Chapter 7. Permutations and Combinations

Question-1

In a class there are 27 boys and 14 girls. The teacher wants to select 1 boy and 1 girl to represent a competition. In how many ways can the teacher make this selection?

Solution:
1 boy among 27 boys is being selected. Hence the number of ways of selecting boys is 27.
1 girl among 14 girls is being selected. Hence the number of ways of selecting girls is 14.
Total number of ways the teacher can make this selection is $27 \times 14 = 378$.

Question-2

Given 7 flags of different colours, how many different signals can be generated if a signal requires the use of two flags, one below the other?

Solution:
The top position can be occupied by 7 flags and the bottom position can be occupied by 6 flags. Hence the total number of signals that can be generated using two flags one below the other is $7 \times 6 = 42$.

Question-3

A person wants to buy one fountain pen, one ball pen and one pencil from a stationery shop. If there are 10 fountain pen varieties, 12 ball pen varieties and 5 pencil varieties, in how many ways can he select these articles?

Solution:
Number of ways of selecting 1 fountain pen among 10 pens is 10.
Number of ways of selecting 1 ball pen among 12 ball pens is 12.
Number of ways of selecting 1 pencil among 5 pencil pens is 5.
Therefore total number of ways that he can select these articles is $10\times 12\times 5 = 600$.

Question-4

Twelve students compete in a race. In how many ways can the first three prizes be given?

Solution:

In a race of twelve students first prize can be given to any one of the 12 students, second prize can be given to any one of the 11 students and third prize can be given to any one of the 10 students.

Therefore total number of ways that first three prizes can be given is $12 \times 11 \times 10 = 1320$

Question-5

From among the 36 teachers in a college, one principal, one vice-principal and one teacher-in charge are to be appointed. In how many ways can this be done?

Solution:

In 36 ways one principal can be appointed, 35 ways one vice – principal post can be appointed and 34 ways one teacher-incharge can be appointed.

Therefore number of ways the selection can be made is $36 \times 35 \times 34 = 42840$ ways.

Question-6

There are 6 multiple-choice questions in an examination. How many sequences of answers are possible, if the first three questions have 4 choices each and the next three have 2 each?

Solution:

Each of the first three questions can be answered in 4 ways. Remaining each of the three questions can be answered in 2 ways.

Therefore total number of ways of choosing the answers are $4 \times 4 \times 4 \times 2 \times 2 \times 2 = 512$

Question-7

How many numbers are there between 500 and 1000, which have exactly one of their digits as 8?

Solution:

Digits which can occupy hundreds place are 5, 6, 7, 8, 9.
If the digits that occupy hundreds place are 5, 6, 7 and 9.
If 8 takes the units place then the tens place can be occupied by 9 digits.
Similarly if 8 takes the tens place then the units place can be occupied by 9 digits.
Totally units and tens place can be represented in 18ways.
Therefore total number of ways without 8 in hundreds place are $4 \times 18 = 72$.

If 8 takes up hundreds place then units and tens place can be occupied by 9 digits.
Therefore total number of ways with 8 in hundreds place are $9 \times 9 = 81$.
Therefore total number between 500 and 1000 which have exactly one of their digits as 8 are $72 + 81 = 153$

Question-8

How many five-digit number license plates can be made if
(i) first digit cannot be zero and the repetition of digits is not allowed.
(ii) the first digit cannot be zero, but the repetition of digits is allowed?

Solution:

(i) There are totally 10 digits they are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
Ten thousands place can be filled by 9 ways
Remaining places can be filled in 9, 8, 7 and 6 ways.
Therefore total number of ways are $9 \times 9 \times 8 \times 7 \times 6 = 27216$.

(ii) There are totally 10 digits they are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
Ten thousands place can be filled by 9 ways.
Remaining places can be filled in 10, 10, 10 and 10 ways.
Therefore total number of ways are $9 \times 10 \times 10 \times 10 \times 10 = 90000$.

Question-9

How many different numbers of six digits can be formed from the digits 2, 3, 0, 7, 9, 5 when repetition of digits is not allowed?

Solution:

The lakh's place can be filled in by 5 ways. The remaining 5 places can be filled in 5, 4, 3, 2, 1 ways.

Therefore the total number of six digits that can be formed is $5 \times 5 \times 4 \times 3 \times 2 \times 1 = 600$

Question-10

How many odd numbers less than 1000 can be formed by using the digits 0, 3, 5, 7 when repetition of digits is not allowed?

Solution:

Number of odd digits with one digit are 3.

Number of odd digits with two digits are $3 \times 2 = 6$.

Number of odd digits with three digits are $3 \times 3 \times 2 = 18$

Therefore total number of odd numbers less than 1000 that can be formed by using the digits 0, 3, 5, 7 when repetition of digits is not allowed = $3 + 6 + 18 = 27$.

Question-11

In how many ways can an examinee answer a set of 5 true/false type questions?

Solution:

For each set there are two answers, true/false.

Total number of answers an examinee can get are $2 \times 2 \times 2 \times 2 \times 2 = 32$

Question-12

How many 4-digit numbers are there?

Solution:

The thousands place cannot be filled by zeros.

Therefore the number of ways thousands place can be filled is 9.

The remaining three places can be filled by 10 ways.

Hence the total number of ways 4-digit numbers can be formed = $9 \times 10 \times 10 \times 10 = 9000$

Question-13

How many three-letter words can be formed using a, b, c, d, e if:
(i) repetition is allowed (ii) repetition is not allowed?

Solution:
(i) Total number of ways of forming three-letter word are $5 \times 5 \times 5 = 125$
(ii) Total number of ways of forming three-letter word without repetition are
$5 \times 4 \times 3 = 60$

Question-14

A coin is tossed five times and outcomes are recorded. How many possible outcomes are there?

Solution:
Each time a coin is tossed there are two outcomes head/tail.
Total number of possible outcomes = $2 \times 2 \times 2 \times 2 \times 2 = 32$

Question-15

Evaluate the following:
(i) $_5P_3$
(ii) $_{15}P_3$
(iii) $_5P_5$
(iv) $_{25}P_{20}$
(v) $_9P_5$

Solution:
:(i) $_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{(5\times 4\times 3)\times 2!}{2!} = 60$
(ii) $_{15}P_3 = \frac{15!}{(15-3)!} = \frac{15!}{12!} = \frac{(15\times 14\times 13)\times 12!}{12!} = 2730$
(iii) $_5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5\times 4\times 3\times 2\times 1}{1} = 120$
(iv) $_{25}P_{20} = \frac{25!}{(25-20)!} = \frac{25!}{5!} = \frac{25\times 24\times 23!}{5\times 4\times 3\times 2} = 5 \times 23!$
(v) $_9P_5 = \frac{9!}{(9-5)!} = \frac{9!}{4!} = \frac{9\times 8\times 7!}{4\times 3\times 2\times 1} = 3 \times 7!$

Question-16

If $_nP_4 = 20.\ _nP_3$, find n.

Solution:

$_nP_4 = 20.\ _nP_3$

$\frac{n!}{(n-4)!} = 20.\ \frac{n!}{(n-3)!}$

$\frac{n!}{(n-4)!} = 20.\ \frac{n!}{(n-3)(n-4)!}$

$n - 3 = 20$

$n = 23$

Question-17

If $_{10}P_r = 5040$, find the value of r.

Solution:

$_{10}P_r = 5040$

$\frac{10!}{(10-r)!} = 7!$

$\frac{10!}{(10-r)!} = \frac{10!}{6!}$

$(10 - r)! = 6!$

$10 - r = 6$

$r = 4$

Question-18

If $_{56}P_{r+6} : {}_{54}P_{r+3} = 30800 : 1$,find r.

Solution:

$_{56}P_{r+6} : {}_{54}P_{r+3} = 30800 : 1$

$\frac{56!}{(56-r-6)!} : \frac{54!}{(54-r-3)!} = 30800$

$\frac{56!}{(56-r-6)!} \times \frac{(54-r-3)!}{54!} = 30800$

$\frac{56 \times 55 \times 54!}{(50-r)!} \times \frac{(51-r)!}{54!} = 30800$

$56 \times 55\ (51 - r) = 30800$

$51 - r = 10$

$r = 41$

$= 1 + [2! - 1!] + [3! - 2!] \ldots\ldots [+(n+1)! - n!]$

$= 1 + (n+1)! - 1$

$=(n + 1)!$

Question-20

Prove that $_nP_r = {}_{(n-1)}P_r + r.{}_{(n-1)}P_{(r-1)}$.

Solution:

$_{(n-1)}P_r + r.{}_{(n-1)}P_{(r-1)} = \frac{(n-1)!}{(n-1-r)!} + \frac{r.(n-1)!}{((n-1)-(r-1))!}$

$= \frac{(n-1)!}{(n-1-r)!} + \frac{r.(n-1)!}{(n-r)!}$

$= \frac{(n-r)(n-1)!}{(n-r)(n-1-r)!} + \frac{r.(n-1)!}{(n-r)!}$

$= \frac{(n-r)(n-1)!}{(n-r)!} + \frac{r.(n-1)!}{(n-r)!}$

$= \frac{(n-1)! \times (n-r+r)}{(n-r)!}$

$= \frac{(n-1)! \times n}{(n-r)!}$

$= \frac{n!}{(n-r)!}$

$= {}_nP_r$

Question-21

Three men have 4 coats, 5 waistcoats and 6 caps. In how many ways can they wear them?

