Unit 10 (Straight Lines)

Short Answer Type Questions

Q1. Find the equation of the straight line which passes through the point (1, -2)

Sol. Equation of line in intercept form is: $\frac{x}{a}+\frac{y}{b}=1$

Here, we have $a = b$.

So, equation of the line reduces to: $\frac{x}{a}+\frac{y}{a}=1$ or $x+y=a$

Since the point $(1, -2)$ lies on the line, we get $1-2=a$ or $a=-1$.

Therefore, equation of the line is: $x+y=-1$ or $x+y+1=0$

Q2. Find the equation of the line passing through the point (5,2) and perpendicular to the line joining the points (2, 3) and (3, -1)

Sol: We have points A(5, 2), B(2, 3) and C(3, -1).

Slope of the line BC, $m_{BC}=\frac{-1-3}{3-2}=-4$

Line through A is perpendicular to BC.

$\therefore$ Slope of required line $=\frac{1}{4}$

The equation of line passing through the point $A(5, 2)$ and having slope $\frac{1}{4}$ is:

$$y-2=\frac{1}{4}(x-5) \text{ or } x-4y+3=0$$

Q3. Find the angle between the lines y = (2 -√3) (x + 5) and y = (2 + -√3) {x – 7)

Sol: Slope of the line = (2 -√3)(x + 5) is: m_l = (2 -√3)

Slope of the line $y=(2+\sqrt{3})(x-7)$ is: $m_2=(2+\sqrt{3})$

Let θ be the angle between these lines. Then

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{(2-\sqrt{3})-(2+\sqrt{3})}{1+(2-\sqrt{3})(2+\sqrt{3})}\right|=\left|\frac{-2\sqrt{3}}{1+4-3}\right|=\sqrt{3}$$

$\therefore$ $\theta=\frac{\pi}{3}=60°$, which is an acute angle

Thus, obtuse angle between the lines = 180°– 60° = 120°

Q4. Find the equation of the lines which passes through the point (3, 4) and cuts off intercepts from the coordinate axes such that their sum is 14.

Sol. Equation of line in intercept form is $\frac{x}{a}+\frac{y}{b}=1$

Given that, $a + b = 14 \Rightarrow b = 14 - a$

So, equation of line is: $\frac{x}{a}+\frac{y}{14-a}=1$

Since it passes through the point (3, 4), we have

$$\frac{3}{a}+\frac{4}{14-a}=1$$

$\Rightarrow \quad a^2 - 13a + 42 = 0 \quad \Rightarrow \quad (a-7)(a-6) = 0$

$\therefore \quad a = 7$ or $a = 6$

When $a = 7$, then $b = 7$

When $a = 6$, then $b = 8$

Thus, equation of line is: $\frac{x}{7}+\frac{y}{7}=1$, i.e., $x + y = 7$ or $\frac{x}{6}+\frac{y}{8}=1$

Q5. Find the points on the line x+y = 4 which lie at a unit distance from the line 4x + 3y= 10

Sol. Let the required point be (h, k) lies on the line x + y = 4

i.e., $\quad h + k = 4 \quad$ (i)

The distance of the point (h, k) from the line $4x + 3y = 10$ is:

$$\left|\frac{4h+3k-10}{\sqrt{16+9}}\right|=1 \quad \text{(given)}$$

$\Rightarrow \quad 4h + 3k - 10 = \pm 5$

This gives two results:

$$4h + 3k = 15 \qquad \text{(ii)}$$

$$4h + 3k = 5 \qquad \text{(iii)}$$

Solving (i) and (ii), we get $(h, k) \equiv (3, 1)$.

Solving (i) and (iii), we get $(h, k) \equiv (-7, 11)$.

6. Show that the tangent of an angle between the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{a}-\frac{y}{b}=1$ is $\frac{2ab}{a^2-b^2}$.

Sol. Slope of the line $\frac{x}{a}+\frac{y}{b}=1$ is: $m_1 = -\frac{b}{a}$

Slope of the line $\frac{x}{a}-\frac{y}{b}=1$ is: $m_2 = \frac{b}{a}$

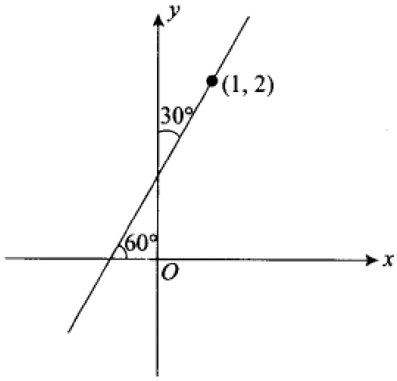
Let θ be angle between the given lines. Then

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right| = \left|\frac{-\frac{b}{a}-\frac{b}{a}}{1+\left(\frac{-b}{a}\right)\left(\frac{b}{a}\right)}\right| = \left|\frac{\frac{-2b}{a}}{\frac{a^2-b^2}{a^2}}\right|$$

Then, $\tan\theta = \frac{2ab}{a^2-b^2}$

Q7. Find the equation of lines passing through (1,2) and making angle 30° with y-axis.

y-axis.



Sol. As shown in the figure line makes an angle 30° with y-axis, then it makes an angle 60° with x-axis.

$\therefore$ Slope of the line = $\tan 60° = \sqrt{3}$

So, the equation of line passing through (1, 2) and having slope $\sqrt{3}$ is:

$$y-2=\sqrt{3}\,(x-1)$$

$$\Rightarrow \quad y-\sqrt{3}\,x-2+\sqrt{3}=0$$

Q8. Find the equation of the line passing through the point of intersection of lx + y = 5 and x + 3 y +8 = 0 and parallel to the line 3x + 4y = 1.

Sol. Given lines are:

$$2x+y=5 \qquad \text{(i)}$$

$$x+3y=-8 \qquad \text{(ii)}$$

Solving (i) and (ii), we get their point of intersection as $\left(\frac{23}{5},\frac{-21}{5}\right)$.

Slope of line $3x + 4y = 7$ is $\frac{-3}{4}$. So, the line parallel to this line has slope $\frac{-3}{4}$.

