	5

∴ Total number of possible ways $= 2[{}^6C_5 \times {}^6C_2 + {}^6C_4 \times {}^6C_3]$

$= 2[6 \times 15 + 15 \times 20] = 2 \times 390 = 780$

Q4. Out of 18 points in a plane, no three are in the same line except five points which are collinear. Find the number of lines that can be formed joining the point.

Sol: There are 18 point in a plane, of which 5 points are collinear.

Number of straight lines formed by joining the 18 points taking 2 at a time = ${}^{18}C_2$

Now, number of straight lines formed by joining 5 points which are collinear taking 2 at a time = ${}^5C_2$

But 5 collinear points, when joined pairwise, give only one line.

∴ Required number of straight lines $= {}^{18}C_2 - {}^5C_2 + 1 = 153 - 10 + 1 = 144$

Q5. We wish to select 6 persons from 8 but, if the person A is chosen, then B must be chosen. In how many ways can selections be made?

Sol: Total number of persons = 8

Number of person to be selected = 6

It is given that, if A is chosen then, B must be chosen. Therefore, following cases arise:

Case I: When *A* is chosen, *B* must be chosen.

Number of ways = Number of ways of selecting 4 more persons from remaining 6 persons

$= {}^{8-2}C_{6-2} = {}^6C_4 = 15$

Case II: When *A* is not chosen.

∴ Number of ways = Number of ways of selecting 6 persons from remaining 7 persons ${}^7C_6 = 7$

∴ Total number of ways = 15 + 7 = 22

Q6. How many committee of five persons with a chairperson can be selected form 12 persons?

Sol: Total number of persons =12

Number of persons to be selected = 5

Out of 12 persons, a chairperson can be selected in $^{12}C_1 = 12$ ways.
Remaining 4 persons can be selected out of 11 persons in $^{11}C_4 = 330$ ways.
$\therefore$ Total number of committee $= 12 \times 330 = 3960$

Q7. How many automobile license plates can be made, if each plate contains two different letters followed by three different digits?

Sol: There are 26 English alphabets and 10 digits (0 to 9).

It is given that each plate contains two different letters followed by three different digits.

$\therefore$ Arrangement of 26 letters, taken 2 at a time $= {}^{26}P_2 = 26 \times 25 = 650$
And arrangement of 10 digits, taken three at a time $= {}^{10}P_3 = 10 \times 9 \times 8 = 720$
$\therefore$ Total number of license plates $= 650 \times 720 = 468000$

Q8. A bag contains 5 black and 6 red balls. Determine the number of ways in • which 2 black and 3 red balls can be selected from the lot.

Sol: The bag contains 5 black and 6 red balls.

Now, 2 black balls can be selected from 5 black balls in $^5C_2$ ways.
And 3 red balls can be selected from 6 red balls in $^6C_3$ ways.
$\therefore$ Total number of ways in which 2 black and 3 red balls can be selected
$$= {}^5C_2 \times {}^6C_3 = 10 \times 20 = 200$$

Q9. Find the number of permutations of n distinct things taken r together, in which 3 particular things must occur together.

Sol: Total number of things. = n

We have to arrange r things out of n in which three particular things must occur together.

Therefore, combination of n things taken r at a time in which 3 things always occur $= {}^{n-3}C_{r-3}$
Three things taken together will be considered as 1 group.
Number of arrangements of three things = 3!
Now, we have to arrange $r - 3 + 1 = (r - 2)$ objects.
$\therefore$ Number of arrangements of $(r - 2)$ objects $= (r - 2)!$
$\therefore$ Total number of arrangements $= {}^{n-3}C_{r-3} \times (r-2)! \times 3!$

Q10. Find the number of different words that can be formed from the letters of the word TRIANGLE, so that no vowels are together.

Sol: Given word is: TRIANGLE Consonants are: T, R, N, G, L Vowels are: I, A, E

Since we have to form words in such a way that no two vowels are together, we first arrange consonants.

