Unit 9 (Sequence And Series)

Short Answer Type Questions:

Q1. The first term of an A.P. is a and the sum of the first p terms is zero, show that the sum of its next q terms

$\frac{-a(p+q)q}{p-1}$

Sol: Let the common differeence of the given A.P be d

Given that $S_p = 0$

$$\Rightarrow \quad \frac{p}{2}[2a+(p-1)d]=0 \quad \Rightarrow 2a+(p-1)d=0 \quad \Rightarrow d=\frac{-2a}{p-1}$$

Now, sum of next q terms

$$= S_{p+q} - S_p = S_{p+q} - 0$$

$$= \frac{p+q}{2}[2a+(p+q-1)d] = \frac{p+q}{2}[2a+(p-1)d+qd]$$

$$= \frac{p+q}{2}\left[0+\frac{q(-2a)}{p-1}\right] = \frac{-a(p+q)q}{(p-1)}$$

Q2. A man saved Rs. 66000 in 20 years. In each succeeding year after the first year, he saved Rs. 200 more than what he saved in the previous year. How much did he save in the first year?

Sol: Let us assume that the man saved Rs.a in the first year.

In each succeeding year, an increment of Rs. 200 is made. So, it forms an A.P. whose

First term = a, Common difference, d = 200 and n=20

$$\therefore \quad S_{20} = \frac{20}{2}[2a+(20-1)d]$$

=> 6600 = 2a + 19 x 200 => 2a = 2800

∴a = 1400

Q3. A man accepts a position with an initial salary of Rs. 5200 per month. It is understood that he will receive an automatic increase of Rs. 320 in the very next month and each month thereafter.

(i) Find his salary for the tenth month.

(ii) What is his total earnings during the first year?

Sol: The man gets a fixed increment of Rs. 320 each month. Therefore, this forms an A.P. whose

First term, a = 5200 and Common difference, d = 320

(i) Salary for 10th month will be given by a_n, where $n = 10$.

$\therefore$ Salary for 10th month $= a_{10}$

$$= a + (n-1)d$$
$$= 5200 + (10-1) \times 320$$
$$= 5200 + 9 \times 320 = 5200 + 2880 = ₹8080$$

(ii) Total earnings during the first year is equal to the sum of 12 terms of the A.P.

$\therefore$ Total earnings $= S_{12}$

$$= \frac{12}{2}[2 \times 5200 + (12-1)\,320]$$
$$= 6[10400 + 11 \times 320]$$
$$= 6[10400 + 3520] = 6 \times 13920 = ₹83520$$

4. If the pth and qth terms of a G.P. are q and p respectively, then show that its $(p+q)$th term is $\left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$.

Sol. Let the first term and common ratio of G.P. be a and r, respectively.

Given that, pth term $= q \Rightarrow ar^{p-1} = q$ ...(i)

and qth term $= p \Rightarrow ar^{q-1} = p$ ...(ii)

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{ar^{p-1}}{ar^{q-1}} = \frac{q}{p} \Rightarrow r^{p-q} = \frac{q}{p} \Rightarrow r = \left(\frac{q}{p}\right)^{\frac{1}{p-q}}$$

On substituting the value of r in Eq. (i), we get

$$a\left(\frac{q}{p}\right)^{\frac{p-1}{p-q}} = q \Rightarrow a = q\left(\frac{p}{q}\right)^{\frac{p-1}{p-q}}$$

$\therefore$ $(p+q)$th term, $T_{p+q} = a \cdot r^{p+q-1}$

$$= q\left(\frac{p}{q}\right)^{\frac{p-1}{p-q}}\left(\frac{q}{p}\right)^{\frac{p+q-1}{p-q}} = q\left(\frac{p}{q}\right)^{\frac{p-1}{p-q}-\frac{p+q-1}{p-q}}$$

$$= q\left(\frac{q}{p}\right)^{\frac{q}{p-q}} = \frac{q^{\frac{q}{p-q}+1}}{p^{\frac{q}{p-q}}} = \frac{q^{\frac{p}{p-q}}}{p^{\frac{q}{p-q}}} = \left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$$

Q5. A carpenter was hired to build 192 window frames. The first day he made five frames and each day, thereafter he made two more frames than he made the day before. How many days did it take him to finish the job?

Sol: Here, a = 5 and d = 2

Let the carpenter finish the job in n days.

