Unit 16 (Probability)

Short Answer Type Questions

Q1. If the letters of the word ALGORITHM are arranged at random in a row what is the probability the letters GOR must remain together as a unit?

Sol: We have word ALGORITHM Number of letters = 9

$\therefore$ Total number of words = $9! = n(S)$

If 'GOR' remain together, then we consider it as one group.

$\therefore$ Number of letters = $6 + 1 = 7$

Number of words, if 'GOR' remain together in the order = $7! = n(E)$

$$\therefore \quad \text{Required probability} = \frac{n(E)}{n(S)} = \frac{7!}{9!} = \frac{1}{72}$$

Q2. Six new employees, two of whom are married to each other, are to be assigned six desks that are lined up in a row. If the assignment of employees to desks is made randomly, what is the probability that the married couple will have nonadjacent desks?

Sol: Six employees can be arranged in 6! ways.

n(S) = 6!

Two adjacent desks for married couple can be selected in 5 ways viz.,(l, 2), (2, 3), (3,4), (4, 5), (5,6).

This couple can be arranged in the two desks in 2! ways.

Other four persons can be arranged in 4! ways.

So, number of ways in which married couple occupy adjacent desks

= 5×2! x4! =2×5!

So, number of ways in which married couple occupy non-adjacent desks = 6! – 2 x 5! = 4 x 5! = n(E)

$$\therefore \text{Probability that the couple has non-adjacent desk} = \frac{4 \times 5!}{6!} = \frac{2}{3}$$

Q3. Suppose an integer from 1 through 1000 is chosen at random, find the probability that the integer is a multiple of 2 or a multiple of 9.

Sol: We have integers 1,2, 3,...1000

We have integers 1,2, 3,...1000

n(S) = 1000

Number of integers which are multiple of 2 = 500 Let the number of integers which are multiple of 9 be n.

nth term = 999 => 9 + (n -1)9 = 999

=> 9 + 9n – 9 = 999

=> n = 111

From 1 to 1000, the number of multiples of 9 is 111.

The multiple of 2 and 9 both are 18, 36,..., 990.

Let m be the number of terms in above series.

.'. mth term = 990

=> 18 + (m- 1)18 = 990

=> 18+18m-18 = 990

=> m = 55

Number of multiples of 2 or 9 = 500 +111-55 = 556 = n(E)

$$\therefore \quad \text{Required probability} = \frac{n(E)}{n(S)} = \frac{556}{1000} = 0.556$$

Q4. An experiment consists of rolling a die until a 2 appears.

(i) How many elements of the sample space correspond to the event that the 2 appears on the Ath roll of the die?

(ii) How many elements of the sample space correspond to the event that the 2 appears not later than the Ath roll of the die?

Sol: Number of outcomes when die is thrown is '6'.

(i) If 2 appears on the Ath roll of the die.

So, first (k -1) roll have 5 outcomes each and Kth roll results 2

Number of outcomes = 5^{k-1}

(ii) If we consider that 2 appears not later than K th roll of the die, then 2 comes before Ath roll.

If 2 appears in first roll, number of ways = 1 If 2 appears in second roll, number of ways = 5 x 1 (as first roll does not result in 2)

If 2 appears in third roll, number of ways

= 5 x 5 x 1 (as first two rolls do not result in 2)

Similarly if 2 appears in (k – l)th roll, number of ways = [5x5x5… (k- 1) times] x 1 = 5^{k-1} Possible outcomes if 2 appears before kth roll = 1 +5 + 5^2 + 5^3+ … +5^{k-l}

$$= \frac{1(5^k - 1)}{5-1} = \frac{5^k - 1}{4}$$

Q5. A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find P(G), where G is the event that a number greater than 3 occurs on a single roll of the die.

