DCAM classes
Dynamic Classes for Academic Mastery

Chapter 1. Sets

Question-1

Which of the following are sets?

(i) The collection of all months of a year beginning with letter J.
(ii) The collection of most talented writers of India.
(iii) A team of eleven best cricket batsmen of the world.
(iv) The collection of all boys in your class.
(v) The collection of all natural numbers less than 100.
(vi) The collection of novels written by the river Prem Chand.
(vii) The collection of all even integers.
(viii) The collection of different problems in this chapter.
(ix) A collection of most dangerous animals of the world.

Solution:

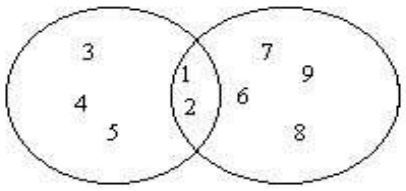
(i), (iv), (v), (vi), (vii) and (viii) are sets.

Question-2

$A \cup B = \{1,2,3,4,5,6,7,8,9\}$
$A \cap B \{1,2\}$ and $A = \{1, 2, 3, 4, 5\}$ find the set B.

Solution:

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$A \cap B = \{1, 2\}$ and
$A = \{1, 2, 3, 4, 5\}$



$\therefore B = \{1, 2, 6, 7, 8, 9\}$

Question-3

If U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, find the complements of the following sets:

(i) A = {2, 4, 6, 8}
(ii) B = {1, 3, 5, 7, 9}

(iii) C = { 2, 3, 5, 7}

(iv) φ
(v) U

Solution:

(i) A′ = {1, 3, 5, 7, 9}
(ii) B′ = {2, 4, 6, 8}
(iii) C′ = {1, 4, 6, 8, 9}
(iv) φ′ = U
(v) U′ = φ

Question-4

Let A = {1, 2, 3, 4, 5, 6}. Insert the appropriate symbol ∈ or ∉ in the blank spaces:

(i) 5 _____ A
(ii) 8 _____ A
(iii) 0 _____ A
(iv) 4 _____A
(v) 2 _____ A
(vi) 10 _____ A

Solution:

(i) 5 __∈ __ A

(ii) 8 __∉ __ A
(iii) 0 __∉__ A
(iv) 4 __∈ __ A
(v) 2 __∈ __ A
(vi) 10 __∉ __ A

Question-5

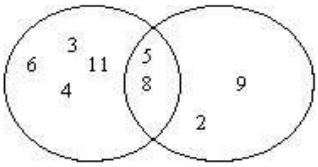
If A ∪ B ={2, 3, 4, 5, 6, 8, 9, 11}, A ∩ B {5,8} and B = {2, 5, 8, 9} find the set A – B

Solution:

A ∪ B ={2, 3, 4, 5, 6, 8, 9, 11},

A ∩ B = {5,8}

B = {2, 5, 8, 9}



∴ A = {3, 4, 5, 6, 8, 11}

∴ A – B = {3, 4, 6, 11}

Question-6

If U is the set of all natural numbers and A' is the set of all composite numbers, what is A?

Solution:

U = {1, 2, 3, 4, 5, 6, ……}

A' = {4, 6,……..}

Then A = {1, 2, 3, 5, ……..}

Question-7

Write the following sets in the roaster form:

(i) A = {x : x is an integer and −3 ≤ x < 7}

(ii) B = {x : x is a natural number less than 6}

(iii) C = {x : x is two digit natural number such that sum of its digits is 8}

(iv) D = {x : x is a prime number which is a divisor of 60}

(v) E = the set of all letters in the word TRIGONOMETRY

(vi) F = the set of all letters in the word SETS.

Solution:

(i) A = {-3, -2, -1, 0, 1, 2, 3, 4, 5, 6}

(ii) B = {1, 2, 3, 4, 5}

(iii) C = {17, 26, 35, 44, 53, 62, 71}

(iv) D = {2, 3, 5}

(v) E = {T, R, I, G, O, N, M, E, R, Y}

(vi) F = {S, E, T}

Question-8

If A = {p, q, r, s} find A ∩ A and A ∪ A.

Solution:

A = {p, q, r, s}

∴ A ∩ A = {p, q, r, s}

∴ A ∪ A = {p, q, r, s}

Question-9

Which of the following statements are true and which are false?

(i) U' = φ

(ii) φ ' = U

(iii) For any two subsets, X and Y of U,
(X ∪ Y)' = X' ∪ Y'

(iv) For any two subsets, X and Y of U,
(X ∩ Y)' = X' ∩ Y'

(v) For any two subsets, S and T of U,
(S ∪ T)' = S' ∩ T'

(vi) For any two subsets S and T of U,
(S ∩ T)' = S' ∪ T'

Solution:

(i) True

(ii) True

(iii) False

(iv) False

(v) True

(vi) True

Question-10

If A = {x : x is a letter in the word, 'follow'} and
B = {x : x is a letter in the word, 'wolf'}, show that A = B.

Solution:

Clearly, A = {f, o , l, w} and B = {w, o, l, f}.

Since every element of A is in B and every element of B is in A, so A = B.

Question-11

Express the following sets by using the set builder method:

(i) A = { 1, 3, 5, 7, 9}

(ii) B = {2, 4, 6, 8}

(iii) C = {-1, 1}

(iv) D = {1, 5, 10, 15, ...}

(v) E = {14, 21, 28, 35, 42, ...,98}

Solution:

(i) A = { x : x is an odd natural number, x £ 9}

(ii) B = { x : x is an even natural number, x ≤ 8}

(iii) C= {x : x is an odd natural number and |x| < 2}

(iv) D = {x : x is a natural number multiple of 5 and x = 1}

(v) E = {x : x is a multiple of 7 and 7<x<100}

Question-12

Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60^0, what is A?

Solution:

U = The set of all triangles in a plane.

A = The set of all triangles with at least one angle different from 60^0.

A' = The set of all equilateral triangles.

Question-13

List all the elements of the following sets:

(i) A = {x : x is an odd natural number}

(ii) B = {x : x is an integer, -1/2<x<9/2}

(iii) C = {x : x is an integer, $x^2 \leq 4$}

(iv) D = {x : x is a letter in the word "LOYAL"}

(v) E = {x : x is a month of a year not having 31 days}

(vi) F = {x : x is a consonant in the English alphabet which precedes k}

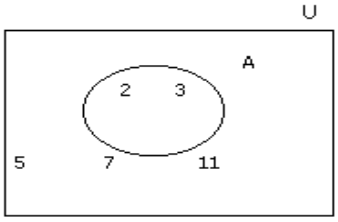
Solution:

(i) A = {1, 3, 5, 7, 9, 11, 13….}
(ii) B = {0, 1, 2, 3, 4}
(iii) C = {-2, -1, 1, 2}
(iv) D = {L, O, Y, A}
(v) E = {February, April, June, September, November}
(vi) F = {b, c, d, f, g, h, j}

Question-14

Represent the following sets in a Venn diagram: U = {2, 3, 5, 7, 11}, A = {2, 3}

Solution:



Question-15

Match each of the sets on the left described in the roster form with the same set on the right described in set builder form:

(i) {1, 2, 3, 6} **(a) { x : x is a prime number and a divisor of 6 }**

(ii) {2, 3} **(b) {x : x is an odd natural number less than 10}**

(iii) {H, A, Y, R, N} **(c) {x : x is a natural number and divisor of 6.}**

(iv) {1, 3, 5, 7, 9} **(d) {x : x is a letter of the word 'HARYANA'.}**

Solution:

(i) {1, 2, 3, 6} (c) {x : x is a natural number and divisor of 6.}

(ii){2, 3} (a) { x : x is a prime number and a divisor of 6 }

(iii) {H, A, Y, R, N} (d) {x : x is a letter of the word 'HARYANA'.}

(iv) {1, 3, 5, 7, 9} (b) {x : x is an odd natural number less than 10}

Question-16

Which of the following sets is finite or infinite?

(i) The set of the months of a year.

(ii) {1, 2, 3, ……}

(iii) {1, 2, 3, ……., 99, 100}

(iv) The set of positive integers greater than 100.

(v) The set of prime numbers less that 99.

