DCAM classes

Dynamic Classes for Academic Mastery

Sets

1. Set

A well-defined collection of objects, is called a set. Sets are denoted by capital letters A, B, C, X, Y, Z etc. and elements of a set are denoted by a, b, c, x, y, z etc.

If a is an element of set A, then we say that a belongs to A and written as $a \in A$ and b does not belongs to set A is written as $b \notin A$.

2. Representation of Sets

There are two ways of representing a set

(i) **Roster form or Tabular form or Listing method** In the roster form, we list all the elements of the set within curly braces {} and separate them by commas.

(ii) **Set-builder form or Rule method** In the set-builder form, we list the property or properties satisfied by all the elements of the sets.

3. Types of Sets

(i) **Empty set** A set which does not contain any element is called an empty set or the void set or the null set and it is denoted by {} or ϕ.

(ii) **Singleton set** A set consisting of a single element, is called a singleton set.

(iii) **Finite and Infinite sets** A set which is empty or consists of a finite number of elements is called a finite set, otherwise, the set is called an infinite set.

(iv) **Equivalent sets** Two finite sets A and B are said to be equal, if they have equal number of elements, i.e. $n(A) = n(B)$.

(v) **Equal sets** Two sets A and B are said to be equal, if they have exactly the same elements and we write $A = B$. Otherwise, the sets are said to be unequal and we write $A \neq B$.

4. Subset

A set A is said to be a subset of a set B, if every element of A is also an element of B. In symbols, we can write

$$A \subset B, \text{ if } x \in A \Rightarrow x \in B$$

Also, if $A \subset B$ and $A \neq B$, then A is called a proper subset of B and B is called superset of A.

Note (i) *Every set is a subset of itself.*

(ii) *The empty set is a subset of every sets.*

(iii) *The total number of subsets of a finite set containing n elements is 2^n.*

5. Subsets of the Set of Real Numbers

We know that, every real number is either a **rational** or an **irrational** number and the set of real numbers is denoted by R. *There are many important subsets of set of real numbers which are given below*

(i) **Natural numbers** The numbers being used in counting as 1, 2, 3, 4,..., called natural numbers.

The set of natural numbers is denoted by N.

Thus, $N = \{1, 2, 3, 4, ...\}$

(ii) **Whole numbers** The natural numbers along with number 0 (zero) form the set of whole numbers i.e. 0, 1, 2, 3, ... , are whole numbers. The set of whole numbers is denoted by W.

Thus, $W = \{0, 1, 2, 3, ...\}$

(iii) **Integers** The natural numbers, their negatives and zero make the set of integers and it is denoted by Z.

$$Z = \{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

(iv) **Rational numbers** A number of the form $\frac{p}{q}$, where p and q both are integers and $q \neq 0$ (division by 0 is not permissible), is called a rational number.

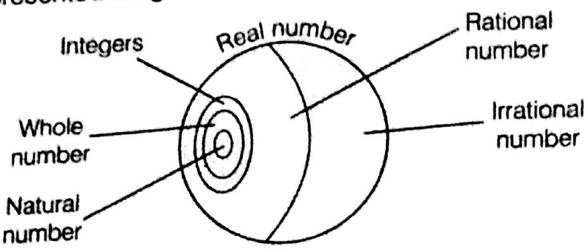
The set of rational numbers is generally denoted by Q. Thus, $Q = \left\{\frac{p}{q} : p, q \in Z \text{ and } q \neq 0\right\}$

(v) **Irrational numbers** A number which cannot be written in the form p/q, where p and q both are integers and $q \neq 0$, is called an irrational number.

The set of irrational numbers is denoted by T. Thus,

$$T = \{x : x \in R \text{ and } x \notin Q\}$$

Diagrammatical Representation All the subsets can be represented diagrammatically as given below



6. Intervals as Subsets of *R*

Let a and b be two given real numbers such that $a < b$, then

(i) the set of real numbers $\{x : a < x < b\}$ is called an **open interval** and is denoted by (a, b).

(ii) the set of real numbers $\{x : a \le x \le b\}$ is called a **closed interval** and is denoted by $[a, b]$.

(iii) intervals closed at one end and open at the other are known as **semi-open** or **semi-closed** intervals. $[a, b) = \{x : a \le x < b\}$ is an open interval from a to b which includes a but excludes b. $(a, b] = \{x : a < x \le b\}$ is an open interval from a to b which excludes a but includes b.

