Chapter 2. Relations and Functions

Question-1

Find x and y, if (2x, x+y) = (6, 2).

Solution:

$2x = 6$

$\therefore x = 3$

$x + y = 2$

$3 + y = 2$

$\therefore y = -1$

Question-2

Find the domain of the following function : $f(x) = x|x|$

Solution:

The domain of the function $f(x) = x|x|$ is R.

Question-3

Let A = {a, b, c} and B = {p, q}. Find

(i) A × B
(ii) B × A
(iii) A × A
(iv) B × B

Solution:

(i) A × B = {(a,p), (a,q), (b,p), (b,q), (c,p), (c,q)}
(ii) B × A = {(p,a), (q,a), (p,b), (q,b), (p,c), (q,c)}
(iii) A × A = {(a,a), (a,b), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c)}
(iv) B × B = {(p,p), (p,q), (q,p), (q,q)}

Question-4

Find the domain of the following function : $f(x) = \frac{1}{\sqrt{x+|x|}}$

Solution:

$x+|x| = 0$ for $x < 0$ or $x = 0$.

$\therefore$ The domain of the function $f(x) = \frac{1}{\sqrt{x+|x|}}$ is $(0, \infty)$

Question-5

Let A = {1, 2, 3}, B = {2, 3, 4} and C = {4, 5}. Verify that

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Solution:

(i) L.H.S = $A \times (B \cap C) = \{1, 2, 3\} \times \{4\} = \{(1,4), (2,4), (3,4)\}$

R.H.S = $(A \times B) \cap (A \times C)$

$= \{(1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\} \cap \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)\}$

$= \{(1,4), (2,4), (3,4)\}$ $\therefore A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) L.H.S = $A \times (B \cup C)$

$= \{1, 2, 3\} \times \{2, 3, 4, 5\}$

$= \{(1,2), (1, 3), (1, 4), (1,5), (2, 2), (2,3), (2,4), (2,5), (3,2), (3,3), (3,4), (3,5)\}$

R.H.S = $(A \times B) \cup (A \times C)$

$= \{(1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\} \cup \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)\}$

$= \{(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5) (3,2), (3,3),(3,4), (3,5)\}$

$\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$

Question-6

If R is the relation "less than" from A = {1, 2, 3, 4, 5} to B = {1, 4, 5}, write down the set of ordered pairs corresponding to R. Find the inverse relation to R.

Solution:

$R = \{(x, y) / x \in A, y \in B \text{ and } x < y\}$

$= \{(1,4),(1, 5),(2,4), (2,5), (3,4), (3,5), (4,5)\}$

$\therefore$ Inverse relation corresponds to the Cartesian product {(4,1),(5, 1),(4,2), (5,2), (4,3), (5,3), (5,4)} and corresponds to the relation 'greater than' from B to A.

Question-7

Prove that $A \cap (B - C) = (A \cap B) - (A \cap C)$.

Solution:

Let $x \in A \cap (B - C)$

$\Rightarrow x \in A$ and $x \in (B - C)$

$\Rightarrow x \in A$ and $\{x \in B$ and $x \notin C\}$

$\Rightarrow x \in A$ and $\{x \in B$ and $x \notin C\}$

$\Rightarrow x \in A$ and $x \in B$ or $x \in A$ and $x \notin C$

$\Rightarrow (A \cap B) - (A \cap C)$

Question-8

If A= {1, 2, 3} , B = {4}, C = {5}, then verify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B - C) = (A \times B) - (A \times C)$

Solution:

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

$A \times (B \cup C) = \{1, 2, 3\} \times \{4, 5\}$

$= \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$

$(A \times B) \cup (A \times C) = \{(1, 4), (2, 4), (3, 4)\} \cup \{(1, 5), (2,5), (3, 5)\}$

$= \{(1, 4), (2, 4), (3, 4), (1, 5), (2,5), (3, 5)\}$

$\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B - C) = (A \times B) - (A \times C)$

$A \times (B - C) = \{1, 2, 3\} \times \{4\}$

$= \{(1,4), (2,4), (3,4)\}$

$(A \times B) - (A \times C) = \{(1,4), (2,4), (3,4)\} - \{(1, 5), (2, 5), (3, 5)\}$

$= \{(1,4), (2,4), (3,4)\}$

Question-9

If R is the relation in N x N defined by (a,b) R (c,d) if and only if a + d = b + c, show that R is an equivalence relation.

Solution:

Reflexive

$(a,b)\ R\ (a,b) \Leftrightarrow a + b = b + a$ for $a,b \in N$

$\Leftrightarrow b + a = a + b$ (Transposing)

$\Leftrightarrow (a,b)\ R\ (a,b)$ for $a,b \in N$

$\therefore (a,b)\ R\ (a,b) \Leftrightarrow (a,b)\ R\ (a,b)$ for $a,b \in N$

Symmetric

If $(a,b)\ R\ (c,d) \Leftrightarrow a + d = b + c$ for $a,b,c,d \in N$

$\Leftrightarrow b + c = a + d$ (transposing)

$\Leftrightarrow (c,d)\ R\ (a,b)$ for $a,b,c,d \in N$

$\therefore (a,b)\ R\ (c,d) \Leftrightarrow (c,d)\ R\ (a,b)$ for $a,b,c,d \in N$

Transitive

If $(a,b)\ R\ (c,d) \Leftrightarrow a + d = b + c$ for $a,b,c,d \in N$

and $(c,d)\ R\ (e,f) \Leftrightarrow c + f = d + e$ for $c,d,e,f \in N$

then $(a,b)\ R\ (c,d) \Leftrightarrow a + d = b + c$ for $a,b,c,d \in N$

$\Leftrightarrow a + d + e + f = b + c + e + f$

$\Leftrightarrow a + (d + e) + f = b + c + e + f$ (since $c + f = d + e$)

$\Leftrightarrow a + \cancel{c} + f + \cancel{f} = b + \cancel{c} + e + \cancel{f}$

$\Leftrightarrow a + f = b + e$

$\Leftrightarrow (a,b)\ R\ (e,f)$ for $a,b,e,f \in N$

$\therefore (a,b)\ R\ (c,d) \Leftrightarrow (a,b)\ R\ (e,f)$ for $a,b,c,d,e,f \in N$

$\therefore$ the relation defined by (a,b) R (c,d) if and only if a + d = b + c is an equivalence relation.

Question-10

Find the domain of the following function : $f(x) = \frac{x}{x^2 - 3x + 2}$

Solution:

$x^2-3x+2 = 0$ for $x = 2,1$. $\therefore$ The domain of the function : $f(x) = \frac{x}{x^2 - 3x + 2}$ is R- {1,2}.

Question-11

Let A = {1, 2, 3, 4} and S = {(a, b): a ∈ A, b ∈ A, a divides b}. Write S explicitly.

Solution:

S = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)}

Question-12

Find the domain of the following function : f(x) = $e^{x+\sin x}$

Solution:

The domain of the function : f(x) = $e^{x+\sin x}$ is R .

Question-13

Find the domain of the following function : f(x) = $\frac{x+7}{x^2-8x+4}$

Solution:

$x^2 - 8x+4 = 0$ for $x = \frac{8 \pm \sqrt{64-16}}{2} = \frac{8 \pm \sqrt{48}}{2} = \frac{8 \pm 4\sqrt{3}}{2} = 4 \pm 2\sqrt{3}$

The domain of the function: f(x) = $\frac{x+7}{x^2-8x+4}$ is $R - \left\{4 \pm 2\sqrt{3}\right\}$

Question-14

Let A = {1, 2} and B = {3, 4}. Write all subsets of A × B.

Solution:

A × B = {(1,3), (1,4), (2,3), (2,4)}.

∴ The subsets of A × B are φ , {(1,3)}, {(1,4)}, {(2,3)}, {(2,4)},

{(1,3), (1,4)}, {(1,3), (2,3)}, {(1,3), (2,4)}, {(1,4), (2,3)}, {(1,4), (2,4)}, {(2,3), (2,4)},

{(1,3), (1,4), (2,3)}, {(1,3), (1,4), (2,4)}, {(1,3), (2,3), (2,4)}, {(1,4), (2,3), (2,4)},

{(1,3), (1,4), (2,3), (2,4)}.

Question-15

Find the domain of the following function : f(x) = [x] + x

Solution:

The domain of the function f(x) = [x] + x is R.

Question-16

Let A and B be two sets such that n(A) = 3 and n(B) = 2. If (x, 1), (y, 2), (z, 1) are in A × B, find A and B, where x, y, z are distinct elements.

Solution:

A = {x, y, z} and B = {1, 2}

Question-17

Find the domain of the following function : f(x) = $\frac{\sin^{-1} x}{x}$

Solution:

The domain of the function : f(x) = $\frac{\sin^{-1} x}{x}$ is $[-1,1]-\{0\}$

Question-18

Let A = {1,2}, B = {1, 2, 3, 4}, C = {5, 6} and D= {5, 6, 7, 8}. Verify that A × C ⊂ B × D.

Solution:

A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}

B × D = {(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}

∴ A × C ⊂ B × D.

Question-19

Find the range of each of the following function: f(x) = $|x-3|$

Solution:

f(x) =|x-3| is positive for all values of x in R.

The range of the function f(x) = $|x-3|$ is(0, ∞).

Question-20

Let A be a non empty set such that A × B = A × C. show that B = C.

Solution:

Let a ∈ A. Since B ≠ φ , there exists b ∈ B. Now, (a, b) ∈ A × B = A × C implies b ∈ C.

∴ every element in B is in C giving B ⊂ C. Similarly, C ⊂ B. Hence B = C.

Question-21

Find the range of the following function: $f(x) = 1 - |x-2|$

Solution:

$|x-2| \geq 0 \Rightarrow 1-|x-2| \leq 1$

The range of the function $f(x) = 1-|x-2|$ is $(-\infty,1)$

Question-22

Find the range of the following function: $f(x) = \frac{|x-4|}{x-4}$

Solution:

$f(x) = \frac{|x-4|}{x-4} = 1$, if $x - 4 > 0$

$= -1$, if $x - 4 < 0$

$\therefore$ *The range of the function* $f(x) = \frac{|x-4|}{x-4}$ *is* $(-1,1)$

Question-23

Let A = {1, 2, 3, 4} and B = {x, y, z}. Let R be a relation from A to B defined by R = {(1, x), (1, z), (3, x), (4, y)}. Find the domain and range of R.

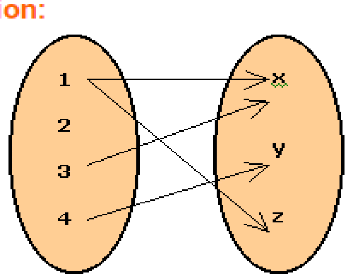
Solution:

Domain of R = {1, 3, 4} and Range R = {x, y, z} = B

Question-24

Let A = {1, 2, 3, 4} and B = {x, y, z}.
Let R be a relation from A to B defined by R = {(1, x), (1, z), (3, x), (4, y)}.
Draw the arrow diagram of relation R.

Solution:



Question-25

Find the range of the following function: $f(x) = \sqrt{16 - x^2}$

Solution:

The range of the function : $f(x) = \sqrt{16-x^2}$ is $[0,4]$.

Question-26

Find the range of the following function: f(x)= $\frac{1}{\sqrt{x-5}}$

Solution:
The range of the function is (0, ∞)

Question-27

In N×N, show that the relation defined by (a,b)R(c,d) if and only if ad = bc is an equivalence relation.