Sol: If is given that 2 x Probability of even number = Probability of odd

$$\Rightarrow \quad P(O) = 2P(E)$$

Now, $P(O) + P(E) = 1$

$$\Rightarrow \quad 2P(E) + P(E) = 1$$

$$\Rightarrow \quad P(E) = \frac{1}{3} \text{ and } P(O) = \frac{2}{3}$$

$$\therefore \quad P(G) = P(\text{number greater than 3})$$

$$= P(\text{number is 4, 5 or 6})$$

$$= 2 \times P(E) \times P(0) = 2 \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{9}$$

Q6. In a large metropolitan area, the probabilities are .87, .36, .30 that a family (randomly chosen for a sample survey) owns a colour television set, a black and white television set, or both kinds of sets. What is the probability that a family owns either anyone or both kinds of sets?

Sol: Let C be the even that family own colour television set and B be the event that family owns a black and white television set It is given that,

P(C) = 0.87, P{B) = 0.36 and P(C∩B) = 0.30 We have to find probability that a family owns either anyone or both kind of sets i.e., P(B ∪ C)

Now, P(B∪C) = P(B) + P(C)-P(C∩ B)

= 0.87 + 0.36-0.30= 0.93

Q7. If A and B are mutually exclusive events, P(A) =35 and P(B) = 0.45, find

(a) P(A')

(b) P(B')

(c) P(A∪ B)

(d) P(A∩ B)

(e) P(A∪ B')

(f) P(A'∩B')

Sol. Since, it is given that, A and B are mutually exclusive events.

$\therefore \quad P(A \cap B) = 0 \qquad [\because A \cap B = \phi]$

and $P(A) = 0.35, P(B) = 0.45$

(a) $P(A') = 1 - P(A) = 1 - 0.35 = 0.65$

(b) $P(B') = 1 - P(B) = 1 - 0.45 = 0.55$

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.35 + 0.45 - 0 = 0.80$

(d) $P(A \cap B) = 0$

(e) $P(A \cap B') = P(A) - P(A \cap B) = 0.35 - 0 = 0.35$

(f) $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.8 = 0.2$

Q8. A team of medical students doing their internship have to assist during surgeries at a city hospital. The probabilities of surgeries rated as very complex, complex, routine, simple or very simple are respectively, 0.15,0.20, 0.31, 0.26, .08. Find the probabilities that a particular surgery will be rated

(a) complex or very complex;

(b) neither very complex nor very simple;

(c) routine or complex

(d) routine or simple

Sol. Let E_1, E_2, E_3, E_4 and E_5 be the event that surgeries are rated as very complex, complex, routine, simple or very simple, respectively.

$\therefore \quad P(E_1) = 0.15, P(E_2) = 0.20, P(E_3) = 0.31, P(E_4) = 0.26, P(E_5) = 0.08$

(a) $P(\text{complex or very complex}) = P(E_1 \text{ or } E_2) = P(E_1 \cup E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2) = 0.15 + 0.20 - 0 = 0.35$

(b) $P(\text{neither very complex nor very simple})$,

$P(E'_1 \cap E'_5) = P(E_1 \cup E_5)' = 1 - P(E_1 \cup E_5)$

$= 1 - [P(E_1) + P(E_5)] = 1 - (0.15 + 0.08) = 1 - 0.23 = 0.77$

(c) $P(\text{routine or complex}) = P(E_3 \cup E_2) = P(E_3) + P(E_2) = 0.31 + 0.20 = 0.51$

(d) $P(\text{routine or simple}) = P(E_3 \cup E_4) = P(E_3) + (E_4) = 0.31 + 0.26 = 0.57$

Q9. Four candidates A, B, C, ZJhave applied for the assignment to coach a school cricket team. If A is twice as likely to be selected as B, and B and C are given about the same chance of being selected, while C is twice as likely to be selected as D, what are the probabilities that

(a) C will be selected? (b) A will not be selected?

Sol: It is given that A is twice as likely to be selected as B.

P(A) = 2P(B)

B and C are given about the same chance of being selected.

P(B) = P(C)

C is twice as likely to be selected as D.