Solution:

(i) Finite set

(ii) Infinite set

(iii) Finite set

(iv) Infinite set

(v) Finite set

Question-17

Represent the following sets in a Venn diagram:

U = { x : x is a natural number and 2 ≤ x ≤ 8.}

A = {x : x ∈ U and x divides 18}

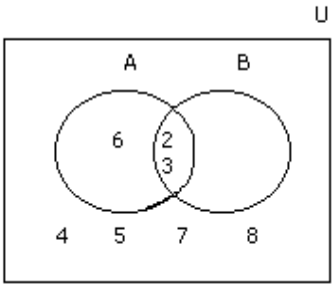
B = {x: x ∈ U and x is a prime divisor of 18}

Solution:

U = {2, 3, 4, 5, 6, 7, 8}

A = {2, 3, 6}

and B = {2, 3}



Question-18

Which of the following sets is finite or infinte?

(i) The set of lines which are parallel to the x-axis.

(ii) The set of letters in the English alphabet.

(iii) The set of numbers which are multiples of 5.

(iv) The set of animals living on earth.

(v) The set of circles in plane passing through the origin.

Solution:

(i) Infinite set

(ii) Finite set

(iii) Infinite set

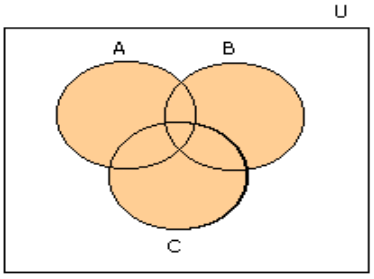
(iv) Finite set

(v) Infinite set

Question-19

If A, B and C are three subsets of the universal set U, draw a venn diagram showing A ∪ (B ∪ C)

Solution:



Question-20

Which of the following are examples of the null set?

(i) Set of odd natural numbers divisible by 2.

(ii) Set of even prime numbers.

(iii) {x : x is a natural number, x<5 and simultaneously x>7}

(iv) {y : y is a point common to any parallel lines}

Solution:

(i) Null set.

(ii) It is not a null set because 2 is a even prime number.

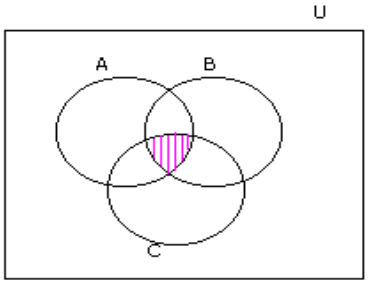
(iii) Null set.

(iv) Null set.

Question-21

If A, B and C are three subsets of the universal set U, draw a venn diagram showing (A ∩ B) ∩ C.

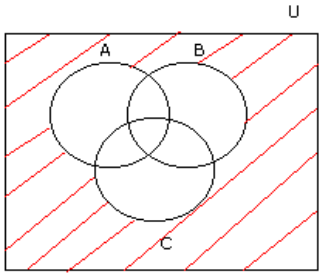
Solution:



Question-22

If A, B and C are three subsets of the universal set U, draw a venn diagram showing [(A ∪ B) ∪ C]'

Solution:



Question-23

In the following, state whether A = B or not:

(i) A = {a, b, c, d} B = {d, c, b, a}

(ii) A = {4, 8, 12, 16} B = {8, 4, 16, 18}

(iii) A = {2, 4, 6, 8, 10} B = {x :x is positive even integer less than 10}

(iv) A = {x : x is a multiple of 10} B = {10, 15, 20, 25, 30,.....}

Solution:

(i) A = B

(ii) A ≠ B, because element 12 of set A is not present in set B and element 18 of set B is not present in set A.

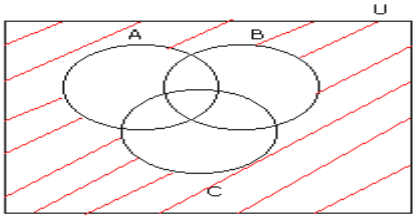
(iii) A = B

(iv) A ≠B, because set B consists of elements that are multiple of 5.

Question-24

If A, B and C are three subsets of the universal set U, draw a venn diagram showing (A' ∩ B') ∩ C'

Solution:



Question-25

Are the following pair of sets equal? Give reasons.

(i) A = {2, 3}
B = {x: x is a solution of $x^2 + 5x + 6 = 0$}

(ii) A = {x:x is a letter in the word FOLLOW}
B = {y:y is a letter in the word WOLF}

Solution:

(i) A = {2, 3}

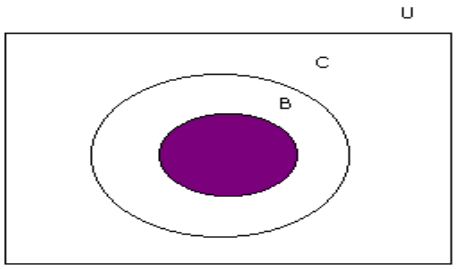
B = {x: x is a solution of $x^2 + 5x + 6 = 0$} = {2, 3}
Therefore the above pair are equal sets.

(ii) A = {x:x is a letter in the word FOLLOW} = {F, O, L, W}
B = {y:y is a letter in the word WOLF} = {W, O, L, F}
Therefore the above pair are equal sets.

Question-26

If A, B, and C are three subsets of the universal set U, draw Venn diagrams for the following:B ∩ C, when B ⊂ C.

Solution:



Question-27

From the sets given below, select equal sets and equivalent sets

A = {0, a}	**B = {1, 2, 3, 4}**
C = {4, 8, 12}	**D = {3, 1, 2, 4}**
E = {1, 0}	**F = {8, 4, 12}**
G = {1, 5, 7, 11}	**H = {a, b}**

Solution:

Equal sets:
(i) B = D
(ii) C = F;
Equivalent sets:

(i) A, E, H;
(ii) D, G;

Question-28

Given that A = {6, 7, 8, 9, 10} and B = {2, 3, 4, 5}. Write down all ordered pairs (a, b) such that a is divisible by b and hence write down the set ordered pairs given the relation 'is a multiple of' from A and B.

Solution:

A = {6, 7, 8, 9, 10} and B = {2, 3, 4, 5}

List of all ordered pairs : (6, 2), (6, 3), (6, 4), (6, 5), (7, 2), (7, 3), (7, 4), (7, 5), (8, 2), (8, 3), (8, 4), (8, 5), (9, 2), (9, 3), (9, 4), (9, 5), (10, 2), (10, 3), (10, 4), (10, 5)

The ordered pairs (a, b) such that a is divisible by b: (6, 2), (6, 3), (8, 2), (8, 4), (9, 3), (10, 2), (10,5)

Question-29

Which of the following statements are true?

(i) The set of all cats is contained in the set of all animals.
(ii) The set of all isosceles triangles is contained in the set of all equilateral triangles.
(iii) The set of all rectangles is contained in the set of all squares.
(iv) The sets A = {1} and B = {{1}} are equal.
(v) The sets A = {x : x is a letter in the word "TITLE"} and B = {x : x is a letter in the word "LITTLE"} are equal.

Solution:

(i) True
(ii) False
(iii) False
(iv) False
(v) True

Question-30

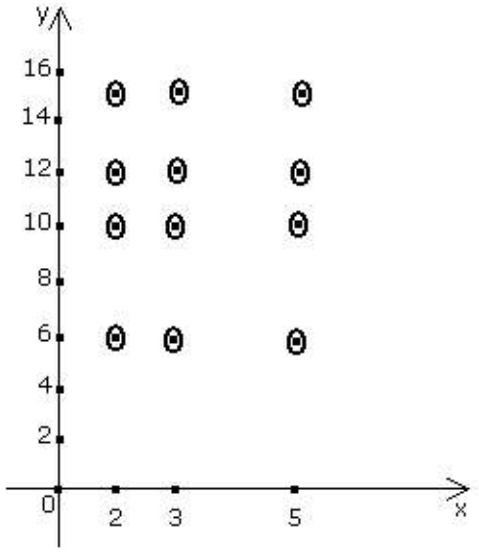
Let R be the relation defined by 'divides" from A = {2, 3, 5} to B = {6, 10, 12, 15}. Represent R (i) as a set of ordered pairs (ii) as a graph and (iii) by an arrow diagram.