7. Universal Set

If there are some sets under consideration, then there happens to be a set which is a superset of each one of the given sets. Such a set is known as the universal set and is denoted by U.

8. Power Set

The collection of all subsets of a set A is called the power set of A. It is denoted by $P(A)$. If the number of elements in A, i.e. $n(A) = m$, then the number of elements in $P(A) = 2^m$.

Properties of Power Sets

(i) If $A \subseteq B$, then $P(A) \subseteq P(B)$.

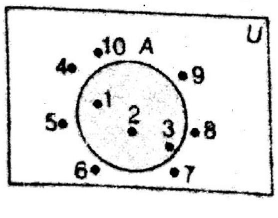
(ii) $P(A) \cap P(B) = P(A \cap B)$

(iii) $P(A \cup B) \ne P(A) \cup P(B)$

9. Venn Diagrams

Venn diagrams are the diagrams, which represent the relationship between sets. In Venn diagrams, the universal set is represented usually by a **rectangular region** and its subset are represented usually by **circle** or a **closed geometrical figure** inside the universal set. Also, an element of a set is represented by a **point** within the circle of set.

e.g. If $U = \{1, 2, 3, 4, \dots, 10\}$ and $A = \{1, 2, 3\}$, *then its Venn diagram is as shown in the figure*



10. Operations on Sets

(i) **Union of sets** The union of two sets A and B is the set of all those elements which belong to either in A or in B or in both A and B. It is denoted by $A \cup B$.
Thus, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

(ii) **Intersection of sets** The intersection of two sets A and B is the set of all those elements, which are common to both A and B. It is denoted by $A \cap B$.
Thus, $A \cap B = \{x : x \in A \text{ and } x \in B\}$

(iii) **Disjoint sets** Two sets A and B are said to be disjoint sets, if they have no common elements i.e. if $A \cap B = \phi$

(iv) **Difference of sets** For any two sets A and B, their difference $(A - B)$ is defined as the set of elements which belong to A but not to B.
Thus, $A - B = \{x : x \in A \text{ and } x \notin B\}$
Similarly, $B - A = \{x : x \in B \text{ and } x \notin A\}$

(v) **Symmetric difference of two sets** For any two sets A and B, their symmetric difference is the set $(A - B) \cup (B - A)$. It is denoted by $A \Delta B$.
Thus, $A \Delta B = (A - B) \cup (B - A)$
$= \{x : x \in A \text{ or } x \in B \text{ but } x \notin A \cap B\}$

11. Complement of a Set

Let U be the universal set and A is a subset of U. Then, the complement of A with respect to U is the set of all elements of U which are not the elements of A.
Thus, $A' = U - A = \{x : x \in U \text{ and } x \notin A\}$

Some Properties of Complement Sets

(i) $A \cup A' = U$ (ii) $A \cap A' = \phi$

(iii) $U' = \phi$ (iv) $\phi' = U$ (v) $(A')' = A$

12. Laws of Algebra of Sets

(i) **Idempotent laws** For any set A, we have
(a) $A \cup A = A$
(b) $A \cap A = A$

(ii) **Identity laws** For any set A, we have
(a) $A \cup \phi = A$
(b) $A \cap U = A$

(iii) **Commutative laws** For any two sets A and B, we have
(a) $A \cup B = B \cup A$
(b) $A \cap B = B \cap A$

(iv) **Associative laws** For any three sets A, B and C, we have
(a) $A \cup (B \cup C) = (A \cup B) \cup C$
(b) $A \cap (B \cap C) = (A \cap B) \cap C$

(v) **Distributive laws** If A, B and C are three sets, then
(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(vi) **De-Morgan's laws** If A and B are two sets, then
(a) $(A \cup B)' = A' \cap B'$
(b) $(A \cap B)' = A' \cup B'$

13. Results on Number of Elements in Sets

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) If $(A \cap B) = \phi$, then $n(A \cup B) = n(A) + n(B)$

(iii) $n(A - B) = n(A) - n(A \cap B)$

(iv) $n(B - A) = n(B) - n(A \cap B)$

(v) $n(A' \cup B') = n[(A \cap B)'] = n(U) - n(A \cap B)$

(vi) $n(A' \cap B') = n[(A \cup B)'] = n(U) - n(A \cup B)$

(vii) Let A, B and C be any three finite sets, then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$