Then, $S_n = 192$

$$\Rightarrow \quad 192 = \frac{n}{2}[2a + (n-1)d] \quad \Rightarrow \quad 192 = \frac{n}{2}[2 \times 5 + (n-1)2]$$

$$\Rightarrow \quad 192 = n[5 + n - 1] \quad \Rightarrow \quad n^2 + 4n - 192 = 0 \quad \Rightarrow \quad (n-12)(n+16) = 0$$

$$\therefore \quad n = 12$$

Q6. The sum of interior angles of a triangle is 180°. Show that the sum of the interior angles of polygons with 3, 4, 5, 6, … sides form an arithmetic progression. Find the sum of the interior angles for a 21 sided polygon.

Sol: We know that, sum of interior angles of a polygon of side n is (n – 2) x 180°.

Let $t_n = (n-2) \times 180°$

Since t_n is linear in n, it is nth term of some A.P.

$$t_3 = a = (3-2) \times 180° = 180°$$

Common difference, $d = 180°$

Sum of the interior angles for a 21 sided polygon is:

$$t_{21} = (21-2) \times 180° = 3420°$$

Q7. A side of an equilateral triangle is 20 cm long. A second equilateral triangle is inscribed in it by joining the mid-points of the sides of the first triangle. This process is continued for third, fourth, fifth, triangles. Find the perimeter of the sixth inscribed equilateral triangle.

Sol: Let the given equilateral triangle be Δ ABC with each side of 20 cm.

By joining the mid-points of this triangle, we get another equilateral triangle of side equal to half of the length of side of ΔABC.

Continuing in this way, we get a set of equilateral triangles with side equal to half of the side of the previous triangle.

Now,

Perimeter of first triangle = 20 x 3 = 60 cm;

Perimeter of second triangle = 10 x 3 = 30 cm;

Perimeter of third triangle = 5×3 = 15 cm;

Clearly, 60, 30, 15, …, form a G.P. with $a = 60$ and $r = \frac{30}{60} = \frac{1}{2}$.

We have, to find perimeter of sixth inscribed triangle i.e., we have to find the sixth term of the G.P.

$\therefore$ Perimeter of sixth inscribed triangle

$$= a_6 = ar^{6-1} = 60 \times \left(\frac{1}{2}\right)^5 = \frac{60}{32} = \frac{15}{8} \text{cm}$$

Q8. In a potato race 20 potatoes are placed in a line at intervals of 4 m with the first potato 24 m from the starting point. A contestant is required to bring the potatoes back to the

starting place one at a time. How far would he run in bringing back all the potatoes?

Sol: Distance travelled to bring first potato = 24 + 24 = 2 x 24 = 48 m

Distance travelled to bring second potato = 2(24 + 4) = 2 x 28 = 56 m

Distance travelled to bring third potato = 2(24 + 4 + 4) = 2 X 32 = 64 m; and so on…

Clearly, 48, 56, 64,… is an A.P. with first term 48 and common difference 8. Also, number of terms is 20.

Total distance run in bringing back all the potatoes,

$$S_{20} = \frac{20}{2}[2 \times 48 + (20 - 1) \times 8] = 20[48 + 76] = 20 \times 124 = 2480 \text{ m}$$

Q9. In a cricket tournament 16 school teams participated. A sum of Rs. 8000 is to be awarded among themselves as prize money. If the last placed team is awarded Rs. 275 in prize money and the award increases by the same amount for successive finishing places, how much amount will the first place team receive?

Sol: Let the first place team get Rs. a as the prize money.

Since award money increases by the same amount for successive finishing places, we get an A.P.

Let the constant amount be d.

Here, $t_{16} = 275$, $n = 16$ and $S_{16} = 8000$

$\therefore$ $t_{16} = a + (16 - 1)(-d)$

$\Rightarrow$ $275 = a - 15d$ …(i)

Also, $S_{16} = \frac{16}{2}[2a + (n - 1)(-d)] \Rightarrow 8000 = 8[2a + (16 - 1)(-d)]$

$\Rightarrow$ $1000 = 2a - 15d$ …(ii)

Solving Eqs. (i) and (ii), we get $a = 725$.

Hence, first place team receives ₹725.