Solution:

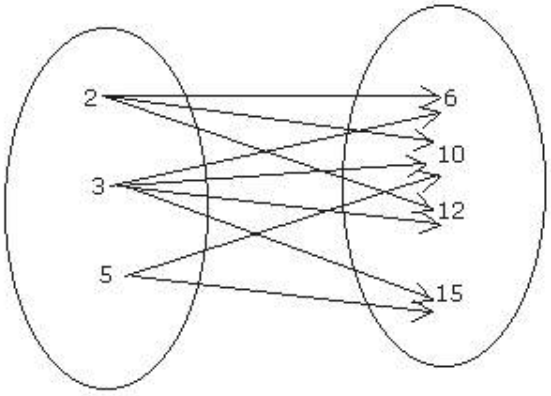
A = {2, 3, 5} to B = {6, 10, 12, 15}

(i) R = {(2, 6), (2, 10), (2, 12), (2, 15), (3, 6), (3, 10), (3, 12), (3, 15), (5, 6), (5, 10), (5, 12), (5, 15)}

(ii)



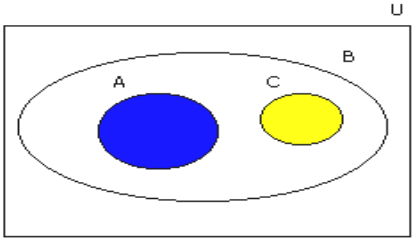
(iii)



Question-31

A and C are disjoint sets and both A and C are subsets of B, draw Venn diagrams for the following.

Solution:



Question-32

Make correct statements by filling in the symbols $\subset$ or $\not\subset$ in the blank spaces:

(i) {2, 3, 4} ..... {1, 2, 3, 4, 5,}

(ii) {a, b, c}..... {b, c, d}

(iii) {x : x is a student of Class 9 of your school.} .....{x: x is a student of your school}

(iv) {x : x is a circle in the plane} .....{x : x is a circle with radius 1}

(v) {x : x is a triangle in the plane} ..... { x : x is a rectangle in the plane}

(vi) {x : x is an equilateral triangle in the plane}.....{ x : x is a triangle in the plane}

(vii) {x : x is an even natural number} ..... {x : x is an integer}

Solution:

(i) {2, 3, 4} ...$\subset$.... {1, 2, 3, 4, 5,}

(ii) {a, b, c}...$\not\subset$..... {b, c, d}

(iii) {x : x is a student of Class 9 of your school.} ...$\subset$....{x; x is a student of your school.}

(iv) {x : x is a circle in the plane.} ...$\not\subset$... {x : x is a circle with radius 1.}

(v) {x : x is a triangle in the plane.} ...$\not\subset$... { x : x is a rectangle in the plane.}

(vi) {x : x is an equilateral triangle in the plane .}... $\subset$..... { x : x is a triangle in the plane.}

(vii) {x : x is an even natural number.} ...$\subset$... {x : x is an integer.}

Question-33

If A and B are two sets such that A has 21 elements, B has 17 elements, and A ∪ B has 21 elements, how many elements does A ∩ B have?

Solution:

We have n(A) = 12

n(B) = 17

n(A ∪ B) = 21

By using the formula,

n(A ∪ B) = n(A) + n(B) - n(A ∩ B)

We have 21 = 12 + 17 - n(A ∩ B)

∴ n(A ∩ B) = 29 -21

∴ n(A ∩ B) = 8

Question-34

If P = {2, 3, 4, 8, 9} write down the sets of ordered pairs representing the relations (i) is a factor of (ii) is divisible by (iii) is a multiple of 2 on P.

Solution:

P = {2, 3, 4, 8, 9}

(i) The sets of ordered pairs representing "is a factor of" : {(2, 2), (2, 4), (2, 8), (3, 3), (3, 9), (4, 4), (4, 8), (8, 8), (9, 9)}

(ii) The sets of ordered pairs representing "is divisible by" : {(2, 2), (4, 2), (8, 2), (4, 4), (8, 4), (8, 8), (9, 3), (9, 9), (3, 3)}

(iii) The sets of ordered pairs representing "is a multiple of 2 on P" : {((2, 2), (4, 2)), (8, 2)}

Question-35

Examine whether the following statements are true or false:

(i) {a, b} ⊄ {b, c, a}

(ii) {a, e} ⊂ { x : x is a vowel in the English alphabet.}

(iii) {1, 2, 3} ⊂ {1, 2, 3}

(iv) {a} ⊂ {a, b, c}

(v) {a} ∈ {a, b, c}

(vi) {x : x is an even natural number less than 6.} ⊂ { x : x is a natural number which divides 36}

Solution:

(i) False, because elements a and b are present in that set.

(ii) True

(iii) True

(iv) True

(v) False

(vi) True

Question-36

Describe the relation R defined from A to B where A = {- 1, 2, 3, 4} to B = {-2, 4, 6} by the set R = {(- 1, - 2), (2, 4), (3, 6)}.

Solution:

A = {- 1, 2, 3, 4} to B = {-2, 4, 6}
set R = {(- 1, - 2), (2, 4), (3, 6)} is the relation "is half of"

Question-37

If A and B are disjoint sets, show that n(A ∪ B) = n(A) + n(B)

Solution:

We know that, if A and B are disjoint sets then $n(A \cap B) = \varphi$.
Hence, by using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
We have $n(A \cup B) = n(A) + n(B) - \varphi$
$\therefore n(A \cup B) = n(A) + n(B)$

Example: Let A = {1, 2} and B = {3, 4},
then $A \cup B = \{1, 2, 3, 4\}$ and $A \cap B = \varphi$
Now $n(A) = 2, n(B) = 2, n(A \cup B) = 4$ and $n(A \cap B) = \varphi$
Hence, $n(A \cup B) = n(A) + n(B)$

Question-38

Let A = {1, 2, {3, 4}, 5}. Which of the following statements are false and why?

(i) $\{3, 4\} \subset A$

(ii) $\{3, 4\} \in A$

(iii) $\{\{3, 4\}\} \subset A$

(iv) $1 \in A$

(v) $1 \subset A$

(vi) $\{1, 2, 5\} \subset A$

(vii) $\{1, 2, 5\} \in A$

(viii) $\{1, 2, 3\} \subset A$

(ix) $\varphi \in A$

(x) $\{\varphi\} \subset A$

Solution:

(i) False, {3,4} is an element not a set.
(ii) True
(iii) True
(iv) True
(v) False, 1 is an element not a set.
(vi) True
(vii) False, {1, 2, 5} is a set not an element.
(viii) False, 3 is an element of set contained in A.
(ix) False, φ is not an element of A.
(x) False, φ is not an element of A.

Question-39

Write the power set of A = {3,6,9}.

Solution:

P(A)= {φ , {3}, {9},{6},{3,6},{3,9},{6,9},{3,6,9}}.

Question-40

Describe the relation, domain and range if (i) R = {(1, 1), (8, 2), (27, 3), (64, 4)} (ii) R = {(Delhi, India), (Paris, France), (Karachi, Pakistan)} (iii) R = {(4, - 2), (9, - 3), (1, 1), (4, 2), (1, - 1), (9, 3)}

Solution:

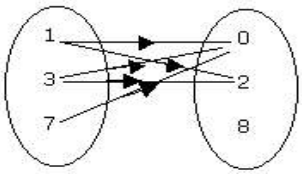
(i) R = {(1, 1), (8, 2), (27, 3), (64, 4)}
R is the relation "is the cube of "
Domain = {1, 8, 27, 64}
Range = {1, 2, 3, 4}
(ii) R = {(Delhi, India), (Paris, France), (Karachi, Pakistan)}
R is the relation "is the capital of"
Domain = {Delhi, Paris, Karachi}
Range = {India , France, Pakistan}
(iii) R = {(4, - 2), (9, - 3), (1, 1), (4, 2), (1, - 1), (9, 3)}
R is the relation "is the square of"
Domain = {1, 4, 9}
Range = (- 3, - 2, - 1, 2 , 3}

Question-41

If $x \in \{1, 3, 7\}$, $y \in \{0, 2, 8\}$ and R is the relation such that $x + y < 8$, represent R (i) as a set of ordered pairs and (ii) by an arrow diagram.

Solution:

(i) $x \in \{1, 3, 7\}$, $y \in \{0, 2, 8\}$, R is such that $x + y < 8$.
The set of ordered pairs = {(1, 0), (1, 2), (3, 0), (3, 2), (7, 0)}
(ii)



Question-42

Which of the following sets are equal ?

A = {x : x ∈ N, x < 3}, B = {1, 2}, C = {3, 1}

D = {x : x ∈ N, x is odd, x < 5}, E = {1, 2, 1}, F = {1, 1, 3}

Solution:

A = {1, 2}, B = {1, 2}, C = {3, 1},
D = {1, 3}, E = {1, 2, 1}, F = {1, 1, 3}
A, B, E and C, D, F are equal sets.

