Unit 2 (Relations And Functions)

Short Answer Type Questions

Q1. If A = {-1, 2, 3 } and B = {1, 3}, then determine

(i) AxB (ii) BxC (c) BxB (iv) AxA

Sol: We have A = {-1,2,3} and B = {1,3}

(i) A x B = {(-1, 1), (-1, 3), (2, 1), (2, 3), (3,1), (3, 3)}

(ii) BxA = {(1, -1), (1, 2), (1,3), (3,-1), (3,2), (3, 3)}

(iii) BxB= {(1,1), (1,3), (3,1), (3, 3)}

(iv) A xA = {(-1, -1), (-1, 2), (-1, 3), (2, -1), (2, 2), (2, 3), (3, -1), (3, 2), (3,3)}

Q2. If P = {x : x < 3, x e N}, Q= {x : x≤2,x ∈ W}. Find (P∪ Q) x (P∩ Q), where W is the set of whole numbers.

Sol: We have, P={x: x<3,x ∈ N} = {1,2}

And Q = {x :x≤ 2,x∈ W] = {0,1,2}

P∪Q= {0, 1,2} and P ∩ Q= {1,2}

(P ∪ Q) x (P ∩ Q) = {0,1, 2} x {1,2}

= {(0,1), (0, 2), (1,1), (1,2), (2,1), (2, 2)}

Q3. IfA={x:x∈ W,x < 2}, 5 = {x : x∈N, 1 <.x < 5}, C= {3, 5}. Find

(i) Ax(B∩Q) (ii) Ax(B∪C)

Sol: We have, A = {x :x∈ W,x< 2} = {0, 1};

B = {x : x ∈ N, 1 <x< 5} = {2, 3,4}; and C= {3, 5}

(i) B∩ C = {3}

A x (B ∩ C) = {0, 1} x {3} = {(0, 3), (1, 3)}

(ii) (B ∪ C) ={2,3,4, 5}

A x (B ∪ C) = {0, 1} x {2, 3,4, 5}

= {(0,2), (0,3), (0,4), (0,5), (1,2), (1,3), (1,4), (1, 5)}

Q4. In each of the following cases, find a and b. (2a + b, a – b) = (8, 3) (ii) {a/4, a – 2b) = (0, 6 + b)

Sol: (i) We have, (2a + b,a-b) = (8,3)

=> 2a + b = 8 and a – b = 3

On solving, we get a = 11/3 and b = 2/3

(ii) We have, $\left(\frac{a}{4}, a-2b\right) = (0, 6+b)$

$\Rightarrow \quad \frac{a}{4} = 0 \Rightarrow a = 0$

and $\quad a - 2b = 6 + b$

$\Rightarrow \quad 0 - 2b = 6 + b$

$\Rightarrow \quad b = -2$

$\therefore \quad a = 0, b = -2$

Q5. Given A = {1,2,3,4, 5}, S= {(x,y) :x∈ A,y∈ A}.Find the ordered pairs which satisfy the conditions given below

x+y = 5 (ii) x+y<5 (iii) x+y>8

Sol: We have, A = {1,2, 3,4, 5}, S= {(x,y) : x ∈ A,y∈ A}

(i) The set of ordered pairs satisfying x + y= 5 is {(1,4), (2,3), (3,2), (4,1)}

(ii) The set of ordered pairs satisfying x+y < 5 is {(1,1), (1,2), (1,3), (2, 1), (2,2), (3,1)}

(iii) The set of ordered pairs satisfying x +y > 8 is {(4, 5), (5,4), (5, 5)}.

Q6. Given R = {(x,y) : x,y ∈ W, $x^2 + y^2$ = 25}. Find the domain and range of R

Sol: We have, R = {(x,y):x,y∈ W, $x^2 + y^2 = 25$}

= {(0,5), (3,4), (4, 3), (5,0)}

Domain of R = Set of first element of ordered pairs in R = {0,3,4, 5}

Range of R = Set of second element of ordered pairs in R = {5,4, 3, 0}

Q7. If R_1 = {(x, y)| y = 2x + 7, where x∈ R and -5 ≤ x ≤ 5} is a relation. Then find the domain and range of R_1.