10. If $a_1, a_2, a_3, \ldots, a_n$ are in A.P., where $a_i > 0$ for all i, show that

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \cdots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} = \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}}$$

Sol. Given that, $a_1, a_2, \ldots, a_n$ are in A.P., $\forall\, a_i > 0$

$\therefore$ $a_1 - a_2 = a_2 - a_3 = \ldots = a_{n-1} - a_n = -d$ (constant) (1)

Now, $\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \cdots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}}$

$$= \frac{\sqrt{a_1} - \sqrt{a_2}}{a_1 - a_2} + \frac{\sqrt{a_2} - \sqrt{a_3}}{a_2 - a_3} + \cdots + \frac{\sqrt{a_{n-1}} - \sqrt{a_n}}{a_{n-1} - a_n} \quad \text{(rationalizing)}$$

$$= \frac{\sqrt{a_1} - \sqrt{a_2}}{-d} + \frac{\sqrt{a_2} - \sqrt{a_3}}{-d} + \cdots + \frac{\sqrt{a_{n-1}} - \sqrt{a_n}}{-d}$$

$$= \frac{1}{-d}[\sqrt{a_1} - \sqrt{a_n}]$$

$$= \frac{a_1 - a_n}{-d(\sqrt{a_1} + \sqrt{a_n})} \quad \text{(rationalizing)}$$

$$= \frac{-(n-1)d}{-d(\sqrt{a_1} + \sqrt{a_n})} \quad (\text{as } a_n = a_1 + (n-1)d)$$

$$= \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}}$$

Q11. Find the sum of the series $(3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + \ldots$ to

(i) n terms

(ii) 10 terms

Sol: Given series is: $(3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + \ldots$ n terms

$$T_n = (2n+1)^3 - (2n)^3 = (2n+1-2n)[(2n+1)^2 + (2n+1)2n + (2n)^2]$$
$$= 12n^2 + 6n + 1$$

(i) Sum of n terms,

$$S_n = \sum_{n=1}^{n}(12n^2 + 6n + 1) = 12 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{6n(n+1)}{2} + n$$
$$= 2n(n+1)(2n+1) + 3n(n+1) + n$$
$$= 2n(2n^2 + 3n + 1) + 3n^2 + 3n + n$$
$$= 4n^3 + 9n^2 + 6n$$

(ii) Sum of 10 terms, $S_{10} = 4 \times (10)^3 + 9 \times (10)^2 + 6 \times 10$
$$= 4000 + 900 + 60 = 4960$$

Q12. Find the rth term of an A.P. sum of whose first n terms is $2n + 3n^2$

Sol: Sum of k terms of A.P., $S_n = 2n + 3n^2$

$$\therefore \quad T_n = S_n - S_{n-1} = (2n + 3n^2) - [2(n-1) + 3(n-1)^2]$$
$$= [2n - 2(n-1)] + [3n^2 - 3(n-1)^2]$$
$$= 2 + 3(n - n + 1)(n + n - 1)$$
$$= 2 + 3(2n - 1) = 6n - 1$$

$\therefore$ rth term $T_r = 6r - 1$

Long Answer Type Questions

Q13. If A is the arithmetic mean and G_1 , G_2 be two geometric means between any two numbers, then prove that 2A = $\frac{G_1^2}{G_2} + \frac{G_2^2}{G_2}$

Sol: Let the numbers be a and b.

Then, $A = \dfrac{a+b}{2}$ or $2A = a + b$ (i)

Also, G_1 and G_2 are geometric means between a and b, then a, G_1, G_2, b are in G.P.

Let r be the common ratio.

Then, $b = ar^{4-1} = ar^3 \Rightarrow \dfrac{b}{a} = r^3 \Rightarrow r = \left(\dfrac{b}{a}\right)^{1/3}$

$$\therefore \quad G_1 = ar = a\left(\frac{b}{a}\right)^{1/3} = a^{2/3}b^{1/3}$$

and $$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{2/3} = a^{1/3}b^{2/3}$$

$$\therefore \quad \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1} = \frac{G_1^3 + G_2^3}{G_1G_2} = \frac{a^2b + ab^2}{ab} = a + b = 2\text{ A}$$

14. If $\theta_1, \theta_2, \theta_3, \ldots, \theta_n$ are in A.P., whose common difference is d, show that

$$\sec\theta_1 \sec\theta_2 + \sec\theta_2 \sec\theta_3 + \ldots + \sec\theta_{n-1} \sec\theta_n = \frac{\tan\theta_n - \tan\theta_1}{\sin d}.$$

Sol. Since $\theta_1, \theta_2, \theta_3, \ldots, \theta_n$ are in A.P., we get

$$\theta_2 - \theta_1 = \theta_3 - \theta_2 = \ldots = \theta_n - \theta_{n-1} = d \qquad \text{(i)}$$

Now, $\sec\theta_1 \sec\theta_2 = \frac{1}{\sin d} \cdot \frac{\sin d}{\cos\theta_1 \cos\theta_2} = \frac{1}{\sin d} \cdot \frac{\sin(\theta_2 - \theta_1)}{\cos\theta_1 \cos\theta_2}$

$$= \frac{1}{\sin d} \cdot \frac{(\sin\theta_2 \cos\theta_1 - \cos\theta_2 \sin\theta_1)}{\cos\theta_1 \cos\theta_2} = \frac{\tan\theta_2 - \tan\theta_1}{\sin d}$$

Similarly, $\sec\theta_2 \sec\theta_3 = \frac{\tan\theta_3 - \tan\theta_2}{\sin d}$; $\sec\theta_3 \sec\theta_4 = \frac{\tan\theta_4 - \tan\theta_3}{\sin d}$;

and so on.