Solution:
<u>Reflexive</u>
(a,b)R(a,b) Û ab = ba for a,bÎN
Û ba = ab (Transposing)
Û (a,b)R(a,b) for a,bÎN

∴ (a,b)R(a,b) ⇔ (a,b)R(a,b) for a,bÎN.

<u>Symmetric</u>
(a,b)R(c,d) Û ad = bc a,b,c,dÎN
Û bc = ad (Transposing)
Û (c,d)R(a,b) for a,b,c,dÎN

∴ (a,b)R(c,d) ⇔ (c,d)R(a,b) for a,b,c,dÎN

<u>Transitive</u>
If (a,b)R(c,d) ⇔ ad = bc a,b,c,dÎN
and (c,d)R(e,f) ⇔ c = de c,d,e,fÎN

Then (a,b)R(c,d) ⇔ ad = bc a,b,c,dÎN
Û adef = bcef (Multiplying both sides by ef)
Û adef = be(cf)
Û adef = bede (Since cf = de)
Û af = be
Û (a,b) R (e,f) a,b,e,fÎN

∴ (a,b)R(c,d) Û (a,b) R (e,f) a,b,e,fÎN

∴ the relation defined by (a,b)R(c,d) if and only if ad = bc is an equivalence relation.

Question-28

Find the domain and the range of the following function : f(x) = $\frac{1}{\sqrt{x-[x]}}$

Solution:

We know that $0 \le x - [x] \le 1$ for all $x \in R$. Also, $x - [x] = 0$ for $x \in Z$.

$f(x) = \frac{1}{\sqrt{x-[x]}}$ is defined if $x - [x] > 0$

i.e., $x \in R - Z$.

Hence the domain of the function is R − Z.

Question-29

Find the domain and the range of the following function : f(x)= $\frac{1}{\sqrt{4+3\sin x}}$

Solution:

$-1 \le \sin x \le 1 \Rightarrow -3$ £ $3 \sin x \le 3$

i.e $-1 \le 4 + 3 \sin x \le 7$

$\therefore \frac{1}{\sqrt{7}} \le y \le 1$

The domain of the function is R; Range : $\frac{1}{\sqrt{7}} \le y \le 1$

Question-30

Let R be the relation on Z defined by a R b if and only if a − b is an even integer. Find (i) R, (ii) domain R, (iii) range of R.

Solution:

(i) R = {(a, b): a and b are even integers} ∪ {(c, d) : c and d are odd integers}

(ii) Domain = Z

(iii) Range = Z

Question-31

Find the domain and the range of the following function : f(x) = 1-|x−3|

Solution:

The domain of the function is R ; Range : (- ∝ ,1)

Question-32

Let R be the relation on Z defined by R = {(a, b): a ∈ Z, b ∈ Z, $a^2 = b^2$}. Find (i) R, (ii) domain R, (iii) range of R.

Solution:

(i) R = {(a, a): a ∈ Z} ∪ {(a, -a): a ∈ Z}

(ii) Domain = Z

(iii) Range = Z

Question-33

Find the domain and the range of the following function : f(x) = x!

Solution:

The domain of the function is N ∪ {0}; Range : {n! : n = 0,1,2…..}

Question-34

Determine the domain and the range of the relation R defined by R = {(x+1, x+5): x ∈ {0,1 ,2, 3, 4,5}}

Solution:

Domain = {1, 2, 3, 4, 5, 6}, Range = {5, 6, 7, 8, 9 ,10}

Question-35

Determine the domain and the range of the relation R, where R = {(x, x^3) : x is a prime number less than 10}.

Solution:

Domain = {2, 3, 5, 7}, Range = {8, 27, 125, 343}

Question-36

Find the domain and the range of the following function : f(x) = $\sin^2(x^3) + \cos^2(x^3)$

Solution:

$\sin^2(x^3) + \cos^2(x^3) = 1$

∴ The domain of the function is R ; Range : {1}

Question-37

Is inclusion of a subset in another, i.e., ARB if and only if $A \subset B$, in the context of a universal set, an equivalence relation in the class of subsets of the universal set? Justify your answer.

Solution:

Let U be the universal set .Let R be the relation ' is a subset of' or 'is included in ' between the subsets of U.

Since every set is a subset of itself i.e., for every subset A in U, $A \subseteq A$ or A R A.

$\therefore$ R is reflexive.

Now let A and B be two subsets of U such that $A \subseteq B$, then it is not necessary that B must also be a subset of A.

$\therefore$ A R B need not imply B R A.

$\therefore$ R is not symmetric.

Hence R is not an equivalence relation.

Question-38

Find the domain and the range of the following function : $f(x) = \frac{x^2-9}{x-3}$

Solution:

$f(x) = \frac{x^2-9}{x-3} = x+3$

The domain of the function is R ; Range : R .

Question-39

Determine the domain and range of the following relations

(i) {(1, 2), (1, 4), (1, 6), (1, 8)}

(ii) $\{(x, y) : x \in N, y \in N$ and $x + y = 10\}$

(iii) $\{(x, y) : x \in N, x<5, y = 3\}$

(iv) $\{(x, y) : y = |x - 1|, x \in Z$ and $|x| \leq 3\}$

Solution:

(i) Domain = {1}, Range = {2, 4, 6, 8}

(ii) Domain = {1, 2, 3, 4, 5, 6, 7, 8, 9}, Range = {9, 8, 7, 6, 5 ,4, 3, 2, 1}

(iii) Domain = {1, 2, 3, 4}, Range = {3}

(iv) Domain = {-3, -2, -1, 0, 1, 2, 3}, Range = {4, 3, 2, 1, 0}

Question-40

How many relations are possible from a set A of m elements to another set B of n elements? Why?

Solution:

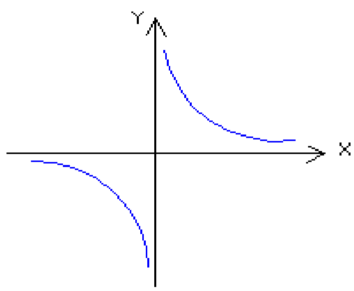
Number of elements in A = m.
Number of elements in B = n
∴ Number of elements in A×B = mn
Number of subsets of A×B = 2^{mn}

Since every subset of A×B is a relation from A to B therefore 2^{mn} relations are possible from A to B.

Question-41

Draw the graph of the following function: $f(x) = \frac{1}{x}, x \neq 0$

Solution:



Question-42

Let A = {1, 2}. List all the relations on A.

Solution:

$A \times A = \{(1,1),(1,2),(2,1),(2,2)\}$

The relations on A are $\phi, \{(1,1)\}, \{(1,2)\}, \{(2,1)\}, \{(2,2)\},$

$\{(1,1),(1,2)\}, \{(1,1),(2,1)\}, \{(1,1),(2,2)\},$
$\{(1,2),(2,1)\}, \{(1,2),(2,2)\}\{(2,1),(2,2)\},$
$\{(1,1),(1,2),(2,1)\}, \{(1,1),(1,2),(2,2)\},$
$\{(1,1),(2,1),(2,2)\}, \{(1,2),(2,1),(2,2)\},$
$\{(1,1),(1,2),(2,1),(2,2)\}$

Question-43

Let A = {x, y, z} and B = {1, 2}. Find the number of relations from A into B.

Solution:

n (A) = 3 and n (B) = 2

∴n (A × B) = 2 × 3 = 6

∴the number of relations from A into B are $2^6 = 64$.

Question-44

Which of the following relations are functions? If it is a function, determine its domain and range:

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}
(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}
(iii) {(0, 0), (1, 1), (1, -1), (4, 2), (4, -2), (9, 3), (9, -3), (16, 4), (16, -4)}
(iv) {(1, 2), (1, 3), (2, 5)}
(v) {(2, 1), (3, 1), (5, 2)}
(vi) {(1, 2), (2, 2), (3, 2)}

Solution:

(i) Domain = {2, 5, 8, 11, 14, 17}, Range = {1}

(ii) Domain = {2, 4, 6, 8, 10, 12, 14}, Range = {1, 2, 3, 4, 5, 6, 7}

(iii) No, As there are four pairs of ordered pairs which have the same first element.

(iv) No, As two ordered pairs which have the same first element.

(v) Domain = {2, 3, 5}, Range = {1, 2}

(vi) Domain = {1, 2, 3}, Range = {2}

Question-45

If A = {1,2,3}, B = {a, b}, find A × A .

Solution:

A × A = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

Question-46

Find the domain and range of the following functions:

(i) $\left\{\left(x, \frac{x^2-1}{x-1}\right): x \in R, x \neq 1\right\}$

(ii) {(x, -|x|): x ∈ R}

Solution:

(i) Domain = R − {1}, Range = R − {2}

(ii) Domain = R, Range = {y: y ∈ R and y ≤ 0}

Question-47

If A = {1,2,3}, B = {a, b}, find A × B

Solution:

A × B = {(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)}

Question-48

Find the domain and range of the following functions:

(i) $\{(x, \sqrt{9-x^2}): x \in R\}$

(ii) $\left\{\left(x, \frac{1}{1-x^2}\right): x \in R, x \neq 1\right\}$

Solution:

(i) Domain = {x: x ∈ R and −3 ≤ x ≤ 3}, Range = {y: y ∈ R and −3 ≤ y ≤ 3}

(ii) Domain = R − {1, -1}, Range = {y: y ∈ R, y ≠ 0, y<0 and y ≥ 1}

Question-49

If A = {1,2,3}, B = {a, b}, find B × B.

Solution:

B × B = {(a, a), (a, b), (b, a), (b, b)}

Relations & Functions

1. Let $A = \{-1, 2, 3\}$ and $B = \{1, 3\}$. Determine

   (i) $A \times B$ (ii) $B \times A$

   (iii) $B \times B$ (iv) $A \times A$

2. If $P = \{x : x < 3, x \in N\}, Q = \{x : x \leq 2, x \in W\}$. Find $(P \cup Q) \times (P \cap Q)$, where W is the set of whole numbers.

3. If $A = \{x : x \in W, x < 2\}$ $B = \{x : x \in N, 1 < x < 5\}$ $C = \{3, 5\}$ find

   (i) $A \times (B \cap C)$ (ii) $A \times (B \cup C)$

4. In each of the following cases, find a and b.

   (i) $(2a + b, a - b) = (8, 3)$ (ii) $\left(\frac{a}{4}, a-2b\right) = (0, 6 + b)$

5. Given $A = \{1, 2, 3, 4, 5\}$, $S = \{(x, y) : x \in A, y \in A\}$. Find the ordered pairs which satisfy the conditions given below:

   (i) $x + y = 5$ (ii) $x + y < 5$ (iii) $x + y > 8$

6. Given $R = \{(x, y) : x, y \in W, x^2 + y^2 = 25\}$. Find the domain and Range of R.

7. If $R_1 = \{(x, y) \mid y = 2x + 7$, where $x \in \mathbf{R}$ and $-5 \leq x \leq 5\}$ is a relation. Then find the domain and Range of R_1.

8. If $R_2 = \{(x, y) \mid x$ and y are integers and $x^2 + y^2 = 64\}$ is a relation. Then find R_2.

9. If $R_3 = \{(x, |x|) \mid x$ is a real number$\}$ is a relation. Then find domain and range of R_3.

10. Is the given relation a function? Give reasons for your answer.

    (i) $h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$

    (ii) $f = \{(x, x) \mid x$ is a real number$\}$

    (iii) $g = \left\{\left(n, \frac{1}{n}\right) \mid n \text{ is a positive integer}\right\}$

    (iv) $s = \{(n, n^2) \mid n$ is a positive integer$\}$

    (v) $t = \{(x, 3) \mid x$ is a real number.