Five consonants can be arranged in 5! ways.

$$\times C \times C \times C \times C \times C \times$$

Arrangements of consonants (in the fig. marked as C) creates six gaps marked as '×'
Now three vowels can be arranged in any three of these 6 gaps in $^6P_3$ ways.
So, total number of arrangements $= 5! \times {}^6P_3 = 120 \times 120 = 14400$

Q11. Find the number of positive integers greater than 6000 and less than 7000 which are divisible by 5, provided that no digit is to be repeated.

Sol: We have to form 4-digit numbers which are greater than 6000 and less than 7000.

We know that a number is divisible by 5, if at the unit place of the number there is 0 or 5.

So, unit digit can be filled in 2 ways.

The thousandth place can be filled by '6' only.

The hundredth place and tenth place can be filled together in 8 x 7 = 56 ways. So, total number of ways = 56 x 2 = 112

Q12. Thereare 10persons named $P_1, P_2, P_3, \ldots, P_{!0}$.Outof 10 persons, 5 persons are to be arranged in a line such that in each arrangement P, must occur whereas P_4 and P_5 do not occur. Find the number of such possible arrangements.

Sol. Given that, P_1, P_2, ..., P_{10}, are 10 persons, out of which 5 persons are to be arranged but P, must occur whereas P_4 and P_5 never occurs.

As P, is already occurring w'e have to select now 4 out of 7 persons.

.·. Number of selections = $^7C_4$ = 35 Number of arrangements of 5 persons = 35 x 5! = 35 x 120 = 4200

Q13. There are 10 lamps in a hall each one of them can be switched on independently. Find the number of ways in which the hall can be illuminated.

Sol: There are 10 lamps in a hall.

The hall can be illuminated if at least one lamp is switched.

.·. Total number of ways = $^{10}C_1 + {}^{10}C_2 + {}^{l0}C_3 \ldots + {}^{10}C_{]0}$

= $2^{10} - 1$ = 1024- 1 = 1023

Q14. A box contains two white, three black and four red balls. In how many ways can three balls be drawn from the box, if at least one black ball is to be included in the draw?

Sol: There are two white, three black and four red balls.

We have to draw 3 balls, out of these 9 balls in which at least one black ball is included.

So we have following possibilities:

Black balls	1	2	3
Other than black	2	1	0

∴ Number of selections = $^3C_1 \times {}^6C_2 + {}^3C_2 \times {}^6C, + {}^3C_3 \times {}^6C_0$

= 3×15+ 3×6+1= 45+ 18 + 1= 64

Q15. If $^nC_{r-1}$= 36 ^nC_r = 84 and $^nC_{r+1}$= 126, then find the value of $^rC_2$.

Sol. We know that $\frac{^nC_r}{^nC_{r-1}} = \frac{n-r+1}{r}$.

$$\therefore \quad \frac{n-r+1}{r} = \frac{84}{36} \text{ (given)}$$

$$\Rightarrow \quad \frac{n-r+1}{r} = \frac{7}{3} \quad \Rightarrow 3n - 3r + 3 = 7r$$

$$\Rightarrow \quad 10r - 3n = 3 \qquad \text{(i)}$$

$$\frac{^nC_{r+1}}{^nC_r} = \frac{n-(r+1)+1}{r+1} = \frac{126}{84} \text{ (given)}$$

$$\therefore \quad \frac{n-r}{r+1} = \frac{3}{2} \quad \Rightarrow 2n - 2r = 3r + 3$$

$$\Rightarrow \quad 2n - 5r = 3 \qquad \text{(ii)}$$

Solving (i) and (ii), we get $n = 9$ and $r = 3$.

$$\therefore \quad {}^rC_2 = {}^3C_2 = 3$$

Q16. Find the number of integers greater than 7000 that can be formed with the digits 3, 5, 7, 8 and 9 where no digits are repeated.