Then the equation of the line passing through the point $\left(\frac{23}{5},\frac{-21}{5}\right)$ having slope $\frac{-3}{4}$ is:

$$y+\frac{21}{5}=\frac{-3}{4}\left(x-\frac{23}{5}\right)$$

$$\Rightarrow \quad 4y+\frac{84}{5}=-3x+\frac{69}{5} \quad \Rightarrow \quad 3x+4y=\frac{84-69}{5}=3$$

$$\Rightarrow \quad 3x+4y+3=0$$

Q9. For what values of a and b the intercepts cut off on the coordinate axes by the line ax + by + 8 = 0 are equal in length but opposite in signs to those cut off by the line 2x − 3y + 6 = 0 on the axes.

Sol: Given line is:

$$ax + by + 8 = 0 \quad \Rightarrow \quad \frac{x}{\frac{-8}{a}} + \frac{y}{\frac{-8}{b}} = 1$$

So, the intercepts are $\frac{-8}{a}$ and $\frac{-8}{b}$.

Another given line is:

$$2x - 3y + 6 = 0 \quad \Rightarrow \quad \frac{x}{-3} + \frac{y}{2} = 1$$

So, the intercepts are –3 and 2.

According to the question, we have

$$\frac{-8}{a} = 3 \text{ and } \frac{-8}{b} = -2$$

$$\therefore \quad a = -\frac{8}{3}, b = 4$$

Q10. If the intercept of a line between the coordinate axes is divided by the point (-5,4) in the ratio 1 : 2, then find the equation of the line.

Sol: Let the line through the point P(-5, 4) meets axis at A(h, 0) and B(0, k)

According to the question, we have AP: BP =1:2

$$\therefore \quad (-5, 4) \equiv \left(\frac{1 \times 0 + 2 \times h}{1+2}, \frac{1 \times k + 2 \times 0}{1+2}\right)$$

$$\therefore \quad -5 = \frac{2h}{3} \text{ and } 4 = \frac{k}{3}$$

$$\Rightarrow \quad h = -15/2 \text{ and } k = 12$$

Thus, equation of the line using intercept form is:

$$\frac{x}{-15/2} + \frac{y}{12} = 1 \quad \Rightarrow \quad 8x - 5y + 60 = 0$$

Q11. Find the equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of 120° with the positive direction of x-axis.

Sol: Given that the line makes and angle 120° with positive direction of x-axis.

$\therefore$ Slope of the line is $\tan 120° = -\sqrt{3}$

So, equation of the required line is: $y = -\sqrt{3}x + c \Rightarrow \sqrt{3}x + y - c = 0$.

Now distance of this line from (0, 0) is 4 units.

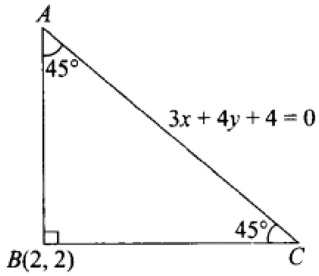
$$\therefore \quad \frac{|\sqrt{3}(0) + 0 - c|}{\sqrt{3+1}} = 4$$

$$\Rightarrow \quad |c| = 8 \quad \Rightarrow \quad c = \pm 8$$

Thus, equation of the required lines is $\sqrt{3}x + y \pm 8 = 0$.

Q12. Find the equation of one of the sides of an isosceles right angled triangle whose hypotenuse is given by 3x + Ay = 4 and the opposite vertex of the hypotenuse is (2, 2).

Sol. As shown in the figure, hypotenuse is along the line $3x + 4y + 4 = 0$.



$\therefore$ Slope of $AC = \frac{-3}{4}$.

Since ABC is isosceles right angled triangle, $\angle BAC = \angle ACB = 45°$.

Now, let the slope of the line making an angle 45° with AC be m.

$$\therefore \quad \tan 45° = \left| \frac{m - \left(-\frac{3}{4}\right)}{1 + m\left(-\frac{3}{4}\right)} \right| \Rightarrow \frac{4m+3}{4-3m} = \pm 1$$

$$\Rightarrow \quad 4m + 3 = 4 - 3m \text{ or } 4m + 3 = 3m - 4 \quad \Rightarrow \quad m = 1/7 \text{ or } m = -7$$

So, if the slope of line BC is 1/7 then the slope of line AB is –7.

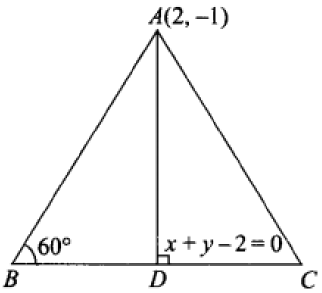
So, equation of BC is: $y - 2 = (1/7)(x - 2) \Rightarrow x - 7y + 12 = 0$.

Equation of AB is: $y - 2 = -7(x - 2) \Rightarrow 7x + y - 16 = 0$.

Long Answer Type Questions

Q13. If the equation of the base of an equilateral triangle is x + y – 2 and the vertex is (2, -1), then find the length of the side of the triangle.

Sol. As shown in the figure in equilateral ΔABC, BC is base along the line $x + y - 2 = 0$ and vertex A is $(2, -1)$.



Altitude from A on BC meets at the mid point of BC.

Now, distance AD = distance of A from BC

$$= \left| \frac{2 + (-1) - 2}{\sqrt{1^2 + 1^2}} \right| = \frac{1}{\sqrt{2}}$$

Also in ΔABD,

$$\sin 60° = \frac{AD}{AB} \quad \Rightarrow \quad AB = \frac{AD}{\sin 60°} = \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{6}}$$

Q14. A variable line passes through a fixed point P. The algebraic sum of the perpendiculars drawn from the points (2, 0), (0, 2) and (1, 1) on the line is zero. Find the coordinates of the point P

Sol. Let the variable line through the fixed point P is $ax + by + c = 0$ (i)

Perpendicular distance from $A(2, 0) = \dfrac{2a+0+c}{\sqrt{a^2+b^2}}$

Perpendicular distance from $A(0, 2) = \dfrac{0+2b+c}{\sqrt{a^2+b^2}}$

Perpendicular distance from $A(1, 1) = \dfrac{a+b+c}{\sqrt{a^2+b^2}}$

According to the question, we have

$$\frac{2a+0+c}{\sqrt{a^2+b^2}}+\frac{0+2b+c}{\sqrt{a^2+b^2}}+\frac{a+b+c}{\sqrt{a^2+b^2}}=0$$

$\Rightarrow$ $3a + 3b + 3c = 0$ or $a + b + c = 0$ (ii)

From (i) and (ii), variable line passes through the fixed point (1, 1).