11. If f and g are real functions defined by $f(x) = x^2 + 7$ and $g(x) = 3x + 5$, find each of the following

    (a) $f(3) + g(-5)$ (b) $f\left(\frac{1}{2}\right) \times g(14)$

(c) $f(-2) + g(-1)$ (d) $f(t) - f(-2)$

(e) $\dfrac{f(t)-f(5)}{t-5}$, if $t \neq 5$

12. Let f and g be real functions defined by $f(x) = 2x + 1$ and $g(x) = 4x - 7$.
    (a) For what real numbers x, $f(x) = g(x)$?
    (b) For what real numbers x, $f(x) < g(x)$?

13. If f and g are two real valued functions defined as $f(x) = 2x + 1$, $g(x) = x^2 + 1$, then find.

    (i) $f + g$ (ii) $f - g$ (iii) fg (iv) $\dfrac{f}{g}$

14. Express the following functions as set of ordered pairs and determine their range. $f: \mathbf{X} \to \mathbf{R}$, $f(x) = x^3 + 1$, where $\mathbf{X} = \{-1, 0, 3, 9, 7\}$

15. Find the values of x for which the functions $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal

16. Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ a function? Justify. If this is described by the relation, $g(x) = \alpha x + \beta$, then what values should be assigned to α and β?

17. Find the domain of each of the following functions given by

    (i) $f(x) = \dfrac{1}{\sqrt{1-\cos x}}$ (ii) $f(x) = \dfrac{1}{\sqrt{x+|x|}}$

    (iii) $f(x) = x\,|x|$ (iv) $f(x) = \dfrac{x^3-x+3}{x^2-1}$

    (v) $f(x) = \dfrac{3x}{2x-8}$

18. Find the range of the following functions given by

(i) $f(x) = \frac{3}{2-x^2}$ (ii) $f(x) = 1 - |x-2|$

(iii) $f(x) = |x-3|$ (iv) $f(x) = 1 + 3\cos 2x$

(Hint : $-1 \le \cos 2x \le 1 \Rightarrow -3 \le 3\cos 2x \le 3 \Rightarrow -2 \le 1 + 3\cos 2x \le 4$)

19. Redefine the function $f(x) = |x-2| + |2+x|$, $-3 \le x \le 3$

20. If $f(x) = \frac{x-1}{x+1}$, then show that

(i) $f\left(\frac{1}{x}\right) = -f(x)$ (ii) $f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$

21. Let $f(x) = \sqrt{x}$ and $g(x) = x$ be two functions defined in the domain $R^+ \cup \{0\}$. Find

(i) $(f+g)(x)$ (ii) $(f-g)(x)$

(iii) $(fg)(x)$ (iv) $\left(\frac{f}{g}\right)(x)$

22. Find the domain and Range of the function $f(x) = \frac{1}{\sqrt{x-5}}$.

23. If $f(x) = y = \frac{ax-b}{cx-a}$, then prove that $f(y) = x$

Choose the correct answers in Exercises from 24 to 35 (M.C.Q.)

24. Let $n(A) = m$, and $n(B) = n$. Then the total number of non-empty relations that can be defined from A to B is

(A) m^n (B) $n^m - 1$

(C) $mn - 1$ (D) $2^{mn} - 1$

25. If $[x]^2 - 5[x] + 6 = 0$, where [.] denote the greatest integer function, then

(A) $x \in [3, 4]$ (B) $x \in (2, 3]$

(C) $x \in [2, 3]$

(D) $x \in [2, 4)$

26. Range of $f(x) = \dfrac{1}{1-2\cos x}$ is

(A) $\left[\dfrac{1}{3}, 1\right]$

(B) $\left[-1, \dfrac{1}{3}\right]$

(C) $(-\infty, -1] \cup \left[\dfrac{1}{3}, \infty\right)$

(D) $\left[-\dfrac{1}{3}, 1\right]$

27. Let $f(x) = \sqrt{1+x^2}$, then

(A) $f(xy) = f(x) \,.\, f(y)$

(B) $f(xy) \geq f(x) \,.\, f(y)$

(C) $f(xy) \leq f(x) \,.\, f(y)$

(D) None of these

[Hint : find $f(xy) = \sqrt{1+x^2y^2}$, $f(x) \,.\, f(y) = \sqrt{1+x^2y^2+x^2+y^2}$]

28. Domain of $\sqrt{a^2 - x^2}$ $(a > 0)$ is

(A) $(-a, a)$

(B) $[-a, a]$

(C) $[0, a]$

(D) $(-a, 0]$

29. If $f(x) = ax + b$, where a and b are integers, $f(-1) = -5$ and $f(3) = 3$, then a and b are equal to

(A) $a = -3, b = -1$

(B) $a = 2, b = -3$

(C) $a = 0, b = 2$

(D) $a = 2, b = 3$

30. The domain of the function f defined by $f(x) = \sqrt{4-x} + \dfrac{1}{\sqrt{x^2-1}}$ is equal to

(A) $(-\infty, -1) \cup (1, 4]$

(B) $(-\infty, -1] \cup (1, 4]$

(C) $(-\infty, -1) \cup [1, 4]$

(D) $(-\infty, -1) \cup [1, 4)$

31. The domain and range of the real function f defined by $f(x) = \dfrac{4-x}{x-4}$ is given by

(A) Domain = $\mathbf{R}$, Range = $\{-1, 1\}$

(B) Domain = $\mathbf{R} - \{1\}$, Range = $\mathbf{R}$

(C) Domain = $\mathbf{R} - \{4\}$, Range = $\{-1\}$

(D) Domain = $\mathbf{R} - \{-4\}$, Range = $\{-1, 1\}$

32. The domain and range of real function f defined by $f(x) = \sqrt{x-1}$ is given by

(A) Domain = $(1, \infty)$, Range = $(0, \infty)$

(B) Domain = $[1, \infty)$, Range = $(0, \infty)$

(C) Domain = $[1, \infty)$, Range = $[0, \infty)$

(D) Domain = $[1, \infty)$, Range = $[0, \infty)$

33. The domain of the function f given by $f(x) = \dfrac{x^2+2x+1}{x^2-x-6}$

(A) $\mathbf{R} - \{3, -2\}$ (B) $\mathbf{R} - \{-3, 2\}$

(C) $\mathbf{R} - [3, -2]$ (D) $\mathbf{R} - (3, -2)$

34. The domain and range of the function f given by $f(x) = 2 - |x-5|$ is

(A) Domain = $\mathbf{R}^+$, Range = $(-\infty, 1]$

(B) Domain = $\mathbf{R}$, Range = $(-\infty, 2]$

(C) Domain = $\mathbf{R}$, Range = $(-\infty, 2)$

(D) Domain = $\mathbf{R}^+$, Range = $(-\infty, 2]$

35. The domain for which the functions defined by $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal is

(A) $\left\{-1, \frac{4}{3}\right\}$ (B) $\left[-1, \frac{4}{3}\right]$

(C) $\left(-1, \frac{4}{3}\right)$ (D) $\left[-1, \frac{4}{3}\right)$

CBSE Class 11 Mathematics

Important Questions

Chapter 2

Relations and Functions

1 Marks Questions

1. Find a and b if (a – 1, b + 5) = (2, 3)If A = {1,3,5}, B = {2,3} find : (Question-2, 3)

Ans. a = 3, b = –2

2. A × B

Ans. A × B = {(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)}

3. B × A Let A = {1,2}, B = {2,3,4}, C = {4,5}, find (Question- 4,5)

Ans. B × A = { (2,1), (2,3), (2,5), (3,1), (3,3), (3,5)}

4. A × (B ∩ C)

Ans. {(1,4), (2,4)}

5. A × (B ∪ C)

Ans. {(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5)}

6. If P = {1,3}, Q = {2,3,5}, find the number of relations from A to B

Ans. 2^6 = 64

7. If A = {1,2,3,5} and B = {4,6,9}, R = {(x, y) : |x – y| is odd, x ∈ A, y ∈ B} Write R in roster form

Which of the following relations are functions. Give reason.

Ans. R = { (1,4), (1,6), (2,9), (3,4), (3,6), (5,4), (5,6)}

8. R = { (1,1), (2,2), (3,3), (4,4), (4,5)}

Ans. Not a function because 4 has two images.

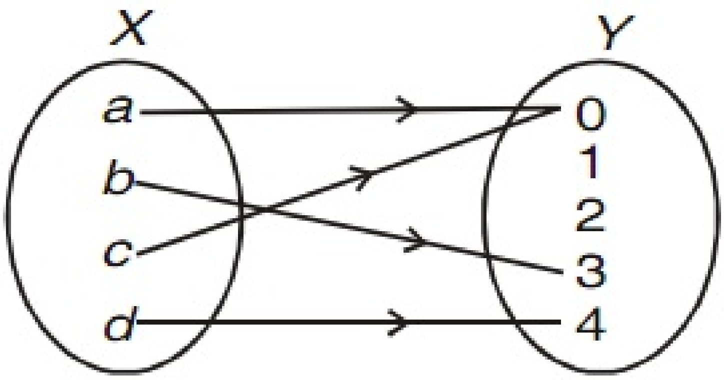
9. R = { (2,1), (2,2), (2,3), (2,4)}

Ans. Not a function because 2 does not have a unique image.

10. R = { (1,2), (2,5), (3,8), (4,10), (5,12), (6,12)} Which of the following arrow diagrams represent a function? Why?

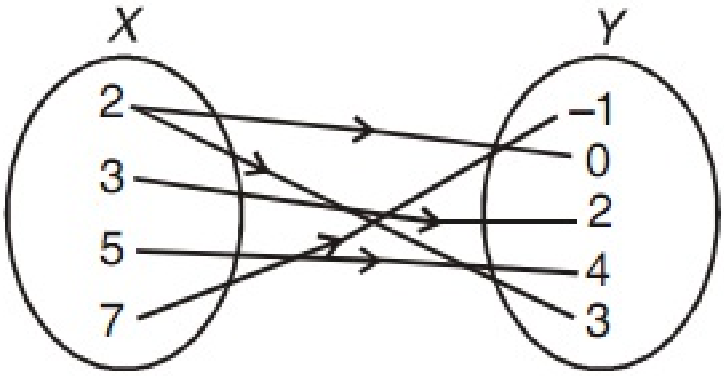
Ans. Function

11.



Ans. Function

12.



Let f and g be two real valued functions, defined by, f(x) = x2, g(x) = 3x +2.