Sol: We have, R_1 = {(x, y)|y = 2x + 7, where x∈ R and -5 ≤x ≤ 5}

Domain of R_1 = {-5 ≤ x ≤ 5, x ∈ R} = [-5, 5]

x ∈ [-5, 5]

=> 2x ∈ [-10,10]

=>2x + 7∈ [-3, 17]

Range is [-3, 17]

Q8. If R_2 = {(x, y) | x and y are integers and $x^2 + y^2$ = 64} is a relation. Then find R_2

Sol: We have, R_2 = {(x, y) | x and y are integers and $x^2 + y^2 – 64$}

Clearly, $x^2 = 0$ and $y^2 = 64$ or $x^2 = 64$ and$y^2 = 0$

x = 0 and y = ±8

or x = ±8 and y = 0

R_2 = {(0, 8), (0, -8), (8,0), (-8,0)}

Q9. If R_3 = {(x, |x|) | x is a real number} is a relation. Then find domain and range

Sol: We have, R_3 = {(x, |x)) | x is real number}

Clearly, domain of R_3 = R

Now, $x \in R$ and $|x| \geq 0$.

Range of R_3 is $[0,\infty)$

Q10. Is the given relation a function? Give reasons for your answer.

(i) h={(4,6), (3,9), (-11,6), (3,11)}

(ii) f = {(x, x) | x is a real number}

(iii) g = {(n, 1 ln)| nis a positive integer}

(iv) s= {(n, n^2) | n is a positive integer}

(v) t= {(x, 3) | x is a real number}

Sol: (i) We have, h = {(4,6),(3,9), (-11,6), (3,11)}.

Since pre-image 3 has two images 9 and 11, it is not a function.

(ii) We have, f = {(x, x) | x is a real number}

Since every element in the domain has unique image, it is a function.

(iii) We have, g= {(n, 1/n) | nis a positive integer}

For n, it is a positive integer and 1/n is unique and distinct. Therefore,every element in the domain has unique image. So, it is a function.

(iii) We have, s = {(n, n^2) | n is a positive integer}

Since the square of any positive integer is unique, every element in the domain has unique image. Hence, ibis a function.

(iv) We have, t = {(x, 3)| x is a real number}.

Since every element in the domain has the image 3, it is a constant function.

Q11. If f and g are real functions defined byf(x) = x^2 + 7 and g(x) = 3x + 5, find each of the following

(i) $f(3) + g(-5)$ **(ii)** $f(1/2) \times g(14)$

(iii) $f(-2) + g(-1)$ **(iv)** $f(t) - f(-2)$

(v) $\dfrac{f(t) - f(5)}{t-5}$, if $t \neq 5$

Sol. Given that, f and g are real functions defined by $f(x) = x^2 + 7$ and $g(x) = 3x + 5$.

(i) $f(3) = (3)^2 + 7 = 9 + 7 = 16$

and $g(-5) = 3(-5) + 5 = -15 + 5 = -10$

$\therefore \quad f(3) + g(-5) = 16 - 10 = 6$

(ii) $f(1/2) = (1/2)^2 + 7 = (1/4) + 7 = 29/4$
and $g(14) = 3(14) + 5 = 42 + 5 = 47$
$\therefore \quad f(1/2) \times g(14) = (29/4) \times 47 = 1363/4$

(iii) $f(-2) = (-2)^2 + 7 = 4 + 7 = 11$
and $g(-1) = 3(-1) + 5 = -3 + 5 = 2$
$\therefore \quad f(-2) + g(-1) = 11 + 2 = 13$

(iv) $f(t) = t^2 + 7$ and $f(-2) = (-2)^2 + 7 = 4 + 7 = 11$
$\therefore \quad f(t) - f(-2) = t^2 + 7 - 11 = t^2 - 4$

(v) $f(t) = t^2 + 7$ and $f(5) = 5^2 + 7 = 25 + 7 = 32$
$\therefore \quad \frac{f(t) - f(5)}{t-5}$, if $t \neq 5$

$$= \frac{t^2 + 7 - 32}{t - 5}$$

$$= \frac{t^2 - 25}{t - 5} = \frac{(t-5)(t+5)}{(t-5)} = t + 5 \quad [\because t \neq 5]$$

Q12. Let f and g be real functions defined by f(x) = 2x+ 1 and g(x) = 4x – 7.

