DCAM classes
Dynamic Classes for Academic Mastery

Chapter 14. Statistics

Question-1

Find the mean deviation from the mean for the following data:
4, 7, 8, 9, 10, 12, 13, 17

Solution:

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{80}{8} = 10$

$\sum_{i=1}^{8} |x_1 - \bar{x}| = 6 + 3 + 2 + 1 + 0 + 2 + 3 + 7 = 24$

$M.D.(\bar{x}) = 24/8 = 3$

Question-2

Find the mean deviation from the mean for the following data:
6.5, 5, 5.25, 5.5, 4.75, 4.5, 6.25, 7.75, 8.5

Solution:

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{54}{9} = 6$

$\sum_{i=1}^{8} |x_1 - \bar{x}| = 0.5 + 1 + 0.75 + 0.5 + 1.25 + 1.5 + 0.25 + 1.75 + 2.5 = 10$

$M.D.(\bar{x}) = 10/9 = 1.1$

Question-3

Find the mean deviation from the mean for the following data:
38, 70, 48, 40, 42, 55, 63, 46, 54, 44

Solution:

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{500}{10} = 50$

$\sum_{i=1}^{8} |x_1 - \bar{x}| = 12 + 20 + 2 + 10 + 8 + 5 + 13 + 4 + 4 + 6 = 84$

$M.D.(\bar{x}) = 84/10 = 8.4$

Question-4

Find the mean deviation from the mean for the following data:
13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Solution:

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{168}{12} = 14$

$\sum_{i=1}^{8} | x_1 - \bar{x} | = 1 + 3 + 2 + 0 + 3 + 1 + 4 + 2 + 3 + 4 + 2 + 3 = 28$

M.D.($\bar{x}$) = 28/12 = 2.33

Question-5

Find the mean deviation from the mean for the following data:
36, 72, 46, 42, 60, 45, 53, 46, 51, 49

Solution:

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{500}{10} = 50$

$\sum_{i=1}^{8} | x_1 - \bar{x} | = 14 + 22 + 4 + 8 + 10 + 5 + 3 + 4 + 1 + 1 = 72$

M.D.($\bar{x}$) = 72/10 = 7.2

Question-12

Find the mean deviation from the median for the following data:
34, 66, 30, 38, 44, 50, 40, 60, 42, 51

Solution:

No of observations n = 10

Arrangement in ascending order are as follows:

30, 34, 38, 40, 42, 44, 50, 51, 60, 66.

Median is 5th and 6th term i.e 42 and 44.

Therefore the median is (42 + 44)/2 = 43

$\sum |x_i - \text{Median}| = 13 + 9 + 5 + 3 + 1 + 1 + 7 + 8 + 17 + 23$

Hence M.D (Median) = $|x_i - \text{Median}|/n$ = 87/10 = 8.7

Question-13

Find the mean deviation from the median for the following data:
22, 24, 30, 27, 29, 31, 25, 28, 41, 42

Solution:
No of observations n = 10
Arrangement in ascending order are as follows:

22, 24, 25, 27, 28, 29, 30, 31, 41, 42
Median is 5th and 6th term i.e 28 and 29.

Therefore the median is (28 + 29)/2 = 28.5
$\sum |x_i - \text{Median}|$ = 6.5 + 4.5 + 3.5 + 1.5 + 0.5 + 0.5 + 1.5 + 2.5 + 12.5 + 13.5
Hence M.D (Median) = $|x_i - \text{Median}|/n$ = 47/10 = 4.7

Question-14

Find the mean deviation from the median for the following data:
38, 70, 48, 34, 63, 42, 55, 44, 53, 47

Solution:
No of observations n = 10
Arrangement in ascending order are as follows:
34, 38, 42, 44, 47, 48, 55, 53, 63, 70,
Median is 5th and 6th term i.e 47 and 48.

Therefore the median is (47 + 48)/2 = 47.5
$\sum |x_i - \text{Median}|$ = 13.5 + 9.5 + 5.5 + 3.5 + 0.5 + 0.5 + 7.5 + 5.5 + 15.5 + 22.5
Hence M.D (Median) = $|x_i - \text{Median}|/n$ = 84/10 = 8.4

Question-17

Find the arithmetic mean of the series 1, 2, 2^2, ..., 2^{n-1}.

Solution:

$\sum x = 1 + 2 + 2^2 + \ldots\ldots + 2^{n-1}$

Sum are in G.P

$\therefore \sum x = \frac{1(2^n - 1)}{2-1} = 2^n - 1$

A.M = $\sum x/n = (2^n-1)/n$

Question-19

Find the mean and variance for the following data:

6, 7, 10, 12, 13, 4, 8, 12

Solution:

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9.$

The respective $(x_i - \bar{x})^2$ are $3^2, 2^2, 1^2, 3^2, 4^2, 5^2, 1^2, 3^2$.

$\sum (x_i - \bar{x})^2 = 9 + 4 + 1 + 9 + 16 + 25 + 1 + 9 = 74$

Hence variance $(\sigma^2) = 74/8 = 9.25$

Question-20

Find the mean and variance for the following data:
2, 4, 5, 6, 8, 17

Solution:

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{2+4+5+6+8+17}{6} = \frac{42}{6} = 7.$$

The respective $(x_i - \bar{x})^2$ are $5^2, 3^2, 2^2, 1^2, 1^2, 10^2$.

$\sum (x_i - \bar{x})^2 = 25 + 9 + 4 + 1 + 1 + 100 = 140$

Hence variance $(\sigma^2) = 140/6 = 23.33$

Question-21

Find the mean for the following data:
First *n* natural numbers

Solution:

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{1+2+3.........+n}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

Question-28

[Hint: First make the data continuous by making the classes as 32.5-36.5, 36.5-40.5,40.5-44.5,44.5-48.5, 48.5-52.5 and the proceed]

Solution:

Classes	x_i	$y_i = (x_i - 42.5)/4$	f_i	$f_i y_i$	$f_i y_i^2$
32.5-36.5	34.5	-2	15	-30	60
36.5-40.5	38.5	-1	17	-17	17
40.5-44.5	42.5	0	21	0	0
44.5-48.5	46.5	1	22	22	22
48.5-52.5	50.5	2	25	50	100
Total			100	25	199

Mean diameter of the circles = $\bar{x}$ = $\left[42.5 + \frac{25}{100} \times 4\right]$ = 43.5

Variance (σ^2) = $[(4)^2/100][199 - 625/100]$ = 30.84

Hence the Standard Deviation is (σ) = $\sqrt{30.84}$ =5.55

Question-29

[Hint: Compare the variance of two groups. The group with greater variance is more variable]

Solution:

classes	x_i	$y_i = (x_i-45)/10$	Group A			Group B		
			f_i	$f_i y_i$	$f_i y_i^2$	f_i	$f_i y_i$	$f_i y_i^2$
10-20	15	-3	9	-27	81	18	-54	162
20-30	25	-2	17	-34	68	22	-44	88
30-40	35	-1	32	-32	32	40	-40	40
40-50	45	0	23	0	0	18	0	0
50-60	55	1	40	40	40	32	32	32
60-70	65	2	18	36	72	8	16	32
70-80	75	3	1	3	9	2	6	18
Total			140	-14	302	140	-84	372

Group A

Variance (σ^2) = $[(10)^2/140][302 - 196/140]$ = 214.7

Group B

Variance (σ^2) = $[(10)^2/140][372 - 7056/140]$ = 229.7

The variance group B is more than group A. Therefore group B has more variable.

Question-31

The mean and variance of 8 observations are 9 and 9.25, respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

Solution:

Let the remaining two observations be x and y.

