DCAM classes
Dynamic Classes for Academic Mastery

Chapter 9. Sequences and Series

Question-1

Write the first 5 terms of each of the following sequences:

(i) $a_n = (-1)^{n-1} 5^{n+1}$

(ii) $a_n = \frac{n(n^2+5)}{4}$

(iii) $a_n = -11n + 10$

(iv) $a_n = \frac{n+1}{n+2}$

(v) $a_n = \frac{1-(-1)^n}{3}$

(vi) $a_n = \frac{n^2}{3^n}$

Solution:

(i) $a_n = (-1)^{n-1} 5^{n+1}$

$a_1 = (-1)^0 5^2 = 5^2;$

$a_2 = (-1)^1 5^3 = -5^3$

$a_3 = (-1)^2 5^4 = 5^4;$

$a_4 = (-1)^3 5^5 = -5^5$

$a_5 = (-1)^4 5^6 = 5^6$

(ii) $a_n = \frac{n(n^2+5)}{4}$

$a_1 = \frac{1(1^2+5)}{4} = \frac{6}{4} = \frac{3}{2}$

$a_2 = \frac{2(4+5)}{4} = \frac{18}{4} = \frac{9}{2}$

$a_3 = \frac{3(9+5)}{4} = \frac{21}{2}$

$a_4 = \frac{4(16+5)}{4} = 21$

$a_5 = \frac{5(25+5)}{4} = \frac{75}{2}$

(iii) $a_n = -11n + 10$

$a_1 = -11 + 10 = -1$

$a_2 = -22 + 10 = -12$

$a_3 = -33 + 10 = -23$

$a_4 = -44 + 10 = -34$

$a_5 = -55 + 10 = -45$

(iv) $a_n = \frac{1+1}{1+2} = \frac{2}{3}$

$a_2 = \frac{2+1}{2+2} = \frac{3}{4}$

$a_3 = \frac{3+1}{3+2} = \frac{4}{5}$

$a_4 = \frac{4+1}{4+2} = \frac{5}{6}$

$a_5 = \frac{5+1}{5+2} = \frac{6}{7}$

(v) $a_n = \frac{1-(-1)^n}{3}$

$a_1 = \frac{1-(-1)^1}{3} = \frac{2}{3}$

$a_2 = \frac{1-(-1)^2}{3} = 0$

$a_3 = \frac{1-(-1)^3}{3} = \frac{2}{3}$

$a_4 = \frac{1-(-1)^4}{3} = 0$

$a_5 = \frac{1-(-1)^5}{3} = \frac{2}{3}$

(vi) $a_n = \frac{n^2}{3^n}$

$a_1 = \frac{1^2}{3^1}$; $a_2 = \frac{2^2}{3^2}$; $a_3 = \frac{3^2}{3^3}$; $a_4 = \frac{4^2}{3^4}$; $a_5 = \frac{5^2}{3^5}$

Question-2

Find the first terms of the following sequences whose n^{th} term is

(i) $a_n = 2+\frac{1}{n}$; a_5, a_7

(ii) $a_n = \cos\left[\frac{n\pi}{2}\right]$; a_4, a_5

(iii) $a_n = \frac{(n+1)^2}{n}$; a_7, a_{10}

(iv) $a_n = (-1)^{n-1} 2^{n+1}$; a_5, a_8

Solution:

(i) $a_n = 2 + \frac{1}{n}$

$a_5 = 2 + \frac{1}{5}$

$= \frac{11}{5}$;

$a_7 = 2 + \frac{1}{7}$

$= \frac{15}{7}$

(ii) $a_n = \cos\left[\frac{n\pi}{2}\right]$;

$a_4 = \cos\left(\frac{4\pi}{2}\right) = \cos 2\pi = 1$

$a_5 = \cos\left[\frac{5\pi}{2}\right] = \cos\left[2\pi + \frac{\pi}{2}\right] = \cos\frac{\pi}{2} = 0$

(iii) $a_n = \frac{(n+1)^2}{n}$

$a_7 = \frac{(7+1)^2}{7}$

$= \frac{64}{7}$;

$a_{10} = \frac{(10+1)^2}{10}$

$= \frac{121}{10}$

(iv) $a_n = (-1)^{n-1} 2^{n+1}$

$a_5 = (-1)^4 2^{5+1}$

$= 2^6$

$= 64$;

$a_8 = (-1)^7 2^{8+1}$

$= -2^9$

$= -512$

Question-3

Find the first 6 terms of the sequence whose general term is
$a_n = \{n^2 - 1$ if n is odd $\frac{n^2+1}{2}$ if n is even$\}$

Solution:

$a_1 = 1^2 - 1 = 0$

$a_2 = \frac{2^2+1}{2} = \frac{5}{2}$

$a_3 = 3^2 - 1 = 8$

$a_4 = \frac{4^2+1}{2} = \frac{17}{2}$

$a_5 = 5^2 - 1 = 24$

$a_6 = \frac{6^2+1}{2} = \frac{37}{2}$

Question-4

Write the first five terms of the sequence given by

(i) $a_1 = a_2 = 2,\ a_n = a_{n-1} - 1,\ n>2$

(ii) $a_1 = 1,\ a_2 = 2,\ a_n = a_{n-1} + a_{n-2},\ n>2$

(iii) $a_1 = 1,\ a_n = na_{n-1},\ n \geq 2$

(iv) $a_1 = a_2 = 1,\ a_n = 2a_{n-1} + 3a_{n-2},\ n>2$

Solution:

(i) Put $n = 3 \Rightarrow a_3 = a_2 - 1 = 2\text{-}1 = 1$

$n = 4 \Rightarrow a_4 = a_3 - 1 = 1\text{-}1 = 0$

$n = 5 \Rightarrow a_5 = a_4 - 1 = 0\text{-}1 = -1$

(ii) Put $n = 3 \Rightarrow a_3 = a_2 + a_1 = 2 + 1 = 3$

$n = 4 \Rightarrow a_4 = a_3 + a_2 = 3 + 2 = 5$

$n = 5 \Rightarrow a_5 = a_4 + a_3 = 5 + 3 = 8$

(iii) Put $n = 2 \Rightarrow a_2 = 2\ a_1 = 2.1 = 2$

$n = 3 \Rightarrow a_3 = 3\ a_2 = 3.2 = 6$

$n = 4 \Rightarrow a_4 = 4\ a_3 = 4.6 = 24$

$n = 5 \Rightarrow a_5 = 5\ a_4 = 5.24 = 120$

(iv) Put $n = 3 \Rightarrow a_3 = 2a_2 + 3a_1 = 2(1) + 3(1) = 5$

$n = 4 \Rightarrow a_4 = 2a_3 + 3a_2 = 2(5) + 3(1) = 13$

$n = 5 \Rightarrow a_5 = 2a_4 + 3a_3 = 2(13) + 3(5) = 41$

Question-5

Find the n^{th} partial sum of the series $\sum_{n=1}^{\infty} \frac{1}{3^n}$

Solution:

$\sum_{n=1}^{\infty} \frac{1}{3^n}$

$S_n = \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots\ldots\ldots\ldots \frac{1}{3^n}$

$S_{n+1} = \frac{1}{3} + \frac{1}{3^2} + \ldots\ldots\ldots\ldots + \frac{1}{3^n} + \frac{1}{3^{n+1}}$

$S_{n+1} = s_n + \frac{1}{3^{n+1}}$

$S_{n+1} = \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots\ldots\ldots\ldots \frac{1}{3^n} + \frac{1}{3^{n+1}}$

$= \frac{1}{3}\left[1 + \frac{1}{3} + \frac{1}{3^2} + \ldots + \frac{1}{3^n}\right] = \frac{1}{3}\left[1 + s_n\right]$

