Unit 15 (Statistics)

Short Answer Type Questions

Q1. Find the mean deviation about the mean of the distribution:

Size	20	21	22	23	24
Frequency	6	4	5	1	4

Sol.

Size	Frequency	$f_i x_i$	$d_i = \|x_i - \bar{x}\|$	$f_i d_i$
20	6	120	1.65	9.90
21	4	84	0.65	2.60
22	5	110	0.35	1.75
23	1	23	1.35	1.35
24	4	96	2.35	9.40
Total	20	433		25

Now, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{433}{20} = 21.65$

$\therefore$ $\text{MD} = \frac{\Sigma f_i |x_i - \bar{x}|}{\Sigma f_i} = \frac{25}{20} = 1.25$

Q2. Find the mean deviation about the median of the following distribution:

Marks obtained	10	11	12	14	15
Number of students	2	3	8	3	4

Sol.

Marks obtained	f_i	cf	$d_i = \|x_i - M_e\|$	$f_i d_i$
10	2	2	2	4
11	3	5	1	3
12	8	13	0	0
14	3	16	2	6
15	4	20	3	12
Total	$\Sigma f_i = 20$			$\Sigma f_i d_i = 25$

Now, $M_e = \left(\frac{20+1}{2}\right)$ item $= \left(\frac{21}{2}\right) = 10.5^{\text{th}}$ item

$\therefore$ $M_e = 12$

$\therefore$ $\text{MD} = \frac{\Sigma f_i d_i}{\Sigma f_i} = \frac{25}{20} = 1.25$

Q3. Calculate the mean deviation about the mean of the set of first n natural numbers when n is an odd number.

Sol: Consider first natural number when n is an odd i.e., 1, 2, 3,4,... , n [odd].

$$\text{Mean } \bar{x} = \frac{1+2+3+\cdots+n}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

$$\therefore \text{MD} = \frac{\left|1-\frac{n+1}{2}\right| + \left|2-\frac{n+1}{2}\right| + \left|3-\frac{n+1}{2}\right| + \cdots + \left|n-\frac{n+1}{2}\right|}{n}$$

$$= \frac{\left|-\frac{n+1}{2}\right| + \left|2-\frac{n+1}{2}\right| + \cdots + \left|\frac{n-1}{2}-\frac{n+1}{2}\right| + \left|\frac{n+1}{2}-\frac{n+1}{2}\right| + \left|\frac{n+3}{2}-\frac{n+1}{2}\right| \cdots + \left|\frac{2n-2}{2}-\frac{n+1}{2}\right| + \left|n-\frac{n+1}{2}\right|}{n}$$

$$= \frac{2}{n}\left[1+2+\cdots+\frac{n-3}{2}+\frac{n-1}{2}\right]\left(\frac{n-1}{2}\right)\text{terms}$$

$$= \frac{2}{n}\left[\frac{\left(\frac{n-1}{2}\right)\left(\frac{n-1}{2}+1\right)}{2}\right] \qquad \left[\because \text{Sum of first } n \text{ natrual numbers} = \frac{n(n+1)}{2}\right]$$

$$= \frac{2}{n}\cdot\frac{1}{2}\left[\left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)\right] = \frac{1}{n}\left(\frac{n^2-1}{4}\right) = \frac{n^2-1}{4n}$$

Q4. Calculate the mean deviation about the mean of the set of first n natural numbers when n is an even number.

Sol: Consider first **n** natural number, when **n** is even i.e., 1, 2, 3,4..n.

$\therefore \quad$ Mean $\bar{x} = \dfrac{1+2+3+\cdots+n}{n} = \dfrac{n(n+1)}{2n} = \dfrac{n+1}{2}$

$$MD = \frac{1}{n}\left[\left|1-\frac{n+1}{2}\right| + \left|2-\frac{n+1}{2}\right| + \left|3-\frac{n+1}{2}\right|\right] + \left|\frac{n-2}{2}-\frac{n+1}{2}\right|$$

$$+\left|\frac{n}{2}-\frac{n+1}{2}\right| + \left|\frac{n+2}{2}-\frac{n+1}{2}\right| + \cdots + \left|n-\frac{n1}{2}\right|$$

$$= \frac{1}{n}\left[\left|\frac{1-n}{2}\right| + \left|\frac{3n-n}{2}\right| + \left|\frac{5-n}{2}\right| + \cdots + \left|\frac{-3}{2}\right| + \left|\frac{1}{2}\right| + \cdots + \left|\frac{n-1}{2}\right|\right]$$

$$= \frac{2}{n}\left[\frac{1}{2}+\frac{3}{2}+\cdots+\frac{n-1}{2}\right]\left(\frac{n}{2}\right)\text{terms}$$

$$= \frac{1}{n}\cdot\left(\frac{n}{2}\right)^2 \qquad [\because \text{ Sum of first } n \text{ natural numbers} = n^2]$$

Q5. Find the standard deviation of the first n natural numbers.

Sol.

x_i	1	2	3	4	5	...	...	n
x_i^2	1	4	9	16	25	...	...	n^2

Now, $\Sigma x_i = 1+2+3+4+\ldots+n = \dfrac{n(n+1)}{2}$

and $\Sigma x_i^2 = 1^2+2^2+3^2+\ldots+n^2 = \dfrac{n(n+1)(2n+1)}{6}$

$$\therefore \quad \alpha = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2} = \sqrt{\frac{n(n+1)(2n+1)}{6n} - \frac{n^2(n+1)^2}{4n^2}}$$

$$= \sqrt{\frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}} = \sqrt{\frac{2(2n^2+3n+1)-3(n^2+2n+1)}{12}}$$

$$= \sqrt{\frac{4n^2+6n+2-3n^2-6n-3}{12}} = \sqrt{\frac{n^2-1}{12}}$$

Q6. The mean and standard deviation of some data for the time taken to complete . a test are calculated with the following results:
Number of observations = 25, mean = 18.2 seconds, standard deviation = 3.25 s.