Then mean = $\frac{6+7+10+12+12+13+x+y}{8}=9$

$60 + x + y = 72$

$x + y = 12$ ....(i)

Variance = $\frac{(6-9)^2+(7-9)^2+(10-9)^2+(12-9)^2+(12-9)^2+(13-9)^2+(x-9)^2+(y-9)^2}{8} = 9.25$

$(-3)^2 + (-2)^2 + (1)^2 + (3)^2 + (3)^2 + (4)^2 + x^2 + y^2 - 18(x + y) + 2\times 9^2 = 9.25\times 8$

$x^2 + y^2 - 216 + 210 = 74$

$x^2 + y^2 = 80$ ....(ii)

But from (i)

$x^2 + y^2 = 144 - 2xy$ ....(iii)

$\therefore 144 - 2xy = 80$

$2xy = 64$ ....(iv)

Subtracting (iv) from (ii)

$x^2 + y^2 - 2xy = 80 - 64$

$(x - y)^2 = 16$

$x - y = \pm 4$ ....(v)

Hence solving (i) and (v)

$x = 8, y = 4$ and $x = 4, y = 8$

Therefore the remaining two observations are 4 and 8.

Question-32

The mean and variance of 7 observations are 8 and 16, respectively. If five of the observations are 2, 4, 10, 12, 14, find the remaining two observations.

Solution:

Let the remaining two observations be x and y.

Then mean = $\frac{2+4+10+12+14+x+y}{7}=8$

$42 + x + y = 56$

$x + y = 14$ ....(i)

Variance = $\frac{(2-8)^2+(4-8)^2+(10-8)^2+(12-8)^2+(14-8)^2+(x-8)^2+(y-8)^2}{7} = 16$

$(-6)^2 + (-4)^2 + (2)^2 + (4)^2 + (6)^2 + x^2 + y^2 - 16(x + y) + 2 \times 8^2 = 16 \times 7$

$x^2 + y^2 - 224 + 236 = 112$

$x^2 + y^2 = 100$ ....(ii)

But from (i)

$x^2 + y^2 = 196 - 2xy$ ....(iii)

$\therefore 196 - 2xy = 100$

$2xy = 96$ ....(iv)

Subtracting (iv) from (ii)

$x^2 + y^2 - 2xy = 100 - 96$

$(x - y)^2 = 4$

$x - y = \pm 2$ ....(v)

Hence solving (i) and (v)

x = 8, y = 6 and x = 6, y = 8

Therefore the remaining two observations are 8 and 6.

Question-33

The mean and variance of 6 observations are 8 and 4, respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

Solution:

Let the observations be x_1 , x_2, x_3, …., x_{20} and $\bar{x}$ be their mean. Then

$$8 = \frac{1}{6}\sum_{i=1}^{6}(x_i - \bar{x})^2$$

$$\text{or } \sum_{i=1}^{6}(x_i - \bar{x})^2 = 48$$

If each observation is multiplied by 3, the resulting observations are $3x_1$, $3x_2$, $3x_3$, …., $3x_{20}$.

Their new mean $\bar{x} = \frac{3(x_1 + x_2 + x_3 + \dots + x_n)}{n} = 3\bar{x} = 3 \times 8 = 24$

and new variance $\frac{1}{6}\sum_{i=1}^{6}(3x_i - \bar{X})^2 = \frac{1}{6}\sum_{i=1}^{6}(3x_i - 3\bar{x})^2 = \frac{3}{6}\sum_{i=1}^{6}(x_i - \bar{x})^2 = 3 \times 48 = 144$

Therefore the new standard deviation is $\sqrt{144} = 12$

Question-34

Given that $\bar{x}$ is the mean and σ^2 is the variance of *n* observations x_1, x_2, x_3, ……x_n. Prove that the mean and variance of the observations ax_1, ax_2, ax_3, …..ax_n are a $\bar{x}$ and $a^2 \sigma^2$, respectively, ($a \neq 0$).

Solution:

Let the observations be x_1 , x_2, x_3, …., x_n and $\bar{x}$ be their mean. Then $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

If each observation is multitplied by a, the resulting observations are ax_1, ax_2, ax_3,…..ax_n

Their new mean $\bar{x} = \frac{a(x_1 + x_2 + x_3 + \dots + x_n)}{n} = a\bar{x}$

And new variance $\frac{1}{n}\sum_{i=1}^{n}(ax_i - \bar{X})^2 = \frac{1}{n}\sum_{i=1}^{n}(ax_i - a\bar{x})^2 = \frac{a}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = a\sigma^2$

Hence proved.

Question-35

The mean of 20 observations are found to be 10. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean in each of the following cases:
(i) If the wrong item is omitted.
(ii) If it is replaced by 12.

Solution:

Let the observations be x_1, x_2, x_3, ..., x_{20} and $\bar{x}$ be their mean. Then $\bar{x} = 10$
$2 = \frac{1}{20}\sum_{i=1}^{20}(x_i - \bar{x})^2$ or $\sum_{i=1}^{20}(x_i - \bar{x})^2 = 40$

(i) Observation 8 is omitted.

New mean = $\bar{x} = \frac{20\times 10 - 8}{19} = 10.11$

(ii) Observation 8 is replaced by 12.

Difference = 12 – 8 = 4

New mean = $\bar{x} = \frac{20\times 10 + 4}{20} = 10.2$

Question-36

Prove that $(x_1 - \bar{x}) + (x_2 - \bar{x}) + \ldots\ldots (x_n - \bar{x}) = 0$ where $\bar{x} = \frac{x_1 + x_2 + \ldots\ldots + x_n}{n}$

Solution:

$$(x_1 - \bar{x}) + (x_2 - \bar{x}) + \ldots\ldots\ldots(x_n - \bar{x}) = x_1 + x_2 + x_3 \ldots\ldots x_n - n\frac{(x_1 + x_2 + x_3 + \ldots\ldots\ldots x_n)}{n} = 0.$$

Question-37

Prove the identity $\sum_{i=1}^{n}(x_i-\bar{x})^2 = \sum_{i=1}^{n}x_i^2 - n\bar{x}^2 = \sum_{i=1}^{n}x^2 - \frac{(\sum_{i=1}^{n}x_i)^2}{n}$.

Solution:

$\sum_{i=1}^{n}(x_i-\bar{x})^2$

$= \sum_{i=1}^{n}\left(x_i^2 - 2x_i\bar{x} + \bar{x}^2\right)$

$= \sum_{i=1}^{n}x_i^2 - 2\bar{x}\sum_{i=1}^{n}x_i + n\bar{x}^2$

$= \sum_{i=1}^{n}x_i^2 - 2\bar{x}n\bar{x} + n\bar{x}^2$ (Since $\sum_{i=1}^{n}x_i = n\bar{x}$)

$= \sum_{i=1}^{n}x_i^2 - n\bar{x}^2$

$= \sum_{i=1}^{n}x_i^2 - n(\sum_{i=1}^{n}\frac{x_i}{n})^2$

$= \sum_{i=1}^{n}x_i^2 - \frac{1}{n}(\sum_{i=1}^{n}x_i)^2$

Question-38

The mean of 9 items is 15. If one more item is added to this series, the mean becomes 16. Find the value of the 10th item.