P(C) = 2 P(D)

$\Rightarrow \qquad P(A) = 4P(D) \Rightarrow P(D) = \frac{P(A)}{4}$

Now, sum of probability = 1

$P(A) + P(B) + P(C) + P(D) = 1$

$\therefore \qquad P(A) + \frac{P(A)}{2} + \frac{P(A)}{2} + \frac{P(A)}{4} = 1$

$\Rightarrow \qquad \frac{4P(A) + 2P(A) + 2P(A) + P(A)}{4} = 1$

$\Rightarrow \qquad 9P(A) = 4 \Rightarrow P(A) = \frac{4}{9}$

(a) $P(C \text{ will be selected}) = P(C) = P(B) = \frac{P(A)}{2} = \frac{4}{9 \times 2} = \frac{2}{9}$

(b) $P(A \text{ will not be selected}) = P(A') = 1 - P(A) = 1 - \frac{4}{9} = \frac{5}{9}$

Q10. One of the four persons John, Rita, Aslam or Gurpreet will be promoted next month. Consequently the sample space consists of four elementary outcomes S = {John promoted, Rita promoted, Aslam promoted, Gurpreet promoted}. You are told that the chances of John's promotion is same as that of Gurpreet, Rita's chances of promotion are twice as likely as Johns. Aslam's chances are four times that of John.

(a) Determine

P(John promoted)

P(Rita promoted)

P(Aslam promoted)

P(Gurpreet promoted)

(b) If A = {John promoted or Gurpreet promoted}, find P(A).

Sol: Let Event: J = John promoted

R = Rita promoted

A = Aslam promoted

G = Gurpreet promoted

Given sample space, S = {John promoted, Rita promoted, Aslam promoted, Gurpreet promoted}

i.e. S={J,R,A,G)

It is given that, chances of John's promotion is same as that of Gurpreet.

P(J) = P(G)

Rita's chances of promotion are twice as likely as John.

P(R) = 2P(J)

And Aslam's chances of promotion are four times that of John.

P(A) = 4P(J)

Now, P(J) + P(R) + P(A) + P(G) = 1 => P(J) + 2P(J) + 4P(J) + P(J) = 1

=> 8P(J) = 1

P(J) = 1/8

(a) $P(\text{John promoted}) = P(J) = \frac{1}{8}$

$P(\text{Rita promoted}) = P(R) = 2P(J) = 2 \times \frac{1}{8} = \frac{1}{4}$

$P(\text{Aslam promoted}) = P(A) = 4P(J) = 4 \times \frac{1}{8} = \frac{1}{2}$

$P(\text{Gurpreet promoted}) = P(G) = P(J) = \frac{1}{8}$

(b) A = John promoted or Gurpreet promoted

$\therefore \quad A = J \cup G$

$$P(A) = P(J \cup G)$$
$$= P(J) + P(G) - P(J \cap G)$$
$$= \frac{1}{8} + \frac{1}{8} - 0 \qquad [\because P(J \cap G) = 0]$$
$$= \frac{1}{4}$$

Q11. The accompanying Venn diagram shows three events, A, B, and C, and also the probabilities of the various intersections (for instance, P(A∩B) = .07).

Determine

(a) $P(A)$

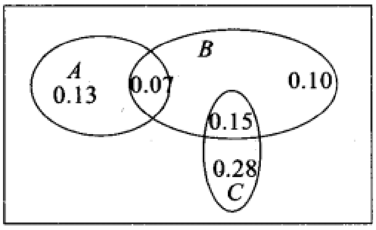
(b) $P(B \cap \bar{C})$

(c) $P(A \cup B)$

(d) $P(A \cap \bar{B})$

(e) $P(B \cap C)$

(f) Probability of exactly one of the three occurs.



Sol. From the above Venn diagram,

(a) $P(A) = 0.13 + 0.07 = 0.20$

(b) $P(B \cap \bar{C}) = P(B) - P(B \cap C) = 0.07 + 0.10 + 0.15 - 0.15 = 0.07 + 0.10 = 0.17$

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.13 + 0.07 + 0.07 + 0.10 + 0.15 - 0.07$
$= 0.13 + 0.07 + 0.10 + 0.15 = 0.45$

(d) $P(A \cap \bar{B}) = P(A) - P(A \cap B) = 0.13 + 0.07 - 0.07 = 0.13$

(e) $P(B \cap C) = 0.15$

(f) $P(\text{exactly one of the three occurs}) = 0.13 + 0.10 + 0.28 = 0.51$

Q12. One urn contains two black balls (labelled Bx and B2) and one white ball. A second urn contains one black ball and two white balls (labelled W_1 and W_2). Suppose the following experiment is performed. One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then a second ball is chosen at random from the same urn without replacing the first ball.