Q15. In what direction should a line be drawn through the point (1, 2) so that its point of intersection with the line x+y = 4 is at a distance √6/3 from the given point.

Sol. Let slope of the line be m. Also, the line passes through the point $A(1, 2)$

$\therefore$ Equation of line is $y - 2 = m(x - 1)$ or $mx - y + 2 - m = 0$ (i)

Also the equation of the given line is $x + y - 4 = 0$ (ii)

Let these lines meet at point B.

Solving (i) and (ii), we get $B \equiv \left(\dfrac{m+2}{m+1}, \dfrac{3m+2}{m+1}\right)$

Now, given that $AB = \dfrac{\sqrt{6}}{3}$

$\Rightarrow$ $$AB^2 = \frac{6}{9} \Rightarrow \left(\frac{m+2}{m+1}-1\right)^2+\left(\frac{3m+2}{m+1}-2\right)^2=\frac{6}{9}$$

$\Rightarrow$ $$\left(\frac{1}{m+1}\right)^2+\left(\frac{m}{m+1}\right)^2=\frac{2}{3} \Rightarrow \frac{1+m^2}{(1+m)^2}=\frac{2}{3}$$

$\Rightarrow$ $3 + 3m^2 = 2 + 2m^2 + 4m \Rightarrow m^2 - 4m + 1 = 0$

$\therefore$ $$m=\frac{4\pm\sqrt{16-4}}{2}=2\pm\sqrt{3}=2+\sqrt{3} \text{ or } 2-\sqrt{3}$$

$\therefore$ $\tan\theta = 2+\sqrt{3}$ or $2-\sqrt{3}$

$\therefore$ $\theta = 75°$ or $\theta = 15°$

Q16. A straight line moves so that the sum of the reciprocals of its intercepts made on axes is constant. Show that the line passes through a fixed point.

Sol. Equation of line in intercept form is $\dfrac{x}{a}+\dfrac{y}{b}=1$.

Given that, $\dfrac{1}{a}+\dfrac{1}{b}=\text{constant}=\dfrac{1}{k}(\text{say})$

$\therefore$ $\dfrac{k}{a}+\dfrac{k}{b}=1$

So, (k, k) lies on $\dfrac{x}{a}+\dfrac{k}{b}=1$

Hence, the line passes through the fixed point (k, k).

Q17. Find the equation of the line which passes through the point (-4, 3) and the portion of the line intercepted between the axes is divided internally in the ratio 5 : 3 by this point.

Sol: Let the line through the point P(-A, 3) meets axis at A(h, 0) and 0(0, k)

Now according to the question AP : BP =5:3

$$\therefore \qquad (-4, 3) \equiv \left(\frac{3\times h+5\times 0}{5+3}, \frac{3\times 0+5\times k}{5+3}\right) \equiv \left(\frac{3h}{8}, \frac{5k}{8}\right)$$

$$\Rightarrow \qquad -4 = \frac{3h}{8} \text{ and } 3 = \frac{5k}{8}$$

$$\Rightarrow \qquad h = -\frac{32}{3} \text{ and } k = \frac{24}{5}$$

So, equation of the required line in intercept form is:

$$\frac{x}{-32/3} + \frac{y}{24/5} = 1 \quad \Rightarrow \quad 9x - 20y + 96 = 0$$

Q18. Find the equations of the lines through the point of intersection of the lines x-y+ 1=0 and 2x – 3y + 5 = 0 and whose distance from the point (3, 2) is 7/5

Sol. Given lines are: $x - y + 1 = 0$ (i)

and $2x - 3y + 5 = 0$ (ii)

Solving these lines, we get point of intersection as (2, 3).

Let slope of the required line be m

So, equation of line is:

$$y - 3 = m(x - 2) \quad \Rightarrow \quad mx - y + 3 - 2m = 0$$

Distance of this line from (3, 2) $= \frac{7}{5}$

$$\therefore \qquad \frac{7}{5} = \left|\frac{3m - 2 + 3 - 2m}{\sqrt{1+m^2}}\right| \Rightarrow \frac{49}{25} = \frac{(m+1)^2}{1+m^2}$$

$$\Rightarrow \qquad 49 + 49m^2 = 25(m^2 + 2m + 1) \quad \Rightarrow \quad 24m^2 - 50m + 24 = 0$$

$$\Rightarrow \qquad 12m^2 - 25m + 12 = 0 \quad \Rightarrow \quad (3m - 4)(4m - 3) = 0$$

$$\therefore \qquad m = \frac{4}{3} \text{ or } \frac{3}{4}$$

So, the equation of the line can be:

$$y - 3 = \frac{4}{3}(x - 2) \text{ or } y - 3 = \frac{3}{4}(x - 2)$$

$$\Rightarrow \qquad 4x - 3y + 1 = 0 \text{ or } 3x - 4y + 6 = 0$$

Q19. If the sum of the distances of a moving point in a plane from the axes is 1, then find the locus of the point.

then find the locus of the point.

Sol. Let the coordinates of moving point P be (x, y).

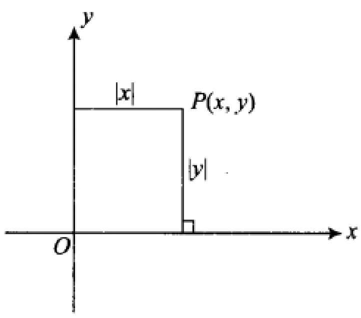
Given that, the sum of distances of this point in a plane from the axes is 1.

$$\therefore \qquad |x| + |y| = 1$$

$$\Rightarrow \qquad \pm x \pm y = 1$$

$$\Rightarrow \qquad x + y = 1, -x - y = 1,$$
$$-x + y = 1, x - y = 1$$

These equations form a square.