Solution:

Total number of ways that the three men can wear = 4 × 5 × 6 = 120

Question-22

How many 4-letter words, with or without meaning, can be formed, out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?

Solution:

Total number of 4-letter words that can be formed with or without meaning are $_{10}P_4$

$= \frac{10!}{6!}$

$= 10 \times 9 \times 8 \times 7$

$= 5040$

Question-23

How many 3-digit numbers are there, with distinct digits, with each digit odd?

Solution:

Total number of 3-digit numbers that can be formed with distinct odd digit numbers = ${}_5P_3$

= $\frac{5!}{2!}$

= 5× 4× 3

= 60

Question-24

Find the sum of all the numbers that can be formed with the digits 2, 3, 4, 5 taken all at a time.

Solution:

There are 4 digits and 4 places. Therefore number of permutation is $4P_4$ = 4! = 24.

There are six numbers ending with 2, 3, 4, 5.(Each of the digits 2, 3, 4, 5 occurs in 3! times in the unit's place)

Therefore totality of unit places = (6× 2) + (6× 3) + (6× 4) + (6× 5) = 6(2 + 3 + 4 + 5) = 84

Similarly the sum in the ten's place, hundreds place and thousands place each is 84.

Therefore total 84 + 840 + 8400 + 84000 = 93324.

Question-25

How many different words can be formed with the letters of the word 'MISSISSIPPI'?

Solution:

Number of letters = 11

Number of I's, S's and P's are 4, 4 and 2.

Therefore number of permutation = 11!/4!4!2!

Question-26

(i) How many different words can be formed with letters of the word 'HARYANA'?
(ii) How many of these begin with H and end with N?

Solution:
(i) Total number of letters = 7
Number of A's = 3
Therefore required number of permutation = 7!/3! = 840

(ii) Let the first place be H and the last place be N.
The remaining letters are five namely ARYAA
Number of A's = 3
Therefore number of permutation = 5!/3! = 20.

Question-27

How many 4-digit numbers are there, when a digit may be repeated any number of times?

Solution:
The thousands place cannot be filled by zeros.(Then it becomes a three digit number.)
Therefore the number of ways thousands place can be filled is 9.
The remaining three places can be filled by 10 ways.
Hence the total number of ways 4-digit numbers can be formed = 9 × 10 × 10 × 10 = 9000

Question-28

In how many ways can 5 rings of different types be worn in 4 fingers?

Solution:
Number of permutation is 5^4 = 625

Question-29

In how many ways can 9 students are seated in a (i) line (ii) circle?

Solution:
(i) The number of ways in which 9 students can be arranged in a line = ${}_9P_9$ = 9!
(ii) The number of ways in which 9 students can be arranged in a circle = (9 - 1)! = 8!

Question-30

In how many ways can a garland of 20 flowers be made?

Solution:

Number of ways a garland of 20 flowers can be made is 19!/2.

Question-31

Evaluate the following:

(i) $_{10}C_8$

(ii) $_{100}C_{98}$

(iii) $_{75}C_{75}$

Solution:

(i) $_{10}C_8 = \frac{10!}{8!2!} = \frac{10 \times 9 \times 8!}{8!2!} = 45.$

(ii) $_{100}C_{98} = \frac{100!}{98!2!} = \frac{100 \times 99 \times 98!}{98!2!} = 4950.$

(iii) $_{75}C_{75} = \frac{75!}{75!0!} = 1$

Question-32

If $_nC_{10} = {}_nC_{12}$, find $_{23}C_n$.

Solution:

$_nC_{10} = {}_nC_{12}$

$n = 10 + 12 = 22$

$_{23}C_n = {}_{23}C_{22} = \frac{23!}{22!1!} = 23$

Question-33

If $_8C_r - {}_7C_3 = {}_7C_2$, find r.

Solution:

$_8C_r - {}_7C_3 = {}_7C_2$

$_8C_r = {}_7C_3 + {}_7C_2$

$_8C_r = {}_8C_3$ (Since $_nC_r + {}_nC_{r-1} = {}_{n+1}C_r$)

$r = 3$

Question-34

If $_{16}C_4 = {}_{16}C_{r+2}$, find $_rC_2$.

Solution:

$_{16}C_4 = {}_{16}C_{r+2}$

$r + 2 + 4 = 16$ (Since $_nC_x = {}_nC_y$ and $x \neq y$, then $x + y = n$)

$r = 10$

$_rC_2 = {}_{10}C_2 = \frac{10!}{2!8!} = \frac{10\times 9\times 8!}{2!8!} = 45$

Question-35

Find n if

(i) $_nC_3 = \frac{20}{3}\ {}_nC_2$

(ii) $_nC_{(n-4)} = 70$

Solution:

(i) $_nC_3 = \frac{10}{3}\ {}_nC_2$

$\frac{nC3}{nC2} = \frac{10}{3}$

$\frac{n-3+1}{3} = \frac{10}{3}$ (Since $\frac{nCr}{nCr\text{-}1} = \frac{n-r+1}{r}$)

$n - 2 = 10$

$n = 12$.

(ii) $_nC_{(n-4)} = 70$

$\frac{n!}{(n-4)!4!} = 70$

$\frac{n(n-1)(n-2)(n-3)\times(n-4)!}{(n-4)!4!} = 70$

$n(n\text{-}1)(n\text{-}2)(n\text{-}3) = 70 \times 24$

$(n^2 - 3n)(n^2 - 3n + 2) = 1680$

$n^2 - 3n = y$

$y(y + 2) = 1680$

$y^2 + 2y - 1680 = 0$

$y(y + 42) - 40(y + 42) = 0$

$(y + 42)(y - 40) = 0$

Therefore $y = -42$ or 40

therefore $n^2 - 3n = -42$

$n^2 - 3n + 42 = 0$

$n = \frac{3 \pm \sqrt{9-168}}{2} = \frac{3 \pm \sqrt{-159}}{2}$

$n^2 - 3n = 40$

$n^2 - 3n - 40 = 0$

$n^2 - 8n + 5n = 40$

$n^2 - 3n - 40 = 0$

$n^2 - 8n + 5n = 40$

$n(n - 8) + 5(n - 8) = 0$

$(n + 5)(n - 8) = 0$

$n = 8$ or -5

Therefore n = 8.

Question-36

If $_{(n+2)}C_8 : _{(n-2)}P_4 = 57 : 16$, find n.

Solution:

$$\frac{(n+2)!}{8!(n-6)!} \times \frac{(n-6)!}{(n-2)!} = \frac{57}{16}$$

$$\frac{(n+2)(n+1)n(n-1)}{8!} = \frac{57}{16}$$

$$(n+2)(n+1)n(n-1) = 57 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times \frac{2}{16}$$

$$(n+2)(n+1)n(n-1) = 57 \times 7 \times 6 \times 5 \times 4 \times 3$$

$$(n^2 + n)(n^2 + n - 2) = 143640$$

$$y = n^2 + n$$

$$y(y - 2) = 143640$$

$$y^2 - 2y - 143640 = 0$$

$$y = \frac{2 \pm \sqrt{4 + 4(143640)}}{2}$$

$$= \frac{2 \pm \sqrt{574564}}{2}$$

$$= \frac{2 \pm 758}{2}$$

$$= \frac{760}{2}$$

or $\frac{-756}{2} = 380$ or -378

$$n^2 + n - 380 = 0$$

or $n^2 + n + 378 = 0$

$$(n + 20)(n - 19) = 0$$

or $n^2 + n + 378 = 0$

$$n = -20 \text{ or } 19$$

Therefore n = 19

$n^2 + n + 378 = 0$

Since the roots are imaginary, n = 19.

Question-37

Prove that the product of 2n consecutive negative integers is divisible by (2n)!

Solution:

Let the numbers be $-(x + 1), -(x + 2), \ldots\ldots -(x + 2n)$

Their product is $(-1)^{2n}(x + 1)(x + 2)\ldots\ldots\ldots\ldots(x + 2n)$

$$= (x + 1)(x + 2)\ldots\ldots\ldots\ldots(x + 2n)$$

$$= \frac{1.2.3\ldots\ldots x.(x+1)\ldots\ldots(x+2n)}{1.2.3\ldots\ldots\ldots x}$$

$$= \frac{(x+2n)!}{x!}$$

$$\frac{\text{Product}}{2n!} = \frac{(x+2n)!}{x!2n!} = x + 2nC_x$$

Product is divisible by (2n)!

Question-38

If $_{28}C_{2r} : {_{24}C_{2r-4}} = 225 : 11$, find r.