Solution:

Let the value of 9 items be x_1 , x , x_2 ...... x_9

$15 = \frac{x_1 + x_2 + \ldots\ldots x_9}{9}$ ∴ $x_1+x_2+\ldots\ldots x_9 = 15 \times 9 = 135$

Let x_{10} be the 10th item

AM of $x_1, x_2, \ldots\ldots x_9, x_{10} = 16$

$16 = \frac{x_1 + x_2 \ldots\ldots x_9 + x_{10}}{10}$ ∴ $x_1+x_2 \ldots\ldots\ldots\ldots x_9+x_{10} = 160$

$135 + x_{10} = 160$

$\Rightarrow x_{10} = 25$

Question-39

The average weight of a group of 25 items was calculated to be 78.4kg. It was later discovered that a weight was misread as 69kg instead of 96kg. Calculate correct average.

Solution:

No. of items = 25

Incorrect average = 78.4kg

Incorrect reading of weight of an item = 69kg

Correct reading of weight of an item = 96kg

Let the variable weight be denoted by 'x'

$\bar{x} = \frac{\sum x}{n}$

Incorrect $\bar{x} = \frac{\text{Incorrect} \sum x}{25}$

$78.4 = \frac{\text{Incorrect} \sum x}{25}$

Incorrect $\sum x = 78.4 \times 25 = 1960\text{kg}$

New correct $\sum x = \text{Incorrect} \sum x$ - incorrect weight of an item + correct weight of an item

Correct $\bar{x} = \frac{\text{correct} \sum x}{25} = \frac{1987}{25} = 79.48$ kg

Question-40

The mean of 9 items is 15. If one more item is added to this series, the mean becomes 16. Find the value of the 10th item.

Solution:

Let the value of 9 items be $x_1, x, x_2 \ldots\ldots x_9$

$15 = \frac{x_1 + x_2 + \ldots\ldots x_9}{9} \therefore x_1 + x_2 + \ldots\ldots x_9 = 15 \times 9 = 135$

Let x_{10} be the 10^{th} item

AM of $x_1, x_2, \ldots\ldots x_9, x_{10} = 16$

$16 = \frac{x_1 + x_2 \ldots\ldots x_9 + x_{10}}{10} \therefore x_1 + x_2 \ldots\ldots\ldots\ldots x_9 + x_{10} = 160$

$135 + x_{10} = 160$

$\Rightarrow x_{10} = 25$

Statistics

1. Find the mean deviation about the mean of the distribution:

Size	20	21	22	23	24
Frequency	6	4	5	1	4

2. Find the mean deviation about the median of the following distribution:

Marks obtained	10	11	12	14	15
No. of students	2	3	8	3	4

3. Calculate the mean deviation about the mean of the set of first n natural numbers when n is an odd number.
4. Calculate the mean deviation about the mean of the set of first n natural numbers when n is an even number.
5. Find the standard deviation of the first n natural numbers.
6. The mean and standard deviation of some data for the time taken to complete a test are calculated with the following results:

   Number of observations = 25, mean = 18.2 seconds, standard deviation = 3.25 seconds.

   Further, another set of 15 observations $x_1, x_2, ..., x_{15}$, also in seconds, is now available and we have $\sum_{i=1}^{15} x_i = 279$ and $\sum_{i=1}^{15} x_i^2 = 5524$. Calculate the standard derivation based on all 40 observations.
7. The mean and standard deviation of a set of n_1 observations are $\bar{x}_1$ and s_1, respectively while the mean and standard deviation of another set of n_2 observations are $\bar{x}_2$ and s_2, respectively. Show that the standard deviation of the combined set of $(n_1 + n_2)$ observations is given by

$$\text{S.D.} = \sqrt{\frac{n_1(s_1)^2 + n_2(s_2)^2}{n_1 + n_2} + \frac{n_1 n_2(\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$$

8. Two sets each of 20 observations, have the same standard derivation 5. The first set has a mean 17 and the second a mean 22. Determine the standard deviation of the set obtained by combining the given two sets.

9. The frequency distribution:

x	A	2A	3A	4A	5A	6A
f	2	1	1	1	1	1

where A is a positive integer, has a variance of 160. Determine the value of A.

10. For the frequency distribution:

x	2	3	4	5	6	7
f	4	9	16	14	11	6

Find the standard distribution.

11. There are 60 students in a class. The following is the frequency distribution of the marks obtained by the students in a test:

Marks	0	1	2	3	4	5
Frequency	$x-2$	x	x^2	$(x+1)^2$	$2x$	$x+1$

where x is a positive integer. Determine the mean and standard deviation of the marks.

12. The mean life of a sample of 60 bulbs was 650 hours and the standard deviation was 8 hours. A second sample of 80 bulbs has a mean life of 660 hours and standard deviation 7 hours. Find the overall standard deviation.

13. Mean and standard deviation of 100 items are 50 and 4, respectively. Find the sum of all the item and the sum of the squares of the items.

14. If for a distribution $\sum(x-5)=3$, $\sum(x-5)^2=43$ and the total number of item is 18, find the mean and standard deviation.

15. Find the mean and variance of the frequency distribution given below:

x	$1 \le x < 3$	$3 \le x < 5$	$5 \le x < 7$	$7 \le x < 10$
f	6	4	5	1

16. Calculate the mean deviation about the mean for the following frequency distribution:

Class interval	0 - 4	4 - 8	8 - 12	12 - 16	16 - 20
Frequency	4	6	8	5	2

17. Calculate the mean deviation from the median of the following data:

Class interval	0 - 6	6 - 12	12 - 18	18 - 24	24 - 30
Frequency	4	5	3	6	2

18. Determine the mean and standard deviation for the following distribution:

Marks	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Frequency	1	6	6	8	8	2	2	3	0	2	1	0	0	0	1

19. The weights of coffee in 70 jars is shown in the following table:

Weight (in grams)	Frequency
200 - 201	13
201 - 202	27
202 - 203	18
203 - 204	10
204 - 205	1
205 - 206	1

Determine variance and standard deviation of the above distribution.

20. Determine mean and standard deviation of first n terms of an A.P. whose first term is a and common difference is d.

21. Following are the marks obtained, out of 100, by two students Ravi and Hashina in 10 tests.

Ravi	25	50	45	30	70	42	36	48	35	60
Hashina	10	70	50	20	95	55	42	60	48	80

Who is more intelligent and who is more consistent?

22. Mean and standard deviation of 100 observations were found to be 40 and 10, respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively, find the correct standard deviation.

23. While calculating the mean and variance of 10 readings, a student wrongly used the reading 52 for the correct reading 25. He obtained the mean and variance as 45 and 16 respectively. Find the correct mean and the variance.

CBSE Class 11 Mathematics

Important Questions

Chapter 15

Statistics

1 Marks Questions

1. In a test with a maximum marks 25, eleven students scored 3,9,5,3,12,10,17,4,7,19,21 marks respectively. Calculate the range.

Ans. The marks can be arranged in ascending order as 3,3,4,5,7,9,10,12,17,19,21.

Range = maximum value – minimum value

=21-3

= 18

2. Coefficient of variation of two distributions is 70 and 75, and their standard deviations are 28 and 27 respectively what are their arithmetic mean?