(i) For what real numbers x,f(x)= g(x)?

(ii) For what real numbers x,f (x) < g(x)?

Sol: We have,f(x) = 2x + 1 and g(x) = 4x-7

(i) Now f (x) = g(x)

=> 2x+l=4x-7

=> 2x = 8 =>x = 4

(ii) f (x) < g(x)

=> 2x + 1 < 4x – 7

=> 8 < 2x

=> x > 4

Q13. If f and g are two real valued ftmctions defined as f(x) = 2x + 1, g(x) = x^2 + 1, then find.

(i) $f + g$ (ii) $f - g$ (iii) fg (iv) $\frac{f}{g}$

Sol. We have, $f(x) = 2x + 1$ and $g(x) = x^2 + 1$

(i) $(f + g)(x) = f(x) + g(x)$
$= 2x + 1 + x^2 + 1 = x^2 + 2x + 2$

(ii) $(f - g)(x) = f(x) - g(x)$
$= (2x + 1) - (x^2 + 1) = 2x + 1 - x^2 - 1 = 2x - x^2$

(iii) $(fg)(x) = f(x) \cdot g(x)$
$= (2x + 1)(x^2 + 1) = 2x^3 + 2x + x^2 + 1 = 2x^3 + x^2 + 2x + 1$

(iv) $\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{2x+1}{x^2+1}$

Q14. Express the following functions as set of ordered pairs and determine their range.

f:X->R,f{x) = x^3 + 1, where X= {-1,0, 3, 9, 7}

Sol: We have, f:X→ R,flx) = x^3 + 1.

Where X = {-1, 0, 3, 9, 7}

Now f (-l) = $(-l)^3$+1 =-l + 1 =0

f(0) = $(0)^3$+l=0+l = l

f(3) = $(3)^3$ + 1 = 27 + 1 = 28

f(9) = $(9)^3$ + 1 = 729 + 1 = 730

f(7) = $(7)^3$ + 1 = 343 + 1 = 344

f= {(-1, 0), (0, 1), (3, 28), (9, 730), (7, 344)}

Range of f= {0, 1, 28, 730, 344}

Q15. Find the values of x for which the functions f(x) = 3x ^{2} -1 and g(x) = 3+ x are equal.

Sol: f(x) = g(x)

=> $3x^2$-l=3+x => $3x^2$-x-4 = 0 => (3x − 4)(x+ 1) − 0

x= -1,4/3

Q16. Is g = {(1, 1), (2, 3), (3, 5), (4, 7)} a function? Justify. If this is described by the relation, g(x) = x +, then what values should be assigned to and ?

Sol: We have, g = {(1, 1), (2, 3), (3, 5), (4,7)}

Since, every element has unique image under g. So, g is a function.

Now, g(x) = x + For (1,1), g(l) = a(l) + P

=> l = + (i)

For (2, 3), g(2) = (2) +

=> 3 = 2 + (ii)

On solving Eqs. (i) and (ii), we get = 2, = -l

f(x) = 2x-1

Also, (3, 5) and (4, 7) satisfy the above function.

Q17. Find the domain of each of the following functions given by

(i) $f(x) = \dfrac{1}{\sqrt{1-\cos x}}$

(ii) $f(x) = \dfrac{1}{\sqrt{x+|x|}}$

(iii) $f(x) = x|x|$

(iv) $f(x) = \dfrac{x^3 - x + 3}{x^2 - 1}$

(v) $f(x) = \dfrac{3x}{28 - x}$

Sol. (i) We have, $f(x) = \dfrac{1}{\sqrt{1-\cos x}}$

Now $-1 \le \cos x \le 1$

$\Rightarrow \quad -1 \le -\cos x \le 1$

$\Rightarrow \quad 0 \le 1 - \cos x \le 1$

So, $f(x)$ is defined, if $1 - \cos x \ne 0$

$\therefore \quad \cos x \ne 1$

$\therefore \quad x \ne 2n\pi, n \in Z$

$\therefore$ **Domain of f is $R - \{2n\pi : n \in Z\}$**

(ii) We have, $f(x) = \dfrac{1}{\sqrt{x+|x|}}$

If $x > 0, x + |x| = x + x = 2x > 0$

If $x < 0, x + |x| = x - x = 0$

Clearly, $x = 0$ is not possible.