Ans. Not a function

13. (f + g)(–2)

Ans. 0

14. (f – g)(1)

Ans. -4

15. (fg)(–1)

Ans. -1

16. $\left(\frac{f}{g}\right)(0)$

Ans. 0

17. If f(x) = x3, find the value of, $\frac{f(5) - f(1)}{5-1}$

Ans. 31

18. Find the domain of the real function, f(×) = $\sqrt{x^2 - 4}$

Ans. $(-\infty, -2] \cup [2, \infty)$

19. Find the domain of the function, f (×) = $\frac{x^2+2x+3}{x^2-5x+6}$ **Find the range of the following functions, (Question- 20,21)**

Ans. R – {2,3}

20. f (x) = $\frac{1}{1-x^2}$

Ans. $(-\infty, -0] \cup [1, \infty)$

21. f(x) = x^2 **+ 2**

Ans. $[2,\infty)$

22. Find the domain of the relation, R = { (x, y) : x, y ∊ Z, xy = 4} Find the range of the following relations : (Question-23, 24)

Ans. {–4, –2, –1,1,2,4}

23. R = {(a,b) : a, b ∊ N and 2a + b = 10}

Ans. {2,4,6,8}

24.R = $\left\{\left(x, \frac{1}{x}\right) : x \in z,\ 0 < x < 6\right\}$

Ans. $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$

25.If the ordered Pairs $(x-1, y+3)$ **and** $(2, x+4)$ **are equal, find** x **and** y

$(i)\ (3,3)\quad (ii)\ (3,4)\quad (iii)\ (1,4)\quad (iv)\ (1,0)$

Ans. $(3,4)$

26. If, $n(A)=3, n(B)=2, A$ **And** B **are two sets Then no. of relations of** $A\times B$ **have.**

$(i)\ (6)\quad (ii)\ (12)\quad (iii)\ (32)\quad (iv)\ (64)$

Ans. 64

27.Let $f(x)=-|x|$ **then Range of function**

$(i)\ (0,\infty)\quad (ii)\ (-\infty,\infty)\quad (iii)\ (-\infty,0)\quad (iv)$ none of there

Ans. $(-\infty,0)$

28.A real function f **is defined by** $f(x)=2x-5$. **Then the Value of** $f(-3)$

(i) -11 $\quad (ii)$ 1 $\quad (iii)$ 0 $\quad (iv)$ none of there

Ans. -11

29.If $P=\{a,b,c\}$ **and** $Q=\{d\}$, **form the sets** $P\times Q$ **and** $Q\times P$ **are these two Cartesian products equal?**

Ans. Given $P=\{a,b,c\}$ and $Q=\{d\}$, by definition of cartesion product, we set

$P\times Q=[(a,d),(b,d),(c,d)]$ and $Q\times P=[(d,a),(d,b),(d,c)]$

By definition of equality of ordered pains the pair (a,d) is not equal to the pair (d,a) therefore $p\times Q\neq Q\times P$.

30..If A and B are finite sets such that $n(A)=m$ and $n(B)=k$ find the number of relations from A to B

Ans. Linen $n(A)=n$ and $n(B)=k$

$\therefore n(A\times B)=\bigcap(A)\times\bigcap(B)=mk$

$\therefore$ the number of subsets of $A\times B=2mk$

$\because n(A)=m,$ then the number of subsets of $A=2^m$

Since every subset of $A\times B$ is a relation from A to B therefore the number of relations from A to B = 2^{mk}

31.Let $f=\{(1,1),(2,3),(0,-1),(-1,3),......\}$ be a function from z to z defined by $f(x)=ax+b,$ for same integers a and b determine a and b.

Ans. Given $f(x)=ax+b$

Since $(1,1)\in f_1 f(1)=1\Rightarrow a+b=1......(i)$

$(2,3)\in f.f(2)=3\Rightarrow 2a+b=3........(ii)$

Subtracting (i) from(ii) we set a=2

Substituting a=2 is (ii) we get 2+b=1

$\Rightarrow$ b = -1

Hence a = 2, b = -1

32.Express $\{(x,y): y+2x=5, xy\in w\}$ as the set of ordered pairs

Ans. Since $y+2x=5$ and $x,y\in w,$

Put $x=0, y+0=5\Rightarrow y=5$

$x=1, y+2\times 1=5 \Rightarrow y=3$
$x=2, y+2\times 2=5 \Rightarrow y=1$

For anther values of $x \in w$, we do not get $y \in w$.

Hence the required set of ordered peutes is $\{(0'5),(1,3),(2,1)\}$

33.If $A=\{1,2\}$, find $(A\times A\times A)$

Ans. We have

$A\times A\times A=\{(1,1,1),(1,1,2),(1,2,1),(2,1,1),(2,2,1),(2,2,2)\}$

34. A Function f is defined by $f(x)=2x-3$ find $f(5)$

Ans. $Here\ f(x)=2x-3$

$f(x)=(2\times 5-3)=7$

35.Let $f=\{(0,-5),(1,-2),(2,1),(3,4),(4,7)\}$ be a linear function from z into z find f

Ans. $f(x)=3x-5$

36.If the ordered pairs $(x-2,2y+1)$ and $(y-1,x+2)$ are equal, find x & y

Ans. $x=3,\qquad y=2$

37.Let $A=\{-1,2,5,8\}$, $B=\{0,1,3,6,7\}$ and R be the relation, is one less than from A to B then find domain and Range of R

Ans. Given $A=\{-1,2,5,8\}$, $B=\{0,1,3,6,7\}$, and R is the relation 'is one less than' from

A to B therefore $R = [(-1,0),(2,3),(5,6)]$

Domain of $R = \{-1,2,5\}$ and range of $R = \{0,3,6\}$

38.Let R be a relation from N to N define by $R = [(a,b): a,b \in N \text{ and } a = b^2]$.

Is the following true $a,b \in R$ implies $(b,a) \in R$

Ans. No; let $a = 4, b = 2$. As $4 = 2^2$, so $(4,2) \in R$ but $2 \neq 4^2$ so $(2,4) \in R$

39.Let N be the set of natural numbers and the relation R be define in N by R = $[(x,y): y = 2x, x,y \in N]$. what is the domain, co domain and range of R ? Is this relation a function?

Ans. Given $R = [(x,y): y = 2x, x,y \in N]$

$\therefore$ Domain of $R = N$, co domain of $R = N$, and Range of R is the set of even natural numbers.

Since every natural number x has a unique image $2x$ therefore, the relation R is a function.

40.Let $R = \{(x,y): y = x+1\}$ and $y \in \{0,1,2,3,4,5\}$ list the element of R

Ans. $R = \{(-1,0),(0,1),(1,2),(2,3),(3,4),(4,5)\}$

41.Let f be the subset of $Q \times Z$ defined by

$f = \left\{\left(\frac{m}{n}, m\right): mn \in Z, n \neq 0\right\}$ **. Is f a function from Q to Z? Justify your answer**

Ans. f Is not a function from Q to Z

$f\left(\frac{1}{2}\right) = 1 \; and \; f\left(\frac{2}{4}\right) = 2$

$But \; \frac{1}{2} = \frac{2}{4}$

$\therefore$ One element $\frac{1}{2}$ have two images

$\therefore$ f is not function

42.The function $'f'$ which maps temperature in Celsius into temperature in Fahrenheit is defined by $f(c) = \frac{9}{5}c + 32$ find $f(0)$

Ans. $f(0) = \frac{9}{5} \times 0 + 32$

$f(0) = 32$

43.If $f\left(x = x^3 - \frac{1}{x^3}\right)$ **Prove that** $f(x) + f\left(\frac{1}{x}\right) = 0$

Ans. $f(x) = x^3 - \frac{1}{x^3}$

$f\left(\frac{1}{x}\right) = \frac{1}{x^3} - x^3$

$f(x) + f\left(\frac{1}{x}\right) = \cancel{x^3} - \cancel{\frac{1}{x^3}} + \cancel{\frac{1}{x^3}} - \cancel{x^3}$

$= 0$

44.If A and B are two sets containing m and n elements respectively how many different relations can be defined from A to B ?

Ans. 2^{m+n}

CBSE Class 12 Mathematics

Important Questions

Chapter 2

Relations and Functions

4 Marks Questions

1. Let A = {1,2,3,4}, B = {1,4,9,16,25} and R be a relation defined from A to B as, R = {(x, y) : x ∈ A, y ∈ B and y = x^2}

(a) Depict this relation using arrow diagram.

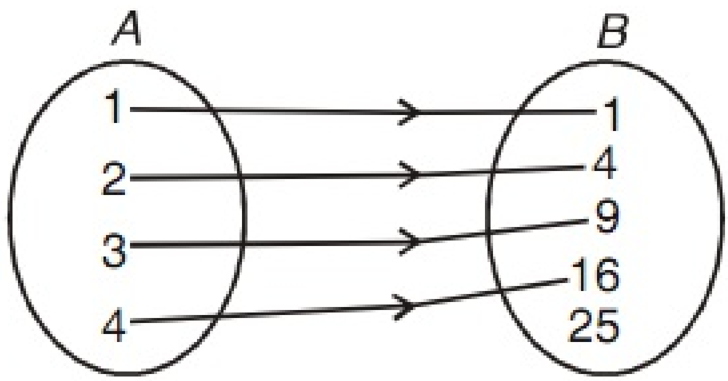
(b) Find domain of R.

(c) Find range of R.

(d) Write co-domain of R.

Ans.

(a)



(b) {1,2,3,4}

(c) {1,4,9,16}

(d) {1,4,9,16,25}

2. Let R = { (x, y) : x, y ∈ N and y = 2x} be a relation on N. Find :

(i) Domain

(ii) Codomain

(iii) Range

Is this relation a function from N to N

Ans. (i) N

(ii) N

(iii) Set of even natural numbers

yes, R is a function from N to N.

3. Find the domain and range of, f(x) = |2x – 3| – 3

Ans. Domain is R

Range is [–3, ∞)

4. Draw the graph of the Constant function, f : R ∊ R; f(x) = 2 x ∊ R. Also find its domain and range.

Ans. Domain = R

Range = {2}

5.Let $R = \{(x, -y) : x, y = \in W, 2x + y = 8\}$ **then**

(i) Find the domain and the range of R **(ii)** Write R as a set of ordered pairs.

Ans. (i) Given $2x + y = 8$ and $x.y \in w$

Put

$$x = 0, 2\times 0 + y = 8 \Rightarrow y = 8,$$
$$x = 1, 2\times 1 + y = 8 \Rightarrow y = 6,$$

$x = 2, 2\times2 + y = 8 \Rightarrow y = 4,$

$x = 3, 2\times3 + y = 8 \Rightarrow y = 2,$

$x = 4, 2\times4 + y = 8 \Rightarrow y = 0$

for all other values of $x \in w$, we do not get $y \in w$

$\therefore$ Domain of $R = \{0,1,2,3,4,\}$ and range of $R = \{8,6,4,2,0\}$

(ii) R as a set of ordered pairs can be written as

$R = \{(0,8),(1,6),(2,4),(3,2),(4,0)\}$

6.Let R be a relation from Q to Q defined by $R = \{(a,b) : a, b \in Q \text{ and } a - b \in z,\}$ **show that** $(i)(a,a) \in R$ for all $a \in Q$ $(ii)(a,b) \in R$ implies that $(b,a) \in R$

$(iii)(a,b) \in R$ and (b,c) $\in R$ implies that $(a,c) \in R$

Ans. $R = [(a,b) : a, b \in Q \text{ and } a - b \in z]$

(i) For all $a \in Q, a - a = 0$ and $0 \in z$, it implies that $(a,a) \in R$.