Solution:

$$\frac{28!}{2r!(28-2r)!} \times \frac{(2r-4)!(24-2r+4)!}{24!} = \frac{225}{11}$$

$$\frac{28!}{2r!(28-2r)!} \times \frac{(2r-4)!(28-2r)!}{24!} = \frac{225}{11}$$

$$\frac{28 \times 27 \times 26 \times 25}{2r(2r-1)(2r-2)(2r-3)} = \frac{225}{11}$$

$9(2r)(2r - 1)(2r - 2)(2r - 3) = 28 \times 27 \times 26 \times 11$

$2r(2r - 1)(2r - 2)(2r - 3) = 28 \times 3 \times 26 \times 11$

Put $x = 2r$

$x(x - 1)(x - 2)(x - 3) = 28 \times 3 \times 26 \times 11$

$(x^2 - 3x)(x^2 - 3x + 2) = 28 \times 3 \times 26 \times 11$

Let $y = x^2 - 3x$

$y(y + 2) = 24024$

$y^2 + 2y - 24024 = 0$

$(y + 156)(y - 154) = 0$

$y = -156$ or 154

$x^2 - 3x = -156$

$x^2 - 3x + 156 = 0$

$x = \frac{3 \pm \sqrt{9 - 624}}{2}$

$x^2 - 3x = 154$

$x^2 - 3x - 154 = 0$

$(x - 14)(x - 11) = 0$

$x = 14$ or -11

Therefore $2r = 14$

$r = 7$.

Question-39

If there are 12 persons in a party, and if each two of them shake hands with each other, how many handshakes happen in the party?

Solution:

Number of persons in a party = 12.

Number of handshakes = ${}_{12}C_2 = \frac{12!}{10!2!} = \frac{12\times 11\times 10!}{10!2!} = 66$

Question-40

In how many ways a committee of 5 members can be selected from 6 men and 5 women, consisting of 3 men and 2 women?

Solution:

Number of men = 6

Number of women = 5$\therefore$ Required number of ways = ${}_6C_3 \times {}_5C_2 = \frac{6!}{3!3!} \times \frac{5!}{2!3!} = \frac{6\times 5\times 4\times 3!}{3\times 2\times 3!} \times \frac{5\times 4\times 3!}{2\times 3!} = 20\times 10 = 200.$

Question-41

How many triangles can be obtained by joining 12 points, five of, which are collinear?

Solution:

Number of points = 12

Number of points collinear = 5

$\therefore$ Number of triangles = ${}_{12}C_3 - {}_5C_3 = \frac{12\times 11\times 10}{1\times 2\times 3} - \frac{5\times 4\times 3}{1\times 2\times 3} = 220 - 10 = 210$

Question-42

A box contains 5 different red and 6 different white balls. In how many ways can 6 balls be selected so that there are atleast two balls of each other?

Solution:

Number of red balls = 5

Number of white balls = 6

$\therefore$ Required number of ways = ${}_5C_4 \times {}_6C_2 + {}_5C_3 \times {}_6C_3 + {}_5C_2 \times {}_6C_4$

$= \frac{5!}{4!1!} \times \frac{6!}{2!4!} + \frac{5!}{3!2!} \times \frac{6!}{3!3!} + \frac{5!}{2!3!} \times \frac{6!}{4!2!}$

$= 5 \times \frac{6\times 5\times 4!}{2!4!} + \frac{5\times 4\times 3!}{3!2!} \times \frac{6\times 5\times 4\times 3!}{3!3!} + \frac{5\times 4\times 3!}{2!3!} \times \frac{6\times 5\times 4!}{4!2!}$

$= 5 \times 15 + 10 \times 20 + 10 \times 15$

$= 75 + 200 + 150$

$= 425$

Question-43

In how many ways can a cricket team of eleven be chosen out of a batch of 15 players if
(i) there is no restriction on the selection
(ii) a particular player is always chosen;
(iii) a particular player is never chosen?

Solution:

Number of players = 15

(i) Number of ways team selected when there is no restriction on the selection = ${}_{15}C_{11}$

$= \frac{15!}{11!4!} = \frac{15\times14\times13\times12\times11!}{11!4\times3\times2} = 1365.$

(ii) Number of ways team selected when a particular player is always chosen = ${}_{14}C_{10}$

$= \frac{14!}{10!4!} = \frac{14\times13\times12\times11\times10!}{10!4\times3\times2} = 1001.$

(iii) Number of ways team selected when a particular player is never chosen = ${}_{14}C_{11}$

$= \frac{14!}{11!3!} = \frac{14\times13\times12\times11!}{11!3\times2} = 364.$

Permutations & Combinations

1. Eight chairs are numbered 1 to 8. Two women and 3 men wish to occupy one chair each. First the women choose the chairs from amongst the chairs 1 to 4 and then men select from the remaining chairs. Find the total number of possible arrangements.

   [Hint: 2 women occupy the chair, from 1 to 4 in ${}^4P_2$ ways and 3 men occupy the remaining chairs in ${}^6P_3$ ways.]

2. If the letters of the word RACHIT are arranged in all possible ways as listed in dictionary. Then what is the rank of the word RACHIT ?

   [Hint: In each case number of words beginning with A, C, H, I is 5!]

3. A candidate is required to answer 7 questions out of 12 questions, which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. Find the number of different ways of doing questions.

4. Out of 18 points in a plane, no three are in the same line except five points which are collinear. Find the number of lines that can be formed joining the point.

   [Hint: Number of straight lines = ${}^{18}C_2 - {}^5C_2 + 1$.]

5. We wish to select 6 persons from 8, but if the person A is chosen, then B must be chosen. In how many ways can selections be made?

6. How many committee of five persons with a chairperson can be selected from 12 persons.

   [Hint: Chairman can be selected in 12 ways and remaining in ${}^{11}C_4$.]

7. How many automobile license plates can be made if each plate contains two different letters followed by three different digits?

8. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected from the lot.

9. Find the number of permutations of n distinct things taken r together, in which 3 particular things must occur together.

10. Find the number of different words that can be formed from the letters of the word 'TRIANGLE' so that no vowels are together.

11. Find the number of positive integers greater than 6000 and less than 7000 which are divisible by 5, provided that no digit is to be repeated.

12. There are 10 persons named $P_1, P_2, P_3, \ldots P_{10}$. Out of 10 persons, 5 persons are to be arranged in a line such that in each arrangement P_1 must occur whereas P_4 and P_5 do not occur. Find the number of such possible arrangements.

    [Hint: Required number of arrangement = ${}^7C_4 \times 5!$]

13. There are 10 lamps in a hall. Each one of them can be switched on independently. Find the number of ways in which the hall can be illuminated.

[Hint: Required number = $2^{10} - 1$].

14. A box contains two white, three black and four red balls. In how many ways can three balls be drawn from the box, if atleast one black ball is to be included in the draw.

[Hint: Required number of ways = ${}^3C_1 \times {}^6C_2 + {}^3C_2 \times {}^6C_2 + {}^3C_3$.]

15. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then find ${}^rC_2$.

[Hint: Form equation using $\frac{{}^nC_r}{{}^nC_{r+1}}$ and $\frac{{}^nC_r}{{}^nC_{r-1}}$ to find the value of r.]

16. Find the number of integers greater than 7000 that can be formed with the digits 3, 5, 7, 8 and 9 where no digits are repeated.

[Hint: Besides 4 digit integers greater than 7000, five digit integers are always greater than 7000.]

17. If 20 lines are drawn in a plane such that no two of them are parallel and no three are concurrent, in how many points will they intersect each other?

18. In a certain city, all telephone numbers have six digits, the first two digits always being 41 or 42 or 46 or 62 or 64. How many telephone numbers have all six digits distinct?

19. In an examination, a student has to answer 4 questions out of 5 questions; questions 1 and 2 are however compulsory. Determine the number of ways in which the student can make the choice.

20. A convex polygon has 44 diagonals. Find the number of its sides.

[Hint: Polygon of n sides has (${}^nC_2 - n$) number of diagonals.]

Long Type Questions

21. 18 mice were placed in two experimental groups and one control group, with all groups equally large. In how many ways can the mice be placed into three groups?

22. A bag contains six white marbles and five red marbles. Find the number of ways in which four marbles can be drawn from the bag if (a) they can be of any colour (b) two must be white and two red and (c) they must all be of the same colour.

23. In how many ways can a football team of 11 players be selected from 16 players? How many of them will

(i) include 2 particular players?

(ii) exclude 2 particular players?

24. A sports team of 11 students is to be constituted, choosing at least 5 from Class XI and atleast 5 from Class XII. If there are 20 students in each of these classes, in how many ways can the team be constituted?

25. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has

(i) no girls

(ii) at least one boy and one girl

(iii) at least three girls.

Objective Type Questions

26. If ${}^{n}C_{12} = {}^{n}C_{8}$, then n is equal to

(A) 20 (B) 12 (C) 6 (D) 30

27. The number of possible outcomes when a coin is tossed 6 times is

(A) 36 (B) 64 (C) 12 (D) 32

28. The number of different four digit numbers that can be formed with the digits 2, 3, 4, 7 and using each digit only once is

(A) 120 (B) 96 (C) 24 (D) 100

29. The sum of the digits in unit place of all the numbers formed with the help of 3, 4, 5 and 6 taken all at a time is

(A) 432 (B) 108 (C) 36 (D) 18

30. Total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is equal to

(A) 60 (B) 120 (C) 7200 (D) 720

31. A five digit number divisible by 3 is to be formed using the numbers 0, 1, 2, 3, 4 and 5 without repetitions. The total number of ways this can be done is

(A) 216 (B) 600 (C) 240 (D) 3125

[Hint: 5 digit numbers can be formed using digits 0, 1, 2, 4, 5 or by using digits 1, 2, 3, 4, 5 since sum of digits in these cases is divisible by 3.]