$s_n + \frac{1}{3^{n+1}} = \frac{1}{3} + \frac{1}{3} s_n$

$3 s_n + \frac{1}{3^n} = 1 + s_n$

$2s_n = 1 - \frac{1}{3^n}$

$s_n = \frac{1}{2}\left[1 - \frac{1}{3^n}\right]$

Question-6

Find the sum of first n terms of the series $\sum_{n=1}^{\infty} 5^n$

Solution:

$\sum_{n=1}^{\infty} 5^n = 5 + 5^2 + 5^3 + \ldots\ldots\ldots\ldots + 5^n + \ldots\ldots\ldots$

$s_n = 5 + 5^2 + 5^3 + \ldots\ldots\ldots\ldots + 5^n$

$s_{n+1} = 5 + 5^2 + 5^3 + \ldots\ldots\ldots\ldots + 5^n + 5^{n+1}$

$= s_n + 5^{n+1}$

Also $s_{n+1} = 5 + 5^2 + 5^3 + \ldots\ldots\ldots\ldots + 5^n + 5^{n+1}$

$= 5[1 + 5 + 5^2 + \ldots\ldots\ldots\ldots + 5^n]$

$= 5[1 + s_n]$

$s_n + 5^{n+1} = 5 + 5 s_n$

$4s_n = 5^{n+1} - 5$

$\therefore s_n = \frac{5(5^n - 1)}{4}$

Question-7

Find the sum of 101th term to 200th term of the series $\sum_{n=1}^{\infty} \frac{1}{2^n}$

Solution:

$\sum_{n=1}^{\infty} \frac{1}{2^n}$

To find $S_{200} - S_{100}$

To find S_{200}: $S_{200} = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots\ldots + \frac{1}{2^{200}}$

$S_{201} = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots\ldots + \frac{1}{2^{200}} + \frac{1}{2^{201}}$

$= S_{200} + \frac{1}{2^{201}}$

also, $S_{201} = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots\ldots + \frac{1}{2^{200}} + \frac{1}{2^{201}}$

$= \frac{1}{2}\left[1 + \frac{1}{2} + \frac{1}{2^2} + \ldots\ldots \frac{1}{2^{200}}\right]$

$S_{200} + \frac{1}{2^{201}} = \frac{1}{2}[1 + S_{200}]$

$2S_{200} + \frac{1}{2^{200}} = 1 + S_{200}$

$S_{200} = 1 - \frac{1}{2^{200}}$

Similarly $S_{100} = 1 - \frac{1}{2^{100}}$

Hence $S_{200} - S_{100} = \left[1 - \frac{1}{2^{200}}\right] - \left[1 - \frac{1}{2^{100}}\right]$

$= \frac{1}{2^{100}} - \frac{1}{2^{200}}$

Question-8

Find five arithmetic means between 1 and 19.

Solution:

Let 1, x_1, x_2, x_3, x_4, x_5, 19 be in A.P.

Let d be the common difference

$19 = 1 + (n-1)d$

$19 = 1 + 6d$

$\therefore d = 3$

$\therefore x_1 = 1 + 3 = 4$

$x_2 = 4 + 3 = 7$

$x_3 = 7 + 3 = 10$

$x_4 = 10 + 3 = 13$

$x_5 = 13 + 3 = 16$

The arithmetic means are 4, 7, 10, 13, 16.

Question-9

Find six arithmetic mean between 3 and 17.

Solution:

Let 3, x_1, x_2, x_3, x_4, x_5, x_6, 17 be in A.P

Then 17 = 3 + (n-1)d

17 = 3 + 7d

14 = 7d

d = 2

$x_1 = 3 + 2 = 5$

$x_2 = 5 + 2 = 7$

$x_3 = 7 + 2 = 9$

$x_4 = 9 + 2 = 11$

$x_5 = 11 + 2 = 13$

$x_6 = 13 + 2 = 15$

The arithmetic means are 5, 7, 9, 11, 13, 15.

Question-10

Find the single A.M. between

(i) 7 and 13

(ii) 5 and −3

(iii) (p + q) and (p - q)

Solution:

(i) A.M. between 7 and 13 = $\frac{7+13}{2}$ = 10

(ii) A.M. between 5 and −3 = $\frac{5-3}{2}$ = 1

(iii) A.M. between (p + q) and (p − q) = $\frac{p+q+p-q}{2}$ = p

Question-11

If b is the G.M. of a and c and x is the A.M of a and b and y is the A.M of b and c, prove that $\frac{a}{x}+\frac{c}{y} = 2$.

Solution:

b = G.M. of a and c $\Rightarrow \sqrt{ac} = b$ ....(1)

x = A.M. of a and b $\Rightarrow x = \frac{a+b}{2}$ ....(2)

y = A.M. between b and c $\Rightarrow y = \frac{b+c}{2}$ ....(3)

To prove that $\frac{a}{x}+\frac{b}{y} = 2$

From (1) $b^2 = ac \Rightarrow c = \frac{b^2}{a}$

$$\frac{a}{x}+\frac{c}{y} = \frac{2a}{a+b}+\frac{2c}{b+c}$$

$$= \frac{2a}{a+b}+\frac{\frac{2b^2}{a}}{b+\frac{b^2}{a}}$$

$$= \frac{2a}{a+b}+\frac{2bb}{b(a+b)}$$

$$= \frac{2a}{a+b}+\frac{2b}{a+b}$$

$$= \frac{2(a+b)}{a+b}$$

$$= 2$$

Question-12

The first and second terms of H.P are $\frac{1}{3}$ and $\frac{1}{5}$ respectively, find the 9^{th} term.

Solution:

Let the H.P are $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}+$..........

$$\frac{1}{a} = \frac{1}{3} \Rightarrow a = 3$$

$$\frac{1}{a+d} = \frac{1}{5} \Rightarrow 5 = a + d;\ d = 2$$

9^{th} term $= \frac{1}{a+8d} = \frac{1}{3+16} = \frac{1}{19}$

Question-13

If a, b, c are in H.P., prove that $\frac{b+a}{b-a} + \frac{b+c}{b-c} = 2$.

Solution:

If a, b, c are in H.P then $b = \frac{2ac}{a+c}$

$\frac{b}{a} = \frac{2c}{a+c}$

$\Rightarrow \frac{b+a}{b-a} = \frac{2c+a+c}{2c-a-c} = \frac{3c+a}{c-a}$ ....(1)

Also $\frac{b}{c} = \frac{2a}{a+c}$

$\frac{b+c}{b-c} = \frac{2a+a+c}{2a-a-c} = \frac{3a+c}{a-c}$ ....(2)

Adding (1) and (2)

$\therefore \frac{b+a}{b-a} + \frac{b+c}{b-c} = \frac{3c+a}{c-a} + \frac{3a+c}{a-c}$

$= \frac{3c+a}{c-a} - \frac{3a+c}{c-a}$

$= \frac{3c+a-3a-c}{c-a}$

$= \frac{2c-2a}{c-a}$

$= 2$

Hence $\frac{b+a}{b-a} + \frac{b+c}{b-c} = 2$.

Question-14

The difference between two positive numbers is 18, and 4 times their G.M. is equal to 5 times their H.M. find the numbers.

Solution:

Let the two numbers be a and b

$b - a = 18$

$4\sqrt{ab} = 5\left[\frac{2ab}{a+b}\right]$

$2\sqrt{ab} = \frac{5ab}{a+b}$

$2(a+b) = 5\sqrt{ab}$

$4(a+b)^2 = 25ab$

$4(a^2 + b^2 + 2ab) = 25ab$

$4a^2 + 4b^2 - 17ab = 0$

$4a^2 + 4(18+a)^2 - 17a(18+a) = 0$

$4a^2 + 4(324 + 36a + a^2) - 306a - 17a^2 = 0$

$4a^2 + 1296 + 144a + 4a^2 - 306a - 17a^2 = 0$

$-9a^2 - 162a + 1296 = 0$

$a^2 + 18a - 144 = 0$

$(a+24)(a-6) = 0$

$a = -24$ (or) 6

If a = 6 then b is 24.