Ans. Given C.V (first distribution) = 70

Standard deviation = σ_1 = 28

C.V $\frac{\sigma 1}{\bar{x}1}\times 100$

= $70=\frac{28}{\bar{x}1}\times 100$

$\bar{x}=\frac{28}{70}\times 100$

$\bar{x}=40$

Similarly for second distribution

C.V $= \frac{\sigma_2}{x_2} \times 100$

$75 = \frac{27}{\bar{x}_2} \times 100$

$\bar{x}_2 = \frac{27}{75} \times 100$

$\bar{x}_2 = 36$

3. Write the formula for mean deviation.

Ans.MD$\left(\bar{x}\right) = \frac{\Sigma f_i \,|\, x_i - \bar{x} \,|}{\Sigma f_i} = \frac{1}{x} \Sigma f_i \,|\, x_i - \bar{x} \,|$

4. Write the formula for variance

Ans. Variance $\sigma^2 = \frac{1}{n} \Sigma f_i \left(x_i - \bar{x}\right)^2$

5. Find the median for the following data.

x_i **579101215**

f_i **862226**

Ans.

x_i	**5**	**7**	**9**	**10**	**12**	**15**
f_i	8	6	2	2	2	6
$c.f$	8	14	16	18	20	26

$n = 26$. Median is the average of 13th and 14th item, both of which lie in the c.f 14

$\therefore x_i = 7$

$$\therefore \text{ median} = \frac{\text{13observation} + \text{14th observation}}{2}$$

$$= \frac{7+7}{2} = 7$$

6. Write the formula of mean deviation about the median

Ans. $MD.(M) = \frac{\sum f_i \,| x_i M |}{\sum f_i} = \frac{1}{n} \sum f_i \,| x_i - M |$

7. Find the rang of the following series 6,7,10,12,13,4,8,12

Ans. Range = maximum value – minimum value

= 113-4

=9

8. Find the mean of the following data 3,6,11,12,18

Ans. Mean = $\frac{\text{sun of observation}}{\text{Total no of observation}}$

$$= \frac{50}{5} = 10$$

9. Express in the form of a + ib (3i-7) + (7-4i) – (6+3i) + i^{23}

Ans. Let

Z = $\cancel{3i} - \cancel{7} + \cancel{7} - 4i - 6 - \cancel{3i} + \left(i^4\right)^5 i^3$

$= -4i - 6 - i \left[\begin{array}{l} \because i^4 = 1 \\ i^3 = -i \end{array} \right.$

$= -5i - 6$

$= -6 + (-5i)$

10. Find the conjugate of $\sqrt{-3} + 4i^2$

Ans. Let $z = \sqrt{-3} + 4i^2$

$= \sqrt{3}\ i - 4$

$\bar{z} = -\sqrt{3}\ i - 4$

11. Solve for x and y, 3x + (2x-y) i= 6 – 3i

Ans.3x = 6

x = 2

2x – y = - 3

2 × 2 – y = - 3

- y = - 3 – 4

y = 7

12. Find the value of 1+i^2 + i^4 + i^6 + i^8 + ---- + i^{20}

Ans. $1 + i^2 + (i^2)^2 + (i^2)^3 + (i^2)^4 + - - - - - + (i^2)^{10} = 1 \left[\because i^2 = -1 \right.$

13. Multiply 3-2i by its conjugate.

Ans. Let z = 3 – 2i

$\bar{z} = 3 + 2i$

$z\,\bar{z} = (3 - 2i)(3 + 2i)$

$= 9 + \cancel{6i} - \cancel{6i} - 4i^2$

$= 9 - 4(-1)$

$= 13$

14. Find the multiplicative inverse 4 – 3i.

Ans. Let z = 4 – 3i

$\bar{z} = 4 + 3i$

$|z| = \sqrt{16 + 9} = 5$

$z^{-1} = \frac{\bar{z}}{|z|^2}$

$= \frac{4 + 3i}{25}$

$= \frac{4}{25} + \frac{3}{25}i$

15. Express in term of a + ib $\frac{(3 + i\sqrt{5})(3 - i\sqrt{5})}{(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2})}$

Ans. $= \frac{(3)^2 - (i\sqrt{5})^2}{\cancel{\sqrt{3}} + \sqrt{2}i - \cancel{\sqrt{3}} + i\sqrt{2}}$

$= \frac{9 + 5}{2\sqrt{2}i} = \frac{\cancel{14}\ 7}{\cancel{2}\sqrt{2}i}$

$= \frac{7}{\sqrt{2}i} \times \frac{\sqrt{2}i}{\sqrt{2}i} = \frac{7\sqrt{2}i}{-2}$

16. Evaluate $i^n + i^{n+1} + i^{n+2} + i^{n+3}$

Ans. $= i^n + i^n.i^1 + i^n.i^2 + i^n.i^3$

$= i^n + i^n.i - i^n + i^n.(-i) \quad \begin{bmatrix} i^3 = -i \\ i^2 = -1 \end{bmatrix}$

$= 0$

17. If 1, w, w^2 are three cube root of unity, show that (1 – w + w^2) (1 + w – w^2) = 4

Ans. (1 – w + w^2) (1 + w – w^2)

(1 + w^2 - w) (1 + w – w^2)

$(-w-w)(-w^2-w^2)\begin{bmatrix} \because 1+w=-w^2 \\ 1+w^2=-w \end{bmatrix}$

$(-2w)(-2w^2)$

$4w^3 \left[w^3 = 1 \right.$

4×1

$= 4$

18. Find that sum product of the complex number $-\sqrt{3}+\sqrt{-2}$ and $2\sqrt{3}-i$

Ans. $z_1 + z_2 = -\sqrt{3} + \sqrt{2}i + 2\sqrt{3} - i$

$= \sqrt{3} + \left(\sqrt{2} - 1\right)i$

$z_1 z_2 = \left(-\sqrt{3} + \sqrt{2}i\right)\left(2\sqrt{3} - i\right)$

$= -6 + \sqrt{3}i + 2\sqrt{6}i - \sqrt{2}\ i^2$

$= -6 + \sqrt{3}\ i + 2\sqrt{6}\ i + \sqrt{2}$

$$=\left(-6+\sqrt{2}\right)+\left(\sqrt{3}+2\sqrt{6}\right)i$$

19. Write the real and imaginary part 1 – 2i^2

Ans. Let z = 1 – 2i^2

=1 – 2 (-1)

= 1 + 2

= 3

= 3 + 0.i

Re (z) = 3, Im (z) = 0

20. If two complex number z_1, z_2 are such that $|z_1| = |z_2|$, is it then necessary that z_1 = z_2

Ans. Let z_1 = a + ib

$$|z_1|=\sqrt{a^2+b^2}$$
$$z_2=b+ia$$

$$|z_2|=\sqrt{b^2+a^2}$$
Hence $|z_1|=|z_2|$ but $z_1 \neq z_2$

21. Find the conjugate and modulus of $\overline{9-i}+\overline{6+i^3}-\overline{9+i^2}$

Ans. Let $z=\overline{9-i}+\overline{6-i}\ -\overline{9-1}$

$$=9+i+6+i-0$$
$$=5+2i$$

$$\bar{z}=5-2i$$

$|z| = \sqrt{(5)^2 + (-2)^2}$

$= \sqrt{25+4}$

$= \sqrt{29}$

22. Find the number of non zero integral solution of the equation |1-i|^x = 2^x

Ans. $|1-i|^x = 2^x$

$\left(\sqrt{(1)^2 + (-1)^2}\right)^x = 2^x$

$\left(\sqrt{2}\right)^x = 2^x$

$(2)^{\frac{1}{2}x} = 2^x$

$\frac{1}{2}x = x$

$\frac{1}{2} = 1$

$1 = 2$

Which is false no value of x satisfies.