32. Every body in a room shakes hands with everybody else. The total number of hand shakes is 66. The total number of persons in the room is

(A) 11 (B) 12 (C) 13 (D) 14

33. The number of triangles that are formed by choosing the vertices from a set of 12 points, seven of which lie on the same line is

(A) 105 (B) 15 (C) 175 (D) 185

34. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines is

(A) 6 (B) 18 (C) 12 (D) 9

35. The number of ways in which a team of eleven players can be selected from 22 players always including 2 of them and excluding 4 of them is

(A) ${}^{16}C_{11}$ (B) ${}^{16}C_{5}$ (C) ${}^{16}C_{9}$ (D) ${}^{20}C_{9}$

36. The number of 5-digit telephone numbers having atleast one of their digits repeated is

(A) 90,000 (B) 10,000 (C) 30,240 (D) 69,760

37. The number of ways in which we can choose a committee from four men and six women so that the committee includes at least two men and exactly twice as many women as men is

(A) 94 (B) 126 (C) 128 (D) None

38. The total number of 9 digit numbers which have all different digits is

(A) 10! (B) 9 ! (C) 9 × 9! (D) 10×10!

39. The number of words which can be formed out of the letters of the word ARTICLE, so that vowels occupy the even place is

(A) 1440 (B) 144

(C) 7! (D) ${}^4C_4 \times {}^3C_3$

40. Given 5 different green dyes, four different blue dyes and three different red dyes, the number of combinations of dyes which can be chosen taking at least one green and one blue dye is

(A) 3600 (B) 3720 (C) 3800 (D) 3600

[Hint: Possible numbers of choosing or not choosing 5 green dyes, 4 blue dyes and 3 red dyes are 2^5, 2^4 and 2^3, respectively.]

Fill in the Blanks Type Questions

41. If ${}^nP_r = 840$, ${}^nC_r = 35$, then r = ______.

42. ${}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$ = ______.

43. The number of permutations of n different objects, taken r at a line, when repetitions are allowed, is ______.

44. The number of different words that can be formed from the letters of the word INTERMEDIATE such that two vowels never come together is ______.

[Hint: Number of ways of arranging 6 consonants of which two are alike is $\frac{6!}{2!}$ and number of ways of arranging vowels $= {}^7P_6 \times \frac{1}{3!} \times \frac{1}{2!}$.]

45. Three balls are drawn from a bag containing 5 red, 4 white and 3 black balls. The number of ways in which this can be done if at least 2 are red is ______

46. The number of six-digit numbers, all digits of which are odd is ______.

47. In a football championship, 153 matches were played . Every two teams played one match with each other. The number of teams, participating in the championship is ______.

48. The total number of ways in which six '+' and four '–' signs can be arranged in a line such that no two signs '–' occur together is ______.

49. A committee of 6 is to be chosen from 10 men and 7 women so as to contain atleast 3 men and 2 women. In how many different ways can this be done if two particular women refuse to serve on the same committee.

[Hint: At least 3 men and 2 women: The number of ways = ${}^{10}C_3 \times {}^{7}C_3 + {}^{10}C_4 \times {}^{7}C_2$. For 2 particular women to be always there: the number of ways = ${}^{10}C_4 + {}^{10}C_3 \times {}^{5}C_1$. The total number of committees when two particular women are never together = Total – together.]

50. A box contains 2 white balls, 3 black balls and 4 red balls. The number of ways three balls be drawn from the box if at least one black ball is to be included in the draw is ______.

True or False Type Questions

51. There are 12 points in a plane of which 5 points are collinear, then the number of lines obtained by joining these points in pairs is ${}^{12}C_2 - {}^{5}C_2$.

52. Three letters can be posted in five letterboxes in 3^5 ways.

53. In the permutations of n things, r taken together, the number of permutations in which m particular things occur together is ${}^{n-m}P_{r-m} \times {}^{r}P_m$.

54. In a steamer there are stalls for 12 animals, and there are horses, cows and calves (not less than 12 each) ready to be shipped. They can be loaded in 3^{12} ways.

55. If some or all of n objects are taken at a time, the number of combinations is $2^n – 1$.

56. There will be only 24 selections containing at least one red ball out of a bag containing 4 red and 5 black balls. It is being given that the balls of the same colour are identical.

57. Eighteen guests are to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on other side of the table. The number of ways in which the seating arrangements

can be made is $\frac{11!}{5!6!}(9!)(9!)$.

[Hint: After sending 4 on one side and 3 on the other side, we have to select out of 11; 5 on one side and 6 on the other. Now there are 9 on each side of the long table and each can be arranged in 9! ways.]

58. A candidate is required to answer 7 questions out of 12 questions which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. He can choose the seven questions in 650 ways.

59. To fill 12 vacancies there are 25 candidates of which 5 are from scheduled castes. If 3 of the vacancies are reserved for scheduled caste candidates while the rest are open to all, the number of ways in which the selection can be made is ${}^{5}C_3 \times {}^{20}C_9$.

Match Type Questions

In each if the Exercises from 60 to 64 match each item given under the column C_1 to its correct answer given under the column C_2.

60. There are 3 books on Mathematics, 4 on Physics and 5 on English. How many different collections can be made such that each collection consists of :

C_1	C_2
(a) One book of each subject;	(i) 3968
(b) At least one book of each subject :	(ii) 60
(c) At least one book of English:	(iii) 3255

61. Five boys and five girls form a line. Find the number of ways of making the seating arrangement under the following condition:

C_1	C_2
(a) Boys and girls alternate:	(i) $5! \times 6!$
(b) No two girls sit together :	(ii) $10! - 5!6!$
(c) All the girls sit together	(iii) $(5!)^2 + (5!)^2$
(d) All the girls are never together :	(iv) $2!5!5!$

62. There are 10 professors and 20 lecturers out of whom a committee of 2 professors and 3 lecturer is to be formed. Find :

C_1	C_2
(a) In how many ways committee : can be formed	(i) ${}^{10}C_2 \times {}^{19}C_3$
(b) In how many ways a particular : professor is included	(ii) ${}^{10}C_2 \times {}^{19}C_2$
(c) In how many ways a particular : lecturer is included	(iii) ${}^{9}C_1 \times {}^{20}C_3$
(d) In how many ways a particular : lecturer is excluded	(iv) ${}^{10}C_2 \times {}^{20}C_3$

63. Using the digits 1, 2, 3, 4, 5, 6, 7, a number of 4 different digits is formed. Find

C_1	C_2
(a) how many numbers are formed?	(i) 840
(b) how many numbers are exactly divisible by 2?	(ii) 200
(c) how many numbers are exactly divisible by 25?	(iii) 360
(d) how many of these are exactly divisble by 4?	(iv) 40

64. How many words (with or without dictionary meaning) can be made from the letters of the word MONDAY, assuming that no letter is repeated, if

C_1	C_2
(a) 4 letters are used at a time	(i) 720
(b) All letters are used at a time	(ii) 240
(c) All letters are used but the first is a vowel	(iii) 360

CBSE Class 11 Mathematics

Important Questions

Chapter 7

Permutations and Combinations

1 Marks Questions

1. Evaluate $4! - 3!$

Ans. $4!-3!=4.3!-3!$

$=(4-1).3!$

$=3.3!=3\times3\times2\times1$

$=18$

2.If $^{n}c_{a}=^{n}c_{b}$ **find** n

Ans. $^{n}C_{a}=^{n}C_{b}\Rightarrow{}^{n}C_{a}=^{n}C_{n-b}$

$a=n-b$

$n=a+b$

3.The value of 0! Is?

Ans. $0!=1$

4.Given 5 flags of different colours here many different signals can be generated if each signal requires the use of 2 flags. One below the other

Ans. First flag can be chosen is 5 ways

Second flag can be chosen is 4 ways

By $F.P.C.$ total number of ways $=5\times4=20$

5.How many 4 letter code can be formed using the first 10 letter of the English alphabet, if no letter can be repeated?

Ans. First letter can be used in 10 ways

Second letter can be used in 9 ways

Third letter can be used in 8 ways

Forth letter can be used in 7 ways

By $F.P.C.$ total no. of ways $=10.9.8.7$

$=5040$

6.A coin is tossed 3 times and the outcomes are recorded. How many possible out comes are there?

Ans. Total no. of possible out comes $=2\times2\times2=8$

7.Compute $\frac{8!}{6!\times2!}$

Ans. $\frac{8!}{6!2!}=\frac{8.7.\cancel{6!}}{\cancel{6!}.2.1}$

$=4\times7=28$

8.If ${}^{n}C_{8}={}^{n}C_{2}$. find ${}^{n}C_{2}$.

Ans. Given

${}^{n}C_{8}={}^{n}C_{2}\Rightarrow {}^{n}C_{n-8}={}^{n}C_{2}$

$n-8=2$

$n=10$

$\therefore {}^nC_2 = {}^{10}C_2 = \frac{\lfloor 10}{\lfloor 10-2 \lfloor 2}$

$= \frac{10.9.\lfloor 8}{\lfloor 8 \times 2.1} = 5 \times 9 = 45$

9.In how many ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colours.