Therefore the numbers are 6 and 24.

Question-15

If the A.M. between two numbers is 1, prove that their H.M. is the square of their G.M.

Solution:

A.M. between two numbers a and b is 1.

$\frac{a+b}{2} = 1$

$\Rightarrow a + b = 2$

$HM = (GM)^2$

$HM = \frac{2ab}{a+b} = \frac{2ab}{2} = ab$

$GM = \sqrt{ab}$

$\therefore (GM)^2 = ab$

Hence $HM = GM^2$

Question-16

If a, b, c are in A.P and a, mb, c are in G.P then prove that a, m^2b, c are in H.P.

Solution:

Given

a, b, c are in A.P.

$\Rightarrow b = \frac{a+c}{2}$ ....(1)

a, mb, c are in G.P.

$\Rightarrow mb = \sqrt{ac}$ ....(2)

To prove

a,m^2b,c are in H.P.

i.e., $m^2b = \frac{2ac}{a+c}$

Proof

$R.H.S = \frac{2ac}{a+c} = \frac{2m^2b^2}{2b}$ from (2) and (1)

$= m^2 b = LHS$

Question-17

If the p^{th} and q^{th} terms of a H.P. are q and p respectively, show that $(pq)^{th}$ term is 1.

Solution:

Given

p^{th} and q^{th} terms of a H.P. are q and p.

Therefore $\frac{1}{a+(p-1)d} = q$ ....(1)

and $\frac{1}{a+q-1d} = p$ ....(2)

To prove

pq^{th} term, ie., $\frac{1}{a+(pq-1)d} = 1$

Proof

From (1) $a + pd - d = \frac{1}{q}$

From (2) $a + qd - d = \frac{1}{p}$

Subtracting $(p - q)d = \frac{1}{q} - \frac{1}{p} = \frac{p-q}{pq}$

$\therefore d = \frac{1}{pq}$

$\therefore a + p\frac{1}{pq} - \frac{1}{pq} = \frac{1}{q}$

$a = \frac{1}{pq}$

$\therefore a + (pq - 1)d = \frac{1}{pq} + (pq - 1)\frac{1}{pq}$

$= \frac{1 + pq - 1}{pq}$

$= \frac{pq}{pq} = 1$

$\therefore \frac{1}{a + (pq-1)d} = 1$

$\Rightarrow pq^{th}$ term is 1.

Question-18

Three number form a H.P. the sum of the numbers is 11 and the sum of the reciprocals is one. Find the numbers.

Solution:

Let $\frac{1}{a-d}, \frac{1}{a}, \frac{1}{a+d}$ be in H.P.

Their sum is $\frac{1}{a-d} + \frac{1}{a} + \frac{1}{a+d} = 11$

The sum of their reciprocal is $a - d + a + a + d = 1$

$3a = 1$

$\Rightarrow a = \frac{1}{3}$

$$\therefore \frac{1}{\frac{1}{3}-d} + \frac{1}{\frac{1}{3}} + \frac{1}{\frac{1}{3}+d} = 11$$

$$\frac{3}{1-3d} + 3 + \frac{3}{1+3d} = 11$$

$$\frac{3}{1-3d} + \frac{3}{1+3d} = 8$$

$$\frac{3(1+3d+1-3d))}{1-9d^2} = 8$$

$$\frac{6}{1-9d^2} = 8$$

$$6 = 8 - 72d^2$$

$$72d^2 = 2$$

$$\Rightarrow d^2 = \frac{1}{36}$$

$$\Rightarrow d = \frac{1}{6}$$

The numbers are $\frac{1}{\frac{1}{3}-\frac{1}{6}} + \frac{1}{\frac{1}{3}} + \frac{1}{\frac{1}{3}+\frac{1}{6}} = \frac{1}{\frac{1}{6}}, \frac{1}{\frac{1}{3}}, \frac{1}{\frac{1}{2}}$

The numbers are 6, 3, 2.

Question-19

Write the first four terms in the expansions of the following:

(i) $\frac{1}{[2+x]^4}$ where |x| >2

(ii) $\frac{1}{\sqrt[3]{6-3x}}$ where |x|<2

Solution:

(i) $\frac{1}{(2+x)^4} = \frac{1}{16\left(1+\frac{x}{2}\right)^4} = \frac{1}{16}\left[1+\frac{x}{2}\right]^{-4}$

$= \frac{1}{16}\left[1-4\left(\frac{x}{2}\right)+\frac{4.5}{1.2}\left(\frac{x}{2}\right)^2-\frac{4.5.6}{1.2.3}\left(\frac{x}{2}\right)^3+...\right]$

$= \frac{1}{16}\left[1-2x+\frac{5}{2}x^2-\frac{5}{2}x^3+....\right]$

(ii) $\frac{1}{\sqrt[3]{6-3x}} = \frac{1}{(6-7x)^{\frac{1}{3}}} = \frac{1}{6^{\frac{1}{3}}}\left[1-\frac{x}{2}\right]^{\frac{1}{3}}$

$= \frac{1}{6^{\frac{1}{3}}}\left[1+\frac{1}{3}\left(\frac{x}{2}\right)+\frac{\left(\frac{1}{3}\right)\left(\frac{4}{3}\right)}{1.2}\left(\frac{x}{2}\right)^2+\frac{\left[\frac{1}{3}\right]\left[\frac{4}{3}\right]\left[\frac{7}{3}\right]}{1.2.3}\left[\frac{x}{2}\right]^3\right]$

$= \frac{1}{6^{\frac{1}{3}}}\left[1+\frac{x}{6}+\frac{x^2}{18}+\frac{7}{326}x^3+....\right]$

Question-20

Evaluate the following:

(i) $\sqrt[3]{1003}$ correct to 4 places of decimals.

(ii) $\frac{1}{\sqrt[3]{128}}$ correct to 4 places of decimals.

(iii) $\sqrt[3]{1003} - \sqrt[3]{997}$ correct to 3 place of decimals.

Solution:

(i) $\sqrt[3]{1003} = (1003)^{\frac{1}{3}}$

$$= (1000 + 3)^{\frac{1}{3}}$$

$$= (1000)^{1/3}\left[1 + \frac{3}{1000}\right]^{\frac{1}{3}}$$

$$= 10\,[1+0.003]^{\frac{1}{3}}$$

$$= 10\left[1+\frac{1}{3}(0.003)+\frac{\left[\frac{1}{3}\right]\left[\frac{-2}{3}\right]}{1.2}(0.003)^2+....\right]$$

$$= 10\,[1+0.001-0.000001+....1]$$

$$= 10.00999$$

(ii) $\frac{1}{\sqrt[3]{128}} = \frac{1}{(128)^{\frac{1}{3}}} = \frac{1}{(125+3)^{\frac{1}{3}}} = \frac{1}{5\left[1+\frac{3}{125}\right]^{\frac{1}{3}}} = \frac{1}{5}\left[1+\frac{3}{125}\right]^{\frac{1}{3}}$

$$= \frac{1}{5}\,(1 + 0.024)^{-\frac{1}{3}} = \frac{1}{5}\left[1-\frac{1}{3}(0.024)+\frac{\frac{1}{3}\left[\frac{4}{3}\right]}{1.2}(0.024)^2.....\right]$$

$$= \frac{1}{5}\,[1-0.008+0.000128] = 0.1984256$$

(iii) $\sqrt[3]{1003} - \sqrt[3]{997} = (1000 + 3)^{1/3} - (1000 - 3)^{1/3}$

$$= 10\left[1+\frac{3}{1000}\right]^{\frac{1}{3}} - 10\left[1-\frac{3}{1000}\right]^{\frac{1}{3}}$$

$$= 10\,[1 + 0.003]^{\frac{1}{3}} - 10[1 - 0.003]^{\frac{1}{3}}$$

$$= 10\left[1+\frac{1}{3}(0.003)+\frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)}{1.2}(0.003)^2+....\right]$$

$$-10\left[1-\frac{1}{3}(0.003)+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{1.2}(0.003)^2...\right]$$

$$= 10\left[1+\frac{0.003}{3}-\frac{(0.003)^2}{9}-1+\frac{0.003}{3}+\frac{(0.003)^2}{9}....\right]$$

$$= 10[0.002] = 0.02$$

Question-21

If x is so small show that

(i) $\frac{\sqrt{1-x}}{1+x} = 1 - x + \frac{x^2}{2}$ (app)

(ii) $\frac{1}{(1+x)^2\sqrt{1+4x}} = 1 - 4x$ (app.)