Sol: We have to find the number of integers greater than 7000 with the digits 3,5, 7, 8 and 9.

So, with these digits, we can make maximum five-digit numbers because repetition is not allowed.

Since all the five-digit numbers are greater than 7000, we have Number of five-digit integers =

5x4x3x2x1 = 120 A four-digit integer is greater than 7000 if thousandth place has any one of 7, 8 and 9.
Thus, thousandth place can be filled in 3 ways. The remaining three places can be filled from remaining four digits in $^4P_3$ ways.
So, total number of four-digit integers = 3x $^4P_3$ = 3x4x3x2 = 72 Total number of integers = 120 + 72 = 192

Q17. If 20 lines are drawn in a plane such that no two of them are parallel and no three are concurrent, in how many points will they intersect each other?
Sol: It is given that no two lines are parallel which means that all the lines are intersecting and no three lines are concurrent.
One point of intersection is created by two straight lines.
Number of points of intersection = Number of combinations of 20 straight lines taken two at a time

$$= {}^{20}C_2 = \frac{20!}{2!18!} = \frac{20\times 19}{2\times 1} = 190$$

Q18. In a certain city, all telephone numbers have six digits, the first two digits always being 41 or 42 or 46 or 62 or 64. How many telephone numbers have all six digits distinct?
Sol: If first two digit is 41, the remaining 4 digits can be arranged in $^8P_4$ = 8 x 7 x 6×5 = 1680 ways.
Similarly, if first two digit is 42, 46, 62, or 64, the remaining 4 digits can be arranged in $^8P_4$ ways i.e., 1680 ways.
.'. Total number of telephone numbers having all six digits distinct = 5x 1680 = 8400

Q19. In an examination, a student has to answer 4 questions out of 5 questions, questions 1 and 2 are however compulsory. Determine the number of ways in which the student can make the choice.
Sol: It is given that 2 questions are compulsory out of 5 questions.
So, the other 2 questions can be selected from the remaining 3 questions in $^3C_2$ = 3 ways.

Q20. A convex polygon has 44 diagonals. Find the number of its sides.
[Hint: Polygon of n sides has ($^nC_2$ – n) number of diagonals.]
Sol: Let the convex polygon has n sides.
Number of diagonals=Number of ways of selecting two vertices – Number of sides = $^nC_2$ – n
It is given that polygon has 44 diagonals.

$$\therefore \quad {}^nC_2 - n = 44$$
$$\Rightarrow \quad \frac{n!}{2!(n-2)!} - n = 44 \Rightarrow \frac{n(n-1)}{2} - n = 44 \Rightarrow n(n-1) - 2n = 88$$
$$\Rightarrow \quad n^2 - 3n - 88 = 0 \quad \Rightarrow (n-11)(n+8) = 0 \quad \Rightarrow n = 11$$

Long Answer Type Questions
Q21. 18 mice were placed in two experimental groups and one control group with all groups equally large. In how many ways can the mice be placed into three groups?
Sol: It is given that 18 mice were placed equally in two experimental groups and one control group i.e., three groups.
Each group is of 6 mice.

So, number of ways forming three groups each of size '6' = ${}^{18}C_6\,{}^{12}C_6\,{}^{6}C_6 = \frac{18!}{6!6!6!}$

But in this counting the order in which three particular groups are formed is also counted.

Clearly, when we form groups the order must not be counted.

Now three particular groups can occur in 3! ways when order is counted.