$\therefore$ **Domain of $f = R^+$**

(iii) We have, $f(x) = x|x|$

We know that 'x' and '$|x|$' are defined for all real values.

Clearly, $f(x)$ is defined for and $x \in R$.

$\therefore$ **Domain of $f = R$**

(iv) We have, $f(x) = \dfrac{x^3 - x + 3}{x^2 - 1}$

$f(x)$ is not defined, if $x^2 - 1 = 0$

$\Rightarrow \quad (x - 1)(x + 1) = 0$

$\Rightarrow \quad x = -1, 1$

$\therefore \quad$ Domain of $f = R - \{-1, 1\}$

(v) We have, $f(x) = \dfrac{3x}{28 - x}$

Clearly, $f(x)$ is not defined, if $28 - x = 0$

$\Rightarrow \quad x \neq 28$

$\therefore \quad$ Domain of $f = R - \{28\}$

Q18. Find the range of the following functions given by

(i) $f(x) = \dfrac{3}{2 - x^2}$ (ii) $f(x) = 1 - |x - 2|$

(iii) $f(x) = |x - 3|$ (iv) $f(x) = 1 + 3 \cos 2x$

Sol. (i) We have, $f(x) = \dfrac{3}{2 - x^2} = y$ (let)

$\Rightarrow \quad 2 - x^2 = \dfrac{3}{y} \quad \Rightarrow \quad x^2 = 2 - \dfrac{3}{y}$

Since $x^2 \geq 0$, $2 - \dfrac{3}{y} \geq 0$

$\Rightarrow \quad \dfrac{2y - 3}{y} \geq 0$

$\Rightarrow \quad 2y - 3 \geq 0$ and $y > 0$ or $2y - 3 \leq 0$ and $y < 0$

$\Rightarrow \quad y \geq 3/2$ or $y < 0$

$\Rightarrow \quad y \in (-\infty, 0) \cup [3/2, \infty)$

$\therefore \quad$ Range of f is $(-\infty, 0) \cup [3/2, \infty)$

(ii) We know that, $|x - 2| \geq 0$

$\Rightarrow \quad -|x - 2| \leq 0$

$\Rightarrow \quad 1 - |x - 2| \leq 1$

$\Rightarrow \quad f(x) \leq 1$

$\therefore \quad$ Range of f is $(-\infty, 1]$

(iii) We know that, $|x - 3| \geq 0$

$\Rightarrow \quad f(x) \geq 0$

$\therefore \quad$ Range of $f = [0, \infty)$

(iv) We know that, $-1 \leq \cos 2x \leq 1$

$\Rightarrow \quad -3 \leq 3 \cos 2x \leq 3$

$\Rightarrow \quad -2 \leq 1 + 3 \cos 2x \leq 4$

$\Rightarrow \quad -2 \leq f(x) \leq 4$

$\therefore \quad$ Range of $f = [-2, 4]$

Q19. Redefine the function f(x) = |x-2| + |2+x| , -3 ≤x ≤3

Sol. $f(x)=\begin{cases}-(x-2)-(2+x), & -3\le x<-2\\ -(x-2)+(2+x), & -2\le x<2\\ (x-2)+(2+x), & 2\le x\le 3\end{cases}$

$=\begin{cases}-2x, & -3\le x<-2\\ 4, & -2\le x<2\\ 2x, & 2\le x\le 3\end{cases}$

When $-3\le x\le -2,\ 4\le -2x\le 6$

When $2\le x\le 3,\ 4\le 2x\le 6$

Thus range is [4, 6].