Further, another set of 15 observations $x_1, x_2, \ldots, x_{15}$, also in seconds, is now available and we have $\sum_{i=1}^{15} x_i = 279$ and $\sum_{i=1}^{15} x_i^2 = 5524$. Calculate the standard deviation based on all 40 observations.

Sol. Given, $n_1 = 25$, $\bar{x}_i = 18.2$, $\sigma_1 = 3.25$,

$$n_2 = 15, \sum_{i=1}^{15} x_i = 279 \text{ and } \sum_{i=1}^{15} x_i^2 = 5524$$

For first set $\Sigma x_i = 25 \times 18.2 = 455$

$$\therefore \quad \sigma_1^2 = \frac{\Sigma x_i^2}{25} - (18.2)^2$$

$$\Rightarrow \quad (3.25)^2 = \frac{\Sigma x_i^2}{25} - (18.2)^2 \Rightarrow 10.5625 + 331.24 = \frac{\Sigma x_i^2}{25}$$

$$\Rightarrow \quad \Sigma x_i^2 = 25 \times (10.5625 + 331.24) = 25 \times 341.8025 = 8545.0625$$

For combined SD of the 40 observations, $n = 40$.

$$\text{Now} \quad \sum_{i=1}^{40} x_i^2 = 5524 + 8545.0625 = 14069.0625$$

$$\text{and} \quad \sum_{i=1}^{40} x_i = 455 + 279 = 734$$

$$\therefore \quad \text{SD} = \sqrt{\frac{14069.0625}{40} - \left(\frac{734}{40}\right)^2} = \sqrt{351.1726 - (18.35)^2}$$

$$= \sqrt{351.726 - 336.7225} = \sqrt{15.0035} = 3.87$$

7. The mean and standard deviation of a set of n_1 observations are $\bar{x}_1$ and s_1, respectively while the mean and standard deviation of another set of n_2 observations are $\bar{x}_2$ and s_2, respectively. Show that the standard deviation of the combined set of $(n_1 + n_2)$ observations is given by

$$\therefore \quad \text{SD} = \sqrt{\frac{n_1(s_1)^2 + n_2(s_2)^2}{n_1 + n_2} + \frac{n_1 n_2(\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$$

Sol. We have two sets of observations,

$$x_i, i = 1, 2, 3 \ldots, n_1 \text{ and } y_j, j = 1, 2, 3, \ldots, n_2$$

$$\therefore \quad \bar{x}_1 = \frac{1}{n_i}\sum_{i=1}^{n_1} x_i \text{ and } \bar{x}_2 = \frac{1}{n_2}\sum_{j=1}^{n_2} y_j$$

$$\Rightarrow \quad \sigma_1^2 = \frac{1}{n_1}\sum_{i=1}^{n_1}(x_i - \bar{x}_1)^2 \text{ and } \sigma_2^2 = \frac{1}{n_2}\sum_{j=1}^{n_2}(y_i - \bar{x}_2)^2$$

Now, mean $\bar{x}$ of the given series is given by

$$\bar{x} = \frac{1}{n_1 + n_2}\left[\sum_{i=1}^{n_1} x_i + \sum_{j=1}^{n_2} y_j\right] = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

The variance σ^2 of the combined series is given by

$$\sigma^2 = \frac{1}{n_1 + n_2}\left[\sum_{i=1}^{n_1}(x_i - \bar{x})^2 + \sum_{j=1}^{n_2}(y_j - \bar{x})^2\right]$$

$$\text{Now, } \sum_{i=1}^{n_1}(x_i - \bar{x})^2 = \sum_{i=1}^{n_1}(x_i - \bar{x}_j + \bar{x}_j - \bar{x})^2$$

$$\text{But } \sum_{i=1}^{n_1}(x_i - \bar{x}_i) = 0$$ [Algebraic sum of the deviation of values of first series from their mean is zero]

$$\text{Also, } \sum_{i=1}^{n_1}(x_i - \bar{x})^2 = n_1 s_1^2 = n_1(\bar{x}_1 - \bar{x})^2 = n_1 s_1^2 + n_1 d_1^2$$

Where, $d_1 = (\bar{x}_1 - \bar{x})$

Similarly, $\sum_{j=1}^{n_2} (y_j - \bar{x})^2 = \sum_{j=1}^{n_2} (y_j - \bar{x}_i + \bar{x}_i - \bar{x})^2 = n_2 s_2^2 + n_2 d_2^2$

where, $d_2 = \bar{x}_2 - \bar{x}$

Combined SD, $\sigma = \sqrt{\dfrac{[n_1(s_1^2 + d_1^2) + n_2(s_2^2 + d_2^2)]}{n_1 + n_2}}$

where, $d_1 = \bar{x}_1 - \bar{x} = \bar{x}_1 - \left(\dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}\right) = \dfrac{n_2(\bar{x}_1 - \bar{x}_2)}{n_1 + n_2}$

and $d_2 = \bar{x}_2 - \bar{x} = \bar{x}_2 - \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \dfrac{n_1(\bar{x}_2 - \bar{x}_1)}{n_1 + n_2}$

$\therefore \quad \sigma^2 = \dfrac{1}{n_1 + n_2}\left[n_1 s_1^2 + n_2 s_2^2 + \dfrac{n_1 n_2(\bar{x}_1 + \bar{x}_2)^2}{(n_1 + n_2)^2} + \dfrac{n_2 n_1(\bar{x}_2 - \bar{x}_1)^2}{(n_1 + n_2)^2}\right]$

Also, $\sigma = \sqrt{\dfrac{n_1 s_2^2 + n_2 s_2^2}{n_1 + n_2} + \dfrac{n_1 n_2(\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$

Q8. Two sets each of 20 observations, have the same standard derivation 5. The first set has a mean 17 and the second a mean 22. Determine the standard deviation of the set obtained by combining the given two sets.