(ii) Given $(a,b) \in R \Rightarrow a - b \in z \Rightarrow -(a-b) \in z$

$\Rightarrow b - a \in z \Rightarrow (b,a) \in R.$

(iii) Given $(a,b) \in R$ and $(b,c) \in R \Rightarrow a - b \in z$ and $b - c \in z \Rightarrow (a-b)+(b-c) \in z$

$\Rightarrow a - c \in z \Rightarrow (a-c) \in R.$

7. If $f(x) = \dfrac{x^2 - 3x + 1}{x - 1}$, find $f(-2) + f\left(\dfrac{1}{3}\right)$

Ans. Given $f(x) = \dfrac{x^2 - 3x + 1}{x - 1}$, $Df = R - \{1\}$

$$\therefore f(-2)=\frac{(-2)^2-3(-2)+1}{-2-1}-\frac{4+6+1}{-3}=1\frac{1}{3} \text{ and}$$

$$f\left(\frac{1}{3}\right)=\frac{\left(\frac{1}{3}\right)^2-3\times\frac{1}{3}+1}{\frac{1}{3}-1}=\frac{\frac{1}{9-1+1}}{-\frac{2}{3}}=\frac{\frac{1}{9}}{-\frac{2}{3}}=\frac{1}{9}\times\left(-\frac{3}{2}\right)=-\frac{1}{6}$$

$$\therefore f(-2)+f\left(\frac{1}{3}\right)=-\frac{11}{3}-\frac{1}{6}=\frac{-22-1}{6}=\frac{-23}{6}=3\frac{5}{6}.$$

8.Find the domain and the range of the function $f(x)=3x^2-5$. Also find $f(-3)$ and the numbers which are associated with the number 43 m its range.

Ans. Given $f(x)=3x^2-5$

For Df, $f(x)$ must be real number

$\Rightarrow 3x^2-5$ must be a real number

Which is a real number for every $x \in R$

$\Rightarrow Df = R$..........(i)

$for\ Rf$, let $y=f(x)=3x^2-5$

We know that for all $x \in R, x^2 \geq 0 \Rightarrow 3x^2 \geq 0$

$\Rightarrow 3x^2-5 \geq -5 \Rightarrow y \geq -5 \Rightarrow Rf=[-5,\infty]$

Funthes, as $-3 \in Df$, $f(-3)$ exists is and $f(-3)$

$=3(-3)^2-5=22.$

As $43 \in Rf$ on putting $y=43$ is (i) weget

$3x^2\text{-}5=43 \Rightarrow 3x^2=48 \Rightarrow x^2=16 \Rightarrow x=-4,4.$

There fore -4 and 4 are number

$(\text{is } Df)$ which are associated with the number 43 in Rf

9.If $f(x) = x^2 - 3x + 1$, find x such that $f(2x) = f(x)$

Ans. Given $f(x) = x^2 - 3x + 1, Df = R$

$\therefore f(2x) = (2x)^2 - 3(2x) + 1 = 4x^2 - 6x + 1$

As $f(2x) = f(x)$ (Given)

$\Rightarrow 4x^2 - 6x + 1 = x^2 - 3x + 1$

$\Rightarrow 3x^2 - 3x = 0 \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0$

$\Rightarrow x = 0, 1.$

10.Find the domain and the range of the function $f(x) = \sqrt{x-1}$

Ans. Given $f(x) = \sqrt{x-1},$

for Df, $f(x)$ must be a real number

$\Rightarrow \sqrt{x-1}$ must be a real number

$\Rightarrow x - 1 \geq 0 \Rightarrow x \geq 1$

$\Rightarrow Df = [1, \infty]$

for Rf, let $y = f(x) = \sqrt{x-1}$

$\Rightarrow \sqrt{x-1} \geq 0 \Rightarrow y \geq 0$

$\Rightarrow Rf = [0, \infty]$

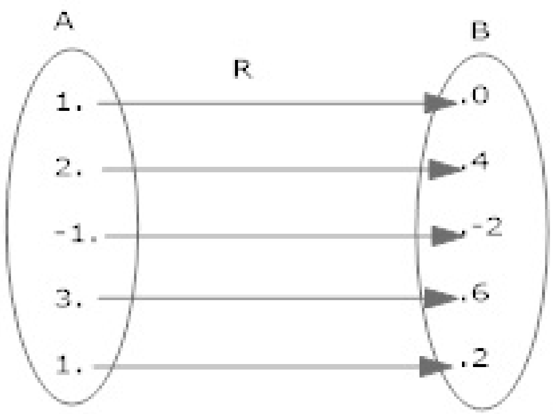
11.Let a relation $R = \{(0,0), (2,4), (-1,2), (3,6), (1,2)\}$ **then**

(i) write domain of R

(ii) write range of R

(iii) write R the set builder form

(iv) represent R by an arrow diagram



Ans. Given $R = \left[(0,0),(2,4),(-1,-2,),(3,6),(1,2)\right]$

(i) Domain of $R = \left[0,2,-1,3,1\right]$

(ii) Rang of $R = \left[0,4,-2,6,2\right]$

(iii)R in the builder from can be written as

$$R = \left[(x,y): x \in I, -1 \le x \le 3, y = 2x\right]$$

(iv) The reaction R can be represented by the arrow diagram are shown.

12.Let $A = \{1,2,3\}$, $B = \{1,2,3,4\}$ **and** $R = \{(x,y):(x,y) \in A \times B, y = x+1\}$

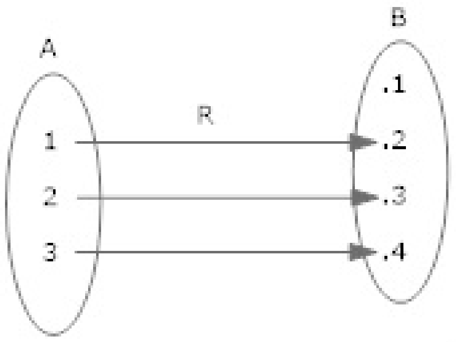
(i) find $A \times B$

(ii) write R in roster form

(iii) write domain & range of R

(iv) represent R by an arrow diagram

Ans. (i) $\{(1,1),(1,2),(1,3),(1,4)$



$(2,1),(2,2),(2,3),(2,4)$
$(3,1),(3,2),(3,3),(3,4)\}$

(ii) $R = [(1,2),(2,3),(3,4)]$

(iii) Domain of $R = \{1,2,3\}$ and range of $R = \{2,3,4\}$

(iv) The relation R can be represented by the are arrow diagram are shown.

13.The cartesian product $A \times A$ has a elements among which are found $(-1,0)$ and $(0,1)$. find the set and the remaining elements of $A \times A$

Ans. Let $n(A) = m$

Given $n(A \times A) = 9 \Rightarrow n(A)n(A) = 9$

$\Rightarrow m.m = 9 \Rightarrow m^2 = 9 \Rightarrow m = 3 \quad (\because m > 0)$

Given $(-1,0) \in A \times A \Rightarrow -1 \in A$ and $0 \in A$

Also $(0,1) \in A \times A \Rightarrow 0 \in A$ and $1 \in A$

This $-1,0,1 \in A$ but $n(A) = 3$

Therefore $A=[-1,0,1]$

The remaining elements of $A\times A$ are $(-1,-1),(-1,1),(0,-1),(0,0),(1,-1),(1,0),(1,1)$

14.Find the domain and the range of the following functions $f(x)=\dfrac{1}{\sqrt{5-x}}$

Ans. Given $f(x)=\dfrac{1}{\sqrt{5-x}}$

For $D_F, f(x)$ must be a real number

$\Rightarrow \dfrac{1}{\sqrt{5-x}}$ Must be a real number

$\Rightarrow 5-x>0 \Rightarrow 5>x \Rightarrow x<5$

$\Rightarrow D_F=(-\infty,5)$

For R_F let $y=\dfrac{1}{\sqrt{5-x}}$

As $x<5, 0<5-x$

$\Rightarrow 5-x>0 \Rightarrow \sqrt{5-x}>0$

$\Rightarrow \dfrac{1}{\sqrt{5x}}>0 \left(\because \dfrac{1}{a}>0 \text{ if and only if } a>0\right)$

$\Rightarrow y>0$

$\Rightarrow R_F=(0,\infty)$

15.Let $f(x)=x+1$ **and** $g(x)=2x-3$ **be two real functions. Find the following functions** $(i)\ f+g \quad (ii)\ f-g \quad (iii)\ fg \quad (iv)\ \dfrac{f}{g} \quad (v)\ f^2-3g$

Ans. Given $f(x)=x+1$ and $g(x)=2x-3$ we note that $D_F=R$ and $D_g=R$ so there functions have the same Domain R

(i) $(f+g)(x)=f(x)+g(x)=(x+1)+(2x-3)=3x-2$, for $x\in R$

(ii) $(f-g)(x)=f(x)-g(x)=(x+1)-(2x-3)\ -x+4$, for all $x\in R$

(iii) $(fg)(x)=f(x)=(x+1)(2x-3)=2x^2-x-3$, for all $x\in R$

(iv) $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{x+1}{2s-3}, x\neq\frac{3}{2}, x\in R$

(v) $(f^2-3g)(x)=(f^2)(x)-(3f)(x)=(f(x))^2-3g(x)$

$=(x+1)^2-3(2x-3)=x^2+2x+1-6x+9$

$=x^2-4x+10$, for all $x\in R$

16.Find the domain and the range of the following functions

$(i)\ f(x)=\frac{x-3}{2x+1}\ (ii)\ f(x)=\frac{x^2}{1+x^2}\ (iii)\ f(x)=\frac{1}{1-x^2}$

Ans. (i) Given $f(x)=\frac{x-3}{2x+1}$

For D_F, $f(x)$ must be a real number

$\Rightarrow\frac{x-3}{2x+1}$ must be a real number

$\Rightarrow 2x+1\neq 0\Rightarrow x\neq-\frac{1}{2}$

$\Rightarrow D_F=$ set of all real number except

$-\frac{1}{2}$ $i.e. R-\left[-\frac{1}{2}\right]$

For R_F, let $y=\frac{x-3}{2x+1} \Rightarrow 2xy+y=x-3$

$\Rightarrow (2y-1)x=-y-3 \Rightarrow x=\frac{y+3}{1-2y}$ but $x \in R$

$\Rightarrow \frac{y+3}{1-2y}$ Must be a real number $\Rightarrow 1-2y \neq 0 \Rightarrow y \neq \frac{1}{2}$

$\Rightarrow R_F =$ Set of all real number except $\frac{1}{2}$ $R-\left[\frac{1}{2}\right]$

(ii) Given $f(x)=\frac{x^2}{1+x^2}$

For D_F, $f(x)$ must be a real number $\Rightarrow \frac{x^2}{1+x^2}$

Must be a real number

$\Rightarrow D_F = R \qquad (\because x^2+1 \neq 0 \text{ for all } x \in R)$

For R_F let $y=\frac{x^2}{1+x^2} \Rightarrow x^2y+y=x^2$

$\Rightarrow (y-1)x^2=-y \Rightarrow x^2=\frac{-y}{y-1}, y \neq 1$

But $x2 \geq 0$ for all $x \in R \Rightarrow \frac{-y}{y-1} \geq 0, y \neq 1$

Multiply both sides by $(y-1)^2$, a positive real number

$\Rightarrow -y(y-1) \geq 0$

$\Rightarrow y(y-1) \leq 0 \Rightarrow (y-0)(y-1) \leq 0$

$\Rightarrow 0 \leq y \leq 1$ but $y \neq 1$

$\Rightarrow 0 \leq y < 1$

$\Rightarrow R_F = (0,1)$

(iii) Given $f(x) = \frac{1}{1-x^2}$

For $D_{F,} f(x)$ must be a real number

$\Rightarrow \frac{1}{1-x^2}$ Must be a real number

$\Rightarrow 1-x^2 \neq 0 \Rightarrow x \neq -1,1$

$\Rightarrow D_F =$ Set of all real number except $-1,1 i.e D_F = R - [-1,1]$

For R_F let $y = \frac{1}{1-x^2}, y \neq 0$

$\Rightarrow 1-x^2 = \frac{1}{y} \Rightarrow x^2 = 1 - \frac{1}{y} \neq 0$

But $x^2 \geq 0$ for all $\in D_F \Rightarrow 1 - \frac{1}{y} \geq 0$

But $y^2 > 0, y \neq 0$

Multicity bath sides by y^2 a positive real number

$\Rightarrow y2\left(1-\frac{1}{y}\right) \geq 0 \Rightarrow y.(y-1) \geq 0 \Rightarrow (y-0)(y-1) \geq 0$