Solution:

(i) $\frac{\sqrt{1-x}}{1+x} = (1-x)^{\frac{1}{2}}(1+x)^{-\frac{1}{2}}$

$$= \left[1 - \frac{1}{2}x + \frac{\left[\frac{1}{2}\right]\left[\frac{-1}{2}\right]}{1.2}x^2 + \ldots\right]\left[1 - \frac{1}{2}x + \frac{\frac{1}{2}\frac{3}{2}}{1.2}x^2 + \ldots\right]$$

$$= \left[1 - \frac{x}{2} - \frac{x^2}{8}\right]\left[1 - \frac{x}{2} + \frac{3x^2}{8}\right]$$

$$= 1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x}{2} + \frac{x^2}{4} + \frac{3x^2}{8} + \ldots$$

$$= 1 - x + \frac{x^2}{2} \text{ (app.)}$$

(ii) $\frac{1}{(1+x)^2\sqrt{1+4x}} = (1+x)^{-2}(1+4x)^{-1/2}$

$$= \left[1 - 2x + \frac{2.3}{1.2}x^2 + \ldots\right]\left[1 - \frac{1}{2}(4x) + \ldots\right]$$

$$= (1 - 2x + \ldots\ldots)(1 - 2x + \ldots\ldots)$$

$$= 1 - 2x - 2x + 4x^2 + \ldots = 1 - 4x \text{ (app.)}$$

Question-22

If x is so large prove that $\sqrt{x^2+25} - \sqrt{x^2+9} = \frac{8}{x}$ nearly.

Solution:

$$\sqrt{x^2+25} - \sqrt{x^2+9} = x\left[1 + \frac{25}{x^2}\right]^{\frac{1}{2}} - x\left[1 + \frac{9}{x^2}\right]^{\frac{1}{2}}$$

$$= x\left[1 + \frac{1}{2}\left[\frac{25}{x^2}\right] + \frac{\frac{1}{2}\left[\frac{-1}{2}\right]}{1.2}\left[\frac{25}{x^2}\right]^2 + \ldots\right] - x\left[1 + \frac{1}{2}\left[\frac{9}{x^2}\right] + \frac{\frac{1}{2}\left[\frac{-1}{2}\right]}{1.2}\left[\frac{9}{x^2}\right]^2 + \ldots\right]$$

$$= x + \frac{25}{2x} - \frac{625}{8x^3} + \ldots - x - \frac{9}{2x} + \frac{81}{8x^3} + \ldots$$

$$= \frac{16}{2x} + \frac{81}{8x^3} + \ldots$$

$$= \frac{16}{2x} = \frac{8}{x} \text{ approximately}$$

Question-23

If c is small compared to l, show that $\left[\frac{1}{l+c}\right]^{\frac{1}{2}} + \left[\frac{1}{l-c}\right]^{\frac{1}{2}} = 2 + \frac{3c^2}{4l^2}$ (app)

Solution:

$$\left[\frac{l}{l+c}\right]^{\frac{1}{2}} + \left[\frac{1}{l-c}\right]^{\frac{1}{2}} = \frac{1}{\left[1+\frac{c}{l}\right]^{\frac{1}{2}}} + \frac{1}{\left[1-\frac{c}{l}\right]^{\frac{1}{2}}}$$

$$= \left[1+\frac{c}{l}\right]^{\frac{1}{2}} + \left[1-\frac{c}{l}\right]^{\frac{-1}{2}}$$

Since c is small in comparison with l then $\left|\frac{c}{l}\right| < 1$,∴ binomial expansion is valid.

$$= 1+\left[\frac{-1}{2}\right]\left[\frac{c}{l}\right]+\frac{\left[\frac{-1}{2}\right]\left[\frac{-1}{2}-1\right]}{1.2}\left[\frac{-c}{l}\right]^2+\ldots\ldots\ldots$$

$$+1+\left[\frac{-1}{2}\right]\left[\frac{-c}{l}\right]+\frac{\left[\frac{-1}{2}\right]\left[\frac{-1}{2}-1\right]}{1.2}\left[\frac{-c}{l}\right]^2+\ldots\ldots\ldots$$

$$= 1 - \frac{c}{2l} + \frac{3c^2}{8l^2} + \ldots\ldots + 1 + \frac{c}{2l} + \frac{3c^2}{8l^2} + \ldots\ldots$$

$= 2 + \frac{3c^2}{4l^2}$ approximately.

Question-24

Find the 5^{th} term in the expansion of $(1 - 2x^3)^{11/2}$.

Solution:

$$(1 - 2x^3)^{11/2} = 1 + \left[\frac{11}{2}\right](-2x^3) + \frac{\left[\frac{11}{2}\right]\left[\frac{2}{9}\right]}{1.2}(-2x^3)^2 + \frac{\left[\frac{11}{2}\right]\left[\frac{2}{9}\right]\left[\frac{7}{2}\right]}{1.2.3}(-2x^3)^3 + \frac{\left[\frac{11}{2}\right]\left[\frac{2}{9}\right]\left[\frac{7}{2}\right]\left[\frac{5}{2}\right]}{1.2.3.4}(-2x^3)^4$$

5^{th} term is $\frac{\left[\frac{11}{2}\right]\left[\frac{2}{9}\right]\left[\frac{7}{2}\right]\left[\frac{5}{2}\right]}{1.2.3.4}(-2x^3)^4 = \frac{1155}{8}x^{12}$

Question-25

Find the $(r + 1)^{th}$ term in the expansion of $(1 - x)^{-4}$.

Solution:

T_{r+1} in $(1 - x)^{-4}$

$(1-x)^{-4} = \frac{1}{6}[1.2.3 + 2.3.4.x + \ldots\ldots\ldots\ldots(r + 1)(r + 2)(r + 3)x^r+\ldots\ldots\ldots]$

$\therefore\ T_{r+1} = \frac{(r+1)(r+2)(r+3)}{6}x^r$

Question-26

Show that $x^n = 1 + n\left[1-\frac{1}{x}\right] + \frac{n(n+1)}{1.2}\left[1-\frac{1}{x}\right]^2 + \ldots..$

Solution:

R.H.S $= 1 + n\left[1-\frac{1}{x}\right] + \frac{n(n+1)}{1.2}\left[1-\frac{1}{x}\right]^2 + \ldots..$

Put $y = 1-\frac{1}{x}$

$= 1 + ny + \frac{n(n+1)}{1.2}y^2 + \ldots.$

$= (1-y)^{-n}$

$= \left[1-\left[1-\frac{1}{x}\right]\right]^{-n}$

$= \left[\frac{1}{x}\right]^{-n}$

$= x^n$

= L.H.S

Question-27

Find the sum to infinity of the series

(i) $1+\frac{9}{8}+\frac{9.15}{8.16}+\frac{9.15.21}{8.16.24}+\ldots$

(ii) $1 - \frac{1}{5}+\frac{1.4}{5.10}-\frac{1.4.7}{5.10.15}+\ldots.$

(iii) $\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\ldots$

Solution:

(i) Let $S = 1+\frac{9}{8}+\frac{9.15}{8.16}+\frac{9.15.21}{8.16.24}+\ldots$

$= 1+\frac{9}{6}\left(\frac{6}{8}\right)+\frac{\left(\frac{9}{6}\right)\left(\frac{5}{6}\right)}{1.2}\left(\frac{6}{8}\right)^2+\ldots\ldots.$