Question-43

If A and B are two sets such that A∨ B has 25 elements, A has 10 elements, and B has 37 elements, how many elements does A ∧ B have?

Solution:

$n(A \cup B) = 25; n(A) = 10 ; n(B) = 37$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\therefore 25 = 10 + 37 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 12$

$\therefore A \wedge B$ has 12 elements.

Question-44

In a group of 52 persons, 16 drink tea but not coffee and 33 drink tea. Find :

(i) how many drink tea and coffee both:

(ii) how many drink coffee but not tea.

Solution:

Let A be the set of those persons who drink tea and let B be the set of those persons who drink coffee. Then,

$A \cap B$ = set of persons who drink both tea and coffee.

$A - B$ = set of persons who drink tea but not coffee.

$B - A$ = set of persons who drink coffee but not tea

$\therefore n(A \cup B) = 52, n(A-B) = 16$ and $n(A) = 33$

Now, $n(A-B) + n(A \cap B) = n(A)$

$\therefore n(A \cap B) = n(A) - n(A - B) = (33 - 16) = 17$

Thus, 17 persons drink tea and coffee both.

Now, $n(A) = 33$, $n(A \cup B) = 52$ and $n(A \cap B) = 17$

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\therefore n(A) = 33$, $n(A \cup B) = 52$ and $n(A \cap B) = 17$.

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow n(B) = n(A \cup B) + n(A \cap B) - n(A)$

$\Rightarrow n(B) = (52 + 17 - 33) = 36$

Also, $n(B-A) + n(A \cap B) = n(B)$

$\Rightarrow n(B-A) = n(B) - n(A \cap B) = (36 - 17) = 19$

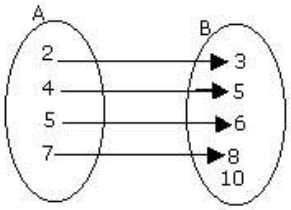
$\therefore$ 19 persons drink coffee but not tea.

Question-45

Represent the relation R from A = {2, 4, 5, 7} to B = {3, 5, 6, 8, 10} by an arrow diagram given a R b if b = a + 1 where a ∈ A and b ∈ B.

Solution:

A = {2, 4, 5, 7} to B = {3, 5, 6, 8, 10}



Sets

1. Write the following sets in the roaster from
   (i) $A = \{x : x \in \mathbf{R}, 2x + 11 = 15\}$ (ii) $B = \{x \mid x^2 = x, x \in \mathbf{R}\}$
   (iii) $C = \{x \mid x$ is a positive factor of a prime number $p\}$
2. Write the following sets in the roaster form :
   (i) $D = \{t \mid t^3 = t, t \in \mathbf{R}\}$ (ii) $E = \{w \mid \frac{w-2}{w+3} = 3, w \in \mathbf{R}\}$
   (iii) $F = \{x \mid x^4 - 5x^2 + 6 = 0, x \in \mathbf{R}\}$
3. If $Y = \{x \mid x$ is a positive factor of the number $2^{p-1}(2^p - 1)$, where $2^p - 1$ is a prime number$\}$. Write Y in the roaster form.
4. State which of the following statements are true and which are false. Justify your answer.
   (i) $35 \in \{x \mid x$ has exactly four positive factors$\}$.
   (ii) $128 \in \{y \mid$ the sum of all the positive factors of y is $2y\}$
   (iii) $3 \notin \{x \mid x^4 - 5x^3 + 2x^2 - 112x + 6 = 0\}$
   (iv) $496 \notin \{y \mid$ the sum of all the positive factors of y is $2y\}$.
5. Given $L = \{1, 2, 3, 4\}$, $M = \{3, 4, 5, 6\}$ and $N = \{1, 3, 5\}$
   Verify that $L - (M \cup N) = (L - M) \cap (L - N)$
6. If A and B are subsets of the universal set U, then show that
   (i) $A \subset A \cup B$ (ii) $A \subset B \Leftrightarrow A \cup B = B$
   (iii) $(A \cap B) \subset A$
7. Given that $N = \{1, 2, 3, \ldots, 100\}$. Then write
   (i) the subset of N whose elements are even numbers.
   (ii) the subset of N whose element are perfect square numbers.
8. If $X = \{1, 2, 3\}$, if n represents any member of X, write the following sets containing all numbers represented by
   (i) $4n$ (ii) $n + 6$ (iii) $\frac{n}{2}$ (iv) $n - 1$

9. If $Y = \{1, 2, 3, \ldots 10\}$, and a represents any element of Y, write the following sets, containing all the elements satisfying the given conditions.

   (i) $a \in Y$ but $a^2 \notin Y$ (ii) $a + 1 = 6, a \in Y$

   (iii) a is less than 6 and $a \in Y$

10. A, B and C are subsets of Universal Set U. If $A = \{2, 4, 6, 8, 12, 20\}$ $B = \{3, 6, 9, 12, 15\}$, $C = \{5, 10, 15, 20\}$ and U is the set of all whole numbers, draw a Venn diagram showing the relation of U, A, B and C.

11. Let U be the set of all boys and girls in a school, G be the set of all girls in the school, B be the set of all boys in the school, and S be the set of all students in the school who take swimming. Some, but not all, students in the school take swimming. Draw a Venn diagram showing one of the possible interrelationship among sets U, G, B and S.

12. For all sets A, B and C, show that $(A - B) \cap (C - B) = A - (B \cup C)$

Determine whether each of the statement in Exercises 13 – 17 is true or false. Justify your answer.

13. For all sets A and B, $(A - B) \cup (A \cap B) = A$

14. For all sets A, B and C, $A - (B - C) = (A - B) - C$

15. For all sets A, B and C, if $A \subset B$, then $A \cap C \subset B \cap C$

16. For all sets A, B and C, if $A \subset B$, then $A \cup C \subset B \cup C$

17. For all sets A, B and C, if $A \subset C$ and $B \subset C$, then $A \cup B \subset C$.

Using properties of sets prove the statements given in Exercises 18 to 22

18. For all sets A and B, $A \cup (B - A) = A \cup B$

19. For all sets A and B, $A - (A - B) = A \cap B$

20. For all sets A and B, $A - (A \cap B) = A - B$

21. For all sets A and B, $(A \cup B) - B = A - B$

22. Let $T = \left\{x \mid \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}\right\}$. Is T an empty set? Justify your answer.

23. Let A, B and C be sets. Then show that
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

24. Out of 100 students; 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science; 4 in English and Science; 4 in all the three. Find how many passed
   (i) in English and Mathematics but not in Science
   (ii) in Mathematics and Science but not in English
   (iii) in Mathematics only
   (iv) in more than one subject only

25. In a class of 60 students, 25 students play cricket and 20 students play tennis, and 10 students play both the games. Find the number of students who play neither?

26. In a survey of 200 students of a school, it was found that 120 study Mathematics, 90 study Physics and 70 study Chemistry, 40 study Mathematics and Physics, 30 study Physics and Chemistry, 50 study Chemistry and Mathematics and 20 none of these subjects. Find the number of students who study all the three subjects.

27. In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers. Find
   (a) The number of families which buy newspaper A only.
   (b) The number of families which buy none of A, B and C

28. In a group of 50 students, the number of students studying French, English, Sanskrit were found to be as follows:
French = 17, English = 13, Sanskrit = 15
French and English = 09, English and Sanskrit = 4

French and Sanskrit = 5, English, French and Sanskrit = 3. Find the number of students who study

   (i) French only
   (ii) English only
   (iii) Sanskrit only
   (iv) English and Sanskrit but not French
   (v) French and Sanskrit but not English
   (vi) French and English but not Sanskrit
   (vii) at least one of the three languages
   (viii) none of the three languages

Choose the correct answers from the given four options in each Exercises 29 to 43 (M.C.Q.).