23. If (a + ib) (c + id) (e + if) (g + ih) = A + iB then show that

$\left(a^2+b^2\right)\left(c^2+d^2\right)\left(e^2+f^2\right)\left(g^2+h^2\right) = A^2 + B^2$

Ans. $(a+ib)(c+id)(e+if)(g+ih) = A+iB$

$\Rightarrow |(a+ib)(c+id)(e+if)(g+ih)| = |A+iB|$

$|a+ib||c+id||e+if||g+ih| = |A+iB|$

$$\left(\sqrt{a^2+b^2}\right)\left(\sqrt{c^2+d^2}\right)\left(\sqrt{e^2+f^2}\right)\left(\sqrt{g^2+h^2}\right)=\sqrt{A^2+B^2}$$

sq. both side

$$\left(a^2+b^2\right)\left(c^2+d^2\right)\left(e^2+f^2\right)\left(g^2+h^2\right)=A^2+B^2$$

CBSE Class 12 Mathematics
Important Questions
Chapter
Statistics

4 Marks Questions

1.The mean of 2,7,4,6,8 and p is 7. Find the mean deviation about the median of these observations.

Ans. Observations are 2, 7, 4, 6, 8 and p which are 6 in numbers $\therefore n = 6$

The near of these observations is 7

$$\frac{2+7+4+6+8+p}{6} = 7$$

$$= 27 + p = 42$$

$$= p = 15$$

Arrange the observations in ascending order 2,4,6,7,8,15

$\therefore$ Medias (M) = $\dfrac{\frac{n}{2} \text{ th observation} + \left(\frac{n}{2}+1\right) \text{ th observation}}{2}$

$$= \frac{\text{3rd observation} + \text{4th observation}}{2}$$

$$= \frac{6+7}{2} = \frac{13}{2}$$

$$= 6.5$$

Calculation of mean deviation about Median.

xi	xi-M	|xi-M|

2	-4.5	4.5
4	-2.5	2.5
6	-0.5	0.5
7	0.5	0.5
8	1.5	1.5
15	8.5	8.5
Total		18

$\therefore$ Media's deviation about median $= \frac{\cancel{18}^{3}}{\cancel{6}} = 3.$

2.Find the mean deviation about the mean for the following data!

x_i **1030507090**

f_i **42428168**

Ans. To calculate mean, we require $f_i xi$ values then for mean deviation, we require $| xi - \bar{x}$ | values and $f_i \, | xi - \bar{x} |$ values.

xi	f_i	$f_i xi$	$\| xi - \bar{x} \|$	$fi \, \| xi - \bar{x} \|$
10	4	4	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	80	4000		1280

$n = \sum f_i = 80 \quad \sigma d \sum f_i xi = 4000$

$$\bar{x} = \frac{\sum f_i xi}{n} = \frac{4000}{80} = 50$$

Mean deviation about the mean

MD $\left(\bar{x}\right) = \frac{\sum f_i \,|\, xi - \bar{x}\,|}{n} = \frac{1280}{80} = 16$

3.Find the mean, standard deviation and variance of the first n natural numbers.

Ans. The given numbers are 1, 2, 3, ……, n

Mean

$$\bar{x} = \frac{\sum n}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

Variance

$$\sigma 2 = \frac{\sum xi^2}{n} - \bar{x}$$

$$= \frac{\sum n^2}{n} - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{n(n+1)(2n+1)}{6n} - \frac{(n+1)^2}{4}$$

$$= (n+1)\left[\frac{2n+1}{6} - \frac{n+1}{4}\right]$$

$$= (n+1)\left(\frac{n-1}{12}\right) = \frac{n^2-1}{12}$$

$\therefore$ Standard deviation $\sigma = \frac{\sqrt{n^2-1}}{12}$

4.Find the mean variance and standard deviation for following data

Ans.

x_i	**4**	**8**	**11**	**17**	**20**	**24**	**32**
f_i	3	5	9	5	4	3	1

Note: - 4th, 5th and 6th columns are filled in after calculating the mean.

xi	f_i	f_ix_i	$xi-\bar{x}$	$\left(xi-\bar{x}\right)^2$	$f_ix_i(xi-\bar{x}$
4	3	12	-10	100	300
8	5	40	-6	36	180
11	9	99	-3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
Total	30	402			1374

Here $n=\sum f_i=30,\quad \sum f_ix_i=420$

$\therefore$ Mean $\bar{x}=\frac{\sum f_ix_i}{n}=\frac{420}{30}=14$

$\therefore$ Variance $\sigma^2=\frac{1}{n}\sum f_i\left(x_i-\bar{x}\right)^2$

$=\frac{1}{30}\times 1374$

$=45.8$

$\therefore$ Standard deviation $\sigma=\sqrt{45.8}$

= 6.77

5.The mean and standard deviation of 6 observations are 8 and 4 respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

Ans. Let $x_i, x_2 ......x_6$ be the six given observations

Then $\bar{x} = 8$ and $\sigma = 4$

$$\bar{x} = \frac{\sum x_i}{n} = 8 = \frac{x_1 + x_2 + ... + x_6}{6}$$

$$x_1 + x_2 + ...x_6 = 48$$

Also $\sigma^2 \frac{\sum x_1^2}{n} - \left(\bar{x}\right)^2$

$$= 4^2 = \frac{x_1^2 + x_2^2 ...... + x_6^2}{6} - (8)^2$$

$$= x_1^2 + x_2^2 + ...x_6^2$$

$$= 6 \times (16 + 64) = 480$$

As each observation is multiplied by 3, new observations are

$$3x_1, 3x_2, ...3x_6$$

New near $\overline{X} = \frac{3x_1 + 3x_2 + ...3x_6}{6}$

$$= \frac{3(x_1 + x_2 + ...x_6)}{6}$$

$$= \frac{3 \times 48}{6}$$

$$= 24$$

Let σ_1 be the new standard deviation, then

$$\sigma_1^2 = \frac{(3x_1)^2 + (3x_2)^2 + ... + (3x_6)^2}{6} - \left(\overline{X}\right)^2$$

$$= \frac{9\left(x_1^2 + x_2^2 + \ldots\ldots x_6^2\right)}{6} - (24)^2$$

$$= \frac{9 \times 480}{6} - 576$$

$$= 720 - 576$$

$$= 144$$

$$\sigma_1 = 12$$

6.Prove that the standard deviation is independent of any change of origin, but is dependent on the change of scale.

Ans. Let us use the transformation $u = ax + b$ to change the scale and origin

Now $u = ax + b$

$$= \sum u = \sum (ax + b) = a\sum x + b.n$$

Also $\sigma u^2 = \frac{\sum\left(u - \bar{u}\right)^2}{n} = \frac{\sum\left(ax + b - a\bar{x} - b\right)^2}{n}$

$$= \frac{\sum a^2\left(x - \bar{x}\right)^2}{n} = \frac{a^2\sum\left(x - \bar{x}\right)^2}{n}$$

$$= a^2\sigma x^2$$

$$\therefore \quad \sigma^2 u = a2\sigma^2 u$$

$$= \sigma u = |a|\, \sigma x$$

Both σu , σx are positive which shows that standard deviation is independent of choice of origin, but depends on the scale.

7.Calculate the mean deviation about the mean for the following data

Expenditure0-100100-200200-300300-400400-500500-600600-700700-800

persons 489107543

Ans.

Expenditure	**No. of persons** f_i	**Mid point** x_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
0-100	4	50	200	308	1232
100-200	8	150	1200	208	1664
200-300	9	250	2250	108	972
300-400	10	350	3500	8	80
400-500	7	450	3150	92	644
500-600	5	550	2750	192	960
600-700	4	650	2600	292	1168
700-800	3	750	2250	392	1176
	50		17900		7896

$n = \sum f_i = 50$

$\sum f_i x_i = 17900$

$\therefore$ mean = $\frac{1}{n} \sum f_i x_i = \frac{17900}{50} = 358$

$MD(\bar{x}) = \frac{1}{n} \sum fi \lvert x_i - \bar{x} \rvert$

$= \frac{7896}{50} = 157.92$

8.Find the mean deviation about the median for the following data

Marks 0-1010-2020-3030-4040-5050-60

No. of boys 810101642

Ans.