(a) Write the sample space showing all possible outcomes

(b) What is the probability that two black balls are chosen?

(c) What is the probability that two balls of opposite colour are chosen?

Sol:It is given that one of the two urn is chosen, then a ball is randomly chosen from the urn, then a second ball is chosen at random from the same urn without replacing the first ball.

(a) Sample space $S = \{B_1B_2, B_2B_1, B_1W, WB_1, B_2W, WB_2, BW_1, W_1B, BW_2, W_2B, W_1W_2, W_2W_1\}$

$\therefore \quad n(S) = 12$

(b) If two black ball are chosen, then the favourable cases are B_1B_2 and B_2B_1.

$\therefore \quad \text{Required probability} = \frac{2}{12} = \frac{1}{6}$

(c) If two balls of opposite colours are chosen, the favourable cases are $B_1W, WB_1, B_2W, WB_2, BW_1, W_1B, BW_2$ and W_2B.

$\therefore \quad \text{Required probability} = \frac{8}{12} = \frac{2}{3}$

Q13. A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the Probability that

(a) All the three balls are white

(b) All the three balls are red

(c) One ball is red and two balls are white

Sol: Number of red balls = 8

and number of white balls = 5

Three balls are draw at random

$\therefore \quad n(S) = {}^{13}C_3$

(a) $P(\text{All the three balls are white}) = \frac{{}^5C_3}{{}^{13}C_3} = \frac{\frac{5!}{3!2!}}{\frac{13!}{3!10!}} = \frac{10}{\frac{13\times 12\times 11}{6}} = \frac{5}{143}$

(b) $P(\text{all the three balls are red}) = \frac{{}^8C_3}{{}^{13}C_3} = \frac{\frac{8\times 7\times 6}{6}}{\frac{13\times 12\times 11}{6}} = \frac{8\times 7\times 6}{13\times 12\times 11} = \frac{28}{143}$

(c) $P(\text{one ball is red and two balls are white}) = \frac{{}^8C_1\times {}^5C_2}{{}^{13}C_3} = \frac{8\times 10}{\frac{13\times 12\times 11}{6}} = \frac{40}{143}$

14. If the letters of the word ASSASSINATION are arranged at random. Find the Probability that

(a) Four S's come consecutively in the word

(b) Two I's and two N's come together

(c) All A's are not coming together

(d) No two A's are coming together.

Sol. We have word 'ASSASSINATION'.

Number of letters = 13

Letters are $3A$'s, $4S$'s, $2I$'s, $2N$'s, 1 T's and 1 O's

Total number of ways these letters can be arranged $= n(S) = \frac{13!}{3!4!2!2!}$

(a) If for S's come consecutively in the word, then we considers these 4 S's as 1 group.

So, now number of letters is 10 i.e., $(SSSS), A, A, A, I, I, N, N, T, O$

$$\therefore \quad n(E) = \frac{10!}{3!2!2!}$$

$$\therefore \quad \text{Required probability} = \frac{\frac{10!}{3!2!2!}}{\frac{13!}{3!4!2!2!}} = \frac{4!}{13 \times 12 \times 11} = \frac{2}{143}$$

(b) If 2 I's and 2N's come together, then there as 10 alphabets.

i.e., (IINN), $A, A, A, S, S, S, S, T, O$

Number of words when 2 I's and 2 N's are come together

$$= \frac{10!}{3!4!} \times \frac{4!}{2!2!}$$

$$\therefore \quad \text{Required probability} = \frac{\frac{10!4!}{3!4!2!2!}}{\frac{13!}{3!4!2!2!}} = \frac{4!}{13 \times 12 \times 11} = \frac{2}{143}$$