29. Suppose $A_1, A_2, \ldots, A_{30}$ are thirty sets each having 5 elements and $B_1, B_2, \ldots, B_n$ are n sets each with 3 elements, let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$ and each element of S belongs to exactly 10 of the A_i's and exactly 9 of the B_j'S. then n is equal to
(A) 15 (B) 3 (C) 45 (D) 35

30. Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of the second set. The values of m and n are, respectively,
(A) 4, 7 (B) 7, 4 (C) 4, 4 (D) 7, 7

31. The set $(A \cap B')' \cup (B \cap C)$ is equal to
(A) $A' \cup B \cup C$ (B) $A' \cup B$ (C) $A' \cup C'$ (D) $A' \cap B$

32. Let F_1 be the set of parallelograms, F_2 the set of rectangles, F_3 the set of rhombuses, F_4 the set of squares and F_5 the set of trapeziums in a plane. Then F_1 may be equal to
(A) $F_2 \cap F_3$ (B) $F_3 \cap F_4$
(C) $F_2 \cup F_5$ (D) $F_2 \cup F_3 \cup F_4 \cup F_1$

33. Let S = set of points inside the square, T = the set of points inside the triangle and C = the set of points inside the circle. If the triangle and circle intersect each other and are contained in a square. Then
(A) $S \cap T \cap C = \phi$ (B) $S \cup T \cup C = C$
(C) $S \cup T \cup C = S$ (D) $S \cup T = S \cap C$

34. Let R be set of points inside a rectangle of sides a and b $(a, b > 1)$ with two sides along the positive direction of x-axis and y-axis. Then
(A) $R = \{(x, y) : 0 \le x \le a, 0 \le y \le b\}$
(B) $R = \{(x, y) : 0 \le x < a, 0 \le y \le b\}$
(C) $R = \{(x, y) : 0 \le x \le a, 0 < y < b\}$
(D) $R = \{(x, y) : 0 < x < a, 0 < y < b\}$

35. In a class of 60 students, 25 students play cricket and 20 students play tennis, and 10 students play both the games. Then, the number of students who play neither is

(A) 0 (B) 25 (C) 35 (D) 45

36. In a town of 840 persons, 450 persons read Hindi, 300 read English and 200 read both. Then the number of persons who read neither is

(A) 210 (B) 290 (C) 180 (D) 260

37. If $X = \{8^n - 7n - 1 \mid n \in N\}$ and $Y = \{49n - 49 \mid n \in N\}$. Then

(A) $X \subset Y$ (B) $Y \subset X$ (C) $X = Y$ (D) $X \cap Y = \phi$

38. A survey shows that 63% of the people watch a News Channel whereas 76% watch another channel. If x% of the people watch both channel, then

(A) $x = 35$ (B) $x = 63$ (C) $39 \leq x \leq 63$ (D) $x = 39$

39. If sets A and B are defined as

$A = \{(x, y) \mid y = \frac{1}{x}, 0 \neq x \in R\}$ $B = \{(x, y) \mid y = -x, x \in R\}$, then

(A) $A \cap B = A$ (B) $A \cap B = B$ (C) $A \cap B = \phi$ (D) $A \cup B = A$

40. If A and B are two sets, then $A \cap (A \cup B)$ equals

(A) A (B) B (C) ϕ (D) $A \cap B$

41. If $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$ $B = \{2, 4, \ldots, 18\}$ and N the set of natural numbers is the universal set, then $A' \cup (A \cup B) \cap B')$ is

(A) ϕ (B) N (C) A (D) B

42. Let $S = \{x \mid x$ is a positive multiple of 3 less than 100$\}$

$P = \{x \mid x$ is a prime number less than 20$\}$. Then $n(S) + n(P)$ is

(A) 34 (B) 31 (C) 33 (D) 30

43. If X and Y are two sets and X′ denotes the complement of X, then $X \cap (X \cup Y)'$ is equal to

(A) X (B) Y (C) ϕ (D) $X \cap Y$

CBSE Class 11 Mathematics

Important Questions

Chapter 1

Sets

1 Marks Questions

Which of the following are sets? Justify your answer.

1. The collection of all the months of a year beginning with letter M

Ans. Set

2. The collection of difficult topics in Mathematics. Let A = {1,3,5,7,9}. Insert the appropriate symbol $\in$ or $\in$ in blank spaces :– (Question- 3,4)

Ans. Not a set

3. 2—A

Ans. $\in$

4. 5 – A

Ans. $\in$

5. Write the set A = { x : x is an integer, –1 $\leq$ x < 4} in roster form

Ans. A = {–1, 0, 1, 2, 3}

6. List all the elements of the set,

$A = \{ x : x \in Z, -1/2 < x < 11/2\}$

Ans. A = { 0,1,2,3,4,5}

7. Write the set B = {3,9,27,81} in set-builder form. Which of the following are empty sets? Justify.

Ans. B = { x : x = 3^n , n ∈ N and 1 ≤ n ≤ 4}

8. A = { x : x ∈ N and 3 <x <4}

Ans. Empty set

9. B = { x : x ∈ N and x^2 = x}Which of the following sets are finite or Infinite? Justify.

Ans. Non-empty set

10. The set of all the points on the circumference of a circle.

Ans. Infinite set

11. B = { x : x ∈ N and x is an even prime number}

Ans. Finite set

12. Are sets A = { –2,2}, B = { x : x ∈ Z, x^2 –4 = 0} equal? Why?

Ans. Yes

13. Write (–5,9] in set-builder form

Ans. { x : x ∈ R, –5 < x ≤ 9}

14. Write { x : –3 ≤ x <7} as interval.

Ans. [–3,7)

15. If A = { 1,3,5}, how many elements has P(A)?

Ans. $2^3 = 8$

16. Write all the possible subsets of A = {5,6}. If A = {2,3,4,5}, B = { 3,5,6,7}

Ans. ∅, { 5}, {6}, {5,6}

17. A ∪ B

Ans. A ∪ B = {2,3,4,5,6,7}

18. A ∩ B

Ans. A ∩ B = {3, 5}

19. If A = {1,2,3,6} , B = {1, 2, 4, 8} find B – A

Ans. B – A = {4,8}

20. If A = {p, q}, B = {p, q, r}, is B a superset of A? Why?

Ans. Yes, because A is a subset of B

21. Are sets A = {1,2,3,4}, B = { x : x ∈ N and 5 ≤ x ≤ 7} disjoint? Why?

Ans. es, because A ∩ B = ∅

22. If X and Y are two sets such that n(X) = 19, n(Y) = 37 and n(X ∩ Y) = 12, find n(X ∪ Y).

Ans. n(X ∪ Y) = 44

23.Describe the set in Roster form

$\{x: x \text{ is a two digit number such that the sum of its digit is } 8\ \}$

Ans. { 17, 26, 35, 44, 53, 62, 71, 80 }

24.Are the following pair of sets equal? Give reasons.

A = { x:x is a letter in the word FOLLOW}

B = { y:y is a letter in the word WOLF}

Ans. A= {F, O, L, W}

B = {W, O, L, F }

Hence A=B

25.Write down all the subsets of the set {1,2,3}

Ans. $\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

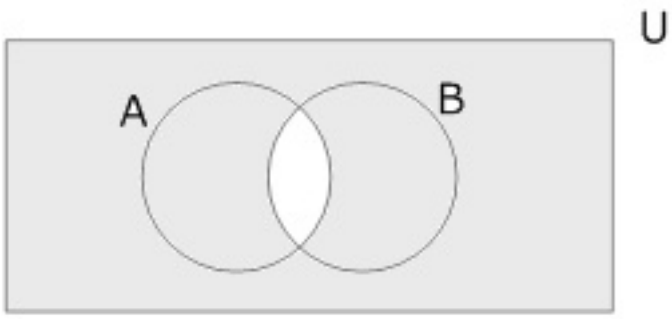
26.Let A = { 1,2,{3,4,},5} is $\{\{3,4\}\} \in A$ **is incorrect. Give reason.**

Ans. {3,4} is an elements of sets A, therefore $\{\{3,4\}\}$ is a set containing element {3,4} which is belongs to A

Hence $\{\{3,4\}\} \in A$ is correct

27.Draw venn diagram for $(A \cap B)'$

Ans. $(A \cap B)'$ = U- (A∩B)



28.Write the set in roster form A = The set of all letters in the word T R I G N O M E T R Y

Ans. A = {T, R, I, G, N, O, M, E, Y}

29.Are the following pair of sets equal? Give reasons

A, the set of letters in "ALLOY" and B, the set of letters in "LOYAL".

Ans. A = {A, L, O, Y}

B = {L, O, Y, A}

Hence A = B

30.Write down the power set of A , A = {1, 2, 3}

Ans. $P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

31.A = {1, 2, {3, 4}, 5} which is incorrect and why. (i) {3, 4} $\subset$ A (ii) {3, 4} $\in$ A

Ans.{3, 4} is an element of set A.

Hence $\{3,4\} \in A$ is correct and

$\{3,4\} \subset A$ is incorrect.

32.Fill in the blanks.

(i) $A \cup A' =$--------

(ii) $(A')'$ = ---------

(iii) $A \cap A' =$ --------

Ans. (i)U

(ii) A

(iii) ϕ

33.Write the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ in the set builder form.