$\therefore \quad \sec\theta_1 \sec\theta_2 + \sec\theta_2 \sec\theta_3 + \ldots + \sec\theta_{n-1} \sec\theta_n = \frac{\tan\theta_n - \tan\theta_1}{\sin d}$

Q15. 1f the sum of p terms of an AP. is q and the sum of q terms isp, then show that the sum ofp + q terms is —(p + q). Also, find the sum of first p — q terms (where, p > q).

Sol: Let first term and common difference of the A.P. be a and d, respectively. Given, $S_p = q$

$$\Rightarrow \quad \frac{p}{2}[2a + (p-1)d] = q \quad \Rightarrow \quad 2a + (p-1)d = \frac{2q}{p} \qquad \text{(i)}$$

Also, $S_q = p$

$$\Rightarrow \quad \frac{q}{2}[2a + (q-1)d] = p \quad \Rightarrow \quad 2a + (q-1)d = \frac{2p}{q} \qquad \text{(ii)}$$

On subtracting Eq. (ii) from Eq. (i), we get

$$(p-q)d = \frac{2q}{p} - \frac{2p}{q} \text{ or } (p-q)d = \frac{2(q^2 - p^2)}{pq}$$

$$\therefore \quad d = \frac{-2(p+q)}{pq} \qquad \text{(iii)}$$

On substituting the value of d into Eq. (i), we get

$$2a + (p-1)\left(\frac{-2(p+q)}{pq}\right) = \frac{2q}{p}$$

$$\Rightarrow \quad a = \frac{q}{p} + \frac{(p+q)(p-1)}{pq} \qquad \text{(iv)}$$

$$\text{Now, } S_{p+q} = \frac{p+q}{2}[2a + (p+q-1)d]$$

$$= \frac{p+q}{2}\left[\frac{2q}{p} + \frac{2(p+q)(p-1)}{pq} - \frac{2(p+q-1)(p+q)}{pq}\right]$$

$$= (p+q)\left[\frac{q}{p} + \frac{(p+q)(p-1-p-q+1)}{pq}\right]$$

$$= (p+q)\left[\frac{q}{p} + \frac{(p+q)(-q)}{pq}\right]$$

$$= (p+q)\left[\frac{q}{p} - \frac{p+q}{p}\right] = -(p+q)$$

$$\text{Also, } S_{p-q} = \frac{p-q}{2}[2a + (p-q-1)d]$$

$$= \frac{p-q}{2}\left[\frac{2q}{p} + \frac{2(p+q)(p-1)}{pq} - \frac{(p-q-1)2(p+q)}{pq}\right]$$

$$= (p-q)\left[\frac{q}{p} + \frac{(p+q)(p-1-p+q+1)}{pq}\right]$$

$$= (p-q)\left[\frac{q}{p} + \frac{(p+q)q}{pq}\right]$$

$$= (p-q)\left[\frac{q}{p} + \frac{p+q}{p}\right] = (p-q)\frac{(p+2q)}{p}$$

Q16. If pth, qth and rth terms of an A.P. and G.P. are both a, b and c, respectively, then show that $a^{b-c}\, b^{c-a} \cdot c^{a-b} = 1$.

Sol: Let A and d be the first term and common difference of A.P., respectively. Also, let B and R be the first term and common ratio of G.P., respectively.

It is given that,

$$A + (p-1)d = a \quad \text{(i)}$$
$$A + (q-1)d = b \quad \text{(ii)}$$
$$A + (r-1)d = c \quad \text{(iii)}$$

Also,

$$a = BR^{p-1} \quad \text{(iv)}$$
$$b = BR^{q-1} \quad \text{(v)}$$
$$c = BR^{r-1} \quad \text{(vi)}$$

On subtracting Eq. (ii) from Eq. (i), we get

$$a - b = d(p-q)$$

On subtracting Eq. (iii) from Eq. (ii), we get

$$b - c = d(q-r)$$

On subtracting Eq. (i) from Eq. (iii), we get

$$c - a = d(r-p)$$

$$\therefore \quad a^{b-c} \cdot b^{c-a} \cdot c^{a-b}$$

$$= (BR^{(p-1)})^{d(q-r)} (BR^{(q-1)})^{d(r-p)} (BR^{(r-1)})^{d(p-q)}$$

$$= B^{d[(q-r)+(r-p)+(p-q)]} R^{d[(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)]}$$

$$= B^0 R^0 = 1$$

Objective Type Questions:

Q17. If the sum of n terms of an A.P. is given by $S_n = 3n + 2n^2$, then the common difference of the A.P. is

(a) 3

(b) 2

(c) 6

(d) 4

Sol: (d) Given, $S_n = 3n + 2n^2$

$S_1 = 3(1) + 2(1)^2 = 5 = t_1$

$S_2 = 3(2) + 2(2)^2 = 14 = t_1 + t_2$

$\therefore \quad S_2 - S_1 = 9 = t_2$

$\therefore \quad d = t_2 - t_1 = 9 - 5 = 4$

Q18. If the third term of G.P. is 4, then the product of its first 5 terms is

(a) 4^3

(b) 4^4

(c) 4^5

(d) none of these

Sol: (c) Let a and r the first term and common ratio, respectively.

Given that the third term is 4.

$\therefore \quad ar^2 = 4$

Product of first 5 terms $= a \cdot ar \cdot ar^2 \cdot ar^3 \cdot ar^4 = a^5 r^{10} = (ar^2)^5 = 4^5$

Q19. If 9 times the 9th term of an A.P. is equal to 13 times the 13th term, then the 22nd term of the A.P. is

(a) 0

(b) 22

(c) 198

(d) 220

Sol: (a) Let the first term and coiqmon difference of given A.P. be a and d, respectively.

It is given that $9 \cdot t_9 = 13 \cdot t_{13}$

$\Rightarrow \quad 9(a + 8d) = 13(a + 12d) \quad \Rightarrow \quad 9a + 72d = 13a + 156d$

$\Rightarrow \quad 4a + 84d = 0 \quad \Rightarrow \quad 4(a + 21d) = 0$

$\Rightarrow \quad t_{22} = 0$

Q20. If x, 2y and 3z are in A.P. where the distinct numbers x, y and z are in G.P., then the common ratio of the G.P.is

Sol: Since x, 2y and 3z are in A.P., we get

$$2y = \frac{x + 3z}{2}$$

$\Rightarrow \quad 4y = x + 3z \qquad$ (i)

Also, x, y and z are in G.P.

Therefore, $y = xr$ and $z = xr^2$, where 'r' is the common ratio.

$\therefore \quad 4xr = x + 3xr^2 \qquad$ [Using (i)]

$\Rightarrow \quad 4r = 1 + 3r^2 \quad \Rightarrow \quad 3r^2 - 4r + 1 = 0 \quad \Rightarrow \quad (3r - 1)(r - 1) = 0$

$\Rightarrow \quad r = \frac{1}{3}$ (For $r = 1$; x, y, z are not distinct)

Q21. If in an A.P., $S_n = qn^2$ and $S_m = qm^2$, where S_r denotes the sum of r terms of the AP, then S_q equals

(a) $\frac{q^3}{2}$ (b) mnq (c) q^3 (d) $(m+n)q^2$

Sol. **(c)** Given, $S_n = qn^2$ and $S_m = qm^2$

$\therefore \quad S_1 = q, S_2 = 4q, S_3 = 9q$ and $S_4 = 16q$

Now, $\quad t_1 = q$

$\therefore \quad t_2 = S_2 - S_1 = 4q - q = 3q$

$t_3 = S_3 - S_2 = 9q - 4q = 5q$

$t_4 = S_4 - S_3 = 16q - 9q = 7q$

So, the A.P. is: $q, 3q, 5q, 7q, \ldots$

Thus, first term is q and common difference is $3q - q = 2q$.

$$\therefore \quad S_q = \frac{q}{2}[2 \times q + (q-1)2q] = \frac{q}{2} \times [2q + 2q^2 - 2q] = \frac{q}{2} \times 2q^2 = q^3$$

Q22. Let S_n denote the sum of the first n terms of an A.P. If $S_{2n} = 3S_n$, then $S_{3n} : S_n$ is equal to

(a) 4

(b) 6

(c) 8

(d) 10

Sol: (b) Let first term be a and common difference be d.