Either $y \leq 0$ or $y \geq 1$ but $y \neq 0$

$\Rightarrow R_F = (-\infty, 0) \bigcup (1, \infty).$

17.If $A = \{1, 2, 3\}$ $B = \{3, 4\}$ **and** $c = \{4, 5, 6\}$

find (i) $A \times (B \bigcup C)$ (ii) $A \times (B \bigcap C)$ (iii) $(A \times B) \bigcap (B \times C)$

Ans. We have

(i) $(B \bigcup C) = \{3, 4\} \bigcup \{4, 5, 6\} = \{3, 4, 5, 6\}$

$\therefore A \times (B \bigcup C)$

$= \{1, 2, 3\} \times \{3, 4, 5, 6\}$

$= \{(1,3), (1,4), (1,5), (1,6), (2,3), (2,4),$

$(2,5), (2,6), (3,3), (3,4), (3,5), (3,6)\}$

(ii) $(B \bigcap C) = \{3, 4\} \bigcap \{4, 5, 6\} = \{4\}$

$\therefore A \times (B \bigcap C) = \{1, 2, 3\} \times \{4\} = \{(1, 4), (2, 4), (3, 4)\}$

(iii) $(A \times B) = \{1, 2, 3\} \times \{3, 4\}$

$= \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$

$(B \times C) = \{3, 4\} \times \{4, 5, 6\}$

$= \{(3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\}$

$\therefore (A \times B) \bigcap (B \times C) = \{(3, 4)\}$

18.For non empty sets A **and** B **prove that** $(A \times B) = (B \times A) \Leftrightarrow A = B$

Ans. First we assume that $A = B$

Then $(A \times B) = (A \times A)$ and $(B \times A) = (A \times A)$

$\therefore (A \times B) = (B \times A)$

This, when $A = B$, then $(A \times B) = (B \times A)$

Conversely, Let $(A \times B) = (B \times A)$, and let be $x \in A$.

Then, $x \in A \Rightarrow (x, b) \in A \times B$ for same $b \in B$

$\Rightarrow (x, b) \in B \times A \qquad [\because A \times B = B \times A]$

$\Rightarrow x \in B$.

$\therefore A \subseteq B$

similarly, $B \subseteq A$

Hence, $A = B$

19.Let m be a given fixed positive integer. let $R = [(a, b) : a, b \in z \text{ and} (a-b) \text{is divisible by } m]$ show that R is an equivalence relation on Z.

Ans. $R = \{(a, b) : a, b \in Z \text{ and} (a-b) \text{ is divisible by } m\}$

(i) Let $a \in Z$. Then,

$a - a = 0$, which is divisible by m

$\therefore (a, a) \in R$ for all $a \in Z$

so R is refleseive

(ii) Let $(a, b) \in R$ Then

$(a, b) \in R \Rightarrow (a-b)$ is divisible by m

$\Rightarrow -(a-b)$ is divisible by m

$\Rightarrow (b-a)$ is divisible m

$\Rightarrow (b, a) \in R$

Then $(a, b) \in R \Rightarrow (b, a) \in R$.

So R is symmetric.

(iii) Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow (a-b)$ is divisible by m and $(b-c)$ is divisible by m

$\Rightarrow [(a-b)+(b-c)]$ is divisible by m

$\Rightarrow (a-c)$ is divisible by m

$\Rightarrow (a, c) \in R$

$\therefore (a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.

So, R is transitive this R is reflexive symmetric and transitive Hence, R is an equivalence relation and Z .

20.Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3, 4\}$ let R be the relation, is greater than from A to B. Write R as a a set of ordered pairs. find domain (R) and range (R)

Ans. $R = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,1), (5,2), (5,3), (5,4)\}$

Domain of R $= \{2, 3, 4, 5\}$ Range of R $= \{1, 2, 3, 4\}$

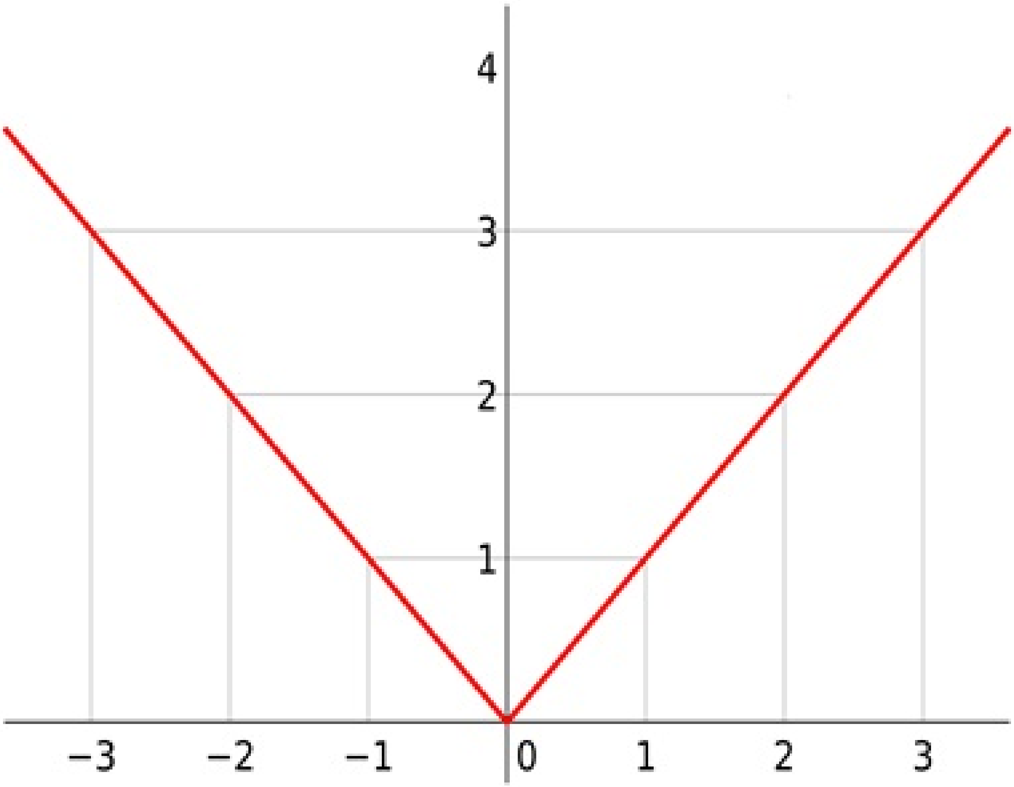
21.Define modulus function Draw graph.

Ans. let $f : R \to R : f(x) = |x|$ for each $x \in R$. then $f(x) = |x| = \begin{cases} x, \text{ when } x \geq 0 \\ -x, \text{when } x < 0 \end{cases}$

we know that $|x| > 0$ for all x

$\therefore \text{dom}(f) = R$ and $\text{range}(f) =$ set of non negative real number

Drawing the graph of modulus function defined by



$$f : R \to R : f(x) = |x| = \begin{cases} x \text{ when } x \geq 0 \\ -x \text{ when } x < 0 \end{cases}$$

We have

x	**3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**
$f(x)$	3	2	1	0	1	2	3	4

Scale: 5 small divisions = 1 unit

On a graph paper, we plot the points
$A(-3,3), B(-2,2), C(-1,1), o(0,0), D(1,1), \in(2,2), F(3,3)$ and $G(4,4)$

Join them successively to obtain the graph lines AO and OG, as show in the figure above.

22.Let $f(x)=\begin{cases}x^2, \text{when } 0\le x\le 3\\ 3x, \text{when } 3\le x\le 10\end{cases}$ $g(x)\begin{cases}x^2, 0\le x\le 3\\ 2x, 3\le x\le 10\end{cases}$ **Show that** f **is** a **function, while g is not** a **function.**

Ans. Each element in $\{0,10\}$ has a unique image under f.

But, $g(3)=3^2=9$ and

$g(3)=(2\times 3)=6$

So g is not a function

23.Let $A=\{1,2\}$ **and** $b=\{3,4\}$ **write** $A\times B$ **how many subsets will** $A\times B$ **have? List them.**

Ans. $A\times B=\{(1,3),(1,4),(2,3),(2,4)\};$ 16 Subsets of $A\times B$ have

$\text{Subsets}=\phi,\{(1,3)\},\{(1,4)\},\{(2,3)\},\{(2,4)\},$
$\{(1,4)\},\{(1,3),(2,3)\},\{(1,3),(2,4)\},$

$\{(1,4),(2,3)\},\{(1,4),(2,4)\},\{(2,3)\},$
$\{(2,4)\},\{(1,3),(1,4),(2,3)\},\{(1,3),(1,4),\}$

$\{(2,4),\},\{(1,3),(2,3),(2,4)\},\{(1,4),(2,3)\},$
$\{(2,4)\};$

24.Let $A=\{1,2\}, B=\{1,2,3,4\}, C=\{5,6\}$ **and** $D=\{5,6,7,8\}$ **verify that**
$(i)A\times(B\cap C)=(A\times B)\cap(A\times C)$ $(ii)A\times C$ is subset of $B\times D$

Ans. $L.H.S.$ $B\cap C=\phi$

Part-I

L.H.S $A\times(B\cap C)=\phi$

$R.H.S.$ $A\times B=\begin{Bmatrix}(1,1),(1,2),(1,3),(1,4)\\(2,1),(2,2),(2,3),(2,4)\end{Bmatrix}$

$A\times C=\{(1,5),(1,6),(2,5),(2,6)\}$

$(A\times B)\cap(A\times C)=\phi$ $\quad L.H.S=R.H.S$

Part-II

$B\times D=\begin{Bmatrix}(1,5),(1,6),(1,7),(1,8)\\(2,5),(2,6),(2,7),(2,8)\end{Bmatrix}$

$(A\times C)\subset(B\times D)$

25.Find the domain and the range of the relation R defined by
$R=[(x+1,x+3):x\in(0,1,2,3,4,5)]$

Ans. Given $x\in\{0,1,2,3,4,5\}$

put $x=0, x+1=0+1=1$ and $x+3=0+3=3$
$x=1, x+1=1+1=2$ and $x+3=1+3=4,$

$x=2, x+1=2+1=3$ and $x+3=2+3=5,$
$x=3, x+1=3+1=4$ and $x+3=3+3=6,$

$x=4, x+1=4+1=5$ and $x+3=4+3=7$
$x=5, x+1=6$ and $x+3=5+3=8$

Hence $R=[(1,3),(2,4),(3,5),(4,6),(5,7),(6,8)]$
$\therefore$ Domain of $R=[1,2,3,4,5,6]$ and range of $R=[3,4,5,6,7,8]$

26.Find the linear relation between the components of the ordered pairs of the relation R where $R = \{(2,1),(4,7),(1,-2),......\}$

Ans. Given $R = \{(2,1),(4,7),(1,-2),....\}$

Let $y = ax + b$ be the linear relation between the components of R

Since $(2,1) \in R, \therefore y = ax + b \Rightarrow 1 = 2a + b........(i)$

Also $(4,7) \in R, \therefore y = ax + b \Rightarrow 7 = 4a + b........(ii)$

Subtracting (i) from (ii), we get $2a = 6 \Rightarrow a = 3$

Subtracting $a = 3$ is (i), we get $1 = 6 + b \Rightarrow b = -5$

Subtracting there values of a and b in $y = ax + b,$ we get

$y = 3x - 5,$ which is the required linear relation between the components of the given relation.