So, actual number of ways = $\frac{18!}{6!6!6!3!}$

Q22. A bag contains six white marbles and five red marbles. Find the number of ways in which four marbles can be drawn from the bag, if (i) they can be of any colour (ii) two must be white and two red and (iii) they must all be of the same colour.
Sol:Total number of marbles = 6 white +- 5 red = 11 marbles

(a) If they can be of any colour means we have to select 4 marbles out of 11
∴ Required number of ways = ${}^{11}C_4$
(b) Two white marbles can be selected in ${}^{6}C_2$
Two red marbles can be selected in ${}^{5}C_2$ ways.
∴ Total number of ways = ${}^{6}C_2$ x ${}^{5}C_2$ = 15 x 10 = 150
(c) If they all must be of same colour,
Four white marbles out of 6 can be selected in ${}^{6}C_4$ ways.
And 4 red marbles out of 5 can be selected in ${}^{5}C_4$ ways.
∴ Required number of ways = ${}^{6}C_4$ + ${}^{5}C_4$ = 15 + 5 = 20

Q23. In how many ways can a football team of 11 players be selected from 16 players? How many of them will

- **include 2 particular players?**
- **exclude 2 particular players?**

Sol: Total number of players = 16
We have to select a team of 11 players
So, number of ways = ${}^{16}C_{11}$
(i) If two particular players are included then more 9 players can be selected from remaining 14 players in ${}^{14}C_9$
(ii) If two particular players are excluded then all 11 players can be selected from remaining 14 players in ${}^{14}C_{11}$

Q24. sports team of 11 students is to be constituted, choosing at least 5 from class XI and at least 5 from class XII. If there are 20 students in each of these classes, in how many ways can the team be constituted?
Sol: Total number of students in each class = 20
We have to select at least 5 students from each class.
So we can select either 5 students from class XI and 6 students from class XII or 6 students from class XI and 5 students from class XII.
∴ Total number of ways of selecting a team of 11 players = ${}^{20}C_5$ x ${}^{20}C_6$ + ${}^{20}C_6$ x ${}^{20}C_5$ = 2 x ${}^{20}C_5$ x ${}^{20}C_6$

Q25. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected, if the team has
(i) no girls
(ii) at least one boy and one girl
(iii) at least three girls
Sol: Number of girls = 4;

Number of boys = 7

We have to select a team of 5 members provided that

(i) Team having no girls

$\therefore$ Required number of ways = ${}^7C_5 = \frac{7 \times 6}{2!} = 21$

(ii) Team having at least one boy and one girl

$\therefore$ Required number of ways

$= {}^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$

$= 7 \times 1 + 21 \times 4 + 35 \times 6 + 35 \times 4 = 7 + 84 + 210 + 140 = 441$

(iii) Team having at least three girls

Required number of ways = ${}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1$

$= 4 \times 21 + 7 = 84 + 7 = 91$

Objective Type Questions

Q26. If ${}^nC_{12} = {}^nC_8$, then n is equal to

(a) 20

(b) 12

(c) 6

(d) 30

Sol. **(a)** We have, ${}^nC_{12} = {}^nC_8$

$\Rightarrow \quad {}^nC_{n-12} = {}^nC_8 \quad [\because {}^nC_r = {}^nC_{n-r}]$

$\Rightarrow \quad n - 12 = 8 \Rightarrow n = 20$

Q27. The number of possible outcomes when a coin is tossed 6 times is

(a) 36

(b) 64

(c) 12

(d) 32

Sol: (b) Number of outcomes when a coin tossed = 2 (Head or Tail)

∴Total possible outcomes when a coin tossed 6 times = 2x2x2x2x2x 2 = 64

Q28. The number of different four-digit numbers that can be formed with the digits 2, 3, 4, 7 and using each digit only once is

(a) 120

(b) 96

(c) 24

(d) 100

Sol: (c) Given digits 2,3,4 and 7, we have to form four-digit numbers using these digits.

∴Required number of ways = ${}^4P_4$ = 4!=4x3x2x1 = 24

Q29. The sum of the digits in unit place of all the numbers formed with the help of 3,4, 5 and 6 taken all at a time is

(a) 432

(b) 108

(c) 36

(c) 18

Sol: (b) If the unit place is '3' then remaining three places can be filled in 3! ways.

Thus '3' appears in unit place in 3! times.

Similarly each digit appear in unit place 3! times.