Ans. No. of ways of selecting 9 balls

$= {}^6C_3 \times {}^5C_3 \times {}^5C_3$

$= \frac{\lfloor 6}{\lfloor 3 \lfloor 3} \times \frac{\lfloor 5}{\lfloor 2 \lfloor 3} \times \frac{\lfloor 5}{\lfloor 2 \lfloor 3}$

$= \frac{6.5.4\lfloor 3}{6.\lfloor 3} \times \frac{5.4.\lfloor 3}{2.\lfloor 3} \times \frac{5.4.\lfloor 3}{2.\lfloor 3}$

$= 20 \times 10 \times 10 = 2000$

10.Find r, **if** $5.{}^4P_r = 6.{}^5P_{r-1}$

Ans. $5.\ {}^4p_r = 6.\ {}^5p_{r-1}$

$\Rightarrow 5.\frac{\lfloor 4}{\lfloor 4-r} = 6.\frac{\lfloor 5}{\lfloor 5-r+1}$

$\Rightarrow \frac{5.\lfloor 4}{\lfloor (4-r)} = \frac{6.5.\lfloor 4}{\lfloor 6-r}$

$\Rightarrow \frac{1}{\lfloor 4-r} = \frac{6}{(6-r)(5-r)\lfloor 4-r}$

$\Rightarrow (6-r)(5-r) = 6$

$\Rightarrow 30-6r-5r+r^2=6$

$\Rightarrow r^2-11r-5r+r^2=6$

$\Rightarrow r^2-8r-3r+24=0$

$\Rightarrow r(r-8)-3(r-8)=0$

$\Rightarrow (r-3)(r-8)=0$

$r=3$ or $r=8$

$\therefore$ r = 3

$r=8$ Rejected. Because if we put $r=8$ the no. in the factorial is –ve.

11.If $\frac{1}{6!}+\frac{1}{7!}+=\frac{x}{8!}$ find x

Ans. $\frac{1}{6!}+\frac{1}{7.6!}=\frac{x}{8.7.6!}$

$\frac{1}{6!}\left[1+\frac{1}{7}\right]=\frac{x}{56.6!}$

$\frac{8}{7}=\frac{x}{56}$

$x=8\times 8=64$

12.Write relation between $^{n}p_{r}$. and $^{n}C_{r}$

Ans. $^{n}P_{r}={}^{n}C_{r}\times \lfloor r$

13.What is $\lfloor n$

Ans. $\lfloor n$ Is multiplication of n consecutive natural number

$\lfloor n = n(n-1)(n-2)(n-3)......5.4.3.2.1$

14.If $^{n}C_{o} = 1$ what is the value of $^{99}C_{o}$

Ans. $^{n}C_{o} = 1$ then $^{99}C_{o} = 1$

15.How many words, with or with not meaning each of 2 vowels and 3 consonants can be flamed from the letter of the word DAUGHTER?

Ans. In the word DAUGHTER There are 3 vowels and 5 consonants out of 3 vowels, 2 vowels can be selected in $^{3}C_{2}$ ways and 3 consonants can be selected in $^{5}C_{3}$ ways 5 letters 2 vowel and 3 consonant can be arranged in 5! Ways

$\therefore$ Total no. of words $= {}^{3}C_{2} \times {}^{5}C_{3} \times 5!$

$$= \frac{\lfloor 3}{\lfloor 1 \lfloor 2} \times \frac{\lfloor 5}{\lfloor 2 \times \lfloor 3} \times \lfloor 5$$

$$= \frac{3.\lfloor 2}{\lfloor 2} \times \frac{5.4.\lfloor 3}{\lfloor 2.\lfloor 3} \times 5.4.3.2.1$$

$$= 3600$$

16.Convert the following products into factorials $5 \times 6 \times 7 \times 8 \times 9$

Ans. $\dfrac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9}{1 \times 2 \times 3 \times 4}$

$$= \frac{\lfloor 9}{\lfloor 4}$$

17.Evaluate $\dfrac{n!}{(n-r)!}$, when $n = 5$, $r = 2$

Ans. $\frac{\lfloor n}{\lfloor n-r} = \frac{\lfloor 5}{\lfloor 5-2} = \frac{\lfloor 5}{\lfloor 3}$

$= \frac{5.4.\lfloor 3}{\lfloor 3} = 20$

18.Evaluate $^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$

Ans. $\left({}^{15}C_8 + {}^{15}C_9\right) - \left({}^{15}C_6 + {}^{15}C_7\right) \quad \left[{}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r\right]$

$= {}^{16}C_9 - {}^{16}C_7 = 0 \qquad \left[\because {}^{16}C_9 = {}^{16}C_7\right]$

19.What is the value of $^{n}C_o + {}^{n}C_1 + {}^{n}C_2 + \ldots\ldots + {}^{n}C_n$

Ans. 2^n

20.Find n **if** $^{2n}C_3 : {}^{n}C_3 = 11:1$

Ans. Given $^{2n}C_3 : {}^{n}C_3 = 11:1 \Rightarrow \frac{\lfloor 2n}{\lfloor 3 \lfloor 2n-3} \times \frac{\lfloor 3 \lfloor n-3}{\lfloor n} = \frac{11}{1}$

$\Rightarrow \frac{2n(2n-1)(2n-2)\lfloor 2n-3}{\lfloor 2n-3} \times \frac{\lfloor n-3}{n(n-1)(n-2)\lfloor n-3} = \frac{11}{1}$

$\Rightarrow \frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = \frac{11}{1}$

$\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)} = \frac{11}{1} \Rightarrow \frac{4(2n-1)}{n-2} = \frac{11}{1}$

$\Rightarrow 11n - 22 = 8n - 4 \Rightarrow 3n = 18$

$\Rightarrow n = 6$

21.Evaluate $^{10}C_7 + {}^{10}C_6$

Ans. $^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

$^{10}C_7 + {}^{10}C_6 = {}^{10+1}C_7$

$= {}^{11}C_7 = \frac{\lfloor 11}{\lfloor 11-7 \lfloor 7}$

$= \frac{\lfloor 11}{\lfloor 4 \lfloor 7} = \frac{11.10.9_3.8.7.\lfloor 7}{4.3.2.1.\lfloor 7} = 330$

22.If $1 \le r \le n$ **then what is the value of** $\frac{n}{r}\ {}^{n-1}C_{r-1}$

Ans. $^{n}C_r$

H	T	U
7	8	9

23.How many 3 digit numbers can be formed by using the digits 1 to 9 if no. digit is repeated

Ans.

$9 \times 8 \times 7 = 504$

24.Convert into factorial 2.4.6.8.10.12

Ans. $2\times1.\ 2\times2.\ 2\times3.\ 2\times4.\ 2\times5.\ 2\times6$

$= 2^6 \left[1.2.3.4.5.6\right] = 2^6\ \lfloor 6$

25.How many words with or without meaning can be formed using all the letters of the word 'EQUATION' at a time so that vowels and consonants occur together

Ans. In the word 'EQUATION' there are 5 vowels [A.E.I.O.U.] and 3 consonants [Q.T.N]

Total no. of letters = 8

Arrangement of 5 vowels = $\lfloor 5$

Arrangements of 3 consonants = $\lfloor 3$

Arrangements of vowels and consonants = $\lfloor 2$

$\therefore$ Total number of words $= \lfloor 5 \times \lfloor 3 \times \lfloor 2$

$= 5.4.3.2.1 \times 3.2.1 \times 2.1 = 1440$

CBSE Class 12 Mathematics

Important Questions

Chapter 7

Permutations and Combinations

4 Marks Questions

1. How many words, with or without meaning can be made from the letters of the word MONDAY. Assuming that no. letter is repeated, it

(i) 4 letters are used at a time

(ii) All letters are used but first letter is a vowel?

Ans. Part-I In the word MONDAY there are 6 letters

$\therefore n = 6$

4 letters are used at a time

$\therefore r = 4$

Total number of words $= {}^{n}P_{r}$

$$= {}^{6}P_{4} = \frac{\lfloor 6}{\lfloor 6-4}$$

$$= \frac{\lfloor 6}{\lfloor 2} = \frac{6.5.4.3.\lfloor 2}{\lfloor 2} = 360$$

Part-II All letters are used at a time but first letter is a vowel then OAMNDY

2 vowels can be arranged in 2! Ways

4 consonants can be arranged in 4! Ways

$\therefore$ Total number of words $= 2! \times 4!$

$= 2 \times 4.3.2.1 = 48$

2.Prove that ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

Ans. Proof L.H.S.

$${}^{n}C_{r} + {}^{n}C_{r-1} = \frac{\lfloor n}{\lfloor n-r \lfloor r} + \frac{\lfloor n}{\lfloor n-r+1 \lfloor r-1}$$

$$= \frac{\lfloor n}{\lfloor (n-r)\ r \lfloor r-1} + \frac{\lfloor n}{(n-r+1)\lfloor n-r \lfloor r-1}$$

$$= \frac{\lfloor n}{\lfloor n-r \lfloor r-1} \qquad \left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

$$= \frac{\lfloor n}{\lfloor n-r \lfloor r-1} \qquad \left[\frac{n-\cancel{r}+1+\cancel{r}}{r(n-r+1)}\right]$$

$$= \frac{\lfloor n (n+1)}{\lfloor n-r (n-r+1) \lfloor r-1\ r}$$

$$= \frac{\lfloor n+1}{\lfloor n+1-r \lfloor n-r} = {}^{n+1}C_{r}$$

3. A bag contains 5 black and 6 red balls determine the number of ways in which 2 black and 3 red balls can be selected.

Ans. No. of black balls =5

No. of red balls = 6

No. of selecting black balls = 2

No. of selecting red balls = 3

Total no. of selection $= {}^{5}C_{2} \times {}^{6}C_{3}$

$$= \frac{\lfloor 5}{\lfloor 5-2 \lfloor 2} \times \frac{\lfloor 6}{\lfloor 6-3 \ \lfloor 3}$$

$$\frac{5\times 4\times 3!}{3!\times 2}\times\frac{6\times 5\times 4\times 3!}{3!\times 3\times 2}=200$$

4.In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together?