Sol. Given, $n_1 = 20$, $\sigma_1 = 5$, $\bar{x}_1 = 17$ and $n_2 = 20$, $\sigma_2 = 5$, $\bar{x}_2 = 22$

We know that, $\sigma = \sqrt{\dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} + \dfrac{n_1 n_2(\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$

$= \sqrt{\dfrac{20 \times (5)^2 + 20 \times (5)^2}{20 + 20} + \dfrac{20 \times 20(17 - 22)^2}{(20 + 20)^2}}$

$= \sqrt{\dfrac{1000}{40} + \dfrac{400 \times 25}{1600}} = \sqrt{25 + \dfrac{25}{4}} = \sqrt{\dfrac{125}{4}} = \sqrt{31.25} = 5.59$

Q9. The frequency distribution:

X	A	2A	3 A	4A	5 A	6A
f	2	1	1	1	1	1

where A is a positive integer, has a variance of 160. Determine the value of A.

Sol.

x	f_i	f_1x_i	$f_ix_i^2$
A	2	$2A$	$2A^2$
$2A$	1	$2A$	$4A^2$
$3A$	1	$3A$	$9A^2$
$4A$	1	$4A$	$16A^2$
$5A$	1	$5A$	$25A^2$
$6A$	1	$6A$	$36A^2$
Total	$n = 7$	$\Sigma f_ix_i = 22A$	$\Sigma f_ix_i^2 = 92A^2$

$$\therefore \quad \sigma^2 = \frac{\Sigma f_i x_i^2}{n} - \left(\frac{\Sigma f_i x_i}{n}\right)^2$$

$$\Rightarrow \quad 160 = \frac{92A^2}{7} - \left(\frac{22A}{7}\right)^2 \Rightarrow 160 = \frac{92A^2}{7} - \frac{484A^2}{49}$$

$$\Rightarrow \quad 160 = (644 - 484)\frac{A^2}{49} \Rightarrow 160 = \frac{160A^2}{49}$$

$$\Rightarrow \quad A^2 = 49 \quad \therefore \quad A = 7$$

Q10. For the frequency distribution:

X	2	3	4	5	6	7
f	4	9	16	14	11	6

Find the standard deviation.

Sol.

x_i	f_i	$d_i = x_i - 4$	$f_i\, d_i$	$f_id_i^2$
2	4	–2	–8	16
3	9	–1	–9	9
4	16	0	0	0
5	14	1	14	14
6	11	2	22	44
7	6	3	18	54
Total	$n = 60$		$\Sigma f_i = 37$	$\Sigma f_id_i^2 = 137$

$$\therefore \quad \text{SD} = \sqrt{\frac{\Sigma f_i d_i^2}{n} - \left(\frac{\Sigma f_i d_i}{n}\right)^2} = \sqrt{\frac{137}{60} - \left(\frac{37}{60}\right)^2} = \sqrt{2.2833 - (0.616)^2}$$

$$= \sqrt{2.2833 - 0.3794} = \sqrt{1.9037} = 1.38$$

Q11. There are 60 students in a class. The following is the frequency distribution of the marks obtained by the students in a test:

Marks	0	i	2	3	4	5
Frequency	$x - 2$	X	x^2	$(x+1)^2$	2x	$x + 1$

where x is a positive integer. Determine the mean and standard deviation of the marks.

Sol. Sum of frequencies,

$$x-2+x+x^2+(x+1)^2+2x+x+1=60 \qquad \text{(given)}$$

$\Rightarrow$ $2x^2+7x-60=0$

$\Rightarrow$ $(2x+15)(x-4)=0$

$\Rightarrow$ $x=4$

x_i	f_i	$d_i = x_i - 3$	$f_i d_i$	$f_i d_i^2$
0	2	−3	−6	18
1	4	−2	−8	16
2	16	−1	−16	16
$A=3$	25	0	0	0
4	8	1	8	8
5	5	2	10	20
Total	$\Sigma f_i = 60$		$\Sigma f_i d_i = -12$	$\Sigma f_i d_i^2 = 78$

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i} = 3 + \left(\frac{-12}{60}\right) = 2.8$$

Standard Deviation,

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i d_i}{\Sigma f_i}\right)^2} = \sqrt{\frac{78}{60} - \left(\frac{-12}{60}\right)^2} = \sqrt{1.3 - 0.04} = \sqrt{1.26} = 1.12$$

Q12. The mean life of a sample of 60 bulbs was 650 hours and the standard deviation was 8 hours. A second sample of 80 bulbs has a mean life of 660 hours and standard deviation 7 hours. Find the overall standard deviation.

Sol. Here, $n_1 = 60$, $\bar{x}_1 = 650$, $s_1 = 8$ and $n_2 = 80$, $\bar{x}_2 = 660$, $s_2 = 7$

$$\therefore \qquad \sigma = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} + \frac{n_1 n_2 (\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$$

$$= \sqrt{\frac{60 \times (8)^2 + 80 \times (7)^2}{60 + 80} + \frac{60 \times 80 (650 - 660)^2}{(60 + 80)^2}}$$

$$= \sqrt{\frac{6 \times 64 + 8 \times 49}{14} + \frac{60 \times 80 \times 100}{140 \times 140}}$$

$$= \sqrt{\frac{192 + 196}{7} + \frac{1200}{49}} = \sqrt{\frac{388}{7} + \frac{1200}{49}}$$

$$= \sqrt{\frac{2716 + 1200}{49}} = \sqrt{\frac{3915}{49}} = \sqrt{79.9} = 8.9$$

Q13. Mean and standard deviation of 100 items are 50 and 4, respectively. Then find the sum of all the item and the sum of the squares of the items.