(c) If all A's are coming together, then there are 11 alphabets

i.e., $(AAA), S, S, S, S, I, I, N, N, T, O$

$\therefore$ Number of words when all A's come together $= \frac{11!}{4!2!2!}$

$\therefore$ Probability when all A's come together

$$= \frac{\frac{11!}{4!2!2!}}{\frac{13!}{4!3!2!2!}} = \frac{3!}{13 \times 12} = \frac{1}{26}$$

Then the probability that all A's does not come together

$$= 1 - \frac{1}{26} = \frac{25}{26}$$

(d) If no two A's are together, then first we arrange the alphabets other than A's.

i.e. $S, S, S, S, I, I, N, N, T, O$

These letters can be arranged in $\frac{10!}{4!2!2!}$ ways.

$$\times S \times S \times S \times S \times I \times I \times N \times N \times T \times O \times$$

Arrangement of these letters creates eleven gaps shown as '×'

Three gaps for three A's can be selected in ${}^{11}C_3$ ways.

$\therefore$ Total number of words when no two A's together

$$= {}^{11}C_3 \times \frac{10!}{4!2!2!} = \frac{11!}{3!8!} \times \frac{10!}{4!2!2!}$$

$\therefore$ The probability that no two A's come together

$$= \frac{\frac{11! \times 10!}{3!8!4!2!2!}}{\frac{13!}{4!3!2!2!}} = \frac{10!}{8! \times 13 \times 12} = \frac{10 \times 9}{13 \times 12} = \frac{15}{26}$$

Q15. A card is drawn from a deck of 52 cards. Find the probability of getting a king or a heart or a red card.

Sol: Number of cards = 52 .-. n(S) = 52

4 king + 13 heart + 26 red – 13 – 2 = 28 = n{E)

∴ Required probability = 28/52 = 7/13

16. A sample space consists of 9 elementary outcomes $E_1, E_2, ..., E_9$ whose probabilities are

$$P(E_1) = P(E_2) = 0.08, P(E_3) = P(E_4) = P(E_5) = 0.1,$$
$$P(E_6) = P(E_7) = 0.2, P(E_8) = P(E_9) = 0.07$$

Suppose $A = \{E_1, E_5, E_8\}, B = \{E_2, E_5, E_8, E_9\}$

(a) Calculate $P(A)$, $P(B)$, and $P(A \cap B)$

(b) Using the addition law of probability, calculate $P(A \cup B)$

(c) List the composition of the event $A \cup B$, and calculate $P(A \cup B)$ by adding the probabilities of the elementary outcomes.

(d) Calculate $P(\bar{B})$ from $P(B)$, also calculate $P(\bar{B})$ directly from the elementary outcomes of $\bar{B}$

Sol. (a) $P(A) = P(E_1) + P(E_5) + P(E_8) = 0.08 + 0.1 + 0.07 = 0.25$

(b) $P(B) = P(E_2) + P(E_5) + P(E_8) + P(E_9) = 0.08 + 0.1 + 0.07 + 0.07 = 0.32$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Now, $A \cap B = \{E_5, E_8\}$

$\therefore \quad P(A \cap B) = P(E_5) + P(E_8) = 0.1 + 0.07 = 0.17$

$\therefore \quad P(A \cup B) = 0.25 + 0.32 - 0.17 = 0.40$

(c) $A \cup B = \{E_1, E_2, E_5, E_8, E_9\}$

$$P(A \cup B) = P(E_1) + P(E_2) + P(E_5) + P(E_8) + P(E_9)$$
$$= 0.08 + 0.08 + 0.1 + 0.07 + 0.07 = 0.40$$

(d) $\because \quad P(\bar{B}) = 1 - P(B) = 1 - 0.32 = 0.68$

and $\bar{B} = \{E_1, E_3, E_4, E_6, E_7\}$

$\therefore \quad P(\bar{B}) = P(E_1) + P(E_3) + P(E_4) + P(E_6) + P(E_7)$

$$= 0.08 + 0.1 + 0.1 + 0.2 + 0.2 = 0.68$$

Q17. Determine the probability p, for each of the following events.