So, sum of digits in unit place = 3!(3 + 4 + 5 + 6) = 18 x 6 = 108

Q30. The total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is

(a) 60

(b) 120

(c) 7200

(d) 720

Sol: (c) Given number of vowels = 4 and number of consonants = 5 We have to form words by 2 vowels and 3 consonants.

So, lets first select 2 vowels and 3 consonants.

Number of ways of selection = ${}^4C_2 \times {}^5C_3 = 6 \times 10 = 60$ Now, these letters can be arranged in 5! ways.

So, total number of words = 60 x 5! = 60 x 120 = 7200

Q31. A five-digit number divisible by 3 is to be formed using the numbers 0, 1,2,4, and 5 without repetitions. The total number of ways this can be done is

(a) 216

(b) 600

(c) 240

(d) 3125

[Hint: 5 digit numbers can be formed using digits 0, 1, 2, 4, 5 or by using digits 1, 2, 3, 4, 5 since sum of digits in these cases is divisible by 3.]

Sol:(a) We know that a number is divisible by 3 if the sum of its digits is divisible by 3.

Now sum of the given six digits is 15 which is divisible by 3. So to form a number of five-digit which is divisible by 3 we can remove either 'O' or '3'. If digits 1, 2, 3,4, 5 are used then number of required numbers = 5!

If digits 0, 1,2,4, 5 are used then first place from left can be filled in 4 ways and remaining 4 places can be filled in 4! ways. So in this case required numbers are 4 x 4! ways.

So, total number of numbers = 120 + 96 = 216

Q32. Everybody in a room shakes hands with everybody else. If the total number of hand shakes is 66, then the total number of persons in the room is

(a) 11

(b) 12

(c) 13

(d) 14

Sol: (b) Between any two person there is one hand shake.

So, number of hand shakes $= {}^nC_2 = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2} = 66$ (given)

$\Rightarrow \quad n(n-1) = 132 \quad \Rightarrow n^2 - n - 132 = 0 \quad \Rightarrow (n-12)(n+11) = 0$

$\therefore \quad n = 12$

Q33. The number of triangles that are formed by choosing the vertices from a set of 12 points, seven of which lie on the same line is

(a) 105 (b) 15 (c) 175 (d) 185

Sol: (d) Number of ways of selecting 3 points from given 12 points = ${}^{12}C_3$

But any three points selected from given seven collinear points does not form triangle.

Number of ways of selecting three points from seven collinear points = ${}^7C_3$Required number of triangles = ${}^{12}C_3 - {}^7C_3 = 220 - 35 = 185$

Q34. The number of parallelograms that can be formed form a set of four parallel lines intersecting another set of three parallel lines is

(a) 6

(b) 18

(c) 12

(d) 9

Sol: (b) To form parallelogram we required a pair of line from a set of 4 lines and another pair of line from another set of 3 lines.

Required number of parallelograms = $^4C_2$ x $^3C_2$ = 6×3 = 18

Q35. The number of ways in which a team of eleven players can be selected from 22 players always including 2 of them and excluding 4 of them is

(a) $^{16}C_{11}$

(b) $^{16}C_5$

(c) $^{16}C_9$

(d) $^{20}C_9$

Sol: (c) Total number of players = 22

We have to select a team of 11 players.

We have to exclude 4 particular of them, so only 18 players are now available. Also from these 2 particular players are always included. Therefore we have to select 9 more players from the remaining 16 players.

So, required number of ways = $^{16}C_9$

Q36. The number of 5-digit telephone numbers having at least one of their digits repeated is

(a) 900000

(b) 10000

(c) 30240

(d) 69760

Sol: (d) Total number of telephone numbers when there is no restriction = 10^5 Also number of telephone numbers having all digits different = $^{l0}P_5$ Required number of ways = $10^5 - {}^{l0}P_5$ = 1000000 -10x9x8x7x6 = 1000000-30240 = 69760

Q37. The number of ways in which we.can choose a committee from four men and six women, so that the committee includes at least two men and exactly twice as many women as men is

(a) 94

(b) 126

(c) 128

(d) none of these

Sol: (a) Number of men = 4; Number of women = 6

It is given that committee includes at least two men and exactly twice as many women as men.