Sol. Here, $\bar{x} = 50$, $n = 100$ and $\sigma = 4$

$$\therefore \quad \frac{\Sigma x_i}{100} = 50$$

$$\Rightarrow \quad \Sigma x_i = 5000$$

and $$\sigma^2 = \frac{\Sigma f_i x_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i x_i}{\Sigma f_i}\right)^2$$

$$\Rightarrow \quad (4)^2 = \frac{\Sigma f_i x_i^2}{100} - (50)^2$$

$$\Rightarrow \quad 16 = \frac{\Sigma f_i x_i^2}{100} - 2500$$

$$\Rightarrow \quad \frac{\Sigma f_i x_i^2}{100} = 16 + 2500 = 2516$$

$$\therefore \quad \Sigma f_i x_i^2 = 251600$$

Q14. If for a distribution **Σ** (x -5)= 3,**Σ** (x -5)^2= 43 and the total number of item is 18, find the mean and standard deviation.

Sol: Given, n = 18, Σ (x − 5) = 3 and Σ (x − 5)^2 = 43

$$\therefore \quad \text{Mean} = A + \frac{\Sigma(x-5)}{18} = 5 + \frac{3}{18} = 5 + 0.1666 = 5.1666 = 5.17$$

and $$\text{SD} = \sqrt{\frac{\Sigma(x-5)^2}{n} - \left(\frac{\Sigma(x-5)}{n}\right)^2} = \sqrt{\frac{43}{18} - \left(\frac{3}{18}\right)^2}$$

$$= \sqrt{2.3889 - (0.166)^2} = \sqrt{2.3889 - 0.0277} = 1.53$$

Q15. Find the mean and variance of the frequency distribution given below:

x	$1 \le x < 3$	$3 \le x < 5$	$5 \le x < 7$	$7 \le x < 10$
f	6	4	5	1

Sol.

x	f_i	x_i	$f_i x_i$	$f_i x_i^2$
1 – 3	6	2	12	24
3 – 5	4	4	16	64
5 – 7	5	6	30	180
7 – 10	1	8.5	8.5	72.25
Total	$n = 16$		$\Sigma f_i x_i = 66.5$	$\Sigma f_i x_i^2 = 340.25$

$$\therefore \quad \text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{66.5}{16} = 4.13$$

And $$\text{variance} = \sigma^2 = \frac{\Sigma f_i x_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i x_i}{\Sigma f_i}\right)^2 = \frac{340.25}{16} - (4.13)^2$$

$$= 21.2656 - 17.0569 = 4.21$$

Long Answer Type Questions

Q16. Calculate the mean deviation about the mean for the following frequency distribution:

Class interval	0-4	4-8	8-12	12-16	16-20
Frequency	4	6	8	5	2

Sol.

Class interval	f_i	x_i	f_ix_i	$d_i = \|x_i - \bar{x}\|$	f_id_i
0 – 4	4	2	8	7.2	28.8
4 – 8	6	6	36	3.2	19.2
8 – 12	8	10	80	0.8	6.4
12 – 16	5	14	70	4.8	24.0
16 – 20	2	18	36	8.8	17.6
Total	$\Sigma f_i = 25$		$\Sigma f_ix_i = 230$		$\Sigma f_id_i = 96$

$\therefore$ Mean, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{230}{25} = 9.2$

And Mean deviation $= \frac{\Sigma f_i d_i}{\Sigma f_i} = \frac{96}{25} = 3.84$

Q17. Calculate the mean deviation from the median of the following data

Class interval	0 – 6	6 – 12	12 -18	18 -24	24 -30
Frequency	4	5	3	6	2

Sol.

Class interval	f_i	x_i	cf	$d_i = \|x_i - \bar{m}_d\|$	f_id_i
0 – 6	4	3	4	11	44
6 – 12	5	9	9	5	25
12 – 18	3	15	12	1	3
18 – 24	6	21	18	7	42
24 – 30	2	27	20	13	26
Total	$N = 20$		$\Sigma f x_i = 230$		$\Sigma f d_i = 140$

$\because \quad \frac{N}{2} = \frac{20}{2} = 10$

So, the median class is 12 – 18

$\therefore \quad \text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h = 12 + \frac{10-9}{3} \times 6 = 12 + \frac{6}{3}(10-9) = 14$

$\text{MD} = \frac{\Sigma f_i d_i}{\Sigma f_i} = \frac{140}{20} = 7$

Q18. Determine the mean and standard deviation for the following distribution:

Marks	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Frequency	1	6	6	8	8	2	2	3	0	2	1	0	0	0	1

Sol.