Then, $S_{2n} = 3S_n$

$$\Rightarrow \quad \frac{2n}{2}[2a + (2n-1)d] = \frac{3n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad 2[2a + (2n-1)d] = 3[2a + (n-1)d]$$

$$\Rightarrow \quad 4a + (4n-2)d = 6a + (3n-3)d$$

$$\Rightarrow \quad 2a = (n+1)d \qquad \text{(i)}$$

$$\text{Now,} \quad \frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}[2a + (3n-1)d]}{\frac{n}{2}[2a + (n-1)d]} = \frac{3[2a + (3n-1)d]}{[2a + (n-1)d]}$$

$$= \frac{3[(n+1)d + (3n-1)d]}{[(n+1)d + (n-1)d]} = \frac{3[4nd]}{2nd} = 6$$

23. The minimum value of $4^x + 4^{1-x}, x \in R$ is

(a) 2 (b) 4 (c) 1 (d) 0

Sol. (b) $4^x + 4^{1-x} = 4^x + \frac{4}{4^x} = \left(2^x - \frac{2}{2^x}\right)^2 + 4$

$\therefore$ $4^x + 4^{1-x} \geq 4$

24. Let S_n denote the sum of the cubes of the first n natural numbers and s_n denote the sum of the first n natural numbers.

Then $\sum_{r=1}^{n} \frac{S_r}{s_r}$ equals to

(a) $\frac{n(n+1)(n+2)}{6}$ (b) $\frac{n(n+1)}{2}$

(c) $\frac{n^2+3n+2}{2}$ (d) none of these

Sol. (a) $\sum_{r=1}^{n} \frac{S_r}{s_r} = \sum_{r=1}^{n} \frac{\left[\frac{r(r+1)}{2}\right]^2}{\frac{r(r+1)}{2}} = \sum_{r=1}^{n} \frac{r(r+1)}{2} = \frac{1}{2}\left[\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r\right]$

$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$

$= \frac{1}{2} \cdot \frac{n(n+1)}{2}\left[\frac{2n+1}{3} + 1\right] = \frac{n(n+1)}{4}\left[\frac{2n+4}{3}\right]$

$= \frac{n(n+1)(n+2)}{6}$

Q25. If t_n denotes the nth term of the series 2 + 3+ 6+11 + 18+..., then t_{50} is

(a) $49^2 - 1$

(b) 49^2

(c) 50^2+l

(d) $49^2 + 2$

Sol. (d) $S_{50} = 2 + 3 + 6 + 11 + 18 + \ldots + t_{49} + t_{50}$ (i)

$\therefore$ $S_{50} = 0 + 2 + 3 + 6 + 11 + 18 + \ldots + t_{49} + t_{50}$ (ii)

On subtracting Eq. (ii) from Eq. (i), we get

$0 = (2 + 1 + 3 + 5 + 7 + \ldots \text{up to 50 terms}) - t_{50}$

$\Rightarrow$ $t_{50} = 2 + [1 + 3 + 5 + 7 + \ldots \text{ upto 49 terms}]$

$= 2 + \frac{49}{2}[2 \times 1 + (49-1) \times 2] = 2 + 49(1 + 48) = 2 + 49^2$

Q26. The lengths of three unequal edges of a rectangular solid block are in G.P. If the volume of the block is 216 cm^3 and the total surface area is 252 cm^2, then the length of the longest edge is

(a) 12 cm

(b) 6 cm

(c) 18 cm

(d) 3 cm

Sol: (a) Let the length, breadth and height of rectangular solid block be a/r, a and ar, respectively.

$\therefore$ Volume $= \frac{a}{r} \times a \times ar = 216 \text{ cm}^3$

$\Rightarrow$ $a^3 = 216 = 6^3 \Rightarrow a = 6$

Also, Surface area $= 2\left(\frac{a}{r} \cdot a + a \cdot ar + \frac{a}{r} \cdot ar\right) = 252$

$\Rightarrow$ $2a^2\left(\frac{1}{r} + r + 1\right) = 252 \Rightarrow 2 \times 36\left(\frac{1 + r^2 + r}{r}\right) = 252$

$\Rightarrow$ $2(1 + r^2 + r) = 7r \Rightarrow 2r^2 - 5r + 2 = 0 \Rightarrow (2r - 1)(r - 2) = 0$

$\therefore$ $r = \frac{1}{2}, 2$

For $r = \frac{1}{2}$: Length $= \frac{a}{r} = \frac{6 \times 2}{1} = 12$, Breadth $= a = 6$,

Height $= ar = 6 \times \frac{1}{2} = 3$

For $r = 2$: Length $= \frac{a}{r} = \frac{6}{2} = 3$, Breadth $= a = 6$, Height $= ar = 6 \times 2 = 12$

Fill in the Blanks

Q27. If a, b and c are in G.P., then the value of $\frac{a-b}{b-c}$ is equal to ________

Sol: Given that, a, b and c are in G.P.