Ans. $\left\{x: x=\frac{n}{n+1}, \text{ where n is a natural no. and } 1 \leq n \leq 6\right\}$

34.Is set C = { x : x – 5 = 0} and E = {x : x is an integral positive root of the equation x^2 – 2x – 15 = 0} are equal?

Ans. C = {5}

$x^2-2x-15=0$

$x^2-5x+3x-15=0$

$x(x-5)+3(x-5)=0$

$(x-5)(x+3)=0$

$x=5$

$x=-3 \quad [x=-3 \text{ reject}]$

$x=5$

$E=\{5\}$

Henc C = E.

35.Write down all possible proper subsets of the set {1, {2}}.

Ans. $\phi, \{1\}, \{\{2\}\}, \{1,\{2\}\}$

36.State whether each of the following statement is true or false.

(i) {2, 3, 4, 5} and {3, 6} are disjoint sets.

(ii) {2, 6, 10} and {3, 7, 11} are disjoint sets

Ans. (i) $\{2,3,4,5\} \cap \{3,6\} = \{3\} \neq \phi$

Hence false

(ii) $\{2,6,10\} \cap \{3,7,11\} = \phi$

true

37.Fill in the blanks

(i) (A ∪ B)′ = ---------

(ii) (A ∩ B)′ = ---------

Ans. $(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

38.Write the set of all vowels in the English alphabet which precede k in roster Form

Ans. A = {b, c, d, f, g, h, j}

39.Is pair of sets equal? Give reasons.

A = {2, 3} B = x : x is solution of $x^2 + 5x + 6 = 0$}

Ans. A = {2, 3}

B = {- 2, - 3}

A ≠ B

[∴ $x^2 + 5x + 6 = 0$

$x^2 + 3x + 2x + 6 = 0$

x = - 2 – 3

40.Write the following intervals in set builder form: (-3, 0) and [6, 12]

Ans. (- 3, 0) = {x : x ∈ R, - 3 < x < 0}

[6, 12] = {x : x ∈ R, 6 ≤ x ≤ 12}

41.If X = {a, b, c, d}

Y = {f, b, d, g}

Find X – Y and Y – X

Ans. X – Y = {a, b, c, d} – {f, b, d, g}

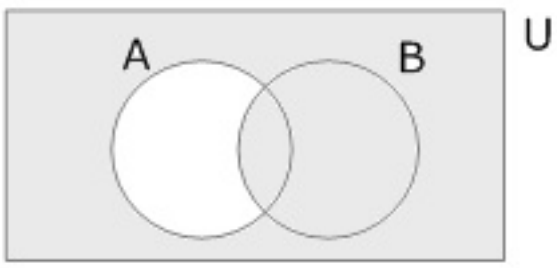
= {a, c}

Y – X = {f, b, d, g} – {a, b, c, d}

= {f, g}

42.If A and B are two given sets, Then represent the set $(A - B)'$, using Venn diagram.

Ans. $(A - B)'$ = ∪ - (A – B)



43.List all the element of the set A = { x : x is an integer $x^2 \leq 4$}

Ans. {-2, -1, 0, 1, 2}

44.From the sets given below pair the equivalent sets.

A = { 1, 2, 3}, B = {x, y, z, t}, C = {a, b, c} D = {0, a}

Ans. A = {1, 2, 3} B = {a, b, c} are equivalent sets $\left[\because n(A) = n(B)\right]$

45.Write the following as interval

(i) {x : x ∈ R, - 4 < x ≤ 6}

(ii) {x : x ∈ R, 3 ≤ x ≤ 4}

Ans. (i) (-4, 6]

(ii) [3, 4]

46.If A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13}, C = {11, 13, 15} Find (A ∩ B) ∩ (B ∪ C)

Ans. A ∩ B = {7, 9, 11}

B ∪ C = {7, 9, 11, 13, 15}

(A ∩ B) ∩ (B ∪ C) = {7, 9, 11}

47.Write the set $\left\{\frac{1}{3}, \frac{3}{5}, \frac{5}{7}, \frac{7}{9}, \frac{9}{11}, \frac{11}{13}\right\}$ **in set builder form.**

Ans. $\left\{\frac{2n-1}{2n+1}:\ \text{n is a natural no. less than } 7\right\}$

CBSE Class 12 Mathematics

Important Questions

Chapter 1

Sets

4 Marks Questions

1. In a group of 800 people, 500 can speak Hindi and 320 can speak English. Find

(i) How many can speak both Hindi and English?

(ii) How many can speak Hindi only?

Ans. (i) 20 people can speak both Hindi and English

(ii) 480 people can speak Hindi only

2. A survey shows that 84% of the Indians like grapes, whereas 45% like pineapple. What percentage of Indians like both grapes and pineapple?

Ans. 29% of the Indians like both grapes and pineapple.

3. In a survey of 450 people, it was found that 110 play cricket, 160 play tennis and 70 play both cricket as well as tennis. How many play neither cricket nor tennis?

Ans. Hint : ∪ – set of people surveyed

A – set of people who play cricket

B – set of people who play tennis

Number of people who play neither cricket nor tennis

= $n[(A \cup B)']$ = $n(U) – n(A \cup B)$

= 450 – 200

= 250

4. In a group of students, 225 students know French, 100 know Spanish and 45 know both. Each student knows either French or Spanish. How many students are there in the group?

Ans. There are 280 students in the group.

5. If A = [–3, 5), B = (0, 6] then find (i) A – B, (ii) A ∪ B

Ans. (i) [–3, 0]; (ii) [–3, 6]

6. In a survey of 400 students in a school, 100 were listed as taking apple juice, 150 as taking orange juice and 75 were listed as taking both apple as well as orange juice. Find how many students were taking neither apple juice nor orange juice.

Ans. Let A denote the set of students taking apple juice and B denote the set of students taking orange juice

n(U) = 400, n(A) = 100, n (B) = 150 n(A ∩ B) =75

n ($A' \cap B'$) = n(A ∪ B)′

=n(∪) – n(A ∪ B)

= n(∪) – [n(A) + n(B) – n(A ∩ B)]

=400-100-150+75=225

7. A survey shows that 73% of the Indians like apples, whereas 65% like oranges. What % Indians like both apples and oranges?

Ans. Let A=set of Indian who like apples

B= set of Indian who like oranges

n (A)= 73, n (B) = 65

n(A ∪ B)=100

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

=73+65-100

=38

38% like both

8. In a school there are 20 teachers who teach mathematics or physics. Of these 12 teach mathematics and 4 teach both physics and mathematics. How many teach physics?

Ans. $n(M \cup P) = 20, n(M)=12$

$n(M \cap P) =4$

$n(M \cup P) = n(M)+n(P)- n(M \cap P)$

n(P)=12

9. Let U = {1, 2, 3, 4, 5, 6} A = {2, 3} and B = {3, 4, 5}

Find $A' \cap B'$, $A \cup B$ and hence show that $(A \cup B)' = A' \cap B'$.

Ans. A' = U – A

= { 1, 4, 5, 6}

B' = U – B

={1, 2, 6}

A $\cup$ B = {2, 3, 4, 5}

$(A \cap B)'$ = U – (A $\cup$ B)

= {1, 6}

$A' \cap B'$ = {1, 6}

Hence proved.

10. For any two sets A and B prove by using properties of sets that:

$$(A \cap B) \cup (A - B) = A$$

Ans. L. H. S. = (A ∩B) ∪ (A – B)

= (A ∩ B) ∪ (A $\cap B'$) $\{\because (A-B)=(A \cap B')\}$

$= A \cap (B \cup B')$ {Distributine Law}

$= A \cap (U)$ $\{\because B \cup B' = U\}$

$= A$

11. If A and B, are two sets and U is the universal set such that n(U) = 1000, n(A) = 300, n (B) 300 and n(A ∩ B) = 200 find n $(A' \cap B')$.

Ans. n ($A' \cap B'$) = n ($A \cup B)'$

= n(U) – n (A ∪ B)

= n (U) – [n(A) + n (B) – n (A ∩B)]

= 1000 – [300+300-200]

= 1000 – 400

= 600

12. There are 210 members in a club. 100 of them drink tea and 65 drink tea but not coffee, each member drinks tea or coffee.

Find how many drink coffee, How many drink coffee, but not tea.