Marks	No. of boys	Cumulative Frequency	Mid points	$\lvert x_i - M \rvert$	$f_i \lvert x_i - M \rvert$
0-10	8	8	5	22	176
10-20	10	18	15	12	120
20-30	10	28	25	2	20
30-40	16	44	35	8	128
40-50	4	48	45	18	72
50-60	2	50	55	28	56
total	50				572

$\frac{n}{2}^{th}$ or 25^{th} item $= 20-30,$ which is the median class.

Median $= l + \frac{\frac{n}{2}-c}{f} \times c = 20 + \frac{25-18}{10} \times 10$

= 27

$$MD(M) = \frac{1}{n}\Sigma f_i \lvert x_i - M \rvert = \frac{572}{50} = 11.44$$

9.An analysis of monthly wages point to workers in two firms A and B, belonging to the same industry, given the following result. Find mean deviation about median.

Firm AFirm B

No of wages earns586648

Average monthly wagesRs 5253Rs 5253

Ans. For firm A, number of workers = 586

Average monthly wage is Rs 5253

Total wages = Rs 5253 $\times$ 586

= Rs 3078258

For firm B, total wages = Rs 253 $\times$ 648

=Rs 3403944

Hence firm B pays out amount of monthly wages.

10.Find the mean deviation about the median of the following frequency distribution

Class 0-66-1212-1818-2424-30

Frequency8101295

Ans.

Class	**Mid value**	**Frequency**	$C.f$	$\|x_i - 14\|$	$f_i \|x_i - 14\|$
0-6	3	8	8	11	88
6-12	9	10	18	5	50
12-18	15	12	30	1	12
18-24	21	9	39	7	63
21-30	27	5	44	13	65
			$N = \sum f_i = 44$		$\sum f_i \|x_i - 14\| = 278$

$$N = 44 = \frac{N}{2}$$

12-18 is the medias class

Medias = $l + \frac{\frac{N}{2} - F}{f} \times h$

$$h = 6,\ l = 12,\ f = 12,\ F = 18$$

Medias

$$=12+\frac{22-18}{12}\times 6$$

$$=12+\frac{4\times 6}{12}$$

$$=14$$

Mean deviation about median = $\frac{1}{N}\sum f_i\,|x_i-14|$

$$=\frac{278}{74}=6.318$$

11.Calculate the mean deviation from the median from the following data

Salary per week(in Rs) 10-2020-3030-4040-5050-6060-70

no. of workers 461020106

Ans.

Salary per Week (in Rs)	**Mid value** x_i	**Frequency** f_i	Cf	$\|d_i\|=x_i-45$	$f\,\|d_i\|$
10-20	15	4	4	30	120
20-30	25	6	10	20	120
30-40	35	10	20	10	100
40-50	45	20	40	0	0
50-60	55	10	50	10	100
60-70	65	6	56	20	120
70-80	75	4	60	30	120
		$N=\sum f_i=60$			$\sum f_i\,\|d_i\|=680$

$N=60$ $=\frac{N}{2}=30$

40-50 is the median class

$l = 40,\ f = 20,\ h = 10,\ F = 20$

Medias = $\dfrac{l - \frac{N}{2} - F}{f} \times h$

$= \dfrac{40 + 30 - 20}{20} \times 10 = 45$

Mean deviation = $\dfrac{\sum f_i |d_i|}{N}$ = $\dfrac{680}{60} = 11.33$

12.Let $x_1, x_2 ...... x_n$ values of a variable Y and let 'a' be a non zero real number. Then prove that the variance of the observations $ay_1, ay_2 ...... ay_n$ is $a^2 \text{ var}(Y)$. also, find their standard deviation.

Ans. Let $v_1, v_2 ...... v_n$ value of variables v such that $v_1 = ay_i, 1, 2 ..... n,$ then

$$\overline{V} = \frac{1}{n}\sum_{i=1}^{n} v_i = \frac{1}{n}\sum_{i=1}^{n}(ayi) = a\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right) = a\overline{y}$$

$$v_i - \overline{V} = ay_i - a\overline{y}$$

$$v_i - \overline{V} = a\left(y_i - \overline{Y}\right)$$

$$\left(v_i - \overline{V}\right)^2 = a^2\left(y_i - \overline{Y}\right)^2$$

$$\sum_{i=1}^{n}\left(v_i - \overline{V}\right)^2 = a^2 \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \overline{Y}\right)^2$$

$$\text{Var }(V) = a^2 \text{ Var }(Y)$$

$$\sigma_u = \sqrt{\text{var}(v)} = \sqrt{a^2 \text{ var}(Y)} = |a| \sqrt{\text{var}(Y)}$$

$$= |a|\, \sigma_y$$

13.If $a+ib=\frac{(x+i)^2}{2x^2+1}$ Prove that $a^2+b^2=\frac{(x^2+1)^2}{(2x^2+1)^2}$

Ans. $a+ib=\frac{(x+i)^2}{2x^2+1}$ (i) (Given)

Taking conjugate both side

$$a-ib=\frac{(x-i)^2}{2x^2+1} \quad \text{(ii)}$$

(i) × (ii)

$$(a+ib)(a-ib)=\left(\frac{(x+i)^2}{2x^2+1}\right)\times\left(\frac{(x-i)^2}{2x^2+1}\right)$$

$$(a)^2-(ib)^2=\frac{(x^2-i^2)^2}{(2x^2+1)^2}$$

$$a^2+b^2=\frac{(x^2+1)^2}{(2x^2+1)^2} \quad \text{proved.}$$

14.If $(x+iy)^3=u+iv$ then show that $\frac{u}{x}+\frac{v}{y}=4(x^2-y^2)$

Ans. $(x+iy)^3=4+iv$

$$x^3+(iy)^3+3x^2(iy)+3.x(iy)^2=u+iv$$

$$x^3-iy^3+3x^2yi-3xy^2=u+iv$$

$$x^3-3xy^2+(3x^2y-y^3)i=u+iv$$

$$x\left(x^2-3y^2\right)+y\left(3x^2-y^2\right)i=u+iv$$

$$x\left(x^2-3y^2\right)=u,\ y\left(3x^2-y^2\right)=v$$

$$x^2-3y^2=\frac{u}{x}\ \text{(i)}\ \Big|\ 3x^2-y^2=\frac{v}{y}\ \text{(ii)}$$

(i) + (ii)

$$4x^2-4y^2=\frac{u}{x}+\frac{v}{y}$$

$$4\left(x^2-y^2\right)=\frac{u}{x}+\frac{v}{y}$$

15.Solve $\sqrt{3}x^2-\sqrt{2}x+3\sqrt{3}=0$

Ans. $\sqrt{3}\ x^2-\sqrt{2}\ x+3\ \sqrt{3}=0$

$$a=\sqrt{3},b=-\sqrt{2},c=3\sqrt{3}$$

$$D=b^2-4ac$$

$$=\left(-\sqrt{2}\right)^2-4\times\sqrt{3}\left(3\sqrt{3}\right)$$

$$=2-36$$

$$=-34$$

$$x=\frac{-b\pm\sqrt{D}}{2a}$$

$$=\frac{-\left(-\sqrt{2}\right)\pm\sqrt{-34}}{2\times\sqrt{3}}$$

$$= \frac{\sqrt{2} \pm \sqrt{34}\ i}{2\sqrt{3}}$$

16.Find the modulus $i^{25} + \left(1+3i\right)^3$

Ans. $i^{25} + (1+3i)^3$

$$= \left(i^4\right)^6 .i + 1 + 27i^3 + 3(1)(3i)(1+3i)$$

$$= i + \left(1 - 27i + 9i + 27i^2\right)$$

$$= i + 1 - 18i - 27$$

$$= -26 - 17i$$

$$\left|i^{25} + \left(1+3i\right)^3\right| = \left|-26 - 17i\right|$$

$$= \sqrt{(-26)^2 + (-17)^2}$$

$$= \sqrt{676 + 289}$$

$$= \sqrt{965}$$

17.If $a + ib = \frac{(x+i)^2}{2x-i}$ prove that $a^2 + b^2 = \frac{\left(x^2+1\right)^2}{4x^2+1}$