20. If $f(x)=\frac{x-1}{x+1}$, then show that

(i) $f\left(\frac{1}{x}\right)=-f(x)$ (ii) $f\left(-\frac{1}{x}\right)=\frac{-1}{f(x)}$

Sol. We have, $f(x)=\frac{x-1}{x+1}$

(i) $f\left(\frac{1}{x}\right)=\frac{\frac{1}{x}-1}{\frac{1}{x}+1}=\frac{\frac{1-x}{x}}{\frac{1+x}{x}}=\frac{1-x}{1+x}=-f(x)$

(ii) $f\left(-\frac{1}{x}\right)=\frac{-\frac{1}{x}-1}{-\frac{1}{x}+1}=\frac{-1-x}{-1+x}=\frac{1+x}{1-x}=\frac{-1}{f(x)}$

$\therefore\quad f\left(-\frac{1}{x}\right)=-\frac{1}{f(x)}$

Q21. Let f (x) = √x and g(x) = xbe two functions defined in the domain R^{+} ∪ {0}. Find

(i) (f+g)(x)

(ii) (f-g)(x)

(iii) (fg)(x)

(iv) f/g(x)

Sol. We have, $f(x) = \sqrt{x}$ and $g(x) = x$ be two function defined in the domain $R^+ \cup \{0\}$

(i) $(f+g)(x) = f(x) + g(x) = \sqrt{x} + x$

(ii) $(f-g)(x) = f(x) - g(x) = \sqrt{x} - x$

(iii) $(fg)(x) = f(x).g(x) = \sqrt{x} \cdot x = x^{\frac{3}{2}}$

(iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}$

22. Find the domain and Range of the function $f(x) = \frac{1}{\sqrt{x-5}}$.

Sol. We have, $f(x) = \frac{1}{\sqrt{x-5}}$

Clearly, $f(x)$ is defined, if $x - 5 > 0 \Rightarrow x > 5$

Thus, domain of f is $(5, \infty)$.

For $x - 5 > 0$, $\sqrt{x-5} > 0$

$$\therefore \quad \frac{1}{\sqrt{x-5}} > 0$$

Hence, range of f is $(0, \infty)$

Q23. If f(x)= y = ax-b/ cx-a then prove that f (y) = x

Sol. We have, $f(x) = y = \frac{ax-b}{cx-a}$

$$\therefore \quad f(y) = \frac{ay-b}{cy-a} = \frac{a\left(\frac{ax-b}{cx-a}\right) - b}{c\left(\frac{ax-b}{cx-a}\right) - a}$$

$$= \frac{a(ax-b) - b(cx-a)}{c(ax-b) - a(cx-a)}$$

$$= \frac{a^2x - ab - bcx + ab}{acx - bc - acx + a^2} = \frac{a^2x - bcx}{a^2 - bc} = \frac{x(a^2-bc)}{(a^2-bc)}$$

$$\therefore \quad f(y) = x$$

Objective Type Questions

Q24. Let n(A) = m, and n(B) = n. Then the total number of non-empty relations that can be defined from A to B is

(a) m^n

(b) $n^m - 1$

(c) mn – 1

(d) $2^{mn} - 1$

Sol: (d) We have, n(A) = m and n(B) = n

n(A xB) = n(A). n(B) = mn

Total number of relation from A to B = Number of subsets of AxB = 2^{mn}

So, total number of non-empty relations = $2^{mn} - 1$

Q25. If $[x]^2 - 5[x] + 6 = 0$, where [.] denote the greatest integer function, then

(a) $x \in [3,4]$

(b) $x \in (2, 3]$

(c) $x \in [2, 3]$

(d) $x \in [2, 4)$

Sol: (d) We have $[x]^2 - 5[x] + 6 = 0 \Rightarrow [(x - 3)([x] - 2) = 0$

$\Rightarrow [x] = 2,3$.

For $[x] = 2, x \in [2, 3)$

For $[x] = 3, x \in [3,4)$

$x \in [2, 3) \cup [3,4)$

Or $x \in [2,4)$

26. Range of $f(x) = \dfrac{1}{1-2\cos x}$ is

(a) $\left[\frac{1}{3}, 1\right]$ (b) $\left[-1, \frac{1}{3}\right]$

(c) $(-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$ (d) $\left[-\frac{1}{3}, 1\right]$