(a) An odd number appears in a single toss of a fair die.

(b) At least one head appears in two tosses of a fair coin.

(c) The sum of 6 appears in a single toss of a pair of fair dice.

Sol: (a) When a die is thrown the possible outcomes are

S = {1, 2, 3,4, 5, 6} out of which 1, 3, 5 are odd,

$\therefore \quad$ Required probability $= \frac{3}{6} = \frac{1}{2}$

(b) When a fair coin is tossed two times the sample space is

$S = \{HH, HT, TH, TT\}$

If at least one head appears then the favourable cases are HH, HT and TH.

$\therefore \quad$ Required probability $= \frac{3}{4}$

(c) When a pair of dice is rolled, total number of cases $= 6 \times 6 = 36$

If sum is 6 then possible outcomes are $(1, 5), (5, 1), (2, 4), (4, 2)$ and $(3, 3)$.

$\therefore \quad$ Required probability $= \frac{5}{36}$

Objective type Questions

Q18. In a non-leap year, the probability of having 53 Tuesdays or 53 Wednesdays is

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) none of these

Sol. **(b)** In a non-leap year' there are 365 days which have 52 weeks and 1 day.

If this day is a Tuesday or Wednesday, then the year will have 53 Tuesday or 53 Wednesday.

$\therefore \quad$ Required probability $= \frac{1}{7} + \frac{1}{7} = \frac{2}{7}$

Q19. Three numbers are chosen from 1 to 20. Find the probability that they are not consecutive

(a) $\frac{186}{190}$ (b) $\frac{187}{190}$ (c) $\frac{188}{190}$ (d) $\frac{18}{^{20}C_3}$

Sol. **(b)** Since, the set of three consecutive numbers from 1 to 20 are (1, 2, 3), (2, 3, 4), (3, 4, 5), …, (18, 19, 20), i.e., 18.

$$P(\text{numbers are consecutive}) = \frac{18}{^{20}C_3} = \frac{18}{\frac{20 \times 19 \times 18}{3!}} = \frac{3}{190}$$

$$P(\text{three number are not consecutive}) = 1 - \frac{3}{190} = \frac{187}{190}$$

Q20. While shuffling a pack of 52 playing cards, 2 are accidentally dropped. Find the probability that the missing cards to be of different colours

(a) $\frac{29}{52}$ (b) $\frac{1}{2}$ (c) $\frac{26}{51}$ (d) $\frac{27}{51}$

Sol. **(c)** In a back of 52 cards 26 are red colour and 26 are black colour.

$$\therefore P(\text{both cards of opposite colour}) = \frac{^{26}C_1 \times {^{26}C_1}}{^{52}C_2} = \frac{26 \times 26}{\frac{52 \times 51}{2}} = \frac{26}{51}$$

Q21. Seven persons are to be seated in a row. The probability that two particular persons sit next to each other is

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{2}{7}$ (d) $\frac{1}{2}$

Sol. **(c)** If two persons sit next to each other, then consider these two person as 1 group.

Now, we have to arrange 6 persons.

$\therefore$ Number of arrangement $= 2! \times 6!$

Total number of arrangement of 7 persons $= 7!$

$\therefore$ Required probability $= \frac{2!6!}{7!} = \frac{2}{7}$

Q22. Without repetition of the numbers, four digit numbers are formed with the numbers 0, 2, 3, 5. The probability of such a number divisible by 5 is

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$ (c) $\frac{1}{30}$ (d) $\frac{5}{9}$

Sol. **(d)** We have digits 0, 2, 3, 5.

Number of divisible by 5 if unit place digit is '0' or '5'

If unit place is '0' then first three places can be filled in 3! Ways

If unit place is '5' then first place can be filled in two ways and second and third place can be filled in 2! ways.