Ans. $a + ib = \frac{(x+i)^2}{2x-i}$ (i) (Given)

$a - ib = \frac{(x-i)^2}{2x+i}$ (ii) [taking conjugate both side

(i) × (ii)

$$(a+ib)(a-ib)=\frac{(x+i)^2}{(2x-i)}\times\frac{(x-i)^2}{(2x+i)}$$

$$a^2+b^2=\frac{(x^2+1)^2}{4x^2+1} \quad \text{proved.}$$

18.Evaluate $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

Ans. $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

$$\left[\left(i^4\right)^4.i^2+\frac{1}{i^{25}}\right]^3$$

$$\left[i^2+\frac{1}{\left(i^4\right)^6.i}\right]^3$$

$$\left[-1+\frac{1}{i}\right]^3$$

$$\left[-1+\frac{i^3}{i^4}\right]^3$$

$$[-1-i]^3=-(1+i)^3$$

$$=-\left[1^3+i^3+3.1.i(1+i)\right]$$

$$=-\left[1-i+3i+3i^2\right]$$

$$=-[1-i+3i-3]$$

$= -[-2+2i] = 2-2i$

19.Find that modulus and argument $\frac{1+i}{1-i}$

Ans. $\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

$= \frac{(1+i)^2}{1^2 - i^2}$

$= \frac{1+i^2+2i}{1+1}$

$= \frac{2i}{2}$

$= i$

$z = 0 + i$

$r = |z| = \sqrt{(0)^2 + (1)^2} = 1$

Let α be the acute $\angle$s

$\tan \alpha = \left|\frac{1}{0}\right|$

$\alpha = \pi/2$

$\arg(z) = \pi/2$

$r = 1$

20.For what real value of x and y are numbers equal (1+i) y^2 + (6+i) and (2+i) x

Ans.(1+i) y^2 + (6 + i) = (2 + i) x

$y^2 + iy^2 + 6 + i = 2x + xi$

$(y^2 + 6) + (y^2 + 1)\, i = 2x + xi$

$y^2 + 6 = 2x$

$y^2 + 1 = x$

$y^2 = x - 1$

$x - 1 + 6 = 2x$

$5 = x$

$y = \pm 2$

21.If x + iy = $\sqrt{\dfrac{1+i}{1-i}}$, prove that $x^2 + y^2 = 1$

Ans. $x + iy = \sqrt{\dfrac{1+i}{1-i}}$ (i) (Given)

taking conjugate both side

$x - iy = \sqrt{\dfrac{1-i}{1+i}}$ (ii)

(i) × (ii)

$(x+iy)(x-iy) = \sqrt{\dfrac{1+i}{1-i}} \times \sqrt{\dfrac{1-i}{1+i}}$

$(x)^2 - (iy)^2 = 1$

$x^2 + y^2 = 1$

Proved.

22.Convert in the polar form $\frac{1+7i}{(2-i)^2}$

Ans. $\frac{1+7i}{(2-i)^2}=\frac{1+7i}{4+i^2-4i}=\frac{1+7i}{3-4i}$

$=\frac{1+7i}{3-4i}\times\frac{3+4i}{3+4i}$

$=\frac{3+4i+21i+28i^2}{9+16}$

$=\frac{25i-25}{25}=i-1$

$=-1+i$

$r=|z|=\sqrt{(-1)^2+1^2}=\sqrt{2}$

Let α be the acute $\angle$s

ten $\alpha=\left|\frac{1}{-1}\right|$

$\alpha=\pi/4$

since Re (z) < 0, Im (z) > 0

$\theta=\pi-\alpha$

$=\pi-\frac{\pi}{4}=3\pi/4$

$z=r(\text{Cos}\,\theta+\text{i Sin}\,\theta)$

$=\sqrt{2}\left(\text{Cos}\,\frac{3\pi}{4}+i\,Sin\,\frac{3\pi}{4}\right)$

23.Find the real values of x and y if (x - iy) (3 + 5i) is the conjugate of – 6 – 24i

Ans.

(x – iy) (3 + 5i) = - 6 + 24i

3x + 5xi – 3yi – 5yi^2 = - 6 + 24i

$$(3x+5y)+(5x-3y)i=-6+24i$$

$$3x+5y=-6$$

$$5x-3y=24$$

$$x=3$$
$$y=-3$$

24.If $|z_1|=|z_2|=1$, prove that $\left|\frac{1}{z_1}+\frac{1}{z_2}\right|=|z_1+z_2|$

Ans. If $|z_1|=|z_2|=1$ (Given)

$$\Rightarrow |z_1|^2=|z_2|^2=1$$

$$\Rightarrow z_1\ \overline{z_1}=1$$

$$\overline{z_1}=\frac{1}{z_1} \quad (1)$$

$$z_2\ \overline{z_2}=1$$

$$\overline{z_2}=\frac{1}{z_2} \quad (2)$$

$[\ \because z\ \overline{z}=|z|^2$

$$\left|\frac{1}{z_1}+\frac{1}{z_2}\right| = \left|\overline{z_1}+\overline{z_2}\right|$$

$$= \left|\overline{z_1+z_2}\right|$$

$$= \left|z_1+z_2\right|$$

$\left[\because \left|\overline{z}\right| = \left|z\right|\right.$ proved.

CBSE Class 12 Mathematics

Important Questions

Chapter

Statistics

6 Marks Questions

1.Calculate the mean, variance and standard deviation of the following data:

Classes	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**
Frequency	**3**	**7**	**12**	**15**	**8**	**3**	**2**

Ans.

Classes	**Frequency**	**Mid Point**	$f_i xi$	$\left(x_i - \overline{x}\right)^2$	$f_i\left(x_i - \overline{x}\right)^2$
30-40	3	35	105	729	2187
40-50	7	45	315	289	2023
50-60	12	55	660	49	588
60-70	15	65	975	9	135
70-80	8	75	600	169	1352
80-90	3	85	255	529	1587
90-100	2	95	190	1089	2178
Total	50		3100		10050

Here $n = \sum f_i = 50, \sum f_i x_i = 3100$

$\therefore$ Mean $\overline{x} = \frac{\sum f_i x_i}{n} = \frac{3100}{50} = 62$

Variance $\sigma^2 = \frac{1}{n}\sum f_i\left(xi - \overline{x}\right)^2$

$= \frac{1}{50} \times 10050$

$= 201$

Standard deviation $\sigma = \sqrt{201} = 14.18$

2.The mean and the standard deviation of 100 observations were calculated as 40 and 5.1 respectively by a student who mistook one observation as 50 instead of 40. What are the correct mean and standard deviation?