Sol. **(c)** We know that, $-1 \le \cos x \le 1$

$\Rightarrow \quad -1 \le -\cos x \le 1$

$\Rightarrow \quad -2 \le -2\cos x \le 2$

$\Rightarrow \quad -1 \le 1 - 2\cos x \le 3$

Now $f(x) = \dfrac{1}{1-2\cos x}$ is defined if

$-1 \le 1 - 2\cos x < 0$ or $0 < 1 - 2\cos x \le 3$

$$\Rightarrow \quad -1 \ge \frac{1}{1-2\cos x} > -\infty \text{ or } \infty > \frac{1}{1-2\cos x} \ge \frac{1}{3}$$

$$\Rightarrow \quad \frac{1}{1-2\cos x} \in (-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$$

27. Let $f(x) = \sqrt{1+x^2}$, then

(a) $f(xy) = f(x) \times f(y)$ (b) $f(xy) \ge f(x) \times f(y)$

(c) $f(xy) \le f(x) \times f(y)$ (d) None of these

Sol. **(c)** We have, $f(x) = \sqrt{1+x^2}$

$$f(xy) = \sqrt{1+x^2y^2}$$

$$f(x)\cdot f(y) = \sqrt{1+x^2}\cdot\sqrt{1+y^2} = \sqrt{(1+x^2)(1+y^2)} = \sqrt{1+x^2+y^2+x^2y^2}$$

Now $\sqrt{1+x^2y^2} \le \sqrt{1+x^2+y^2+x^2y^2}$

$\Rightarrow \quad f(xy) \le f(x) \times f(y)$

28. Domain of $\sqrt{a^2-x^2}$ $(a>0)$ is

(a) $(-a, a)$ (b) $[-a, a]$ (c) $[0, a]$ (d) $(-a, 0]$

Sol. **(b)** We have $f(x) = \sqrt{a^2-x^2}$

Clearly $f(x)$ is defined, if $a^2 - x^2 \ge 0$

$\Rightarrow \quad x^2 \le a^2$

$\Rightarrow \quad -a \le x \le a$ $\quad [\because a > 0]$

$\therefore$ Domain of f is $[-a, a]$

Q29. If fx) ax+ b, where a and b are integers,f(-1) = -5 and f(3) – 3, then a and b are equal to

(a) a = -3, b =-1

(b) a = 2, b =-3

(c) a = 0, b = 2

(d) a = 2, b = 3

Sol. **(b)** We have, $f(x) = ax + b$

$\therefore \quad f(-1) = a(-1) + b$

$\Rightarrow \quad -5 = -a + b$ $\quad$ (i)

Also, $\quad f(3) = a(3) + b$

$\Rightarrow \quad 3 = 3a + b$ $\quad$ (ii)

On solving Eqs. (i) and (ii), we get

$$a = 2 \text{ and } b = -3$$

30. The domain of the function f defined by $f(x) = \sqrt{4-x} + \dfrac{1}{\sqrt{x^2-1}}$ is equal to

(a) $(-\infty, -1) \cup (1, 4]$ (b) $(-\infty, -1] \cup (1, 4]$

(c) $(-\infty, -1) \cup [1, 4]$ (d) $(-\infty, -1) \cup [1, 4)$

Sol. **(a)** We have, $f(x) = \sqrt{4-x} + \dfrac{1}{\sqrt{x^2-1}}$

$f(x)$ is defined if $4 - x \ge 0$ and $x^2 - 1 > 0$

$\Rightarrow \quad x - 4 \le 0$ and $(x+1)(x-1) > 0$

$\Rightarrow \quad x \le 4$ and ($x < -1$ or $x > 1$)

$\therefore$ Domain of $f = (-\infty, -1) \cup (1, 4]$

31. The domain and range of the real function f defined by $f(x) = \dfrac{4-x}{x-4}$ is

(a) Domain $= R$, Range $= \{-1, 1\}$

(b) Domain $= R - \{1\}$, Range $= R$

(c) Domain $= R - \{4\}$, Range $= \{-1\}$

(d) Domain $= R - \{-4\}$, Range $= \{-1, 1\}$

Sol. **(c)** We have, $f(x) = \dfrac{4-x}{x-4} = -1$, for $x \ne 4$

32. The domain and range of real function f defined by $f(x) = \sqrt{x-1}$ is given by

(a) Domain = $(1, \infty)$, Range = $(0, \infty)$ (b) Domain = $[1, \infty)$, Range = $(0, \infty)$