Marks	f_i	$f_i x_i$	$d_i = x_i - \bar{x}$	$f_i d_i$	$f_i d_i^2$
2	1	2	−4	−4	16
3	6	18	−3	−18	54
4	6	24	−2	−12	24
5	8	40	−1	−8	8
6	8	48	0	0	0
7	2	14	1	2	2
8	2	16	2	4	8
9	3	27	3	9	27
10	0	0	4	0	0
11	2	22	5	10	50
12	1	12	6	6	36
13	0	0	7	0	0
14	0	0	8	0	0
15	0	0	9	0	0
16	1	16	10	10	100
Total	$\Sigma f_i = 40$	$\Sigma f_i x_i = 239$		$\Sigma f_i d_i = -1$	$\Sigma f_i x_i^2 = 325$

$\therefore$ Mean $\bar{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{239}{40} = 5.975 \approx 6$

and $$\sigma = \sqrt{\frac{\Sigma f_i x_i}{\Sigma f_i} - \left(\frac{\Sigma f_i d_i}{\Sigma f_i}\right)^2} = \sqrt{\frac{325}{40} - \left(\frac{-1}{40}\right)^2}$$

$$= \sqrt{8.125 - 0.000625} = \sqrt{8.124375} = 2.85$$

Q19. The weights of coffee in 70 jars are shown in the following table:

Weight (in grams)	Frequency
200-201	13
201-202	27
202 – 203	18
203-204	10
204-205	1
205-206	1

Determine variance and standard deviation of the above distribution.

Sol.

Class interval	f_i	x_i	$d_i = x_i - \bar{x}$	$f_i d_i$	$f_i d_i^2$
200 – 201	13	200.5	–2	–26	52
201 – 202	27	201.5	–1	–27	27
202 – 203	18	202.5	0	0	0
203 – 204	10	203.5	1	10	10
204 – 205	1	204.5	2	2	4
205 – 206	1	205.5	3	3	9
Total	$\Sigma f_i = 70$			$\Sigma f_i d_i = -38$	$\Sigma f_i d_i^2 = 102$

$$\therefore \quad \sigma^2 = \frac{\Sigma f_i d_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i d_i}{\Sigma f_i}\right)^2 = \frac{102}{70} - \left(\frac{-38}{70}\right)^2 = 1.4571 - 0.2916 = 1.1655$$

$$\therefore \quad \sigma = \sqrt{1.1655} = 1.08 \text{ g}$$

Q20. Determine mean and standard deviation of first n terms of an A.P. whose first term is a and common difference is d.

Sol.

x_i	$x_i - a$	$(x_i - a)^2$
a	0	0
$a + d$	d	d^2
$a + 2d$	$2d$	$4d^2$
...	...	$9d^2$
...	...	...
...	...	...
$a + (n-1)d$	$(n-1)d$	$(n-1)^2 d^2$
$\Sigma x_i = \frac{n}{2}[2a + (n-1)d]$		

$$\because \quad \text{Mean} = \frac{\Sigma x_i}{n} = \frac{1}{n}\left[\frac{n}{2}(2a + (n-1)d\right] = a + \frac{(n-1)}{2}d$$

$$\therefore \quad \Sigma(x_i - a) = d\,[1 + 2 + 3 + \ldots + (n-1)\,d] = d\frac{(n-1)n}{2}$$

$$\text{and} \quad \Sigma(x_i - a)^2 = d^2 \cdot [1^2 + 2^2 + 3^2 + \ldots + (n-1)^2] = \frac{d^2 n(n-1)(2n-1)}{6}$$

$$\sigma = \sqrt{\frac{(x_i - a)^2}{n} - \left(\frac{x_i - a}{n}\right)^2}$$

$$= \sqrt{\frac{d^2 n(n-1)(2n-1)}{6n} - \left[\frac{d(n-1)n}{2n}\right]^2} = \sqrt{\frac{d^2(n-1)(2n-1)}{6} - \frac{d^2(n-1)^2}{4}}$$

$$= d\sqrt{\frac{(n-1)}{2}\left(\frac{2n-1}{3} - \frac{n-1}{2}\right)} = d\sqrt{\frac{(n-1)}{2}\left[\frac{4n-2-3n+3}{6}\right]}$$

$$= d\sqrt{\frac{(n-1)(n+1)}{12}} = d\sqrt{\frac{n^2-1}{12}}$$

Q21. Following are the marks obtained, out of 100, by two students Ravi and Hashinain 10 tests.

Ravi	25	50	45	30	70	42	36	48	35	60

Hashina	10	70	50	20	95	55	42	60	48	80

Who is more intelligent and who is more consistent?

Sol. For Ravi,

x_i	$d_i = x_i - 45$	d_i^2
25	–20	400
50	5	25
45	0	0
30	–15	225
70	25	625
42	–3	9
36	–9	81
48	3	9
35	–10	100
60	15	225
Total	$\Sigma d_i = -9$	$\Sigma d^2 i = 1699$

$$\sigma = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2} = \sqrt{\frac{1699}{10} - \left(\frac{-9}{10}\right)^2}$$
$$= \sqrt{169.9 - 0.81} = \sqrt{169.09} = 13.003$$

Now, $\quad \bar{x} = A + \frac{\Sigma d_i}{\Sigma f_i} = 45 - \frac{14}{10} = 43.6$

For Hashina,

x_i	$d_i = x_i - 55$	d_i^2
10	–45	2025
70	15	225
50	–5	25
20	–35	1225
95	40	1600
55	0	0
42	–13	169
60	5	25
48	–7	49
80	25	625
Total	$\Sigma d_i = -20$	$\Sigma d^2 i = 5968$

$$\therefore \quad \sigma = \sqrt{\frac{5968}{10} - \left(\frac{-20}{10}\right)^2} = \sqrt{596.8 - 4} = \sqrt{592.8} = 24.46$$

For Ravi, CV $= \frac{\sigma}{\bar{x}} \times 100 = \frac{13.003}{43.6} \times 100 = 29.82$

For Hashina, CV $= \frac{\sigma}{\bar{x}} \times 100 = \frac{24.46}{55} \times 100 = 44.47$

Hence, Hashina is more consistent and intelligent.

Q22. Mean and standard deviation of 100 observations were found to be 40 and 10,respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively, find the correct standard deviation.