Q20. P_1, P_2 are points on either of the two lines $y - \sqrt{3}\,|x| = 2$ at a distance of 5 units from their point of intersection. Find the coordinates of the foot of perpendiculars drawn from P_1,

P_2 on the bisector of the angle between the given lines.

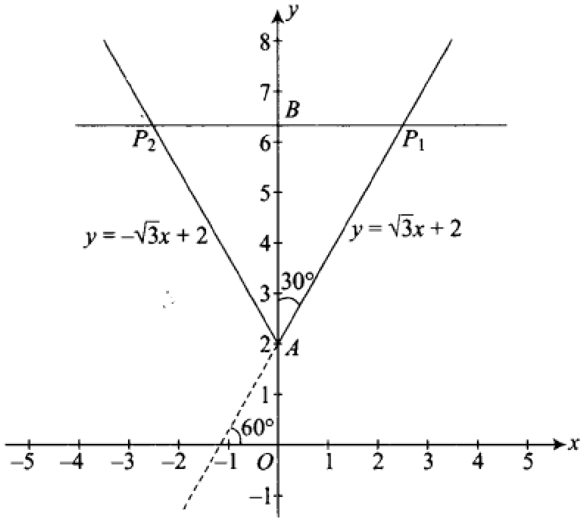
Sol. Given lines are: $y - \sqrt{3}x = 2$, for $x \geq 0$ (i)

and $y + \sqrt{3}x = 2$, for $x \leq 0$ (ii)

Clearly, lines intersect at $A(0, 2)$.

Line (i) is inclined at an angle of 60° with $+ve$ direction of x-axis.

Line (ii) is inclined at an angle of 120° with $+ve$ direction of x-axis.



P_1 and P_2 are points at distance 5 units from point A on the lines.

Clearly, angle bisector of lines is y-axis.

Foot of perpendicular from P_1 and P_2 on y-axis is B.

Now, $AP_1 = 5$

$\therefore$ In ΔABP_1, $\dfrac{AB}{AP_1} = \cos 30°$

$\therefore$ $AB = \dfrac{5\sqrt{3}}{2}$

$\therefore$ $OB = 2 + \dfrac{5\sqrt{3}}{2}$

So, the coordinates of the foot of perpendicular are $\left(0, 2 + \dfrac{5\sqrt{3}}{2}\right)$.

Q21. If p is the length of perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b}$ and a^2,p^2 and are in the A.P , then show that $a^4+b^4 = 0$

Sol. Since p is the length of perpendicular from the origin on the line $\dfrac{x}{a} + \dfrac{y}{b} = 1$, we have

$$p = \frac{|0+0-1|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = \frac{ab}{\sqrt{a^2+b^2}} \Rightarrow p^2 = \frac{a^2b^2}{a^2+b^2}$$

Given that, a^2, p^2 and b^2 are in AP.

$\therefore$ $2p^2 = a^2 + b^2$

$\Rightarrow$ $\dfrac{2a^2b^2}{a^2+b^2} = a^2 + b^2 \Rightarrow 2a^2b^2 = (a^2+b^2)^2$

$\Rightarrow$ $2a^2b^2 = a^4 + b^4 + 2a^2b^2$

$\Rightarrow$ $a^4 + b^4 = 0$

Objective Type Questions

Q22. A line cutting off intercept -3 from the y-axis and the tangent of angle to the x-axis is 3/5, its equation is

(a) $5y - 3x + 15 = 0$ (b) $3y - 5x + 15 = 0$
(c) $5y - 3x - 15 = 0$ (d) None of these

Sol. **(a)** Let the equation of the line be $y = mx + c$.

Given that, $c = -3$ and $m = \frac{3}{5}$.

So, equation of the line is: $y = \frac{3}{5}x - 3 \quad \Rightarrow \quad 5y - 3x + 15 = 0$

Q23. Slope of a line which cuts off intercepts of equal lengths on the axes is
(a) -1
(b) 0
(c) 2
(d) √3

Sol: (a) Equation of the according to the question is $\frac{x}{a} + \quad \frac{y}{a}$
=> x+y = a
Required slope = -1

Q24. The equation of the straight line passing through the point (3, 2) and perpendicular to the line y = x is
(a) x-y = 5
(b) x+y = 5
(c)x+y=l
(d)x-y=1
Sol:(b) Slope of the given line y = x is 1.
Thus, slope of line perpendicular to y = x is -1.
Line passes through the point (3, 2).
So, equation of the required line is:y-2=-l (x – 3) => x + y = 5

Q25. The equation of the line passing through the point (1,2) and perpendicular to the line x +y + 1 = 0 is
(a) y-x +1=0
(b) y — x—1=0
(c) y-x + 2 = 0
(d) y — x — 2=0
Sol: (b) Slope of the given line +1=0 is-1.
So, slope of line perpendicular to above line is 1.
Line passes through the point (1,2).
Therefore, equation of the required linens:
y-2 = 1(x- 1) => y-x-1=0.

Q26. The tangent of angle between the lines whose intercepts on the axes are a, -b and b, -a, respectively, is

(a) $\dfrac{a^2-b^2}{ab}$ (b) $\dfrac{b^2-a^2}{2}$ (c) $\dfrac{b^2-a^2}{2ab}$ (d) None of these

Sol. **(c)** Intercepts of line are a and $-b$; i.e., line passes through the points $(a, 0)$, $(0, -b)$.

$\therefore$ Slope of line, $m_1 = \dfrac{-b-0}{0-a} = \dfrac{b}{a}$

Intercepts of line are $b, -a$; i.e., line passes through the points $(b, 0)$, $(0, -a)$.