Ans. Let us first seat 0 the 5 girls. This can be done in 5! Ways

X G X G X G X G X G X

There are 6 cross marked placer and the three boys can be seated in $^{6}P_{3}$ ways

Hence by multiplication principle

The total number of ways

$$=5!\ \times\ ^{6}P_{3}=5!\ \times\ \frac{6!}{3!}$$

$$=4\times 5\times 2\times 3\times 4\times 5\times 6$$
$$=14400$$

5. How many words, with or without meaning, each of 3 vowels and 2 consonants can be formed from the letters of the word INVOLUTE

Ans. In the INVOLUTE there are 4 vowels, namely I.O.E.U and 4 consonants namely M.V.L and T

The number of ways of selecting 3 vowels

Out of $4 = {}^{4}C_{3} = 4$

The number of ways of selecting 2 consonants

Out of $4 = {}^{4}C_{2} = 6$

$\therefore$ No of combinations of 3 vowels and 2 consonants $= 4 \times 6 = 24$

5 letters 2 vowel and 3 consonants can be arranged in 5! Ways

Therefore required no. of different words $= 24 \times 5! = 2880$

6. Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements

(i) do the words start with P

(ii) do all the vowels always occur together

Ans. The number of letters in the word INDE PENDENCE are 12 In which E repeated in 4 times. N repeated 3 times. D repeated 2 times

(i) If the word starts with P The position of P is fined

Then the no. of arrangements $= \frac{11!}{4!3!2!} = 138600$

(ii) All the vowels always occur together There are 5 vowels in which 4 E's and 1 I

EEEEI NDPNDNC

$\therefore$ Total letters are 8 letters Can be arranged in $= \frac{8!}{3!2!}$ ways

Also 5 vowels can be arranged in $= \frac{5!}{4!}$ ways.

$\therefore$ required number of arrangements

$$= \frac{8!}{3!2!} \times \frac{5!}{4!} = 16800$$

7.Find n if ${}^{n-1}P_3 : {}^{n}P_4 = 1 : 9$

Ans. Given

$^{n-1}P_3 : {}^nP_4 = 1:9$

$\frac{\lfloor n-1}{\lfloor n-1-3} : \frac{\lfloor n}{\lfloor n-4} = 1:9$

$\frac{\frac{\lfloor n-1}{\lfloor n-4}}{\frac{\lfloor n}{\lfloor n-4}} = \frac{1}{9}$

$\frac{\lfloor n-1}{n\lfloor n-1} = \frac{1}{9}$

$n = 9$ s

8. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Ans. Given total no. of players =17

5 players can bowl.

$\therefore$ no. of bat's man =17-5=12

Out of 5 bowlers 4 can be choose in $^5C_4$ ways

Out of 12 bat's man (11-4)=7 bat's man can be choose in $^{12}C_7$ ways

Total no. of selection of 11 players

$= {}^5C_4 \times {}^{12}C_7$

$= \frac{\lfloor 5}{\lfloor 5-4 \lfloor 4} \times \frac{\lfloor 12}{\lfloor 12-7 \lfloor 7}$.

$$5 \times \frac{12.11.10.9.8.\underline{|7}}{5.4.3.2.1\underline{|7}} = 3960$$

9. How many numbers greater than 1000000 can be formed by using the digits 1,2,0,2,4,2,4?

Ans. Given digits $1, 2, 0, 2, 4, 2, 4$ Are 7

The no. of arrangements of 7 digits $= \frac{7!}{3!21.} = 420$

If 0 is in extreme left position.

The no. of arrangements of 6 digits $= \frac{6!}{3!2!} = 60$

$\therefore$Required numbers $= 420 - 60 = 360$

10. In how many ways can the letters of the word ASSA SS IN ATION be arranged so that all the S's are together?

Ans. In the word ASSA SSI N ATION there are 13 letters

But all S are together. Then no. of letters 10 [4 S take 1]then required no. of arrangements

$$= \frac{10!}{3!2!2!} \quad = 151200$$

11. The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be flamed from the alphabet?

Ans. No. of vowels = 5

No. of consonants = 21

2 vowels can be selected in $^5C_2$ ways

2consonents can be selected in ${}^{21}C_2$ ways

No. of arrangements of 4 letters [2 vowel and 2 consonants] =4!

$\therefore$ Required no. of wards $= {}^5C_2 \times {}^{21}C_2 \times 4!$

$$= \frac{\lfloor 5}{\lfloor 3 \lfloor 2} \times \frac{\lfloor 21}{\lfloor 19 \lfloor 2} \times 4.\lfloor 3$$

$$= \frac{5.4.3.\lfloor 2 \times 21.20.\lfloor 19 \times 4.\lfloor 3}{\lfloor 3.\lfloor 2 \quad \lfloor 19 \times 2}$$

$$= 50400$$

12. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I’s not come together?

Ans. In the word MISSISSIPPI no. of letters = 11

In which 4 I’s, 4 S’s and 2P

$\therefore$ Total no. of words $= \frac{11!}{4!4!2!} = 34650$

When four I’s come together

Then four I’s as one letter

And other letters are 7

Then no. of words when four I’s come together

$$= \frac{8!}{4!2!} = 840$$

Then the no. of permutations when four I’s do not come together

$$= 34650 - 840 = 33810$$

13. In how many ways can 4 red, 3 yellow and 2 green discs be arranged in a row if the discs of the same colors are in distinguishable?

Ans. Total no. of discs $=4+3+2=9$

Out of 9 discs 4 are of some kind, 3are of same kind, 2 are of same kind

There for the number of arrangements $=\dfrac{9!}{4!3!2!}=1260$

14. Find the number of permutations of the letters of the word ALLAHABAD.

Ans. In the word ALLAHABAD no. of letters = 9

In which four A's and two L's

$\therefore$ The no. of permutations $=\dfrac{9!}{4!2!}=7560$

15. How many 4 letter code can be formed using the first 10 letters of the English alphabet if no letter can be repeated?

Ans. There are 10 letters of English alphabet

For making 4 letter code

First letter can be choose in 10 ways

Second letter can be choose in 9 ways

Third letter can be choose in 8 ways

Forth letter can be choose in 7 ways

By fundamental principle of multiplication

Total no. of code $=10\times9\times8\times7=5040$

16. Determine the number of ways of choosing 5 cards out of a deck of 52 cards which

include exactly one ace.

Ans. In a deck of 52 cards, there are 4 aces and 48 other cards. Here, we have to choose exactly one ace 4 other cards.

The number of ways of choosing one ace out of 4 aces $= {}^4C_1$.

The number of ways of choosing 4 cards out of the other 48 cards $= {}^{48}C_4$.

Corresponding to one way of choosing an ace, there are ${}^{48}C_4$ ways of choosing 4 other cards. But there are ${}^4C_1$ ways of choosing aces, there fore, the required number of ways

$$= {}^4C_1 \times {}^{48}C_4 = \frac{4}{1} \times \frac{48\times47\times46\times45}{1\times2\times3\times4} = 778320$$

17.How many numbers greater than 56000 and farmed by using the digits 4,5,6,7,8, no digit being repeated in any number?

Ans. Number greater than 56000 and farmed by using the digits 4,5,6,7,8 are of types

$$5(6/7/8)\times\times\times \text{ or } (6/7/8)\times\times\times$$

Now numbers of type $5(6/7/8)\times\times\times$ are $1\times3\times3\times2\times1=18$ in number

Number of type $(6/7/8)\times\times\times\times$ are $3\times4\times3\times2\times1=72$ in number

Hence required number of numbers is $18+72=90$

18.Find n, if $\frac{\lfloor n}{\lfloor 2\ \lfloor n-2}$ and $\frac{\lfloor n}{\lfloor 4 \lfloor n-4}$ are in the ratio 2:1

Ans. Given $\frac{\lfloor n}{\lfloor 2\ \lfloor n-2} : \frac{\lfloor n}{\lfloor 4\ \lfloor n-4} = 2:1$

$$\Rightarrow \frac{\lfloor n}{\lfloor 2\ \lfloor n-2} \times \frac{\lfloor 4\ \lfloor n-4}{\lfloor n} = \frac{2}{1}$$

$$\Rightarrow \frac{4\times3\times\lfloor 2\times\lfloor n-4}{\lfloor 2\times(n-2)\times(n-3)\times\lfloor n-4}=\frac{2}{1}$$

$$\Rightarrow \frac{4\times3}{(n-2)(n-3)}=\frac{2}{1}\Rightarrow(n-2)(n-3)=6$$

$$\Rightarrow n^2-5n=0\Rightarrow n(n-5)=0$$

$$\Rightarrow n=0 \text{ or } n=5$$

But, for $n=0$, $\lfloor n-2$ and $\lfloor n-4$ are not meaningful, therefore, $n=5$.