(c) Domain = $[1, \infty)$, Range = $[0, \infty)$ (d) Domain = $[1, \infty)$, Range = $[0, \infty)$

Sol. **(d)** We have, $f(x) = \sqrt{x-1}$

Clearly, $f(x)$ is defined if $x - 1 \geq 0$

$\Rightarrow \quad x \geq 1$

$\therefore \quad$ Domain of $f = [1, \infty)$

Now for $x \geq 1$, $x - 1 \geq 0$

$\Rightarrow \quad \sqrt{x-1} \geq 1$

$\Rightarrow \quad$ Range of $f = [0, \infty)$

33. The domain of the function f given by $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - x - 6}$ is

(a) $R - \{3, -2\}$ (b) $R - \{-3, 2\}$ (c) $R - [3, -2]$ (d) $R - (3, -2)$

Sol. **(a)** We have, $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - x - 6}$

$f(x)$ is not defined, if $x^2 - x - 6 = 0$

$\Rightarrow \quad (x - 3)(x + 2) = 0$

$\therefore \quad x = -2, 3$

$\therefore \quad$ Domain of $f = R - \{-2, 3\}$

34. The domain and range of the function f given by $f(x) = 2 - |x - 5|$ is

(a) Domain = $R+$, Range = $(-\infty, 1]$ (b) Domain = R, Range = $(-\infty, 2]$

(c) Domain = R, Range = $(-\infty, 2)$ (d) Domain = $R+$, Range = $(-\infty, 2]$

Sol. **(b)** We have, $f(x) = 2 - |x - 5|$

Clearly, $f(x)$ is defined for all $x \in R$.

$\therefore \quad$ Domain of $f = R$

Now, $|x - 5| \geq 0, \forall x \in R$

$\Rightarrow \quad -|x - 5| \leq 0$

$\Rightarrow \quad 2 - |x - 5| \leq 2$

$\therefore \quad f(x) \leq 2$

$\therefore \quad$ Range of $f = (-\infty, 2]$

35. The domain for which the functions defined by $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal is

(a) $\left\{-1, \dfrac{4}{3}\right\}$ (b) $\left[-1, \dfrac{4}{3}\right]$ (c) $\left(-1, -\dfrac{4}{3}\right)$ (d) $\left[-1, -\dfrac{4}{3}\right)$

Sol. **(a)** We have, $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$

$f(x) = g(x)$

$\Rightarrow \quad 3x^2 - 1 = 3 + x \Rightarrow 3x^2 - x - 4 = 0 \Rightarrow (3x - 4)(x + 1) = 0$

$\therefore \quad x = -1, \dfrac{4}{3}$

Fill in the Blanks Type Questions

Q36. Let f and g be two real functions given by f= {(0, 1), (2,0), (3,-4), (4,2), (5, 1)} g= {(1,0), (2,2), (3,-1), (4,4), (5, 3)} then the domain of f x g is given by________ .

Sol: We have, f = {(0, 1), (2, 0), (3, -4), (4, 2), (5,1)} and g= {(1, 0), (2, 2), (3, 1), (4,4), (5, 3)}

Domain of f = {0,2, 3, 4, 5}

And Domain of g= {1, 2, 3,4, 5}

Domain of (f x g) = (Domain of f) ∩ (Domain of g) = {2, 3,4, 5}

Matching Column Type Questions

Q37. Let f= {(2,4), (5,6), (8, -1), (10, -3)} andg = {(2, 5), (7,1), (8,4), (10,13), (11, 5)} be two real functions. Then match the following:

Column I		Column II	
(a)	$f-g$	(i)	$\left\{\left(2,\frac{4}{5}\right),\left(8,\frac{-1}{4}\right),\left(10,\frac{-3}{13}\right)\right\}$
(b)	$f+g$	(ii)	{(2, 20), (8, –4), (10, –39)}
(c)	$f\times g$	(iii)	{(2, –1), (8, –5), (10, –16)}
(d)	$\frac{f}{g}$	(iv)	{(2, 9), (8, 3), (10, 10)}

Sol. Domain of $f(x)$ is {2, 5, 8, 10}.