So, number of numbers ending with digit '5' is $2 \times 2! = 4$

$\therefore$ Total number of numbers divisible by $5 = 3! + 4 = 10 = n(E)$

Also total number of numbers $= 3 \times 3! = 18$

$\therefore$ Required probability $= \frac{10}{18} = \frac{5}{9}$

Q23. If A and B are mutually exclusive events, then

(a) $P(A) \leq P(\bar{B})$ (b) $P(A) \geq P(\bar{B})$
(c) $P(A) < P(B)$ (d) none of these

Sol. **(a)** For mutually exclusive events,

$$P(A \cap B) = 0$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\Rightarrow \quad P(A \cup B) = P(A) + P(B)$$
$$\Rightarrow \quad P(A) + P(B) \leq 1$$
$$\Rightarrow \quad P(A) \leq 1 - P(B)$$
$$\therefore \quad P(A) \leq P(\bar{B})$$

Q24. If P(A ∪B) = P(A n B) for any two events A and B, then
(a) P(A) = P(B) (b) P (A) > P (B)
(c) P(A) < P(B) (d) none of these

Sol: (a) We have, P(A ∪ B) = P(A n B)
P(A) + P(B) − P(A ∩ B) = P(A ∩ B)

$$\Rightarrow \quad [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)] = 0$$
But $\quad P(A) - P(A \cap B) \geq 0$
and $\quad P(B) - P(A \cap B) \geq 0 \qquad [\because P(A \cap B) \leq P(A) \text{ or } P(B)]$
$$\Rightarrow \quad P(A) - P(A \cap B) = 0$$
and $\quad P(B) - P(A \cap B) = 0$
$$\Rightarrow \quad P(A) = P(A \cap B)$$
and $\quad P(B) = P(A \cap B)$
$$\therefore \quad P(A) = P(B)$$

Q25. If 6 boys and 6 girls sit in a row at random. The probability that all the girls sit together is

(a) $\frac{1}{432}$ (b) $\frac{12}{431}$ (c) $\frac{1}{132}$ (d) none of these

Sol. **(c)** If all the girls sit together, then considered it as 1 group.
Number of arrangements of 6 + 1 = 7 persons in a row is 7! and the girls interchanges their seats in 6! ways.

$$\therefore \quad \text{Required probability} = \frac{6!7!}{12!} = \frac{1}{132}$$

Q26. A single letter is selected at random from the word 'PROBABILITY'. The probability that it is a vowel is

(a) $\frac{1}{3}$ (b) $\frac{4}{11}$ (c) $\frac{2}{11}$ (d) $\frac{3}{11}$

Sol. **(b)** Total number of alphabet in the word probability = 11
Number of vowels = 4(*O*, *A*, *I*, *I*)

$$P(\text{letter is vowel}) = \frac{4}{11}$$

Q27. If the probabilities for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails is

(a) >0.5 (b) 0.5 (c) ≤ 0.5 (d) 0

Sol. **(c)** Given, $P(A \text{ fail}) = 0.2$
and $\quad P(B \text{ fail}) = 0.3$
$$\therefore \quad P(\text{either } A \text{ or } B \text{ fail}) \leq P(A \text{ fail}) + P(B \text{ fail})$$
$$\leq 0.2 + 0.3$$
$$\leq 0.5$$

28. The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\bar{A}) + P(\bar{B})$ is

(a) 0.4 (b) 0.8 (c) 1.2 (d) 1.6

Sol. **(c)** We have, $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.2$

$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad 0.6 = P(A) + P(B) - 0.2$

$\Rightarrow \quad P(A) + P(B) = 0.8$

$\therefore \quad P(\bar{A}) + P(\bar{B}) = 1 - P(A) + 1 - P(B) = 2 - [P(A) + P(B)] = 2 - 0.8 = 1.2$

Q29. If M and N are any two events, tlie probability that at least one of them occurs is .

(a) P(M) + P(N) – 2 P(M ∩N)

(b) P(M) + P(N) – P(M ∩ N)

(c) P(M) + P(N) + P(M ∩ N)

(d) P(M) + P(N) + 2P(M∩N)

Sol: (B) If M and N are any two events.

.-. P(M ∪N) = P(M) + P(N) – P(M ∩ N) .

True/False Type Questions

Q30. The probability that a person visiting a zoo will see the giraffe is 0.72, the probability that he will see the bears is 0.84 and the probability that he will see both is 0.52.