So, we can select either 2 men and 4 women or 3 men and 6 women.

∴ Required number of committee formed = $^4C_2$ x $^6C_4$ + $^4C_3$ x $^6C_6$

= 6×15 + 4×1=94

Q38. The total number of 9-digit numbers which have all different digits is

(a) 10!

(b) 9!

(c) 9×9!

(d) 10×10!

Sol: (c) We have to form 9-digit number which has all different digit.

First digit from the left can be filled in 9 ways (excluding '0').

Now nine digits are left including 'O'.

So remaining eight places can be filled with these nine digits in ^9P_S ways.

So, total number of numbers = 9 x $^9P_8$ = 9×9!

Q39. The number of words which can be formed out of the letters of the word ARTICLE, so that vowels occupy the even place is

(a) 1440

(b) 144

(c) 7!

(d) $^4C_4 \times {}^3C_3$

Sol: (b) We have word ARTICLE.

Vowels are A, I, E and consonants are R, T, C, L.

Now vowels occupy three even places (2^{nd}, 4^{th} and 6^{th}) in 3! ways.

In remaining four places four consonants can be arranged in 4! ways.

So, total number of words = 3! x4! = 6×24= 144

Q40. Given five different green dyes, four different blue dyes and three different red dyes, the number of combinations of dyes which can be chosen taking at least one green and one blue dye is

(a) 3600

(b) 3720

(c) 3800

(d) 3600

[Hint: Possible numbers of choosing or not choosing 5 green dyes, 4 blue dyes and 3 red dyes are 2^5, 2^4 and 2^3, respectively.]

Sol. **(b)** At least one green dye can be chosen in $^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 2^5 - 1$ ways.

At least one blue dye can be chosen in $^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - 1$ ways.

Any number of red dyes can be chosen in $^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3 = 2^3$ ways.

So, total number of required selection = $(2^5 - 1) \times (2^4 - 1) \times 2^3 = 3720$

Fill in the Blanks Type Questions

41. If $^nP_r = 840$ and $^nC_r = 35$, then r is equal to _________.

Sol. We have, $^nP_r = 840$ and $^nC_r = 35$

Now $^nP_r = {}^nC_r \cdot r!$

$\Rightarrow \quad 840 = 35 \times r! \quad \Rightarrow \quad r! = 24 \qquad \therefore r = 4$

42. $^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$ is equal to _________.

Sol. $^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7 = {}^{15}C_8 + {}^{15}C_9 - {}^{15}C_{15-6} - {}^{15}C_{15-7}$ $[\because {}^nC_r = {}^nC_{n-r}]$

$= {}^{15}C_8 + {}^{15}C_9 - {}^{15}C_9 - {}^{15}C_8 = 0$

43. The number of permutation of n different objects, taken r at a time, when repetitions are allowed, is ______.

Sol. Number of permutations of n different things taken r at a time when repetition is allowed = filling r places with the help of n different objects when repetition is allowed = $n \times n \times n \ldots r$ times = n^r

44. The number of different words that can be formed from the letters of the word INTERMEDIATE such that two vowels never come together is ______.

[**Hint:** Number of ways of arranging 6 consonants of which two are alike is $\frac{6!}{2!}$ and number of ways of arranging vowels = $^7P_6 \times \frac{1}{3!} \times \frac{1}{2!}$.]

Sol. Letters of the word INTERMEDIATE are: Vowels (I, E, E, I, A, E) and consonants (N, T, R, M, D, T)

Now we have to arrange these letters if no two vowels come together.

So, first arrange six consonants in $\frac{6!}{2!}$ ways.

Arrangement of six consonants creates seven gaps.

Six vowels can be arranged in these gaps in $^7C_6 \times \frac{6!}{2!3!}$ ways.