$\therefore$ Slope of line, $m_2 = \dfrac{-a-0}{0-b} = \dfrac{a}{b}$

If θ is the angle between the lines, then

$$\tan = \theta = \frac{\frac{b}{a}-\frac{a}{b}}{1+\frac{a}{b}\cdot\frac{b}{a}} = \frac{\frac{b^2-a^2}{ab}}{2} = \frac{b^2-a^2}{2ab}$$

Q27. If the line $\frac{x}{a}+\frac{y}{b}$ passes through the points (2, -3) and (4, -5), then (a, b) a b is

(a) (1,1)

(b) (-1,1)

(c) (1,-1)

(d) (-1,-1)

Sol. **(d)** Given, line is: $\dfrac{x}{a}+\dfrac{y}{b}=1$

Since the points $(2, -3)$ and $(4, -5)$ lie on this line, we have

$$\frac{2}{a}-\frac{3}{b}=1 \qquad \text{(i)}$$

and
$$\frac{4}{a}-\frac{5}{b}=1 \qquad \text{(ii)}$$

Using $2\times$(i) – (ii), we get $-\dfrac{6}{b}+\dfrac{5}{b}=1 \Rightarrow \dfrac{-1}{b}=1 \quad \therefore \quad b=-1$

On putting $b=-1$ in Eq. (i), we get $\dfrac{2}{a}+3=1$ or $a=-1$

$\therefore \quad (a, b) = (-1, -1)$

Q28. The distance of the point of intersection of the lines 2x – 3y + 5 = 0 and 3x + 4y = 0 from the line 5x – 2y = 0 is

(a) $\dfrac{130}{17\sqrt{29}}$ (b) $\dfrac{13}{7\sqrt{29}}$ (c) $\dfrac{130}{7}$ (d) None of these

Sol. **(a)** Given lines are:

$$2x-3y+5=0 \qquad \text{(i)}$$

and
$$3x+4y=0 \qquad \text{(ii)}$$

Solving these lines, we get point of intersection as $\left(\dfrac{-20}{17}, \dfrac{15}{17}\right)$.

$\therefore$ Distance of this point from the line '$5x-2y=0$'

$$=\frac{\left|5\times\left(-\frac{20}{17}\right)-2\left(\frac{15}{17}\right)\right|}{\sqrt{25+4}} = \frac{\left|\frac{-100}{17}-\frac{30}{17}\right|}{\sqrt{29}} = \frac{130}{17\sqrt{29}}$$

Q29. The equations of the lines which pass through the point (3, -2) and are inclined at 60° to the line √3 x + y = 1 is

(a) $y+2=0,\ \sqrt{3}x-y-2-3\sqrt{3}=0$

(b) $x-2=0,\ \sqrt{3}x-y+2+3\sqrt{3}=0$

(c) $\sqrt{3}x-y-2-3\sqrt{3}=0$

(d) None of these

Sol. **(a)** Slope of the given line $\sqrt{3}x+y=1$ is, $m_1=-\sqrt{3}$.

Let the slope of the required line which makes an angle of 60° with the above line is m.

$$\therefore \quad \tan 60^\circ=\left|\frac{-\sqrt{3}-m}{1-\sqrt{3}m}\right| \Rightarrow \left|\frac{-\sqrt{3}-m}{1-\sqrt{3}m}\right|=\sqrt{3}$$

$$\Rightarrow \quad -\sqrt{3}-m=\sqrt{3}-3m \text{ or } -\sqrt{3}-m=-\sqrt{3}+3m$$

$$\Rightarrow \quad m=\sqrt{3} \text{ or } m=0$$

Line is passing through the point (3, –2).

Thus, the equation of the required line is: $y+2=\sqrt{3}(x-3)$ or $y+2=0$

$$\Rightarrow \quad \sqrt{3}x-y-2-3\sqrt{3}=0 \text{ and } y+2=0$$

Q30. The equations of the lines passing through the point (1,0) and at a distance √3/2 from the origin, are

(a) $\sqrt{3}x+y-\sqrt{3}=0, \sqrt{3}x-y-\sqrt{3}=0$

(b) $\sqrt{3}x+y+\sqrt{3}=0, \sqrt{3}x-y+\sqrt{3}=0$

(c) $x+\sqrt{3}y-3=0, x-\sqrt{3}y-\sqrt{3}=0$

(d) None of these.

Sol. **(a)** Let the slope of the line be 'm'

Then equation of line passing through (1, 0) is:

$$y-0=m(x-1)$$

$$\Rightarrow \quad y-mx+m=0 \qquad \text{(i)}$$

It is given that the distance of the line from origin is $\frac{\sqrt{3}}{2}$.

$$\therefore \quad \frac{\sqrt{3}}{2}=\frac{|0-0+m|}{\sqrt{1+m^2}} \Rightarrow \frac{\sqrt{3}}{2}=\frac{|m|}{\sqrt{1+m^2}}$$

$$\Rightarrow \quad 3+3m^2=4m^2 \Rightarrow m^2=3 \Rightarrow m=\pm\sqrt{3}$$

So, equation of line is: $\sqrt{3}x+y-\sqrt{3}=0$ or $\sqrt{3}x-y-\sqrt{3}=0$.

31. The distance between the lines $y = mx + c_1$ and $y = mx + c_2$ is

(a) $\dfrac{c_1 - c_2}{\sqrt{m^2+1}}$ (b) $\dfrac{|c_1 - c_2|}{\sqrt{1+m^2}}$ (c) $\dfrac{c_2 - c_1}{\sqrt{1+m^2}}$ (d) 0

Sol. **(b)** Let any point on the line $y = mx + c_1$ be $P(x_1, y_1)$.

The equation of the other line is: $y = mx + c_2 \Rightarrow mx - y + c_2 = 0$

Distance of point P from this line, $d = \dfrac{|mx_1 - y_1 + c_2|}{\sqrt{m^2+1}}$

Since P lies on the first line, we get

$$y_1 = mx_1 + c_1 \quad \Rightarrow mx_1 - y_1 = -c_1$$

$$\therefore \quad d = \frac{|c_1 - c_2|}{\sqrt{m^2+1}}$$

32. The coordinates of the foot of perpendicular from the point (2, 3) on the line $y = 3x + 4$ is given by

(a) $\left(\dfrac{37}{10}, \dfrac{-1}{10}\right)$ (b) $\left(-\dfrac{1}{10}, \dfrac{37}{10}\right)$ (c) $\left(\dfrac{10}{37}, -10\right)$ (d) $\left(\dfrac{2}{3}, -\dfrac{1}{3}\right)$

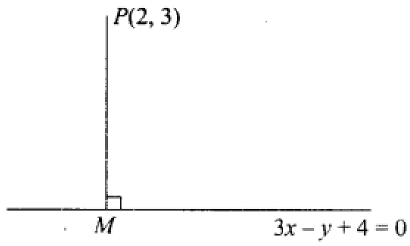
Sol. **(b)**

Let the foot of perpendicular from the point $P(2, 3)$ on the line $3x - y + 4 = 0$ be $M(h, k)$.