19. Prove that $\lfloor 2n = 1.3.5......(2n-1).2^n\ .\lfloor n$

Ans. $\lfloor 2n = 1.2.3.4.5.6......(2n-1)(2n)$

$$=[1.3.5......(2n-1)][2.4.6......2n]$$

$$=[1.3.5.......(2n-1)][(2.1)(2.2)(2.3)......(2n)]$$

$$=1.3.5.......(2n-1).2^n.(1.2.3......n)$$

$$=1.3.5.......(2n-1).2^n.\lfloor n, \text{ as desired.}$$

20. How many 4 letter words with or without meaning, can be formed out of the letters of the word 'LOGARITHMS', if repetition of letters is not allowed?

Ans. There are 10 letters in the word 'LOGARITHMS'

For making 4 letter word we take 4 at a time

$\therefore$ No. of arrangements 10 letters taken 4 at a time

$$= {}^{10}P_4 = 5040$$

21. From a class of 25 students 10 are to be chosen for an excursion Party. There are 3

students who decide that either all of them will join or none of them will join. In how many ways can excursion party be chosen?

Ans. Total no. of students = 25

No. of students to be selected = 10

I case :

3 students all of them will join the excursion party.

Then remaining 7 students will be selected out of (25-3 = 22) in $^{22}C_7$ ways

II case :

All 3 students will not join the party then 10 students will be selected in $^{22}C_{10}$ ways

Total no. of selection $= {}^{22}C_7 + {}^{22}C_{10}$

$$= \frac{\lfloor 22}{\lfloor 15 \lfloor 7} + \frac{\lfloor 22}{\lfloor 12 \lfloor 10} = 817190$$

22. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

Ans. No. of red balls = 6

No. of white balls = 5

No. of blue balls = 5

No. of selecting each colour balls = 3

$\therefore$ required no. of selection = ${}^{6}C_3 \times {}^{5}C_3 \times {}^{5}C_3$

$$= \frac{\lfloor 6}{\lfloor 3 \lfloor 3} \times \frac{\lfloor 5}{\lfloor 2 \lfloor 3} \times \frac{\lfloor 5}{\lfloor 2 \lfloor 3}$$

$$= \frac{6.5.4.\lfloor 3}{\lfloor 3.3.2.1} \times \frac{5.4^2.\lfloor 3}{2.1.\lfloor 3} \times \frac{5.^2 4.\lfloor 3}{2.1.\lfloor 3}$$

$$= 5 \times 4 \times 5 \times 2 \times 5 \times 2$$

$$= 20 \times 10 \times 10 = 2000$$

23. Find the number of 3 digit even number that can be made using the digits 1, 2, 3, 4, 5, 6, 7, if no digit is repeated?

Ans. For making 3 digit even numbers unit place of digit can be filled in 3 ways. Ten's place of digit can be filled in 5 ways. Hundred place of digit can be filled in 4 ways. $\therefore$ Required number of 3 digit even number $= 3 \times 5 \times 4 \ = 60$

24. If $^nP_r = {}^nP_{r+1}$ and $^nC_r = {}^nC_{r-1}$ **find the values of** n and r

Ans. Given that $^nP_r = {}^nP_{r+1}$

$$\Rightarrow \frac{\lfloor n}{\lfloor n-r} = \frac{\lfloor n}{\lfloor n-r-1}$$

$$\Rightarrow \frac{1}{(n-r)\ \lfloor n-r-1} = \frac{1}{\lfloor n-r-1}$$

$$= n-r = 1 ......(i)$$

And $^nC_r = {}^nC_{r-1}$

$$\Rightarrow \frac{\lfloor n}{\lfloor n-r\ \lfloor r} = \frac{\lfloor n}{\lfloor n-r+1\ \lfloor r-1}$$

$$\Rightarrow \frac{1}{r} = \frac{1}{n-r+1}$$

$$\Rightarrow n-r+1 = r \Rightarrow n-2r = -1 ......(ii)$$

Solving eq. (i) and (ii)

$n=3, \quad r=2$

25. Prove that the product r of consecutive positive integer is divisible by $\lfloor r$

Ans. Suppose & consecutive positive integers are $(n+1)$, $(n+2)$, ...$(n+r)$

Then product $=(n+1).(n+2).(n+3)......(n+r)$

$$=\frac{\lfloor n \ (n+1).(n+2).(n+3)......(n+r)}{\lfloor n}$$

$$=\frac{1.2.3......n.(n+1)(n+2)(n+3)......(n+r)}{\lfloor n}$$

$$=\frac{\lfloor n+r}{\lfloor n}=\frac{\lfloor n+r}{\lfloor r \ \lfloor n+r-r} \quad \lfloor r$$

$=\left({}^{n+r}C_r\right) \lfloor r$ which is divisible by $\lfloor r$

CBSE Class 12 Mathematics
Important Questions
Chapter 7
Permutations and Combinations

6 Marks Questions

1. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has :

(i) no girl? (ii) at least one boy and one girl? (iii) at least 3 girls?

Ans. Number of girls = 4

Number of boys = 7

Number of selection of members = 5

(i) If team has no girl

We select 5 boys

∴ Number of selection of 5 members

$$= {}^7C_5 = \frac{\lfloor 7}{\lfloor 5 \lfloor 2} = 21$$

(ii) At least one boy and one girl the team consist of

Boy	Girls
1	4
2	3
3	2
4	1

The required number of ways

$= {}^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$

$= 7 + 84 + 210 + 140$

$= 441$

(iii) At least 3 girls

Girls	Boys
3	2
4	1

The required number of ways

$= {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1 = 84 + 7$

$= 91$

2. Find the number of words with or without meaning which can be made using all the letters of the word. AGAIN. If these words are writer as in a dictionary, what will be the 50th word?

Ans. In the word 'AGAIN' there are 5 letters in which 2 letters (A) are repeated

Therefore total no. of words $\frac{5!}{2!} = 60$

If these words are written as in a dictionary the number of words starting with Letter A. [A A G I N] $= 4! = 24$

The no. of wards starting with G [G A A I N] $= \frac{4!}{2!} = 12$

The no. of words starting with I [I A A G N] $= \frac{4!}{2!} = 12$

Now

Total words $= 24 + 12 + 12 = 48$

49^{th} Words = N A A G I

50^{th} Words = N A A I G

3. What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of there

(i) Four cards one of the same suit

(ii) Four cards belong to four different suits

(iii) Are face cards.

(iv) Two are red cards & two are black cards.

(v) Cards are of the same colour?

Ans. The no. of ways of choosing 4 cards form 52 playing cards.

$$^{52}C_4 = \frac{52!}{4!48!} = 270725$$

(i) If 4 cards are of the same suit there are 4type of suits. [diamond club, spade and heart] 4 cards of each suit can be selected in $^{13}C_4$ ways

$\therefore$ Required no. of selection $= {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4$

$= 4 \times {}^{13}C_4 = 2860$

(ii) If 4 cards belong to four different suits then each suit can be selected in $^{13}C_1$ ways required no. of selection $= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = 13^4$

(iii) If all 4 cards are face cards. Out of 12 face cards 4 cards can be selected in $^{12}C_4$ ways.

$\therefore$ required no. of selection $^{12}C_4 = \frac{\underline{|12}}{\underline{|8}\,\underline{|4}} = 495$

(iv) If 2 cards are red and 2 are black then. Out of 26 red card 2 cards can be selected in $^{26}C_2$

ways similarly 2 black card can be selected in $^{26}C_2$ ways

$\therefore$ required no. of selection $= {}^{26}C_2 \times {}^{26}C_2$

$$= \frac{26!}{2!4!} \times \frac{26!}{2!4!} = (325)^2$$

$$= 105625$$

(v) If 4 cards are of the same colour each colour can be selected in $^{26}C_4$ ways

Then required no. of selection

$$= {}^{26}C_4 + {}^{26}C_4 = 2 \times \frac{26!}{4!22!}$$

$= 29900$ s

4. If $^{n}P_{r} = {}^{n}P_{r+1}$ and $^{n}c_{r} = {}^{n}c_{r-1}$ find the value of n and r

Ans. Given that

$^nP_r = {}^nP_{r+1}$

$\Rightarrow \frac{\lfloor n}{\lfloor n-r} = \frac{\lfloor n}{\lfloor n-r-1}$

$\Rightarrow \frac{1}{(n-r)\lfloor n-r-1} = \frac{1}{\lfloor n-r-1}$

$\Rightarrow n-r=1..........(i)$

also $^nC_r = {}^nC_{r-1}$

$\Rightarrow \frac{\lfloor n}{\lfloor n-r \lfloor r} = \frac{\lfloor n}{\lfloor n-r+1 \lfloor r-1}$