Ans. Given that $n = 100$

Incorrect mean $\bar{x} = 40,$

Incorrect S.D $(\sigma) = 5.1$

As $\bar{x} = \frac{\sum x_i}{n}$

$$40 = \frac{\sum x_i}{100} = \sum x_i = 4000$$

= incorrect sum of observation =4000

= correct sum of observations = 4000-50+40

= 3990

So correct mean = $\frac{3990}{100} = 39.9$

Also $\sigma = \sqrt{\frac{1}{n}\sum x_i^2 - \left(\bar{x}\right)^2}$

Using incorrect values,

$$5.1 = \sqrt{\frac{1}{100}\sum x_i^2 - (40)^2}$$

$$= 26.01 = \left[\frac{1}{100}\sum x_i^2 - 1600\right]$$

$= \sum x_i^2 = 2601 + 160000$

= 162601

= incorrect $\sum x_i^2 = 162601$

= correct $\sum x_i^2 = 162601 - (50)^2 + (40)^2$

= 162601-2500+1600=161701

$\therefore$ Correct $\sigma = \sqrt{\frac{1}{100}\text{correct} \sum x_i^2 - \left(\text{correct } \bar{x}\right)^2}$

$= \sqrt{\frac{1}{100}(161701) - (39.9)^2} \quad = \sqrt{1617.01 - 1592.01}$

$= \sqrt{25} = 5$

Hence, correct mean is 39.9 and correct standard deviation is 5.

3.200 candidates the mean and standard deviation was found to be 10 and 15 respectively. After that if was found that the scale 43 was misread as 34. Find the correct mean and correct S.D

Ans. $n = 200, \overline{X} = 40, \sigma = \overline{15}$

$\overline{X} = \frac{1}{n}\sum x_i = \sum x_i = n\overline{X} = 200 \times 40 = 8000$

Corrected $\sum x_i$ = Incorrect $\sum x_i -$ (sum of incorrect +sum of correct value)

= 8000-34+43= 8009

$\therefore$ Corrected mean = $\frac{\text{corrected} \sum x_i}{n} = \frac{8009}{200} = 40.045$

$\sigma = 15$

$$15^2 = \frac{1}{200}\left(\sum x_i^{\,2}\right) - \left(\frac{1}{200}\sum x_i\right)^2$$

$$225 = \frac{1}{200}\left(\sum x_i^{\,2}\right) - \left(\frac{8000}{200}\right)^2$$

$$225 = \frac{1}{200} \times 1825 = 365000$$

Incorrect $\sum x_i^{\,2} = 365000$

Corrected $\sum x_i^{\,2}$ = (incorrect $\sum x_i^{\,2}$) – (sum of squares of incorrect values) + (sum of square of correct values)

$= 365000 - (34)^2 + (43)^2 = 365693$

Corrected σ = $\sqrt{\frac{1}{n}\sum x_i^{\,2} - \left(\frac{1}{n}\sum x_i\right)^2} = \sqrt{\frac{365693}{200} - \left(\frac{8009}{200}\right)^2}$

$\sqrt{1828.465 - 1603.602} = 14.995$

4.Find the mean deviation from the mean 6,7,10,12,13,4,8,20

Ans. Let $\overline{X}$ be the mean

$$\overline{X} = \frac{6+7+10+12+13+4+8+20}{8} = 10$$

x_i	$\lvert d_i \rvert = \lvert x_i - \overline{X} \rvert = \lvert x_i - 10 \rvert$
6	4
7	3
10	0

12	2
13	3
4	6
8	2
20	10
Total	$\sum d_i$ = 30

$\sum d_i$ = 30 and n = 8

$$\therefore MD = \frac{1}{n}\sum |d_i| = \frac{30}{8} = 3.75$$
$$\therefore MD = 3.75$$

5.Find two numbers such that their sum is 6 and the product is 14.

Ans. Let x and y be the no.

x + y = 6

xy = 14

$$x^2 - 6x + 14 = 0$$

$$D = -20$$

$$x = \frac{-(-6) \pm \sqrt{-20}}{2 \times 1}$$

$$= \frac{6 \pm 2\sqrt{5}\ \mathrm{i}}{2}$$

$$= 3 \pm \sqrt{5}\ \mathrm{i}$$

$$\mathrm{x} = 3 + \sqrt{5}\ \mathrm{i}$$

$$y = 6 - \left(3 + \sqrt{5}\ \mathrm{i}\right)$$

$$= 3 - \sqrt{5}\ \mathrm{i}$$

when $\mathrm{x} = 3 - \sqrt{5}\ \mathrm{i}$

$$y = 6 - \left(3 - \sqrt{5}\ \mathrm{i}\right)$$

$$= 3 + \sqrt{5}\ \mathrm{i}$$

6.Convert into polar form $z = \dfrac{i-1}{\cos\dfrac{\pi}{3} + i\ \mathrm{Sin}\ \dfrac{\pi}{3}}$

Ans. $z = \dfrac{i-1}{\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i}$

$$= \frac{2(i-1)}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i}$$

$$z = \frac{\sqrt{3}-1}{2} + \frac{\sqrt{3}+1}{2}i$$

$$r = |z| = \left(\frac{\sqrt{3}-1}{2}\right)^2 + \left(\frac{\sqrt{3}+1}{2}\right)^2$$

$r = 2$

Let α be the acule $\angle$s

$$\tan\alpha = \left| \frac{\frac{\sqrt{3}+1}{2}}{\frac{\sqrt{3}-1}{2}} \right|$$

$$= \left| \frac{\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\left(1-\frac{1}{\sqrt{3}}\right)} \right|$$

$$= \left| \frac{\tan\frac{\pi}{4}+\tan\frac{\pi}{6}}{1-\tan\frac{\pi}{4}\tan\frac{\pi}{6}} \right|$$

$$\tan\alpha = \left| \tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right) \right|$$

$$\alpha = \frac{\pi}{4}+\frac{\pi}{6} = \frac{5\pi}{12}$$

$$z = 2\left(Cos\frac{5\pi}{12} + i\ \text{Sin}\ \frac{5\pi}{12} \right)$$

7.If α and β are different complex number with |β| = 1 Then find $\left| \frac{\beta-\alpha}{1-\bar{\alpha}\beta} \right|$

Ans. $\left| \frac{\beta-\alpha}{1-\bar{\alpha}\beta} \right|^2 = \left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta} \right)\left(\overline{\frac{\beta-\alpha}{1-\bar{\alpha}\beta}} \right) \qquad \left[\because |z|^2 = z\bar{z}\right.$

$$=\left(\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right)\left(\frac{\overline{\beta}-\overline{\alpha}}{1-\alpha\overline{\beta}}\right)$$

$$=\left(\frac{\beta\overline{\beta}-\beta\overline{\alpha}-\alpha\overline{\beta}+\alpha\overline{\alpha}}{1-\alpha\overline{\beta}-\overline{\alpha}\beta+\alpha\overline{\alpha}\beta\overline{\beta}}\right)$$

$$=\left(\frac{|\beta|^{2}-\beta\overline{\alpha}-\alpha\overline{\beta}+|\alpha|^{2}}{1-\alpha\overline{\beta}-\overline{\alpha}\beta+|\alpha|^{2}|\beta|^{2}}\right)$$

$$=\left(\frac{1-\beta\overline{\alpha}-\alpha\overline{\beta}+|\alpha|^{2}}{1-\alpha\overline{\overline{\beta}}-\overline{\overline{\alpha}}\beta+|\alpha|^{2}}\right)\left[\because |\beta|=1\right.$$

$$=1$$

$$\left|\frac{\beta-\alpha}{1-\alpha\beta}\right|=\sqrt{1}$$

$$\left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right|=1$$