$M(h, k)$ lies on the given line,

$\therefore \quad 3h - k + 4 = 0 \quad$ (i)

Also, slope of the given line is 3.



$\therefore$ Slope of PM $= -\dfrac{1}{3} = \dfrac{k-3}{h-2}$ or $h + 3k - 11 = 0 \quad$ (ii)

Solving (i) and (ii), we get $(h, k) \equiv \left(-\dfrac{1}{10}, \dfrac{37}{10}\right)$

Q33. If the coordinates of the middle point of the portion of a line intercepted between the coordinate axes is (3, 2), then the equation of the line will be

(a) 2x + 3y = 12

(b) 3x + 2y=l2

(c) 4x-3y = 6

(d) 5x- 2y=10

Sol: (a) Since, the middle point is P(3, 2), then line meets axes at A(6, 0) and B (0, 4).

$\therefore$ Equation of the line using intercept form is:

$$\frac{x}{6} + \frac{y}{4} = 1 \quad \Rightarrow \quad 2x + 3y = 12$$

Q34. Equation of the line passing through (1,2) and parallel to the line y = 3x – 1 is

(a)y + 2=x+l

(b) y + 2 = 3(x + 1)

(c) y -2 = 3(x — 1)

(d) y-2=x-l

Sol: (c) Line is parallel to the line y = 3x – 1.

So, slope of the line is'3'.

Also, line passes through the point (1,2).

So, equation of the line is: y – 2 = 3(x – 1)

Q35. Equations of diagonals of the square formed by the lines x = 0, y = 0, x = 1 and y = 1 are

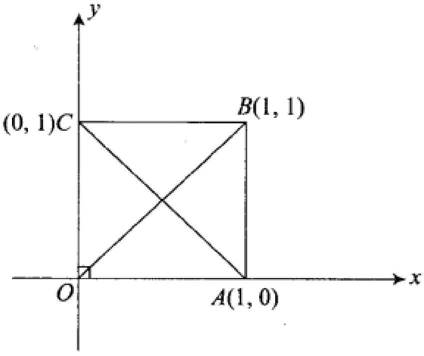
(a) $y=x, y+x=1$ (b) $y=x, x+y=2$

(c) $2y=x, y+x=\frac{1}{3}$ (d) $y=2x, y+2x=1$

Sol. **(a)** Given lines are plotted on coordinate plane as shown in the adjacent figure.

From the figure, equation of diagonal OB is $y=x$.

Equation of the diagonal AC is $x+y=1$ (using intercept form).



36. For specifying a straight line, how many geometrical parameters should be known?

(a) 1 (b) 2 (c) 4 (d) 3

Sol. **(b)** General equation of straight line or linear equation in two variables is $ax+by+c=0$.

We know that at least one of a and b must be non-zero.

Let $a \neq 0$. Then equation of the line is:

$$x+\frac{b}{a}y+\frac{c}{a}=0 \text{ or } x+py+q=0, \text{ where } p=\frac{b}{a} \text{ or } q=\frac{c}{a}$$

Thus for getting the equation of the fixed straight line two parameters should be known.

Q37. The point (4, 1) undergoes the following two successive transformations:

(i) Reflection about the line y = x

(ii) Translation through a distance 2 units along the positive x-axis Then the final coordinates of the point are

(a) (4,3)

(b) (3,4)

(c) (1,4)

(d) (7/2,7/2)

Sol: (b) Reflection of A (4, 1) in y = x is 5(1,4).

Now translation of point B through a distance '2' units along the positive x-axis shifts B to C(1 + 2,4) or C(3,4).

Q38. A point equidistant from the lines 4x + 3y+ 10 = 0, 5x – 12y + 26 = 0 and lx + 24y – 50 = 0 is

(a) (1,-1)

(b) (1, 1)

(c) (0,0)

(d) (0, 1)

Sol: (c) Clearly distance of each of three lines from (0, 0) is 2 units.

Q39. A line passes through (2, 2) and is perpendicular to the line 3x + y = 3. Its y-intercept is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$

Sol. **(d)** Slope of given line $3x+y=3$ is -3.

$\therefore$ Slope of perpendicular line $=\frac{1}{3}$

Thus, equation of the required line is: $y-2=\frac{1}{3}(x-2) \Rightarrow x-3y+4=0$

For y-intercept, put $x=0$.

$$0-3y+4=0 \Rightarrow y=\frac{4}{3}, \text{ which is } y\text{-intercept.}$$

Q40. The ratio in which the line 3x + 4y + 2 = 0 divides the distance between the lines 3x + 4y + 5 = 0 and 3x + 4y – 5 = 0 is
(a) 1:2 (b) 3:7 (c) 2:3 (d) 2:5

Sol: (b) Given lines are:

$$3x + 4y + 5 = 0 \quad \text{(i)}$$

$$3x + 4y - 5 = 0 \quad \text{(ii)}$$

The third line is: $3x + 4y + 2 = 0$ (iii)

Distance between the line (i) and (iii) $= \frac{|5-2|}{\sqrt{9+16}} = \frac{3}{5}$

Distance between the line (ii) and (iii) $= \frac{|-5-2|}{\sqrt{9+16}} = \frac{7}{5}$

Hence, the required ratio is $\frac{3}{5}:\frac{7}{5}$ or 3 : 7.

Q41. One vertex of the equilateral triangle with centroid at the origin and one side asx + y- 2 = 0is
(a) (-1,-1) (b) (2,2) (c) (-2,-2) (d) (2,-2)

Sol. (c)

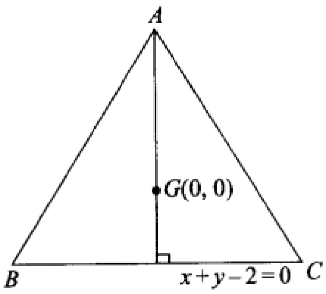
Let ABC be the equilateral triangle with vertex $A(h, k)$.