Sol. Given, $n = 100$, $\overline{x} = 40$ and $\sigma = 10$

$$\therefore \quad \frac{\Sigma x_i}{n} = 40$$

$$\Rightarrow \quad \frac{\Sigma x_i}{100} = 40$$

$$\Rightarrow \quad \Sigma x_i = 4000$$

Now, Corrected $\Sigma x_i = 4000 - 30 - 70 + 3 + 27 = 3930$

Corrected mean $= \dfrac{2930}{100} = 39.3$

Now, $$\sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 = \frac{\Sigma x_i^2}{n} - (40)^2$$

$$\Rightarrow \quad 100 = \frac{\Sigma x_i^2}{100} - 1600$$

$$\Rightarrow \quad \Sigma x_i^2 = 170000$$

Now, Corrected $\Sigma x_i^2 = 170000 - (30)^2 - (70)^2 + 3^2 + (27)^2 = 164938$

$$\therefore \quad \text{Corrected } \sigma = \sqrt{\frac{164938}{100} - (39.3)^2} = \sqrt{1649.38 - 1544.49} = \sqrt{104.9}$$
$$= 10.24$$

Q23. While calculating the mean and variance of 10 readings, a student wrongly used the reading 52 for the correct reading 25. He obtained the mean and variance as 45 and 16 respectively. Find the correct mean and the variance.

Sol. Given $n = 10$, $\overline{x} = 45$ and $\sigma^2 = 16$

$$\overline{x} = 45 \Rightarrow \frac{\Sigma x_i}{n} = 45$$

$$\Rightarrow \quad \frac{\Sigma x_i}{10} = 45 \quad \Rightarrow \quad \Sigma x_i = 450$$

Corrected $\Sigma x_i = 450 - 52 + 25 = 423$

$$\therefore \quad \text{Corrected mean, } \overline{x} = \frac{423}{10} = 42.3$$

$$\Rightarrow \quad \sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$

$$\Rightarrow \quad 16 = \frac{\Sigma x_i^2}{10} - (45)^2$$

$$\Rightarrow \quad \Sigma x_i^2 = 20410$$

$$\therefore \quad \text{Corrected } \Sigma x_i^2 = 20410 - (53)^2 + (25)^2 = 18331$$

And Corrected $\sigma^2 = \dfrac{18331}{10} - (42.3)^2 = 43.81$

Objective Type Questions

Q24. The mean deviation of the data 3,10, 10,4, 7, 10, 5 from the mean is (a) 2 (b) 2.57 (c) 3 (d) 3.75

Sol: (b) Given, observations are 3, 10, 10, 4, 7, 10 and 5.

$\therefore \qquad \bar{x} = \frac{3+10+10+4+7+10+5}{7} = \frac{49}{7} = 7$

x_i	$d_i = \lvert x_i - \bar{x} \rvert$
3	4
10	3
10	3
4	3
7	0
10	3
5	2
Total	$\Sigma d_i = 18$

Now, $\text{MD} = \frac{\Sigma d_i}{N} = \frac{18}{7} = 2.57$

25. Mean deviation for n observations $x_1, x_2, ..., x_n$ from their mean $\bar{x}$ is given by

(a) $\sum_{i=1}^{n}(x_i - \bar{x})$ (b) $\frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$

(c) $\sum_{i=1}^{n}(x_i - \bar{x})^2$ (d) $\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

Sol. **(b)** $\text{MD} = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$

Q26. When tested, the lives (in hours) of 5 bulbs were noted as follows: 1357, 1090, 1666, 1494, 1623 The mean deviations (in hours) from their mean is (a) 178 (b) 179 (c) 220 (d) 356

Sol. **(a)** Since, the lives of 5 bulbs are 1357, 1090, 1666, 1494 and 1623

$\therefore \qquad \text{Mean} = \frac{1357+1090+1666+1494+1623}{5} = \frac{7230}{5} = 1446$

x_i	$d_i = \lvert x_i - \bar{x} \rvert$
1357	89
1090	356
1666	220
1494	48
1623	177
Total	$\Sigma d_i = 890$

$\text{MD} = \frac{\Sigma d_i}{N} = \frac{890}{5} = 178$

Q27. Following are the marks obtained by 9 students in a mathematics test:
50, 69,20, 33, 53, 39,40, 65, 59 The mean deviation from the median is:
(a) 9 (b) 10.5 (c) 12.67 (d) 14.76

Sol: (c) Since, marks obtained by 9 students in Mathematics are 50,69,20,33,53, 39,40, 65 and 59.

Rewrite the given data in ascending order.

20, 33, 39,40, 50, 53, 59, 65, 69,

Here, n = 9 [odd]

$\therefore$ Median = 5^{th} term

$M_e = 50$

x_i	$d_i = \lvert x_i - M_e \rvert$
20	30
33	17
39	11
40	10
50	0
53	3
59	9
65	15
69	19
$N = 2$	$\Sigma d_i = 114$

$$\therefore \quad \text{MD} = \frac{114}{9} = 12.67$$

28. The standard deviation of the data 6, 5, 9, 13, 12, 8, 10 is

(a) $\sqrt{\frac{52}{7}}$ (b) $\frac{52}{7}$ (c) $\sqrt{6}$ (d) 6

Sol. **(a)** Given, data are 6, 5, 9, 13, 12, 8, and 10

x_i	x_i^2
6	36
5	25
9	81
13	169
12	144
8	64
10	100
$\Sigma x_i = 63$	$\Sigma x_i^2 = 619$

$$\therefore \quad \text{SD} = \sigma = \sqrt{\frac{\Sigma x_i^2}{N} - \left(\frac{\Sigma x_i}{N}\right)^2} = \sqrt{\frac{619}{7} - \left(\frac{63}{7}\right)^2}$$

$$= \sqrt{\frac{7 \times 619 - 3969}{49}} = \sqrt{\frac{4333 - 396}{49}} = \sqrt{\frac{396}{49}} = \sqrt{\frac{52}{7}}$$