$= \left[1-\frac{6}{8}\right]^{-\frac{9}{6}}$

$\left[1-\frac{3}{4}\right]^{-\frac{3}{2}} = \left[\frac{1}{4}\right]^{-\frac{3}{2}} = 4^{\frac{3}{2}} = 4^1\, 4^{\frac{1}{2}} = 4\sqrt{4} = 4(2) = 8$

(ii) Let S = 1 - $\frac{1}{5}+\frac{1.4}{5.10}-\frac{1.4.7}{5.10.15}+\ldots$

$= 1 - \left(\frac{1}{3}\right)\left(\frac{3}{5}\right)+\frac{\left[\frac{1}{3}\right]\left[\frac{4}{3}\right]}{1.2}\left[\frac{3}{5}\right]^2+\ldots$

$= \left[1+\frac{3}{5}\right]^{-\frac{1}{3}}$

$= \left[\frac{8}{5}\right]^{-\frac{1}{3}}$

$= \left[\frac{5}{8}\right]^{\frac{1}{3}}$

(iii) Let S = $\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\ldots$

$S + 1 = 1+\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\ldots$

$= 1+\left(\frac{3}{2}\right)\left(\frac{2}{4}\right)+\frac{\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)}{1.2}\left(\frac{2}{4}\right)^2+\ldots\ldots$

$= \left(1-\frac{2}{4}\right)^{-3/2} = \left(\frac{1}{2}\right)^{-3/2} = 2^{3/2}$

$S + 1 = 2^{3/2}$

Therefore $S = 2^{3/2} - 1$

Question-28

Show that the coefficient of x^n in the infinite series 1 + $\frac{b+ax}{1!}+\frac{(b+ax)^2}{2!}+\frac{(b+ax)^3}{3!}+\ldots\ldots$ is $\frac{e^b a^n}{n!}$.

(ii) Show that $\left(1+\frac{1}{2!}+\frac{1}{4!}+\ldots\ldots\right)^2 = 1+\left(1+\frac{1}{3!}+\frac{1}{5!}+\ldots\ldots\right)^2$.

(iii) Show that $2\left\{1+\frac{(\log n)^2}{2!}+\frac{(\log n)^4}{4!}+\ldots\right\} = n + c$.

Solution:

(i) $1+\frac{y}{1!}+\frac{y^2}{2!}+\frac{y^3}{3!}+\ldots\ldots = e^y = e^{b+ax} = e^b \cdot e^{ax} = e^b\left(1+\frac{ax}{1!}+\frac{(ax)^2}{2!}+\ldots\ldots\right)$

Coefficient of $x^n = e^b \cdot \left(\frac{a^2}{n!}\right)$

(ii) L.H.S = $\left(1+\frac{1}{2!}+\frac{1}{4!}+\ldots\ldots\right)^2 = \left(\frac{e+e^{-1}}{2}\right)^2$

$= 1+\frac{e^2+e^{-2}-2}{4}$

$= \frac{4+e^2+e^{-2}-2}{4}$

$= \frac{e^2+e^{-2}+2}{4}$

= R.H.S

(iii) L.H.S = $2\left\{1+\frac{(\log n)^2}{2!}+\frac{(\log n)^4}{4!}+\ldots\right\}$

Put log n = y

$2\left\{1+\frac{(\log n)^2}{2!}+\frac{(\log n)^4}{4!}+\ldots\right\} = 2\left(\frac{e^y+e^{-y}}{2}\right)$

$= e^y + e^{-y}$

$= e^{\log n} + e^{-\log n}$

$= e^{\log n} + e^{\log 1/n}$

$= n + \frac{1}{n}$

Question-29

Show that log a – log b = $\frac{a-b}{a}+\frac{1}{2}\left(\frac{a-b}{a}\right)^2+\frac{1}{3}\left(\frac{a-b}{a}\right)^3+\ldots\ldots\ldots$

Solution:

R.H.S = $\frac{a-b}{a}+\frac{1}{2}\left(\frac{a-b}{a}\right)^2+\frac{1}{3}\left(\frac{a-b}{a}\right)^3+\ldots\ldots\ldots$

Put $y = \frac{a-b}{a}$

$y + \frac{y^2}{2}+\frac{y^3}{3}+\ldots\ldots\ldots = -\log(1-y)$

$= -\log\left(1-\frac{a-b}{a}\right)$

$= -\log\left(\frac{b}{a}\right)$

$= \log\left(\frac{a}{b}\right)$

= log a – log b

= L.H.S

Question-30

Prove that $\log \frac{n+1}{n-1} = \frac{2n}{n^2+1} + \frac{1}{3}\left(\frac{2n}{n^2+1}\right)^3 + \ldots\ldots\ldots\ldots$

Solution:

R.H.S = $\frac{2n}{n^2+1} + \frac{1}{3}\left(\frac{2n}{n^2+1}\right)^3 + \ldots\ldots\ldots\ldots$

Put $\frac{2n}{n^2+1} = y$

$y + \frac{y^3}{3} + \frac{y^5}{5} \ldots\ldots\ldots\ldots = \frac{1}{2}\log\left(\frac{1+y}{1-y}\right)$

$= \frac{1}{2}\log\left(\frac{1+\frac{2n}{n^2+1}}{1-\frac{2n}{n^2+1}}\right)$

$= \frac{1}{2}\log\left(\frac{n^2+1+2n}{n^2+1-2n}\right)$

$= \frac{1}{2}\log\left(\frac{n+1}{n-1}\right)^2$

$= \log\left(\frac{n+1}{n-1}\right)$

= L.H.S

Question-31

Find the sum to infinity the series $\frac{1}{x-1} + \frac{1}{3}\frac{1}{(x-1)^3} + \frac{1}{5}\frac{1}{(x-1)^5} + \ldots\ldots\ldots\ldots\ldots$

Solution:

$\frac{1}{x-1} + \frac{1}{3}\frac{1}{(x-1)^3} + \frac{1}{5}\frac{1}{(x-1)^5} + \ldots\ldots\ldots\ldots\ldots$

Put y = $\frac{1}{x-1}$

$\frac{1}{y} + \frac{1}{3}\frac{1}{y^3} + \frac{1}{5}\frac{1}{y^5} + \ldots\ldots\ldots\ldots = \frac{1}{2}\log\left(\frac{1+y}{1-y}\right)$

$= \frac{1}{2}\log\left(\frac{1+\frac{1}{1-x}}{1-\frac{1}{1-x}}\right)$

$= \frac{1}{2}\log\left(\frac{1-x+1}{1-x-1}\right)$

$= \frac{1}{2}\log\left(\frac{2-x}{-x}\right)$

$= \frac{1}{2}\log\left(\frac{x+2}{x}\right)$

Question-32

If x is so small show that

(i) $\frac{\sqrt{1-x}}{1+x} = 1 - x + \frac{x^2}{2}$(app)

(ii) $\frac{1}{(1+x)^2\sqrt{1+4x}} = 1 - 4x$(app.)

Solution:

(i) $\frac{\sqrt{1-x}}{1+x} = (1-x)^{\frac{1}{2}}\,(1+x)^{-\frac{1}{2}}$

$$= \left[1 - \frac{1}{2}x + \frac{\left[\frac{1}{2}\right]\left[\frac{-1}{2}\right]}{1.2}x^2 + ...\right]\left[1 - \frac{1}{2}x + \frac{\frac{1}{2}\frac{3}{2}}{1.2}x^2 + ...\right]$$

$$= \left[1 - \frac{x}{2} - \frac{x^2}{8}\right]\left[1 - \frac{x}{2} + \frac{3x^2}{8}\right]$$

$$= 1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x}{2} + \frac{x^2}{4} + \frac{3x^2}{8} + ...$$

$$= 1 - x + \frac{x^2}{2}\text{(app.)}$$

(ii) $\frac{1}{(1+x)^2\sqrt{1+4x}} = (1+x)^{-2}(1+4x)^{-1/2}$

$$= \left[1 - 2x + \frac{2.3}{1.2}x^2 + ...\right]\left[1 - \frac{1}{2}(4x) + ...\right]$$

$$= (1 - 2x + ...)(1 - 2x + ...)$$

$$= 1 - 2x - 2x + 4x^2 + ... = 1 - 4x\text{(app.)}$$

Question-33

If x is so large prove that $\sqrt{x^2+25} - \sqrt{x^2+9} = \frac{8}{x}$ nearly.