Domain of $g(x)$ is {2, 7, 8, 10, 11}.

Thus, domain of $f \pm g, f\times g$ and f/g is {2, 8, 10}.

For function $y = f(x)$, we have $f(2) = 4, f(8) = -1$ and $f(10) = -3$

For function $y = g(x)$, we have $g(2) = 5$, $g(8) = 4$ and $g(10) = 13$

$$(f-g)(2) = f(2) - g(2) = 4 - 5 = -1$$
$$(f-g)(8) = f(8) - g(8) = -1 - 4 = -5$$
$$(f-g)(10) = f(10) - g(10) = -3 - 13 = -16$$

Thus, $(f-g)(x) = \{(2, -1), (8, -5), (10, -16)\}$

$$(f+g)(2) = f(2) + g(2) = 4 + 5 = 9$$
$$(f+g)(8) = f(8) + g(8) = -1 + 4 = 3$$
$$(f+g)(10) = f(10) + g(10) = -3 + 13 = 10$$

Thus, $(f+g)(x) = \{(2, 9), (8, 3), (10, 10)\}$

$$(f\cdot g)(2) = f(2)\cdot g(2) = 4 \cdot 5 = 20$$
$$(f\cdot g)(8) = f(8)\cdot g(8) = (-1)\cdot 4 = -4$$
$$(f\cdot g)(10) = f(10)\cdot g(10) = (-3)\cdot 13 = -39$$

Thus $(f\cdot g)(x) = \{(2, 20), (8, -4), (10, -39)\}$

$$(f/g)(2) = f(2)/g(2) = 4/5 = 4/5$$
$$(f/g)(8) = f(8)/g(8) = (-1)/4 = -1/4$$
$$(f/g)(10) = f(10)/g(10) = (-3)/13 = -3/13$$

Thus $(f/g)(x) = \{(2, 4/5), (8, -1/4), (10, -3/13)\}$

So, correct matching is: (a) – (iii), (b) – (iv), (c) – (ii) and (d) – (i)

True/False Type Questions

Q38. The ordered pair (5,2) belongs to the relation R ={(x,y): y = x – 5, x,y∈Z}

Sol: False

We have, R = {(x, y): y = x – 5, x, y ∈ Z}

When x = 5, then y = 5-5=0 Hence, (5, 2) does not belong to R.

Q39. If P = {1, 2}, then P x P x P = {(1, 1,1), (2,2, 2), (1, 2,2), (2,1, 1)}

Sol: False

We have, P = {1, 2} and n(P) = 2

n(P xPxP) = n(P) x n(P) x n(P) = 2 x 2 x 2
= 8 But given P x P x P has 4 elements.

Q40. If A= {1,2, 3}, 5= {3,4} and C= {4, 5, 6}, then (A x B) ∪ (A x C) = {(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3,4), (3, 5), (3,6)}.

Sol: True

We have.4 = {1,2, 3}, 5= {3,4} andC= {4,5,6}

AxB= {(1, 3), (1,4), (2, 3), (2,4), (3, 3), (3,4)}

And A x C = {(1,4), (1, 5), (1, 6), (2,4), (2, 5), (2, 6), (3,4), (3, 5), (3, 6)}

(A x B)∪(A xC)= {(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3,3), (3,4), (3, 5), (3,6)}

41. If $(x-2, y+5) = \left(-2, \frac{1}{3}\right)$ are two equal ordered pairs, then $x = 4, y = \frac{-14}{3}$.

Sol. False

We have, $(x-2, y+5) = \left(-2, \frac{1}{3}\right)$

$$\Rightarrow \quad x - 2 = -2, y + 5 = \frac{1}{3}$$

$$\Rightarrow \quad x = 0, y = \frac{-14}{3}$$

Q42. If Ax B= {(a, x), (a, y), (b, x), (b, y)}, thenM = {a, b},B= {x, y}.

Sol: True

We have, AxB= {{a, x), {a, y), (b, x), {b, y)}

A = Set of first element of ordered pairs in A x B = {a, b}

B = Set of second element of ordered pairs in A x B = {x, y}