So, total number of words = $\frac{6!}{2!} \times {}^7C_6 \times \frac{6!}{2!3!} = 360 \times 7 \times 60 = 151200$

45. Three balls are drawn form a bag containing 5 red, 4 white and 3 black balls. The number of ways in which this can be done, if at least 2 are red, is ______.

Sol. The possible selection can be either 'two red and one other than red' or 'three red'.

So, the required number of ways = ${}^5C_2 \times {}^7C_1 + {}^5C_3 = 10 \times 7 + 10 = 80$

46. The number of six-digit numbers all digits of which are odd, is ______.

Sol. Odd digits are: 1, 3, 5, 7, 9

$\therefore$ The required number of numbers = $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$

47. In a football championship, 153 matches were played. Every two teams played one match with each other. The number of teams, participating in the championship is _________.

Sol. Let the number of team participating in championship be n.

It is given that every two teams played one match with each other.

$\therefore$ Total match played = ${}^nC_2$

$\Rightarrow \quad {}^nC_2 = 153 \quad \Rightarrow \dfrac{n(n-1)}{2} = 153$

$\Rightarrow \quad n^2 - n - 306 = 0 \quad \Rightarrow (n-18)(n+17) = 0$

$\therefore \quad n = 18$

48. The total number of ways in which six '+' and four '–' signs can be arranged in a line such that no two signs '–' occur together, is ______.

Sol. First arrange six '+' signs.

Number of ways of arrangement is '1' (as signs are identical).

Now, seven gaps are created from which four are to be chosen to put four '–' signs.

Four gaps can be selected in ${}^7C_4$ ways.

In these four gaps, '–' can be arranged in only one way as signs are identical.

$\therefore$ Total number of ways = $1 \times {}^7C_4 \times 1 = 35$

49. A committee of 6 is to be chosen from 10 men and 7 women so as to contain at least 3 men and 2 women. In how many different ways can this be done if two particular women refuse to serve on the same committee.

[**Hint:** At least 3 men and 2 women: The number of ways = ${}^{10}C_3 \times {}^7C_3 + {}^{10}C_4 \times {}^7C_2$. For 2 particular women to be always there: the number of ways = ${}^{10}C_4 + {}^{10}C_3 \times {}^5C_1$. The total number of committees when two particular women are never together = Total – together.]

Sol. Number of men = 10; Number of women = 7

We have to form a committee of 6 persons containing at least 3 men and 2 women containing at least 3 men and 2 women.

$\therefore$ Number of ways = ${}^{10}C_3 \times {}^7C_3 + {}^{10}C_4 \times {}^7C_2 = 120 \times 35 + 210 \times 21 = 8610$

Total number of committee when two particular women are never together

= Total number of ways – Number of ways when two particular women are together

$= ({}^{10}C_3 \times {}^7C_3 + {}^{10}C_4 \times {}^7C_2) - ({}^{10}C_4 + {}^{10}C_3 \times {}^5C_1)$

$= (120 \times 35 + 210 \times 21) - (210 + 120 \times 5)$

$= 4200 + 4410 - (210 + 600) = 7800$

50. A box contains 2 white balls, 3 black balls and 4 red balls. The number of ways three balls be drawn from the box, if at least one black ball is to be included in the draw is ______.

Sol. There are 2 white, 3 black and 4 red balls.

Three balls are drawn and it is given that at least one black ball is to be included in the draw.

$\therefore$ Required number of ways = ${}^3C_1 \times {}^6C_2 + {}^3C_2 \times {}^6C_1 + {}^3C_3$

$= 3 \times 15 + 3 \times 6 + 1 = 45 + 18 + 1 = 64$

True/False Type Questions

Q51. There are 12 points in a plane of which 5 points are collinear, then the number of lines obtained by joining these points in pairs is ${}^{12}C_2 - {}^5C_2$.