$\Rightarrow \frac{1}{\lfloor n-r\ r\lfloor r-1} = \frac{1}{(n-r+1)\lfloor n-r \lfloor r-1}$

$\Rightarrow \frac{1}{r} = \frac{1}{n-r+1}$

$\Rightarrow n-2r=-1.........(ii)$

Solving eq (i) and eq (ii) we get $n=3$ and $r=2$

5. Find the value of n such that

$(i)\ {}^nP_5 = 42\,{}^nP_3,\ n>4$ $\quad\quad (ii)\ \frac{^nP_4}{^{n-1}P_4} = \frac{5}{3},\ n>4$

Ans (i) $^nP_5 = 42\,{}^nP_3$

$\Rightarrow \frac{\lfloor n}{\lfloor n-5} = 42\frac{\lfloor n}{\lfloor n-3}$

$\Rightarrow \frac{1}{\lfloor n-5} = \frac{42}{(n-3)(n-4)\lfloor n-5}$

$\Rightarrow (n-3)(n-4)=42$

$\Rightarrow n^2-4n-3n+12=42$

$\Rightarrow n^2 - 7n - 30 = 0$

$\Rightarrow n^2 - 10n + 3n - 30 = 0$

$\Rightarrow n(n-10) + 3(n-10) = 0$

$\Rightarrow (n+3)(n-10) = 0$

$n = -3$ or $n = 10$

$n = -3$ Is rejected

Because negative factorial is not defined $\therefore n = 10$

(ii)

$$\frac{{}^{n}P_4}{{}^{n-1}P_4} = \frac{5}{3} \qquad n > 4$$

$$\Rightarrow \frac{\frac{\lfloor n}{\lfloor n-4}}{\frac{\lfloor n-1}{\lfloor n-5}} = \frac{5}{3}$$

$$\Rightarrow \frac{\lfloor n}{\lfloor n-4} \times \frac{\lfloor n-5}{\lfloor n-1} = \frac{5}{3}$$

$$\Rightarrow \frac{n \lfloor n-1}{(n-4)\lfloor n-5} \times \frac{\lfloor n-5}{\lfloor n-1} = \frac{5}{3}$$

$\Rightarrow 3n = 5n - 20$

$\Rightarrow -2n = -20 \quad \Rightarrow n = 10$

6. A committee of 7 has to be farmed from 9 boys and 4 girls in how many ways can this be done when the committee consists of

(i) Exactly 3 girls?

(ii) Attest 3 girls?

(iii) Atmost 3 girls?

Ans. No. of boys = 9

No. of girls = 4

But committee has 7 members

(i) When committee consists of exactly 3 girls

	Boys	Girls	
	9	4	
Selecting member	4	3	=7

∴ Required no. of selection $= {}^9C_4 \times {}^4C_3 = 504$

(ii) Attest 3 girls

	Boys	Girls	
	9	4	
Selecting member	4	3	=7
	3	4	=7

The required no. of selections $= {}^9C_4 \times {}^4C_3 + {}^9C_3 \times {}^4C_4$

$= 504 + 84$

$= 588$

(iii) Atmost 3 girls

	Boys	Girls	
	9	4	
	7	0	=7
Selecting member	6	1	=7
	5	2	=7

	4	3	=7

Then required no. of selection

$$= {}^9C_7 \times {}^4C_0 + {}^9C_6 \times {}^4C_1 + {}^9C_5 \times {}^4C_2 + {}^9C_4 \times {}^4C_3$$

$$= 36 + 336 + 756 + 504$$

$$= 1632$$

7. In how many ways can final eleven be selected from 15 cricket players' if

(i) there is no restriction

(ii) one of then must be included

(iii) one of them, who is in bad form, must always be excluded

(iv) Two of then being leg spinners, one and only one leg spinner must be included?

Ans. (i) 11 players can be selected out of 15 in ${}^{15}C_{11}$ ways

$= {}^{15}C_4$ ways $= \dfrac{15\times14\times13\times12}{1\times2\times3\times4}$ ways=1365 ways

(ii) Since a particular player must be included, we have to select 10 more out of remaining 14 players.

This can be done in ${}^{14}C_{10}$ ways ${}^{14}C_4$ ways

$= \dfrac{14\times13\times12\times11}{1\times2\times3\times4}$ ways = 1001 ways

(iii) Since a particular player must be always excluded, we have to choose 11 ways out of remaining 14

This can be done in ${}^{14}C_{11}$ ways $= {}^{14}C_3$ ways

$= \dfrac{14\times13\times12\times11}{1\times2\times3}$ ways $= 364$ ways.

(iv) One leg spinner can be chosen out of 2 in $^{2}C_{1}$ ways. Then we have to select 10 more players get of 13 (because second leg spinner can't be included). This can be done in $^{13}C_{10}$ ways of choosing 10 players. But these are $^{2}C_{1}$ ways of choosing a leg spinner, there fore, by multiplication principle of counting the required number of ways

$= {}^{2}C_{1}\times{}^{13}C_{10}$

$= {}^{2}C_{1}\times{}^{13}C_{3} = \dfrac{2}{1}\times\dfrac{13\times12\times11}{1\times2\times3} = 572$

8. How many four letter words can be formed using the letters of the letters of the word 'FAILURE' so that

(i) F is included in each word

(ii) F is excluded in each word.

Ans. There are 7 letters in the word 'FAILURE'

(i) F is included for making each word using 4 letters

∴ F is already selected

Then other 3 letters can be selected out of 6 are $^{6}C_{3}$ ways

Also arrangements of 4 letters are 4! Ways so:

∴ Total no. of words $= {}^{6}C_{3}\times4! \; = 480$

(ii) F is excluded in each word

∴ Out of 6 letters are choose 4 letters in $^{6}C_{4}$ ways

Also arrangement of 4 letters are 4! Ways so:

$\therefore$ Total no. of words $= {}^6C_4 \times 4! \; = 360$

9. A committee of 5 is to be formed out of 6 gents and 4 Ladies. In how many ways this can be done, when

(i) at least two ladies are included?

(ii) at most two ladies are included?

Ans. No of person to form committee = 5

No. of gents = 6 and No. of ladies = 4

(i) At least two ladies are included

Ladies [4] Gents [6]

Either we Select $\rightarrow$ 2 and 3

or

$\rightarrow$ 3 and 2

Or

$\rightarrow$ 4 and 1

$\therefore$ required number of selection

$$= {}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1$$

$$120 + 60 + 6 = 186$$

(ii) At most two ladies are included?

Ladies [4] Gents [6]

Either we select 0 and 5

Or 1 and 4

Or 2 and 3

$\therefore$ required no. of selection.

$= {}^4C_0 \times {}^6C_5 \times {}^4C_1 \times {}^6C_4 + {}^4C_2 \times {}^6C_3$

$6 + 60 + 120 = 186$

10. In how many ways can the letters of the word PERMUTATIONS be arranged if the

(i) words start with P and with S

(ii) vowels are all together

(iii) There are always 4 letters between P and S?

Ans. In the word PERMUTATIONS there are 12 letters

(i) If the word start with P and end with S then position of P and S will be fixed. Then other 10 letters can be arranged in $\underline{|10}$ ways. But T occurs twice.

$\therefore$ no. of arrangements = $\frac{\underline{|10}}{\underline{|2}}$

=1814400 ways

(ii) Vowels are together?

No. of vowels in the word PERMUTATIONS are 5 which are [A, E, I, O, U]

$\therefore$ Vowels can be arranged is $\underline{|5}$ ways other letters are consonants out of 8 consonants 2 are repeated

$\therefore$ No. of arrangements of consonants $= \frac{\underline{|8}}{\underline{|2}}$

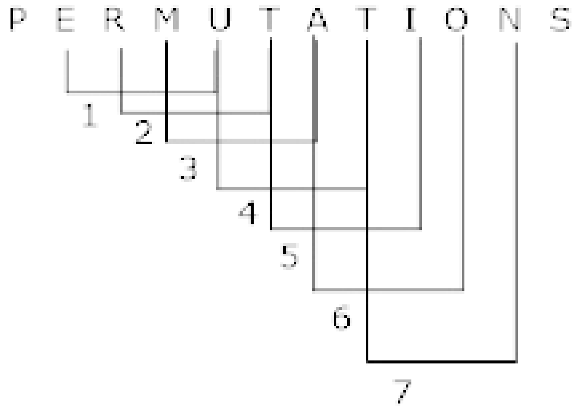
$\therefore$ requires no. arrangements $= \frac{\underline{|8}}{\underline{|2}} \times \underline{|5} = 2419200$

(ii) There are always 4 letters between P and S. in the word 'PERMUTATIONS'

If 4 letters between P and S

Then P and S can be arranged in 2 ways other 10 letters can be arranged in $\frac{\lfloor 10}{\lfloor 2}$ ways

There are 7 pair 4 letters in the words PERMUTATIONS between P and S



∴ Required no. of arrangements = $\frac{\lfloor 10}{\lfloor 2} \times 2 \times 7$

= 2 5401600