Solution:

$$\sqrt{x^2+25} - \sqrt{x^2+9} = x\left[1 + \frac{25}{x^2}\right]^{\frac{1}{2}} - x\left[1 + \frac{9}{x^2}\right]^{\frac{1}{2}}$$

$$= x\left[1 + \frac{1}{2}\left[\frac{25}{x^2}\right] + \frac{\frac{1}{2}\left[\frac{-1}{2}\right]}{1.2}\left[\frac{25}{x^2}\right]^2 + ...\right] - x\left[1 + \frac{1}{2}\left[\frac{9}{x^2}\right] + \frac{\frac{1}{2}\left[\frac{-1}{2}\right]}{1.2}\left[\frac{9}{x^2}\right]^2 + ...\right]$$

$$= x + \frac{25}{2x} - \frac{625}{8x^3} + ... - x - \frac{9}{2x} + \frac{81}{8x^3} + ...$$

$$= \frac{16}{2x} + \frac{81}{8x^3} + ...$$

$$= \frac{16}{2x} = \frac{8}{x}\text{ approximately}$$

Question-34

If c is small compared to l, show that $\left[\frac{1}{l+c}\right]^{\frac{1}{2}} + \left[\frac{1}{l-c}\right]^{\frac{1}{2}} = 2 + \frac{3c^2}{4l^2}$ (app)

Solution:

$$\left[\frac{l}{l+c}\right]^{\frac{1}{2}} + \left[\frac{1}{1-c}\right]^{\frac{1}{2}} = \frac{1}{\left[1+\frac{c}{l}\right]^{\frac{1}{2}}} + \frac{1}{\left[1-\frac{c}{l}\right]^{\frac{1}{2}}}$$

$$= \left[1+\frac{c}{l}\right]^{\frac{1}{2}} + \left[1-\frac{c}{l}\right]^{\frac{-1}{2}}$$

Since c is small in comparison with l then $\left|\frac{c}{l}\right| < 1$, ∴ binomial expansion is valid.

$$= 1+\left[\frac{-1}{2}\right]\left[\frac{c}{l}\right]+\frac{\left[\frac{-1}{2}\right]\left[\frac{-1}{2}-1\right]}{1.2}\left[\frac{-c}{l}\right]^2+\ldots\ldots\ldots$$

$$+1+\left[\frac{-1}{2}\right]\left[\frac{-c}{l}\right]+\frac{\left[\frac{-1}{2}\right]\left[\frac{-1}{2}-1\right]}{1.2}\left[\frac{-c}{l}\right]^2+\ldots\ldots\ldots$$

$$= 1-\frac{c}{2l}+\frac{3c^2}{8l^2}+\ldots\ldots+1+\frac{c}{2l}+\frac{3c^2}{8l^2}+\ldots\ldots$$

$= 2+\frac{3c^2}{4l^2}$ approximately.

Question-35

Find the 5th term in the expansion of $(1 - 2x^3)^{11/2}$.

Solution:

$$(1-2x^3)^{11/2} = 1 + \left[\frac{11}{2}\right](-2x^3) + \frac{\left[\frac{11}{2}\right]\left[\frac{2}{9}\right]}{1.2}(-2x^3)^2 + \frac{\left[\frac{11}{2}\right]\left[\frac{2}{9}\right]\left[\frac{7}{2}\right]}{1.2.3}(-2x^3)^3 + \frac{\left[\frac{11}{2}\right]\left[\frac{2}{9}\right]\left[\frac{7}{2}\right]\left[\frac{5}{2}\right]}{1.2.3.4}(-2x^3)^4$$

5th term is $\frac{\left[\frac{11}{2}\right]\left[\frac{2}{9}\right]\left[\frac{7}{2}\right]\left[\frac{5}{2}\right]}{1.2.3.4}(-2x^3)^4 = \frac{1155}{8}x^{12}$

Question-36

Find the (r + 1)th term in the expansion of $(1 - x)^{-4}$.

Solution:

T_{r+1} in $(1-x)^{-4}$

$(1-x)^{-4} = \frac{1}{6}[1.2.3 + 2.3.4.x + \ldots\ldots\ldots\ldots(r+1)(r+2)(r+3)x^r + \ldots\ldots\ldots]$

$\therefore T_{r+1} = \frac{(r+1)(r+2)(r+3)}{6}x^r$

Question-37

Show that $x^n = 1 + n\left[1-\frac{1}{x}\right] + \frac{n(n+1)}{1.2}\left[1-\frac{1}{x}\right]^2 + \ldots\ldots$

Solution:

R.H.S $= 1 + n\left[1-\frac{1}{x}\right] + \frac{n(n+1)}{1.2}\left[1-\frac{1}{x}\right]^2 + \ldots\ldots$

Put $y = 1-\frac{1}{x}$

$= 1 + ny + \frac{n(n+1)}{1.2}y^2 + \ldots.$

$= (1 - y)^{-n}$

$= \left[1-\left[1-\frac{1}{x}\right]\right]^{-n}$

$= \left[\frac{1}{x}\right]^{-n}$

$= x^n$

$=$ L.H.S

Question-38

Find the sum to infinity of the series

(i) $1+\frac{9}{8}+\frac{9.15}{8.16}+\frac{9.15.21}{8.16.24}+\ldots$

(ii) $1-\frac{1}{5}+\frac{1.4}{5.10}-\frac{1.4.7}{5.10.15}+\ldots.$

(iii) $\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\ldots$

Solution:

(i) Let $S = 1+\frac{9}{8}+\frac{9.15}{8.16}+\frac{9.15.21}{8.16.24}+\ldots$

$= 1+\frac{9}{6}\left(\frac{6}{8}\right)+\frac{\left(\frac{9}{6}\right)\left(\frac{5}{6}\right)}{1.2}\left(\frac{6}{8}\right)^2+\ldots\ldots..$

$= \left[1-\frac{6}{8}\right]^{-\frac{9}{6}}$

$\left[1-\frac{3}{4}\right]^{-\frac{3}{2}} = \left[\frac{1}{4}\right]^{-\frac{3}{2}} = 4^{\frac{3}{2}} = 4^1\, 4^{\frac{1}{2}} = 4\sqrt{4} = 4(2) = 8$

(ii)Let S = 1 - $\frac{1}{5}+\frac{1.4}{5.10}-\frac{1.4.7}{5.10.15}+\ldots.$

$$= 1 - \left(\frac{1}{3}\right)\left(\frac{3}{5}\right)+\frac{\left[\frac{1}{3}\right]\left[\frac{4}{3}\right]}{1.2}\left[\frac{3}{5}\right]^2+\ldots.$$

$$= \left[1+\frac{3}{5}\right]^{-\frac{1}{3}}$$

$$= \left[\frac{8}{5}\right]^{-\frac{1}{3}}$$

$$= \left[\frac{5}{8}\right]^{\frac{1}{3}}$$

(iii) Let S = $\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\ldots$

$$S + 1 = 1+\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\ldots$$

$$= 1+\left(\frac{3}{2}\right)\left(\frac{2}{4}\right)+\frac{\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)}{1.2}\left(\frac{2}{4}\right)^2+\ldots\ldots\ldots$$

$$= \left(1-\frac{2}{4}\right)^{-3/2}=\left(\frac{1}{2}\right)^{-3/2}=2^{3/2}$$

$$S + 1 = 2^{3/2}$$

Therefore S = $2^{3/2} - 1$

Question-39

(i) Show that the coefficient of x^n in the infinite series 1 + $\frac{b+ax}{1!}+\frac{(b+ax)^2}{2!}+\frac{(b+ax)^3}{3!}+\ldots\ldots\ldots$ is $\frac{e^b a^n}{n!}$.