27.Let $A = \{1,2,3,4,5,6\}$ define a relation R from A to A by

$R = \{(x,y) : y = x+1, x, y \in A\}$

(i) write R in the roaster form

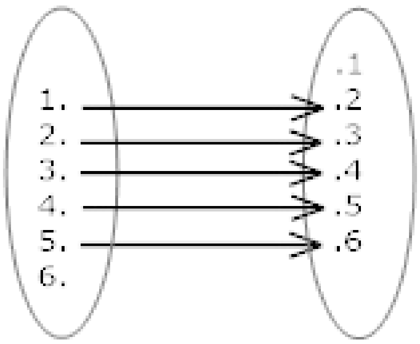
(ii) write down the domain, co-domain and range of R

(iii) Represent R by an arrow diagram

Ans. (i) $\{(1,2),(2,3),(3,4),(4,5),(5,6)\}$

(ii) Domain $= \{1,2,3,4,5\}$ co domain $= A,$ range $= \{2,3,4,5,6\}$

(iii)



28.A relation 'f' is defined by $f : x \to x^2 - 2$ **where** $x \in \{-1, -2, 0, 2\}$

(i) list the elements of f

(ii) is f **a function?**

Ans. Reletion f is defined by $f : x \to x^2 - 2$

(i) is $f(x) = x^2 - 2$ whene $x \in \{-1, -2, 0, 2\}$

$f(-1) = (-1)^2 - 2 = 1 - 2 = -1$

$f(-2) = (-2)^2 - 2 = 4 - 2 = 2$

$f(0) = 0^2 - 2 = 0 - 2 = -2$

$f(2) = 2^2 - 2 = 4 - 2 = 2$

$\therefore f = \{(-1, -1), (-2, 2), (0, -2), (2'2)\}$

(ii) We note that each element of the domain of f has a unique image; therefore, the relation f is a function.

29.If $y = \dfrac{6x-5}{5x-6}$. **Prove that** $f(y) = x,\ x \neq \dfrac{6}{5}$

Ans. $y = \dfrac{6x-5}{5x-6}$

$$y = f(x) = \frac{6x-5}{5x-6}$$

$$f(y) = \frac{6\left[\frac{6x-5}{5x-6}\right]-5}{5\left[\frac{6x-5}{5x-6}\right]-6}$$

$$f(y) = \frac{\frac{36x-\cancel{30}-25x+\cancel{30}}{\cancel{5x-6}}}{\frac{\cancel{30}x-25-\cancel{30}x+36}{\cancel{5x-6}}}$$

$$f(y) = \frac{11x}{11} = x, x \neq \frac{6}{5}$$

30.Let $f: X \to Y$ be defined by $f(x) = x^2$ for all $x \in X$ where $X = \{-2, -1, 0, 1, 2, 3\}$ and $y = \{0, 1, 4, 7, 9, 10\}$ write the relation f in the roster farm. It f a function?

Ans. $f: X \to Y$ defined by

$$f(x) = x^2, x \in X$$
$$and\ X = \{-2, -1, 0, 1, 2, 3\}$$

$$y = \{0, 1, 4, 7, 9, 10\}$$

$$f(-2) = (-2)^2 = 4$$
$$f(-1) = (-1)^2 = 1$$

$$f(0) = 0^2 = 0$$
$$f(1) = 1^2 = 1$$

$$f(2) = 2^2 = 4$$
$$f(3) = 3^2 = 9$$

$\therefore f = \{(-2,4),(-1,1),(0,0),(1,1),(2,4),(3,9)\}$

f is a function because different elements of X have different imager in y

31.Determine a quadratic function `f' defined by

$f(x) = ax^2 + bx + c$ if $f(0) = 6$, $f(2) = 11$ and $f(-3) = 6$

Ans. $f(x) = ax^2 + bx + c$

$f(0) = 6$

$a \times 0^2 + b \times 0 + c = 6$

$c = 6$

$f(2) = 11$

$a \times 2^2 + b \times 2 + c = 11$

$4a + 2b + c = 11$

$4a + 2b + 6 = 11$

$4a + 2b = 11 - 6$

$[4a + 2b = 5] ----(i)$

$(-3) = 6$

$a \times (-3)^2 + b \times (-3) + c = 0$

$9a - 3b + 6 = 0$

$[9a - 3b = -6] ----(ii)$

Multiplying eq. (i) by 3 and eq. (ii) by 2

$12a + \cancel{6b} = 15$

$\underline{18a - \cancel{6b} = -12}$

$30a = 3$

$$a=\frac{3}{30}=\frac{1}{10}$$

$$^{2}\cancel{4}\times\frac{1}{_{5}\cancel{10}}+2b=5$$

$$2b=5\frac{-2}{5}$$

$$2b=\frac{25-2}{5}=\frac{23}{5}$$

$$b=\frac{23}{10}$$

$$\therefore f(x)=\frac{1}{10}x^2+\frac{23}{10}x+6$$

32.Find the domain and the range of the function f defied by $f(x)=\frac{x+2}{|x+2|}$

Ans. $f(x)=\frac{x+2}{|x+2|}$

For Df , $f(x)$ must be a real no.

$$\Rightarrow |x+2|\neq 0\Rightarrow x+2\neq 0\Rightarrow x\neq -2$$

$\therefore$ Domain of f = set of all real numbers

$except-2\,i.e. Df=R-\{-2\}$

$for\ Rf$

$caseI\ \ if\ x+2>0\ then\ |x+2|=x+2$

$$\therefore f(x)=\frac{x+2}{|x+2|}=1$$

$caseII\ \ if\ \ x+2<0,|x+2|=-(x+2)$

$$\therefore f(x) = \left(\frac{\cancel{x+2}}{-x+\cancel{2}}\right) = -1$$

$$\therefore Range\ of\ f = \{-1, 1\}$$

33.Find the domain and the range of $f(x) = \dfrac{x^2}{1+x^2}$

Ans. $f(x) = \dfrac{x^2}{1+x^2}$

$$Domain\ of\ f = all\ real\ no. = R$$

$$for\ Range\ let\ f(x) = y$$

$$y = \frac{x^2}{1+x^2}$$

$$y(1+x^2) = x^2$$

$$y + yx^2 = x^2$$

$$y = x^2 - yx^2$$

$$y = (1-y)x^2$$

$$x^2 = \frac{y}{1-y}$$

$$x = \sqrt{\frac{y}{1-y}}$$

$$\frac{y}{1-y} \geq 0 \qquad 1-y \neq 0$$

$$y \neq 1$$

$$also\ y \geq 0\ and\ 1-y > 0$$

$$y < 1$$

$\therefore Range\ of\ f = [0, 1).$

34. If

$A = \{1, 2, 3\}, B = \{1, 2, 3, 4\}$. and

$R = \{(x, y) : (x, y) \in A \times B, y = x + 1\}$ then

(i) find $A \times B$ (ii) write domain and Range

Ans.

(i)

$A \times B = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4)\}$

(ii) $R = \{(1,2), (2,3), (3,4)\}$

$Domain\ of\ R = \{1.2.3\}$

$Range\ of\ R = \{2, 3, 4\}$

CBSE Class 12 Mathematics

Important Questions

Chapter 2

Relations and Functions

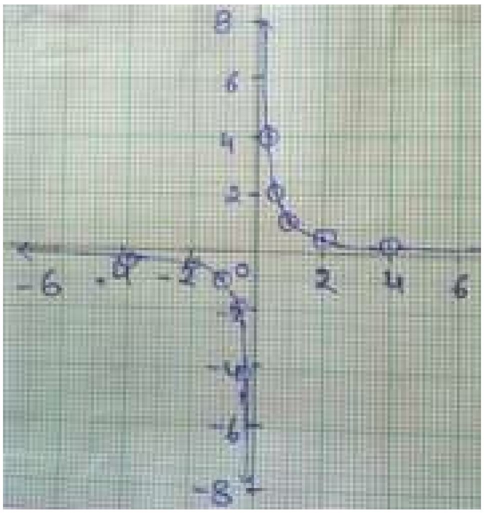
6 Marks Questions

1.Draw the graphs of the following real functions and hence find their range

$f(x)=\frac{1}{x}, x\in R, x\neq 0$

Ans. Given $f(x)=\frac{1}{x}, x\in R, x\neq 0$

Let $y=f(x)=i\ell y=\frac{1}{x}, x\in R, x\neq 0$



(Fig for Answer 11)

x	-4	-2	-1	-0.5	-0.25	0.5	1	2	4
$y=\frac{1}{x}$	-0.25	-0.5	-1	-2	-4	2	1	0.5	0.25

Plot the points shown is the above table and join there points by a free hand drawing.

Portion of the graph are shown the right margin

From the graph, it is clear that $Rf = R - [0]$

This function is called reciprocal function.

2.If $f(x) = x - \frac{1}{x}$, **Prove that** $[f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$

Ans. If $f(x) = x - \frac{1}{x}$, prove that $[f(x)]^3 = f(x^3) + f\left(\frac{1}{x}\right)$

Given $f(x) = x - \frac{1}{x}, Df = R - [0]$

$\Rightarrow f(x^3) = x^3 - \frac{1}{x^3}$ and $f\left(\frac{1}{x}\right) = \frac{1}{x} - \frac{1}{\frac{1}{x}} = \frac{1}{x} - x \ldots\ldots..(i)$

$\therefore [f(x)]^3 = \left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3x.\frac{1}{x}\left(x - \frac{1}{x}\right)$

$= x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$

$= x^3 - \frac{1}{x^3} + 3\left(\frac{1}{x} - x\right)$

$= f(x^3) + 3f\left(\frac{1}{x}\right)[\text{using } (i)]$

3.Draw the graphs of the following real functions and hence find their range

$(i)\ f(x) = 2x - 1 (ii)\ f(x) = \frac{x^2 - 1}{x - 1}$

Ans. (i) Given $f(x)\, i.e. y = x-1$, which is first degree equation in x, y and hence it represents a straight line. Two points are sufficient to determine straight lint uniquely

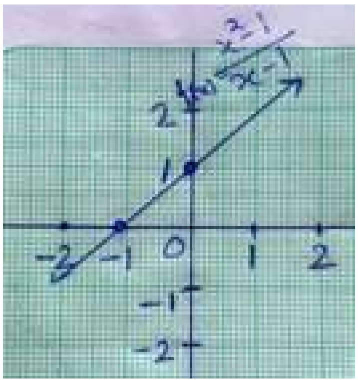
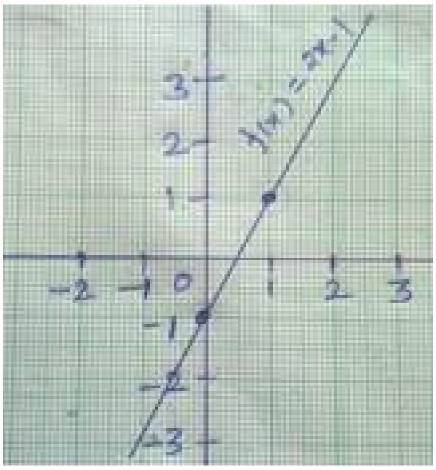


Table of values

x	**0**	**1**
y	-1	1

A portion of the graph is shown in the figure from the graph, it is clear that y takes all real values. It therefore that $R_F = R$

(ii) Given $f(x) = \frac{x^2-1}{x-1} \Rightarrow D_F = R-(1)$



Let $y = f(x) = \frac{x^2-1}{x-1} = x+1 (\because x \neq 1)$

i.e $y = x+1$, which is a first degree equation is x, y and hence it represents a straight line. Two points are sufficient to determine a straight line uniquely

Table of values

x	**-1**	**0**
y	0	1

A portion of the graph is shown is the figure from the graph it is clear that y takes all real values except 2. It fallows that $R_F = R - [2]$.