Also, centroid is $G(0, 0)$.

Now, $AG \perp BC$

Slope of line BC or $x + y - 2 = 0$ is -1.

$\therefore$ Slope of AG, $\frac{k}{h} = 1$ or $h = k$.



Now distance of origin from $BC = \frac{|0+0-2|}{\sqrt{1^2+1^2}} = \sqrt{2}$

$\therefore$ Distance of A form $BC = 3\sqrt{2} = \frac{|h+k-2|}{\sqrt{1^2+1^2}}$

$\therefore$ $|h + k - 2| = 6$

$\Rightarrow$ $h + k - 8 = 0$ or $h + k + 4 = 0$

$\Rightarrow$ $h + h - 8 = 0$ or $h + h + 4 = 0$

$\Rightarrow$ $h = 4$ or $h = -2$

$\therefore$ Vertex is $(-2, -2)$

Fill in the Blanks Type Questions

Q42. If a, b, c are in A.P., then the straight lines ax + by + c = 0 will always pass through

____

Sol. Given line is $ax + by + c = 0$ (i)

Since, a, b and c are in A.P., we get

$$b = \frac{a+c}{2} \text{ or } a - 2b + c = 0 \quad \text{(ii)}$$

On comparing Eqs. (i) and (ii), we get

$$x = 1, y = -2$$

So, $(1, -2)$ lies on the line.

Q43.The line which cuts off equal intercept from the axes and pass through the point (1, -2) is

Sol. Line cutting equal intercepts from the axes is

$$\frac{x}{a}+\frac{y}{a}=1 \quad \text{or} \quad x+y=a \qquad \text{(i)}$$

Since, this line passes through (1, –2), $1-2=a$ or $a=-1$.

So, required equation of the line is: $x+y=-1 \Rightarrow x+y+1=0$

Q44. Equations of the lines through the point (3, 2) and making an angle of 45° with the line x – 2y = 3 are _____

Sol. Slope of the given line $x-2y=3$ is $\frac{1}{2}$.

Let the slope of the required line be m.

$$\therefore \quad \tan 45° = \left|\frac{m-\frac{1}{2}}{1+\frac{1}{2}m}\right| \Rightarrow 1 = \pm\frac{2m-1}{2+m}$$

$$\Rightarrow \quad 2m-1=2+m \text{ or } 1-2m=2+m$$

$$\Rightarrow \quad m=3 \text{ or } m=-\frac{1}{3}$$

Also, the required line passes through the point (3, 2).

So, equation of the line is: $y-2=3(x-3)$ or $y-2=-\frac{1}{3}(x-3)$

$$\therefore \quad 3x-y-7=0 \text{ or } x+3y-9=0$$

Q45. The points (3,4) and (2, -6) are situated on the_____ of the line 3x – 4y – 8= 0.

Sol: Given line is 3x – 4y – 8 = 0

For point (3, 4), 3(3) – 4(4) – 8 = -15 < 0

For point (2, -6), 3(2) – 4(—6) – 8 = 22 > 0

Hence, the points (3,4) and (2, -6) lies on opposite side of the line.

Q46. A point moves so that square of its distance from the point (3, -2) is numerically equal to its distance from the line 5x – 12y = 3. The equation of its locus is______ .

Sol: Let the moving point be P(h, k).

Given point is A(3, -2).

$$AP^2=(h-3)^2+(k+2)^2=d_1^{\,2}$$

Now, distance of the point (h, k) from the line $5x-12y-3=0$ is

$$d_2=\left|\frac{5h-12k-3}{\sqrt{25+144}}\right|=\left|\frac{5h-12k-3}{13}\right|$$

Given that, $d_1^{\,2}=d_2$

$$\Rightarrow \quad (h-3)^2+(k+2)^2=\frac{5h-12k-3}{13} \qquad \text{(Taking +ve sign)}$$

$$\Rightarrow \quad h^2-6h+9+k^2+4k+4=\frac{5h-12k-3}{13}$$

$$\Rightarrow \quad 13h^2+13k^2-78h+52k+169=5h-12k-3$$

$$\Rightarrow \quad 13h^2+13k^2-83h+64k+172=0$$

So, locus of this point is: $13x^2+13y^2-83x+64y+172=0$

Q47. Locus of the mid-points of the portion of the line x sin **θ** + y cos **θ** = p intercepted between the axes is ______

Sol. Line $x \sin\theta + y\cos\theta = p$ meets axes at $A\left(\frac{p}{\sin\theta}, 0\right)$ and $B\left(0, \frac{p}{\cos\theta}\right)$.

Let $P(h, k)$ be the mid-point of AB.

$$\therefore \quad h = \frac{p}{2\sin\theta} \text{ and } k = \frac{p}{2\cos\theta}$$

$$\therefore \quad \sin\theta = \frac{p}{2h} \text{ and } \cos\theta = \frac{p}{2k}$$

Squaring and adding, we get

$$\sin^2\theta + \cos^2\theta = \frac{p^2}{4h^2} + \frac{p^2}{4k^2} \quad \text{or } 1 = \frac{p^2}{4x^2} + \frac{p^2}{4y^2}$$

or $\quad 4x^2y^2 = p^2(x^2 + y^2)$.

True/False Type Questions

Q48. If the vertices of a triangle have integral coordinates, then the triangle can not be equilateral.

Sol: True

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of a triangle ABC, where x_i, y_i, $i = 1, 2, 3$, are integers.

Then, the area of ΔABC is given by

$$\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$= \text{A rational number} \qquad [\because x_i, y_i \text{ are integers}]$

If possible, let the triangle ABC be an equilateral triangle, then its area is given by

$$\Delta = \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(AB)^2 \qquad [\because AB = BC = CA]$$

$$= \frac{\sqrt{3}}{4}(a \text{ positive interger})$$

$[\because \text{vertices are integral } \therefore AB^2 \text{ is a integer}]$

$= \text{an irrational number}$

This is a contradiction to the fact that the area is a rational number. Hence, the triangle cannot be equilateral.