(ii) Show that $\left(1+\frac{1}{2!}+\frac{1}{4!}+\ldots\ldots.\right)^2 = 1+\left(1+\frac{1}{3!}+\frac{1}{5!}+\ldots\ldots.\right)^2$.

(iii) Show that $2\left\{1+\frac{(\log n)^2}{2!}+\frac{(\log n)^4}{4!}+\ldots\right\} = n + c$.

Solution:

(i) $1+\frac{y}{1!}+\frac{y^2}{2!}+\frac{y^3}{3!}+\ldots\ldots\ldots\ldots = e^y = e^{b+ax} = e^b \cdot e^{ax} = e^b\left(1+\frac{ax}{1!}+\frac{(ax)^2}{2!}+\ldots\ldots.\right)$

Coefficient of $x^n = e^b \cdot \left(\frac{a^2}{n!}\right)$

(ii) L.H.S = $\left(1+\frac{1}{2!}+\frac{1}{4!}+\ldots\ldots\right)^2 = \left(\frac{e+e^{-1}}{2}\right)^2$

$$= 1 + \frac{e^2 + e^{-2} - 2}{4}$$

$$= \frac{4 + e^2 + e^{-2} - 2}{4}$$

$$= \frac{e^2 + e^{-2} + 2}{4}$$

= R.H.S

(iii) L.H.S = $2\left\{1 + \frac{(\log n)^2}{2!} + \frac{(\log n)^4}{4!} + \ldots\right\}$

Put log n = y

$$2\left\{1 + \frac{(\log n)^2}{2!} + \frac{(\log n)^4}{4!} + \ldots\right\} = 2\left(\frac{e^y + e^{-y}}{2}\right)$$

$$= e^y + e^{-y}$$

$$= e^{\log n} + e^{-\log n}$$

$$= e^{\log n} + e^{\log 1/n}$$

$$= n + \frac{1}{n}$$

Question-40

Show that log a – log b = $\frac{a-b}{a} + \frac{1}{2}\left(\frac{a-b}{a}\right)^2 + \frac{1}{3}\left(\frac{a-b}{a}\right)^3 + \ldots\ldots\ldots$

Solution:

R.H.S = $\frac{a-b}{a} + \frac{1}{2}\left(\frac{a-b}{a}\right)^2 + \frac{1}{3}\left(\frac{a-b}{a}\right)^3 + \ldots\ldots\ldots$

Put y = $\frac{a-b}{a}$

$y + \frac{y^2}{2} + \frac{y^3}{3} + \ldots\ldots\ldots = -\log(1 - y)$

$$= -\log\left(1 - \frac{a-b}{a}\right)$$

$$= -\log\left(\frac{b}{a}\right)$$

$$= \log\left(\frac{a}{b}\right)$$

= log a – log b

= L.H.S

Question-41

Prove that log $\frac{n+1}{n-1} = \frac{2n}{n^2+1} + \frac{1}{3}\left(\frac{2n}{n^2+1}\right)^3 + \ldots\ldots\ldots\ldots$

Solution:

R.H.S = $\frac{2n}{n^2+1} + \frac{1}{3}\left(\frac{2n}{n^2+1}\right)^3 + \ldots\ldots\ldots\ldots$

Put $\frac{2n}{n^2+1}$ = y

$$y + \frac{y^3}{3} + \frac{y^5}{5} \ldots\ldots\ldots\ldots = \frac{1}{2}\log\left(\frac{1+y}{1-y}\right)$$

$$= \frac{1}{2}\log\left(\frac{1+\frac{2n}{n^2+1}}{1-\frac{2n}{n^2+1}}\right)$$

$$= \frac{1}{2}\log\left(\frac{n^2+1+2n}{n^2+1-2n}\right)$$

$$= \frac{1}{2}\log\left(\frac{n+1}{n-1}\right)^2$$

$$= \log\left(\frac{n+1}{n-1}\right)$$

= L.H.S

Question-42

Find the sum to infinity the series $\frac{1}{x-1} + \frac{1}{3}\frac{1}{(x-1)^3} + \frac{1}{5}\frac{1}{(x-1)^5} + \ldots\ldots\ldots\ldots\ldots$

Solution:

$\frac{1}{x-1} + \frac{1}{3}\frac{1}{(x-1)^3} + \frac{1}{5}\frac{1}{(x-1)^5} + \ldots\ldots\ldots\ldots\ldots$

Put y = $\frac{1}{x-1}$

$$\frac{1}{y} + \frac{1}{3}\frac{1}{y^3} + \frac{1}{5}\frac{1}{y^5} + \ldots\ldots\ldots\ldots = \frac{1}{2}\log\left(\frac{1+y}{1-y}\right)$$

$$= \frac{1}{2}\log\left(\frac{1+\frac{1}{1-x}}{1-\frac{1}{1-x}}\right)$$

$$= \frac{1}{2}\log\left(\frac{1-x+1}{1-x-1}\right)$$

$$= \frac{1}{2}\log\left(\frac{2-x}{-x}\right)$$

$$= \frac{1}{2}\log\left(\frac{x+2}{x}\right)$$

Sequences & Series

Short Type Questions

1. The first term of an A.P.is a, and the sum of the first p terms is zero, show that the sum of its next q terms is $\frac{-a(p+q)q}{p-1}$. [Hint: Required sum = $S_{p+q} - S_p$]
2. A man saved Rs 66000 in 20 years. In each succeeding year after the first year he saved Rs 200 more than what he saved in the previous year. How much did he save in the first year?
3. A man accepts a position with an initial salary of Rs 5200 per month. It is understood that he will receive an automatic increase of Rs 320 in the very next month and each month thereafter.
   (a) Find his salary for the tenth month
   (b) What is his total earnings during the first year?
4. If the pth and qth terms of a G.P. are q and p respectively, show that its $(p+q)^{th}$ term is $\left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$.
5. A carpenter was hired to build 192 window frames. The first day he made five frames and each day, thereafter he made two more frames than he made the day before. How many days did it take him to finish the job?
6. We know the sum of the interior angles of a triangle is 180°. Show that the sums of the interior angles of polygons with 3, 4, 5, 6, ... sides form an arithmetic progression. Find the sum of the interior angles for a 21 sided polygon.
7. A side of an equilateral triangle is 20cm long. A second equilateral triangle is inscribed in it by joining the mid points of the sides of the first triangle. The process is continued as shown in the accompanying diagram. Find the perimeter of the sixth inscribed equilateral triangle.
8. In a potato race 20 potatoes are placed in a line at intervals of 4 metres with the first potato 24 metres from the starting point. A contestant is required to bring the potatoes back to the starting place one at a time. How far would he run in bringing back all the potatoes?
9. In a cricket tournament 16 school teams participated. A sum of Rs 8000 is to be awarded among themselves as prize money. If the last placed team is awarded Rs 275 in prize money and the award increases by the same amount for

successive finishing places, how much amount will the first place team receive?

10. If $a_1, a_2, a_3, \ldots, a_n$ are in A.P., where $a_i > 0$ for all i, show that

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}=\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

11. Find the sum of the series

$(3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + \ldots$ to (i) n terms (ii) 10 terms

12. Find the r^{th} term of an A.P. sum of whose first n terms is $2n + 3n^2$.

[Hint: $a_n = S_n - S_{n-1}$]

13. If A is the arithmetic mean and G_1, G_2 be two geometric means between any two numbers, then prove that

$$2A = \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$$

14. If $\theta_1, \theta_2, \theta_3, \ldots, \theta_n$ are in A.P., whose common difference is d, show that

$$\sec\theta_1 \sec\theta_2 + \sec\theta_2 \sec\theta_3 + \ldots + \sec\theta_{n-1} \sec\theta_n = \frac{\tan\theta_n - \tan\theta_1}{\sin d}.$$

15. If the sum of p terms of an A.P. is q and the sum of q terms is p, show that the sum of $p + q$ terms is $-(p + q)$. Also, find the sum of first $p - q$ terms ($p > q$).