29. Let $x_1, x_2, ..., x_n$ be n observations and x be their arithmetic mean. The formula for the standard deviation is given by

(a) $\Sigma(x_i - \bar{x})^2$ (b) $\frac{\Sigma(x_i - \bar{x})^2}{n}$ (c) $\sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$ (d) $\sqrt{\frac{\Sigma x_i^2}{n} + \bar{x}^2}$

Sol. **(c)** SD is given by $\sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$

Q30. The mean of 100 observations is 50 and their standard deviation is 5. The sum of all squares of all the observations is

(a) 50000 (b) 250000 (c) 252500 (d) 255000

Sol. **(c)** Given, $\bar{x} = 50$, $n = 100$ and $\sigma = 5$

$$\Rightarrow \quad 50 = \frac{\Sigma x_i}{100}$$

$$\Rightarrow \quad \Sigma x_i = 50 \times 100 = 5000$$

Now, $$\sigma^2 = \frac{\Sigma x_i^2}{n} - (\bar{x})^2$$

$$\Rightarrow \quad 25 = \frac{\Sigma x_i^2}{100} - (50)^2$$

$$\Rightarrow \quad 2525 = \frac{\Sigma x_i^2}{100}$$

$$\therefore \quad \Sigma x_i^2 = 252500$$

Q31. Let a, b, c, d, e be the observations with mean m and standard deviation V. The standard deviation of the observations a + k,b + k,c + k,d+k,e + k is

(a) s (b) ks (c) $s+k$ (d) $\frac{s}{k}$

Sol. **(a)** Given observations are a, b, c, d and e.

$$\text{Mean} = m = \frac{a+b+c+d+e}{5}$$

$$\Sigma x_i = a+b+c+d+e = 5\text{ m}$$

Now, $$\text{mean} = \frac{a+k+b+k+c+k+d+k+e+k}{5}$$

$$= \frac{(a+b+c+d+e)+5k}{5} = m+k$$

$$\therefore \quad \text{SD} = \sqrt{\frac{\Sigma(x_i^2+k^2+2kx_i)}{n} - (m^2+k^2+2mk)}$$

$$= \sqrt{\frac{\Sigma x_i^2}{n} - m^2 + \frac{2k\Sigma x_i}{n} - 2mk}$$

$$= \sqrt{\frac{\Sigma x_i^2}{n} - m^2 + 2km - 2mk} \quad \left[\because \frac{\Sigma x_i}{n} = m\right]$$

$$= \sqrt{\frac{\Sigma x_i^2}{n} - m^2}$$

$$= s$$

32. Let x_1, x_2, x_3, x_4, x_5 be the observations with mean m and standard deviation s. The standard deviation of the observations $kx_1, kx_2, kx_3, kx_4, kx_5$ is

(a) $k+s$ (b) $\frac{s}{k}$ (c) ks (d) s

Sol. **(c)** Here, $m = \frac{\Sigma x_i}{5}, s = \sqrt{\frac{\Sigma x_i^2}{5} - \left(\frac{\Sigma x_i}{5}\right)^2}$

So, for observations $kx_1, kx_2, kx_3, kx_4, kx_5$

$$\therefore \quad \text{SD} = \sqrt{\frac{k^2\Sigma x_i^2}{5} - \left(\frac{k\Sigma x_i}{5}\right)^2} = \sqrt{\frac{k^2\Sigma x_i^2}{5} - k^2\left(\frac{\Sigma x_i}{5}\right)^2}$$

$$= k\sqrt{\left(\frac{\Sigma x_i^2}{5}\right) - \left(\frac{\Sigma x_i}{5}\right)^2} = ks$$

33. Let $x_1, x_2, ..., x_n$ be n observations. Let $w_i = lx_i + k$ for $i = 1, 2, ... n$, where l and k are constants. If the mean of x_i's is 48 and their standard deviation is 12,the mean of w_i's is 55 and standard deviation of w_i's is 15, the values of l and k should be

(a) $l = 1.25, k = -5$ (b) $l = -1.25, k = 5$
(c) $l = 2.5, k = -5$ (d) $l = 2.5, k = 5$

Sol. **(a)** Given, $w_i = lx_i + k$, $\bar{x}_i = 48$, $sx_i = 12$, $w_i = 55$ and $sw_i = 15$

Then, $\bar{w}_i = l\bar{x}_i + k$ [where $\bar{w}_i$ is mean w_i's and $\bar{x}_i$ is mean of x_i's]

$\Rightarrow \quad 55 = l \times 48 + k \qquad$ (i)

Now, SD of $w_i = l$(SD of x_i)

$\Rightarrow \quad 15 = l \times 12$

$\Rightarrow \quad l = \frac{15}{12} = 12.5$

From Eq. (i), we get $k = 55 - 1.25 \times 48 = -5$

Q34. Standard deviations for first 10 natural numbers is
(a) 5.5 (b) 3.87 (c) 2.97 (d) 2.87

Sol. **(d)** We know that, SD of first n natural number $= \sqrt{\frac{n^2-1}{12}}$

$\therefore$ SD of first 10 numbers $= \sqrt{\frac{10^2-1}{12}} = \sqrt{\frac{100-1}{12}} = \sqrt{\frac{99}{12}} = \sqrt{8.25} = 2.87$

Q35. Consider the numbers 1,2, 3,4, 5, 6, 7, 8,9,10. If 1 is added to each number, the variance of the numbers so obtained is
(a) 6.5 (b) 2.87 (c) 3.87 (d) 8.25