Q49. The points A(-2, 1), B(0, 5), C(-l, 2) are collinear.

Sol: False

Given points are $A(-2, 1)$, $B(0, 5)$ and $C(-1, 2)$ are collinear.

Slope of $AB = \frac{5-1}{0+2} = 2$

Slope of $BC = \frac{2-5}{-1-0} = 3$

Since the slopes are different, A, B and C are not collinear.

Q50. Equation of the line passing through the point (a cos^3 , a sin^3) and perpendicular to the line

x sec + y cosec = a isx cos -y sin = a sin 2

Sol: False

Line perpendicular to $x \sec \theta + y \operatorname{cosec} \theta = a$ is

$$x \operatorname{cosec} \theta - y \sec \theta = \lambda.$$

This line passes through the point $(a \cos^3 \theta, a \sin^3 \theta)$.

Then $(a \cos^3\theta) \operatorname{cosec} \theta - (a \sin^3\theta) \sec \theta = \lambda$

$$\Rightarrow \qquad \lambda = a\left(\frac{\cos^3 \theta}{\sin \theta} - \frac{\sin^3 \theta}{\cos \theta}\right) = a\frac{\cos 2\theta}{\cos \theta \sin \theta}$$

Then equation of line is $x \cos \theta - y \sin \theta = a \cos 2\theta$

Q51. The straight line 5x + 4y = 0 passes through the point of intersection of the straight lines x + 2y— 10 = 0 and 2x +y + 5 = 0.

Sol: True

Given lines $x + 2y - 10 = 0$ and $2x + y + 5 = 0$ intersect at $\left(-\frac{20}{3}, \frac{25}{3}\right)$.

Now this point satisfies the line $5x + 4y = 0$ as

$$5\left(\frac{-20}{3}\right) + 4\left(\frac{25}{3}\right) = \frac{-100}{3} + \frac{100}{3} = 0$$

52. The vertex of an equilateral triangle is (2, 3) and the equation of the opposite side is $x + y = 2$. Then the other two sides are $y - 3 = (2 \pm \sqrt{3})(x - 2)$.

Sol. True

Let ABC be an equilateral triangle with vertex $A(2, 3)$.

Equation of BC is $x + y = 2$.

Slope of $BC = -1$.

Let slope of line AB be m.

Now, the angle between line AB and BC is 60°.

$$\therefore \qquad \tan 60° = \left|\frac{m+1}{1-m}\right| \quad \Rightarrow \quad \frac{m+1}{1-m} = \pm\sqrt{3} \quad \Rightarrow m = 2 \pm \sqrt{3}$$

Equation of other two sides is $y - 3 = (2 \pm \sqrt{3})(x - 2)$

Q53. The equation of the line joining the point (3, 5) to the point of intersection of the lines 4x +y − 1 = 0 and lx − 3v − 35 = 0 is equidistant from the points (0, 0) and (8, 34).

Sol: True

Given equation of lines are $4x + y - 1 = 0$ and $7x - 3y - 35 = 0$.

Lines intersect at $(2, -7)$

Now, the equation of a line passing through (3, 5) and (2, –7) is:

$$y - 5 = \frac{-7-5}{2-3}(x-3) \quad \Rightarrow \quad y - 5 = 12(x - 3)$$

$$\Rightarrow \qquad 12x - y - 31 = 0 \qquad \text{(i)}$$

Distance from (0, 0) to the line (i), $d_1 = \frac{|-31|}{\sqrt{144+1}} = \frac{31}{\sqrt{145}}$

$\therefore$ Distance from (8, 34) to the line (i), $d_2 = \frac{|96-34-31|}{\sqrt{145}} = \frac{31}{\sqrt{145}}$

So, $d_1 = d_2$

54. The line $\frac{x}{a}+\frac{y}{b}=1$ moves in such a way that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$, where c is a constant. The locus of the foot of the perpendicular from the origin on the given line is $x^2+y^2=c^2$.

Sol. True

We have, equation of line, $\frac{x}{a}+\frac{y}{b}=1$ (i)

Equation of line passing through origin and perpendicular to line (i) is

$$\frac{x}{b}-\frac{y}{a}=0 \quad \text{(ii)}$$

Now, the foot of perpendicular from origin on the line (i) is the point of intersection of lines (i) and (ii).

So, to find its locus we have to eliminate the variable a and b.

On squaring and adding Eqs. (i) and (ii), we get

$$\left(\frac{x}{a}+\frac{y}{b}\right)^2+\left(\frac{x}{a}-\frac{y}{b}\right)^2=1+0 \Rightarrow x^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)+y^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)=1$$

$$\Rightarrow \frac{x^2}{c^2}+\frac{y^2}{c^2}=1 \quad \left[\because \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}\right]$$

$$\Rightarrow x^2+y^2=c^2$$

Q55. The line ax + 2y + 1 = 0, bx + 2y + 1 = 0 and cx + 4y + 1 = 0 are concurrent, if a, b and c are in GP.

Sol: False

Given lines are

$$ax+2y+1=0 \quad \text{(i)}$$
$$bx+3y+1=0 \quad \text{(ii)}$$
$$cx+4y+1=0 \quad \text{(iii)}$$

Solving (i) and (ii) by cross-multiplication method, we get

$$\frac{x}{2-3}=\frac{-y}{a-b}=\frac{1}{3a-2b}$$

So, the point of intersection is $\left(\frac{1}{2b-3a}, \frac{a-b}{2b-3a}\right)$

Since, this point lies on $cx+4y+1=0$, then

$$\frac{c}{2b-3a}+\frac{4(a-b)}{2b-3a}+1=0$$

$$\Rightarrow \quad c+4a-4b+2b-3a=0 \Rightarrow 2b=a+c$$

Hence, a, b, c are in A.P.

Q56. Line joining the points (3, -4) and (-2, 6) is perpendicular to the line joining the points (-3, 6) and (9, -18).

Sol. False

Given points are $A(3,-4)$, $B(-2,6)$, $C(-3,6)$ and $D(9,-18)$.

Now, slope of $AB=\frac{6+4}{-2-3}=-2$

And slope of $CD=\frac{-18-6}{9+3}=-2$

So, line AB is parallel to line CD