16. If p^{th}, q^{th}, and r^{th} terms of an A.P. and G.P. are both a, b and c respectively, show that

$$a^{b-c} \cdot b^{c-a} \cdot c^{a-b} = 1$$

Objective Type Questions

17. If the sum of n terms of an A.P. is given by

$S_n = 3n + 2n^2$, then the common difference of the A.P. is

(A) 3 (B) 2 (C) 6 (D) 4

18. The third term of G.P. is 4. The product of its first 5 terms is

(A) 4^3 (B) 4^4 (C) 4^5 (D) None of these

19. If 9 times the 9^{th} term of an A.P. is equal to 13 times the 13^{th} term, then the 22^{nd} term of the A.P. is

(A) 0 (B) 22 (C) 220 (D) 198

20. If x, $2y$, $3z$ are in A.P., where the distinct numbers x, y, z are in G.P. then the common ratio of the G.P. is

(A) 3 (B) $\frac{1}{3}$ (C) 2 (D) $\frac{1}{2}$

21. If in an A.P., $S_n = q n^2$ and $S_m = qm^2$, where S_r denotes the sum of r terms of the A.P., then S_q equals

(A) $\frac{q^3}{2}$ (B) mnq (C) q^3 (D) $(m+n)q^2$

22. Let S_n denote the sum of the first n terms of an A.P. If $S_{2n} = 3S_n$ then $S_{3n} : S_n$ is equal to

(A) 4 (B) 6 (C) 8 (D) 10

23. The minimum value of $4^x + 4^{1-x}$, $x \in R$, is

(A) 2 (B) 4 (C) 1 (D) 0

24. Let S_n denote the sum of the cubes of the first n natural numbers and s_n denote the sum of the first n natural numbers. Then $\sum_{r=1}^{n} \frac{S_r}{s_r}$ equals

(A) $\frac{n(n+1)(n+2)}{6}$ (B) $\frac{n(n+1)}{2}$

(C) $\frac{n^2+3n+2}{2}$ (D) None of these

25. If t_n denotes the nth term of the series $2 + 3 + 6 + 11 + 18 + \ldots$ then t_{50} is

(A) $49^2 - 1$ (B) 49^2 (C) $50^2 + 1$ (D) $49^2 + 2$

26. The lengths of three unequal edges of a rectangular solid block are in G.P. The volume of the block is 216 cm^3 and the total surface area is 252cm^2. The length of the longest edge is

(A) 12 cm (B) 6 cm (C) 18 cm (D) 3 cm

CBSE Class 11 Mathematics

Important Questions

Chapter 9

Sequences and Series

4 Marks Questions

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$= \frac{1\left[1-\left(\frac{2}{3}\right)^n\right]}{1-\frac{2}{3}}$$

$$= 3\left[1-\left(\frac{2}{3}\right)^n\right]$$

$$S_5 = 3\left[1-\left(\frac{2}{3}\right)^5\right] = \frac{211}{81}$$

3.Find the sum to n terms of the series $1^2+\left(1^2+2^2\right)+\left(1^2+2^2+3^2\right)+----$

Ans. $a_n = 1^2 + 2^2 + 3^2 + \text{---} + n^2$

$$a_n = \frac{n(n+1)(2n+1)}{6}$$

$$S_n = \frac{1}{6}\left[2\sum_{k=1}^{n} k^3 + 3\sum_{k=1}^{n} k^2 + \sum k\right]$$

$$= \frac{1}{6}\left[2.\frac{n^2(n+1)^2}{4} + \frac{3.n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{n(n+1)}{12}\left[n(n+1) + (2n+1) + 1\right]$$

$$= \frac{n(n+1)}{12}\left(n^2 + n + 2n + 1 + 1\right)$$

$$= \frac{n(n+1)(n^2 + 3n + 2)}{12}$$

$$= \frac{n(n+1)^2(n+2)}{12}$$

4.Show that the sum of $(m+n)^{th}$ and $(m-n)^{th}$ terms of an A. P. is equal to twice the m^{th} term.

`

Ans. $a_{m+n} = a + (m+n-1)\ d$

CBSE Class 12 Mathematics

Important Questions

Chapter 9

Sequences and Series

6 Marks Questions

1. 150 workers were engaged to finish a job in a certain no. of days 4 workers dropped out on the second day, 4 more workers dropped out on the third day and so on. It took 8 more days to finish the work find the no. of days in which the work was completed

Ans. a = 150, d = -4

$$S_n = \frac{n}{2}\left[2\times150+(n-1)(-4)\right]$$

If total works who would have worked all n days 150(n-8)

$$\text{A T Q } \frac{n}{2}\left[300+(n-1)(-4)\right]=150(n-8)$$

$$n = 25$$

2. Prove that the sum to n terms of the series

$$11+103+1005+----is\ \frac{10}{9}\left(10^n-1\right)+n^2$$

Ans. Sn = 11 + 103 + 1005 + ---- + n terms

Sn = (10+1) + (102 + 3) + (103 + 5) + ----- + [10n + (2n-1)]

$$S_n = \frac{10(10^n-1)}{10-1}+\frac{n}{2}\left(\not{1}+2n-\not{1}\right)$$

$$= \frac{10}{9}\left(10^n-1\right)+n^2$$

3. The ratio of A M and G. M of two positive no. a and b is m : n show that

$$a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$$

Ans. $\dfrac{\frac{a+b}{2}}{\sqrt{ab}} = \dfrac{m}{n}$

$$\frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$$

by C and D

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{m+n}{m-n}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2} = \frac{m+n}{m-n}$$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{m+n}}{\sqrt{m-n}}$$

by C and D

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$$

Sq both side

$$\frac{a}{b} = \frac{m+\cancel{n}+m-\cancel{n}+2\sqrt{m^2-n^2}}{m+\cancel{n}+m-\cancel{n}-2\sqrt{m^2-n^2}}$$

$$\frac{a}{b} = \frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$$

4. Between 1 and 31, m number have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7th and (m-1)th no. is 5:9 find the value of m.

Ans. $1, A_1, A_2, A_3, ---A_m, 31$ are in AP

$a = 1$

$a_n = 31$

$a_{m+2} = 314$

$a_n = a + (n-1)d$

$31 = a + (m+2-1)d$

$$d = \frac{30}{m+1}$$

$$\frac{A7}{A_{m-1}} = \frac{5}{9} \quad \text{(Given)}$$

$$\frac{1+7\left(\frac{30}{m+1}\right)}{1+(m-1)\left(\frac{30}{m+1}\right)} = \frac{5}{9}$$

$m = 1$

5. The Sum of two no. is 6 times their geometric mean, show that no. are in the ratio (3 + 3 √2) : (3 – 2 √2)

Ans. $a + b = 6\sqrt{ab}$

$$\frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$$

by C and D

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}}=\frac{3+1}{3-1}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2}=\frac{2}{1}$$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{\sqrt{2}}{1}$$

again by C and D

$$\frac{\sqrt{a}+\cancel{\sqrt{b}}+\sqrt{a}-\cancel{\sqrt{b}}}{\cancel{\sqrt{a}}+\sqrt{b}-\cancel{\sqrt{a}}-\sqrt{b}}=\frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$\frac{\cancel{2}\sqrt{a}}{\cancel{2}\sqrt{b}}=\frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$\frac{a}{b}=\frac{\left(\sqrt{2}+1\right)^2}{\left(\sqrt{2}-1\right)^2}$$ (on squaring both side)

$$\frac{a}{b}=\frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$$

$$\frac{a}{b}=\frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

$$a:b=\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$$