4.Let f be a function defined by $F : x \to 5x^2 + 2, x \in R$

(i) find the image of 3 under f

(ii) find $f(3) + f(2)$

(iii) find x **such that** $f(x) = 22$

Ans. Given $f(x) = 5x^2 + 2, x \in R$

(i) $f(3) = 5 \times 3^2 + 2 = 5 \times 9 + 2 = 47$

(ii) $f(2) = 5 \times 2^2 + 2 = 5 \times 4 + 2 = 22$

$\therefore f(3) \times f(2) = 47 \times 22 = 1034$

(iii) $f(x) = 22$

$\Rightarrow 5x^2 + 2 = 22$

$\Rightarrow 5x^2 = 20$

$\Rightarrow x^2 = 4$

$\Rightarrow x = 2, \ -2$

5.The function $f(x) = \frac{9x}{5} + 32$ **is the formula to connect** $x^\circ C$ **to Fahrenheit**

units find $(i)\, f(0)\quad (ii)\, f(-10)\quad (iii)$ the value of x $f(x)=212$ **interpret the result is each case**

Ans. $f(x)=\dfrac{9x}{5}+32(given)$

$$(i)\; f(0)=\left(\frac{9\times 0}{5}+32\right)=32\Rightarrow f(0)=32\Rightarrow 0^{\circ}c=32^{\circ}F$$

$$(ii)\; f(-10)=\left(\frac{9\times(-10)}{5}+32\right)=14\Rightarrow f(-10)=14^{\circ}\Rightarrow(-10)^{\circ}c=14^{\circ}F$$

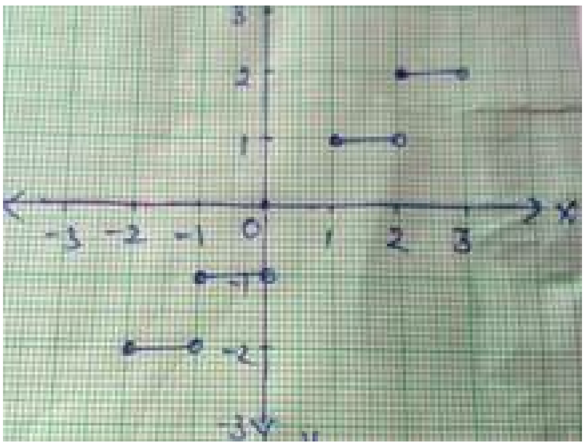
$$(iii)\; f(x)=212\Leftrightarrow\frac{9x}{5}+32=212\Leftrightarrow 9x=5\times(180)$$

$$\Leftrightarrow x=100$$

$$\therefore 212^{\circ}f=100^{\circ}c$$

6.Draw the graph of the greatest integer function, $f(x)=[x]$.

Ans. Clearly, we have



$$f(x)=\begin{cases}-2,\ \text{when } x\in[-2,-1)\\ -1,\text{when } x\in[-1,0)\\ 0,\text{when } x\in[0,1)\\ 1,\text{when } x\in[1,2)\end{cases}$$

x	......	$-2 \le x < 1$	$-1 \le x < 0$	$0 \le x < 1$	$1 \le x < 2$	$2 \le x < 3$	......
y	......	-2	-1	0	1	2	......

7.Find the domain and the range of the following functions:

$(i)\ f(x) = \sqrt{x^2 - 4}$ $\quad (ii)\ f(x) = \sqrt{16 - x^2}$ $(iii) \quad f(x) = \dfrac{1}{\sqrt{9 - x^2}}$

Ans. (i)Given $f(x) = \sqrt{x^2 - 4}$

For $Df_1 f(x)$ must be a real number

$\Rightarrow \sqrt{x^2 - 4}$ Must be a real number

$\Rightarrow x^2 - 4 \ge 0 \Rightarrow (x+2)(x-2) \ge 0$

$\Rightarrow$ either $x \le -2$ or $x \ge 2$

$\Rightarrow D_F = (-\infty, -2] \cup [2, \infty)$.

For R_F, let $y = \sqrt{x^2 - 4}$.........(i)

As square root of a real number is always non-negative, $y \ge 0$

On squaring **(i),** we get $y^2 = x^2 - 4$

$\Rightarrow x^2 = y^2 + 4$ but $x^2 \ge 0$ for all $x \in D_F$

$\Rightarrow y^2 + 4 \ge 0 \Rightarrow y^2 \ge -4$, which is true for all $y \in R$. also $y \ge 0$

$\Rightarrow R_F = [0, \infty)$

(ii) Given $f(x) = \sqrt{16 - x^2}$

For D_F, $f(x)$ must be a real number

$\Rightarrow \sqrt{16-x^2}$ must be a real number

$\Rightarrow \sqrt{16-x^2} \geq 0 \Rightarrow -\left(x^2-16\right) \geq 0$

$\Rightarrow x^2-16 \leq 0$

$\Rightarrow (x+4)(x-4) \leq 0 \Rightarrow -4 \leq x \leq 4$

$\Rightarrow D_F = [-4, 4]$.

For R_F, let $y = \sqrt{16-x^2}$........(i)

As square root of real number is always non-negative, $y \geq 0$

Squaring (i) we get

$y^2 = 16 - x^2$

$\Rightarrow x^2 = 16 - y^2$ but $x^2 \geq 0$ for all $x \in D_f$

$\Rightarrow 16 - y^2 \geq 0 \Rightarrow -\left(y^2-16\right) \geq 0 \Rightarrow y^2 - 16 \leq 0$

$\Rightarrow (y+4)(y-4) \leq 0 \Rightarrow -4 \leq y \leq 4$ but $y \geq 0$

$\Rightarrow R_F = [0, 4]$

(iii) Given $f(x) = \dfrac{1}{\sqrt{9-x^2}}$

For D_F, $f(x)$ must be a real number

$\Rightarrow \dfrac{1}{\sqrt{9-x^2}}$ must be a real number

$$\Rightarrow 9 - x2 > 0 \Rightarrow -\left(x^2 - 9\right) > 0 \Rightarrow x^2 - 9 < 0$$

$$\Rightarrow (x+3)(x-3) < 0 \Rightarrow -3 < x < 3 \Rightarrow D_F = (-3,3)$$

For $R_{f.}$ let $y = \dfrac{1}{\sqrt{9-x^2}}, y \neq 0$.........$(i)$

Also as the square root of a real number is always non-negative, $y > 0$.

on squaring (i) we get

$$y2 = \frac{1}{9-x^2} \Rightarrow 9 - x^2 = \frac{1}{y^2} \Rightarrow x^2 = 9 - \frac{1}{y^2}$$

But $x^2 \geq 0$ for all $x \in D_F \Rightarrow 9 - \dfrac{1}{y^2} \geq 0$

$$y^2 > 0$$

(Multiply bath sides by y^2, a positive real number)

$$\Rightarrow 9y^2 - 1 \geq 0 \Rightarrow y^2 - \frac{1}{9} \geq 0$$

$$\Rightarrow \left(y + \frac{1}{3}\right)\left(y - \frac{1}{3}\right) \geq 0$$

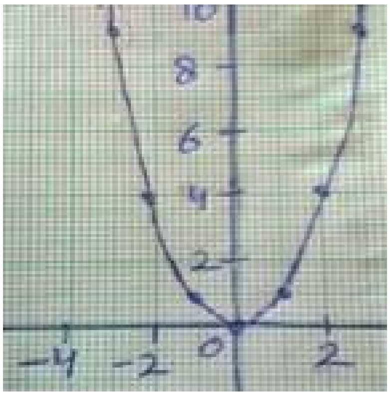
$\Rightarrow$ either $y \leq -\dfrac{1}{3}$ or $y \geq \dfrac{1}{3}$

$$y > 0 \Rightarrow y \geq \frac{1}{3}$$

$$\Rightarrow R_F = [\frac{1}{3}, \infty).$$

8.Draw the graphs of the following real functions and hence find range: $f(x) = x^2$

Ans.



Given $f\left(x\right)=x^2 \Rightarrow D_F=R$

Let $y=f\left(x\right)=x^2, x\in R$

x	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**
$y=x^2$	16	9	4	1	0	1	4	9	16

Plot the points

$\left(-4,16\right),\left(-3,9\right),\left(-2,4\right),\left(-1,1\right),\left(0,0\right),\left(1,1\right)\left(2,4\right),\left(3,9\right),\left(4,16\right).....,$

And join these points by a free hand drawing. A portion of the graph is shown in sigma (next)

From the graph, it is clear that y takes all non-negative real values, if follows that $R_F=\left[0,\infty\right)$

9.Define polynomial function. Draw the graph of $f\left(x\right)=x^3$ find domain and range

Ans. A function $f:R\rightarrow R$ define by

$$f\left(x\right)=a_0+a_1x+a_2x^2+---+a_nx^n$$
$$where\ a_0,a_1,a_2,---a_n \in R$$

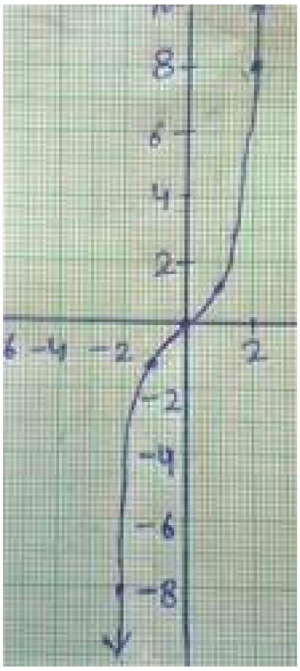
And n is non negative integer is called polynomial function

Graph of $f\left(x\right)=x^3$

x	**0**	**1**	**2**	**-1**	**-2**
$f(x)$	0	1	8	-1	-8

Domain of $f = R$

Range of $f = R$



10.(a) If A, B are two sets such that $n(A\times B)=6$ and some elements of $A\times B$ are $(-1,2),(2,3),(4,3)$, than find $A\times B$ and $B\times A$

(b) Find domain of the function $f(x)=\dfrac{1}{\sqrt{x+[x]}}$

Ans. (a) Given A and B are two sets such that

$n(A\times B)=6$

Some elements of $A\times B$ are

$(-1,2),(2,3)\ and\ (4,3)$

$then\ A=\{-1,2,4\}\ and\ B=\{2,3\}$

$$A \times B = \{(-1,2),(-1,3),(2,2),(2,3),(4,2),(4,3)\}$$

$$B \times A = \{(2,-1),(3,-1),(2,2),(3,2),(2,4),(3,4)\}$$

(b)

$$f(x) = \frac{1}{\sqrt{x+[x]}}$$

we knowe that

$x+[x] > 0$ for all $x > 0$

$x+[x] = 0$ for all $x = 0$

$x+[x] < 0$ for all $x < 0$

also $f(x) = \dfrac{1}{\sqrt{x+[x]}}$ is defined for all

x satisfying $x+[x] > 0$

Hence, Domain $(f) = (0, \infty)$