Ans. n (T) = 100

n (T – C) = 65

$n(T \cup C) = 210$

$n(T - C) = n(T) - n(T \cap C)$

$65 = 100 - n(T \cap C)$

$n(T \cap C) = 35$

$n(T \cup C) = n(T) + n(C) - n(T \cap C)$

$210 = 100 + n(C) - 35$

$n(C) = 145.$

Now,

$$n(C-T) = n(C) - n(C \cap T)$$

$$n(C-T) = 145 - 35$$

$$n(C-T) = 110$$

13. If P (A) = P (B), Show that A = B

Ans. $\forall\ a \in A$

$\Rightarrow \{a\} \subset A$

$\Rightarrow \{a\} \in P(A)$

$\Rightarrow \{a\} \in P(B)\ \ [\because P(A) = P(B)]$

$\Rightarrow \{a\} \in B$

$\Rightarrow a \subset B$

$\Rightarrow A \subset B$

for all $b \in B$

$\Rightarrow \{b\} \subset B$

$\Rightarrow \{b\} \in P(B) \quad [\because P(A) = P(B)]$

$\Rightarrow \{b\} \in P(A)$

$\Rightarrow \{b\} \subset A$

$\Rightarrow b \in A$

$\Rightarrow B \subset A$

Thus $A \subset B$

and $B \subset A$

$\Rightarrow A = B$

14. In a class of 25 students, 12 have taken mathematics, 8 have taken mathematics but not biology. Find the no. of students who have taken both mathematics and biology and the no. of those who have taken biology but not mathematics each student has taken either mathematics or biology or both.

Ans. n (M) = 12, n (M – B) = 8

$n(M \cup B) = 25$

$n(M \cup B) = n(M) + n(B - M)$

$25 = 12 + n(B - M)$

$13 = n(B-M)$

$n(M \cup B) = n(M-B) + n(M \cap B) + n(B-M)$

$25 = 8 + n(M \cap B) + 13$

$n(M \cap B) = 4$

15. A and B are two sets such that n (A – B) = 14 + x , n (B – A) = 3x and n (A ∩ B) = x. Draw a Venn diagram to illustrate this information. If n (A) = n (B), Find (i) the value of

x (ii) n (A ∪ B)

Ans. (i) n (A) = n (A – B) + n (A ∩ B)

= 14 + x + x

= 14 + 2x

N (B) = n (B – A) + n (A ∩ B)

= 3x + x

= 4x

but n (A) = n (B) (Given)

14 + 2x = 4x

x = 7

(ii) n (A ∪ B) = n (A – B) + n (B – A) + n (A ∩ B)

= 14 + x+ 3x + x

= 14 + 5x = 14 + 5 × 7 = 49

16. If A and B are two sets such that A ∪ B= A ∩ B, then prove that A = B

Ans. Let a ∈ A, then a ∈ A ∪ B

Since A ∪ B = A ∩ B

a ∈ A ∩ B . So a ∈ B

Therefore A ⊂ B

Similarly if b ∈ B,

Then b ∈ A ∪ B. Since

A ∪ B = A ∩ B, b ∈ A ∩ B

So b ∈ A

Therefore, B $\subset$ A

Thus A=B

17. Prove that if A ∪ B = C and A ∩ B = ϕ then A = C – B

Ans. C – B = A

$$=(A \cup B)-B$$

= (A ∪ B) ∩ B'

= B' ∩ (A ∪ B)

= (B' ∩ A) ∪ (B' ∩ B)

= (B' ∩ A) ∪ ϕ

= B' ∩ A

= A ∩ B'

= A – B

= A (Proved) $\left[\because A \cap B = \phi\right]$

18. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

Ans. Let C = the set of people who like cricket and

T = the set of people who like tennis.

n (C ∪ T) = 56, n (C) = 40

n (C ∩ T) = 10

n (C ∪ T) = n (C) + n (T) – n (C ∩T)

65 = 40 + n (T) – 10

n (T) = 35

Now,

$$n(T-C)=n(T)-n(T\cap C)$$

$$35-10$$

$$=25$$

19. Let A, B and C be three sets A ∪ B = A ∪ C and A ∩ B = A ∩ C show that B = C

Ans. Let $b \in B \Rightarrow b \in A \cup B$

$\Rightarrow b \in A \cup C$ [$\because A \cup B = A \cup C$

$\Rightarrow b \in A$ or $b \in C$

if $b \in C$ then $B \subset C$

if $b \in A$, then $b \in A \cap B$ [$\because A \cap B = A \cap C$

$\Rightarrow b \in A \cap C$

$\Rightarrow b \in C \Rightarrow B \subset C$

thus in both cases $B \subset C$

Simiularly $C \subset B$

Hence B = C

20. If ∪ = {a, e, i. o. u}

A = {a, e, i}

And B = {e, o, u}

C = {a, i, u}

Then verify that A ∩ (B – C) = (A ∩ B) – (A ∩ C)

Ans. B – C = {e, o}

A ∩ (B – C) = e

A ∩ B = {e}

A ∩ C = {a}

(A ∩ B) – (A ∩ C) = e

Hence proved.

CBSE Class 12 Mathematics

Important Questions

Chapter 1

Sets

6 Marks Questions

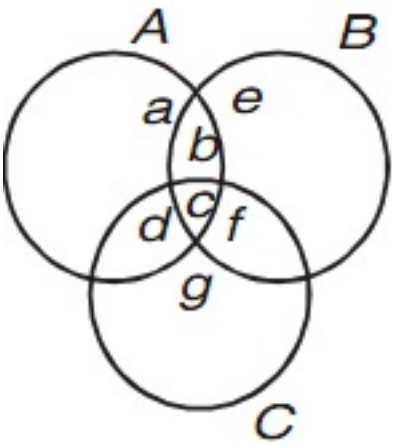
1. In a survey it is found that 21 people like product A, 26 people like product B and 29 like product C. If 14 people like product A and B, 15 people like product B and C, 12 people like product C and A, and 8 people like all the three products. Find

(i) How many people are surveyed in all?

(ii) How many like product C only?

Ans. Hint : Let A, B, C denote respectively the set of people who like product

A, B, C.



a, b, c, d, e, f, g – Number of elements in bounded region

(i) Total number of Surveyed people = a + b + c + d + e + f + g = 43

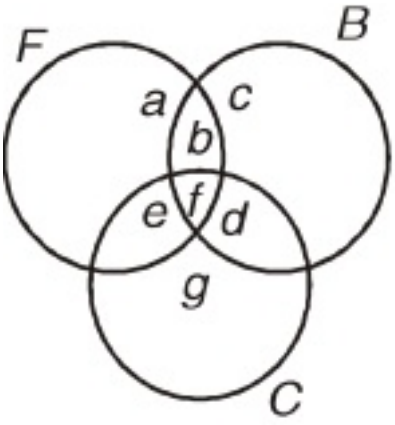
(ii) Number of people who like product C only = g = 10

2. A college awarded 38 medals in football, 15 in basket ball and 20 in cricket. If these medals went to a total of 50 men and only five men got medals in all the three sports,

how many received medals in exactly two of the three sports?

Ans. people got medals in exactly two of the three sports.

Hint :



f = 5

a + b + f + e = 38

b + c + d + f = 15

e + d + f + g = 20

a + b + c + d + e + f + g = 50

we have to find b + d + e

3.There are 200 individuals with a skin disorder, 120 had been exposed to the chemical C_1, 50 to chemical C_2, and 30 to both the chemicals C_1 and C_2. Find the number of individuals exposed to

(1) chemical C_1 but not chemical C_2

(2) chemical C_2 but not chemical C_1

(4) chemical C_1 or chemical C_2

Ans. A denote the set of individuals exposed to the chemical C_1 and B denote the set of

individuals exposed to the chemical C_2

n(U) = 200, n(A) = 120, n(B) = 50, n(A ∩ B) = 30

(i) n(A-B) = n(A) – n(A ∩ B)

=120-30=90

(ii)n(B-A) = n(B) – n(A ∩ B)

=50-30 = 20

(iii)n(A∪B) =n(A)+ n(B)-n(A ∩ B)

=120+50-30

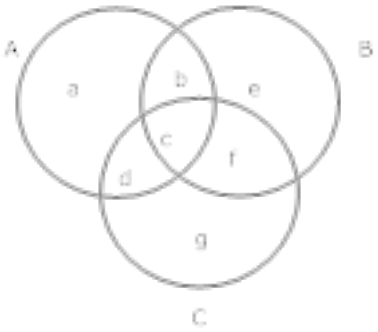
=140

4.In a survey it was found that 21 peoples liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people like C and A, 15 people like B and C and 8 liked all the three products. Find now many liked product C only.