Sol: (d) Given numbers are 1, 2, 3,4, 5, 6, 7, 8, 9 and 10

If 1 is added to each number, then observations will be 2, 3,4, 5, 6,7, 8, 9, 10 and 11.

$$\therefore \quad \Sigma x_i = 2 + 3 + 4 + \dots + 11$$
$$= \frac{10}{2}[2 \times 2 + 9 \times 1] = 5[4+9] = 65$$

and $$\Sigma x_i^2 = 2^2 + 3^2 + 4^2 + 5^2 + \dots + 11^2 = (1^2 + 2^2 + 3^2 + \dots + 11^2) - (1^2)$$
$$= \frac{11 \times 12 \times 23}{6} - 1 = 505$$

$$\therefore \quad s^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 = \frac{505}{10} - \left(\frac{65}{10}\right)^2 = 50.5 - (6.5)^2 = 50.5 - 42.35 = 8.25$$

Q36. Consider the first 10 positive integers. If we multiply each number by -1 and then add 1 to each number, the variance of the numbers so obtained is (a) 8.25 (b) 6.5 (c) 3.87 (d) 2.87

Sol: (a) Since, the first 10 positive integers are 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10.

On multiplying each number by -1, we get

-1, -2, -3, -4, -5, -6, -7, -8, -9, -10 On adding 1 in each number, we get

0, -1, -2, -3, -4, -5, -6, -7, -8, -9

$$\therefore \quad \Sigma x_i = -\frac{9 \times 10}{2} = -45$$

and $$\Sigma x_i^2 = 0^2 + (-1)^2 + (-2)^2 + \cdots + (9)^2 = \frac{9 \times 10 \times 19}{6} = 285$$

$$\therefore \quad SD = \sqrt{\frac{285}{10} - \left(\frac{-45}{10}\right)^2} = \sqrt{\frac{285}{10} - \frac{2025}{100}} = \sqrt{\frac{2850 - 2025}{100}} = \sqrt{8.25}$$

Now, variance $= (SD)^2 = (\sqrt{8.25})^2 = 8.25$

37. The following information relates to a sample of size 60: $\Sigma x^2 = 18000$, $\Sigma x = 960$. The variance is

(a) 6.63 (b) 16 (c) 22 (d) 44

Sol. **(d)** Variance $= \dfrac{\Sigma x_i^2}{n} - \left(\dfrac{\Sigma x_i}{n}\right)^2 = \dfrac{18000}{60} - \left(\dfrac{960}{60}\right)^2 = 300 - 256 = 44$

Q38. Coefficient of variation of two distributions are 50 and 60, and their arithmetic means are 30 and 25 respectively. Difference of their standard deviation is

(a) 0 (b) 1 (c) 1.5 (d) 2.5

Sol. **(a)** Here $CV_1 = 50, CV_2 = 60, \bar{x}_1 = 30$ and $\bar{x}_2 = 25$

$\therefore \quad CV_1 = \dfrac{\sigma_1}{\bar{x}_1} \times 100 \Rightarrow 50 = \dfrac{\sigma_1}{30} \times 100$

$\therefore \quad \sigma_1 = \dfrac{30 \times 50}{100} = 15$

and $\quad CV_2 = \dfrac{\sigma_2}{\bar{x}_2} \times 100$

$\Rightarrow \quad 60 = \dfrac{\sigma_2}{25} \times 100$

$\therefore \quad \sigma_2 = \dfrac{60 \times 25}{100} = 15$

Now, $\sigma_1 - \sigma_2 = 15 - 15 = 0$

Q39. The standard deviation of some temperature data in °C is 5. If the data were converted into °F, the variance would be

(a) 81 (b) 57 (c) 36 (d) 25

Sol. **(A)** Given, $\sigma_C = 5$

Now $F = \dfrac{9C}{5} + 32$

$\therefore \quad \sigma_F = \dfrac{9}{5}\sigma_C = \dfrac{9}{5} \times 5 = 9$

Here, $\sigma_F^2 = (9)^2 = 81$

Fill in the Blanks

40. Coefficient of variation $= \dfrac{\cdots}{\text{Mean}} \times 100$

Sol. $CV = \dfrac{SD}{\text{Meam}} \times 100$

41. If x is the mean of n values of x, then $\sum_{i=1}^{n}(x_i - \bar{x})$ is always equal to ________.

If a has any value other than $\bar{x}$, then $\sum_{i=1}^{n}(x_i - \bar{x})^2$ is ______ than $\Sigma(x_i - a)^2$.

Sol. If $\bar{x}$ is the mean of n values of x, then $\sum_{i=1}^{n}(x_i - \bar{x}) = 0$ and if a has any value other than $\bar{x}$, then $\sum_{i=1}^{n}(x_i - x)^2$ is **less** than $\Sigma(x_i - a)^2$.

42. If the variance of a data is 121, then the standard deviation of the data is ________.

Sol. If the variance of a data is 121, then $SD = \sqrt{\text{Variance}} = \sqrt{121} = 11$

Q43. The standard deviation of a data is _____ of any change in origin, but is ________ on the change of scale.

Sol: The standard deviation of a data is independent of any change in origin but is dependent of charge of scale.

Q44. The sum of the squares of the deviations of the values of the variable is ________ when taken about their arithmetic mean.

Sol: The sum of the squares of the deviations of the values of the variable is minimum when taken about their arithmetic mean.

Q45. The mean deviation of the data is ________ when measured from the median.

Sol: The mean deviation of the data is least when measured from the median.

Q46. The standard deviation is________ to the mean deviation taken from the arithmetic mean.

Sol: The standard deviation is greater than or equal to the mean deviation taken from the arithmetic mean.