$\Rightarrow$ $b = ar$ and $c = ar^2$, where r is the common ratio.

$\Rightarrow$ $\frac{a-b}{b-c} = \frac{a - ar}{ar - ar^2} = \frac{a(1-r)}{ar(1-r)} = \frac{1}{r} = \frac{a}{b}$ or $\frac{b}{c}$

Q28. The sum of terms equidistant from the beginning and end in an A.P. is equal to _____.

Sol: Let a be the first term and d be ihe common difference of the A.P.

t_r = rth term from the beginning $= a + (r - 1)d$

t'_r = rth term from the end

$= (a + (n - 1)d) + (r - 1)(-d)$

(as first term is $t_n = a + (n - 1)d$ and common difference is '$-d$')

Now, $t_r + t'_r = a + (r - 1)d + (a + (n - 1)d) + (r - 1)(-d)$

$= 2a + (n - 1)d$, which is independent of 'r'

Thus, sum of the terms equidistant from the beginning and end in an A.P. is constant

Q29. The third term of a G.P. is 4. The product of the first five terms is

Sol: Let a and r the first term and common ratio, respectively.

Given that the third term is 4.

$\therefore$ $ar^2 = 4$

Product of first 5 terms $= a \cdot ar \cdot ar^2 \cdot ar^3 \cdot ar^4 = a^5 r^{10} = (ar^2)^5 = 4^5$

True/False Type Questions

Q30. Two sequences cannot be in both A.P. and G.P. together.

Sol: False

Consider the sequence 3,3,3; which is A.P. and G.P. both.

Q31. Every progression is a sequence but the converse, i.e., every sequence is also a progression need not necessarily be true.

Sol: True

Consider the progression a, a + d, a + 2d, ... and sequence of prime number 2, 3, 5, 7, 11,...

Clearly, progression is a sequence but sequence is not progression because it does not follow a specific pattern.

Q32. Any term of an A.P. (except first) is equal to half the sum of terms which are equidistant from it.

Sol: True

Let a be the first term and d be the common difference of the A.P.

Consider any term a_r of an A.P.

Now, $a_{r+m} = a_r + (m-1)d$

And $a_{r-m} = a_r + (m-1)(-d)$

$\therefore \quad a_{r+m} + a_{r-m} = a_r + (m-1)d + a_r + (m-1)(-d)$

$\Rightarrow \quad a_{r+m} + a_{r-m} = 2a_r$

$$\Rightarrow \quad a_r = \frac{a_{r+m} + a_{r-m}}{2}$$

Thus, any term of an A.P. (except first) is equal to half the sum of terms which are equidistant from it.

Q33. The sum or difference of two G.P.s, is again a G.P.

Sol: False

Let two G.P.s are $a, ar_1, ar_1^2, ar_1^3, \ldots$; and $b, br_2, br_2^2, br_2^3, \ldots$

Now, sum of two G.P.s is $a+b, (ar_1 + br_2), (ar_1^2 + br_2^2), \ldots$

Clearly, $\dfrac{ar_1 + br_2}{a+b} \neq \dfrac{ar_1^2 + br_2^2}{ar_1 + br_2}$

Similarly, for difference of two G.P.s, we get $\dfrac{ar_1 - br_2}{a-b} \neq \dfrac{ar_1^2 - br_2^2}{ar_1 - br_2}$

So, the sum or difference of two G.P.s is not a G.P.

Q34. If the sum of n terms of a sequence is quadratic expression, then it always represents an A.P.

Sol: False

We know that the sum of n terms of A.P. is

$$S_n = \frac{n}{2}(2a + (n-1)d) = \frac{n}{2}(2a - d + nd) = \left(\frac{2a-d}{2}\right)n + \left(\frac{d}{2}\right)n^2$$

Thus, S_n is of type $An^2 + Bn$.

But general quadratic expression is of the form $An^2 + Bn + C$.

Thus, if the sum of n terms of a sequence is quadratic expression of type $An^2 + Bn + C$, where $C \neq 0$, it does not represents sum of A.P.

Match the questions given under Column I with their appropriate answers given under the column II.

35.

Column I		Column II	
(a)	$4, 1, \frac{1}{4}, \frac{1}{16}$	(i)	A.P.
(b)	2, 3, 5, 7	(ii)	Sequence
(c)	13, 8, 3, –2, –7	(iii)	G.P.

Sol. **(a)** $4, 1, \frac{1}{4}, \frac{1}{16}$ is G.P. with common ratio $\frac{1}{4}$.