Ans. a + b + c + d = 21

b + c + e + f = 26

c + d + f + g = 29



b + c = 14,c + f =15,c + d = 12

c = 8

d = 4,c = 8,f = 7,b = 6,g = 10,e = 5,a = 3

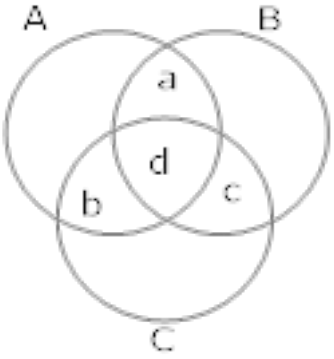
like product c only = g = 10

5.A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medal in all the three sports, how many received medals in exactly two of the three sports?

Ans. Let A, B and C denotes the set of men who received medals in football, basketball and cricket respectively.

n (A) = 38, n (B) = 15, n (C) = 20

n (A $\cup$B $\cup$C) = 58 and n (A $\cap$ B $\cap$ C) = 3



n (A $\cup$B $\cup$C) = n (A) + n (B) + n(C) – n (A $\cap$ B) – n (B $\cap$ C) – n (C $\cap$ A) + n (A $\cap$ B $\cap$ C)

58 = 38 + 15 + 20 – (a + d) – (d + c) – (b + d) + 3

18 = a + d + c + b + d

18 = a + b + c + 3d

18 = a + b + c + 3 $\times$3

9 = a + b + c

6.In a survey of 60 people, it was found that 25 people read news paper H, 26 read newspaper T,

26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three

newspaper. Find

(i) The no. of people who read at least one of the newspapers.

(ii) The no. of people who read exactly one news paper.

Ans. a + b + c + d = 25

b + c + e + f = 26

c + d + f +g = 26

c + d = 9

b + c = 11

c + f = 8

c = 3

f = 5, b = 8, d = 6, c = 3, g = 12

e = 10, a = 8

(i) a + b + c + d + e + f + g = 52

(ii) a + e + g = 30

7.These are 20 students in a chemistry class and 30 students in a physics class. Find the number of students which are either in physics class or chemistry class in the following cases.

(i) Two classes meet at the same hour

(ii) The two classes met at different hours and ten students are enrolled in both the courses.

Ans. Let C be the set of students in chemistry class and P be the set of students in physics class.

n (C) = 20, n (P) = 30

(i) $C \cap P = \phi \Rightarrow n(C \cap P) = 0$

$n(C \cup P) = n(C) + n(P) + n(C \cap P)$

$= 20 + 30 + 0$

$= 50$

(ii) $n(C \cap P) = 10$

$n(C \cup F) = n(C) + n(F) - n(C \cap P)$

$= 20 + 30 - 10$

$= 40$

8.In a survey of 25 students, it was found that 15 had taken mathematics, 12had taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all three subjects.

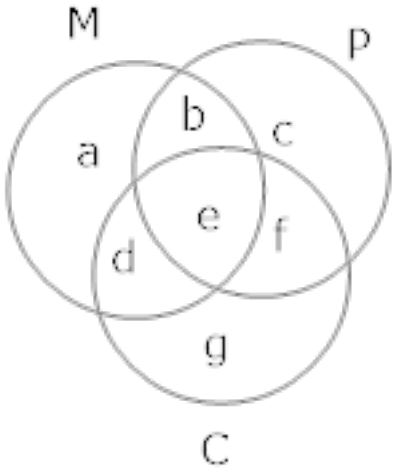
Find the no. of students that had taken

(i) only chemistry (ii) only mathematics (iii) only physics

(iv) physics and chemistry but mathematics (v) mathematics and physics but not chemistry (vi) only one of the subjects (vii) at least one of three subjects

(viii) None of three subjects.

Ans. $n(M)=a+b+d+e=15$



$n(P) = b + c + e + f = 12$

$n(C) = d + e + f + g = 11$

$n(M \cap P) = b + e = 9$

$n(M \cap C) = d + e = 5$

$n(P \cap C) = e + f = 4$

$e = 3$

so $b = 6$, $d = 2$, $f = 1$

$a = 4$, $g = 5$, $c = 2$

(i) $g = 5$,

(ii) $a = 4$,

(iii) $c = 2$

(iv) $f = 1$,

(v) $b = 6$,

(vi) $g + a + c = 11$

(vii) $a + b + c + d + e + f + g + = 23$

(viii) $25 - (a + b + c + d + e + f + g) = 25 - 23 = 2$

9. In a survey of 100 students, the no. of students studying the various languages were found to be English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, no language 24. Find

(i) How many students were studying Hindi?

(ii) How many students were studying English and Hindi?

Ans. ∪ = 100, a = 18

a + e = 23, e + g = 8

a + e + g + d = 26

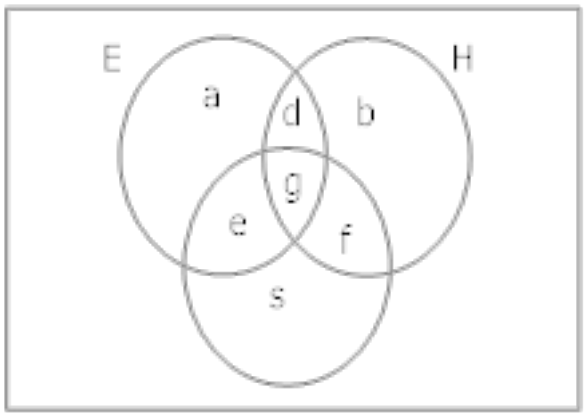
e + g + f + c = 48

g + f = 8

so, e = 5, g = 3, d = 0, f = 5, c = 35

(i) d + g + f + b = 0 + 3 + 5 + 10 = 18

(ii) d + g = 0 + 3 = 3



10. In a class of 50 students, 30 students like Hindi, 25 like science and 16 like both.

Find the no. of students who like

(i) Either Hindi or science

(ii) Neither Hindi nor science.

Ans. Let ∪ = all the students of the class ,H = students who like Hindi

S = Students who like Science

(i) n (H ∪ S) = n (H) + n (S) – n (H ∩ S)

= 30 + 25 – 16

= 39

(ii) n ($H' \cap S'$) = n (H ∪ S $)'$

= ∪ - n (H ∪ S)

= 50 – 39

= 11

11.In a town of 10,000 families, it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C. 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three papers. Find the no. of families which buy

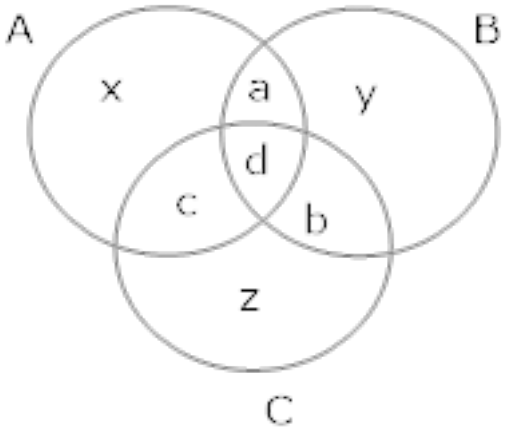
(i) A only (ii) B only (iii) none of A, B, and C.

Ans. x + a + c + d = 4000

y + a + d + b = 2000

z + b + c + d = 1000

a + d = 500, b + d = 300, C + d = 400 d = 200



On Solving a = 300, b = 100, c = 200

(i) x = 4000 – 300 – 200 – 200 = 3300

(ii) y = 2000 – 300 – 200 – 100 = 1400

(iii) z = 1000 – 100 – 200 – 200 = 500

None of these = 10,000 – (3300 + 1400 + 500 + 300+ 100 + 200 + 200)

= 10,000 – 6000

= 4000

12.Two finite sets have m and n elements. The total no. of subsets of the first set is 56 more than the total no. of subsets of second set. Find the value of m and n.

Ans. Let A and B be two sets having m and n elements respectively

no of subsets of A = 2^m

no of subsets of B = 2^n

According to question

$2^m = 56 + 2^n$

$2^m - 2^n = 56$

$2^n (2^{m-n} - 1) = 56$

$2^n (2^{m-n} - 1) = 2^3 (2^3 - 1)$

$2^n = 2^3$

$n = 3$

$m - n = 3$

$m - 3 = 3$

$m = 6$