Sol: False

P(to see giraffe or bear) = P (giraffe) + P (bear) – P(giraffe and bear)

=0.72 + 0.84-0.52= 1.04

which is not possible.

Q31. The probability that a student will pass his examination is 0.73, the probability of the student getting a compartment is 0.13, and the probability that the student will either pass or get compartment is 0.96.

Sol: False

Let A = Student will pass examination

B = Student will getting compartment

P(A) = 0.73, P(B) = 0.13 and P(A or B) = 0.96

P(A or B) = P(A) + P(B) = 0.73 + 0.13 = 0.86

But P(A or B) = 0.96

Hence, given statement is false.

Q32.The probabilities that a typist will make 0, 1,2, 3, 4, 5 or more mistakes in typing a report are, respectively, 0.12, 0.25, 0.36, 0.14, 0.08, 0.11.

Sol: False

Sum of these probabilities must be equal to 1.

P(0) + P(1) + P(2) + P(3) + P{ 4) + P(5)

= 0.12 + 0.25+0.36 + 0.14 + 0.08 + 0.11 = 1.06 which is greater than 1,

So, it is false statement.

Q33. If A and B are two candidates seeking admission in an engineering College. The probability that A is selected is .5 and the probability that both A and B are selected is at most .3. Is it possible that the probability of B getting selected is 0.7?

Sol. False

Given that, P(A) = 0.5, P(A ∩B)< 0.3

Now, P(A) x P(B) ≤ 0.3
=>0.5 x P(B) ≤0.3
=> P(B) ≤0.6
Hence, it is false statement

Q34. The probability of intersection of two events A and B is always less than or equal to those favourable **to the event**
Sol: True
We know that A ∩ B ⊂ A
P(A ∩ B) ≤ P(A)
Hence, it is a true statement.

Q35. The probability of an occurrence of event A is .7 and that of the occurrence of event B is .3 and the probability of occurrence of both is .4.
Sol: False
A ∩B⊆ A, B
P(A ∩B) ≤ P(A), P(B)
But given that P(B) = 0.3 and P(A ∩B) = 0.4, which is not possible.

Q36. The sum of probabilities of two students getting distinction in their final examinations is 1.2.
Sol: True
Probability of each student getting distinction in their final examination is less than or equal to 1, sum of the probabilities of two may be 1.2.
Hence, it is true statement.

Fill in the Blanks Type Questions

Q37. The probability that the home team will win an upcoming football game is 0.77, the probability that it will tie the game is 0.08, and the probability that it will lose the game is ______.

Sol: P(losing) = 1 – (0.77 + 0.08) = 0.15

38. If E_1, E_2, E_3, E_4 are the four elementary outcomes in a sample space and $P(E_1) = 0.1$, $P(E_2) = 0.5$, $P(E_3) = .1$, then the probability of E_4 is _______.

Sol. $P(E_1) + P(E_2) + P(E_3) + P(E_4) = 1$

$\Rightarrow \quad 0.1 + 0.5 + 0.1 + P(E_4) = 1$

$\Rightarrow \quad 0.7 + P(E_4) = 1$

$\therefore \quad P(E_4) = 0.3$

39. Let $S = \{1, 2, 3, 4, 5, 6\}$ and $E = \{1, 3, 5\}$, then $\overline{E}$ is __________.

Sol. Here, $S = \{1, 2, 3, 4, 5, 6\}$

and $\quad E = \{1, 3, 5\}$

$\therefore \quad \overline{E} = S - E = \{2, 4, 6\}$

40. If A and B are two events associated with a random experiment such that $P(A) = 0.3$, $P(B) = 0.2$ and $P(A \cap B) = 0.1$, then the value of $P(A \cap \overline{B})$ is _______.

Sol. $P(A \cap \overline{B}) = P(A) - P(A \cap B) = 0.3 - 0.1 = 0.2$

41. The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then the probability of neither A nor B is _________.

Sol. $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B)]$ [Since, A and B are mutually exclusive]

$= 1 - (0.5 + 0.3) = 1 - 0.8 = 0.2$