(b) 2, 3, 5, 7

$t_2 - t_1 = 3 - 2 = 1$

$t_3 - t_2 = 5 - 3 = 2$

Clearly, $t_2 - t_1 \neq t_3 - t_2$

Hence, it is not an A.P.

Also, $\frac{t_2}{t_1} = \frac{3}{2}$ and $\frac{t_3}{t_2} = \frac{5}{3}$

Clearly, $\frac{t_2}{t_1} \neq \frac{t_3}{t_2}$

So, it is not a GP.

Hence, it is a sequence.

(c) 13, 8, 3, –2, –7

$t_2 - t_1 = 8 - 13 = -5$

$t_3 - t_2 = 3 - 8 = -5$

$t_4 - t_3 = -2 - 3 = -5$

$t_5 - t_4 = -7 - (-2) = -5$

Hence, it is an A.P.

36.

Column I		Column II	
(a)	$1^2 + 2^2 + 3^2 + \ldots + n^2$	(i)	$\left[\frac{n(n+1)}{2}\right]^2$
(b)	$1^3 + 2^3 + 3^3 + \ldots + n^3$	(ii)	$n(n+1)$
(c)	$2 + 4 + 6 + \ldots + 2n$	(iii)	$\frac{n(n+1)(2n+1)}{6}$
(d)	$1 + 2 + 3 + \ldots + n$	(iv)	$\frac{n(n+1)}{2}$

Sol. $1 + 2 + 3 + \ldots + n$ = Sum of n terms of A.P. with first term '1' and common difference '1' $= \frac{n}{2}(1+n)$

Thus, $1 + 2 + 3 + \ldots + n = \frac{n}{2}(n+2)$

$$2 + 4 + 6 + \ldots + 2n = 2(1 + 2 + 3 + \ldots + n)$$

$$= 2 \times \frac{n}{2}(1+n) = n(n+1)$$

Let $S = 1^2 + 2^2 + 3^2 + \ldots + n^2$.

We have, $n^3 - (n-1)^3 = 3n^2 - 3n + 1$;

and by changing n into $n - 1$,

$$(n-1)^3 - (n-2)^3 = 3(n-1)^2 - 3(n-1) + 1;$$
$$(n-2)^2 - (n-3)^3 = 3(n-2)^2 - 3(n-2) + 1;$$
$$\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots$$
$$3^3 - 2^3 = 3.3^2 - 3.3 + 1;$$
$$2^3 - 1^2 = 3.2^2 - 3.2 + 1;$$
$$1^2 - 0^2 = 3.1^2 - 3.1 + 1$$

Hence, by addition,

$$n^3 = 3(1^2 + 2^2 + 3^2 + \ldots + n^2) - 3(1 + 2 + 3 + \ldots + n) + n$$
$$= 3S - \frac{3n(n+1)}{2} + n$$
$$\Rightarrow \quad 3S = n^3 - n + \frac{3n(n+1)}{2} = n(n+1)\left(n - 1 + \frac{3}{2}\right)$$
$$\Rightarrow \quad S = \frac{n(n+1)(2n+1)}{6}$$

Now, let $S = 1^3 + 2^3 + 3^3 + \ldots + n^3$

We have

$$n^4 - (n-1)^4 = 4n^3 - 6n^2 + 4n - 1;$$
$$(n-1)^4 - (n-2)^4 = 4(n-1)^3 - 6(n-1)^2 + 4(n-1) - 1;$$
$$(n-2)^4 - (n-3)^4 = 4(n-2)^3 - 6(n-2)^2 + 4(n-2) - 1;$$
$$\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots$$
$$3^4 - 2^4 = 4.3^3 - 6.3^2 + 4.3 - 1;$$
$$2^4 - 1^4 = 4.2^3 - 6.2^2 + 4.2 - 1;$$
$$1^4 - 0^4 = 4.1^3 - 6.1^2 + 4.1 - 1.$$

Hence, by addition,

$$n^4 = 4S - 6(1^2 + 2^2 + \ldots + n^2) + 4(1 + 2 + \ldots + n) - n;$$
$$\therefore \quad 4S = n^4 + n + 6(1^2 + 2^2 + \ldots + n^2) - 4(1 + 2 + \ldots + n)$$
$$= n^4 + n + n(n+1)(2n+1) - 2n(n+1)$$
$$= n(n+1)(n^2 - n + 1 + 2n + 1 - 2)$$
$$= n(n+1)(n^2 + n);$$
$$\therefore \quad S = \frac{n^2(n+1)^2}{4} = \left\{\frac{n(n+1)}{2}\right\}^2$$