Sol: False

Required number of lines = ${}^{12}C_2 - {}^{5}C_2.+ 1$

Q52. Three letters can be posted in fiv.e letter boxes in 3^5

Sol: False

Each letter can be posted in any one of the five letter boxes.

So, total number of ways of posting three letters = 5x5x5 = 125

Q53. In the permutations of n things r taken together, the number of permutations in which m particular things occur together is

$${}^{n-m}P_{r-m} \times {}^{r}P_{m}.$$

Sol. False

Arrangement of n things, taken r at a time in which m things occur together.

First we select $(r-m)$ objects from $(n-m)$ objects in ${}^{n-m}C_{r-m}$ ways.

Now we consider these m things as 1 group.

Number of objects excluding these m objects = $(r-m)$

Now, first we have to arrange $(r-m+1)$ objects.

Number of arrangements = $(r-m+1)!$

Also m objects which we considered as 1 group, can be arranged in $m!$ ways.

$\therefore$ Required number of arrangements = ${}^{n-m}C_{r-m} \times (r-m+1)! \times m!$

Q54. In a steamer there are stalls for 12 animals and there are horses, cows and calves (not less than 12 each) ready to be shipped. They can be loaded in 3^{12} ways.

Sol: True

In each stall any one of the three animals can be shipped.

So total number of ways of loading = 3x3x3x...xl2 times = 3^{12}

Q55. If some or all of n objects are taken at a time, then the number of combinations is 2^n-1.

Sol: True

If some or all objects taken at a time, then number of combinations would be ${}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n = 2^n - 1$

Q56. There will be only 24 selections containing at least one red ball out of a bag containing 4 red and 5 black balls. It is being given that the balls of the same colour are identical.

Sol: False

Number of ways of selecting any number of objects from given n identical objects is 1.

Now selecting zero or more red ball from 4 identical red balls = 1 + 1 + 1 + 1 + 1=5

Selecting at least 1 black ball from 5 identical black balls =1 + T+1 + 1 + 1= 5 So, total number of ways = 5 x 5 = 25

57. Eighteen guests are to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on other side of the table. The number of ways in which the seating arrangements can be made is $\frac{11!}{5!6!}(9!)(9!)$.

Sol. True

Let two sides of the table be A and B each having 9 seats.

Let on side A, four particular guests and on side B, three particular guests be seated.

Now for side A, five more guests can be selected from remaining eleven guests in ${}^{11}C_5$ ways.

Also on each side nine guests can be arranged in 9! ways.

So total number of ways of arrangements = ${}^{11}C_5 \times 9! \times 9! = \frac{11!}{5!6!}(9!)(9!)$

Q58. A candidate is required to answer 7 questions, out of 12 questions which are divided

into two groups, each containing 6 He is not permitted to attempt more than 5 questions from either group. He can choose the seven questions in 650 ways.

Sol: False

A candidate can attempt questions in following maimer

Group (A)	2	3	4	5
Group (B)	5	4	3	2

Number of ways of attempting 7 questions

$$= {}^6C_2 \times {}^6C_5 + {}^6C_3 \times {}^6C_4 + {}^6C_4 \times {}^6C_3 + {}^6C_5 \times {}^6C_2$$

$$= 2({}^6C_2 \times {}^6C_5 + {}^6C_3 \times {}^6C_4)$$

$$= 2(15 \times 6 + 20 \times 15) = 2(90 + 300) = 2 \times 390 = 780$$

Q59. To fill 12 vacancies there are 25 candidates of which 5 are from scheduled castes. If 3 of the vacancies are reserved for scheduled caste candidates while the rest are open to all, the number of ways in which the selection can be made is ${}^5C_3$ x ${}^{22}C_9$.

Sol: True

We can select 3 scheduled caste candidate out of 5 in ${}^5C_3$ ways.

And we can select 9 other candidates out of 22 in ${}^{22}C_9$ways.

.'. Total number of selections = ${}^5C_3$ x ${}^{22}C_9$